INVESTMENT UNDER UNCERTAINTY AND FINANCIAL MARKET DEVELOPMENT: A *q*-THEORY APPROACH

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Resumen

El artículo presenta un modelo teórico de inversión bajo incertidumbre, incorporando el sector financiero como un factor clave en la determinación del efecto y el nivel de la incertidumbre. A diferencia de trabajos anteriores, el modelo se desarrolla usando el enfoque q de Tobin, analizando los efectos desde una perspectiva de equilibrio — estática comparativa —. El modelo asume reversibilidad parcial en las decisiones de inversión y una firma representativa que enfrenta un mercado no-competitivo. Existe, además, incertidumbre asociada a las condiciones de mercado que la firma enfrenta.

Del modelo se concluye que la incertidumbre afecta negativamente el proceso de acumulación de capital. Asumiendo una condición inicial de equilibrio, a medida que la incertidumbre aumenta, se requerirá un incremento en el crecimiento esperado con el objeto de mantenerse en un estado de equilibrio. Si el requerimiento no es satisfecho, como debiera ocurrir en el corto plazo, q disminuye y, consecuentemente, disminuye el stock de capital. La dinámica de q hace a la empresa converger a un *punto de pseudo-equilibrio*, donde permanecerá mientras no haya cambios en la incertidumbre sobre las condiciones de mercado, mientras no aumenten las expectativas de crecimiento de las condiciones de mercado o mientras no se desarrolle el mercado de capitales.

La profundidad del mercado financiero tiene importantes consecuencias sobre la posibilidad de los agentes de diversificar riesgo, sobre la precisión en la valoración de firmas y en la volatilidad de la economía. Un mercado de capitales mas desarrollado conduce a la economía a un mejor ambiente; menos incertidumbre es percibida por los inversionistas y el efecto de ésta sobre la toma de decisiones se reduce. Como resultado, la inversión aumenta fortaleciendo la capacidad de crecimiento económico.

Abstract

The paper presents a theoretical model of investment under uncertainty, incorporating the financial sector development as a key factor for determining the effect and level of uncertainty. As a difference with previous works, the model is developed using a Tobin's q approach, analyzing the effects from an equilibrium perspective —comparative static—. The model assumes partial reversibility in investment decisions and a representative firm participating in a non-competitive market. Uncertainty is associated to the market conditions faced by the firm in the commercialization of the products or services that offers.

From the model we conclude that uncertainty affects negatively the capital accumulation process. Assuming an initial equilibrium condition, as uncertainty increases, a higher expected growth on market conditions will be required in order to remain in an equilibrium state. If the requirement is not satisfied, as should occur in the short run, q decreases and, as a consequence, the stock of capital. The dynamic on q makes the firm gradually converge to a *pseudo equilibrium point*, where it stays while there is no change in uncertainty about market conditions, increase in the expected growth of market conditions is not verified or the financial market does not become more developed.

Financial market deepness has important effects on economic agents ability to diversify risk, accuracy in firms' valuation and economic volatility. Then, a more developed financial market conducts the economy to an improved environment; less uncertainty is perceived by investors and its effect over decision making is reduced. As a result, investment increases, strengthening the economic growth capacity.

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1 Introduction

Private investment in different countries has shown evidence of a straight relationship with economic stability. The measurement of this effect is crucial in order to support economic policies directed to strengthen investment and, therefore, economic growth capacity. A theoretical and empirical analysis of the uncertainty effect, as well as the role of financial market, are especially important in developing countries, where current investment expenditure and quality are low, and the requirement of a strong economic growth is urgent for their development.

Financial market development has an active role in raising domestic saving and improving the allocative efficiency of capital. This, in turn, would strengthen investment and economic growth[1]. Looking from an investment decision perspective, financial market would have a role for decreasing the risk faced by investors? Would a more developed financial market diminish the effect of risk over investment? Answers to these questions are important in order to measure the benefits of economic policies directed to deepen capital markets in developing countries.

A deeper financial market would incentive the appearance of hedging instruments, capable to reduce the effect of risk. Additionally, considering a more efficient capital market, we observe that less uncertainty is perceived by economic agents, since more stability in macro variables is incorporated. Indeed, capital markets are financially repressed when governments distort them. Usury restrictions on interest rates, heavy reserve requirements on bank deposits, and compulsory credit allocations make deposit interest rates, difficult to predict when inflation is high and unstable and foreign exchange rates also become highly uncertain[2]. On the other hand, a well developed financial sector increase accuracy of value of firms -stock market transparency-, which makes, looking from a Tobin's q perspective, more certain the benefits of investment. Finally, financial development increases availability of funds to finance new investments, considering that, as Cho (1988), McKinnon (1993) and Fry (1995) argue, there is an increase in savings and less discretionary credit assignments.

In order to measure the uncertainty effect on private investment, we may take alternative theoretical approaches. Dixit (1989) analyzes the conditions that a firm considers in order to enter or exit from a market, considering a stochastic behavior of demand. An alternative approach considers the optimal investment decisions of a firm, whose managers maximize the firm's value. Using this approach Pindyck (1982), Abel (1983, 1985) and Abel and Eberly (1994) develop a

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[1] See Cho (1988), McKinnon (1993), Fry (1995) and King and Rose (1993).

[2] See McKinnon (1993).

model considering adjustment costs for capital and uncertainty associated to the price of the final product. Caballero (1991), analyzing above works, argues that symmetry of adjustment costs determines a positive effect of uncertainty over investment, when we consider risk neutral agents. In this case, the positive effect of volatility on the marginal profitability of capital[3] prevails over the negative effect, consequence of more reluctance of firms to invest, in order to avoid getting caught with too much capital.

An irreversible character of investment was incorporated by Pindyck (1988,1991) and Bernanke (1985). In these models disinvestment is not allowed since the cost of this action is higher than the benefit of the alternative use of capital. Also Bertola and Caballero (1991) consider irreversible investment and uncertainty in market conditions, finding a negative relationship between investment and uncertainty. An option theory interpretation for investment decisions is given by Abel, Dixit, Eberly and Pindyck (1995). They show that Tobin's q formulation may be linked to option theory. If irreversibility of investment is incorporated in the formulation (infinite adjustment costs for investment less than zero), higher uncertainty implies a reduction in investment. As we increase symmetry of adjustment cost curve, the effect of uncertainty decreases.

The purpose of this research work is to formulate a new theoretical model for private investment under uncertainty, based on Tobin's q theory (1969). As a difference to previous modeling, it incorporates financial market development as an important factor. According to Tobin's approach, investment is determined by q, a variable that represents the average contribution of one unit of capital to the firm's value. An advantage of this approach is that, for empirical applications, q is an observable variable, that can be approximated by the ratio between the market and accountable values of the firm.

In order to include not only the price of capital as the investment cost, but also other costs associated to new investment, such as capital installation and training of workers, the model considers adjustment costs for changes in the stock of capital. In the model developed in the paper, the cost function considers partial reversibility of investment[4]. Indeed, because buyers of used equipment, unable to evaluate the quality of an item, will generally offer to pay a price that corresponds to the average quality of the market. Sellers who know the quality of the item they are selling will resist unloading above-average merchandise at a reduced price. The average quality of used equipment available in the market will go down and, therefore, so will the market price. As a consequence, equipment is apt to have re-sale values that are well below their original purchase

[3] Convexity of marginal revenue of capital, conseuqence of flexibility of labor, produces that effect.

[4] See for example, Caballero (1991), Pindyck (1988) and Bertola (1988). The last two papers assume irreversible investment.

costs. When investment is specific to a company or to an industry, we have an increase in capital irreversibility.

As a difference to Hartman (1972) and Abel (1983, 1984, 1985) models, from which we capture a positive relationship between uncertainty and investment, the proposed model establishes border conditions for marginal contribution of capital, asymmetric adjustment costs and non-competitive markets. As Dixit and Pindyck (1995) argue, the ability to delay an irreversible or partial irreversible investment expenditure can profoundly affect the decision to invest.

The proposed model considers that uncertainty is exclusively captured by the market conditions that a representative firm faces. Uncertainty in market conditions is interpreted as permanent or transitory shocks in the demand, which may be associated to changes in the market share, economic growth and not anticipated fluctuations of demand.

The financial market development is incorporated by considering that observed q is uncertain, and has an expected value equal to the theoretical q. Additionally, for empirical purposes, the effect of uncertainty over investment expenditure, as well as the uncertainty level, would be decreasing with financial market development. Finally, available financial funds would be a positive function of financial market development.

The paper is structured in four sections, besides this introduction. The principal elements of alternative theoretical approaches for investment modeling are presented in the second section. In the third section, which is divided in three parts, the theoretical model of investment expenditure is developed. In the first part the concept behind Tobin's q investment model is shown. This concept is applied in the second part, in which the proposed investment model under uncertainty is formally developed. In the final part of the section, we analyze the effect on investment of changes in uncertainty, as well as, other economic variables. Additionally, the model is generalized adding a budget constraint, which makes investment determined by the more restricted breach: financial resources or the rate of profitability. The fourth section incorporates financial sector deepness as a determinant of investment. The analysis of the influence's scope of financial development, drives us to reformulate the theoretical model. The summary and conclusions of the paper are included in the fifth and final section, providing a framework for an empirical analysis, natural next step of this paper.

2 Modeling Investment Decisions Under Uncertainty

The principal elements behind stochastic control, critical prices and option approaches for modeling investment under uncertain economic conditions are developed in this section. The purpose of this section is to visualize different alternatives for modeling investment, recognizing the advantages and disadvantages of each one. Critical prices approach determines the conditions over prices that a representative firm looks in order to make decisions about investing or disinvesting in a particular market. Dynamic programming modeling, on the other hand, looks for the optimal investment path over time, in order to maximize the value of the firm, equivalent to shareholders' wealth. Finally, investment decision may be seen as an option, that is exercised when investors estimate is the right moment. Considering partial reversibility of capital, an option to re-sell capital appears. The options to invest or disinvest, as well as other options that could arise, must be valued when the decision to invest is made.

2.1 Critical Prices (S,s Models)

Using equilibrium or non-arbitrage conditions, critical prices or S,s models search for the level that a trigger variable must reach, in order to, an economic agent, make a decision to enter or to exit from a particular market.

This approach is developed by Dixit (1989) for non-financial investment. He finds the conditions over prices, that a firm should evaluate in a market with uncertain demand. Dixit assumes that uncertainty is captured by the final product price, which has a stochastic component, represented by z in the next equation.

$$dP = P(\nu dt + \sigma dz),$$

where ν is the expected growth rate of the price and dz is a Wiener process[5].

[5] A Wiener process or Brownian motion is represented by the next expression:

$$dz = \xi\sqrt{dt}$$

where ξ is a normal random variable, with zero mean and unit variance.

In order to make their decisions, firm's managers evaluate the value of the firm with and without investing. Let $V_0\ (P)$ be the market value of the firm when managers decide to not invest and the final product price is P, and $V_1(P)$ the value when they decide to invest. Using equilibrium arguments, when the managers choose the first alternative, the return of firm's assets must be equal to the expected capital gain. On the other hand, when managers decide to invest, we add to the expected capital gains, the operational profit (π) due to new investment. Therefore:

$$(1/dt)E[dV_0] = \mu V_0,$$
$$(1/dt)E[dV_1] + \pi = \mu V_1,$$

where $(1/dt)$ is the Itô's operator and μ is the market return.

From above expressions we get two differential equations, whose solutions are functions $V_0(P)$ and $V_1(P)$, respectively.

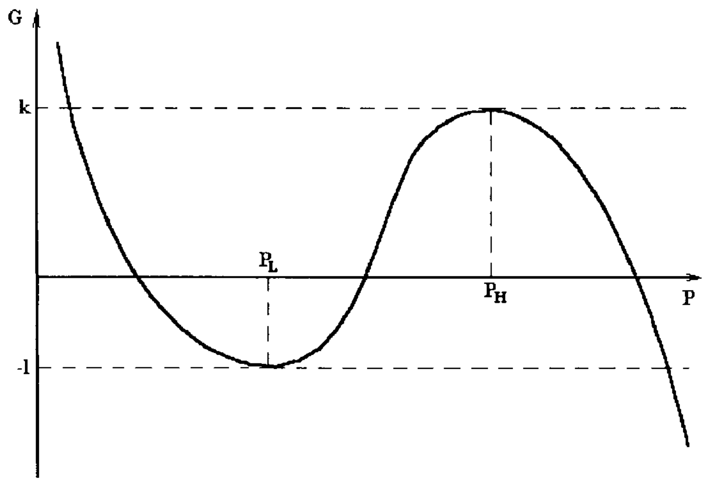
If we denominate k to the payment required to exercise the investment option and l to the disinvesting cost, we obtain the critical values P_H and P_L. These values represent, respectively, the price that the final product should reach in order to make the decision to invest and the price that the product should reach to disinvest. That means, if $P>P_H$ the firm invests in new capital. If $P<P_L$, the firm decides to disinvest. Finally, if the price of the final product remains between P_H and P_L, the firm does not change its current stock of capital.

$$V_0(P_H) = V_1(P_H) - k,$$
$$V_1(P_L) = V_0(P_L) - l.$$

Taking border conditions, we obtain an analytical expression for the critical prices. These prices, as Dixit (1989) shows, are positive functions of uncertainty: as σ^2 increases, P_H and P_L increase. Therefore, higher uncertainty determines less investment.

Defining $G(P)=V_1(P)-V_0(P)$, we have that graphically, from figure 1, we can find the theoretical values of P_H and P_L. We just look for the tangent points of G. Additionally, we have that as k increases, P_L decreases and P_H increases. Similarly, as l increases, P_L decreases and P_H increases. That means, investors become more demanding respect to prices, which reduces total investment.

Figure 1 Finding Critical Prices



Abel and Eberly (1994,1995) present a framework based on Tobin's q theory, in which, as previous analysis, the optimal investment strategy is a two trigger policy. If q, a measure of profitability of investment exceeds a certain upper threshold, the firm exhibits a positive gross investment. If q is below a lower threshold, negative gross investment occurs. When q is between upper and lower threshold, investment is equal to zero.

2.2 Stochastic Control Formulation

For stochastic control models of investment, as opposed to the critical prices approach, we consider an objective function to be maximized by the managers of a representative firm. Assuming rationality in managers' decisions, their objective is to maximize the value of the firm, given by the present value of profits. The formulation is illustrated by considering that uncertainty is associated to the demand faced by the firm.

In the next equation, we express the profit function of a representative firm, in which we subtract from the total revenue, the labor and investment expenditures.

$$\pi(s) = p(x,\theta)x - \omega L - C(I),$$

where π is the price of the final product, x the quantity produced, θ the stochastic component of the price, L is labor, ω is its wage and $C(I)$ a cost function for investment, in which I is the investment expenditure.

Similarly to the critical prices approach, the price may be modeled assuming an expected natural growth and, if the firm has some monopoly power, as a function of the quantity produced by the firm. The production function, which considers the stock of capital K and labor L as the input variables, is assumed to be strictly concave in order to obtain necessary and sufficient conditions from the first order optimality conditions of the problem[6].

Assuming risk neutrality, the optimization problem solved in t is expressed in the following equations, where labor and investment are the decision variables. The firm's value is equal to the present valuation of expected profits, considering a constant interest rate r. The objective function is maximized subject to capital stock's dynamic, which expresses the change of the capital over t as the investment minus the depreciated capital, considering δ as the depreciation rate.

$$J(K,\theta) = Max \int_{t}^{\infty} \pi(s) e^{-rs} ds$$

$$s.t.$$

$$\dot{K} = I - \delta K$$

The above stochastic control problem can be represented by Bellman's equation, in which we have applied Itô's lemma[7].

$$rJ_t = \underset{I}{Max}\left\{ \pi_t + J_K (I - \delta K) + \frac{1}{2}\sigma^2 J_{\theta\theta} \right\}.$$

The first order optimal condition is:

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[6] That means, $F_K, F_L > 0$, $F_{KK}, F_{LL} < 0$ and $F_{KK}F_{LL}\text{-}F_{KL}^2 > 0$, where F is the production function.

[7] Consider the next Itô's process:

$$dx(t) = a(x,t)dt + b(x,t)dz,$$

where z is a standard Wiener process. Suppose also that the process $y(t)$ is defined by $y(t)=F(x,t)$. Then $y(t)$ satisfies the Itô's equation,

$$dy(t) = \left(\frac{\partial F}{\partial x} a + \frac{\partial F}{\partial t} + \frac{1}{2} \frac{\partial^2 F}{\partial x^2} b^2 \right) dt + \frac{\partial F}{\partial x} b dz,$$

where z is the same Wiener process defined in the first equation.

$$J_K = -C'(I).$$

Replacing the above condition in Bellman's equation and solving the resulting differential equation, we would be able to find the optimal path for investment.

Recognizing a high degree of irreversibility in investment decisions, some authors have restricted the modeling to only positive values of I (Arrow, 1968, Pindyck, 1988, 1991, Bernanke 1985). This constraint is equivalent to assume C equal to infinite for negative values of investment. Under this condition, researchers have found that, even for risk neutral agents, uncertainty affects negatively investment levels.

Caballero (1991) shows that the presence of asymmetric and convex adjustment costs, is not sufficient to render a negative relationship between investment and uncertainty. He demonstrates that a degree of imperfect competition is also required, arguing that today's investment decisions depend almost exclusively on the price of capital and the expected profitability of capital. In this case, the convexity of marginal profitability of capital respect to prices is the principal factor for determining the relationship's sign between uncertainty and investment. When we consider asymmetric adjustment costs, having a capital stock over optimal level is worst than having "too little" capital since increasing the stock of capital is cheaper than decreasing it. Additionally, when imperfect competition is introduced, the marginal profitability of capital is significantly affected by the level of capital. An increase in investment today makes more likely for the firm to find its second-period capital "too large," compared to the desired capital stock.

2.3 Option Approach of Investment

When a firm can not adjust its capital stock without incurring in costs, it must consider future opportunities when it makes investment decisions. As it was mentioned in previous section, the literature has considered two alternative ways to interpret this problem. The irreversible investment approach, initiated with Arrow (1968), uses option pricing techniques to derive and characterize optimal investment behavior. The option interpretation of investment has been emphasized by Bertola (1988), Pindyck (1988,1991) and Dixit (1991,1992). It considers that companies have opportunities to invest and they must decide how to exploit those opportunities, making an important analogy with financial options: a company with an opportunity to invest is holding something similar to a financial call option. The company has the right, but not the obligation, to buy an asset at a future time. When the company makes an irreversible investment, it exercises its option. The lost option is an opportunity cost that should be incorporated in the investment decision. As the volatility of possible income and cost increase, the value of the option increases, making costly the decision to invest. Considering partial reversibility of capital, when the firm invests, it acquires a put option since the capital may be later resell. These two options affect the current incentive to invest.

As long as there are some contingencies under which the company would prefer not to invest, that is, when there is some probability to delay the decision, and thus to keep the option alive, has value. The question is then, when to exercise the option. The choice of the most appropriate time is the essence of the optimal investment decision.

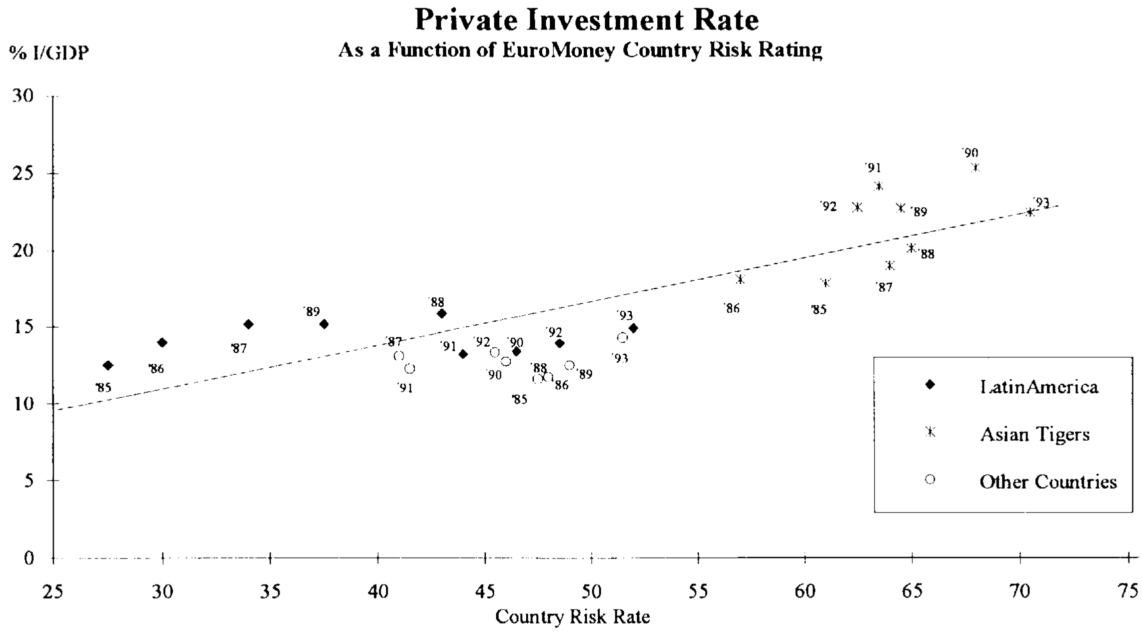
The next equation expresses the benefits of investment (*BI*) that has been separated in the naive net present value (*NPV*) and the value of call and put real options.

$$BI = NPV - V_{Call\ Option} + V_{Put\ Option}.$$

In q-theory approach (Tobin, 1969), the firm faces convex adjustment costs and, along the optimal path, the marginal valuation of a unit of capital, measured by q, is equal to the marginal cost of investment. Abel, Dixit, Eberly and Pindyck (1996), link the q-theory and option pricing approaches. They show how the incentive to invest, summarized by q, can be decomposed into the returns to existing capital, ignoring the possibility of future investment and disinvestment, and the marginal value of the options to invest and disinvest. The option to invest (call option) arises from the expandability of the capital stock, while the option to disinvest (the put option) arises from the reversibility of investment. They conclude that the effect of uncertainty is ambiguous since, as uncertainty increases, call and put option values increase. Higher value of call option increases the incentive for postponing investment, in order to wait for more information. On the other hand, a more expensive put option increases benefits of investment.

3 Theoretical Model of Investment

Empirical evidence from developing countries shows, as our intuition suggests, that there is a negative relationship between investment and uncertainty. Next graph shows investment rate and country risk rate in different groups of countries[8]. Since other factors are not included, it does not show a causality between the variables. Beside each point has been written the corresponding year of data. We observe that "Asian Tigers"[9] have had very high private investment rates, always coming with, relatively, high country risk rates. Latin American countries[10], on the other hand, have shown a small increase in private investment rate, which has been supported by improves in country risk evaluations.



Investment rate behavior of "other countries" is not conclusive. Just for the last four years considered in the sample, we could observe a positive relationship between investment and country risk rate. This particular situation could be explained by having considered a too heterogeneous group[11]. Nevertheless, looking all the context, low investment rates of these

[8] Euromoney and The World Bank.
[9] Indonesia, South Korea, Malaysia, Thailand and Philippines.
[10] Brazil, Chile, Colombia, Mexico, Peru and Venezuela.
[11] Egypt, India, Kenya, South Africa, Syria, Turkey and Tunisia.

countries could be explained, among other factors, by low country risk rates. In appendix A we present the data used to construct the graph.

The theoretical model should be able to capture the observed relationship between uncertainty and investment. For this purpose, we consider an optimal control approach, based on Tobin's q theory. In the first section, the main concepts behind Tobin's theory are briefly described. Then, the theoretical model is developed, which, making some assumptions, allows us to obtain an analytical expression for the investment rate as a function of, among other variables, uncertainty in market conditions.

3.1 Tobin's q Theory

Tobin (1969) developed a fixed capital investment model, which considers that the desired capital stock is reached along time due to the existence of adjustment costs. As a consequence, the short run cost of capital differs from the long run price of capital, equivalent to the market value of the firm. According to this approach, investment is determined by the rate between the market value of the firm (V) and the cost of capital $(p_K K)$, equivalent to the accountable value of the firm. This quotient, q, represents the profitability rate of investment.

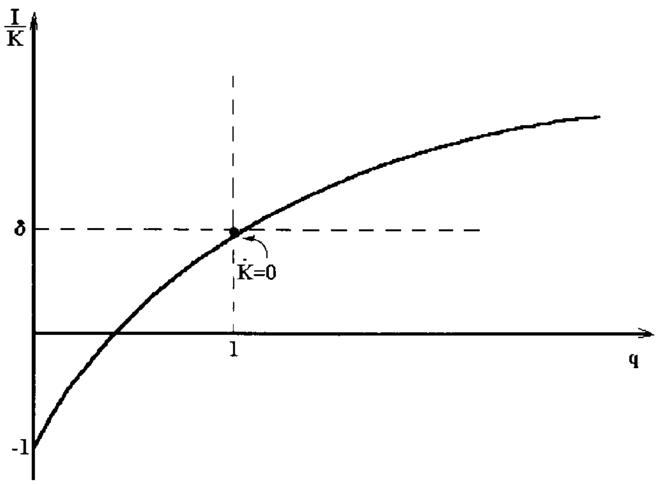
$$q = \frac{V}{P_K K}.$$

According to above definition, a higher q implies more incentives for investing. Only when q is higher than one, we should observe capital accumulation $(I > \delta K)$. Figure 2 illustrates the above condition, showing that for $q=1$, I/K is equal to the depreciation rate δ.

Abel (1980) makes important contributions to q theory, showing that marginal q, instead of average q suggested by Tobin, is the determinant of investment expenditure. That is, the marginal contribution of capital to the firm's value is the relevant variable that firms take into account in order to make their investment decisions.

For empirical purposes, however, it is important to note that marginal q is a non-observable variable, as a difference with average q, which can be measured by the market valuation (stock prices) and the accountable value of the firm. Hayashi (1982) demonstrates that exists a linear relationship between marginal and average q, when production and cost functions satisfy the following conditions[12].

Figure 2 Investment over Capital



(a) The production function is concave, twice differentiable and has constant returns of scale.

(b) The cost function is homogeneous of degree one with respect to investment and capital.

Tax structure has been incorporated in Tobin's q formulation. Models in that direction show that long term or equilibrium q (equal to one in a non-tax economy) changes, modifying the incentives for investing. Summers (1981) obtain that an increase in corporate income tax determines a decrease in long term q and, as a consequence, a smaller desired stock of capital. Additionally, an announced increase in dividend tax does not have a long term effect over investment. For Chilean economy, Lehmann (1991) develops a theoretical and empirical model, finding that only corporate taxes would have an effect over managers' investment decisions.

[12] An extension to this demonstration is presented in appendix C.

3.2 Formulation of the Model

In this section, we formulate the investment model, incorporating uncertainty in market conditions. In the first part, a stochastic control problem is developed looking from a microeconomic perspective, which makes a link with Tobin's q theory. As a first stage, a simple model, without considering budget constraints and financial market depth, is developed. Then, the model is generalized through the inclusion of debt, as an alternative to finance investment. A more elaborated version includes financial market development as a determinant of the level and effect of uncertainty about market conditions.

For the formulation of the model we consider a representative firm, whose managers are risk-neutral and maximize the value of the firm.

Consider a firm that faces an isoelastic demand curve given by equation (1).

$$(1) \qquad Q_s = \tilde{X}_s P_s^{-\varepsilon},$$

where Q_s is the quantity of output demanded, P_s is the price of output, X_s is a stochastic demand shock, and $\varepsilon>1$ is the price elasticity of demand.

The firm produces a non storable output Q_s according to Cobb-Douglas production function, with a scale factor φ.

$$(2) \qquad Q_s = \left(L_s^{1-\beta} K_s^{\beta}\right)^{\varphi},$$

where L_s is labor, K_s is the capital stock. Capital share satisfies $0<\beta<1$.

We obtain that the firm's income is determined by equation (3), which is concave, twice differentiable and homogeneous of degree φ on K and L.

$$(3) \qquad G(K_s, L_s, \tilde{Z}_s) = K_s^{\alpha_1} L_s^{\alpha_2} \tilde{Z}_s,$$

where $\alpha_1=\varphi\beta(1-1/\varepsilon)$, $\alpha_2=\varphi(1-\beta)(1-1/\varepsilon)$, and Z_s is an index that represents market conditions, that, as a function of demand, can be written as:

$$\tilde{Z}_s = \tilde{X}_s^{1/\varepsilon} \tag{4}$$

We assume that market conditions follow a stochastic process, which is represented by equation (5), a Brownian Motion.

$$d\tilde{Z}_s = \tilde{Z}_s \mu ds + \tilde{Z}_s \sigma dW_s, \tag{5}$$

where μ is the rate of growth of Z_S and dW_S is a Wiener Process: $dW_s = \varepsilon\sqrt{ds}$; ε standard normal random variable.

Applying Itô's lemma and defining $\nu = \mu - \frac{1}{2}\sigma^2$, above expression determines that the expected value and the variance of Z_S, given the information available at t, for $s>t$, are[13]:

$$E\left[\ln\left(\tilde{Z}_s/Z_t\right)\right] = \nu(s-t) \quad ; \quad E\left[\tilde{Z}_s\right] = Z_t e^{\mu(s-t)} \ , \tag{6}$$

$$Var\left[\ln\left(\tilde{Z}_s/Z_t\right)\right] = \sigma^2(s-t) \quad ; \quad Var\left[\tilde{Z}_s\right] = Z_t^2 e^{2\mu(s-t)}\left(e^{\sigma^2(s-t)} - 1\right), \tag{7}$$

Looking from a microeconomic perspective, μ is a function of the entrance and exit barriers to the market: low barriers imply that μ is a decreasing function of market conditions improvement, since more incentives to other firms for entering to the market appear. The existence of a non competitive market, as we have assumed, allows us to consider μ as an exogenous variable. From a macroeconomic point of view[14], on the other hand, μ is highly correlated with economic growth and application of responsible economic policies.

Figure 3 represents the behavior of market conditions as a function of time. From equations (6) and (7), we have that the expected value and the variance of market conditions are increasing on time. In any case, however, we assume that μ is not affected by investment decisions.

About labor costs, we assume that the salary is exogenous and certain, equal to ω. Additionally, we assume that labor force, L, is flexible, which implies that the marginal revenue of capital as a function of Z, is convex.

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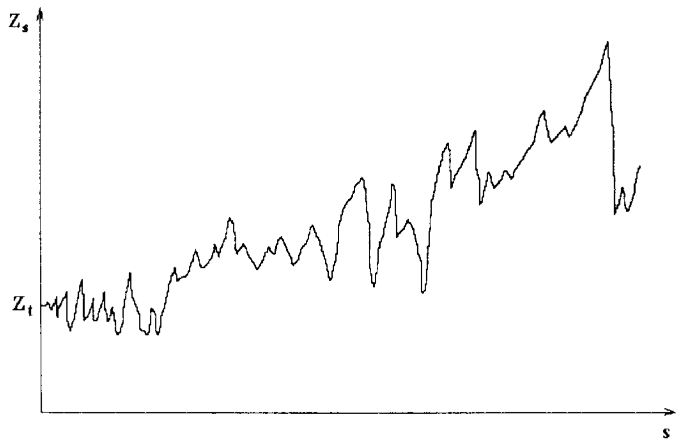
[13] Note that $d\ln\tilde{Z}_s = \nu ds + \sigma dW_s$.

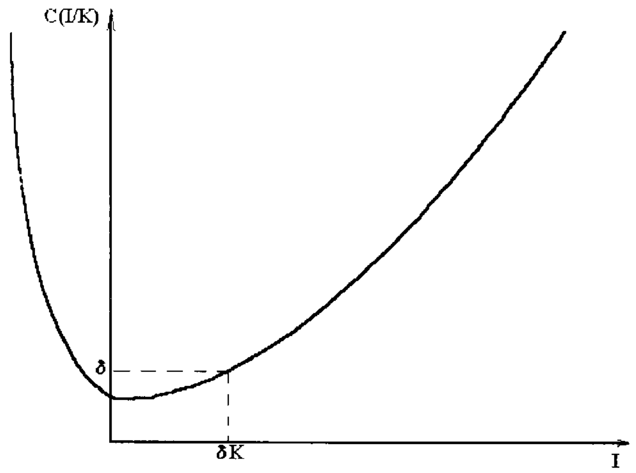
[14] Obtained by the aggregation of the firms.

Figure 3 Market Conditions



On the other hand, in the investment process we consider the existence of adjustment costs represented by function *C(I/K)*, where *I* represents the amount of investment and *K* the stock of capital. This function includes direct cost of investment.

Figure 4 Adjustment Cost Function



Adjustment costs are consequence of the existence of capital installation costs, training of workers, etc. In case of disinvetsment, difficult of buyers to evaluate quality of capital creates an adjustment cost. Indeed, buyers will pay the average quality of the market. Sellers who know the quality of the item will resist loading above average equipments at a reduced price. Therefore, quality and market price of used equipments will go down[15].

The adjustment cost function, C, is a convex function; then $C''>0$. Additionally, assuming that δ represents the depreciation rate of capital, function C verifies that $C(\delta)=\delta$ and $C'(\delta)=1$, meaning that there is no adjustment costs when investment is equal to the depreciated capital and, therefore, K remains constant[16].

Figure 4 represents an adjustment cost function. Note that, when investment is equal to zero the associated cost is non-zero, because the firm's capital stock is changing. Additionally, since capital is partial reversible, the convexity of C is higher when $I<0$.

The adjustment cost function depend on particular conditions of the economy, or, looking from an economic sector perspective, technological or specific peculiarities of the firm or the sector. Especially in developing countries, restrictions on capital flows could constitute an important element in adjustment costs: friction on capital movements increase irreversibility of foreign investment.

On the other hand, quality of labor, different from country to country and a function economic development, is important for the convexity of cost adjustment function. As quality of labor is higher, investors expend lower funds in training.

3.2.1 The Optimization Problem

We stated that managers maximize the market value of the firm, which is equal to the present value of the profits, considering K and L as the decision variables. The profits of the firm are equal to the income $G(K,L,Z)$ minus investment and labor costs. Therefore, the optimization problem is given by:

---

[15] "The market for 'lemons': Quality and the market mechanism", Akerlof (1970).

[16] Convexity of adjustment costs and concavity of production function allow us to generalize Hayashi's prove of existence of a linear relationship between marginal and average q for an optimization problem with uncertainty. For more details, see appendix C.

(8) $$V(K_t, Z_t) = \underset{I,L}{Max}\, E_t \int_t^{\infty} \left[K_s^{\alpha_1} L_s^{\alpha_2} \tilde{Z}_s - C(I_s / K_s) K_s - \omega_s L_s \right] e^{-r(s-t)} ds$$

s.t.

(9) $$\dot{K} = I - \delta K.$$

Equation (9) expresses the dynamic behavior of the stock of capital, given that in period s, investment is equal to I_S and capital depreciation δK.

Using Bellman's equation, which is equivalent to equation (10) but in a differential form, we obtain the first order optimal conditions of the stochastic control problem.

(10) $$rV_t dt = \underset{I,L}{Max}\left\{ \left[K_t^{\alpha_1} L_t^{\alpha_2} \tilde{Z}_t - C(I_s / K_s) K_s - \omega_t L_t \right] dt + E_t\{dV_t\} \right\}.$$

Since the state variable Z is a stochastic variable, Itô's lemma must be used in order to calculate dV. For this purpose, K and Z are the state variables:

(11) $$dV = V_K dK + V_Z dZ + \frac{1}{2} V_{ZZ} dZ^2.$$

According to the evolution of K, expressed in equation (9), we obtain that the expected change in firm's value in a dt interval is[17]:

(12) $$E_t\{dV\} = \left[V_K (I_t - \delta K_t) + V_Z Z_t \mu + \frac{1}{2} V_{ZZ} Z_t^2 \sigma^2 \right] dt.$$

Replacing above expression in equation (10), we get the next fundamental equation of optimality (for more clarity in the presentations, subindex t is eliminated).

(13) $$rV = \underset{I,L}{Max}\left\{ K^{\alpha_1} L^{\alpha_2} Z - \omega L - C(I/K)K + V_K (I - \delta K) + V_Z Z \mu + \frac{1}{2} V_{ZZ} Z^2 \sigma^2 \right\}.$$

---

[17] From (6) and (7), we have that $E\left[V_Z dZ\right] = V_Z Z \mu dt$ and $E\left[V_{ZZ} dZ^2\right] = V_{ZZ} Z^2 \sigma^2 dt$.

Above expression, equivalent to equations (8) and (9), is maximized with respect to investment expenditure and labor, the decision variables of the problem. Taking the derivative of the right side of equation (13) with respect to I and L, the following optimal conditions are obtained[18].

(14) $$L = \left(\frac{\omega K^{-\alpha_1}}{\alpha_2 Z} \right)^{1/(\alpha_2 - 1)},$$

(15) $$V_K = C'(I/K).$$

Replacing equation (15) in (13) we obtain an expression for the firm's value, which still does not consider optimal managers' decisions.

(16) $$rV = K^{\alpha_1} L^{\alpha_2} Z - \omega L - C(I/K)K + C'(I/K)I - V_K \delta K + V_Z Z \mu + \frac{1}{2} V_{ZZ} Z^2 \sigma^2.$$

Taking to above expression the first derivative with respect to K, we get equation (17).

(17) $$(r+\delta)V_K = \alpha_1 Z L^{\alpha_2} K^{\alpha_1 - 1} - C(I/K)K + C'(I/K)\frac{I}{K} - C''(I/K)\frac{I^2}{K^2} - V_{KK}\delta K + V_{KZ} Z \mu + \frac{1}{2} V_{KZZ} Z^2 \sigma^2.$$

Above expression represents the dynamic behavior of V_K, which corresponds to the marginal contribution of capital to the firm's value.

In equation (17), partial derivatives are unknown[19]. Therefore, the next step is to solve the differential equation arising from that equation. In order to obtain an analytical solution, a specific adjustment cost function is proposed.

(18) $$C(I/K) = \chi_{[I>0]} \psi_1 \left(\frac{I}{K} \right)^{\gamma} + \chi_{[I<0]} \psi_2 \left(\left| \frac{I}{K} \right| \right)^{\gamma} + \xi,$$

where, $\gamma > 1$ and χ is a binomial function[20]:

[18] We have assumed that I and L are different from zero.
[19] Note that equation (11) does not give the optimal path for V_K.
[20] As a consequence, if $I=0$, the adjustment cost is $C(I/K)=\xi$.

$$\chi_{[\Im]} = \begin{cases} 1 & \text{if condition } \Im \text{ is satisfied} \\ 0 & \text{o. w.} \end{cases}$$

As we mentioned, the adjustment cost is influenced by particular conditions of the economy. As a consequence γ and ψ_i, $i=1,2$, are function of economic development and economic policies affecting capital movements fluency.

In order to verify the requirements on C, which were established as problem's assumptions, the parameters of the proposed adjustment cost function must verify:

(19) $$\psi_1 = \gamma^{-1}\delta^{1-\gamma} \quad \text{and} \quad \xi = \delta(1-1/\gamma).$$

Additionally, recognizing a partial reversibility of investment, ψ_2 must be higher than ψ_1. The relative difference between those parameters defines the reversibility degree of capital: ψ_2 goes to infinite implies that investment is irreversible. $\psi_2/\psi_1=1$ states that investment and disinvestment decisions face equivalent costs.

Considering the particular adjustment cost function, the differential equation to be solved is:

(20) $$(r+\delta)V_K = \alpha_1 Z L^{\alpha_2} K^{\alpha_1 - 1} - (\gamma-1)^2 \psi_I \left|\frac{I}{K}\right|^{\gamma} - \xi - V_{KK}\delta K + V_{KZ} Z\mu + \frac{1}{2} V_{KZZ} Z^2 \sigma^2$$

In above equation, the value of parameter ψ_I depends on the sign of investment. That means:

(21) $$\psi_I = \begin{cases} \psi_1 & if \quad I>0 \\ \psi_2 & if \quad I<0 \\ 0 & if \quad I=0 \end{cases}$$

The solution of differential equation (20) is expressed in (22), where θ is positive parameter, λ is a parameter such that $\lambda > r+\delta$ and A is a positive function defined in equation (23)[21].

---

[21] For details, see appendix *B*.

(22) $$V_K = \frac{\alpha_1 Z}{r+\alpha_1\delta} L^{\alpha_2} K^{1-\alpha_1} - \frac{(\gamma-1)^2 \psi_I}{\delta(\gamma-1)-r}\left(\frac{I}{K}\right)^{\gamma} - \frac{\xi}{r+\delta} - \theta Z^{-A} K^{\frac{\lambda-r-\delta}{\delta}},$$

(23) $$A = \frac{(\mu-\sigma^2/2)+\sqrt{(\mu-\sigma^2/2)^2+2\lambda\sigma^2}}{\sigma^2}$$

We can verify that A is a decreasing function of σ^2. The limit values of A, when the variance of market conditions goes to extremes, are:

$$\lim_{\sigma^2 \to \infty} A = 0 \qquad \text{and} \qquad \lim_{\sigma^2 \to 0} A = \infty.$$

From the expression of V_K (equation 22), the partial derivatives with respect to Z are evaluated[22]. On the other hand, assuming non-zero investment expenditure, an expression for V_{KK} is obtained from the optimal condition (15).

(24) $$V_{KK} = -\psi\gamma(\gamma-1)\frac{I^{\gamma-1}}{K^{\gamma}},$$

(25) $$V_{KZ} = \theta A Z^{(A+1)} K^{(\lambda-r-\delta)/\delta} + \frac{\alpha_1}{r+\alpha_1\delta-\mu} L^{\alpha_2} K^{\alpha_1-1},$$

(26) $$V_{KZZ} = -\theta A(A+1) Z^{-(A+2)} K^{(\lambda-r-\delta)/\delta}$$

Replacing above expressions and the optimal value of L (equation 14) in equation (20), we obtain the expression (27) for the optimal path of V_K, which is called q in reference to Tobin's q.

(27) $$q = \frac{1}{r+\delta}\left\{\begin{array}{l} \alpha_1\left(\frac{\alpha_2}{\omega}\right)^{\frac{\alpha_2}{1-\alpha_2}} Z^{\frac{1}{1-\alpha_2}} K^{\frac{(\alpha_1+\alpha_2)-1}{1-\alpha_2}} + \psi_I(\gamma-1)\left(\frac{I}{K}\right)^{\gamma} - \xi - \psi_I\gamma(\gamma-1)\left(\frac{I}{K}-\delta\right)\left(\frac{I}{K}\right)^{\gamma-1} + \\ + \frac{\alpha_1\mu Z^{\frac{1}{1-\alpha_2}}}{r+\alpha_1\delta-\mu}\left(\frac{\alpha_2}{\omega}\right)^{\frac{\alpha_2}{1-\alpha_2}} K^{\frac{(\alpha_1+\alpha_2)-1}{1-\alpha_2}} + \mu\theta A Z^{-A} K^{\frac{\lambda-r-\delta}{\delta}} - \frac{1}{2}\theta A(A+1) Z^{-A}\sigma^2 K^{\frac{\lambda-r-\delta}{\delta}} \end{array}\right\}$$

---

[22] Considering that Z is not a decision variable, this calculation, made from a non-optimal path of V_K, is allowed.

3.2.2 Adding a Budget Constraint

In order to incorporate budget constraints, we consider that the firm maintains a fraction b of their capital financed with debt, to which we associate an interest rate r^d. We assume that the total firm's debt can not be higher than a fixed amount B_S.

Therefore, the value of the firm is:

(28) $$V_t = E\int_t^{\infty}\left[K_s^{\alpha_1} L_s^{\alpha_2} \tilde{Z}_s - C(I_s / K_s)K_s - \omega_s L_s - r^d bK_s\right]e^{-r(s-t)}ds.$$

To the optimization problem, we add the next budget constraint:

(29) $$bK_s \leq B_s.$$

Assuming that the available financial resources grow at rate η, we obtain the next expression.

(30) $$rV = K^{\alpha_1} L^{\alpha_2} Z - \omega L - r^d bK - C(I/K)K - V_K(I - \delta K) + V_B B\eta + V_Z Z\nu + \frac{1}{2}V_{ZZ}Z^2\sigma^2.$$

In order to solve this differential equation, we distinguish to cases: financial resources are a non-active or an active constraint. Under the first condition, behavior of q could be expressed by equation (27), adding the associated interest rate payments associated due to firm's debt. This result is consequence of considering $V_B=0$ in expression (30) and marginal contribution of B to q equal to zero. When financial resources become a constraint, on the other hand, we should solve differential equation (30).

An alternative way to obtain the optimal path of investment when budget is a constraint, considers that investment would grow according to availability of financial funds. Then, we can express optimal investment expenditure, according to the next function, which combines the optimal condition (15) and the maximum investment expenditure when financial fund is a limiting resource[23].

(31)
$$\frac{I}{K}=\begin{cases} C'^{-1}(V_K) & if\, V_B=0 \\ \dfrac{\eta B}{bK} & o.w. \end{cases}$$

For empirical purposes, the above condition can be written as a two breach model[24]:

(32)
$$\frac{I}{K}=Min\left\{C'^{-1}(q),\frac{\eta B}{bK}\right\}.$$

Expression (32) states that investment is determined by the active constraint, which, varying on time, could be the rate of profitability of investment expenditure, or the availability of financial funds. In the first case, q is determined by expression (33), in which, as a difference with equation (27), interest payments have been incorporated.

(33)
$$q=\frac{1}{r+\delta}\left\{\begin{array}{l} \alpha_1\left(\dfrac{\alpha_2}{\omega}\right)^{\frac{\alpha_2}{1-\alpha_2}}Z^{\frac{1}{1-\alpha_2}}K^{\frac{(\alpha_1+\alpha_2)-1}{1-\alpha_2}}+\psi_I(\gamma-1)\left(\dfrac{I}{K}\right)^{\gamma}-(\xi+r^d b)-\psi_I\gamma(\gamma-1)\left(\dfrac{I}{K}-\delta\right)\left(\dfrac{I}{K}\right)^{\gamma-1}+ \\ +\dfrac{\alpha_1\mu Z^{\frac{1}{1-\alpha_2}}}{r+\alpha_1\delta-\mu}\left(\dfrac{\alpha_2}{\omega}\right)^{\frac{\alpha_2}{1-\alpha_2}}K^{\frac{(\alpha_1+\alpha_2)-1}{1-\alpha_2}}+\mu\theta AZ^{-A}K^{\frac{\lambda-r-\delta}{\delta}}-\dfrac{1}{2}\theta A(A+1)Z^{-A}\sigma^2K^{\frac{\lambda-r-\delta}{\delta}} \end{array}\right\}$$

Note that in above equation γ and θ are positive parameters, associated to a particular firm, economic sector or all economy (depending on the level of analysis). Both parameters, as is analyzed in next section, could change as a consequence of degree of openness of capital account or financial sector development.

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23 Rama (1987), as this approach suggests, develops a two breach model, in order to explain the behavior of investment.

24 Note that $V_K=q$.

3.3 Long Run Equilibrium

In order to analyze the effect of macroeconomic variables and economic agents' beliefs, we determine the equilibrium conditions, which are characterized by the next set of equations.

$$\text{(34)} \qquad \begin{aligned} &E[\dot{q}] = 0 \\ &\dot{K} = 0 \quad \Rightarrow \quad I = \delta K^{*} \;\; and \;\; q^{*} = 1 \end{aligned}$$

where q^* is the long-run q.

Additionally, we add the transversality condition, which insures convergence to the long run equilibrium.

$$\text{(35)} \qquad \lim_{t \to \infty} E_t\left[qKe^{-rt}\right] = 0.$$

Conditions expressed in (34) state that investment is such that the stock of capital remains constant. From first order conditions, that implies that q must be equal to one.

Replacing (12) into (33), we obtain the next expression of q, where it appears as a function of $E[\dot{q}]$.

$$\text{(36)} \qquad q = \frac{1}{r+\delta}\left\{\alpha_1\left(\frac{\alpha_2}{\omega}\right)^{\frac{\alpha_2}{1-\alpha_2}} Z^{\frac{1}{1-\alpha_2}} K^{\frac{(\alpha_1+\alpha_2)-1}{1-\alpha_2}} + \psi_I(\gamma-1)\left(\frac{I}{K}\right)^{\gamma} - (rb+\xi) + E[\dot{q}]\right\}.$$

Therefore, in order to q be in equilibrium condition (37) must be satisfied, in addition to the relationship between q and K arisen from above equation, when we impose $E[\dot{q}]=0$[25].

---

[25] Note that from (33) and (36) we have:

$$E[\dot{q}] = -\psi_I\gamma(\gamma-1)\left(\frac{I}{K}-\delta\right)\left(\frac{I}{K}\right)^{\gamma-1} + \frac{\alpha_1\mu Z^{\frac{1}{1-\alpha_2}}}{r+\alpha_1\delta-\mu}\left(\frac{\alpha_2}{\omega}\right)^{\frac{\alpha_2}{1-\alpha_2}} K^{\frac{(\alpha_1+\alpha_2)-1}{1-\alpha_2}} + \mu\theta A Z^{-A} K^{\frac{\lambda-r-\delta}{\delta}} - \frac{1}{2}\theta A(A+1)Z^{-A}\sigma^2 K^{\frac{\lambda-r-\delta}{\delta}}$$

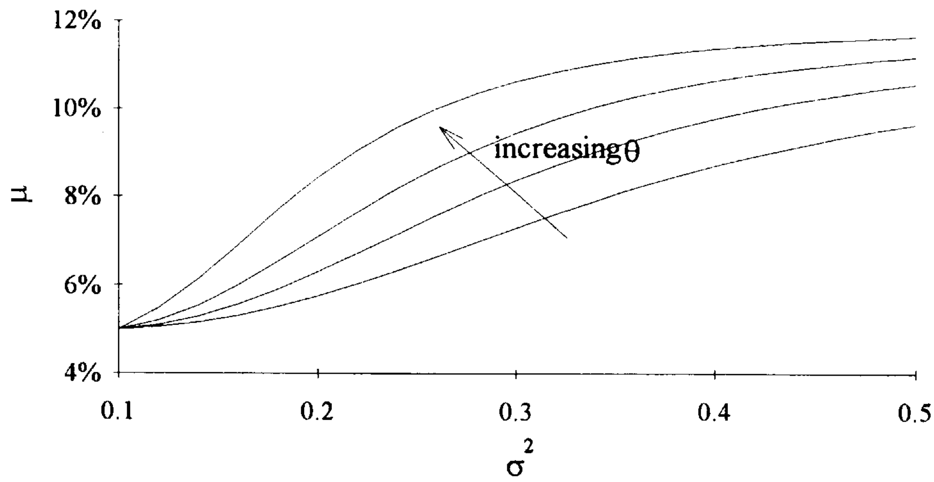
$$(37)\quad -\psi_I\gamma(\gamma-1)\left(\frac{I}{K}-\delta\right)\left(\frac{I}{K}\right)^{\gamma-1}+\frac{\alpha_1\mu Z^{\frac{1}{1-\alpha_2}}}{r+\alpha_1\delta-\mu}\left(\frac{\alpha_2}{\omega}\right)^{\frac{\alpha_2}{1-\alpha_2}}K^{\frac{(\alpha_1+\alpha_2)-1}{1-\alpha_2}}+\mu\theta AZ^{-A}K^{\frac{\lambda-r-\delta}{\delta}}-\frac{1}{2}\theta A(A+1)Z^{-A}\sigma^2K^{\frac{\lambda-r-\delta}{\delta}}=0\cdot$$

That means, we require a minimum expected growth in market conditions in order to be in equilibrium. What is the effect of uncertainty over this required value of μ?

In order to provide an answer, we simulate the required relationship between μ and σ^2, considering different values of θ or characterizations of an economy. For this purpose we assume that initial stock of capital is 10, ψ=400, γ=1.8 and risk free rate 6%. For graphical purposes, σ is normalized. Just the percentage change in σ is relevant for conclusions.

We find that, as uncertainty in market conditions increases, the required growth in market conditions (μ*)is higher. Additionally, for higher values of theta, the sensitivity of μ to uncertainty is greater (see figure 5).

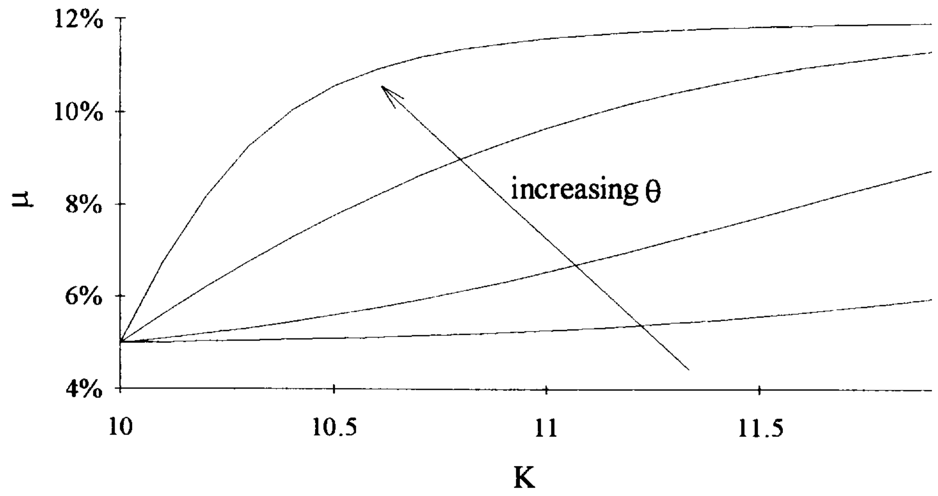
Figure 5 Equilibrium: Required μ as a Function of σ^2



The result got from the simulation is analogous to the classic risk-return trade-off. Indeed, we obtain that an efficient investment is such that, higher expected growth in market conditions — strongly associated to its profitability— compensates the uncertainty faced by the firm. That means, investment increases when we observe an improved risk-return combination.

On the other hand, simulating the required growth in market conditions as a function of the stock of capital, in order to be in equilibrium, we obtain the results presented in figure 6. We observe that as stock of capital increases, the required growth in economic conditions is higher. As parameter theta increases, the sensitivity for low values of K becomes higher. This trend, however, change for higher values of K, where a low theta implies higher sensitivity[26].

Figure 6 Equilibrium: Required μ as a Function of K

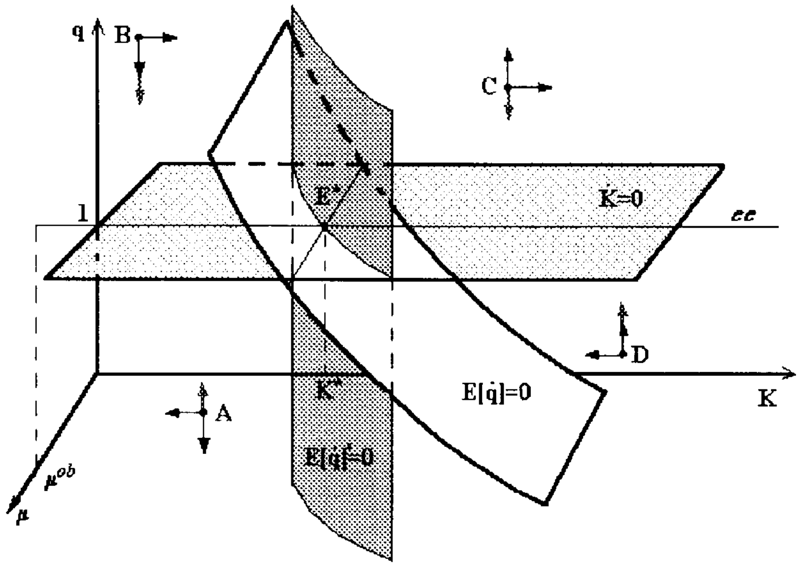


Finally, the requirement on μ, as Z goes up, is lower. That means, for wealthy economies the necessity of economic growing is less strict, in order to maintain a strong investment process, independently of uncertainty perceptions of economic agents.

Figure 7 represents the equilibrium in an instant t. We get that, in addition the relationship between q and capital, a required μ for being in equilibrium is relevant. Therefore, equilibrium's dynamic must be represented in a three dimensional graph. The light gray plane represents the equilibrium in capital stock, conditioned by $q=1$. The white plane represents the relationship between q and K, such that q remains in equilibrium. According to equation (36), as capital stock goes up, q must be lower in order to satisfy $\dot{q}=0$. Otherwise, the firm reaches the *locus* represented by points A or B, whose dynamic moves the firm to line *ee*, as we conclude from the next paragraph.

[26] That means, the curve is convex for high values of θ, and concave for low values of θ. Theoretically, there is a value of θ, for which the equilibrium relationship between μ and K is linear.

Figure 7 Representation of Equilibrium



In addition to equation (36) for q being in equilibrium, we require that μ and K satisfy equation (37), represented by dark gray plane of figure 7. The intersection of planes represents the long run equilibrium, E^* in the figure. Given the current expected growth in market conditions (μ^{ob}), in the long run the firm will be always on line *ee*.

The requirement of two conditions for q being in a long-run equilibrium, could be interpreted, from an analytical perspective, as a separation of $E[\dot{q}]$ into two terms (see equation 36). Each equilibrium plane assumes that the complemented condition is satisfied.

3.3.1 Equilibrium's Dynamic

Note that, since required μ is positive, the market conditions are expected to grow. That means, from equation (36), the white plane is moving to the right, making the equilibrium stock of capital continuously higher. Simultaneously, as Z increases, the required μ for a given K goes down. That means, the vertical plane (dark gray) moves to the right. As a consequence, the equilibrium condition is moving on time, implying a continuos expected growth of the stock of capital. Note that, since changes in Z are uncertain, movements of equilibrium planes, and therefore the changes in the long run equilibrium, are stochastic.

If condition (37) is not satisfied, we move from the equilibrium and, temporarily, we will not be able to reach it. Indeed, if μ becomes under the required growth, the expected change of q on time becomes negative. As a consequence, q starts to decrease, followed by a reduction in investment. Given the equilibrium relationship between μ and K (figure *6*), we will have a decrease in q while the stock of capital, falling from its initial level, has not reached a value, such that the current growth in market conditions, μ^{ob}, is equal to the new required μ. Alternatively, a change in economic conditions, as a decrease in uncertainty or an increase in the expected growth of market conditions, could stop the decrease in q.

Figure 7 expresses with arrows the dynamic of K and q for each section of the graph. It is important to note that in each point we identify two forces over q. In point A, for example, we have that, since we are to the left respect to the white plane, a negative dynamic over q appears. However, K is going down, and then the requirement on μ is decreasing. This phenomenon conducts to a positive dynamic on q, stronger as we move to the left. As a result, at some point, the two opposite forces will annul themselves (necessarily over plane $\dot{K}=0$), reaching a *pseudo-equilibrium* over line *ee*. If the required rate of growth in market conditions is not observed in the economy, the firm will never be able to reach the long term equilibrium E^*.

3.3.2 Determinants of Investment

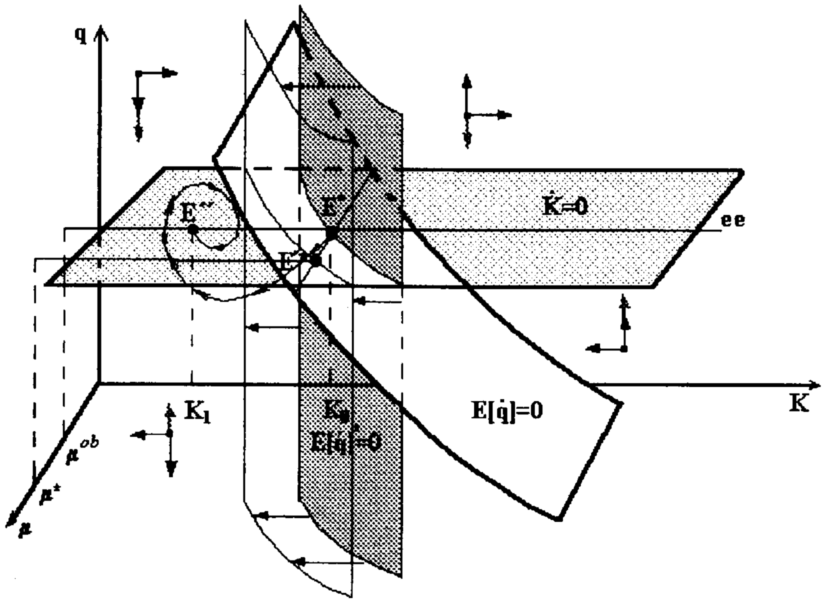
Starting from an equilibrium condition, we analyze the effect of changes in the variables that determine q. Based on those changes, we get the effects over long run equilibrium of capital stock, assuming that financial resources availability is a non-active constraint for investment expenditure. Considering our special interest on the effect of uncertainty, we separate the specific analysis of this variable from the effect of other macroeconomic variables.

(a) Uncertainty

If economic agents perceive higher uncertainty, as we concluded from the simulation that links μ and σ^2, the required economic growth becomes higher. As a consequence, the expected growth of q becomes negative and q starts to decrease, as well as, from equation (32) and assuming that budget is not a constraint, investment[27].

Figure 8 shows a movement of the dark gray plane to the left, carrying the long term equilibrium to $E^{*'}$, in which the required μ is higher.

Figure 8 Increase in Uncertainty



If the expected growth in market conditions remains unchanged, we move from the equilibrium: q becomes less than one and the stock of capital begins to diminish. As a result, the firm falls in the left-bottom area of the graph, where $\dot{K}$ is negative, as well as the dynamic on q. However, as K goes down, the required μ becomes smaller and a force, tending to increase q, appears. This dynamic is opposed to the force that is consequence of being to the left of white plane. At some level of K, this force will be higher to that one driving q in a decreasing path. As a consequence, q increases until K starts to grow, which weakens that force. Then q would re-assumes a decreasing path. After some iterations, a final *pseudo equilibrium*, E'', will be reached. In this point, the forces involved in the dynamic of q are not zero, but they annul each other. As we appreciate in

[27] Indeed, assuming that the firm is initially in or below the long run equilibrium, an increase in required expected growth in market conditions makes the expected change in q to be negative.

figure 8, the capital stock falls from K_0 to K_1, determining a decrease in investment[28]. This effect is a consequence of a decrease of q in the short run. In the long run, q reassumes its equilibrium level, *1*.

It is important to note that the previous analysis, as well as the next examination, is relative to the initial equilibrium. As μ is positive, the market conditions are improving, and then the incentives to invest: We have that the white and dark gray planes are continuously moving to the right.

(b) Other determinants

As was previously mentioned, if the market conditions improve (Z increases), the rate of profitability of new investment increases. As a consequence, considering a positive required expected growth in market conditions, we should have an expected positive movement of white plane. Additionally, since in that case the requirements on μ becomes smaller as Z increases, gray plane moves to the right. Therefore, the equilibrium stock of capital becomes higher, as well as investment in the short and long run.

From equation (37), we conclude that, in addition to direct consequences of better market conditions, the effect of uncertainty over investment decreases. That means, even no considering financial market development, we have a decrease in the effect of uncertainty as a consequence of better market conditions.

On the other hand, an increase in the interest rate determines lower investment expenditures. Indeed, equilibrium planes of q, $E[\dot{q}] = 0$ and $E[\dot{q}]' = 0$, move to the left. The first one, as a consequence of a less rate of profitability of investment. The second one, because the requirement on μ to be in equilibrium increases. As q and K fall down, the requirement over the rate of growth in market conditions decreases. Then, a positive force, tending to increase q, appears in opposition to the decreasing dynamic of q. At a certain point, this force prevails, determining an increase in q. As in the previous case, again, an iteration process is observed until the firm converge to *pseudo-equilibrium* E'' (see figure 9), that is to the right respect to the new long-run equilibrium ($E^{*'}$)[29]. That means, the existence of uncertainty in the economy smoothen the effect of increasing interest rates, assuming that the expected rate of growth of the economy does not change in the short run.

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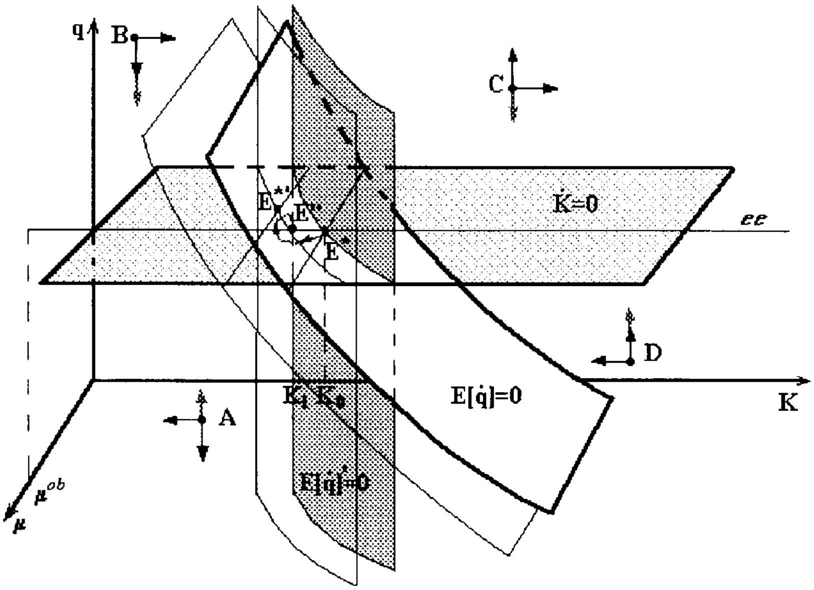
[28] We have that K_1 is such that the current increase in market conditions annuls the two opposite forces on q.

[29] Note that for this new equilibrium, since K has decreased, μ^* is lower than the initial required μ.

Figure 9 Increase in Interest Rate



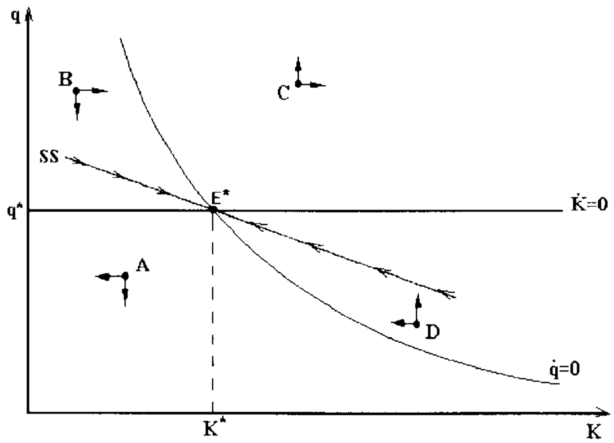
In the problem's presentation, we noted that as quality of labor increases or restrictions in capital flow are relaxed (especially important phenomenon in developing countries), the cost adjustment of investment diminishes. As a result, γ and ψ_I go down, and the negative effect of K over q is reduced. In other words, higher reversibility of investment increase willingness of investors to increase the stock of capital. Graphically, the white plane moves to the right, increasing short run q and, as a consequence, investment.

3.4 Certain World: A Particular Case

In order to evaluate optimal investment for a certain world, we replace $\sigma^2=0$ in expression (28). That implies that q is explained by the next equation, in which we do not use expectations over variables.

(38) $$q = \frac{1}{r+\delta}\left\{\alpha_1\left(\frac{\alpha_2}{\omega}\right)^{\frac{\alpha_2}{1-\alpha_2}} Z^{\frac{1}{1-\alpha_2}} K^{\frac{(\alpha_1+\alpha_2)-1}{1-\alpha_2}} + \psi_I(\gamma-1)\left(\frac{I}{K}\right)^{\gamma} - (rb+\xi) + \dot{q}\right\}.$$

Figure 10 Equilibrium in a Certain World



From equation (37), which is evaluated considering no uncertainty, the requirement of a positive economic growth to be in equilibrium disappears. Therefore, the graphical equilibrium condition becomes represented by a two dimensional graph (figure 10), in which, as a difference with previous versions, the equilibrium capital stock remains unchanged on time. Indeed, in the short run, since transversality condition must be satisfied, the relationship between q and K remains over the saddle path, *SS*. Otherwise, the long run equilibrium is not reached. Additionally, since Z is not allowed to grow in equilibrium, the curve associated to $\dot{q}$ is not moving to the right with time. On the other hand, if interest rate increases, the equilibrium curve of q moves to the left, implying a decrease in K^*[30].

Note that in this simple case, we are able to find a saddle path. However, in the case of having uncertainty in market conditions, the equilibrium's dynamic drives us to a non-determined number of paths which could make the firm converge to a pseudo-equilibrium point, as we called in previous section.

[30] For a deep analysis and conclusions using this model see, for example, Lehmann (1991), Summers (1981) and Rama (1987).

4 Financial Market Development

In this section, the effect over investment of financial market development is analyzed. Most of literature has analyzed the effect of financial development over economic growth, linking the relationship through the effect over savings and interest rates in equilibrium. In this model, however, the analysis is focused in firm's decisions.

In a first section we present the ideas and findings of McKinnon (1993) and Fry (1995), who analyze the role of financial markets from a macroeconomic perspective. They find that liberalization of financial markets strengthen economic growth. In the second section, we generalize the investment model of section, including financial market role, as a determinant of the uncertainty-investment relationship.

4.1 Role of Financial Markets

According to McKinnon (1993), countries that have sustained higher real rates of investment and more stable prices have generally had more robust financial growth. Table 1 presents ratios of the broad money supply (*M3*) - coin and currency, saving deposits, and shorter-term time deposits in banks or other quasi monetary institutions to *GNP*.

McKinnon pays attention to the fact that "even the slower Asian countries tend to be more financially developed than typically more inflationary Latin American countries." However, both group of countries, have fairly low levels of domestically held financial assets, with a low ratio *M3* to *GNP* and its economic growth has been poor. As a difference with those economies, we observe a strong financial development in rapid-growth economies, such Germany, Japan, Korea, Taiwan and Singapore. The large ratio observed for these countries means a large flow of domestic loanable funds for new investment, indicating a well developed financial market. In spite of starting from very different ratios, in 1985, with the exception of Korea, the high speed growth economies reached a *M3* to *GNP* ratio of at least 0.75. The substantial different from Korea's ratio is due to it borrowed heavily abroad in the 1950s, unlike the other four countries, since its initial *M3/GNP* relationship was extremely low. Only in early 1980s Korea reassumed its financial development (McKinnon, 1993).

Table 1 Ratio M3 to GNP as a Determinant of Economic Development

Bank Loanable Funds in Typical Semi-Industrial LDCs
(Ratio of M3 to GNP)

	1960	1965	1970	1975	1980	1985	Mean 1960-85
Argentina	0.245	0.209	0.267	0.168	0.234	0.152	0.213
Brazil	0.148	0.156	0.205	0.164	0.175	0.179	0.171
Chile	0.123	0.130	0.183	0.099	0.208	0.263	0.168
Colombia	0.191	0.204	0.235	-	0.222	0.290	0.228
Mean ratio of M3 to GNP for four Latin American countries							0.195
India	0.283	0.262	0.264	0.295	0.382	0.412	0.316
Philippines	0.186	0.214	0.235	0.186	0.219	0.204	0.207
Sri Lanka	0.284	0.330	0.275	0.255	0.317	0.371	0.305
Turkey	0.202	0.223	0.237	0.222	0.136	0.228	0.208
Mean ratio of M3 to GNP for four Asian countries							0.259

Source: McKinnon (1993)

Bank Loanable Funds in Rapidly Growing Economies
(Ratio of M3 to GNP)

	1955	1969	1965	1970	1975	1980	1985
West Germany	0.331	0.294	0.448	0.583	0.727	0.913	1.019
Japan	0.554	0.737	0.701	0.863	1,026	1.390	1.599
South Korea	0.069	0.114	0.102	0.325	0.323	0.337	0.396
Taiwan	0.115	0.166	0.331	0.462	0.588	0.750	1.264
Singapore	-	-	0.542	0.701	0.668	0.826	0.788
Mean ratio of M3 to GNP for five rapidly growing countries							1.294

Source: McKinnon (1993)

Although a higher rate of financial growth and rising *M3/GNP* ratios are positively correlated with higher growth in real growth domestic product, it does not give us a causality. Looking for it, McKinnon shows that, taking a pool of developing countries, financial assets growth, measured as the sum of monetary and quasi-monetary deposits with the banking sector, is positive correlated to *GDP* growth and interest rate levels. Making simple empirical estimations, and using the World Bank as a reference (the World Bank report, 1989), he finds that higher real deposit rates, consequence of financial liberalization, increase the growth of *GDP*. Additionally, the productivity, as a consequence of higher quality, of investment, as well as its share with respect to total product, becomes higher as real deposit rate increases.

On the other hand, McKinnon finds that the effect of real deposit rate of interest on the variable measuring financial depth, *M3/GNP*, was minor. A much more important effect has the rate of price inflation. Additionally, *M3/GNP*, that naturally tends to be higher in high-income countries, has a positive effect over the rate of growth of the economy.

Fry (1995) suggests, through empirical estimations, that financial conditions have a positive effect over global saving, and a major effect over distribution of financial resources on investment projects. As a consequence, Fry argues that financial liberation and development increases the economic growth.

Cho (1986) extends the discussion about liberalization of financial markets. He argues that credit in markets with imperfect information, to liberalize capital markets is not enough to reach efficiency. Due to adverse selection effect, the government may be able to improve the allocative efficiency of capital by securing loans for particular groups that are potentially very productive. Equity contracts, however, are free from the adverse selection effect.

4.2 Generalized Investment Model

In this section the model is generalized by the incorporation of financial markets, and, as a consequence, the effects of their development on investment decisions.

4.2.1 Dimensions to be Considered

(a) Firm's value

The observed value of the firm, in general, would be different from the theoretical value given by equation (33). Indeed, because of a decrease in information asymmetries, the less developed is the financial market, the more unpredictable is the difference between the theoretical and observed firms' value. For a low developed financial market, it is more difficult and slower to process information coming from firms and economic performance indicators. Additionally, important information asymmetries and speculation appear as a consequence of little transparency of the market.

Equation (39) expresses the observed q, q^{ob}, as a function of the theoretical q, that comes from the model of section 2. In this equation, S represents a stochastic variable, which variance is a function of financial market development.

(39) $$q^{ob} = qe^{S}.$$

In the previous equation, dS is a normally distributed function, with zero mean and standard deviation $\sigma_f \sqrt{dt}$. In this expression σ_f represents the volatility of q, because of having a non well-developed financial market. The *a priory* uncertain value of firms, are function of index f, which reflects financial development.

Expressing (39) in a differential form, we obtain:

(40) $$dq^{ob} = dqe^{S} + q^{ob}dS + \frac{1}{2}q^{ob}dS^{2}.$$

Taking expected values to previous equation, we get[31]:

(41) $$E[\dot{q}] = E\left[\dot{q}^{ob}\right] - \frac{1}{2}E\left[q^{ob}\right]\sigma_f^{2}.$$

We assume that the expression for $E[\dot{q}^{ob}]$ is equivalent to that one obtained for q in previous section. Therefore, rearranging equation (36) we obtain[32]:

[31] We assume that S and q are uncorrelated.
[32] Note that $E[q^{ob}] = q$.

(42) $$E[\dot{q}] = -\psi_I\gamma(\gamma-1)\left(\frac{I}{K}-\delta\right)\left(\frac{I}{K}\right)^{\gamma-1} + \frac{\alpha_1\mu Z^{\frac{1}{1-\alpha_2}}}{r+\alpha_1\delta-\mu}\left(\frac{\alpha_2}{\omega}\right)^{\frac{\alpha_2}{1-\alpha_2}} K^{\frac{(\alpha_1+\alpha_2)-1}{1-\alpha_2}}$$
$$+\mu\theta A Z^{A} K^{\frac{\lambda-r-\delta}{\delta}} - \frac{1}{2}\theta A(A+1)Z^{-A}\sigma^2 K^{\frac{\lambda-r-\delta}{\delta}} - \frac{1}{2}q^{ob}\sigma_f^2$$

From above equation we have that lower σ_f^2 reduces the requirement of growth rate of market conditions[33]. Therefore, as financial market becomes deeper, *ceteris paribus*, q increases, and, therefore, investment expenditure.

An alternative interpretation for the effect of uncertainty about q considers that investment is determined by the original q (q^{ob}) —in case of not having a budget constraint— which is adjusted by its volatility, *vol(q)*. Implicitly, we assume that *vol(q)* is a function of the pre-defined σ_f^2 and interest rate.

(43) $$\frac{I}{K} = f(q, vol(q))$$

(b) Risk perceived by investors

McKinnon (1993) shows that when financial market is not enough developed, we observe usury restrictions in interest rates and heavy reserve requirements on bank deposits. Additionally, deposit interest rates are very difficult to predict when inflation is high. Foreign exchange rates also become highly uncertain.

As a consequence, making a link with theoretical model developed in previous section, we can argue that as the financial market become more developed, σ^2 and σ_f^2 diminish. Graphically, it means that the dark gray plane of figure 7 moves to the right. As a result, we have that financial development incentives firms to increase their investment expenditure and improves conditions for moving to a long run equilibrium condition.

[33] Graphically, it means that the dark gray plane of figure 7 moves to the left.

(c) Effect of uncertainty

With financial development, new financial instruments appear, as well as, more incentives for firms to openly offer stocks in the capital market. As a consequence, there is an increase in the ability of investors to diversify risk, and therefore, the sensitivity of investment to uncertainty becomes smaller. In terms of the theoretical model, the parameter theta diminishes. That means, according to the results expressed in figure 5, the required expected rate of growth in market conditions becomes smaller. If the firm has accumulated stock of capital below equilibrium, in spite of a low economic growth, it would be able to reach the long-run equilibrium, with a stronger growth of investment expenditure.

(d) Availability of financial funds

McKinnon (1993) argues that repressing the monetary system fragments the capital market, with highly adverse consequences for quantity and quality of investment.

As a first consequence, considering an undeveloped financial market, the flow of loanable funds through the organized banking system is reduced, forcing potential borrowers to rely more on self-finance. Additionally, the process of self-financed within enterprises is itself impaired. If the real yield on deposits, as well as on coin and currency, is negative, firms can not easily accumulate liquid assets in preparation for making discrete investments.

Siglitz and Weiss (1981) establish that economic instability, measured by increase of variance in new projects' yield and positive covariance among expected returns, drives to a decrease in optimal interest rates and credit rationing.

The consequence of those effects is that budget constraint becomes more restrictive. That means, in terms of equilibrium conditions represented in figure 7, the availability of financial resources could drive the firm to a permanent desequilibrium condition.

4.2.2 The Model

Summarizing the effects of financial development, we express a generalized version of the investment model:

$$(44)\quad E\left[q^{ob}\right]=\frac{1}{r+\delta}\left\{\alpha_1\left(\frac{\alpha_2}{\omega}\right)^{\frac{\alpha_2}{1-\alpha_2}}Z^{\frac{1}{1-\alpha_2}}K^{\frac{(\alpha_1+\alpha_2)-1}{1-\alpha_2}}+\psi_I(\gamma-1)\left(\frac{I}{K}\right)^{\gamma}-(rb+\xi)+E\left[\dot{q}^{ob}\right]\right\}.$$

Long-run equilibrium conditions:

$$(45)\quad E\left[q^{ob}\right]^{*}=1,$$

$$(46)\quad E\left[q^{ob}\right]=\frac{1}{r+\delta}\left\{\alpha_1\left(\frac{\alpha_2}{\omega}\right)^{\frac{\alpha_2}{1-\alpha_2}}Z^{\frac{1}{1-\alpha_2}}K^{\frac{(\alpha_1+\alpha_2)-1}{1-\alpha_2}}+\psi_I(\gamma-1)\left(\frac{I}{K}\right)^{\gamma}-(rb+\xi)\right\}$$

$$(47)\quad -\psi_I\gamma(\gamma-1)\left(\frac{I}{K}-\delta\right)\left(\frac{I}{K}\right)^{\gamma-1}+\frac{\alpha_1\mu Z^{\frac{1}{1-\alpha_2}}}{r+\alpha_1\delta-\mu}\left(\frac{\alpha_2}{\omega}\right)^{\frac{\alpha_2}{1-\alpha_2}}K^{\frac{(\alpha_1+\alpha_2)-1}{1-\alpha_2}}+\mu\theta_f AZ^{-A}K^{\frac{\lambda-r-\delta}{\delta}}$$
$$-\frac{1}{2}\theta_f A(A+1)Z^{-A}\sigma^2K^{\frac{\lambda-r-\delta}{\delta}}-\frac{1}{2}q^{ob}\sigma_f^2=0$$

The investment rate as a function of q and financial resources, is given by[34]:

$$(48)\quad \frac{I}{K}=Min\left\{C'^{-1}(q),\frac{\eta_f B_f}{bK}\right\}.$$

In the above set of equations, index f of financial development is an exogenous variable. It has an effect over sensitivity of investment to uncertainty (parameter θ), perception of economic agents about uncertainty (σ^2 and σ^2_f), availability of external funds to finance investment (B_f), and the rate of growth of financial resources (η_f). We obtain that as financial sector becomes more developed, θ decreases, which determines an improve in the risk-return trade-off. As a consequence, q increases and then, investment. Additionally, a deeper financial sector makes uncertainty about economic conditions to be lower. Accuracy in firm valuation improve (σ^2_f decrease). Finally, the availability and rate of growth of financial funds, B_f and μ_f respectively,

[34] We could use equation (43) for $C'^{-1}(q)$. However, in that case, we should ignore last term of equation (46).

increase. Then, we obtain that as f goes up (more developed financial market), investment becomes higher.

4.2.3 Aggregation

The investment model concerns to a single representative firm. However, we are interesting in the aggregate investment expenditure of the all economy. Investment decisions are not taken in isolation: there are market interactions among firms, capital equipment is in general heterogeneous, and it could be possible to reconvert capital to new uses, or to sell it to other users. As a consequence, there are linkages across investment decisions that should be recognized in an empirical application of the model.

Individual investments are not synchronized and firms are subject, in addition to aggregate uncertainty, to idiosyncratic uncertainty, related to managerial abilities and the particular market conditions relevant to each firm. As a result, aggregate investment shows smoothness in response to changes in prices or in the economic conditions.

5 Conclusions

The importance of uncertainty in investment decision has been arisen from the stochastic control model developed in the paper. Considering the existence of asymmetric adjustment costs, imperfect competition and linking the investment model to Tobin's q theory, we found that uncertainty in market condition has negative effects on the dynamic of q, a measure of the investment profit rate. Given that investment is a function of q, we conclude that a change in uncertainty decreases investment expenditures.

As a consequence of uncertainty, in order to reach a long term equilibrium, we require a positive expected growth in market conditions. This result captures the classic risk-return trade-off: An efficient investment project requires a minimum expected growth in market condition, close to the rate of profitability of investment, in order to "compensate" market uncertainty. In a completely certain world, there is no requirements on expected economic growth.

Since equilibrium conditions require expected market conditions to be growing, we have, looking from a graphic perspective, positive movements in equilibrium planes, which determine an increasing stock of capital over time, even when firms are not in the equilibrium point.

Looking the effect of other variables, without betraying our intuition, we found that an increase in interest rates determines a decrease in investment. Graphically, it means that equilibrium planes move negatively, implying a smaller equilibrium stock of capital.

Financial market development plays an important role for determining the optimal investment expenditures of firms. Indeed, we found that financial depth affects managers' decision in four aspects. The first one, but not necessarily the most important, considers that in less developed market there are more difficulties to price a firm. Indeed, in low developed markets we have important information asymmetries and difficulties to process new information. As a consequence, looking from a Tobin's q perspective, firm's managers have difficulties in knowing the market valuation of new investment, diminishing the incentives for capital accumulation.

Second effect of financial development over investment: as financial market becomes deeper, the economy becomes more stable. As a consequence, investors will perceive less uncertainty, requiring lower rates of growth in market conditions to reach equilibrium. That means, as uncertainty decreases, we observe an expected increase in q, considering the positive effect over its expected growth.

Third, as financial market development increases, new instruments and the increase of companies' stocks in capital market provide more alternatives for risk diversification. For this reason, the effect of uncertainty in a more developed markets is smaller.

As a difference with previous arguments related to rate of profitability, the forth reason for higher incentives to invest in more developed financial markets is related to the availability of financial resources. Considering a relative consensus of positive effect over savings and more efficient credit allocation in more developed financial markets, we can argue that an increase in financial depth makes funds more abundant to finance investment expenditures.

Uncertainty in market conditions can be associated to political risk, inflation rate, interest rate volatility, frequent variations of the exchange rate and an unstable rate of economic growth. As the model states, variance of market conditions is especially important in investment decisions. Therefore, the economic authorities should take price stabilization as an important objective of their economic policies in order to strengthen the investment process -- increase the investment rate and capital productivity.

In the same direction, and as an important result of the model, incentives for financial development would be crucial in order to promote new and more productive investment expenditures. In this process, nevertheless, we should take into account Cho's (1988) warning, who argues that significant financial deepening outside the repressed banking system becomes impossible when firms are dangerously illiquid and/or inflation is high and unstable. Robust open markets in stocks and bonds and intermediation by trust and insurance require monetary stability.

A significant contribution of the paper, besides the theoretical model itself, is its possible empirical application. Indeed, we have generalized Hayashi's demonstration about marginal and average q linear relationship (see appendix C). Using this result, from a macroeconomic perspective, we are able to approximate the theoretical q model by the quotient between the stock price index and the economy stock of capital.

An empirical application of the model would give us a measurement of stability and financial development benefits. This contribution would be especially important in developing countries, where exists an urgent requirement for "better" and "more" investment. Stabilize and liberalize financial market should be an essential objective in developing countries in order to increase their economic growth capacity.

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Appendix A

Investment Rate and Euromoney Country Risk Rating: A Selective Sample

Table 1 Private Investment over GDP (in current USD)

Country	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
Brazil	20.12%	17.31%	15.89%	15.45%	16.76%	19.84%	20.38%	22.52%	18.51%	16.11%	16.22%	16.35%
Chile	10.89%	8.05%	7.16%	-	-	-	15.63%	19.58%	20.53%	17.67%	19.21%	21.75%
Colombia	9.61%	9.99%	8.82%	8.79%	9.68%	10.76%	11.88%	10.20%	9.53%	9.87%	9.44%	11.54%
Mexico	13.80%	11.69%	12.02%	13.08%	13.75%	13.93%	14.85%	13.89%	14.05%	15.10%	16.82%	16.50%
Peru	22.20%	15.96%	13.76%	12.83%	15.86%	16.15%	19.74%	16.52%	12.87%	12.29%	12.85%	14.11%
Venezuela	7.74%	4.45%	-	-	-	-	12.71%	8.26%	5.06%	8.36%	9.14%	9.24%
Group I	**14.06%**	**11.24%**	**11.53%**	**12.54%**	**14.01%**	**15.17%**	**15.86%**	**15.16%**	**13.42%**	**13.23%**	**13.95%**	**14.91%**
Indonesia	-	-	13.27%	14.17%	16.99%	18.27%	18.62%	19.74%	20.40%	20.71%	19.50%	17.28%
Korea, Rep.of	24.29%	25.20%	25.42%	25.08%	25.14%	25.99%	24.65%	26.72%	30.98%	-	-	-
Malaysia	19.04%	18.84%	17.99%	17.05%	15.31%	14.74%	16.29%	19.62%	21.85%	25.44%	24.27%	-
Philippines	20.26%	24.11%	19.74%	14.12%	14.27%	13.92%	15.06%	17.48%	18.80%	15.68%	15.60%	18.72%
Thailand	19.39%	20.62%	20.53%	18.78%	18.78%	22.06%	26.05%	29.96%	34.55%	34.71%	31.74%	31.39%
Group II	**20.75%**	**22.19%**	**19.39%**	**17.84%**	**18.10%**	**18.99%**	**20.13%**	**22.70%**	**25.32%**	**24.14%**	**22.78%**	**22.46%**
Egypt	9.57%	7.22%	6.09%	6.39%	6.74%	6.59%	3.71%	5.47%	3.48%	9.84%	7.20%	8.21%
India	9.78%	9.56%	9.76%	10.40%	10.05%	11.69%	11.69%	13.06%	14.71%	12.61%	-	-
Kenya	10.88%	11.50%	10.92%	11.12%	11.96%	12.99%	12.31%	11.72%	11.57%	11.27%	10.03%	10.85%
South Africa	16.66%	16.34%	15.22%	13.76%	12.19%	11.66%	13.67%	13.70%	13.42%	12.28%	11.81%	11.10%
Syria	9.03%	7.84%	8.13%	7.88%	10.41%	9.43%	5.74%	8.68%	8.44%	8.82%	-	-
Tunisia	15.98%	14.70%	27.21%	23.22%	20.86%	17.65%	16.57%	19.59%	20.50%	20.74%	20.99%	21.33%
Turkey	7.92%	8.07%	8.54%	8.66%	10.10%	15.74%	18.22%	15.32%	17.04%	16.19%	16.67%	20.02%
Group III	**11.40%**	**10.75%**	**12.27%**	**11.63%**	**11.76%**	**12.25%**	**11.70%**	**12.51%**	**12.74%**	**13.11%**	**13.34%**	**14.30%**

Source: The World Bank data base.

Table 2 Euromoney Country Risk Ratings

Country	1982	1983	1984	1985	1986*	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
Brazil	45.30	26.00	30.10	20.00	35.00	38.00	40.00	32.00	45.00	38.20	37.19	42.61	49.01	53.83	56.81
Chile	53.30	31.00	9.10	11.60	21.00	20.00	54.00	41.00	54.40	50.30	69.55	68.75	70.43	75.32	77.43
Colombia	66.20	67.30	58.00	55.70	42.00	39.00	52.00	37.00	45.00	40.00	53.83	60.68	61.01	60.91	62.37
Mexico	41.90	43.50	56.30	45.55	31.00	42.00	43.00	44.00	58.60	55.80	59.40	60.37	62.00	57.06	60.28
Peru	54.00	43.40	32.80	10.35	21.00	20.00	20.00	25.00	28.90	27.00	19.04	29.85	41.11	41.10	47.63
Venezuela	55.20	38.50	48.40	20.65	31.00	45.00	49.00	47.00	46.10	51.80	51.13	48.96	46.58	45.17	45.37
Group I	**52.65**	**41.62**	**39.12**	**27.31**	**30.17**	**34.00**	**43.00**	**37.67**	**46.33**	**43.85**	**48.36**	**51.87**	**55.02**	**55.57**	**58.32**
Indonesia	72.10	67.40	73.90	63.95	59.00	59.00	61.00	62.00	65.70	57.30	63.53	68.48	68.00	73.32	70.79
Korea, South	65.20	71.50	70.30	70.65	73.00	82.00	83.00	79.00	82.00	76.70	75.45	81.65	82.90	86.12	84.33
Malaysia	74.00	83.80	74.90	78.35	65.00	65.00	70.00	68.00	75.80	76.00	73.61	78.52	81.90	78.63	80.18
Philipines	69.00	65.40	36.90	22.30	25.00	39.00	34.00	38.00	41.40	33.90	33.77	47.32	51.83	58.09	61.49
Thailand	56.00	69.10	70.30	70.60	62.00	75.00	77.00	72.00	72.20	73.30	67.11	75.25	76.43	79.73	77.22
Group II	**67.26**	**71.44**	**65.26**	**61.17**	**56.80**	**64.00**	**65.00**	**63.80**	**67.42**	**63.44**	**62.69**	**70.24**	**72.21**	**75.18**	**74.80**
Egypt	60.00	58.70	56.50	46.55	43.00	29.00	31.00	38.00	39.60	34.50	31.09	43.80	42.79	45.86	45.65
India	72.60	72.60	68.50	58.00	69.00	74.00	74.00	67.00	58.70	45.90	44.07	54.33	59.67	63.61	63.67
Kenya	45.50	38.70	37.80	18.75	38.00	26.00	46.00	43.00	43.20	37.50	37.70	40.40	42.80	39.90	42.28
South Africa	74.40	75.60	72.50	70.95	41.00	49.00	42.00	46.00	55.00	54.00	53.90	60.04	58.96	63.56	62.30
Syria	33.50	39.40	42.10	30.95	38.00	20.00	27.00	30.00	22.30	25.20	34.14	40.85	43.77	28.84	43.77
Turkey	45.40	58.20	59.90	43.20	54.00	45.00	57.00	62.00	54.00	53.10	66.20	64.09	79.21	57.71	57.51
Tunisia	54.10	73.30	73.00	65.95	52.00	46.00	58.00	58.00	50.00	39.10	52.52	58.10	53.95	61.95	53.95
Group IV	**55.07**	**59.50**	**58.61**	**47.76**	**47.86**	**41.29**	**47.86**	**49.14**	**46.11**	**41.33**	**45.66**	**51.66**	**54.45**	**51.63**	**52.73**

Source: Euromoney.

Note: Since 1982 until 1986, the rates for each country were calculated weighting: 20% acces to market; 10% access to trade finance (forfaiting); 15% payment record (wheteher on time or late); 5% difficulties in rescheduling; 20% political risk; 30% selldown - measure of over-subscription.

Since 1987, the calculation considers: 25% Economic performance; 25% Political risk; 10% debt indicators; 10% debt in default; 10 % credit ratings; 5% access to bank finance; 5% access to short-term finance; 5% access to capital markets and 5% discount on forfaiting.

Appendix B

Marginal Contribution of Capital Stock

Differential equation (17) is expressed in (B1), where U was defined as V_K.

$$\text{(B1)} \qquad \frac{1}{2}Z^2\sigma^2 U_{ZZ} + Z\mu U_Z - U_K\delta K - (r+\delta)U = \psi_I(\gamma-1)^2\left(\frac{I}{K}\right)^{\gamma} + \xi - \alpha_1 Z L^{\alpha_2} K^{\alpha_1 - 1}$$

In order to solve the above differential equation, we consider the following variable changes, and function equivalence:

$$\text{(B2)} \qquad k = \log K,$$
$$\text{(B3)} \qquad z = \log Z,$$
$$\text{(B4)} \qquad u(k,z) = U(K,Z).$$

Using these definitions we have that:

$$\text{(B5)} \qquad u_k = U_K K,$$
$$\text{(B6)} \qquad u_z = U_Z Z,$$
$$\text{(B7)} \qquad U_{ZZ}Z^2 = u_{zz} - u_z.$$

Therefore, the differential equation can be written as:

$$\text{(B8)} \qquad \frac{\sigma^2}{2}u_{zz} + \left(\mu - \frac{\sigma^2}{2}\right)u_z - \delta u_k - (r+\delta)u = \psi_I(\gamma-1)^2 I^{\gamma} e^{-\gamma k} + \xi - \alpha_1 L^{\alpha_2} e^{k(\alpha_1 - 1)+z}$$

Particular Solution

In order to find the particular solution, the next functional form is assumed:

$$\text{(B9)} \qquad u_p(k,z) = ae^{-\gamma k} + bL^{\alpha_2}e^{k(\alpha_1 - 1)+z} + c,$$

where a, b, and c must satisfy:

(B10)
$$a = \frac{\psi_I(\gamma-1)^2}{\delta(\gamma-1)-r} I^{\gamma},$$

(B11)
$$b = \frac{\alpha_1}{r+\alpha_1\delta-\mu},$$

(B12)
$$c = -\frac{\xi}{r+\delta}.$$

According to the expression for a, we have that the obtained particular solution would be well defined when $r \neq \delta(\gamma-1)$. In case of $r=\delta(\gamma-1)$, the solution of the differential equations should have a functional form given by:

(B13)
$$u_p(k,z) = aze^{-\gamma k} + bL^{\alpha_2} e^{k(\alpha_1-1)+z} + c.$$

For this functional form there is no change in parameters b and c. However, in this case a should be equal to:

(B14)
$$a = \frac{\psi_I(\gamma-1)^2}{\mu-\frac{\sigma^2}{2}} I^{\gamma}$$

Homogeneous Solution

To obtain the homogeneous solution of the differential equation defined in (B8), we assume that u is a separable function; that is, can be written as $u(k,z)=T(z)S(k)$. Therefore, the homogeneous differential equation to be solved is:

(B15)
$$\frac{\sigma^2}{2}T''S + \left(\mu - \frac{\sigma^2}{2}\right)T'S - \delta TS' - (r+\delta)TS = 0$$

Rewriting above equation, we have:

$$\text{(B16)} \qquad \frac{\frac{\sigma^2}{2} T''S + \left(\mu - \frac{\sigma^2}{2}\right) T'}{T} = \frac{\delta S' - (r+\delta)S}{S} = \lambda,$$

where λ is a constant.

From the above expression, the following differential equations are stated:

$$\text{(B17)} \qquad \frac{\sigma^2}{2} T''S + \left(\mu - \frac{\sigma^2}{2}\right) T' - \lambda T = 0,$$

$$\text{(B18)} \qquad \delta S' + (r + \delta - \lambda) S = 0.$$

The solutions of these differential equations are given, respectively, by expressions (B19) and (B20).

$$\text{(B19)} \qquad T = C_1 e^{-z\left(\mu - \sigma^2/2 + \sqrt{(\mu - \sigma^2/2)^2 + 2\lambda\sigma^2}\right)/\sigma^2},$$

$$\text{(B20)} \qquad S = C_2 e^{k(r+\delta-\lambda)/\delta}.$$

As border conditions we establish that the marginal contribution of Z to V_K is positive and the marginal contribution of K to V_K is negative. That means that $T'>0$, $S>0$ and $S'>0$. Therefore, C_1 is a negative constant, C_2 is a positive constant and parameter λ is such that $\lambda>r+\delta$. Therefore, the solution for the differential equation is:

$$\text{(B21)} \qquad u(k,z) = \frac{\psi_I(\gamma-1)^2}{\delta(\gamma-1)-r}\left(\frac{I}{K}\right)^{\gamma} + \frac{\alpha_1 L^{\alpha_2} e^{k(\alpha_1-1)+z}}{r+\alpha_1\delta-\mu} - \frac{\xi}{r+\delta} - \theta e^{-z\left(\frac{\mu-\sigma^2/2+\sqrt{(\mu-\sigma^2/2)+2\lambda\sigma^2}}{\sigma^2}\right)+k\left(\frac{\lambda-r-\delta}{\delta}\right)},$$

where θ is the product of C_1, C_2 and -1.

Expressing the solution in terms of K and Z we get:

$$\text{(B22)} \qquad U(K,Z) = \frac{\psi_I(\gamma-1)^2}{\delta(\gamma-1)-r}\left(\frac{I}{K}\right)^{\gamma} + \frac{\alpha_1 Z L^{\alpha_2} K^{\alpha_1-1}}{r+\alpha_1\delta-\mu} - \frac{\xi}{r+\delta} - \theta Z^{-\frac{\mu-\sigma^2/2+\sqrt{(\mu-\sigma^2/2)+2\lambda\sigma^2}}{\sigma^2}} K^{\frac{\lambda-r-\delta}{\delta}}$$

In above equation, θ and λ are positive parameters, associated to a particular economy.

Appendix C

Marginal and Average q: An Extension to Hayashi's Finding

Let be π the profits for a representative firm, as is expressed in equation (C1).

(C1) $$\pi = R - C(I/K)K,$$

where R is represented in the next expression:

(C2) $$R = G(K, L, \tilde{Z}) - \omega L.$$

Given that, according to the assumptions, G is homogeneous of degree K and L, we have that:

(C3) $$G_K K + G_L L = G(\alpha_1 + \alpha_2).$$

Replacing on the above equation the first order condition on L $(G_L = \omega)$, we obtain:

(C4) $$G_K = \frac{G(\alpha_1 + \alpha_2)}{K} - \frac{\omega L}{K}.$$

On the other hand,

(C5) $$E_t\left\{\frac{d}{dt}\left[qKe^{-rt}\right]\right\} = E_t\left\{(\dot{q}K + q\dot{K} - rqK)e^{-rt}\right\}.$$

From equation (27) and (9), we know:

(C6) $$E[\dot{q}] = (r + \delta)q - \pi_K - \psi(\gamma - 1)\left(\frac{I}{K}\right)^{\gamma},$$

(C7) $$\dot{K} = I - \delta K.$$

Replacing (C1), (C4), (C6) and (C7) in (C5) we get:

$$\text{(C8)}\quad E_t\left\{\frac{d}{dt}\left[qKe^{-rt}\right]\right\}=\left(-G(\alpha_1+\alpha_2)+\omega L+(r+\delta)q_tK_t-\psi(\gamma-1)K_t\left(\frac{I_t}{K_t}\right)^{\gamma}+q_t(I_t-\delta K_t)-r_tq_tK_t\right)e^{-rt}$$

Replacing the particular expression for adjustment costs defined in (18) and the first order condition (15), we obtain:

$$\text{(C9)}\qquad qI=\psi K\left(\frac{I}{K}\right)^{\gamma}$$

Simplifying expression (C8) and replacing (C9) on it, we get:

$$\text{(C10)}\qquad E_t\left\{\frac{d}{dt}\left[qKe^{-rt}\right]\right\}=E_t\left\{\left[-G(\alpha_1+\alpha_2)+\omega L+C(I/K)K\right]e^{-rt}\right\}.$$

Integrating the above expression we obtain equation (C11).

$$\text{(C11)}\quad q_tK_t=E_t\left[\int_t^{\infty}\pi_te^{-rt}dt+\int_t^{\infty}G_t(\alpha_1+\alpha_2-1)e^{-rt}dt\right]=V_t+E_t\left[\int_t^{\infty}G_t(\alpha_1+\alpha_2-1)e^{-rt}dt\right].$$

Rewriting the above equation,

$$\text{(C12)}\qquad q_t=\frac{V_t+\overline{G}_t}{K_t}=\overline{q}_t+\overline{g}_t$$

From the above equation we conclude that the relationship between marginal and average q is linear.