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BUILDING CONFIDENCE INTERVALS FOR THE BAND-PASS AND HODRICK-PRESCOTT FILTERS: AN APPLICATION USING BOOTSTRAPPING

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BUILDING CONFIDENCE INTERVALS FOR THE BAND-PASS AND HODRICK-PRESCOTT FILTERS: AN APPLICATION USING BOOTSTRAPPING

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Resumen
Este articulo presenta un método que por primera vez en la literatura genera intervalos de confianza para 2 de los filtros estadísticos más popularmente usados en la literatura: el filtro de Hodrick-Prescott y el filtro Band-Pass. Los intervalos de confianza se obtienen usando técnicas de *block-bootstrapping*. Esta técnica se aplica a la construcción de intervalos de confianza para el crecimiento del PIB de tendencia y para el componente cíclico del PIB para los países del G7. Este nuevo concepto puede ampliar la utilidad y las aplicaciones de estos filtros al superar una de sus más citadas limitaciones: no tener intervalos de confianza.

Abstract
This article generates innovative confidence intervals for two of the most popular de trending methods: Hodrick-Prescott and Band-Pass filters. The confidence intervals are obtained using block-bootstrapping techniques for dependent data. As an example, we present GDP trend growth and output gap intervals for the G7 economies. This new methodology will increase the usefulness of these filters by overcoming the absence of confidence intervals.

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1 Introduction

One of the most popular methods of estimating trend and cyclical components of time series is the use of statistical filters, in either their univariate or multivariate form. One of the most common statistical filters is the Hodrick and Prescott (1997)[1] (HP), developed as a mechanical and statistical procedure to extract very low and very high frequencies (fewer than two years and more than eight years approximately) from time series such as GDP. Although this data window seems arbitrary, it comes from the seminal study by Burns and Mitchell (1944), carried out at the National Bureau of Economic Research, which concluded that the US economy presents very clear business cycles lasting up to eight years.[2]

Following a different but related approach, other studies, such as Baxter and King (1995, 1999) and Christiano and Fitzgerald (1999) designed and implemented specific band-pass filters to isolate business-cycle fluctuations in macroeconomic time series. This sort of filter is an approximation of the "ideal band pass filter" that cannot be computed because it requires infinite observations. The band pass filters are designed to isolate fluctuations in the data that persist for periods of 1.5 or 2 through 8 years. Both papers apply the filter to several macroeconomic time series and, as with Hodrick and Prescott, a picture of the U.S. postwar business cycle emerges from their results. The authors claim that this kind of filter can improve two practical problems encountered when using the HP filter: unusual behavior of cyclical components at the sample's beginning and end, and the choice of a smoothing parameter for non-quarterly data. In this paper we use the Christiano and Fitzgerald (1999) (CF) version of the band pass filter. While the HP procedure was included in widely used econometric software, which helped to spread the filter's popularity and applicability, several versions of the band pass filter have been used and several papers and programs are available on the Internet. Hence, nowadays, studies by most academic and

[1] Although this study was published in 1997, the original draft, which is widely cited, was written in 1980.

[2] Recent studies present sophisticated methods for finding the optimal window for any time series, adjusting for data frequency (Ravn and Uhlig, 2001).

official institutions (treasury departments and central banks) that mention any potential output measurements make some reference to the two above-mentioned filters.

This paper presents a new methodology for creating confidence intervals around the cyclical component (that is, the difference between the original series and its estimated trend component) and the series trend growth rate, generated by applying the HP and CF filters to the original series, based on block bootstrap sampling techniques. The procedure is used, as an illustration, to construct confidence intervals for the output gap and trend growth rate. The output gap is an approximation for the cycle of the economy, and is defined as the difference between actual output and the HP or BK output trend generated time series (as a percentage of the filtered series). An application of this new methodology is performed using quarterly GDP data from G7 economies (Canada, France, Germany, Italy, Japan, United Kingdom and United States).

2 The Hodrick-Prescott and Christiano-Fitzgerald Filters

In this section we briefly review the two filters to illustrate how they are applied.

2.1 The Hodrick-Prescott Filter

The HP is a linear filter that creates artificial time series that minimize departures from the original and limit the volatility of this artificially generated vector to some upper bound. This limit is decided based on data frequency (monthly, quarterly, or annual), and in equations (1) and (2) is represented by the parameter λ, using values from 100 to 14,400.[3] The expression to minimize is:

$$\frac{1}{T}\sum_{t=1}^{T}\left(\ln y_t - \ln y_t^*\right)^2 + \frac{l}{T}\sum_{t=1}^{T}\left[\left(\ln y_{t+1}^* - \ln y_t^*\right) - \left(\ln y_t^* - \ln y_{t-1}^*\right)\right]^2 \qquad (1)$$

where $y_t^\star$ represents the HP filtered time series generated from the actual y_t. Its counterpart matrix can

[3] For an analysis of adjusting the HP Filter depending on the data frequency, see Ravn and Uhlig (2001).

be written as:

$$y^{*} = \left(I_{(T)} + l\left(P^{'}P\right)\right)^{-1} y \quad (2)$$

where *y* and $y^{\star}$ are column vectors both with dimension (Tx1), and *P* is the *TxT* matrix represented by a series of concatenated identity and zero matrices:

$$\text{P}=\left\lfloor I_{(T)} \mid O_{(t-2)\times 2}\right\rfloor - 2\left\lfloor O_{(t-2)\times 1} \mid I_{(t-2)} \mid O_{(t-2)\times 1}\right\rfloor + \left\lfloor O_{(t-2)\times 2} \mid I_{(t-2)}\right\rfloor$$

2.2 The Christiano-Fitzgerald Filter

The CF is also a linear filter, like the HP, but in this case it recovers the trend as the component of the series with periodicity between a lower and an upper bound (the default case for CF is 1.5 and 8 years). The main practical problem with this sort of filter is that the *ideal* filter requires an infinite amount of data. Hence, the proposed filter is a linear approximation of the ideal filter.

They approximate the filter as follows. First, it is assumed that the data is generated by a pure random walk (or, if there is a drift, it was previously removed). The authors argue this approximation (likely false) is cost-effective, in the sense that it approximates most macroeconomic series quite well (in particular, output which is used in the examples provided in section 5). Second, to isolate the component of *y* with an oscillation period between p_l and p_u, they proposed $\hat{y}$ as an optimal approximation of y^{*} (the series derived from applying the ideal band-pass filter).

$$\hat{y} = B_0 y_t + B_1 y_{t+1} + ... + B_{T-1-t} y_{T-1} + \tilde{B}_{T-t} y_T + B_1 y_{t-1} + ... + B_{t-2} y_2 + \tilde{B}_{t-1} y_1 , \quad (4)$$

for t = 3,4,...,T-2. Where

$$B_j = \frac{sin(jb) - sin(ja)}{pj}, \; j \geq 1$$

$$B_0 = \frac{b-a}{p}, \; a = \frac{2p}{p_u}, \; b = \frac{2p}{p_l}.$$

$$\tilde{B}_{T-t} = -\frac{1}{2}B_0 - \sum_{j=1}^{T-t-1} B_j \text{, for } t = 3,..., T\text{-2}.$$

And $\tilde{B}_{t-1}$ solves $0 = B_0 + B_1 + ... + B_{T-1-t} + \tilde{B}_{T-t} + B_1 + ... + B_{t-2} + \tilde{B}_{t-1}$

It is worth noting that the CF filter varies with time and is not symmetrical in terms of lead and lagged y.

4 Confidence Intervals and Bootstrapping

To generate confidence intervals we employ a bootstrapping methodology (Hamilton, 1994) described in the following diagram:

INSERT FIGURE 1

This methodology is very appealing for the problem we are trying to assess in this paper – constructing confidence intervals for statistical filters. This approach allows us to evaluate many sources of uncertainty surrounding the estimation of series trend components. To do so, the block bootstrapping procedure re-samples the cyclical component of the series using blocks of observations. Next, it is possible to re-compute the trend component. Hence, it is possible to build a large group of trend and cyclical series estimations.

Given the autocorrelated characteristic of the T-vector elements, the drawing procedure must consider this restriction by sampling sectors or blocks of this time series in such a way that the generated sampled vector preserves the autocorrelation characteristics present in the data. These blocks may be non-overlapping or overlapping, with the bootstrap sample created by sampling random blocks using replacement. The literature of optimal block length for dependent data suggests a size defined by $T^{\frac{1}{3}}$ where T represents the original sample length (Hall *et al.*, 1995).

5 An illustration: confidence intervals for output gap and trend growth in the G7.

This procedure was applied to G7 countries' GDP. We use quarterly information for GDP taken from *International Financial Statistics*, published by the International Monetary Fund (IMF). The data cover periods ranging from 1957 to 2001 in the case of Japan, UK, and USA and from 1986 to 2001 in the case of Canada.

The output gap defined by $\left(\frac{y_t - y_t^*}{y_t^*}\right) \bullet 100$ makes up the generated vector, using an expression for the HP and the CF filters, on which the bootstrapping sampling is done.

Figure 2 presents the estimated output gap generated for these economies by applying the HP filter to quarterly time series data (λ=1,600). The corresponding confidence intervals are defined with upper and lower limits, efficiently reflecting (through wide confidence intervals) the popular and widespread criticism that the HP filter is very end-of-sample data-dependent. This point is crucial for several reasons. Policy makers, for example, are often forced to make on-the-spot evaluations of current economic conditions without a solid tool for building a credible and time consistent argument that will not vanish two or three months hence. Confidence intervals generated using bootstrapping techniques can resolve current limitations of not having real-time data and a non-dependent end-of-sample tool to at least partially evaluate the current economic stance.

INSERT FIGURE 2

The interpretation of figure 2 is straightforward. Once the output gap exceeds any upper or lower limit, we can say that the economy faces a high risk of GDP misalignment with respect to trend GDP. Alternatively, if the output gap indicator rises past the upper bound of the interval and if one believes in a form of Phillips curve, inflationary risks become likely in the near future, whereas if it falls below the lower bound, the risk of a recession or deflationary situation rises. However, it is interesting to notice that confidence intervals are very wide in most cases throughout the sample, suggesting that in most cases output-gaps of –1% to +1% are not different from 0.

In addition, shaded areas in Figure 2 represent the NBER definition for recessionary episodes. It is interesting to notice that in most cases the periods when the negative cycle is different from 0 coincide with periods when the NBER definition implies a recession (namely, in the case of the US). Moreover, the proposed methodology makes it possible to identify both recessionary and expansionary periods.

Figure 3 presents the results of the same exercise, now applying the CF filter. Again, the confidence intervals are very wide, especially at the end of the sample. In terms of the width of the confidence intervals, on average, they move in the range (-1, 1). However, in the case of the CF filter, the volatility of the intervals is smaller than the HP filter in every case. Once more, output gap estimations are statistically different from 0 in a small number of cases only.

INSERT FIGURE 3

Formal comparisons of confidence intervals for both filters are presented in Table 1. Evidently, tests of equality of means and medians are rejected in most cases (except Canada and France), indicating that in general the CF filter confidence intervals are wider than the intervals generated using a HP filter. On the other hand, tests for equality of variance show that, in all cases, confidence intervals generated using the CF filter produce less volatile intervals. In a sense, there is a trade-off involved in the choice of filter: the HP filter provides more precise estimates (in the sense of wide intervals), but the band intervals are more volatile, while the opposite is true of the CF filter.

Now, we will apply our block-bootstrapping procedure to define trend output growth. Figures 4 and 5 present the changes in this indicator with the corresponding confidence interval, again for the G7 economies. In this case we present the median of the bootstrapping estimations for each filter. The picture for both filters is quite similar; there are some slow movements in trend growth, in general with smoother changes in the case of the HP filter.

The case of the US is interesting because that country experienced a steady increase in trend growth during the 1990s, with it rising from the 1% to 3% range in the early 1990s to its current range of from 3% to 5%. The economic interpretation attributes this phenomenon to productivity gains, possibly related to the "new economy", and prosperity arising from information and communications technology (Lansing, 2000). However, it is interesting to highlight that both filters present some differences: while the HP filter found an almost steady increase, with a further rise in the band at the end of the sample, trend growth estimated using the CF filter found it rose until 1995 and then fell.

INSERT FIGURES 4 AND 5

Table 2 compares the confidence intervals generated for both filters. In this case, the results are quite similar to the output gap bands case: the CF-bands are wider and less volatile than the HP-bands.

5 Conclusion

This paper presents a simple methodology that for the first time generates confidence intervals for output gaps and output trend growth based on the application of block bootstrapping techniques to the results of two of the most popular statistical filters. This new procedure provides a measure of the uncertainty surrounding the estimation of trend growth and output gap. This application also has potential applications for policymaking, as it provides authorities with a powerful tool to be used when evaluating current economic conditions. Instead of having only output gap and trend growth point estimates, now they may include confidence intervals that provide considerable macroeconomic insight, as they take into consideration future modifications to monetary policies.

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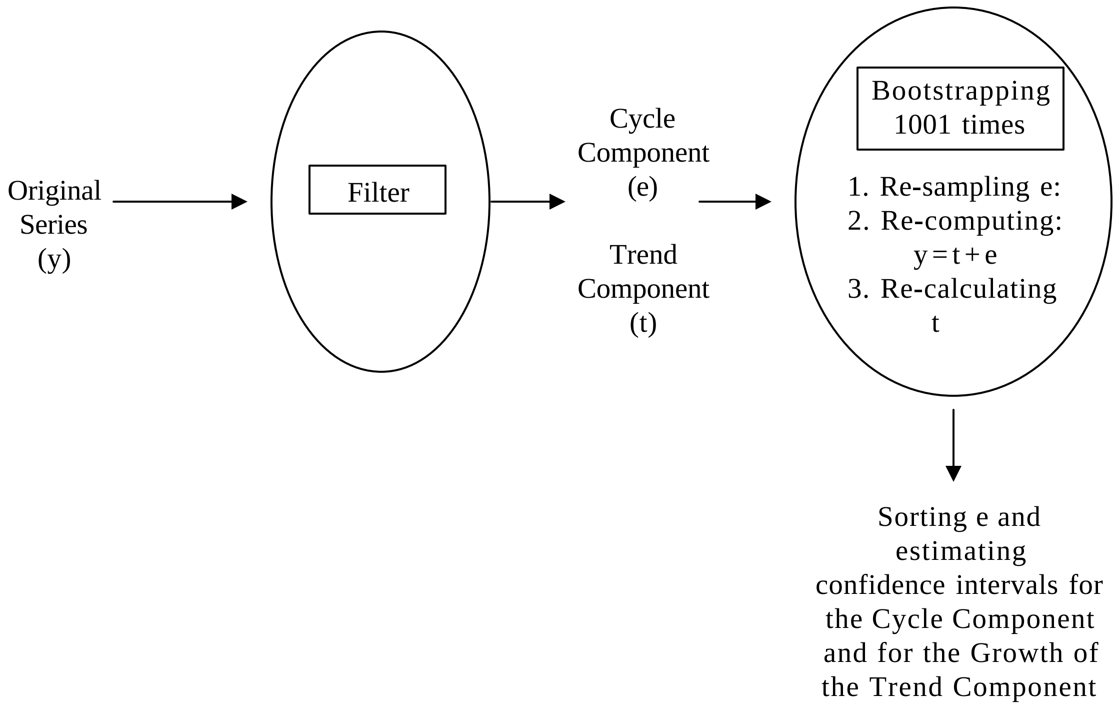

Figure 1. Algorithm to Generate Filter Confidence Intervals

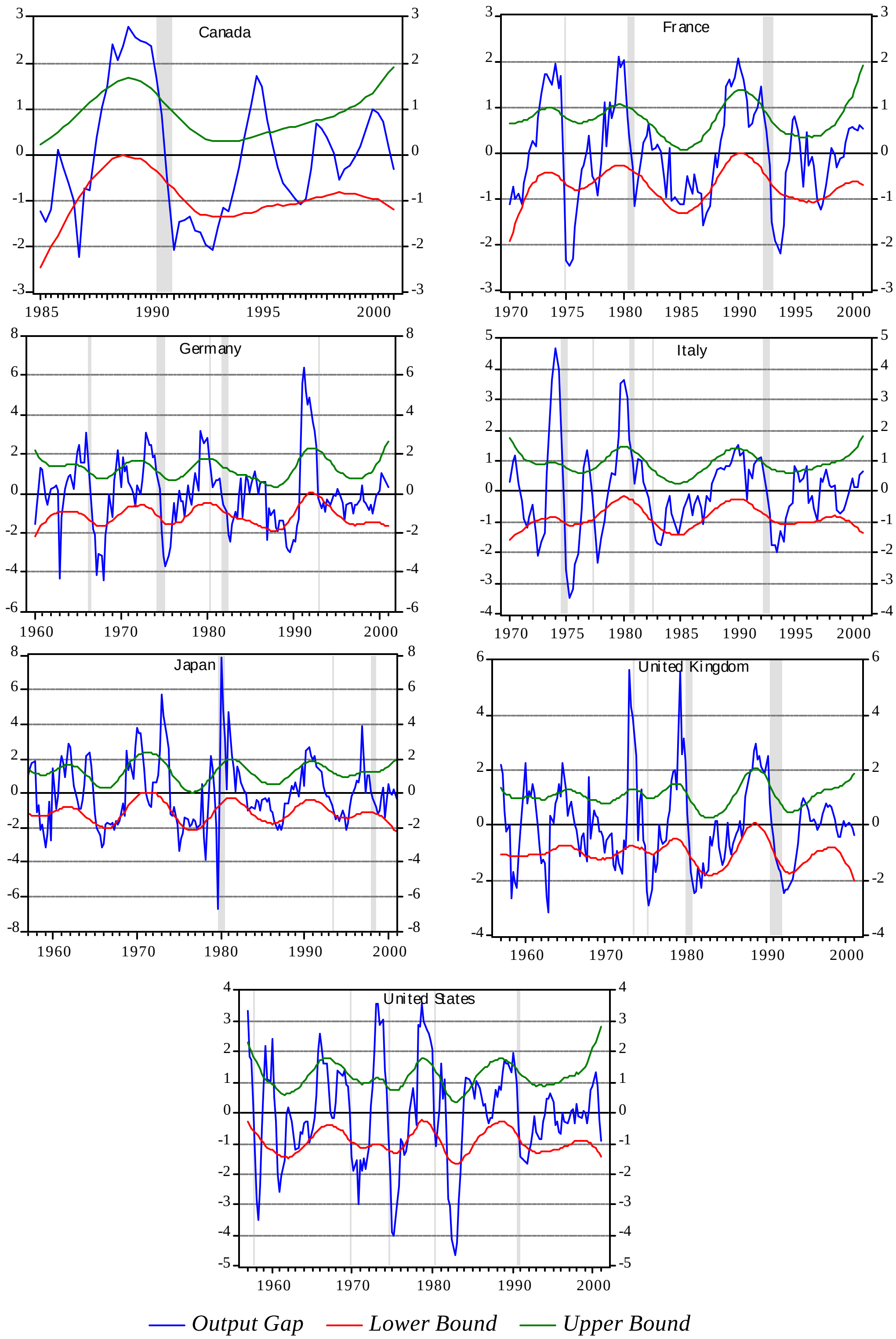

Figure 2. Output Gap Confidence Intervals using the HP filter for G7 Economies

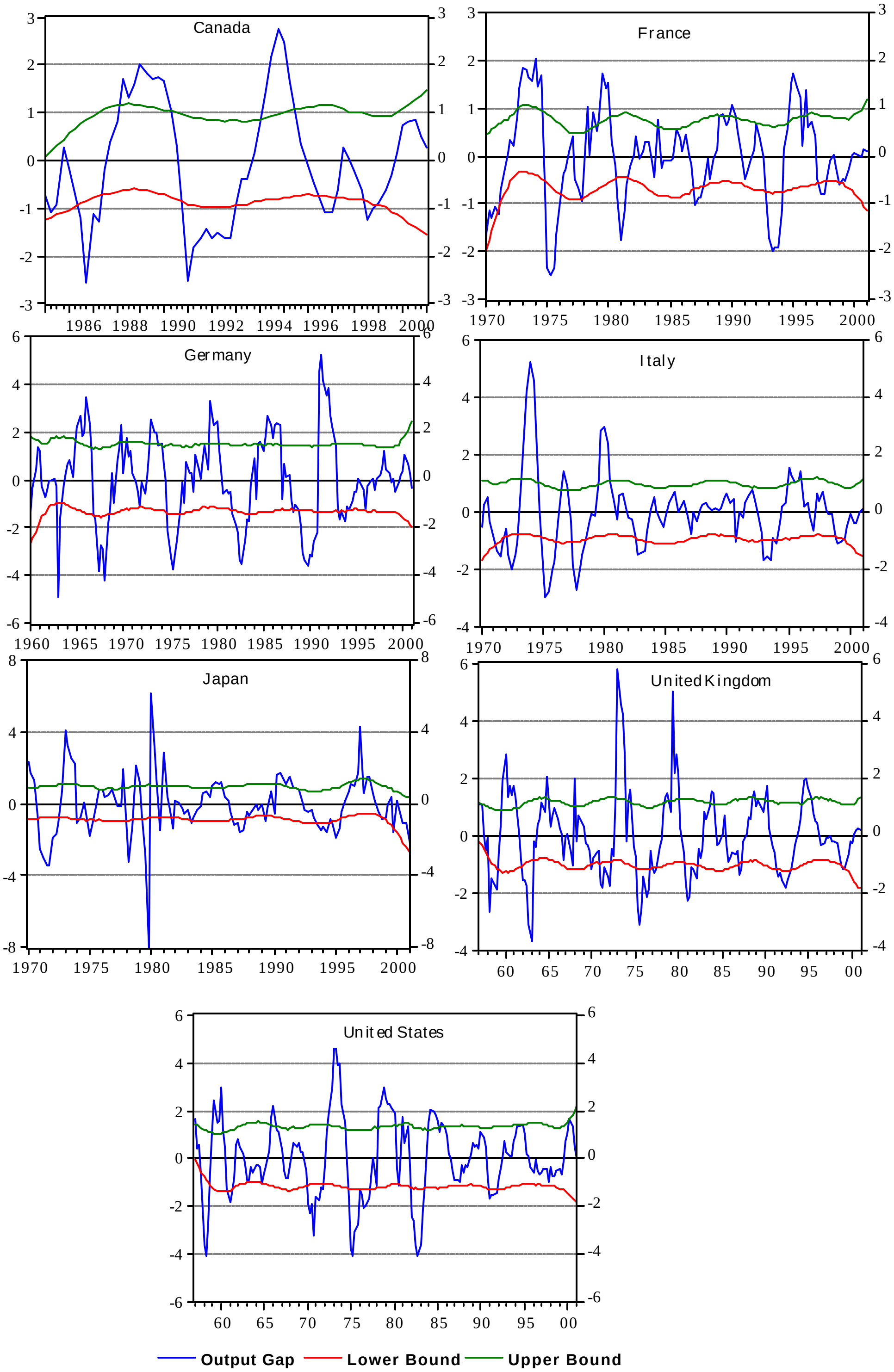

Figure 3. Output Gap Confidence Intervals using the CF filter for G7 Economies

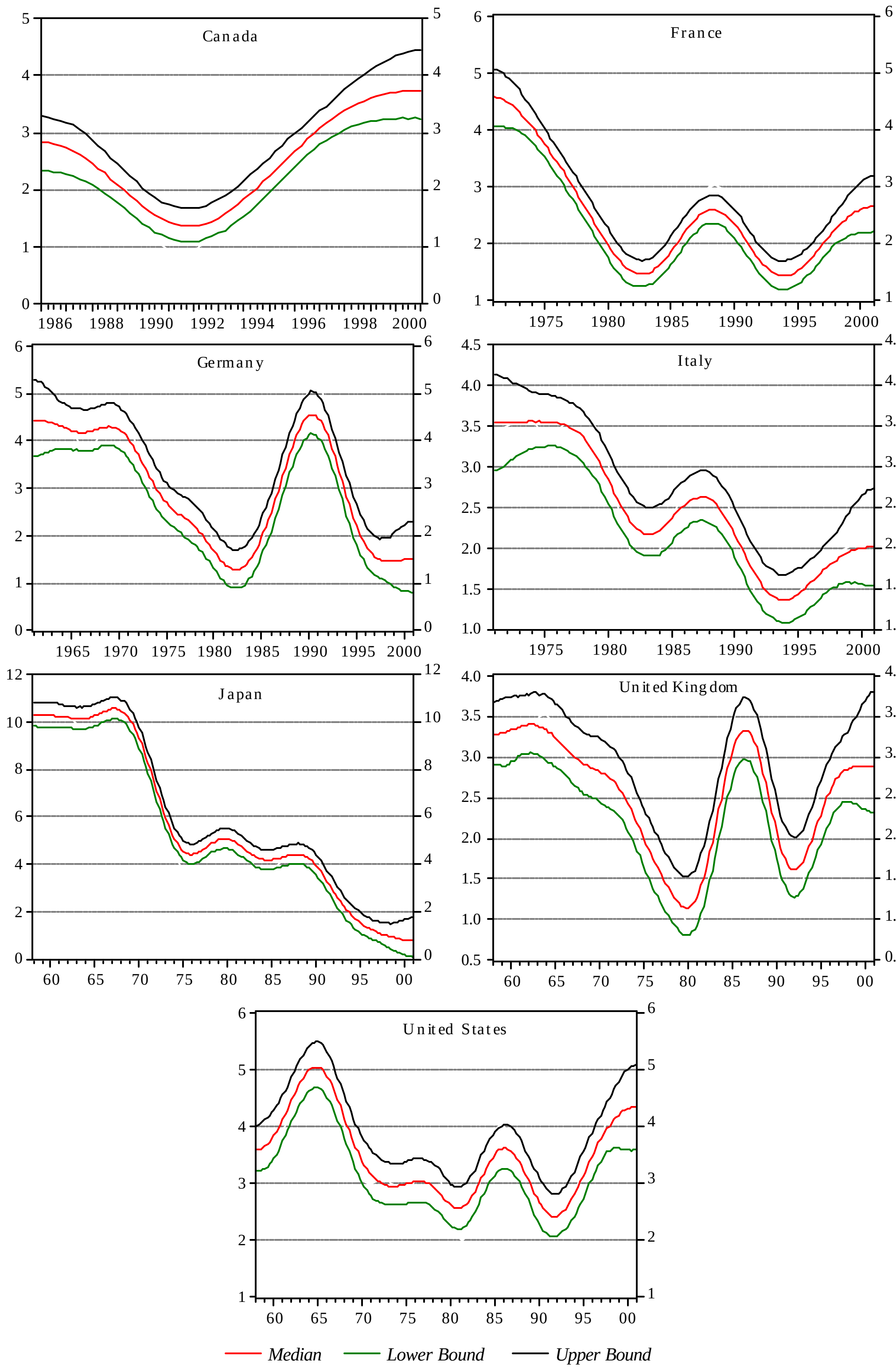

Figure 4. GDP Trend Growth for G7 Economies: Block Bootstrapping Simulations using the HP filter

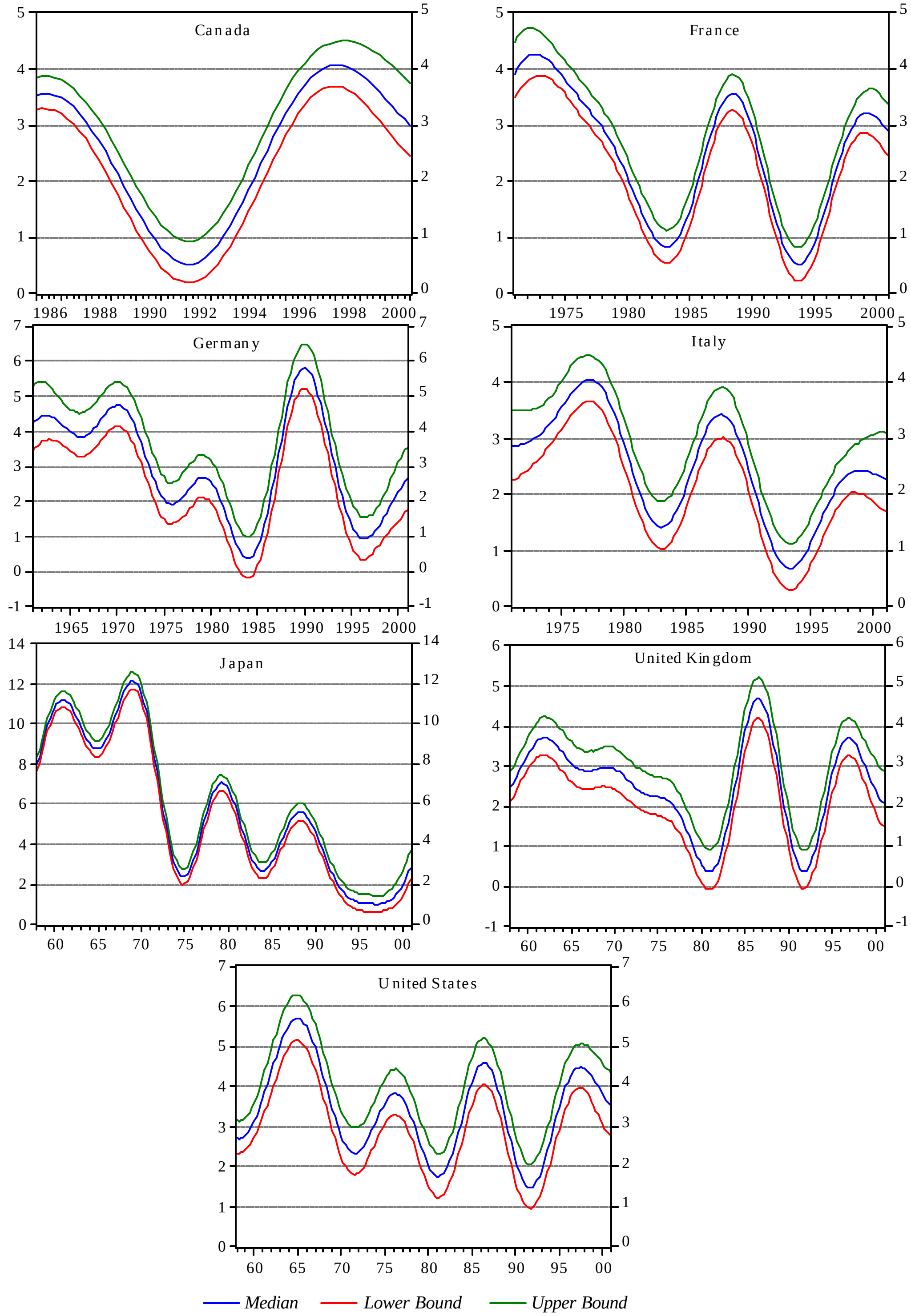

Figure 5. GDP Trend Growth for G7 Economies: Block Bootstrapping Simulations using the CF filter

Table 1. Statistics for Confidence Intervals for Output Gap (in percentage points)

	Canada	France	Germany	Italy	Japan	United Kingdom	United States
HP Filter							
Mean	1.84	1.46	2.40	1.80	2.36	2.12	2.18
Median	1.69	1.39	2.29	1.69	2.31	2.08	2.11
Minimum	3.10	2.59	4.31	3.31	4.21	3.91	4.28
Maximum	1.60	1.31	2.16	1.61	2.19	1.92	2.00
St. Deviation	0.34	0.24	0.37	0.32	0.26	0.25	0.30
CF Filter							
Mean	1.85	1.45	2.85	1.94	1.84	2.21	2.53
Median	1.81	1.40	2.78	1.90	1.82	2.19	2.50
Minimum	2.99	2.42	4.48	2.81	3.07	3.20	4.06
Maximum	1.31	1.30	2.62	1.76	1.47	1.74	1.95
St. Deviation	0.27	0.19	0.29	0.17	0.17	0.15	0.20
Tests of Equality (p-values)							
Mean (t-test)	0.87	0.62	0.00***	0.00***	0.00***	0.00***	0.00***
Median (Chi-square)	0.00***	0.13	0.00***	0.00***	0.00***	0.00***	0.00***
Variance (F-test)	0.08*	0.01***	0.00***	0.00***	0.00***	0.00***	0.00***

Table 2. Statistics for Confidence Intervals for Trend growth (in percentage points)

	Canada	France	Germany	Italy	Japan	United Kingdom	United States
HP Filter							
Mean	0.74	0.56	0.91	0.68	0.91	0.78	0.81
Median	0.63	0.50	0.85	0.61	0.87	0.75	0.78
Minimum	1.21	1.01	1.62	1.20	1.68	1.49	1.49
Maximum	0.58	0.45	0.77	0.58	0.80	0.69	0.72
St. Deviation	0.19	0.14	0.19	0.16	0.15	0.13	0.14
CF Filter							
Mean	0.81	0.65	1.28	0.88	0.84	0.98	1.13
Median	0.77	0.61	1.24	0.84	0.83	0.99	1.13
Minimum	1.32	1.01	1.83	1.39	1.40	1.39	1.57
Maximum	0.58	0.57	1.07	0.77	0.63	0.72	0.79
St. Deviation	0.18	0.11	0.15	0.13	0.11	0.10	0.12
Tests of Equality (p-values)							
Mean (t-test)	0.00***	0.00***	0.00***	0.00***	0.00***	0.00***	0.00***
Median (Chi-square)	0.00***	0.00***	0.00***	0.00***	0.00***	0.00***	0.00***
Variance (F-test)	0.00***	0.00***	0.00***	0.62	0.10	0.16	0.00***

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