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ECONOMY UNDER SEGMENTED ASSET MARKETS
AND STICKY PRICES**

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**OPTIMAL MONETARY POLICY IN A SMALL OPEN
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Resumen

Este trabajo estudia la política monetaria óptima en un modelo de dos sectores para un economía pequeña con mercados financieros segmentados y precios rígidos. Resolvemos el problema de Ramsey bajo completa credibilidad y caracterizamos la política monetaria óptima en una versión del modelo calibrada para Chile. Las principales contribuciones de este trabajo son dos. Primero, bajo la política óptima la volatilidad de la inflación no transable es casi cero. Segundo, estabilizar la inflación no transable es una decisión óptima sin importar la estructura financiera de la economía pequeña. Incluso para un nivel moderado de rigidez de precios, implementar una política monetaria que mitiga la segmentación de mercados es altamente distorsionante. Este último resultado sugiere que las imperfecciones financieras debieran ser corregidas con otros instrumentos.

Abstract

This paper studies optimal monetary policy in a two-sector small open economy model under segmented asset markets and sticky prices. We solve the Ramsey problem under full commitment, and characterize the optimal monetary policy in a version of the model calibrated to the Chilean economy. The contributions of the paper are twofold. First, under the optimal policy the volatility of non-tradable inflation is near zero. Second, stabilizing non-tradable inflation is optimal regardless of the financial structure of the small open economy. Even for a moderate degree of price stickiness, implementing a monetary policy that mitigates asset market segmentation is highly distortionary. This last result suggests that policymakers should resort to other instruments in order to correct financial imperfections.

1 Introduction

The last two decades have been marked by drastic changes in the way monetary policy is conducted. An increasing number of central banks around the world are independent¹, have adopted price stability as the main goal, and have switched to a floating exchange rate regime. The consensus among policymakers is that a low and stable inflation rate is a necessary condition for macroeconomic stability and economic growth². Figure 1 illustrates this point and shows how inflation has converged to a single digit along the 1990s in both developing and developed economies. Although monetary authorities in both groups of countries share a price stability goal, the economic environment in which the monetary policy is implemented is completely different. In particular, developing countries face shallow financial markets that prevent efficient consumption and saving decisions. Figure 2, which compares the ratio of banking deposits to GDP in developing and developed countries, makes this point clear. This measure of financial depth indicates that households and firms in developing countries have limited access to financial instruments to carry out efficient intertemporal decisions.

The magnitude of financial imperfections in emerging market economies has triggered an intense debate among academics and policymakers about the role of monetary policy in this environment³. If households cannot smooth consumption over time, one possible

¹By independent we mean that the monetary authority is not subordinate to the fiscal authority.

²According to the testimony of Alan Greenspan before the Senate Banking Committee on July 2004 : "For twenty-five years, the Federal Reserve has worked to reestablish price stability on a sustained basis. An environment of price stability allows households and businesses to make decisions that best promote the longer-term growth of our economy and with it our nation's continuing prosperity". On the other hand, the Central Bank of Chile described their policy goals in the Monetary Policy Report of May 2004: "The main purpose of the Central Bank of Chile's monetary policy is to keep inflation low and stable, targeted at a range of 2% to 4% per annum, centered on 3%. Controlling inflation is the means by which monetary policy contributes to the population's welfare. Low, stable inflation improves economic performance and growth, while preventing the erosion of personal income". Both quotes reflect the consensus among policymakers about the importance of price stability in promoting economic efficiency and growth.

³For instance, Céspedes et al. (2004) and Gertler et al. (2003) discuss whether it is optimal for the monetary authority to stabilize the exchange rate in order to insure firms that have liabilities

solution to correct this distortion is to implement a monetary policy that increases consumption in bad states of nature and reduces it in good states of nature. Such a policy may improve the intertemporal allocation of households excluded from financial markets. A challenge for the literature of monetary policy in open economies is to study the design of optimal monetary policy in a model that captures the lack of financial development. In particular, it is important to draw clear prescriptions for policymaking in developing countries: Is price stability an optimal policy criterion for a developing small open economy? Is there a role for monetary policy to correct financial imperfections? In this paper we consider these questions in the framework of a dynamic stochastic general equilibrium model.

In the model economy there are two sectors: tradable and non-tradable. The firms in the tradable sector are perfectly competitive and can adjust their prices freely. On the other hand, firms in the non-tradable sector are monopolistically competitive and display price stickiness. Sticky prices are modeled as a quadratic adjustment cost for firms à la Rotemberg(1982)⁴. This cost induces sluggishness in the price level and generates a real effect for monetary injections. We model financial imperfections in developing countries as an asset market segmentation problem. In this environment, only a fraction of the population has access to international capital markets. The households that are excluded from financial markets can only save through the accumulation of real money balances. Even though this assumption cannot capture all types of financial imperfections present in developing countries, it is a tractable way to model the lack of financial assets for a

denominated in foreign currency. Both authors find that a fixed exchange rate is a suboptimal policy that exacerbates the negative effects of an external shock. Caballero and Krishnamurthy (2003), in a different framework, study how monetary policy should be designed in order to provide insurance incentives to households and firms against "sudden stops" (i.e. sudden capital outflows from emerging market economies). They propose a monetary regime in which the private sector may have an incentive to accumulate foreign assets to better deal with sudden stops.

⁴Rotemberg (1982) mentions two reasons why price changes might be costly. First, there is the physical cost of changing posted prices (menu costs). Second, the costs are related to the negative effects on reputation when firms frequently change their prices.

large segment of the population ⁵.

We follow Ramsey (1927) and Lucas and Stokey (1983) in characterizing the optimal monetary policy. In this approach, the Ramsey planner chooses an allocation that maximizes the household's welfare subject to the resource constraints of the economy and additional constraints that capture the equilibrium reactions by firms and households to monetary policy⁶. In addition, we assume that the monetary authority implements the policy under full commitment. This assumption implies that monetary policy is completely credible, and prevents any inflation bias problem in the implementation of the optimal policy.

We calibrate the parameters of the model to the Chilean economy. We chose this country as a benchmark case, since Chile is the first emerging market economy to implement an inflation targeting scheme⁷. The central bank has gained substantial credibility in recent years due to its good reputation in controlling the inflation rate. This is consistent with the lack of inflation bias in the model. Moreover, Chile is a developing country that, though has implemented several structural reforms since the 1970s, displays underdeveloped financial markets and price indexation in the goods markets. Both features are taken into account in the model by assuming asset market segmentation and sticky prices.

The contributions of the paper are twofold. First, the optimal policy largely stabilizes the price of non-tradable goods. A full stabilization of the non-tradable price would be the prescription to completely undo sticky prices⁸. However, since there are additional

⁵Campbell and Mankiw (1989) show empirical evidence that around 50 percent of households in the United States base their consumption decisions on current income, which is consistent with the hypothesis of segmented asset markets. For emerging economies, where financial markets are underdeveloped, it is reasonable to assume that asset market segmentation is at least as severe as in the United States.

⁶This is the primal approach of the Ramsey problem.

⁷Under this framework, the monetary authority announces numerical targets for the inflation rate, and there is a commitment to achieve these targets. Moreover, the central bank is accountable for the policies implemented and there is an increased transparency in the communication of their plans and objectives with the public.

⁸A complete stabilization of the price of non-tradable goods eliminates the distortion associated with

distortions in the model such as monopolistic competition, monetary transactions costs and asset market segmentation, the optimal monetary policy deviates from full price stability in order to minimize the impact of these other frictions on the household's welfare. For a plausible calibration of the distortions mentioned above, we found that the quantitative deviation from price stability is negligible.

Second, we find that a stable non-tradable inflation rate is optimal for any degree of asset market segmentation. In the model there is a tradeoff between undoing the asset market segmentation and sticky price distortions. If the objective is to undo the sticky price distortion, then the monetary policy should be pro-cyclical⁹ when there are productivity shocks in the non-tradable sector. To gain intuition about the procyclicality of the monetary policy we can think about the case when there is an increase in the productivity of the non-tradable sector. The consequence of this productivity shock is an expansion of the non-tradable output and a decrease of the price level in that sector. In order to prevent a decline of the price of non-tradable goods, and hence the resource cost associated with this contraction, it is necessary to stimulate the aggregate demand with a monetary injection. Thereby, a pro-cyclical policy that completely stabilizes the price level will undo the distortion associated to sticky prices.

On the other hand, if the goal is to correct the asset market segmentation distortion, the monetary policy should be counter-cyclical. The key element to understand is the wedge that monetary transaction costs generate. By injecting money in bad states of nature, or when the output is low, the transaction costs are reduced. A lower transaction cost decreases the effective price of consumption, and makes it possible for households to increase their consumption when they are affected by a negative shock. In this way, a counter-cyclical policy improves the intertemporal allocation for those households who

sticky prices. Woodford (2001) shows that in a model economy with sticky prices, price stability is the welfare-maximizing policy. Goodfriend and King (2001) find that price stability is the optimal policy. They call this a *neutral* policy, since it keeps output at the potential level, defined as the outcome of an imperfectly competitive real business cycle model.

⁹Ireland (1996) derives the same result.

do not participate in the asset markets.

When we calibrate the model economy for plausible values of asset market segmentation and price stickiness we find that the tradeoff between correcting these distortions is resolved in favor of undoing price stickiness. Hence, the optimal monetary policy largely stabilizes fluctuations in the non-tradable price. This suggests that correcting the asset market segmentation with monetary policy is highly distortionary. If a monetary authority wants to improve the intertemporal allocation of the households without access to financial markets, it must generate a high volatility in the price of non-tradable goods. Under sticky prices, this volatility generates a deadweight loss in the non-tradable sector that is welfare reducing. For different values of asset market segmentation, this result holds and the optimal monetary policy induces a low volatility of the non-tradable price.

This paper is related to several studies about optimal monetary policy. In a closed economy with flexible prices and perfect competition, Lucas and Stokey (1983), Chari et al (1991) and Chari and Kehoe (1999) followed the Ramsey approach to characterize the optimal monetary policy. They have proven that in this environment the optimal monetary policy should be to set the nominal interest rate to zero (i.e. Friedman's rule). This theoretical result recently has been challenged in models that incorporate monopolistic competition and sticky prices. Khan et al.(2003), Schmitt-Grohé and Uribe (2004) and Siu (2004) find that when the adjustment of prices implies a cost in terms of resources, the Friedman rule is no longer an optimal policy prescription in a closed economy. Nevertheless, when nominal rigidities are present, the monetary policy should approximately stabilize the price level. The presence of monopolistic competition and money distortion induce a deviation from full price stability, however the quantitative magnitude of this departure is minor¹⁰.

¹⁰Adão et al. (2003) show that, in general, in an environment with sticky prices, a cash-in-advance constraint and monopolistic competition it is not optimal to undo the sticky price distortion. They do not, however, evaluate quantitatively how much the allocation under the optimal policy differs from the flexible price allocation.

In an open economy setting, there is an extensive literature about monetary policy under monopolistic competition and sticky prices. However, the Ramsey approach has not been broadly used as in closed economy models¹¹. For the case of small open economy model with sticky prices, Galí and Monacelli (2003) derive the optimal policy but assuming a cashless economy and removing the monopolistic power distortion with an employment subsidy. They find that the optimal policy fully stabilizes the domestic price level. This policy reproduces the flexible price allocation and maximizes the household's welfare.

Asset market segmentation is another friction that has been recently introduced into dynamic general equilibrium monetary models¹². Lahiri et al. (2004) characterize the optimal policy in a small open economy with flexible prices and segmented asset markets. In their model, the optimal monetary policy is a state contingent rule aimed to provide insurance to the agents excluded from asset markets. This policy achieves the first-best allocation and completely undo the asset market segmentation problem in the model economy.

The goal of this paper is to characterize the interaction between sticky prices and asset market segmentation in the design of the optimal monetary policy in a small open economy. In principle, both frictions are present in developing countries, and it is not evident how a monetary policy should deal simultaneously with these distortions. As opposed to most of the open economy macroeconomics literature, we solve the Ramsey problem to characterize the optimal monetary policy in this environment. This approach makes it possible to analyze from a general equilibrium perspective how monetary policy should be implemented to correct multiple distortions in an economy.

The remainder of this paper is organized as follows. Section 2 presents the small

¹¹Schmitt-Grohé and Uribe (2003) solve the Ramsey problem for a small open economy in a flexible price environment. Faia and Monacelli (2004) follow the same approach in a two-country model with sticky prices and monopolistic competition.

¹²See Grossman and Weiss (1983), Rotemberg (1984), Alvarez and Atkeson (1997) and Alvarez et al. (2002).

open economy model. Section 3 describes the Ramsey problem under full commitment. Section 4 discusses the calibration strategy, and analyzes the quantitative results of the model. Section 5 concludes.

2 Model

In this section we describe a simple infinite-horizon production economy with sticky prices and segmented asset markets. This economy consists of two types of households: traders and non-traders. The former type of agent has access to financial markets while the latter one does not participate in them. The non-traders can only save through the accumulation of real money balances. To simplify the model, we suppose that household type (i.e., traders and non-traders) is fixed over time and that the fraction of households participating in the financial market denotes the degree of asset market segmentation in the economy.

The production side of the model has two sectors: tradable and non-tradable. The tradable good sector exhibits flexible prices and takes international prices as given. In contrast, the non-tradable sector displays monopolistic competition and sticky prices, which are modelled as a quadratic adjustment cost à la Rotemberg. The introduction of money in this model is motivated as a device to reduce household's transaction costs. The fiscal policy is characterized by a balanced budget and the government expenditure is financed with lump sum taxes levied on both types of households. Money injections are engineered in the financial markets, so the traders are the only ones who absorb them. The model has three types of exogenous fluctuations: productivity shocks in the tradable and non-tradable sectors, and government expenditure shocks.

2.1 Households

The households decide a sequence of tradable and non-tradable consumption and labor supply with the object to maximize their expected present value utility:

$$U(i) = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t^T(i), c_t^N(i), l_t(i)) \right] (i = tr, nt)$$

where c^T , c^N , and l denote tradable consumption, non-tradable consumption, and labor supply, respectively. β is a subjective discount factor and \mathbb{E}_0 denotes the expectation operator conditional on the information in period 0. The index $i = tr$ stands for allocations for trader households while $i = nt$ is the same but for non-trader households. The two types of households share the same preferences despite having different access to financial markets.

The role of money is to facilitate consumption purchases. In particular, we assume that consumption of both types of goods are subject to a proportional transaction cost, $s(v_t(i))$, that depends on the money velocity:

$$v_t(i) = \frac{p_t^T c_t^T(i) + p_t^N c_t^N(i)}{M_t(i)}$$

where p^T and p^N are the prices of tradable and nontradable goods, respectively. $M(i)$ is the nominal money holdings of type i household.

2.1.1 Traders

The fraction of traders in the economy is denoted by λ . As traders, they have access to two other types of financial assets besides money. They can trade domestic one-period contingent bonds and international one-period non-contingent bonds. The domestic bond delivers one unit of domestic currency in the next period in some particular state.

The international bond delivers one unit of foreign currency in the next period in each state of nature. Consequently, the trader's budget constraint is described by:

$$\begin{aligned}
& (1 + s(v_t(tr)))(p_t^T c_t^T(tr) + p_t^N c_t^N(tr)) + \mathbb{E}_t[q_{t,t+1}d_{t+1}(tr)] \\
& + e_t b_t^*(tr) + M_t(tr) = W_t l_t(tr) + d_t(tr) + e_t R_{t-1}^* b_{t-1}^*(tr) \\
& + M_{t-1}(tr) + \frac{\Pi_t}{\lambda} + \frac{X_t}{\lambda} - T_t - \frac{1-\lambda}{\lambda} p_t^N S
\end{aligned} \tag{1}$$

We explain each of the terms in order. On the left-hand side of the equation, the first term is the expenditure on tradable and nontradable consumption goods including transaction costs. The second one, is the expenditure on domestic contingent bonds. $d_{t+1}(tr)$ is the units of these bonds bought by the trader and $q_{t,t+1}$ is the period t price of these securities normalized by the probability of the occurrence of each state of nature. The trader also buys $b_t^*(tr)$ units of international non-contingent bonds where e_t denotes the nominal exchange rate. The fourth term is the money holdings that the trader chooses to carry over from t to $t + 1$. On the right-hand side of the equation we include the sources of income. The first term is the labor income. W_t is the nominal wage rate and $l_t(tr)$ is the amount of labor supplied. $d_t(tr)$ is the quantity of contingent bonds held by the trader from the previous period that pays at the state in current period t . The third term on the right-hand side is the return on non-contingent international bond holding where R_{t-1}^* is the gross interest rate on this bond in terms of foreign currency. $M_{t-1}(tr)$ is the money holdings from the last period and Π_t is the nominal profits from firms¹³. X_t is the per capita money injections which are carried out in the financial markets and for that reason, are only absorbed by traders. Due to the fact that the size of traders in the economy is λ , Π/λ and X/λ are the dividends and money injections per

¹³Since traders participate in financial markets, they will hold shares of the firms. The fact that firms in the non-tradable sector have market power implies that these profits will be strictly positive.

trader. T is the lump sum taxes which are designed to finance government expenditures and is the same across all type of households.¹⁴ The last term is included to avoid a wealth difference between traders and nontraders in the steady state. S is the subsidy, in terms of nontradable goods, that each non-trader receives which is financed with a tax on trader households¹⁵.

The problem for the traders is to maximize their utility subject to their budget constraint, initial asset holdings $(M_{-1}(tr), d_0(tr), b_0^*(tr))$, and borrowing constraints that prevent Ponzi schemes:

$$d_t(tr) \geq -\bar{d}, b_t^*(tr) \geq -\bar{b}$$

The following are the first-order conditions for their problem:

$$\frac{u_{c^T,t}(tr)}{u_{c^N,t}(tr)} = \frac{p_t^T}{p_t^N} \quad (2)$$

$$-\frac{u_{l,t}(tr)}{u_{c^T,t}(tr)} = \frac{W_t}{p_t^T h(v_t(tr))} \quad (3)$$

where

$$h(v_t(tr)) = 1 + s(v_t(tr)) + s'(v_t(tr))v_t(tr)$$

¹⁴In other words, T_t is the per capita lump sum tax charged to finance the government expenditure in period t .

¹⁵Since traders have more options in terms of assets and receive the firms dividends, they are wealthier than nontraders. This tax on the traders is a subsidy to the nontraders that prevents wealth differences in the steady state.

$$\frac{u_{c^N,t}(tr)}{p_t^N h(v_t(tr))} (1 - s'(v_t(tr))(v_t(tr))^2) = \beta \mathbb{E}_t \left[\frac{u_{c^N,t+1}(tr)}{p_{t+1}^N h(v_{t+1}(tr))} \right] \quad (4)$$

$$\frac{u_{c^T,t}(tr)e_t}{p_t^T h(v_t(tr))} = \beta R_t^* \mathbb{E}_t \left[\frac{u_{c^T,t+1}(tr)e_{t+1}}{p_{t+1}^T h(v_{t+1}(tr))} \right] \quad (5)$$

$$q_{t,t+1} = \beta \frac{u_{c^N,t+1}(tr)}{u_{c^N,t}(tr)} \frac{p_t^N h(v_t(tr))}{p_{t+1}^N h(v_{t+1}(tr))} \quad (6)$$

Equation (2) determines the relative demand of tradable and nontradable goods by the traders as a function of the relative price of tradable goods (p^T/p^N). The traders' labor supply is specified by (3) which equates the marginal rate of substitution between leisure and tradable consumption with the real wage in terms of tradable goods. Since the transaction cost affects the effective price of consumption goods, it introduces a wedge ($h(v_t(tr))$) in the labor supply decision¹⁶.

Equations (4), (5), and (6) define indirectly a money demand function, an interest parity condition, and the market nominal interest rate. To see them clearly, we have to manipulate the equations. First, recall that the gross nominal interest can be written as:

$$R_t = (\mathbb{E}_t[q_{t,t+1}])^{-1}$$

Combining this last expression with (4) implies $R_t(1 - s'(v_t(tr))(v_t(tr))^2) = 1$. Using the definition of velocity and writing $p^T c_t^T(tr) + p_t^N c_t^N(tr)$ as $p_t c_t(tr)$ we obtain:¹⁷

$$\frac{M_t(tr)}{p_t} = \frac{c_t(tr)}{G^{-1}\left(\frac{R_t-1}{R_t}\right)}$$

¹⁶The term ($h(v_t(tr))$) is standard in models with transaction costs. This wedge can be interpreted as an implicit consumption tax.

¹⁷In this case p_t denotes the aggregate price level and $c_t(tr)$ is the composite consumption of traders.

where $G(\cdot)$ is defined as $G(v) = s'(v)v^2$. Also, combining the expression of the gross nominal interest rate with (5) we get an interest parity condition:

$$\frac{R_t}{R_t^*} = \mathbb{E}_t \left[\frac{e_{t+1}}{e_t} \right] + R_t \text{COV} \left(q_{t,t+1}, \frac{e_{t+1}}{e_t} \right)$$

2.1.2 Non-traders

The size of non-trader households in the economy is $(1 - \lambda)$. This type of household does not have access to financial markets and can only use money holdings to transfer resources across time. Their labor supply is a perfect substitute of traders labor supply and therefore they receive the same nominal wage rate W_t . In the same way as traders, they pay T_t in lump sum taxes. As discussed before, they also receive a subsidy S in terms of non-tradable goods such that the wealth difference with traders disappears in the steady state. These elements imply the following budget constraint for non-traders:

$$(1 + s(v_t(nt)))(p_t^T c_t^T(nt) + p_t^N c_t^N(nt)) + M_t(nt) = \tag{7}$$

$$W_t l_t(nt) + M_{t-1}(nt) - T_t + p_t^N S$$

The first-order conditions obtained by maximizing the non-traders utility function subject to their budget constraint are:

$$\frac{u_{c^X,t}(nt)}{u_{c^N,t}(nt)} = \frac{p_t^T}{p_t^N} \tag{8}$$

$$-\frac{u_{l,t}(nt)}{u_{c^X,t}(nt)} = \frac{W_t}{p_t^T h(v_t(nt))} \tag{9}$$

where:

$$h(v_t(nt)) = 1 + s(v_t(nt)) + s'(v_t(nt))v_t(nt)$$

$$\frac{u_{c^N,t}(nt)}{p_t^N h(v_t(nt))} (1 - s'(v_t(nt))(v_t(nt))^2) = \beta \mathbb{E}_t \left[\frac{u_{c^N,t+1}(nt)}{p_{t+1}^N h(v_{t+1}(nt))} \right] \quad (10)$$

Equations (8), (9), and (10) are equivalent to the equations (2), (3), and (4) derived for the traders. Specifically, (8) determines the relative consumption of tradable vis-à-vis non-tradable goods for the non-traders as a function of the relative price of tradable goods. (9) is the labor supply of nontraders and (4) is their implicit money demand. However, since these households do not participate in the asset markets, the implicit money demand does not depend on the nominal interest rate¹⁸.

2.2 Firms in the Tradable Sector

The firms in the tradable sector behave competitively and have a constant returns to scale technology that uses labor and nontradable goods as inputs. In particular, the production of tradable good y^T is described by:

$$y_t^T = z_t^T f^T(l_t^T, N_t) \quad (11)$$

where z_t^T denotes the productivity level displayed in this sector. This variable follows an exogenous stochastic process. l^T and N are the amount of labor and final non-tradable goods used as inputs, respectively. Hence, the firms in the tradable sector solve the following static maximization problem:

$$\max_{l_t^T, N_t} \{ p_t^T z_t^T f^T(l_t^T, N_t) - W_t l_t^T - p_t^N N_t \}$$

which delivers two optimality conditions:

$$W_t = z_t^T p_t^T f_{l^T,t}^T \quad (12)$$

¹⁸This is due to the fact that in absence of asset markets, the intertemporal rate of substitution is not longer linked to the nominal interest rate.

$$p_t^N = z_t^T p_t^T f_{N,t}^T \quad (13)$$

(12) and (13) determine the labor and non-tradable inputs demanded by firms.

2.3 Firms in the Non-tradable Sector

There are two types of firms in the non-tradable sector: retailers and intermediate good producers. The latter use labor to produce a differentiated good while the former combine these intermediate inputs to produce a final good consumed by the households.

2.3.1 Retailers

Retailers create units of nontradable final goods according to a constant elasticity of substitution aggregator of a continuum of nontradable intermediate goods which are indexed along the unit interval $j \in [0, 1]$. Specifically, retailers produce y_t^N units of non-tradable final goods using the following constant returns to scale technology:

$$y_t^N = \left[\int_0^1 y_t^N(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (14)$$

Then the retailers allocate their demands for non-tradable intermediate good $y_t^N(j)$ for all $j \in [0, 1]$, to maximize its profits

$$p_t^N y_t^N - \int_0^1 p_t^N(j) y_t^N(j) dj$$

subject to the constraint (14). The first-order condition of the problem leads to intermediate input demands with constant elasticity:

$$y_t^N(j) = y_t^N \left[\frac{p_t^N(j)}{p_t^N} \right]^{-\varepsilon} \quad (15)$$

The maximization problem also delivers an aggregate price level for the non-tradable goods:

$$p_t^N = \left[\int_0^1 p_t^N(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}} \quad (16)$$

2.3.2 Intermediate Good Producers

The producers of non-tradable intermediate inputs are assumed to be monopolistic competitors and face a cost of adjusting their prices. In particular, we follow Rotemberg (1982) and consider quadratic costs of price adjustment. This creates a sluggish price adjustment by assuming that the intermediate producer of variety j faces a resource cost that is quadratic in the inflation rate of that input (in terms of non-tradable final goods):

$$\frac{\kappa}{2} \left(\frac{p_t^N(j)}{p_{t-1}^N(j)} - 1 \right)^2$$

The parameter κ measures the degree of price stickiness present in the non-tradable sector. The higher κ is, the more sluggish is the adjustment of nominal prices in this sector. The production technology for each non-tradable intermediate input j is given by:

$$y_t^N(j) = z_t^N l_t^N(j)$$

where $l_t^N(j)$ is the labor utilized by this intermediate producer who pays a nominal wage rate W_t . z_t^N is the productivity level of the nontradable sector, which follows an exogenous stochastic process and it is assumed the same for all firms. Therefore, the nominal profits of an intermediate producer of type j in period t are:

$$p_t^N(j)y_t^N(j) - W_t l_t^N(j) - p_t^N \frac{\kappa}{2} \left(\frac{p_t^N(j)}{p_{t-1}^N(j)} \right)^2$$

The adjustment cost in prices generates an intertemporal link in the optimal decisions of intermediate producers since changes in prices in the current period will affect the cost of adjusting them in the next period. Formally, the intermediate producer of variety j will choose a sequence of prices given the demand function (15), the production function, the wage rate, the initial price (p_{-1}^N) and productivity level, maximizing the expected present value of profits¹⁹:

$$\max_{\{p_t^N(j)\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} q_{0,t} \left(p_t^N(j)y_t^N(j) - W_t l_t^N(j) - p_t^N \frac{\kappa}{2} \left(\frac{p_t^N(j)}{p_{t-1}^N(j)} - 1 \right)^2 \right) \right] \quad (17)$$

where $q_{0,t}$ is the price of a nominal contingent security in period 0 that delivers one unit of domestic currency in period t in some particular state normalized by the probability of occurrence. The prices of the securities can be constructed recursively using initially the one period contingent bonds which are priced by the traders ($q_{0,t} = q_{t-1,t}q_{0,t-1}$). The first-order condition for this optimization problem is:

$$\begin{aligned} & -(\varepsilon - 1)z_t^N l_t^N(j) \left(\frac{p_t^N}{p_{t-1}^N(j)} \right)^\varepsilon + \varepsilon \frac{W_t}{z_t^N p_t^N} z_t^N l_t^N(j) \left(\frac{p_t^N}{p_{t-1}^N(j)} \right)^{1+\varepsilon} \\ & - \kappa \frac{p_t^N}{p_{t-1}^N(j)} \left(\frac{p_t^N(j)}{p_{t-1}^N(j)} - 1 \right) + \kappa \mathbb{E}_t \left[q_{t,t+1} \left(\frac{p_{t+1}^N(j)}{p_t^N(j)} - 1 \right) \frac{p_t^N p_{t+1}^N(j)}{(p_t^N(j))^2} \right] = 0 \end{aligned} \quad (18)$$

We will focus on a symmetric equilibrium for the non-tradable intermediate input

¹⁹For simplicity we will assume that the initial prices of all intermediate producers are equal, that is $p_{-1}^N(j) = p_{-1}^N$.

producers which implies that they will charge the same price. As a result, $p_t^N = p_t^N(j)$, $l_t^N = l_t^N(j)$ for all $j \in [0, 1]$. This symmetry translates into the following version of the Phillips curve for the aggregate non-tradable sector:

$$\pi_t^N (\pi_t^N - 1) = \mathbb{E}_t [q_{t,t+1} (\pi_{t+1}^N)^2 (\pi_{t+1}^N - 1)] - \frac{\varepsilon}{\kappa} z_t^N l_t^N \left[\frac{\varepsilon - 1}{\varepsilon} - mc_t \right] \quad (19)$$

where $\pi_t^N = p_t^N / p_{t-1}^N$ is the gross inflation rate of non-tradable goods and mc_t denotes the real marginal cost of producing non-tradable intermediate goods. Due to the fact that intermediate good producers have a constant returns to scale technology, their marginal cost is the same for all of them. The marginal cost is then given by:

$$mc_t = \frac{W_t}{p_t^N z_t^N} \quad (20)$$

Finally, using the symmetry across intermediate good producers and the technology function of final producers of nontradable goods, we can write total final production in this sector as:

$$y_t^N = z_t^N l_t^N \quad (21)$$

2.4 Government

The only distortionary policy instrument available is the nominal interest rate. We abstract from other types of proportional income taxes. Also, government debt is not considered as a way to finance the government expenditures. Hence, we assume that the government faces a balanced budget constraint:

$$M_t^s = M_{t-1}^s + p_t^N g_t^N + X_t - T_t \quad (22)$$

In this equation, T_t is the per capita lump sum taxes charged to households to finance current government expenditure. Another feature of this government is that fiscal

purchases (g_t^N) consist only of non-tradable goods. This variable will follow a stochastic exogenous process.²⁰

The money supply evolves according to:

$$M_t^s = M_{t-1}^s + X_t \quad (23)$$

where X_t denotes per capita money injections in the financial market.

2.5 International Transactions

Regarding trade integration we assume that the law of one price holds for the tradable good:

$$p_t^T = e_t \quad (24)$$

Without loss of generality, we will assume that the foreign price remains constant and equal to one.

In the standard small open economy model the international interest rate is given and international bonds follow a unit root process²¹. This feature prevents the implementation of log-linearization techniques. The unit root implies that deviations from the steady state are permanent, while the log-linearization procedure is accurate only around the steady state. Consequently, in the standard model, log-linearization techniques are unreliable. To overcome this problem, Schmitt-Grohé and Uribe (2003) propose four different methods to induce stationarity in the international bond. In this model, we

²⁰This is done due to the lack of good information regarding nontradable and tradable fiscal expenditure in Chile. However, the relevance of tradable government expenditure is low in overall fiscal expenditure in Chile. Also, simulations not reported here emphasize the small impact of tradable fiscal expenditures since the financial integration of a small open economy allows the agents to hedge against this type of shock.

²¹See Schmitt-Grohé and Uribe (2003).

introduce one of them: an upward-sloping supply of funds. This friction in the international financial markets induces an interest rate premium that is increasing in the total international debt of the economy.²² The functional form we assume for the upward supply of funds is:

$$R_t^* = R^* \left[\frac{B_t^*}{B^*} \right]^\nu \quad (25)$$

B_t^* is the aggregate net foreign assets expressed in foreign currency. This equation has two components. The first one is the steady state value for the gross international interest rate, which is equal to the inverse of the subjective discount factor of households²³. The second one is the risk premium, which depends on the deviation of the foreign debt from its steady state value (B^*). This last value is calibrated to be consistent with the steady state value of net exports over total output.

2.6 Market Clearing Conditions

In each period there are markets for the two type of goods, labor, money, domestic and foreign bonds. The market clearing condition for the labor market is:

$$\lambda l_t(tr) + (1 - \lambda)l_t(nt) = l_t^T + l_t^N \quad (26)$$

We will assume that transaction costs are deadweight losses in the nontradable sector. Also, recalling that N_t is the nontradable input in the tradable sector and $\kappa(\pi_t^N - 1)^2/2$ is the amount of resources used in adjusting prices in that sector, we obtain the equilibrium condition for non-tradable goods:

²²As is common in small open economy models that describe developing countries, we consider the case where the country as a whole is a net debtor. Hence, log-linearization around the steady state will make this upward sloping supply of funds operative.

²³This condition is derived from the steady state of the Euler equation for foreign bonds (5).

$$\begin{aligned}
& \lambda c_t^N(tr) + (1 - \lambda)c_t^N(nt) + \lambda s(v_t(tr)) \left[c_t^N(tr) + \frac{p_t^T}{p_t^N} c_t^T(tr) \right] \\
& + (1 - \lambda)s(v_t(nt)) \left[c_t^N(nt) + \frac{p_t^T}{p_t^N} c_t^T(nt) \right] + N_t \\
& + g_t^N + \frac{\kappa}{2} (\pi_t^N - 1)^2 = y_t^N
\end{aligned} \tag{27}$$

The market clearing condition in the tradable sector can be expressed as:

$$\lambda c_t^T(tr) + (1 - \lambda)c_t^T(nt) + B_t^* \frac{e_t}{p_t^T} = y_t^T + R_{t-1}^* B_{t-1}^* \frac{e_t}{p_t^T} \tag{28}$$

where B_t^* stands for the aggregate net foreign assets held by this economy. Since traders are the only agents that participate in international financial markets, the following equivalence holds:

$$B_t^* = \lambda b_t^*(tr) \tag{29}$$

Because trader households are identical, in equilibrium there is no borrowing or lending in domestic contingent bonds. This implies:

$$d_t(tr) = 0 \tag{30}$$

Finally, the equilibrium condition in the money market is given by:

$$M_t^s = \lambda M_t(tr) + (1 - \lambda)M_t(nt) \tag{31}$$

2.7 Equilibrium

An equilibrium for this economy is a set of (i) Prices: $\{e_t, p_t^T, p_t^N, q_{t,t+1}, W_t, R_t^*, mc_t\}$, and (ii) Allocations: $\{c_t^T(tr), c_t^N(tr), c_t^T(nt), c_t^N(nt), b_t^*(tr), B_t^*, d_t, M_t(tr), M_t(nt), M_t^s,$

$l_t(tr), l_t(nt), l_t^T, l_t^N, N_t, y_t^T, y_t^N$; such that (2) - (13) and (19)-(31) hold, given policies $\{S, X_t, T_t\}$, exogenous process $\{z_t^T, z_t^N, g_t^N\}$, and initial conditions $(M_{-1}(tr), M_{-1}(nt), b_{-1}^*(tr), p_{-1}^N)$.

3 Ramsey Problem

In this section, we characterize the approach applied to find the optimal monetary policy. Our analysis of optimal policy is in the tradition of Ramsey (1927) and draws heavily on modern literature of optimal policy in dynamic economies. We focus on the conditions that describe optimal allocations under full commitment of monetary policy. This approach describes the competitive equilibrium in a primal form and leads to a characterization of the optimal allocations. It recasts all the prices and policy instruments in terms of allocations. Our methodology is built on the work of Khan et al. (2003) and Schmitt-Grohé and Uribe (2004), where they adapt the approach of Stokey and Lucas (1983) to include monopolistic competition and sticky prices in addition to monetary distortions. Unlike these papers, we consider a small open economy and we include another friction: asset market segmentation.

As we saw in the last section, the set of conditions that characterize a competitive equilibrium given a policy include too many equations. Fortunately, we can combine these equations to have a compact set of constraints defining the market equilibrium. The objective of the monetary authority is to achieve an allocation that yields the highest weighted average of the utilities of households. In particular, the Ramsey problem consists of choosing a sequence of plans $\{c_t^T(tr), c_t^N(tr), l_t(tr), v_t(tr), c_t^T(nt), c_t^N(nt), l_t(nt), l_t^T, N_t, \pi_t^N, B_t^*\}$ to maximize the following welfare criterion:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t (\lambda u(c_t^T(tr), c_t^N(tr), l_t(tr)) + (1 - \lambda) u(c_t^T(nt), c_t^N(nt), l_t(nt)))\right]$$

subject to the set of equilibrium conditions explained below. Combining (2) and (8) with (13) we get expressions that relate the marginal rate of substitution between non-tradable and tradable consumption for both types of households with the marginal productivity of non-tradable inputs in the tradable sector:

$$z_t^T f_N^T(l_t^T, N_t) = \frac{u_{c^N,t}(tr)}{u_{c^T,t}(tr)} \quad (32)$$

$$z_t^T f_N^T(l_t^T, N_t) = \frac{u_{c^N,t}(nt)}{u_{c^T,t}(nt)} \quad (33)$$

Likewise, we can obtain equations that link the marginal rate of substitution between leisure and tradable consumption for households with the marginal productivity of labor in the tradable sector. This can be done using (3), (9) and (12):

$$z_t^T f_l^T(l_t^T, N_t) = -\frac{u_{l,t}(tr)h(v_t(tr))}{u_{c^T,t}(tr)} \quad (34)$$

$$z_t^T f_l^T(l_t^T, N_t) = -\frac{u_{l,t}(nt)h(v_t(nt))}{u_{c^T,t}(tr)} \quad (35)$$

Replacing the definition of the non-tradable inflation ($\pi_t^N = p_t^N/p_{t-1}^N$) in (4) and (10) we obtain:

$$\frac{u_{c^N,t}(tr)}{h(v_t(tr))} (1 - s'(v_t(tr))(v_t(tr))^2) = \beta \mathbb{E}_t \left[\frac{u_{c^N,t+1}(tr)}{\pi_{t+1}^N h(v_{t+1}(tr))} \right] \quad (36)$$

$$\frac{u_{c^N,t}(nt)}{h(v_t(nt))} (1 - s'(v_t(nt))(v_t(nt))^2) = \beta \mathbb{E}_t \left[\frac{u_{c^N,t+1}(nt)}{\pi_{t+1}^N h(v_{t+1}(nt))} \right] \quad (37)$$

Also, using (5), (24) and (25) we can derive an expectational equation governing the

portfolio decisions regarding foreign debt:

$$\frac{u_{c^T,t}(tr)}{h(v_t(tr))} = \beta R^* \left[\frac{B_t^*}{B^*} \right]^\nu \mathbb{E}_t \left[\frac{u_{c^T,t+1}(tr)}{h(v_{t+1}(tr))} \right] \quad (38)$$

The Phillips curve derived in (19) can be rearranged to eliminate $q_{t,t+1}$ and mc_t . We do so using (3), (6) and (20):

$$\begin{aligned} \frac{u_{c^N,t}(tr)}{h(v_t(tr))} \pi_t^N (\pi_t^N - 1) &= \beta \mathbb{E}_t \left[\frac{u_{c^N,t+1}(tr)}{h(v_{t+1}(tr))} \pi_{t+1}^N (\pi_{t+1}^N - 1) \right] \\ & - \frac{\varepsilon - 1}{\kappa} \left[\frac{\varepsilon}{\varepsilon - 1} u_{l,t}(tr) + \frac{u_{c^N,t}(tr)}{h(v_t(tr))} z_t^N \right] [\lambda l_t(tr) + (1 - \lambda) l_t(nt) - l_t^T] \end{aligned} \quad (39)$$

In the market-clearing condition for the non-tradable sector (27) we can substitute the definition of the relative price of tradable goods using (2) and (8). Also, combining (11) and (26), we can express the total non-tradable production as function of the total labor supplied and the labor used in the tradable sector. These replacements translate into the following resource constraint:

$$\begin{aligned} & \lambda c_t^N(tr) + (1 - \lambda) c_t^N(nt) + g_t^N + N_t + \lambda s(v_t(tr)) [c_t^N(tr) + \frac{u_{c^T,t}(tr)}{u_{c^N,t}(tr)} c_t^T(tr)] \\ & + \frac{\kappa}{2} (\pi_t^N - 1)^2 + (1 - \lambda) s(v_t(nt)) [c_t^N(nt) + \frac{u_{c^T,t}(nt)}{u_{c^N,t}(nt)} c_t^T(nt)] \\ & = z_t^N [\lambda l_t(tr) + (1 - \lambda) l_t(nt) - l_t^T] \end{aligned} \quad (40)$$

Using the law of one price in the tradable sector (24), we can rewrite the market-clearing condition in the tradable sector as:

$$\lambda c_t^T(tr) + (1 - \lambda)c_t^T(nt) + B_t^* = z_t^T f^T(l_t^T, N_t) + R^* \left[\frac{B_{t-1}^*}{B^*} \right]^\nu B_{t-1}^* \quad (41)$$

Since we have two types of households we need to keep track of one of the household budget constraints. We use the budget constraint of the non-traders (7). We normalize it in terms of non-tradable goods and using (8), the definition of velocity of non-traders, and the fact that in equilibrium $T_t = p_t^N g_t^N$, we obtain:

$$\begin{aligned} & (1 + s(v_t(nt)) + \frac{1}{v_t(nt)}) [c_t^N(nt) + \frac{u_{c^T,t}(nt)}{u_{c^N,t}(nt)} c_t^T(nt)] + g_t^N \\ & = -\frac{u_{l,t}(nt)h_t(nt)}{u_{c^N,t}(tr)} l_t(nt) + S + \left[c_{t-1}^N(nt) + \frac{u_{c^T,t-1}(nt)}{u_{c^N,t-1}(nt)} c_{t-1}^T(nt) \right] \frac{1}{v_{t-1}(nt)\pi_t^N} \end{aligned} \quad (42)$$

The previous constraints can be classified in three groups. (32), (33), (34), (35) and (40) are intratemporal conditions in the sense that they only include only variables dated at t . (41) and (42) which are not intratemporal but predetermined equations because they include variables dated at t and $t - 1$. Finally, (36), (37), (38) and (39) are expectational equations, meaning that they contain expectation of the variables at $t + 1$ based on information at t . This observation is important since we cannot collapse the primal form of the competitive equilibrium into a unique intertemporal implementability condition in period 0 and a set of intratemporal conditions.

As identified by Aiyagari et al.(2003), a real economy without contingent government debt includes forward-looking constraints that must be satisfied each period. In our framework, money distortions and sticky prices also imply forward-looking constraints that must hold every period: the Phillips curve and the implicit money demands. The expectations-augmented Philips curve posits a constrain to the inflation path and it cannot be written as a single constraint in period 0 or as an intratemporal condition. The

implicit money demand restricts the intertemporal behavior of money velocity. Furthermore, asset market segmentation implies the additional presence of equations that define the money demand of non-traders and their budget constraint. The last one contains a intertemporal link given by the fact that the only asset that non-traders may use to smooth consumption is money holdings.

The Ramsey problem described is akin to the framework used by Marcat and Marimon (1999) in their analysis of recursive contracts. Their methodology implies that optimal problems with forward-looking constraints call for a new state variable to be added to the state space to characterize the time-invariant optimal policy rule. In our monetary problem, these variables are the Lagrange multipliers associated with the expectational equations (36)-(39). To see this, it is possible to construct the Lagrangian associated with the optimal problem and rearrange the terms into a recursive *saddle point functional equation*. We relegate this derivation to appendix A.

Defining $\mu_1 - \mu_{11}$ as the Lagrange multipliers associated with equations (32) - (42), respectively, the state variables of the optimal problem are: $x_t = [B_t^*, c_t^T(nt), c_t^N(nt), v_t(nt), \mu_{5,t}, \mu_{6,t}, \mu_{7,t}, \mu_{8,t}]'$. We denote y_t as the vector of all other endogenous variables, i.e., $y_t = [c_t^T(tr), c_t^N(tr), v_t(tr), l_t(tr), l_t(nt), l_t^T, N_t, \pi_t^N, \mu_{1,t}, \mu_{2,t}, \mu_{3,t}, \mu_{4,t}, \mu_{9,t}, \mu_{10,t}, \mu_{11,t}]'$. The exogenous stochastic variables are collected in the vector $z_t = [z_t^T, z_t^N, g_t^N]'$. Then the first order conditions that characterize the optimal policy can be represented as a system of equations of the form:

$$\begin{aligned} G(x_t, x_{t-1}, y_t, z_t) &= 0 \\ \mathbb{E}_t [H(x_{t+1}, x_t, x_{t-1}, y_{t+1}, y_t, z_{t+1}, z_t)] &= 0 \end{aligned}$$

The first set is formed by deterministic equations, while the second one consists of expectational equations. The computational approach involves two steps. First, we compute the steady state which is given by $G(x, x, y, z) = 0$ and $H(x, x, x, y, y, z, z) = 0$. Second, we log-linearize the above system of equations and calculate the local dynamic behavior of endogenous variables given a specified law of motion for exogenous fluctuations. Additionally, when we compute the dynamics of the monetary regimes, such as non-tradable inflation targeting, money peg, and exchange rate peg, we consider the same steady state as the one calculated for the optimal policy. In these last cases, the dynamics are estimated only by log-linearizing equations (32) - (42) and the specific rule of a monetary regime.

4 Dynamics under the Optimal Policy

In this section, we show the numerical results of the model. First, we explain the calibration strategy for the model economy. Then, we describe the dynamics of the optimal monetary policy, and compare them with alternative monetary regimes. We show the impulse responses and the second moments of the simulated economy. Finally, we conduct a sensitivity analysis to evaluate the robustness of the results.

4.1 Calibration

The parameters of the model are chosen to match some features of the Chilean economy and some of them are taken from other studies in the literature of open economies. The time unit is one quarter. We adopt a logarithmic utility function:

$$u(c, l) = \ln c + \psi \log(1 - l) \tag{43}$$

and a C.E.S. function for the composite consumption of tradable and nontradable

goods:

$$c = [\theta(c^T)^\mu + (1 - \theta)(c^N)^\mu]^{\frac{1}{\mu}} \quad (44)$$

We chose a preference weight on leisure consistent with a steady state labor supply of 0.22. For the intratemporal elasticity of substitution $1/(1 - \mu)$ we rely on the estimation of Gonzales-Rozada and Neumeyer (2003) for Argentina and set its value to 0.5²⁴. To the best of our knowledge there are no empirical estimates of the preference weights between tradable and nontradable goods. Thus, we follow Rebelo and Végh (1995) and assume that θ is equal to 0.5.

We specify a C.E.S. production function for the firms in the tradable sector:

$$y_t^T = z_t^T (\alpha_T (l_t^T)^{1-\phi} + (1 - \alpha_T)(N_t)^{1-\phi})^{\frac{1}{1-\phi}} \quad (45)$$

We set the labor weight in the production function to $\alpha_T = 0.4$. This parameter value is taken from Guajardo (2003) and is consistent with the labor share of the tradable sector in Chile. For the elasticity of substitution between labor and the intermediate nontradable input there are no estimates for the Chilean economy. We assume $\phi = 1.5$, which is the value generally used in the international business cycle literature for the elasticity of substitution between domestic and foreign inputs in the production function.²⁵ Based on Bergoing and Piguillem (2003) we set $\varepsilon = 6$, which implies a steady state markup of 20 percent.

We assume the same transaction costs specification as Schmitt-Grohé and Uribe

²⁴We assume that this preference parameter is similar to the one for the Chilean economy.

²⁵See Chari et al. (2002)

(2004):

$$s(v) = \omega v + \frac{\xi}{v} - 2\sqrt{\xi\omega} \quad (46)$$

One particular feature of this transaction technology is that it exhibits a satiation point of real money balances. This is necessary in order to obtain well-defined money demand at the Friedman rule (ie. zero nominal interest rate). With a zero nominal interest rate, transaction costs are nil and the equilibrium consumption velocity is equal to $\bar{v} = \sqrt{\xi/\omega}$. To calibrate the parameters of the transaction costs technology, we estimate an aggregate demand for real money balances based on this specification of the transaction costs:²⁶

$$v_t^2 = \frac{\xi}{\omega} + \frac{1}{\omega} \frac{R_t - 1}{R_t} \quad (47)$$

For the consumption velocity we use the ratio of nominal private consumption to M1. For the estimation, we consider the nominal interest rate on deposits between 90 days and one year. The OLS parameter estimates of equation (47) are $\omega = 0.06$ and $\xi = 0.17$.²⁷

To calibrate the quadratic adjustment cost of prices we follow Galí and Gertler (1999) and estimate the log-linearized version of the expectational augmented Phillips curve assuming zero inflation in the steady state:

²⁶It is not possible to obtain empirical estimates of these parameters for each type of agent, so we assume that both types have the same transaction function. Thus, we estimate an aggregate demand for money.

²⁷The estimated equation is $v_t^2 = 2.68 + 15.64(R_t - 1)/R_t$. The t-statistics for the first and second coefficient are 20.82 and 15.72, respectively. The coefficient of determination is equal to 0.82.

$$\widehat{\pi}_t^N = \beta E_t[\widehat{\pi}_{t+1}^N] + \frac{(\varepsilon - 1)h}{\kappa} \widehat{mc}_t \quad (48)$$

where \widehat{x}_t denotes the log-linearization of variable x_t . This equation resembles the new Phillips curve derived under Calvo's staggered price setting assumptions. We estimate the reduced form of equation (48) using the Generalized Method of Moments.²⁸ The estimator of the marginal costs coefficient, $\frac{(\varepsilon-1)h}{\kappa}$, is equal to 0.084. Given the steady state labor supply and the elasticity between differentiated goods, the implied coefficient for the quadratic cost adjustment is 13.16. This coefficient is consistent with a price stickiness of 4 quarters in the Calvo model.²⁹ This estimate is somewhat higher than the three quarters price stickiness observed in the United States (Sbordone, 2002). Nevertheless, we carry out a sensitivity analysis to evaluate the robustness of the simulation to different assumptions about price stickiness.

We do not have an estimate of the fraction of the population that is excluded from asset markets. Using data from the Survey of Consumer Finance, Mulligan and Sala-i-Martin (2000) show that in 1989, 59 percent of the U.S. households did not invest in interest-bearing assets. This amount of asset market segmentation for a developed economy, suggests that in emerging economies, where capital markets are less developed, this financial friction may be more severe. In the baseline calibration we assume an asset market segmentation of 50 percent in Chile. Also, we conduct a sensitivity analysis to analyze how numerical results may change in response to different degrees of asset market segmentation.³⁰

²⁸We estimate the equation with GMM for the sample period 1990:1 - 2002:4. Instruments used include four lags of non-tradable inflation, wage inflation, real marginal costs, and the non-tradable output gap.

²⁹In Calvo's model, the reduced form parameters of the expectations augmented Phillips curve are the same as in the Rotemberg model. Under the Calvo specification, a fraction θ of the firms cannot adjust their prices every period. The parameter θ for the Chilean nontradable sector is 0.75. This number implies an average stickiness of $1/(1 - \theta)$ periods, which in this case corresponds to 4 quarters.

³⁰With the development of private pension funds in Chile and in most Latin American countries, it is possible to argue that financial markets are not as segmented as shown in the data since workers

For transactions with the rest of the world we assume a highly elastic supply of funds and set $\nu = 0.00001$. As argued by Schmitt-Grohé and Uribe (2001), a small elasticity of the supply of funds schedule reduces fluctuations in the country risk premium. We calibrate the parameter with a low value in order to not modify the short-run properties of the model. This implies that the allocations will be approximately the same with or without the funds schedule. The parameter values are summarized in table 1.

Table 1: Parameter values for the Chilean Economy

Description	Symbol	Value
Discount Factor	β	0.99
Tradable weight in consumption	θ	0.50
Intratemporal Elasticity of Substitution	$\frac{1}{1-\mu}$	0.50
Nontradable Inflation Rate	π^N	1.024
Parameter Transaction Cost Function	ω	0.06
Parameter Transaction Cost Function	ξ	0.17
Markup	$\frac{\varepsilon}{\varepsilon-1}$	1.20
Price adjustment cost	κ	13.16
Labor share in the tradable sector	α_T	0.40
Elasticity of substitution for tradable firms	ϕ	1.5
Foreign interest rate elasticity	ν	10^{-5}
Asset Market Segmentation	λ	0.5

We assume that the exogenous processes in the model economy follow an AR(1) process. First, we remove a linear trend from non-tradable labor productivity, tradable labor productivity, and government expenditure.³¹ Then, we fit the detrended variables

are forced to save for retirement. However, the asset market segmentation considered in this model is with respect to liquid assets. The portfolio of a pension fund can be converted into cash only after the retirement period.

³¹All the variables are expressed in logarithm.

to an AR(1) model. The estimated process are the following (standard errors in parentheses):

$$z_t^T = 0.65z_{t-1}^T + \epsilon_t^T, \quad \epsilon_t^T \sim N(0, \sigma_T^2), \quad \sigma_T = 0.027 \quad (49)$$

(0.11)

$$z_t^N = 0.84z_{t-1}^N + \epsilon_t^N, \quad \epsilon_t^N \sim N(0, \sigma_N^2), \quad \sigma_N = 0.021 \quad (50)$$

(0.08)

$$g_t^N = 0.76g_{t-1}^N + \epsilon_t^G, \quad \epsilon_t^G \sim N(0, \sigma_G^2), \quad \sigma_G = 0.026 \quad (51)$$

(0.10)

4.2 Impulse Responses

In this section we compare the dynamics of the optimal policy against three simple rules: Non-tradable inflation targeting, money peg and exchange rate peg. Formally, we define these rules as follows:

$$\pi_t^N = \bar{\pi}$$

$$e_t/e_{t-1} = \bar{\pi}$$

$$M_t^S / M_{t-1}^S = \bar{\pi}$$

In sum, we set the growth rate of a nominal target equal to the steady state non-tradable inflation rate. In practice, it is difficult for a Central Bank to implement an optimal policy, since it must react contemporaneously to the realization of shocks in the economy. In order to draw a clear policy prescription from the analysis we compare the performance of the Ramsey solution against simple monetary policy rules. The result we find is surprisingly robust: for any shock in the economy, the optimal policy is quantitatively similar to a non-tradable inflation targeting rule. A model economy with this rule implies that the non-tradable price level is constant over every period and state of nature. In turn, this policy undoes all of the costs associated with price stickiness. Under this policy, the monetary authority eliminates all incentives of the firms to change their prices in response to shocks. Hence, this policy rule replicates the flexible price allocation.

The similarity between the allocations of the Ramsey policy and the non-tradable inflation targeting rule reveals that undoing the price stickiness in the nontradable sector is by far the policy with the greatest impact on the household welfare. However, the Ramsey allocation also reduces to some extent the monetary distortions in the model economy. Compared to the non-tradable inflation targeting regime, the social planner smooths the response of the nominal interest rate which in turn mitigates money distortions operating in the economy. Below we examine the impulse response functions for each individual shock.

The Balassa-Samuelson effect holds in the model, hence the real exchange rate appreciates in response to an increase in productivity in the tradable sector. The productivity

shock induces a reduction of the marginal cost of the tradable sector firms and generates a decrease in the relative price of tradable goods. Under sticky prices, a specific monetary policy can affect the dynamics of the economy. Depending on the reaction of the money supply to the productivity shock, a real appreciation can be achieved either with an increase in non-tradable prices or with an appreciation of the nominal exchange rate. Figure 1 shows that the dynamics under the optimal policy are very similar to the one obtained with a non-tradable inflation targeting rule. This result implies that at the Ramsey allocation, the non-tradable price is stabilized and the nominal exchange rate absorbs the real shock. The optimal response to a 1 percent increase in tradable productivity is a real appreciation of the exchange rate by 1 percent, which is achieved almost entirely by a decrease in the nominal exchange rate. Other policy rules induce fluctuations in the non-tradable price that imply a loss of nontradable resources which reduce welfare. The nominal interest rate consistent with the flexible price allocation is highly volatile. In response to a 1 percent productivity shock, the nominal interest rate rises by 110 basis points. Nevertheless, compare to the non-tradable inflation targeting regime, the optimal policy smooths the nominal interest rate to reduce the money distortions associated with transaction costs.

Figure 2 shows the impulse response of consumption and labor for traders and non-traders. We can see that deviations of the Ramsey allocations from the non-tradable inflation targeting rule are quantitatively small. The consumption of tradable and non-tradable goods tend to move together for both types of agents due to the value of the elasticity of intratemporal substitution. For the calibrated elasticity, agents in the economy have a higher aversion to intratemporal substitution compared to intertemporal substitution. This specification generates co-movement between the types of goods among agents.

The two types of agents have different consumption and labor supply volatilities. The

volatility of consumption is affected by the difference in access to financial markets. The traders, who have the possibility to trade bonds with the rest of the world, can smooth consumption over time as opposed to non-traders. In a context of financial autarky, the best response of nontraders is to smooth labor supply to minimize welfare losses. Given that they are unable to smooth consumption over time, they try to smooth as much as possible leisure. In the case of the traders, they have the possibility to choose an optimal combination of consumption and leisure given their access to financial markets. Hence, access to international capital markets implies a highly volatile labor supply and a low volatility of consumption under all policies.

The Balassa-Samuelson effect also operates when the economy is buffeted by a productivity shock in the non-tradable sector. Figure 3 shows that, under the optimal policy, a 1 percent increase in non-tradable productivity entails a depreciation of the real exchange rate by approximately 0.4 percent. Also, in this case, the optimal policy is characterized by the stability of the non-tradable price and a tendency to smooth the nominal interest rate. Overall, compared to the previous case, the non-tradable productivity shock leads to a lower volatility of nominal variables. Figure 4 describes the dynamics of labor and consumption for both agents. The particular dynamics for each type of agent are influenced by the asymmetry in the access to financial markets. In this case the traders have low consumption volatility and high labor volatility, while the converse is true for non-traders.

Figure 5 and 6 shows the response of the main macroeconomic variables to a shock in government expenditure. As Perri (2001) pointed out, the real effects of fiscal shocks tend to be lessened in a small open economy. In the present model, we find that this demand shock generates a nil effect on real activity. There are no significant differences among the evaluated policies in response to a 1 percent increase in non-tradable government expenditure.

4.3 Second Moments and Sensitivity Analysis

In this section we carry out a simulation to study the business cycles properties of the model economy. We abstract from any spillover effects among the productive sectors or any covariance between the exogenous shock processes. We put into perspective the quantitative predictions of the model, and we compare the standard deviations of the simulated data to those of the Chilean economy for the period 1990:1 - 2002:4.

The simulations shown in table 2 reveal the same result as before: the dynamics of the optimal policy resemble those of the non-tradable inflation targeting regime. When we evaluate the dynamics for each type agent, we find that the optimal policy minimizes the consumption and labor supply volatility of non-traders. For the case of traders, we find that for most regimes there is a tradeoff between consumption volatility and labor supply volatility. Considering that traders have access to financial markets, they have the ability to insure themselves against a particular monetary policy, and hence the possibility to choose an optimal combination of consumption and labor for each regime. Conversely, non-traders do not have this opportunity, and suboptimal policy regimes induce a inefficient combination of consumption and leisure.

At the aggregate level, we find that consumption volatility for most of the regimes is relatively low compared to the data. This excessive smoothness of consumption is due to the fact that there is a high negative correlation between the consumption of traders and nontraders, which decreases the volatility of aggregate consumption. In contrast, aggregate output under the optimal policy is slightly more volatile than what is found in the data. This is because in the model labor is highly volatile, due to labor supply decisions made by traders.

The next question we ask is how these results change in response to different assump-

Table 2: Standard deviations under alternative policies (1990:1-2002:4)

Variables	Data	Optimal Policy	Nontradable Inflation T.	Money Peg	Exchange Rate Peg
Nom. Interest Rate	0.718	0.573	0.637	0.365	0.000
Nontradable Inflation	0.903	0.053	0.000	0.293	0.419
Nom. Dep. Rate	3.069	1.892	1.931	1.106	0.000
Money Growth Rate	3.666	2.082	2.635	0.000	2.976
Aggr. Consumption	3.660	0.770	0.857	0.678	1.649
Aggr. Output	2.447	1.956	2.118	2.612	4.293
Real Exchange Rate	3.555	1.963	1.937	1.338	0.737
Labor Supply	1.227	2.614	2.813	3.717	5.997
Cons. Traders		1.140	1.171	0.739	0.303
Cons. Nontraders		1.486	1.637	1.938	3.393
Labor Traders		5.041	5.385	6.641	9.824
Labor Nontraders		0.571	0.650	1.042	2.477

tions regarding price stickiness and asset market segmentation. In figure 7, we plot the volatility of the inflation rate at the optimal policy for different values of κ . The result we obtain is that inflation volatility is near zero for a wide range of parameter values of price stickiness. Even if we consider a moderate sticky price distortion, it is optimal to stabilize the price level of non-tradable goods. Nevertheless, as we decrease the parameter κ , the relative importance of monetary distortions becomes greater. Thus, the optimal policy is redirected to minimize other distortions in the model economy. Figure 8 shows, that the optimal policy in the context of low price stickiness dampens the fluctuations of the nominal interest rate, which mitigate money distortions.

Figure 9 and 10 show a similar sensitivity analysis for different levels of asset market segmentation. From this analysis we conclude that stabilizing non-tradable inflation is optimal regardless of the financial structure of the small open economy. Despite the fact that markets are incomplete for a fraction of the households, the monetary au-

thority should not sacrifice the goal of price stability in order to provide insurance for the nontraders.³² This result suggests that welfare costs associated to sticky prices are substantially larger than those generated by asset market segmentation and monetary transactions. Nevertheless, the nominal interest rate volatility is affected by the magnitude of asset market segmentation. The intuition for this effect is as follows. Since the optimal policy is always aimed at price stabilization of non-tradable goods, the Ramsey allocation implies similar paths for money supply for different degrees of asset market segmentation. However, as asset market segmentation increases, money will be injected to a smaller mass of traders. The traders get rid of the excess money supply by buying goods or assets. This process affects the intertemporal marginal rate of substitution, which defines the market interest rate. Hence, when the traders receive a disproportionate money injection, it will increase the volatility of the interest rate.

5 Concluding Remarks

In this paper we characterize the optimal monetary policy for a small open economy with money distortions, sticky prices, monopolistic competition, and asset market segmentation. Following the Ramsey approach, we have found that in this environment the optimal policy features a volatility of non-tradable inflation near zero. This policy lessens the incentives of non-tradable firms to engage in frequent price adjustment in response to different shocks, and provides an allocation quantitatively similar to the one that arises in an economy with flexible prices. Even though a tension exists to undo all distortions present in the model economy, the optimal policy prioritizes the elimination of sticky prices over other goals. Using monetary instruments to correct other distortions, such as asset market segmentation, is highly distortionary in an environment with sticky prices.

These results have two important implications for policymakers. First, an optimal

³²The insurance role of monetary policy in a small open economy with asset market segmentation is discussed by Lahiri et al. (2004).

monetary policy should target an appropriate price index. Despite the fact that conventional wisdom among policymakers suggests the stabilization of the inflation rate of the consumer price index, this policy can be distortionary. The optimal policy should target only the subset of prices that display stickiness. The empirical evidence³³ shows that the non-tradable sector exhibits more price stickiness than the tradable sector, so stabilizing a price index that puts more weight on the non-tradable sector is welfare-improving.

Second, stabilizing non-tradable inflation is optimal regardless of the financial structure of the economy. This implication is crucial for developing countries, which have shallow financial markets. Even if shallow financial markets increase the volatility of consumption, and hence the welfare cost of business cycle fluctuations, it is not optimal to correct this distortion with monetary policy. A monetary policy aimed at smoothing consumption is highly distortionary since it implies variations in the non-tradable price, which in turn generates a loss of resources in the non-tradable sector. One should interpret this result with caution. The fact that correcting asset market segmentation by monetary means is welfare-reducing does not imply that financial imperfections should not be taken into account by policymakers. As an alternative, we may also think in the possibility of designing an appropriate fiscal policy to achieve a better intertemporal allocation. The benefits of using fiscal instruments to cope with asset market segmentation is an important issue that may be analyzed in the Ramsey policy framework as well.

There are several dimensions in which we can extend our study. We may include financial frictions on the supply side such as in the work of Céspedes et al. (2004). If we introduce a credit channel into the model, the higher volatility of nominal interest rates induced by asset market segmentation would entail higher output volatility. To the extent that a credit channel may increase the volatility of consumption and leisure, the optimal monetary policy may deviate from non-tradable price stability in favor of

³³See Burstein et al. (2003)

stabilizing the nominal interest rate. Another extension would be to add two tradable goods to the model, home and foreign, thereby introducing a role for the terms of trade in the design of monetary policy. In this context, a monetary authority could manipulate the terms of trade in favor of consumers. As shown by Faia and Monacelli (2004), this structure may induce a departure from price stability. The policy implications of these features can also be evaluated in the Ramsey framework. This public finance approach to monetary policy has provided useful insights to understand how multiple distortions should be minimized across time and states of nature.

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6 Appendix A: Lagrangian of the Ramsey Problem

In this appendix, we describe the Lagrangian associated with the Ramsey problem in section 3. To simplify the arguments of the optimization problem we define the following vectors: $d_t = [c_t^T(tr), c_t^N(tr), l_t(tr), v_t(tr), c_t^T(nt), c_t^N(nt), l_t(nt), v_t(nt), l_t^T, N_t, B_t^*, \pi_t^N]'$ and $\mu_t = [\mu_{1,t}, \dots, \mu_{11,t}]'$, where $\mu_1 - \mu_{11}$ are the Lagrange multipliers associated with the constraints (32) - (42). Then the Lagrangian can be written as:

$$\begin{aligned}
& \min_{\{\mu_i\}_{i=0}^{\infty}} \max_{\{d_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t [\lambda u(c_t^T(tr), c_t^N(tr), l_t(tr)) + (1 - \lambda)u(c_t^T(nt), c_t^N(nt), l_t(nt))] \right. \\
& + \mu_{1,t} \left(z_t^T f_N^T(l_t^T, N_t) - \frac{u_{c^N,t}(tr)}{u_{c^T,t}(tr)} \right) + \mu_{2,t} \left(z_t^T f_N^T(l_t^T, N_t) - \frac{u_{c^N,t}(nt)}{u_{c^T,t}(nt)} \right) \\
& + \mu_{3,t} \left(z_t^T f_l^T(l_t^T, N_t) + \frac{u_{l,t}(tr)h(v_t(tr))}{u_{c^T,t}(tr)} \right) \\
& + \mu_{4,t} \left(z_t^T f_l^T(l_t^T, N_t) + \frac{u_{l,t}(nt)h(v_t(nt))}{u_{c^T,t}(nt)} \right) \\
& + \mu_{5,t} \left(\frac{u_{c^N,t}(tr)}{h(v_t(tr))} (1 - s'(v_t(tr))(v_t(tr))^2) - \beta \mathbb{E}_t \left[\frac{u_{c^N,t+1}(tr)}{\pi_{t+1}^N h(v_{t+1}(tr))} \right] \right) \\
& + \mu_{6,t} \left(\frac{u_{c^N,t}(nt)}{h(v_t(nt))} (1 - s'(v_t(nt))(v_t(nt))^2) - \beta \mathbb{E}_t \left[\frac{u_{c^N,t+1}(nt)}{\pi_{t+1}^N h(v_{t+1}(nt))} \right] \right) \\
& + \mu_{7,t} \left(\frac{u_{c^T,t}(tr)}{h(v_t(tr))} - \beta R^* \left[\frac{B_t^*}{B^*} \right]^\nu \mathbb{E}_t \left[\frac{u_{c^T,t+1}(tr)}{h(v_{t+1}(tr))} \right] \right) \\
& + \mu_{8,t} \left(\begin{aligned} & \frac{u_{c^N,t}(tr)}{h(v_t(tr))} \pi_t^N (\pi_t^N - 1) - \beta \mathbb{E}_t \left[\frac{u_{c^N,t+1}(tr)}{h(v_{t+1}(tr))} \pi_{t+1}^N (\pi_{t+1}^N - 1) \right] \\ & + \frac{\varepsilon - 1}{\kappa} \left[\frac{\varepsilon}{\varepsilon - 1} u_{l,t}(tr) + \frac{u_{c^N,t}(tr)}{h(v_t(tr))} z_t^N \right] [\lambda l_t(tr) + (1 - \lambda)l_t(nt) - l_t^T] \end{aligned} \right)
\end{aligned}$$

$$\begin{aligned}
& +\mu_{9,t} \left(\begin{array}{l} z_t^N [\lambda l_t(tr) + (1-\lambda)l_t(nt) - l_t^T] - \lambda c_t^N(tr) - (1-\lambda)c_t^N(nt) \\ -\lambda s(v_t(tr)) \left[c_t^N(tr) + \frac{u_{c^T,t}(tr)}{u_{c^N,t}(tr)} c_t^T(tr) \right] - g_t^N - N_t \\ -(1-\lambda)s(v_t(nt)) \left[c_t^N(nt) + \frac{u_{c^T,t}(nt)}{u_{c^N,t}(nt)} c_t^T(nt) \right] - \frac{\kappa}{2}(\pi_t^N - 1)^2 \end{array} \right) \\
& +\mu_{10,t} \left(z_t^T f^T(l_t^T, N_t) + R^* \left[\frac{B_{t-1}^*}{B^*} \right]^\nu B_{t-1}^* - B_t^* - \lambda c_t^T(tr) - (1-\lambda)c_t^T(nt) \right) \\
& +\mu_{11,t} \left(\begin{array}{l} -\frac{u_{i,t}(nt)h(v_t(nt))}{u_{c^N,t}(nt)} l_t(nt) + S - g_t^N \\ \left[c_{t-1}^N(nt) \frac{u_{c^T,t-1}(nt)}{u_{c^N,t-1}(nt)} c_{t-1}^T(nt) \right] \frac{1}{v_{t-1}(nt)\pi_t^N} \\ - \left[c_t^N(nt) + \frac{u_{c^T,t}(nt)}{u_{c^N,t}(nt)} c_t^T(nt) \right] \left(1 + s(v_t(nt)) + \frac{1}{v_t(nt)} \right) \end{array} \right) \Bigg\}
\end{aligned}$$

given $B_{-1}^*, c_{-1}^T(nt)$, $c_{-1}^N(nt)$, and $v_{-1}(nt)$.

Remark 1. Let $g^1, g^2 : \text{domain}(d_t) \rightarrow \mathbb{R}$ be two functions. We have the following identity:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \chi_t (g^1(d_t) + \beta \mathbb{E}_t[g^2(d_{t+1})]) \right] = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \mathbb{E}_t(\chi_t g^1(d_t) + \chi_{t-1} g^2(d_t)) \right]$$

This can easily be proved rearranging the terms and using the law of iterated expectations. We applied this remark to the Lagrangian above to rewrite:

$$\begin{aligned}
& \min_{\{\mu_i\}_{i=0}^{\infty}} \max_{\{d_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \mathbb{E}_t \left[\lambda u(c_t^T(tr), c_t^N(tr), l_t(tr)) + (1-\lambda)u(c_t^T(nt), c_t^N(nt), l_t(nt)) \right. \right. \\
& \left. \left. + \mu_{1,t} \left(z_t^T f_N^T(l_t^T, N_t) - \frac{u_{c^N,t}(tr)}{u_{c^T,t}(tr)} \right) + \mu_{2,t} \left(z_t^T f_N^T(l_t^T, N_t) - \frac{u_{c^N,t}(nt)}{u_{c^T,t}(nt)} \right) \right] \right\}
\end{aligned}$$

$$\begin{aligned}
& +\mu_{3,t} \left(z_t^T f_l^T(l_t^T, N_t) + \frac{u_{l,t}(tr)h(v_t(tr))}{u_{c^T,t}(tr)} \right) \\
& +\mu_{4,t} \left(z_t^T f_l^T(l_t^T, N_t) + \frac{u_{l,t}(nt)h(v_t(nt))}{u_{c^T,t}(nt)} \right) \\
& +\frac{u_{c^N,t}(tr)}{h(v_t(tr))} \left[(1 - s'(v_t(tr))(v_t(tr))^2)\mu_{5,t} - \frac{1}{\pi_t^N}\mu_{5,t-1} \right] \\
& +\frac{u_{c^N,t}(nt)}{h(v_t(nt))} \left[(1 - s'(v_t(nt))(v_t(nt))^2)\mu_{6,t} - \frac{1}{\pi_t^N}\mu_{6,t-1} \right] \\
& +\frac{u_{c^T,t}(tr)}{h(v_t(tr))} \left[\mu_{7,t} - R^* \left[\frac{B_{t-1}^*}{B^*} \right]^\nu \mu_{7,t-1} \right] \\
& +\frac{u_{c^N,t}(tr)}{h(v_t(tr))} \pi_t^N (\pi_t^N - 1) [\mu_{8,t} - \mu_{8,t-1}] \\
& +\mu_{8,t} \left(\frac{\varepsilon - 1}{\kappa} \left[\frac{\varepsilon}{\varepsilon - 1} u_{l,t}(tr) + \frac{u_{c^N,t}(tr)}{h(v_t(tr))} z_t^N \right] [\lambda l_t(tr) + (1 - \lambda)l_t(nt) - l_t^T] \right) \\
& +\mu_{9,t} \left(\begin{array}{l} z_t^N [\lambda l_t(tr) + (1 - \lambda)l_t(nt) - l_t^T] - \lambda c_t^N(tr) - (1 - \lambda)c_t^N(nt) \\ -\lambda s(v_t(tr)) \left[c_t^N(tr) + \frac{u_{c^T,t}(tr)}{u_{c^N,t}(tr)} c_t^T(tr) \right] - g_t^N - N_t \\ -(1 - \lambda)s(v_t(nt)) \left[c_t^N(nt) + \frac{u_{c^T,t}(nt)}{u_{c^N,t}(nt)} c_t^T(nt) \right] - \frac{\kappa}{2}(\pi_t^N - 1)^2 \end{array} \right) \\
& +\mu_{10,t} \left(z_t^T f^T(l_t^T, N_t) + R^* \left[\frac{B_{t-1}^*}{B^*} \right]^\nu B_{t-1}^* - B_t^* - \lambda c_t^T(tr) - (1 - \lambda)c_t^T(nt) \right)
\end{aligned}$$

$$+ \mu_{11,t} \left(\begin{array}{l} -\frac{u_{l,t}(nt)h(v_t(nt))}{u_{c^N,t}(nt)} l_t(nt) + S - g_t^N \\ + \left[c_{t-1}^N(nt) + \frac{u_{c^T,t-1}(nt)}{u_{c^N,t-1}(nt)} c_{t-1}^T(nt) \right] \frac{1}{v_{t-1}(nt)\pi_t^N} \\ - \left[c_t^N(nt) + \frac{u_{c^T,t}(nt)}{u_{c^N,t}(nt)} c_t^T(nt) \right] \left(1 + s(v_t(nt)) + \frac{1}{v_t(nt)} \right) \end{array} \right) \}}}$$

given $B_{-1}^*, c_{-1}^T(nt), c_{-1}^N(nt), v_{-1}(nt)$, and $\mu_{5,-1} = \mu_{6,-1} = \mu_{7,-1} = \mu_{8,-1} = 0$.

To see the inclusion of $\mu_5 - \mu_8$ as state variables in the characterization of the optimal policy, we follow the framework of Marcet and Marimon (1999), expressing this Lagrangian as a *saddle point function equation*:

$$W(d_{t-1}^x, \mu_{t-1}^x, z_t) =$$

$$\begin{aligned} & \min_{\mu_t} \max_{d_t} \{ \lambda u(c_t^T(tr), c_t^N(tr), l_t(tr)) + (1 - \lambda) u(c_t^T(nt), c_t^N(nt), l_t(nt)) \\ & + \mu_{1,t} \left(z_t^T f_N^T(l_t^T, N_t) - \frac{u_{c^N,t}(tr)}{u_{c^T,t}(tr)} \right) + \mu_{2,t} \left(z_t^T f_N^T(l_t^T, N_t) - \frac{u_{c^N,t}(nt)}{u_{c^T,t}(nt)} \right) \\ & + \mu_{3,t} \left(z_t^T f_l^T(l_t^T, N_t) + \frac{u_{l,t}(tr)h(v_t(tr))}{u_{c^T,t}(tr)} \right) \\ & + \mu_{4,t} \left(z_t^T f_l^T(l_t^T, N_t) + \frac{u_{l,t}(nt)h(v_t(nt))}{u_{c^T,t}(nt)} \right) \\ & + \frac{u_{c^N,t}(tr)}{h(v_t(tr))} \left[(1 - s'(v_t(tr))(v_t(tr))^2) \mu_{5,t} - \frac{1}{\pi_t^N} \mu_{5,t-1} \right] \\ & + \frac{u_{c^N,t}(nt)}{h(v_t(nt))} \left[(1 - s'(v_t(nt))(v_t(nt))^2) \mu_{6,t} - \frac{1}{\pi_t^N} \mu_{6,t-1} \right] \\ & + \frac{u_{c^T,t}(tr)}{h(v_t(tr))} \left[\mu_{7,t} - R^* \left[\frac{B_{t-1}^*}{B^*} \right]^\nu \mu_{7,t-1} \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{u_{c^N,t}(tr)}{h(v_t(tr))} \pi_t^N (\pi_t^N - 1) [\mu_{8,t} - \mu_{8,t-1}] \\
& + \mu_{8,t} \left(\frac{\varepsilon - 1}{\kappa} \left[\frac{\varepsilon}{\varepsilon - 1} u_{l,t}(tr) + \frac{u_{c^N,t}(tr)}{h(v_t(tr))} z_t^N \right] [\lambda l_t(tr) + (1 - \lambda) l_t(nt) - l_t^T] \right) \\
& + \mu_{9,t} \left(\begin{aligned} & z_t^N [\lambda l_t(tr) + (1 - \lambda) l_t(nt) - l_t^T] - \lambda c_t^N(tr) - (1 - \lambda) c_t^N(nt) \\ & - \lambda s(v_t(tr)) \left[c_t^N(tr) + \frac{u_{c^T,t}(tr)}{u_{c^N,t}(tr)} c_t^T(tr) \right] - g_t^N - N_t \\ & - (1 - \lambda) s(v_t(nt)) \left[c_t^N(nt) + \frac{u_{c^T,t}(nt)}{u_{c^N,t}(nt)} c_t^T(nt) \right] - \frac{\kappa}{2} (\pi_t^N - 1)^2 \end{aligned} \right) \\
& + \mu_{10,t} \left(z_t^T f^T(l_t^T, N_t) + R^* \left[\frac{B_{t-1}^*}{B^*} \right]^\nu B_{t-1}^* - B_t^* - \lambda c_t^T(tr) - (1 - \lambda) c_t^T(nt) \right) \\
& + \mu_{11,t} \left(\begin{aligned} & - \frac{u_{l,t}(nt) h(v_t(nt))}{u_{c^N,t}(nt)} l_t(nt) + S - g_t^N \\ & + \left[c_{t-1}^N(nt) + \frac{u_{c^T,t-1}(nt)}{u_{c^N,t-1}(nt)} c_{t-1}^T(nt) \right] \frac{1}{v_{t-1}(nt) \pi_t^N} \\ & - \left[c_t^N(nt) + \frac{u_{c^T,t}(nt)}{u_{c^N,t}(nt)} c_t^T(nt) \right] \left(1 + s(v_t(nt)) + \frac{1}{v_t(nt)} \right) \end{aligned} \right) \\
& + \beta \mathbb{E}_t [W(d_t^x, \mu_t^x, z_{t+1}) | z_t]
\end{aligned}$$

where $W(\cdot)$ is the value function, $d_t^x = [B_t^*, c_t^T(nt), c_t^N(nt), v_t(nt)]'$, $\mu_t^x = [\mu_{5,t}, \mu_{6,t}, \mu_{7,t}, \mu_{8,t}]'$, and $z_t = [z_t^T, z_t^N, g_t^N]'$. Additionally, in the text we collect the first two vectors in $x_t = [d_t^x, \mu_t^x]$ and the rest of the endogenous variables in vector y_t .

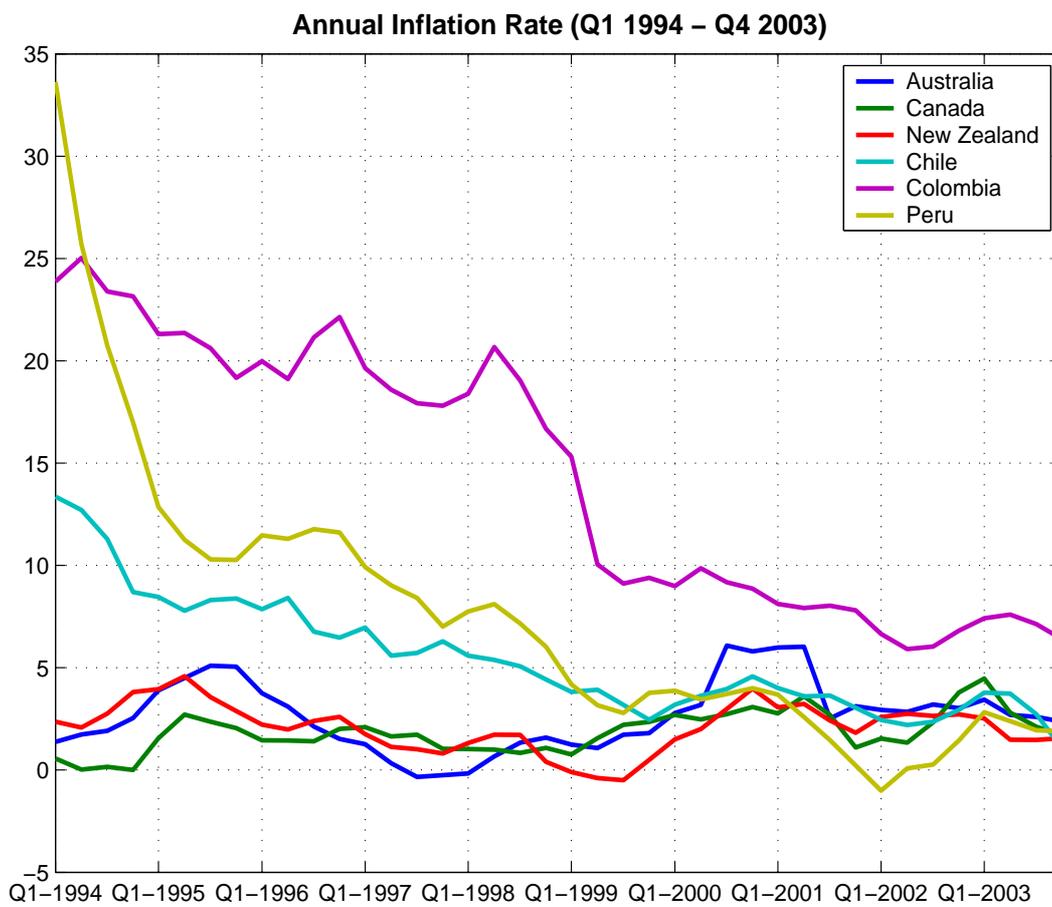
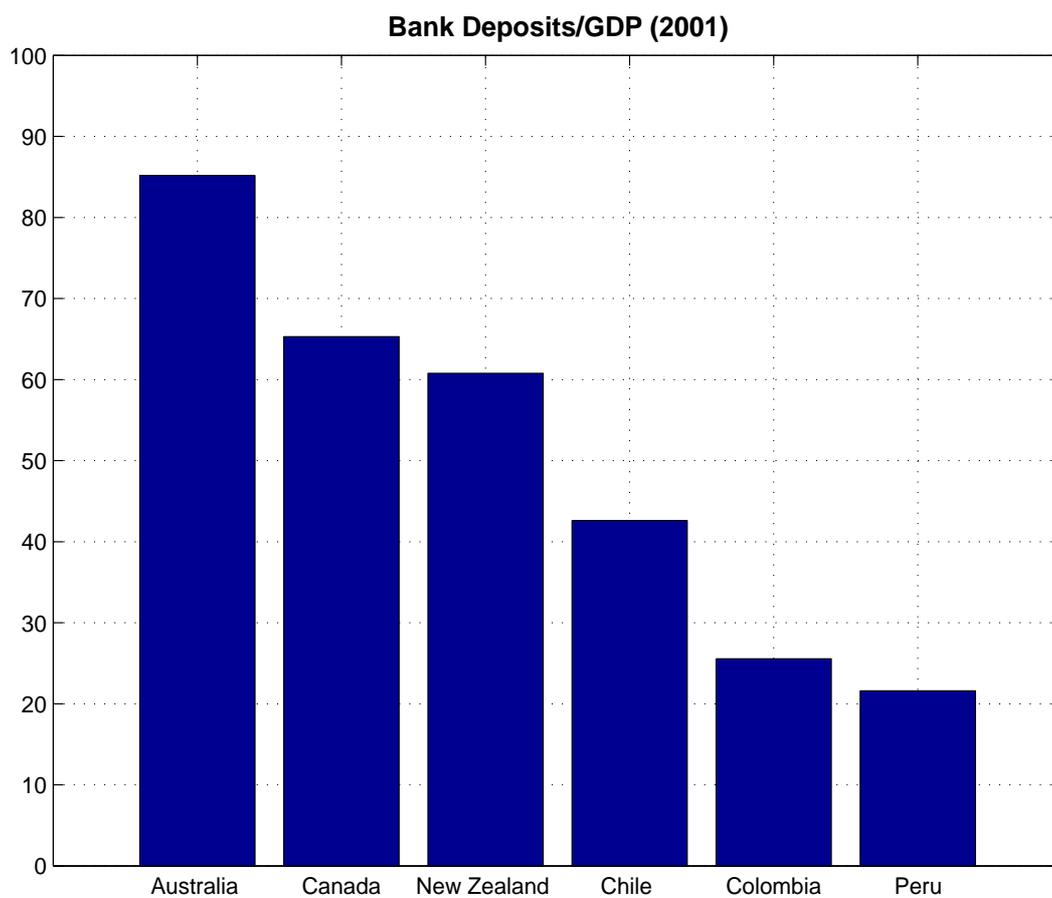


Figure 1: Annual Inflation Rate in a sample of six small open economies



Source: International Financial Statistics.

Figure 2: Financial depth

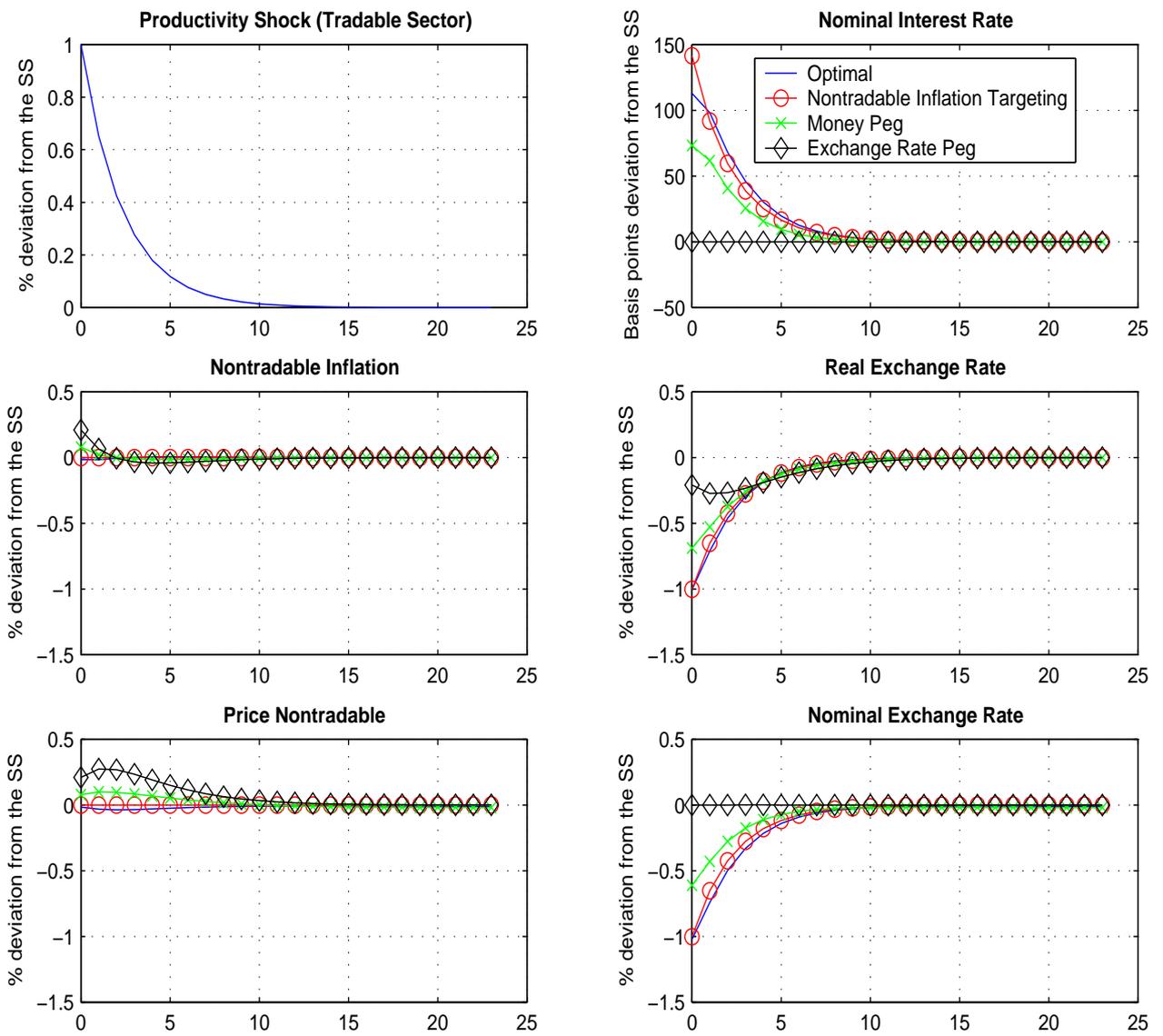


Figure 3: Impulse Responses to a Productivity Shock in the Tradable Sector

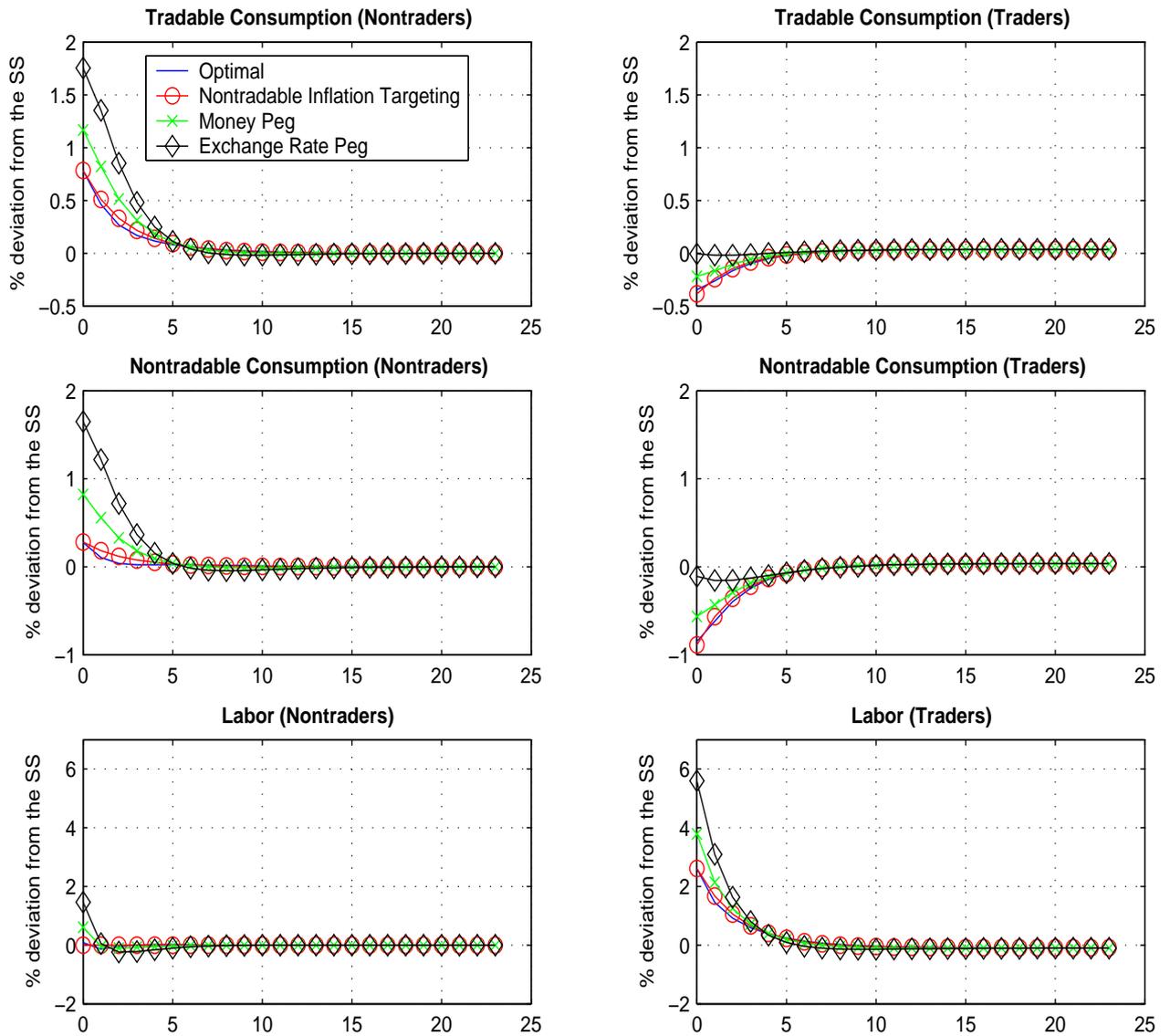


Figure 4: Impulse Responses to a Productivity Shock in the Tradable Sector (cont.)

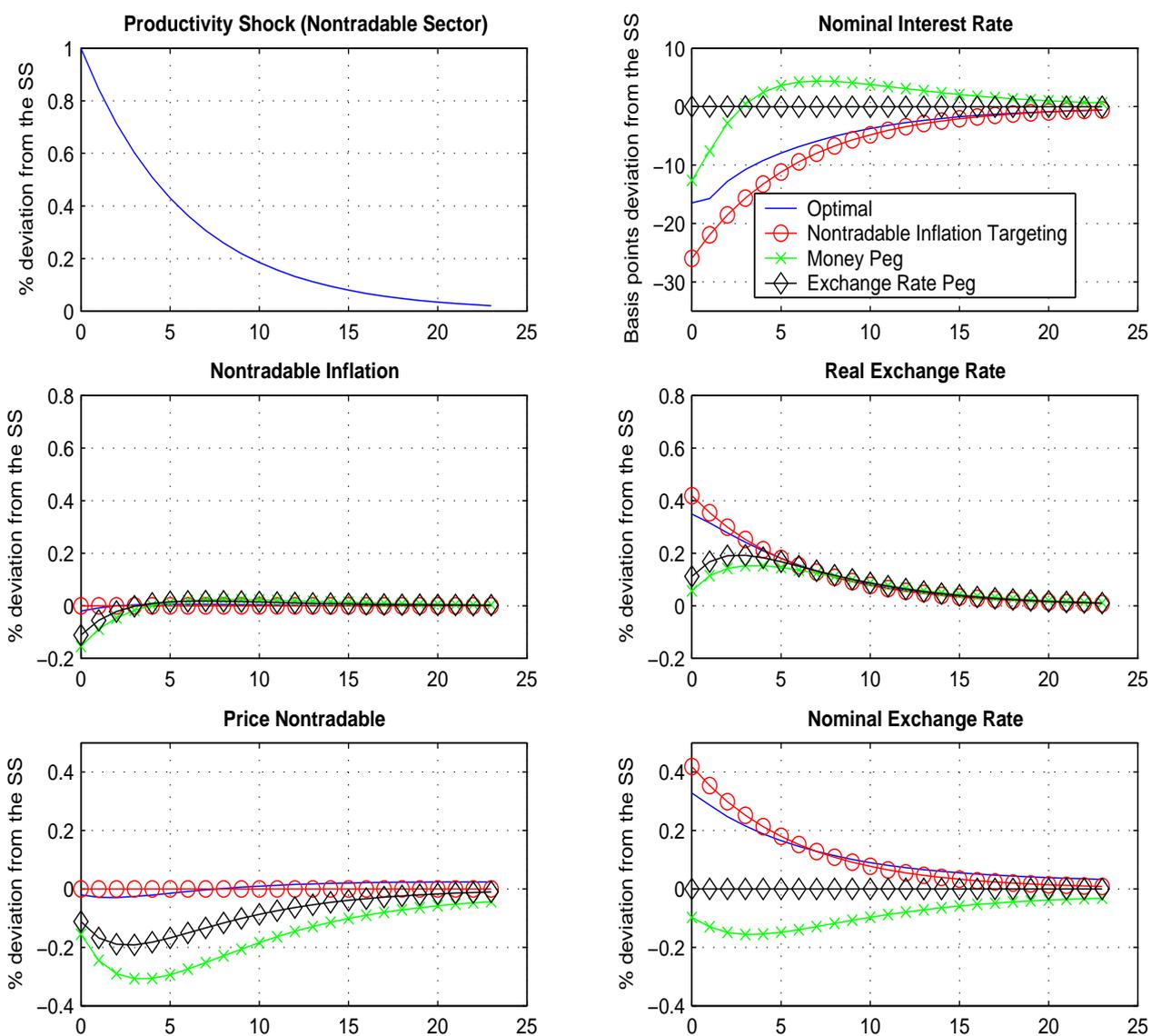


Figure 5: Impulse Responses to a Productivity Shock in the Non-tradable Sector

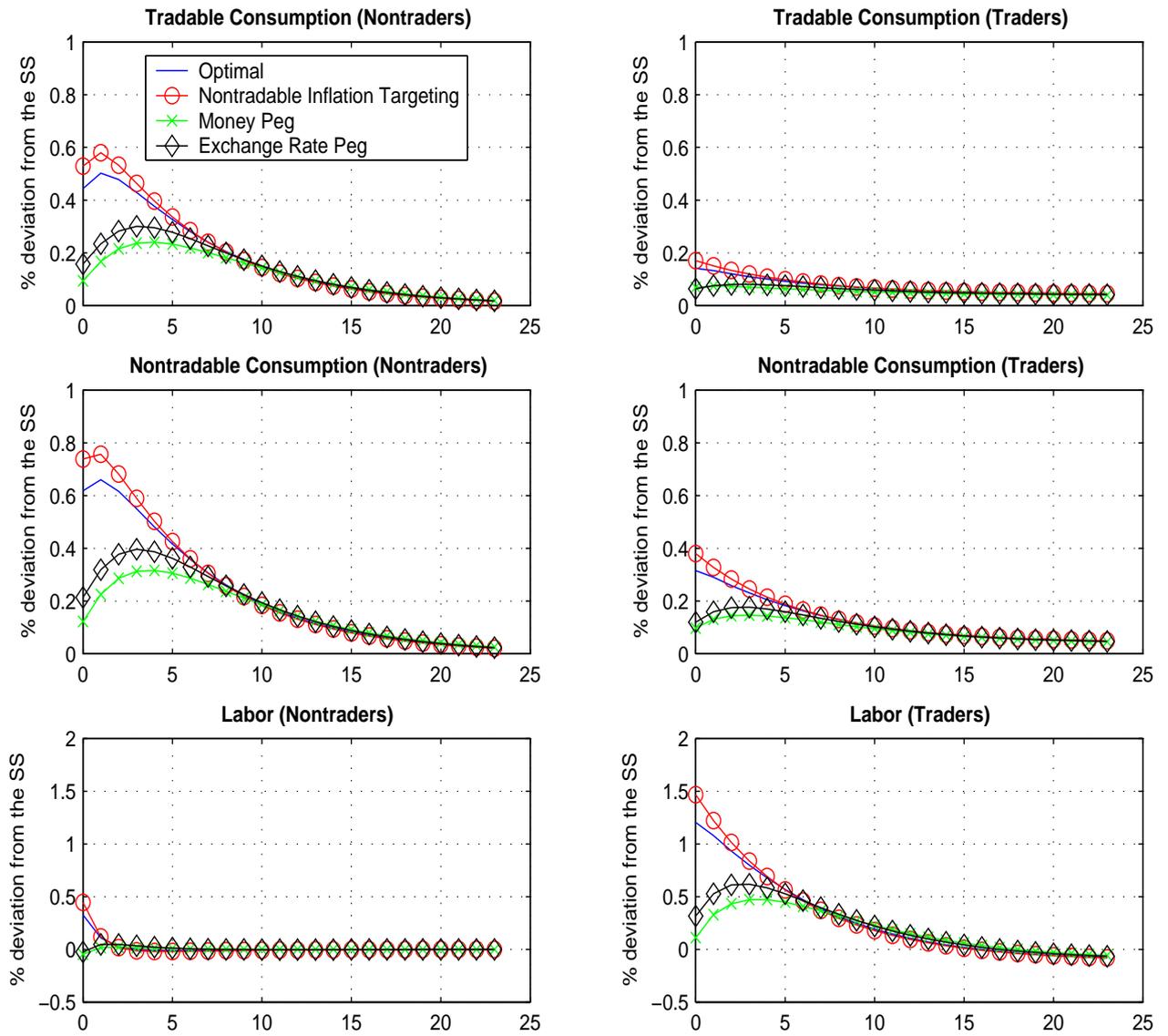


Figure 6: Impulse Responses to a Productivity Shock in the Non-tradable Sector (cont.)

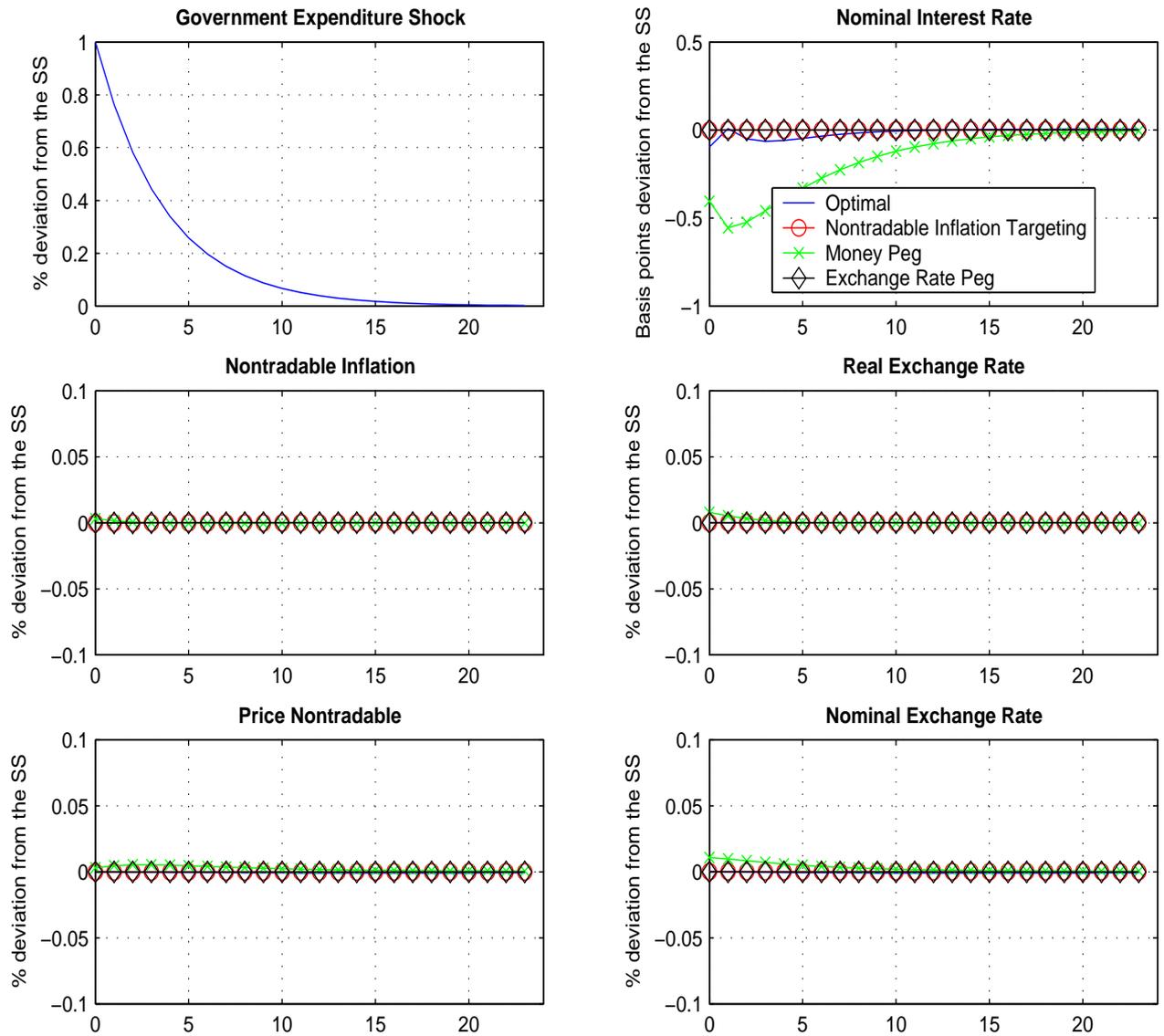


Figure 7: Impulse Responses to a Government Expenditure Shock

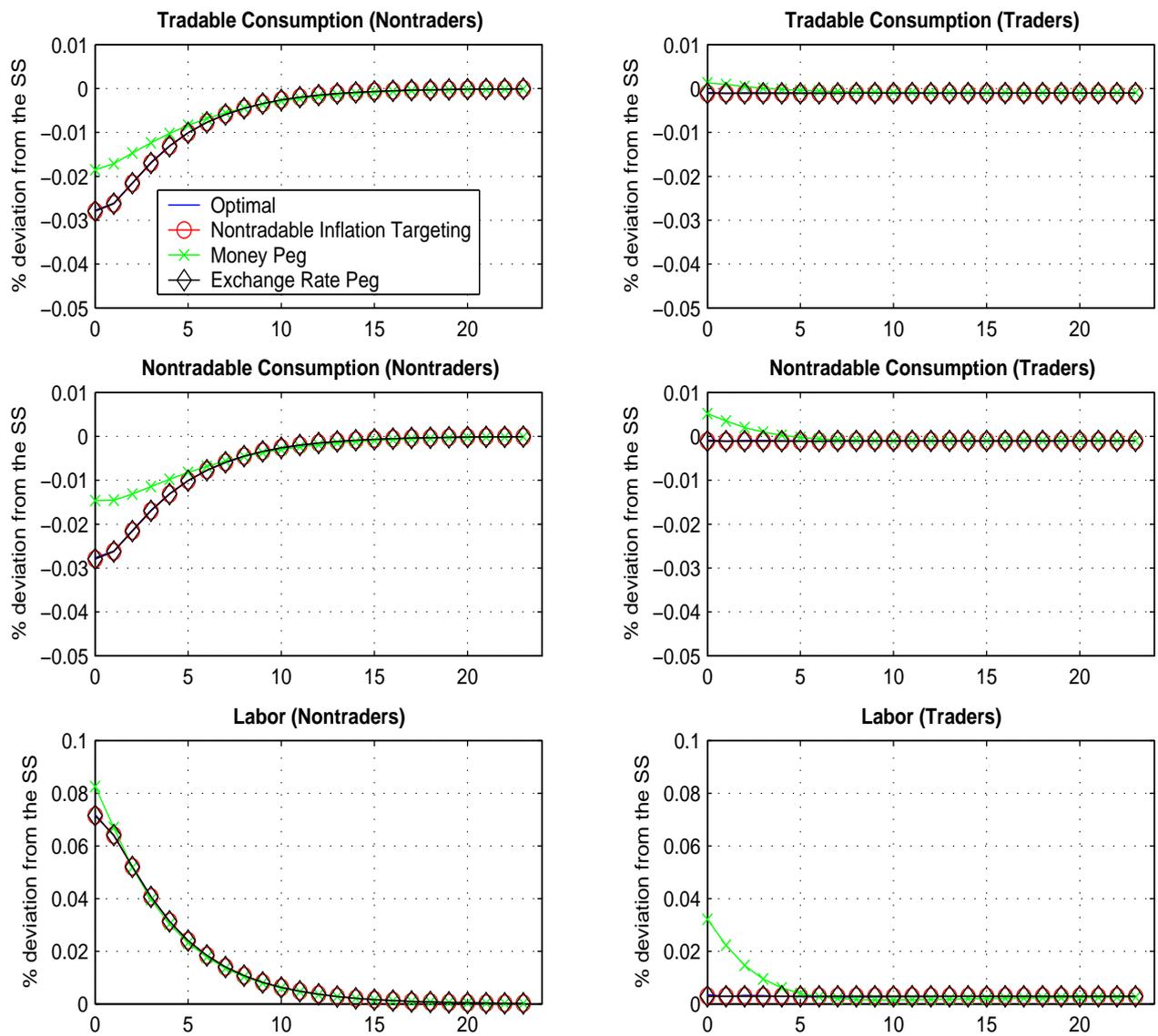


Figure 8: Impulse Responses to a Government Expenditure Shock (cont.)

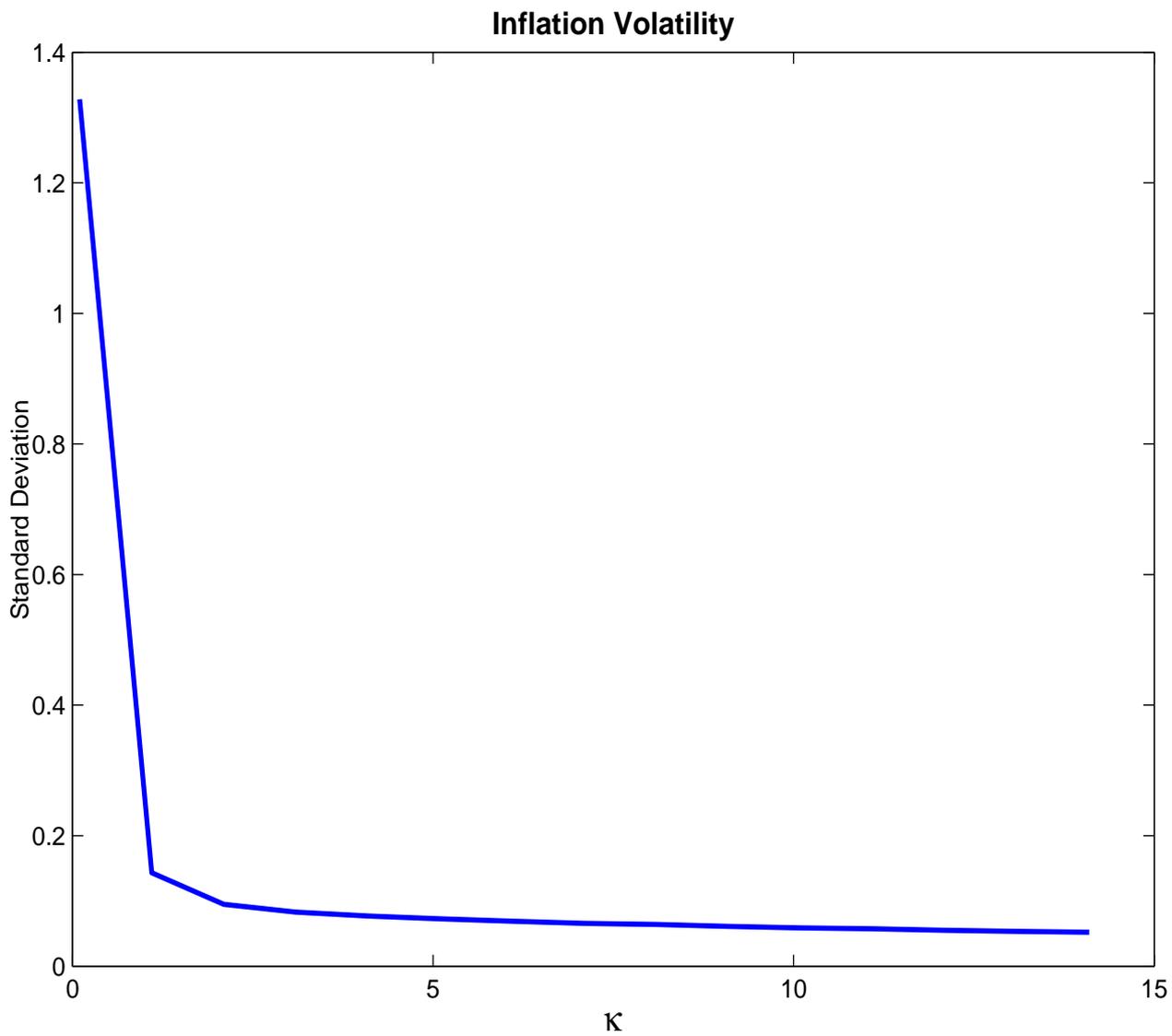


Figure 9: Non-tradable Inflation Volatility and Price Stickiness

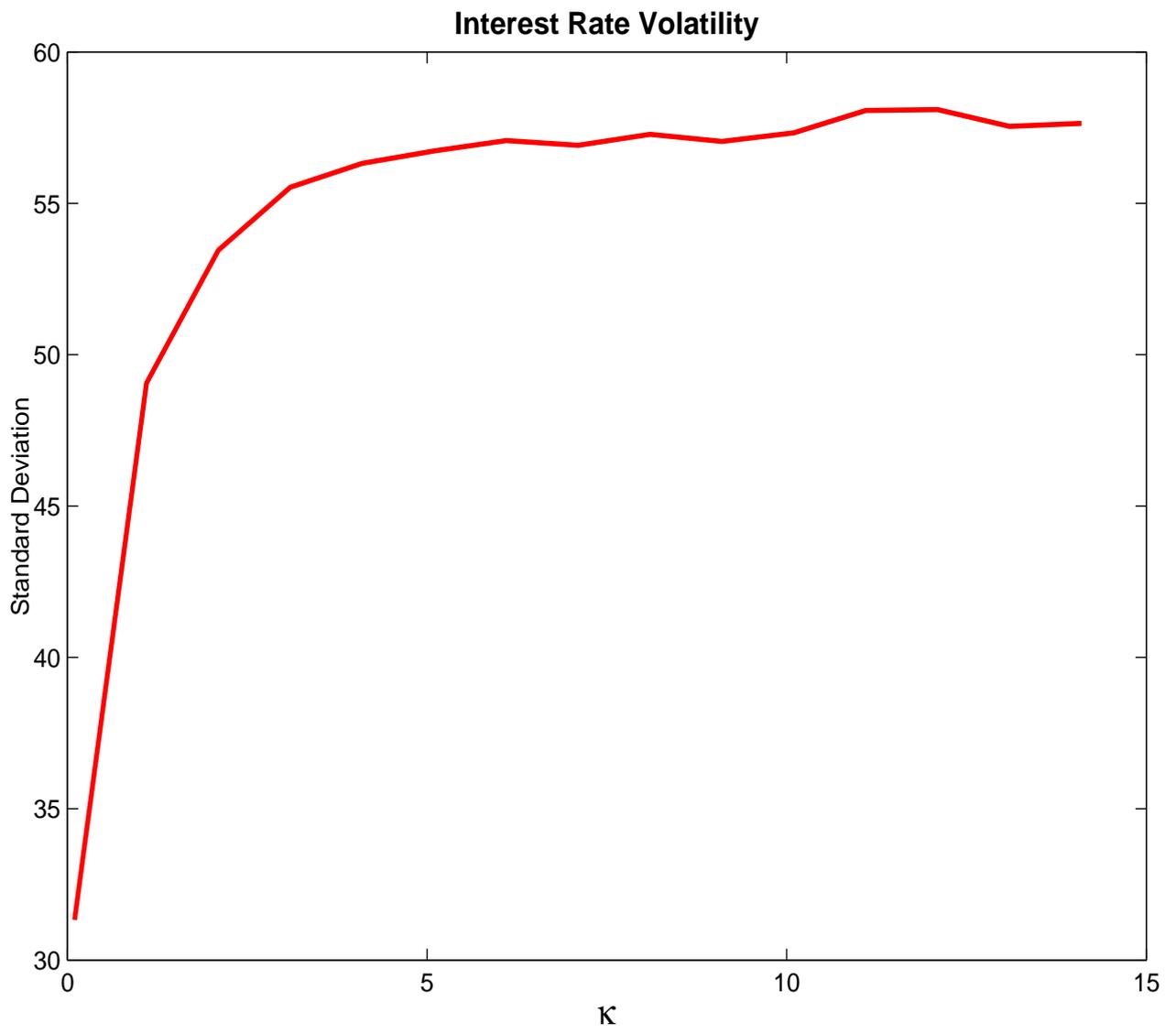


Figure 10: Nominal Interest Rate Volatility and Price Stickiness

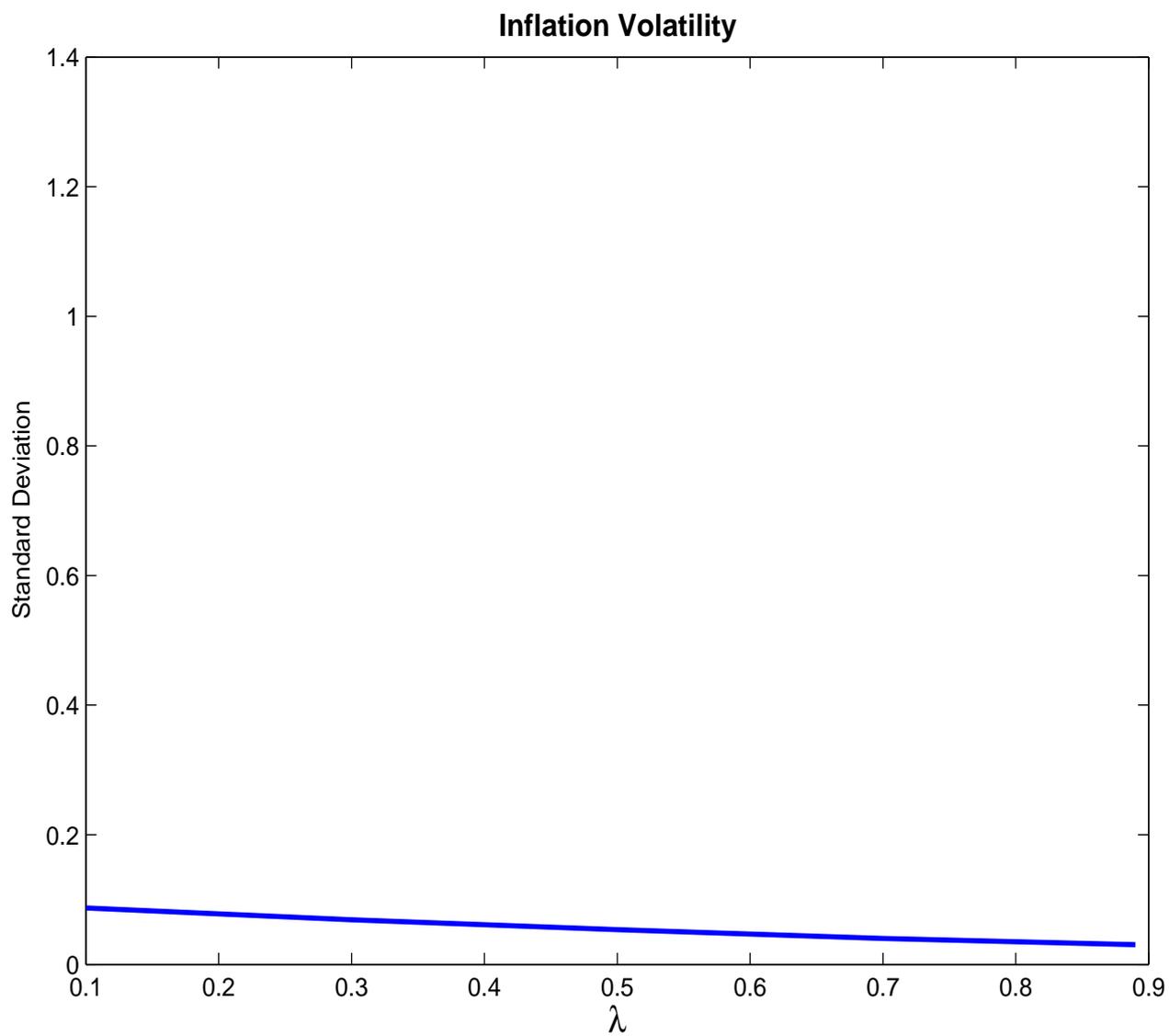


Figure 11: Non-tradable Inflation Volatility and Degree of Asset Market Segmentation

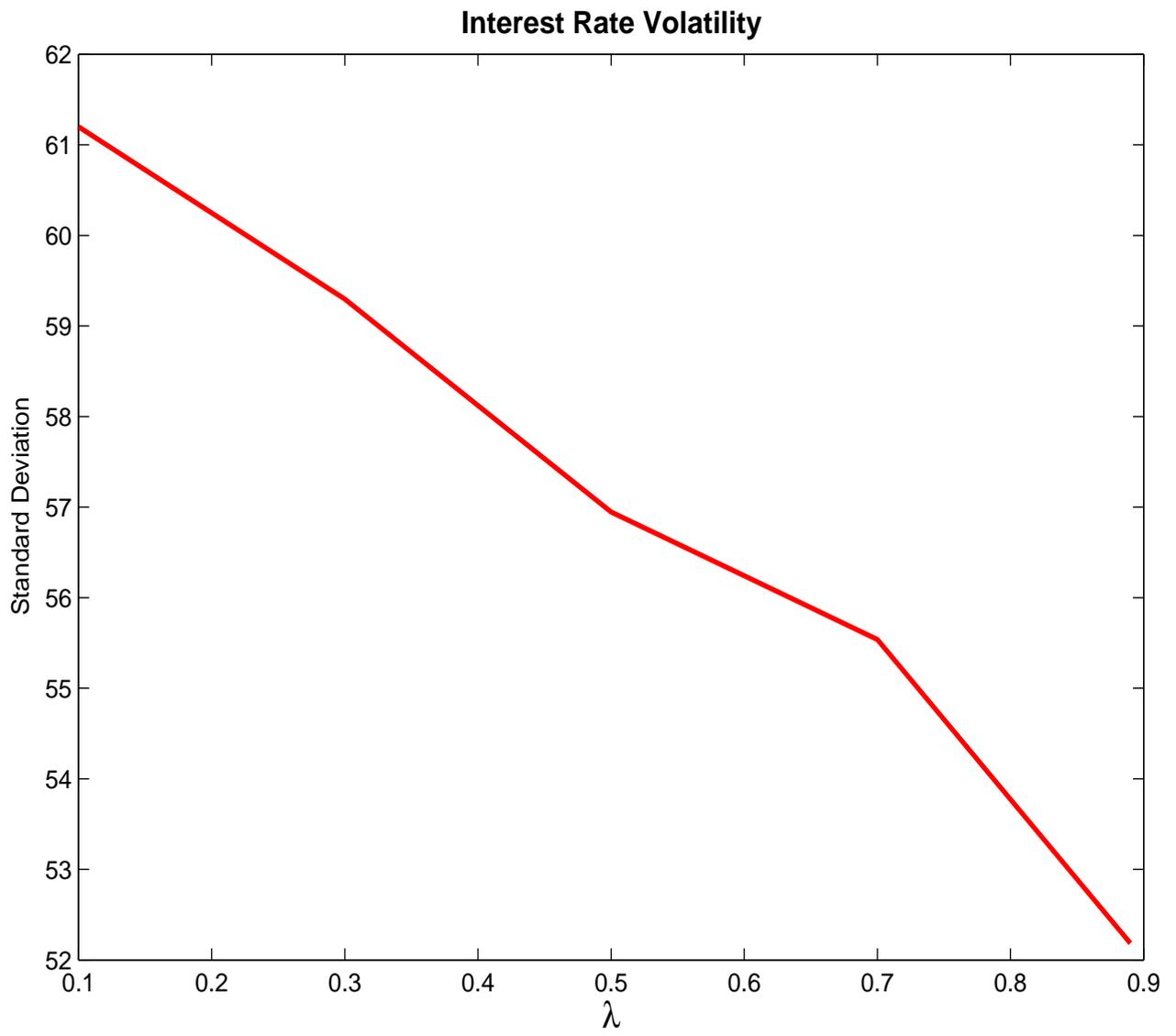


Figure 12: Nominal Interest Rate Volatility and Degree of Asset Market Segmentation

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