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ESTIMATING THE CHILEAN NATURAL RATE OF INTEREST

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Resumen

En este trabajo se utiliza una batería de métodos para estimar la tasa de interés real neutral (TIRN) para Chile. Los métodos se clasificaron en tres categorías: métodos derivados de la teoría económica, la TIRN implícita en los papeles financieros y métodos estadísticos utilizando datos macroeconómicos. Se concluye que la tasa neutral no es constante en el tiempo. Está estrechamente relacionada con la tasa de crecimiento potencial del producto, pero no es equivalente a esta. La aplicación de los métodos entrega resultados bastante similares. La TIRN para Chile se encontraría en un rango entre 2 y 3.6%, con una mediana de 2.8% con datos al cuarto trimestre del 2006.

Abstract

In this paper we use a set of methods to estimate the neutral interest rate for Chile (NRIR). We group the methods into three categories: methods derived from pure economic theory, the NRIR implicit in the price of financial assets, and the rate estimated from a statistical model using macroeconomic data. We learnt that the neutral interest rate is not a time invariant variable. It is closely related to the potential growth of the economy (but it is not equal to the growth rate of the trended output). The application of all methods yields very similar results. The NRIR would be in a range between 2% and 3.6%, with a median equal to 2.8% with data up to the fourth quarter of 2006.

We thank Rómulo Chumacero, Juan Pablo Medina, Klaus Schmidt-Hebbel, Rodrigo Valdés and the participants to internal and open seminars at the Central Bank of Chile for helpful discussions. The views and conclusions presented in the paper are exclusively those of the authors and do not reflect the position of the Central Bank of Chile or of the Board members. Email: rfuentes@bcentral.cl, fgredig@bcentral.cl.

1. Introduction

After a period of low interest rates around the world, over the past two years we have been observing a process known as the normalization of the interest rate. Central banks have been raising their policy interest rates with the goal of reaching the level of a neutral real interest rate (NRIR). Chile has not been the exception in moving toward an interest rate that could be considered neutral. Since the second half of 2004, the Central Bank of Chile (CBC) has been raising its monetary policy interest rate after taking it to a record low. Although the movement toward a more normal or neutral interest rate has been present in many speeches of central banks' governors, the concept of NRIR is a very nebulous one, and there is little knowledge about its "true" level.

The NRIR is a benchmark to measure how expansionary monetary policy is. In that sense, it could be defined as the interest rate that is consistent with an inflation rate aligned with the central bank's implicit or explicit target. The European Central Bank (ECB), for example, has defined the natural rate of interest as "the real short-term interest rate which is consistent in the long-run […] with output at its potential level and a stable rate of inflation".[1] In the context of a New Keynesian DSGE model, the NRIR is the rate that prevails in equilibrium without nominal rigidities (Galí, 2003). Other definition considers the NRIR as the steady-state interest rate, meaning the one that is consistent with a balanced growth path.

Despite the looseness of the concept of neutral rate and the difficulties in measuring it, it is still a key parameter in the way that monetary policy is conducted today. There are different methodologies that consider a variety of theoretical models and empirical approaches to the value of the NRIR.[2] In this paper we use a battery of methods to estimate the neutral interest rate for Chile. Calderón and Gallego (2002) made the first attempt to estimate the neutral interest rate for Chile using different methods. Some of them are used here, and some have been slightly changed based on economic theory and the availability of new methods. Back in 2002, the key point of that paper was that the neutral interest rate was coming down; therefore lowering the monetary policy rate was consistent with the economic situation of the time.

We group the methods into three categories: methods derived from pure economic theory, the NRIR implicit in the price of financial assets, and the rate estimated from a statistical model using macroeconomic data. In the first group we work with two models. One is a traditional consumption based model to estimate the risk-free rate for Chile. The other is based on the conditions for holding the uncovered interest rate parity for a small open economy. In the second group we follow two approaches. First, we estimate the forward interest rate at a medium-term horizon that is implicit in the central bank's indexed bonds (Bomfim, 2001). The second approach is the rate implicit in a state-space model that assumes a common stochastic rate and a term premium between short term and medium term nominal bonds (Basdevant, Björksten and Karagedikli, 2004). The latter method estimates the implicit natural rate of interest in a semi-structural model with unobserved components by using the Kalman filter algorithm. This method allows the joint estimation of the NRIR and the output gap (Laubach and Williams, 2003).

[1] ECB Monthly Bulletin, May 2004.

[2] For a complete revision regarding methods to estimate the natural rate of interest, see Giammarioli and Valla (2004).

It is interesting that the application of all methods yields very similar results. The NRIR would be in a range between 2% and 3.6%. In those models it is possible to see a trajectory of the interest rate following a downward trend. A similar assessment can be found in many studies around the world. In comparison with other countries, actual estimations for the natural real rate of interest indicate that its level approaches 2.5% in USA and Europe (see Appendix).

The paper continues as follows. Section 2 derives the neutral interest rate from economic theory using a closed economy framework and an open economy framework. Section 3 presents the estimation from financial instruments. Section 4 provides estimates using a multivariate statistical filter, and section 5 concludes.

2. Deriving the natural interest rate from economic theory

In this section we analyze the NRIR from two paradigms: the consumption based model and the international interest parity. The first model could be useful to understand the rate of interest in a general equilibrium setting based on Lucas's (1978) model of trees. The second approach assumes a financial open economy for which the uncovered interest parity holds.

2.1 A consumption based model

The traditional benchmark model (see Cochrane, 2001) is based on a representative agent who lives in an endowment economy and maximizes intertemporal utility. She owns an asset (A_t) that pays a gross yield R. The problem could be written as:

$$\max E_0 \sum_{0}^{\infty} \beta^{\,t}\, u(c_t)$$

subject to

$$A_{t+1} = R_t\,(A_t + y_t - c_t) \ ,$$

where c_t is consumption a time t, β is the subjective discount factor, y_t is the endowment at time t that cannot be stored, and $u(.)$ is the utility function with $u'>0$ and $u''<0$. The first-order condition for this problem shows that, in equilibrium, the asset return is equal to the stochastic discount factor.

$$R_t^{-1} = E_t[M_{t+1}] = E_t \left(\frac{\beta \ \ u'(y_{t+1})}{u'(y_t)} \right) , \qquad (1)$$

where M_{t+1} is the stochastic discount factor. Assuming that the utility function is a CRRA, we can rewrite equation (1) as:

$$R_t^{-1} = E_t \left(\frac{\beta \ \ y_{t+1}^{-\gamma}}{y_t^{-\gamma}} \right) , \qquad (2)$$

where γ is the relative risk aversion coefficient. Equation (2) shows a non-linear relationship between the interest rate and the expected growth rate of output. So the NRIR is not equal to the long run growth rate of output, which is a typical rule of thumb followed in practice. To obtain a linear relationship, let us assume that the growth rate of output is distributed as log-normal. We can rewrite (2) as

$$R_t = \left(\frac{\beta \quad E_t \quad y_{t+1}^{-\gamma}}{y_t^{-\gamma}} \right)^{-1} = \left(\exp\{\ln \beta\} \exp\{-\gamma \quad E_t \Delta \ln y_{t+1} + (\gamma^2/2) Var_t(\Delta \ln y_{t+1})\} \right)^{-1}$$

or

$$\ln R_t = r_t = -\ln \beta + \gamma \quad E_t \Delta \ln y_{t+1} - (\gamma^2/2) Var_t(\Delta \ln y_{t+1}) \ , \qquad (3)$$

where r_t is the neutral interest rate implied by the model. In this model, the growth rate of output does not constitute the lower bound for the neutral interest rate as implied by the deterministic Ramsey model. To obtain the neutral interest rate, it is necessary to calibrate the parameters involved: $\beta, \gamma, E_t \Delta \ln c_{t+1}, Var_t(\Delta \ln c_{t+1})$.

The value of the expected growth rate is an unobservable variable. According to Fuentes, Gredig and Larraín (2007), the median estimate of the potential per capita output growth rate[3] for 1995-2006 is 3%. The standard deviation of the growth rate for the same period is estimated at 1.5%. In the following table we present the estimates of the long-run neutral interest rate for different subjective discount factor and risk aversion coefficients. Given the wide range of discount factors and risk aversion coefficients, the NRIR is between 4% and 9%.

Table 1: The neutral interest rate from a consumption based model

	γ		
β	**1**	**1.5**	**2**
0.970	6.03	7.52	9
0.975	5.52	7.01	8.49
0.980	5.01	6.49	7.98
0.985	4.5	5.99	7.47
0.990	3.99	5.48	6.96

Source: Authors' estimates.

As expected, the values obtained using this methodology are relatively high, for instance a plausible value like 6.5%, which is right in the center of the table, is way above the actual market rate.[4] Considering this puzzle later in the literature, Campbell and Cochrane (1999) modified the utility function to introduce the possibility of habit in consumption. In this setting, they work with a utility function given by

$$E_0 \sum_0^{\infty} \beta^t \frac{(c_t - x_t)^{1-\gamma} - 1}{1-\gamma} \ ,$$

[3] This rate is obtained taking 4.6% for potential output and 1.6% for the growth rate of population.
[4] This model is known for yielding high estimates for the interest rate and relatively low risk premium, which are considered a puzzle in the literature. See Cochrane (2001) and Campbell, Lo and MacKinlay (1997) for a summary of this discussion.

where x_t is considered the habit level and is assumed to be exogenous. Note that under this formulation, consumption must be always above the habit and the agents' risk aversion varies with the level of consumption relative to habit. Under this formulation, the marginal utility ratio that generates the stochastic discount factor is:

$$R_t^{-1} = E_t[M_{t+1}] = E_t\beta \left(\frac{S_{t+1}\, c_{t+1}}{S_t\, c_t}\right)^{-\gamma}, \tag{4}$$

where S represents "surplus consumption ratio", (i.e. $S_t = (c_t - x_t)/c_t$). To obtain a reduced form for the risk-free rate, we need to make an assumption on the stochastic process for the surplus consumption ratio. As usual, we assume that habit moves slowly over time as a function of past consumption:

$$x_t \approx \lambda \sum_{j=0}^{\infty} \phi^j c_{t-j} \ ,$$

where ϕ is the weight of past consumption in habit formation. It will also represent the degree of habit persistence that is calibrated by Cochrane y Campbell in 0.87. They work with the "surplus consumption ratio" ($s_t = [\ln(c_t - x_t)/c_t]$) that is assumed to follow an AR(1) process:

$$s_{t+1} = (1-\phi)\overline{s} + \phi s_t + \lambda(s)(c_{t+1} - c_t - g), \tag{5}$$

where g is the long-run growth rate of consumption, $\lambda(s)$ is the sensitivity of the surplus consumption ratio to the deviation of the actual growth rate with respect to the mean and it is a decreasing function of s_t.[5]

Using (4) and (5), we can determine the new stochastic discount factor to evaluate the risk-free rate:

$$\ln R_t = r_t = -\ln\beta + \gamma g - \gamma(1-\phi)(s_t - \overline{s}) - \frac{\gamma^2 Var(\Delta \ln c_t)}{2}[\lambda(s)+1] \tag{6}$$

The first two terms are the same as in the power utility function used in the model without habits; the third term means that the marginal utility has a mean reverting process, meaning that if the consumption surplus is high, the marginal utility is low so it is expected to increase in the future; the forth term is the precautionary saving term: more volatility implies a lower equilibrium interest rate. Campbell and Cochrane (1999) work with a specific function $\lambda(s)$ such that the last two terms collapse in a simpler expression:

$$\ln R_t = r_t = -\ln\beta + \gamma g - \frac{1}{2}\gamma(1-\phi). \tag{7}$$

Now we can evaluate the NRIR in a model with habit, using the same grid as before. Before taking that road, we need to calibrate the parameter ϕ. To do that, we assume that

[5] See Campbell and Cochrane (1999) where they adopt a specific functional form for this parameter, which allows them to simplify things to generate the linear model for the risk-free rate.

the Chilean economy in 1994 (a period of constant monetary policy interest rate) was in equilibrium in the sense that the output gap was zero and the interest rate was equal to the neutral. Using a growth rate of potential per capita output of 5.6% and a real interest rate of 6.5%, with γ=1.5 and β=0.978, we obtain ϕ = 0.945. The neutral interest rate for the same grid of β and γ is shown in table 2.

Table 2: The neutral interest rate from a consumption based model (habit in consumption)

	γ		
β	**1**	**1.5**	**2**
0.970	3.30	3.42	3.55
0.975	2.78	2.91	3.03
0.980	2.27	2.40	2.52
0.985	1.76	1.89	2.01
0.990	1.26	1.38	1.51

Source: Authors' estimates.

In this case, the neutral interest rate would be in the range [1.3 - 3.6]. Taking a medium value from table 2, we can conclude that the NRIR would be around 2.9 under this method.

2.2 Interest rate parity

It is common knowledge that, in the case of an open economy, interest rates are arbitrated. For this reason, we cannot analyze what is the NRIR for Chile without considering the international economy. Starting from an international interest rate corrected by expected depreciation and the risk premium, we can obtain an interest rate for Chile (i) as:

$$i = i^* + \hat{e} + \rho_s + \rho_e \ ,$$

where i^* is the international nominal interest rate, $\hat{e}$ is the expected depreciation of the nominal exchange rate, ρ_s is the country-risk premium and ρ_e is the exchange-rate risk premium. If we take medium-term values for the components of the parity equation, we can derive an estimate for the (nominal) neutral interest rate. For i^* we take the NRIR for the US economy estimated by Kozicki (2005), and we add what is considered the implicit inflation target for the US (2.5%) according to Leigh (2005). To obtain the nominal expected depreciation rate, we use the definition of the real exchange rate:

$$\hat{e} = R\hat{E}R + (\pi - \pi^*) \ ,$$

where $R\hat{E}R$ is the expected depreciation of the real exchange rate, and π and π* are the domestic and international inflation rate targets, respectively. We assume that productivity in the tradable sector relative to non-tradable sector growths at at similar path in Chile respect to the rest of the world; thus we suppose that a real exchange rate appreciation between 0% and 0.5%. The differential between domestic and international inflation target is assumed to be 0.5%, as a result of a 3% target in domestic inflation and 2.5% target in international (USA) inflation. For the country risk premium we take the JP Morgan's EMBI average from the last quarter as the upper bound (80 base points) and suppose a convergence to an inferior level of 40 bp. For the exchange rate risk premium we do not have any good prior better than assuming that exchange rate risk is fully diversified (0 bp). All this information is summarized in table 3. We estimate a nominal interest rate in the range of [5.4% - 6.3%]. Taking the Central Bank of Chile's inflation target as the expected inflation, we have NRIR in the range [2.4 – 3.3].

Table 3: The neutral interest rate from interest rate parity

	Value	*Comment*
i_n*	5%	2.5% (Clark and Kozicki, 2005) + 2.5% (Leigh, 2005)
$\hat{e}^e$	[0%, 0.5%]	0.5% $(\bar{\pi}-\bar{\pi}^*)$+ [0%, -0.5%] $(R\hat{E}R)$
ρ_s	[0.4%, 0.8%]	JP Morgan
ρ_e	0%	Diversicable risk
r_n		[2.4% - 3.3%]
i_n		[5.4% - 6.3%]

Source: Authors' estimates based on cited references.

3. The implicit natural interest rate from financial instruments

Financial instruments contain all the relevant information gathered by agents regarding news and economic perspectives. For instance, long-term interest rates reflect the market's expectations about the future path of short-term rates; then, we could obtain a measure of the natural rate of interest by identifying in financial instruments what the market expects will be the short-term interest rate (monetary policy rate) that would prevail in the long-run, after ongoing temporary imbalances in the economy worked themselves through.[6]

We estimate the implicit NRIR according to the market by two alternative methods. First, we calculate the forward interest rate at a horizon between five and ten years, using two inflation-indexed CBC's bonds, the BCU5 and the BCU10. At that horizon, we expect that eventually the actual cyclical effects will vanish and the prevailing interest rate will be close to the neutral rate. This method was proposed originally by Bomfim (2001) and thereafter applied to Chile by Calderón and Gallego (2002). Second, we estimate the implicit natural interest rate in a state-space model where a stochastic common rate and a term premium are assumed to be between a short-term (PDBC90) and a medium-term (BCP5) CBC nominal bond. This method was proposed originally by Basdevant et al. (2004), and applied to Chile by Baeza (2004).

3.1 Forward curve

Following Calderón and Gallego (2002), we can define the natural rate of interest as:

$$r_t^* = \frac{D_{10} r_{10,t} - D_5 r_{5,t}}{D_{10} - D_5} - \phi \,, \qquad (6)$$

[6] See Bomfim (2001).

where $r_{10,t}$ ($r_{5,t}$) is the BCU10 (BCU5) rate at t, D_{10} (D_5) is the BCU10 (BCU5) duration, and ϕ is a premium coefficient that can include an inflation or a liquidity risk-premium. Since we do not have a precise estimate of this coefficient, we omit it in our estimations and, therefore, our estimations under this method represent an upper bound to the NRIR.

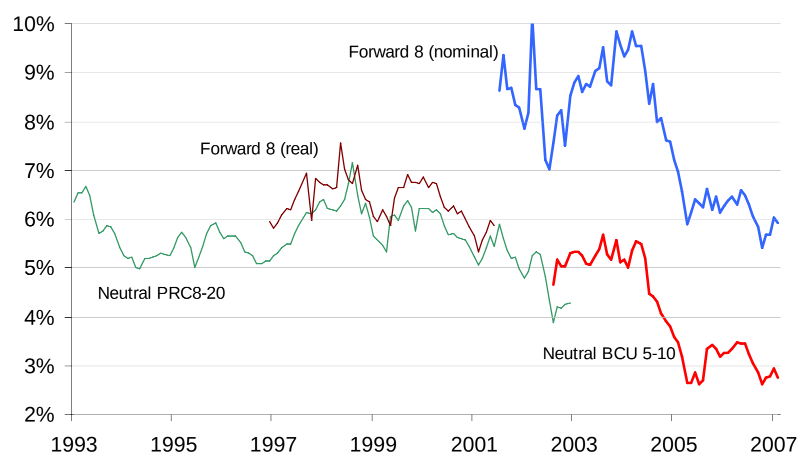
Bond duration corresponds to a weighted mean of the time elapsed when receiving the coupon. This elapsed time is multiplied by the ratio between the coupon's present value, $c/(1+r_{n,t})^i$, and the bond's price, $P_{n,t}$. Then, the bond's duration is calculated as follows:

$$D_n = \sum_{i=1}^{n}\left[i\left(\frac{c}{(1+r_{n,t})^i} \frac{1}{P_{n,t}} \right) \right] \tag{7}$$

with n=5,10.

Figure 1 depicts the natural rate of interest according to the forward curve derived from market expectations. Using the BCU bonds, the natural real rate appears to have fallen from 5% during the 2003-2005 to about 3% during the first quarter of 2007 (red line). Note that the actual nominal 8-year forward rate is about 6% (blue line), indicating an expected inflation rate around the midpoint of the inflation target.

Figure 1: Natural rate of interest, forward curve



Source: Authors' estimates.

3.2 Implicit common stochastic trend in a state-space model

Following Basdevant, et al. (2004), suppose there is a common stochastic trend r* between a short-term (r) and a long-term (R) nominal interest rate and a yield curve spread or term premium (α):

$$r_t = r_t^* + \pi_t^e + \varepsilon_{1,t} \quad (8)$$

$$R_t = r_t^* + \alpha_t + \pi_t^e + \varepsilon_{2,t} \quad (9)$$

Where $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are iid processes of zero mean and constant variances. The term π_t^e represents the expected inflation in period t. Suppose now that the stochastic trend follows a random walk process and the term premium can fluctuate following an AR(1) process:

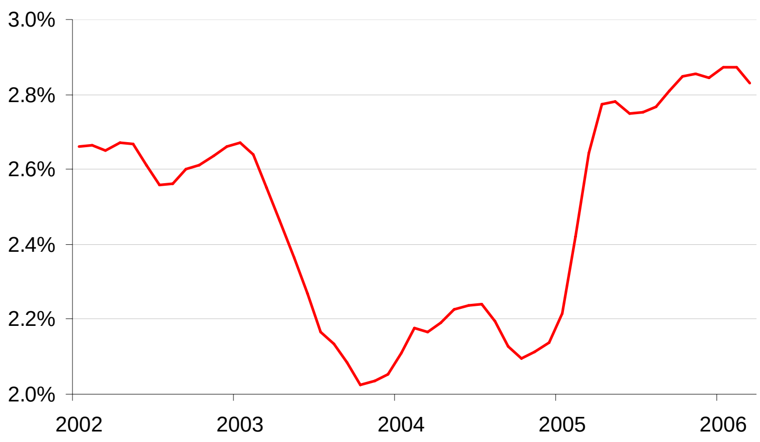
$$r_t^* = r_{t-1}^* + \upsilon_{1,t} \quad (10)$$

$$\alpha_t = \lambda_0 + \lambda_1 \alpha_{t-1} + \upsilon_{2,t} \quad (11)$$

Where $\upsilon_{1,t}$ and $\upsilon_{2,t}$ are iid processes of zero mean and constant variances. Equations (8)-(9), as signal equations, and equations (10)-(11), as transition equations, form the state-space model where the unobserved stochastic trend represents the estimate of the NRIR. This system is estimated by the Kalman filter algorithm[7], using the PDBC90 interest rate for *r*, the BCP interest rate for *R*, and the market 1-year-ahead inflation expectation for π.[8]

Figure 2 depicts the natural rate of interest according to the common stochastic trend method. It shows that the natural rate is actually about 2.8%, but was close to 2% during the last quarter of 2004, when the monetary policy rate achieved minimum levels. The estimated premium term showed low fluctuation and averaged 25 basis points during the 2002-2006 period.

Figure 2: Natural rate of interest, common stochastic trend



Source: Authors' estimates.

[7] More details on the ML estimation and the Kalman filter can be found in Hamilton (1994) and Harvey (1989).

[8] According to a survey conducted by the CBC.

4. The natural rate of interest from macroeconomic data

Although we cannot observe the natural rate of interest, economic theory can tell us how it relates with other variables that we can observe. Then, we can use economic relationships to obtain the implicit NRIR.

Suppose that the economy is characterized by a backward-looking IS curve and a backward-looking Phillips curve:

$$(y_t - y_t^*) = \sum_{s=1}^{S} \alpha_s^y (y_{t-s} - y_{t-s}^*) + \sum_{v=1}^{V} \alpha_v^r (r_{t-v} - r_{t-v}^*) + x_{1,t}'\alpha + \varepsilon_t^y \tag{12}$$

$$\hat{\pi}_t = \sum_{p=1}^{P} \beta_p^\pi \hat{\pi}_{t-p} + \sum_{q=1}^{Q} \beta_q^y (y_{t-q} - y_{t-q}^*) + x_{2,t}'\beta + \varepsilon_t^\pi \ , \tag{13}$$

where y_t is the (log of) GDP, y_t^* the (log of) potential GDP, r_t the real monetary policy rate (MPR), r_t^* the NRIR, $\hat{\pi}_t$ the inflation deviation from the inflation target, $x_{1,t}$ ($x_{2,t}$) a vector comprising other determinants of the output gap (inflation), and ε_t^y (ε_t^π) a white noise process of mean 0 and variance σ_y^2 (σ_π^2).[9]

In this semi-structural macroeconomic model, the NRIR determines the actual position of monetary policy, affecting the evolution of the output gap. The model contains two unobservable variables: the output gap and the NRIR. To estimate the implicit path for these variables, we must assume an underlying process for each one.

Suppose that the potential output grows at rate g, which follows a random walk process:

$$\begin{aligned} y_t^* &= y_{t-1}^* + g_{t-1} \\ g_t &= g_{t-1} + \varepsilon_t^g \end{aligned} \tag{14}$$

where ε_t^g is a residual term of mean 0 and variance σ_g^2.

The assumed process for the potential output implies an I(2) process. This is a specific case of a more complex model of unobserved components where potential output can be affected by a stochastic shock, and the trend growth or the output gap can evolve as autoregressive processes. Our simpler model cannot be statistically rejected, and then the results are not so different from the final estimates resulting from the flexible model.

For r_t^*, we choose two alternatives. In the first case, we depart totally from theory and assume that the natural rate of interest follows a random walk process. In the second case, as in Laubach and Williams (2003), we link the neutral interest rate to the evolution of the growth trend:

[9] For identification purpose, we add the auxiliary equation $y_t = y_t^* + y_t^c$, where y_t^c follows an iid process of zero mean and variance σ_c^2.

$$r^*_{1,t} = r^*_{1,t-1} + \varepsilon^r_{1,t} \tag{15}$$
$$r^*_{2,t} = cg_t + \varepsilon^r_{2,t} \tag{16}$$

where $\varepsilon^r_{1,t}$ ($\varepsilon^r_{2,t}$) is a residual term of mean 0 and variance $\sigma^2_{1,r}$ ($\sigma^2_{2,r}$).

Therefore, we have two models to estimate the natural rate of interest depending on the process chosen for r^*_t: Model 1 (M1), formed by equations (12) to (15); and Model 2 (M2), formed by equations (12) to (14) and (16).

The smoothness of the trend component is controlled by constraining the relative variance of ε^c_t to ε^g_t (σ^2_c / σ^2_g) to be equal to λ_1, whereas the smoothness of r^*_t is controlled by constraining the relative variance of ε^y_t to ε^r_t ($\sigma^2_y / \sigma^2_{jr}$) to be equal to λ^j_2 (with *j*=1,2). Using these restrictions, the alternative models can be estimated by maximum likelihood using the Kalman filter algorithm.

Estimations are carried out using quarterly data from 1986q1 to 2006q4. We use seasonally-adjusted data for the (core) inflation rate (CPIX1), and the GDP.[10] Inflation deviations are computed using the official CBC's inflation targets since 1991. For the previous period we used one-year-ahead inflation forecasts. To eliminate serial correlation in residuals, four lags of inflation deviations are used in equation (13) and one lag for both output and interest rate gaps in equations (12) and (13). As additional controls, in the Phillips curve (in vector $x_{2,t}$) we include the percentage deviation of both the oil-price inflation and the real exchange rate from their respective HP trends. Meanwhile, in the IS curve (in vector $x_{1,t}$) we only include the real exchange rate deviation. The estimations are carried out using λ_1=1600 (as in standard HP), λ^1_2=200, 600, 1000, and λ^2_2=100.[11]

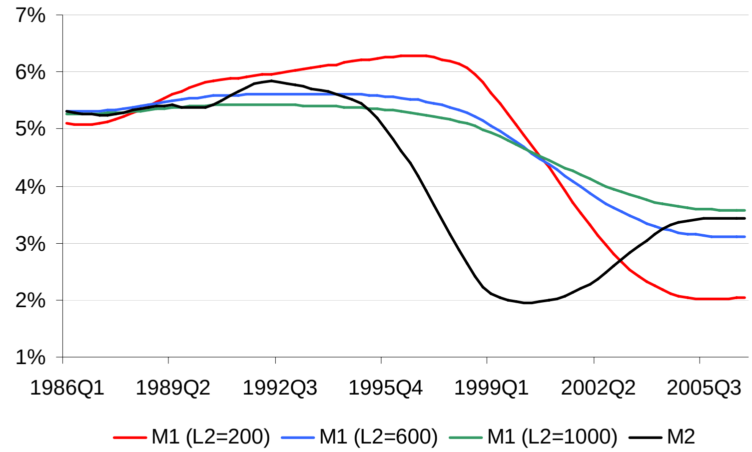
Figure 3 depicts the natural rate of interest according to the semi-structural models M1 and M2, and different smoothness parameters λ^1_2.

Model 1 is quite sensitive to the smoothness parameter; the lower λ^1_2, the lower the natural rate at the end of the sample. Using λ^1_2=200, the natural rate at the fourth quarter of 2006 is about 2%, while if using λ^1_2=1000, the natural rate approaches 3.6%. Note that Model 1 yields a declination of the natural rate since 1999, from levels above 6%, while Model 2 yields a natural rate that started to decline at least four years before and shows recuperation after the 1999 crisis. At the end of the sample, Model 2 yields a natural rate of about 3.5%. Note that at the beginning and at the end of the sample, both methods yield similar results. The main differences arise around the 1999 recession.

[10] CPIX1 inflation excludes oil, perishable goods, and some regulated utilities.

[11] Estimates from Model 2 are less sensitive to λ^2_2.

Figure 3: Natural rate of interest, semi structural models



L2 denotes for λ_2. Source: Authors' estimates.

Of particular interest is parameter *c* that links trend growth with the natural rate of interest. The estimation of M2 yields a value for *c* equal to 2.91, that means that a one-percent increase in trend growth results in an increase of approximately 0.73 percent in the natural rate of interest (see table 4).[12] The rest of parameters, in both the IS and the Phillips curve, have the expected signs and are statistically significant.

Table 4: Model M2, parameter estimates

$\sum_{p=1}^{4}\pi_{t-p}$	β_1^{π}	α_1^{y}	α_1^{r}	c
0.367	0.428	0.747	-0.085	2.913
(0.148)	(0.140)	(0.119)	(0.049)	(0.198)

Standard errors in paretheses. Source: Authors' estimates.

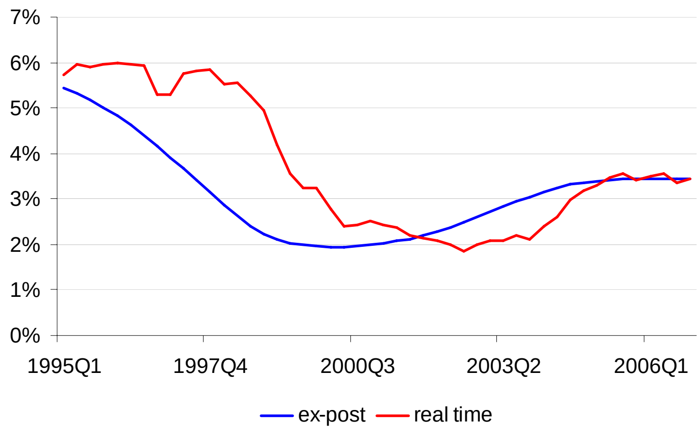
Figure 4 shows how reliable estimations are in real time (for Model 2).[13] Note that before the Asian Crisis, the real-time natural rate of interest was higher by more than 200 basis points than the natural rate calculated ex-post. This reveals that real-time estimation can differ substantially from ex-post estimation, in line with previous studies.[14]

[12] Since estimations are carried out at quarterly frequency, we proxy the effect on annual rates by dividing 2.91 by 4.

[13] This estimation considers quarter-to-quarter rolling estimation. We do not use GDP data revision, then the exercise could be called a quasi-real time estimation.

[14] See Clark and Kozicki (2005).

Figure 4: Natural rate of interest, real-time versus ex-post estimation (M2 model)



Source: Authors' estimates.

5. Summary and conclusions

We learn that the neutral interest rate is not a time invariant variable. It is closely related to the potential growth of the economy (but it is not the growth rate of the trend output). In the case of Chile, a small, open economy, it is also closely related with international financial conditions. This paper has shown that the neutral interest rate could be estimated from a semi-structural model and with the information provided by the domestic financial market. Each of these relations gave different point estimates, but despite the disparities in methodologies, the models gave similar results.

In summary, the neutral real interest rate varies depending on the method used. In table 5 we present the entire estimate with a median equal to 2.8%, and the range goes from 2% to 3.6%.

Table 5: Natural rate of interest, summary of results

Method	Value
1. Consumption based model	2.90%
2. Interest rate parity	2.40%-3.30%
3. Forward curve	2.75%
4. Common stochastic trend	2.80%
5. Macroeconomic model M1	2.00%-3.60%
6. Macroeconomic model M2	3.50%
MEDIAN	**2.83**

Source: Authors' estimates.

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Appendix: Natural rate of interest, survey of studies.

Autor	Institution	Country	Value (%)	Method	Period
European Central Bank (2004)	European Central Bank	EURO	2-4	Not specified	2004
Manrique, Marqués (2004)	Banco de España	USA	2.9	Kalman filter	1964-2003
		USA	2.6	Kalman filter	1993-2003
		USA	1.6-2.3	Kalman filter	2003
		Germany	1.9	Kalman filter	1964-2003
		Germany	1.4	Kalman filter	1993-2003
		Germany	0.5-1.7	Kalman filter	2003
Shadow Council	Shadow Council	EURO	2.25	Not specified	2006
Basdevant, et al. (2004)	Reserve Bank of New Zealand	N. Zealand	3.25-4.25	Yield curve/Kalman filter	2004
Basdevant, et al. (2003)	Reserve Bank of New Zealand	UK	2-2.5	Yield curve/Kalman filter	2003
		Australia	2-2.5	Yield curve/Kalman filter	2003
		Sweden	2-2.5	Yield curve/Kalman filter	2003
		Canada	1	Yield curve/Kalman filter	2003
		USA	1	Yield curve/Kalman filter	2003
		Switzerland	1	Yield curve/Kalman filter	2003
Giammarioli, Valla (2003)	European Central Bank	EURO	3.7	DSGE	1994
		EURO	3	DSGE	2000
Mesonnier, Renne (2004)	Bank of France	France	4	Kalman filter	2000
		France	1.5	Kalman filter	2002
Crespo-Cuaresma, Ritzberger-Gruenwald (2003)	Austrian National Bank	EURO	1.5-2	Cycle-trend decomposition/ Kalman filter	2002
OCDE (2004)	OCDE	USA	2	Kalman filter	2004
Lam,Tkacz (2004)	Bank of Canada	Canada	1.25-2	DSGE	end 2002
Brzoza-Brezezina (2004)		Poland	4	SVAR, Kalman filter	2003
Amato (2004)	BIS	USA, EURO	2.25-2.75	HP, Kalman filter	2004
Bernharsen (2005)	Norges Bank	Norway	3-4	Yield curve	1998-03
		Norway	<3	Yield curve	mid 2003
Norges Bank (2004)	Norges Bank	Norway	3	Taylor rule	2004
Norges Bank (2005)		Norway	2.5-3.5	Interest rate parity	2005
Richard Lambert (2005)	Bank of England	UK	2.5	Yield curve	2005
Clark and Kozicki (2005)	Fed. Res. Bank of Kansas City	USA	2.5	Kalman filter	2005
US Financial Times (2005)	US Financial Times	USA	2.75	Not specified	2005
Robert Parry (2005)	(ex President) San Fco. Fed. Reserve Bank	USA	2.5-3.5	Historical mean	2005
Mickey Levy (2005)	Banc of America Securities	USA	2.5-3	Not specified	2005
Goldman Sachs (2005)	Goldman Sachs	USA	2.5	Not specified	2005
Ted Carmichael (2004)	JPMorgan	USA	1.5	Not specified	end 2002
Gabriel Stein (2003)	Lombar Street Research (London)	USA	1.5	Not specified	end 2003
Anderson, Buol, Rasche (2004)	Bank of St. Louis	USA	3	Trend growth	end 2004
		USA	1-1.5	Trend growth adjusted by risk premium	end 2004
FOMC, public statements (2004)		USA	1.5-3.5	Not specified	end 2004

Source: Authors' elaboration based on cited references.

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