INCOME AND POPULATION GROWTH: A SIMULTANEOUS EQUATION APPROACH

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Resumen

Este artículo estima un modelo de ecuaciones simultáneas de crecimiento del ingreso y de la población. El marco analítico está basado en el modelo neoclásico de crecimiento, incluyendo una ecuación de crecimiento de la población. Se utilizan métodos de estimación no lineales tanto con información completa como parcial. Los resultados muestran que la simultaneidad propuesta genera un importante sesgo en la "tradicional" ecuación de MCO usada en la literatura empírica de crecimiento económico. El parámetro de crecimiento de la población crece en un 50% (en valor absoluto) en la ecuación estándar de ingreso. Finalmente, el artículo muestra que un aspecto cultural como la religión importa dentro de la ecuación de crecimiento de la población, pero no en la ecuación de ingreso.

Abstract

This paper estimates a simultaneous equation model of income and population growth. The framework is based on the neoclassical growth model augmented with a population growth equation. Both limited and full information non-linear estimation methods are used, as well as different specification tests check the consistency of the model. The results show that the proposed simultaneity produces an important bias in the "typical" OLS income equation used in the empirical growth literature. The population growth parameter increases by almost 50\% (in absolute value) in the standard income equation. Finally, the paper shows that a cultural aspect like religion matters in the population growth equation, but not in the income equation.

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1 Introduction

The literature focused on empirical aspects of economic growth and the vast majority of the theoretical developments have considered population growth as an exogenous variable. The typical cross-country empirical growth paper tries to explain per capita income level (or sometimes growth) using population growth and other characteristics such as savings rates as explanatory variables. [1] There are also empirical studies that propose exactly the reverse causation; fertility and population are explained by income and growth.[2] The objective of this paper is to specify and estimate a model of simultaneous determination of income and population growth. For that purpose I combine two well known models of growth determination and fertility behavior. The estimation includes limited and full information procedures and is carried out over a cross-section of countries.[3]

The paper focuses on four different issues. The first one is the estimation of the model itself using limited information and the selection between two alternative specifications. Overidentification and exclusion restriction tests are considered in the discussion. The second issue is the question of how and to what extent the simultaneity of income and fertility affects the "typical" OLS estimation. Inconsistent OLS results are compared with consistent estimates. The third issue is the estimation and evaluation of the model using a full information method. By comparing full and limited information estimates the specification and internal consistency of the mode is checked. Finally, the fourth question is whether a cultural aspect like religion affects the model, especially the estimation of the population growth rate as function of economic variables.

[1] See, e.g., Mankiw, Romer and Weil (1992), Islam (1995), and Knight, Loayza and Villanueva (1993).

[2] Birdsall (1977) and (1988) survey some of these studies. Barro (1991) estimates an equation by OLS in which net fertility is explained with income and other variables. Controlling for human capital he concludes that fertility does not depend on income.

[3] Another venue to tackle the same type of question is to use lagged variables only as instruments. Caselli et al. (1997) follow this procedure estimating growth regressions in a GMM framework .

The results indicate that, consistently with other empirical studies, a model with human capital as a production factor is more appropriate than one without it. Furthermore, the proposed simultaneous model is fairly accurate in describing cross-country variation of both income and population growth and a specification test is not rejected. Another finding is that inconsistent OLS estimates —the typical estimates in the literature— are considerably different from consistent estimates. Finally, I conclude that countries' religion does affect the model, in particular, the population growth equation. After splitting the sample in two groups I show that the population growth equation is different for "liberal" and "conservative" countries, while the income equation is statistically similar.

The paper is organized as follows. In section 2, I derive and put together the two sub-models that conform the basic model to be estimated. The data, sources and samples to be used in the estimation are described in section 3. Section 4 presents limited information estimates, compares consistent estimates to OLS and discusses overidentification and exclusion restriction tests. It also presents full information estimates and a specification test comparing these estimates to the limited information results. Section 5 discusses how a cultural aspect like religion affects the model and presents estimations for "liberal" and "conservative" countries. Section 6 presents some final remarks.

2 Income and Population Growth Model

The model proposed here is based on two well known models. The income (economic growth) sub-model is the Mankiw, Romer and Weil (1992) model, which in turn is based on the Solow model (Solow, 1956) augmented with human capital in the production function. The population growth sub-model basically follows the "household demand" models of fertility of Becker (1960) and Willis (1973).[4]

There are few models that consider simultaneous determination of growth and

[4]See Razin and Sadka for a revision of different theories of population growth.

fertility. One of these is Barro and Becker (1989) where altruistic behavior connects the decision of fertility with consumption (and growth). Another is Becker, Murphy and Tamura (1990). They assume altruistic behavior and a raising return on human capital as the stock of human capital increases. Razin and Sadka (1995, pp. 167–181) propose a model in which child bearing is time consuming and therefore is affected by household labor decisions. In spirit it is similar to the model I follow here, although the production technology they assume follows the endogenous growth tradition. Finally, Galor and Weil (1996) present a model in which the valuation of female labor increases with development and therefore fertility changes with growth. The model contemplated here is much simpler; population growth is affected by income and different prices, such as wages, through income and substitution effects in a static demand model.

2.1 Income Sub-Model

The growth or income determination sub-model assumes a constant returns to scale Cobb-Douglas production function, with production factors physical capital (K), labor (L) and human capital (H). It also assumes labor augmenting technological progress. At time t output is given by

$$Y_t = K_t^{\alpha} H_t^{\beta} \left(A_t L_t\right)^{1-\alpha-\beta}, \tag{1}$$

where A_t represents the level of technology at t. Technological progress grows exogenously at rate g. Population grows at rate n, considered exogenous in this sub-model, but endogenous in the full model. Hence,

$$\begin{aligned} \dot{L}_t &= n L_t \\ A_t &= A_0 e^{gt} \end{aligned} \tag{2}$$

Let $k = K/(AL)$ be the per-effective-unit of labor stock of physical capital, $h = H/(AL)$ the per-effective-unit of labor stock of human capital, and $y = Y/(AL)$ the per-effective-unit of labor product. Assuming that a constant share of income is saved and invested in physical and human capital, the following equations of motion for k and h can be easily derived:

$$
\begin{aligned}
\dot{k}_t &= s_k k_t^{\alpha} h_t^{\beta} - (n+g+\delta) \\
\dot{h}_t &= s_h k_t^{\alpha} h_t^{\beta} - (n+g+\delta)\,,
\end{aligned}
\tag{3}
$$

where s_k and s_h are the shares of income saved (invested) in physical and human capital respectively and δ is the depreciation rate. The depreciation rate is assumed equal for both types of capital as in Mankiw, Romer and Weil (1992). Given the assumption of constant returns to scale in (1), the production function has decreasing returns to accumulable factors and the existence of a steady state level of k and h is guaranteed. That level is given by

$$
\begin{aligned}
k^* &= \left(\frac{s_k^{1-\beta} s_h^{\beta}}{n+g+\delta}\right)^{\frac{1}{1-\alpha-\beta}} \\
h^* &= \left(\frac{s_k^{1-\alpha} s_h^{\alpha}}{n+g+\delta}\right)^{\frac{1}{1-\alpha-\beta}}
\end{aligned}
\tag{4}
$$

Assuming that the economy is in steady state, the replacement of the corresponding values of k and h in the production function (1) in terms of per-effective-unit of labor yields the following per capita income equation:[5]

$$
\begin{aligned}
\ln \frac{Y_t}{L_t} \;=\; & \ln A_0 + gt - \frac{\alpha+\beta}{1-\alpha-\beta}\ln(n+g+\delta) + \frac{\alpha}{1-\alpha-\beta}\ln(s_k) \\
& + \frac{\beta}{1-\alpha-\beta}\ln(s_h)\,.
\end{aligned}
\tag{5}
$$

Equation (5) is the basic equation of income determination. In the cross country set up the term gt is common for all countries. I further assume that the initial level of technology is a constant plus a random country specific shock which in turn is assumed uncorrelated with the saving rates.[6] As in the other studies that use this Solow model approach I will consider the term $g+\delta$ constant across countries and equal to 0.05. Note that the assumption $g+\delta$ being different from zero makes (5) a non-linear equation

[5]This assumption is not trivial for population growth is endogenous. Technical progress will be continually changing per-capita income and the population growth rate. I am implicitly assuming that the adjustment towards the per-effective-unit of labor income steady state occurs at a much higher frequency than movements in the population growth.

[6]Islam (1996) and Knight, Loayza and Villanueva (1993) estimate different technologies across countries through fixed-effects in a panel data set up. They assume exogenous population growth.

in the population growth rate n, the other endogenous variable in the full model. In order to facilitate later discussion it is helpful to re-write (5) in "cross-section format" using also variable names:

$$\begin{aligned} \ln(GDP_i) &= a_0 + a_1 \ln(POPL_i + 0.05) + a_2 \ln(SAVINGS_i) \\ &\quad + a_3 \ln(SCHOOL_i) + e_i. \end{aligned} \tag{6}$$

The sub-index i stands for country i, GDP is per capita income and e is the random shock. Given (5), one expects a_2 and a_3 positive and a_1 negative. Moreover, given the constant returns to scale assumption one expects the sum of the three coefficients to be zero.

A second and simpler specification considered in Mankiw, Romer and Weil (1992) and used by Islam (1995) consists in a production function similar to equation (1) but with only labor and capital as production factors. The same procedure described above to derive equation (5) can be used in this case. The result is

$$\ln \frac{Y_t}{L_t} = \ln A_0 + gt - \frac{\alpha}{1-\alpha} \ln(n + g + \delta) + \frac{\alpha}{1-\alpha} \ln(s_k). \tag{7}$$

In terms of equation (6) this specification restricts the coefficient on human capital (a_3) to be zero and the sum of the other two (a_1 and a_2) to be zero.

2.2 Population Growth Sub-Model

Population growth is determined by fertility behavior, death rates (in turn, closely related to life expectancy) and net migration. I focus here on fertility since it is the only component that has long lasting effects in population growth rates. Malthusian aspects, such as infant and general mortality rate changes, are indirectly considered since the final model presents population growth as a function of per-capita income. According to Malthus it is precisely this variable which would determine the behavior of mortality rates.

The model I describe here is based on the Becker household demand model of fertility.[7] Households have a utility function that depends on consumption of goods,

[7]See Becker (1981) and Birdsall (1988) for a broader perspective. Micro evidence is presented in

number of children and quality of children.[8] Parents maximize this utility function subject to a production function of number and quality of children and income. The input of this production function is not only income (other goods) but also time of the parents. Changes in parents' labor income will produce both income and substitution effects. Under the assumption of all goods being normal the income effect will imply more number and quality of children. Under the assumption that number of children is a more time intensive output than quality of children, the substitution effect will imply more quality and less number of children. Since consumption of goods does not require time in the model, substitution with goods will reinforce less number of children. So long as substitution effects dominate one will observe a negative relation between labor income and population growth. Also, changes between the mother and father wage differential will have impact on the number and quality of children. Under the assumption that women's time is more productive than men's in producing number of children (or, equivalently, more time of men than women is required in order to produce one child) less gender earnings differential will imply less population growth.

Formally, households have a linearly homogenous production function to produce "quality children" of the form $(Q, N) = f(t, x)$, where Q represents quality per child, N is number of children, t is a vector of time devoted to child producing and x other goods required. Households face the following problem:

$$\begin{aligned} &\max U = U(N, Q, Z) \\ &\text{s.t. } p_{QN}QN + p_Q Q + p_N N + p_Z Z = I, \end{aligned} \tag{8}$$

where p_i represents the shadow price of "good" i, Z are consumption goods and I total income (including the value of time devoted to quality-child producing). There exists a shadow price of average quality that is independent of number of children and vice-versa (e.g., "public" goods at home and pregnancy discomfort). The problem has a solution characterized by a demand system. One of the equations of this solution

Rosenzweig (1990).

[8]Quality refers to the child's quality of life; e.g., entertainment and food.

will be a number of children demand function of the form

$$N = N\left(p_{QN}, p_Q, p_N, p_Z, I\right). \tag{9}$$

For the purpose of this paper I assume an isoelastic demand that depends on household income and the differential between men and women earnings. A-priori, both elasticities are negatives. More income, typically associated with larger labor income, will imply a larger shadow price of number of kids. Changes in the gender earnings differential will have a similar effect. With this model as microfoundations I assume the following functional form determining the desired population growth rate n^* at time t:

$$n_t^* = \gamma_0 + \gamma_1 \ln\left(I_t\right) + \gamma_2 DIFF_t, \tag{10}$$

where $DIFF$ is the gender earnings differential. Equation (10) describes the desired (demanded) population growth. I further assume a partial adjustment process in the actual population growth rate. At time t the observed population growth rate will be given by

$$n_t = n_{t-1} + \theta\left(n_t^* - n_{t-1}\right) + \nu_t, \tag{11}$$

where θ represents the partial adjustment coefficient and ν_t a random error uncorrelated with gender earnings differentials and lagged population growth rates. Substitution of equation (10) in (11) and rearrangement yield the basic equation governing the population growth rate:

$$n_t = (1-\theta)\, n_{t-1} + \theta\left\{\gamma_0 + \gamma_1 \ln\left(I_t\right) + \gamma_2 DIFF_t\right\} + \nu_t. \tag{12}$$

Since estimation will be carried out in a cross country setting, the time sub-index is meaningful and the lagged variable can be treated as an exogenous variable. As with the income equation we can re-write (12) in "cross-section format" using the same variable names used in the income-growth model:

$$POPL_i = b_0 + b_1 POPL(-1)_i + b_2 \ln\left(GDP_i\right) + b_3 DIFF_i + \nu_i. \tag{13}$$

From the model presented above one expects a positive coefficient b_1 and negative coefficients b_2 and b_3. Finally, as all partial adjustment models, the coefficient $(1 - b_1)$ allows to differentiate short and long run coefficients.

In sum, the model has two equations that simultaneously determine the per-capita income level and the population growth rate. The specification of the full model is given by equations (6) and (13).

3 Data Description and Samples

The data sources used in empirical growth cross country studies is relatively standard. In this paper I use those same sources, namely the Summers and Heston (1991) Penn World Tables and the World Bank Social Development Indicators. Summers and Heston tables include real GDP, consumption, government expenditure, investment and population among other figures for practically all countries from 1960 on. The main characteristic of this data source is that inter-country figures are comparable because national figures are adjusted through purchasing power parity.

As real per capita income I use the Summers and Heston real average per-capita GDP for the period 1960–1990.[9] There are at least three reasons to use average GDP rather than the GDP of a particular year. The first is that the model assumes that countries are in steady state, an incorrect assumption for a particular year.[10] By taking an average of several years one is offsetting different non-steady state situations. This happens because the steady state changes through out the years and the current state of the economy can be to the right or to the left of the long run equilibrium. In the same fashion an average clearly smooths out business cycles which can be considered noise in the context of growth studies. The second reason has to do with the simul-

[9] 8 countries in the sample have GDP data up to 1988. The remaining years were extrapolated using the country average growth rate.

[10] Mankiw, Romer and Weil (1992) and Islam (1995) estimate also the model outside the steady state linearizing it properly. The result is that the economy reaches halfway to steady state in between 10 and 30 years depending upon the model.

taneity of population growth and economic growth. In order to actually observe such a simultaneity —in terms of contemporaneous effects— the length of the period of time considered as basic unit has to be long enough to permit that changes in population growth effectively have an impact in income through the labor force. An average of several years will insure that changes in fertility behavior have an impact in the size of the labor force. Finally, the third reason is that all other variables considered are averages of twenty-five or thirty years (including those used in the studies mentioned previously) and there is no apparent justification to make a exception with per-capita income.

Population growth is simply calculated as the average population growth rate for the period 1960–1990. Mankiw, Romer and Weil (1992) and other studies use working age population. I choose raw population mainly because the fertility model is focused on total population. Estimations presented in Islam (1995) show that the choice of which population definition to use does not change the results in any significant way. For the purpose of comparison I also present some estimations and specification tests using the data definitions of Mankiw, Romer and Weil (1992) in section 4 and the appendix. Finally, lagged population growth is the average population growth rate during 1950–1960 (from World Population 1983, Bureau of Census).

The data on saving rates in physical and human capital is the one presented in Mankiw, Romer and Weil (1992) appendix. These authors calculated physical capital savings rate as private plus government investment rates. The series considered the average 1960–1985. Although Summers and Heston (1991) present investment rates until 1992, I used the former source because the latter does not include government investment. The human capital savings rate proxy was calculated as the percentage of working age population that is in secondary education. Even though imperfect, it is argued that this proxy is probably proportional to the true rates; only the constant term would be affected. This series also considered the average 1960–1985.

The final series required is a proxy for the differential of gender earnings. I use the World Bank data of female ratio of the labor force (Female L.F. in section 4). Assuming that participation rates depend upon wages, higher earning differential ratios can be associated with lower female labor force participation. This is especially true in a family decision setting as the one described in the population growth sub-model (e.g., see González, 1992).

Two samples of countries are considered in the estimation. The first —named non-oil— has 98 countries and includes basically all world market economies with more than 1 million inhabitants. It excludes oil-producing countries in which a large proportion of the national wealth is generated from oil revenues. The second sample —named intermediate— has 75 countries and includes those countries of the first sample that Summers and Heston considered as having more reliable data (it excludes countries with grade "D" or less). These two same samples are used by Mankiw, Romer and Weil (1992). The list of countries and samples are presented in table 12 in the appendix.

In order to assess how the model is affected by cultural-religious aspects —especially the parameters of the population growth equation— I split the total sample of countries in two sub-samples considering country's major religion. Separate estimations based on each sub-sample are presented in section 4. The first group of countries contains those where more than 50% of the population is Moslem or Roman Catholic. It includes 46 observations and I call this sample "conservative" for obvious reasons. The other group considers all the other 52 countries and is named "liberal". In this sub-sample inhabitants are mainly Christian (non Roman Catholic), animist, have local traditional beliefs, or there is no dominant religion at all. The source for country religion is the World Almanac. Table 12 in the appendix lists to which sample each country belongs.

4 Estimation Results and Related Issues

In this section I present estimation results using both limited and full information methods and discuss some issues regarding the estimation. Since the basic income equation is non-linear in population growth, I use non-linear procedures for its estimation. When estimating the population growth equation using limited information simple linear methods are used. Issues discussed include the selection of the model, specification testing and overidentification restriction testing.

4.1 Limited Information Estimation

Two competing models are considered at a first glance. They are the model without human capital in the production function and the model with human capital as a production factor. Both models include the same population growth equation (13). The income equation is estimated using non-linear two stages least squares (NL2SLS) and the population equation using 2SLS. Naturally, the instruments for each equation are the exogenous variables of the other equation.

Estimation results of the first model using the samples intermediate and non-oil are presented in table 1. The results of both equations are satisfactory in that all the parameter have signs that are as expected and are significant. The only non-significant coefficient —though with the correct sign— is that of female labor force in the larger sample (non-oil). The income equation accounts for a large fraction of the per capita income variation (60%), as it usually does in cross-country studies. The population growth equation remarkably explains approximately 80% of the population growth rate cross-country variation. Finally, the hypothesis resulting from constant returns to scale in the production function (sum of coefficients a_1 and a_2 of equation (6) equal zero) is not rejected in either sample.

Table 2 presents etimation results for the second competing model that includes human capital as a production factor. The results of both equations are again very satisfactory. All signs are correct and only one coefficient is not significant in one sample (as in the other model, female labor force in the non-oil sample). The population growth equation changes very little. Since the only change in its estimation is the use of human capital savings rate (school) as an extra instrument, this result is fairly predictable. The income equation improves the fit significantly. This specification explains approximately 77% of cross-country variation of per capita income depending upon the sample. The hypothesis resulting from constant returns to scale in the production function (sum of coefficients a_1, a_2 and a_3 of equation (6)) is not rejected in either sample. However, in the non-oil sample the null hypothesis is close to be rejected. Assuming that the basic model is correct, this could be a sign of either non constant returns to scale or, more probably, problems in the human capital savings

Table 1: Model Without Human Capital - Limited Information

Sample:	Non-Oil	Intermediate
Observations:	98	75
Income Equation - NL2SLS		
Dependent Variable: Average per capita GDP (1960–1990)		
Constant	5.42	5.13
	(2.10) [2.00]	(2.09) [1.86]
Popl. + 0.05	−1.68	−1.84
	(0.73) [0.70]	(0.71) [0.63]
Savings	1.20	1.22
	(0.15) [0.13]	(0.20) [0.17]
$\bar{R}^2$	0.61	0.60
s.e.e.	0.64	0.63
F-CRS Restriction	0.34	0.56
p-value	0.56	0.46
Population Equation - 2SLS		
Dependent Variable: Population Growth Rate (1960–1990)		
Constant	4.93	5.11
	(0.60) [0.63]	(0.71) [0.81]
Popl. in 50's	0.57	0.58
	(0.05) [0.07]	(0.06) [0.08]
GDP	−0.50	−0.50
	(0.06) [0.06]	(0.08) [0.09]
Female Labor Force	−0.05	−0.11
	(0.04) [0.04]	(0.05) [0.05]
$\bar{R}^2$	0.78	0.82
s.e.e.	0.43	0.43
Long Run Implied Coefficients:		
GDP	−1.15	−1.19
	(0.17)	(0.19)
Female L.F.	−0.11	−0.25
	(0.09)	(0.13)

Standard errors in parenthesis. White robust in brackets. Female L.F. $\times 10$

Table 2: Model With Human Capital - Limited Information

Sample:	Non-Oil	Intermediate
Observations:	98	75
Income Equation - NL2SLS		
Dependent Variable: Average per capita GDP (1960–1990)		
Constant	2.62	3.09
	(1.45) [1.24]	(1.51) [1.19]
Popl. + 0.05	−1.98	−1.74
	(0.53) [0.48]	(0.55) [0.47]
Savings	0.53	0.57
	(0.13) [0.11]	(0.16) [0.14]
School	0.55	0.72
	(0.07) [0.07]	(0.11) [0.10]
$\bar{R}^2$	0.78	0.76
s.e.e.	0.48	0.48
F-CRS Restriction	2.28	0.49
p-value	0.14	0.48
Population Equation - 2SLS		
Dependent Variable: Population Growth Rate (1960–1990)		
Constant	5.38	5.68
	(0.51) [0.55]	(0.59) [0.64]
Popl. in 50's	0.55	0.57
	(0.05) [0.05]	(0.06) [0.07]
GDP	−0.55	−0.57
	(0.05) [0.04]	(0.06) [0.07]
Female Labor Force	−0.06	−0.11
	(0.04) [0.06]	(0.05) [0.05]
$\bar{R}^2$	0.78	0.82
s.e.e.	0.43	0.43
Long Run Implied Coefficients:		
GDP	−1.22	−1.28
	(0.16)	(0.18)
Female L.F.	−0.13	−0.24
	(0.09)	(0.11)

Standard errors in parenthesis. White robust in brackets. Female L.F. ×10

Table 3: Income Equation - Typical Inconsistent OLS Estimations

Sample:	Non-Oil	Intermediate
Observations:	98	75
Income Equation Without Human Capital - *Inconsistent* OLS		
Dependent Variable: Average per capita GDP (1960–1990)		
Constant	–0.67	0.26
	(1.39)	(1.53)
Popl. + 0.05	–3.81	–3.50
	(0.48)	(0.52)
Savings	0.95	0.96
	(0.13)	(0.17)
$\bar{R}^2$	0.68	0.65
s.e.e.	0.58	0.58
Specification Test	14.93	11.80
p-value	0.00	0.01
Income Equation With Human Capital - *Inconsistent* OLS		
Dependent Variable: Average per capita GDP (1960–1990)		
Constant	0.06	0.96
	(1.12)	(1.25)
Popl. + 0.05	–2.95	–2.54
	(0.41)	(0.44)
Savings	0.49	0.51
	(0.12)	(0.16)
School	0.50	0.65
	(0.07)	(0.10)
$\bar{R}^2$	0.79	0.77
s.e.e.	0.47	0.47
Specification Test	7.90	6.25
p-value	0.09	0.18

Standard errors in parenthesis.

tables 8 and 9 in the appendix. The rejection for the non-human capital specifications is similar. The specifications with human capital are not rejected even though the population growth coefficient decreases in approximately 50%.

A second issue that arises from the estimation is the quality of the instruments used for each equation. The model presented in Galor and Weil (1996) relates growth with endogenous fertility through the gender earnings gap. It assumes a production function where "mental" labor is complementary to physical capital while "physical" labor is not. The accumulation process makes "mental" labor more valuable and since in the model women have only this kind of labor skill —in comparison to men who have both— the gender earnings gap decreases with growth (and so does fertility). This kind of model brings the question about potential simultaneity between gender differential and income and its validity as an instrument for the income equation. The same type of doubts can be brought about the quality of human capital savings as an instrument for the population growth equation. For example, schooling could be modeled simultaneously with population growth. Fortunately, since both equations are overidentified, the orthogonality assumption of the instruments and the corresponding equation error can be tested.

Table 4 shows the results of the overidentification restrictions test (Hansen, 1982). Under the null hypothesis of correct specification ("good" instruments), the statistic presented has an asymptotic distribution χ^2 with degrees of freedom equal to the number of overidentification restrictions. The results include tests for both models (with and without human capital) and for both samples. For the income equation with human capital and the population growth equation the null hypothesis is not rejected. For the income equation without human capital the null hypothesis is rejected with 90% of confidence. If the test is performed controlling for heteroskedasticity —using a proper weighting matrix— the results do not change significantly. In such case the null is rejected with 95% of confidence and for the other two equations there is no rejection. The results are reported in table 10 in the appendix.

A plausible explanation for the rejection of the model without human capital is that the schooling variable could be correlated with female labor participation. If human

Table 4: Overidentification Restriction Tests

Sample:	Non-Oil	Intermediate
Observations:	98	75
Income Equation Without Human Capital (NL2SLS)		
Over ID Test	3.19	3.13
p-value	(1.39)	(1.53)
Income Equation With Human Capital (NL2SLS)		
Over ID Test	0.45	0.03
p-value	(0.50)	(0.86)
Population Growth Equation (2SLS)		
Over ID Test	1.90	2.01
p-value	(0.17)	(0.16)

capital is excluded from the income equation it will appear in the error, and female labor participation will no longer be a valid instrument. Anyhow, whatever the correct explanation for the rejection, the model with human capital seems to be a better model than the one without.

In order to provide further evidence about which of the two competing models is more appropriate I perform simple restriction F-tests. The null hypothesis is that the coefficient of human capital (a_3 in equation (6)) is zero. Also, given that previous demographic studies considered education and literacy as explanatory variables for cross-country fertility rates (e.g., see Birdsall, 1977), I test the exclusion restriction of schooling as an explanatory variable in the population growth equation. The null in this case is also that the coefficient is zero. The results, presented in table 11 in the appendix, show that the more appropriate model is the one that includes human capital. The restriction of the coefficient to be zero is rejected in both samples (p-value equal to 0.07 for non-oil and 0.04 for intermediate). For the population equation, on the other hand, the null hypothesis is not rejected in either sample (p-value equal to 0.40 for non-oil and 0.53 for intermediate). In sum, the model with human capital in the income equation and the exclusion restrictions described by equations (6) and (13) appear to be adequate.

4.2 Full Information Estimation

As with the limited information estimation, full information also requires a non-liner method. I use here non-linear three stages least squares (NL3SLS). Considering the results presented above only the model with human capital is estimated. After presenting the estimation results, I present a specification test comparing full to limited information.

Table 5 presents the NL3SLS estimation results. Both equations behave as expected in that all coefficients have correct signs and are significant with confidence of 95% or more. The only exception is again female labor force in the population growth equation for the non-oil sample. In that case the t-test is -1.65 (p-value equal to 0.10).

Again, the adjustment of both equations is good; both $\bar{R}^2$ are around 80%. Finally, the coefficients in both samples are fairly close to each other giving an idea of robust results.

Having both full and limited information estimation results of the model allows one to carry out an specification test comparing both estimates (Hausman, 1978 and 1983). However, one caveat applies: in order to perform such a test an efficient estimate is required and NL3SLS is not necessarily appropriate. In the linear case the efficient full information maximum likelihood estimator (FIML) has the same asymptotic distribution as 3SLS but this is not necessarily true in the case of non-linear models. Nevertheless, given that the non-linearity is only in variables (and not a major one) one can use the following result of asymptotic equivalence between FIML and NL3SLS (Amemiya, 1985 p. 259). In a model of i equations of the form

$$f_{it}\left(y_t, x_t, \alpha_i\right) = u_{it},$$

where y_t is a vector of endogenous variables, x_t a vector of exogenous variables and α_i a vector of parameters, FIML and NL3SLS are asymptotically equivalent if and only if f_{it} can be written in the form

$$f_{it}\left(y_t, x_t, \alpha_i\right) = A_i\left(\alpha_i\right)' z\left(y_t, x_t\right) + B_i\left(\alpha_i x_t\right),$$

where z is an N-vector of surrogate variables. Clearly, equation (6) can be written in this way and since equation (13) is linear, one can use the NL3SLS as the efficient estimate. The test compares the NL3SLS coefficients of each equation to the NL2SLS (and 2SLS) estimates and checks the internal consistency of the full model. Under the null hypothesis of correct specification both estimates are consistent. Under the alternative, only (NL)2SLS is consistent. The statistic distributes asymptotically as χ^2 with degrees of freedom equal to min $[k_i, p - p_i]$ where k_i are RHS variables in equation i and $p - p_i$ are overidentification restrictions of the rest of the system. Rejection would imply that the other equation is misspecified.

The results of the test are presented in table 6. In both equations the null hypothesis is not rejected at the usual levels of confidence. This means that the hypothesis of

Table 5: Model With Human Capital - Full Information

Sample:	Non-Oil	Intermediate
Observations:	98	75
Income Equation - NL3SLS		
Dependent Variable: Average per capita GDP (1960–1990)		
Constant	2.44	2.82
	(1.42)	(1.46)
Popl. + 0.05	−2.02	−1.78
	(0.52)	(0.53)
Savings	0.49	0.50
	(0.12)	(0.15)
School	0.57	0.75
	(0.07)	(0.10)
$\bar{R}^2$	0.79	0.78
s.e.e.	0.47	0.47
Population Equation - NL3SLS		
Dependent Variable: Population Growth Rate (1960–1990)		
Constant	5.44	5.70
	(0.49)	(0.57)
Popl. in 50's	0.55	0.55
	(0.05)	(0.05)
GDP	−0.55	−0.57
	(0.05)	(0.06)
Female Labor Force	−0.07	−0.11
	(0.04)	(0.05)
$\bar{R}^2$	0.79	0.83
s.e.e.	0.42	0.42
Long Run Implied Coefficients:		
GDP	−1.21	−1.27
	(0.15)	(0.21)
Female L.F.	−0.14	−0.25
	(0.08)	(0.10)

Standard errors (GMM) in parenthesis. Female L.F. $\times 10$.

Table 6: Specification Test - Full vs. Limited Information

Sample:	Non-Oil	Intermediate
Observations:	98	75
Income Equation - NL3SLS vs. NL2SLS		
Dependent Variable: Average per capita GDP (1960–1990)		
Test Statistic	1.30	1.38
p-value	0.25	0.24
Population Equation - NL3SLS vs 2SLS		
Dependent Variable: Population Growth Rate (1960–1990)		
Test Statistic	0.38	0.08
p-value	0.54	0.78

correct specification of both equations is not rejected and the NL3SLS estimation is consistent (and efficient).

5 Religion, Income and Population Growth

So far, the model proposed assumes the same constant parameters for all countries regardless of cultural aspects, geographic location and size characteristics. This section discusses and attempts to answer the question of whether a cultural aspects matter in the model. To make this question more specific I focus on countries' religious beliefs and ask whether the model estimates change when groups of countries with different beliefs are considered. In order to do this I divide the sample into two groups; I will refer to the first one as the "conservative" group (Moslem and Roman Catholic) and the other as the "liberal" group.

One could expect that cultural aspects do matter in the model. A-priori, there is no reason to assume the same behavior of culturally different countries regarding income and population growth. There is, however, a very important difference between the income equation and the population growth equation: while the latter has a household behavioral model as origin, the former has a technological support. The coefficients estimated in the income equation have are functions of the parameters of the assumed Cobb-Douglas production function. This is not the case with the population growth model. With these considerations in mind one could expect different estimates in the population growth equation depending upon the religious beliefs. It is by far less plausible though to expect systematic technology differences explained by countries' major religion.

Table 7 presents the NL3SLS estimates of the model for the two described samples. The general results, as in section 4, are satisfactory. Both equations present correct sign and significant coefficients. The only exception is female labor force in the sample "liberal" countries which cannot be considered different from zero. Both equations explain approximately 80% of the respective cross-country variation. As expected, the estimated coefficients for the income equation are fairly similar in both samples. The only exception is the savings rate (physical capital) which is smaller in the "conservative" sample. Nonetheless, the coefficient confidence intervals of each sample overlap. More interestingly, the population growth equation estimates change considerably. All three coefficients are quite different, especially the one for per-capita GDP. As one could have expected, the "conservative" countries population growth rate responds less to income growth than the "liberal" countries.' An unexpected result however is that female labor force only matters in the "conservative" sample. One would have expected a larger (negative) coefficient in the "liberal" countries.

Another interesting feature of the estimation are the long-run coefficients. The partial adjustment coefficients are presented in table 7 together with the long-run estimates of the other coefficients. As one could expect the partial adjustment coefficient of "liberal" countries is larger than the one of "conservative" countries; the population growth rates of the former change more rapidly with economic incentives than the latter's. Finally, the long-run coefficient of income (GDP) is very similar in both samples, meaning similar final behavior even though they present very different short run conduct. In sum, the results presented in this section indicate that the model behaves as expected. While the income equation is relatively similar in both groups of countries, the population growth equation changes significantly, especially regarding

Table 7: Religion, Income and Population Growth - Full Information

Sample:	Liberal	Conservative
Observations:	52	46
Income Equation - NL3SLS		
Dependent Variable: Average per capita GDP (1960–1990)		
Constant	1.88	2.05
	(2.44)	(1.46)
Popl. + 0.05	–2.21	–2.06
	(0.93)	(0.52)
Savings	0.50	0.32
	(0.16)	(0.16)
School	0.60	0.56
	(0.13)	(0.08)
$\bar{R}^2$	0.80	0.79
s.e.e.	0.53	0.40
Population Equation - NL3SLS		
Dependent Variable: Population Growth Rate (1960–1990)		
Constant	5.61	4.02
	(0.49)	(0.57)
Popl. in 50's	0.49	0.65
	(0.07)	(0.07)
GDP	–0.61	–0.37
	(0.06)	(0.09)
Female Labor Force	0.05	–0.13
	(0.09)	(0.05)
$\bar{R}^2$	0.82	0.81
s.e.e.	0.41	0.38
Long Run Implied Coefficients:		
GDP	–1.19	–1.05
	(0.17)	(0.26)
Female L.F.	0.10	–0.38
	(0.18)	(0.15)

Standard errors (GMM) in parenthesis. Female L.F. $\times 10$.

the short-run coefficients.

6 Concluding Remarks

This paper puts together a model of income growth and a model of population growth and attempts its estimation in a simultaneous equation setting. Estimation is done using both limited and full information procedures. Overidentification and exclusion restriction tests indicate that the model of income determination which includes human capital as a production factor is more appropriate than the one which does not include it. The paper also suggests that the typical estimation results of the income equation using OLS —inconsistent under the model proposed here— are considerably different from those using a consistent procedure. Approaches that consider growth and population growth as simultaneously determined should be considered in the growth empirical discussion.

The system estimation (full information) permits to examine the specification and internal consistency of the model through a test comparing full to limited information estimates. The null hypothesis of correct specification is not rejected in either equation. The results show that the proposed simultaneous model is fairly accurate in describing cross-country variation.

A cultural aspect like religion do affect the model. When considering the estimates of two groups of countries, one with "liberal" and the other with "conservative" countries, the population growth equation changes significantly. The short-run coefficient of income in the population growth equation is much larger in "liberal" than in "conservative" countries. However, the long-run coefficient is similar.

The next logical step from the point of view of this paper is to treat the decision of physical and human capital formation as an endogenous variable. For this purpose one should aim to a simple but reach enough savings model. This is not a simple task for exogenous determinants of the savings rate that are uncorrelated with fertility decisions and income level are quite scarce.

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Table 9: Mankiw, Romer and Weil (1992) Data With Human Capital

OLS and NL2SLS Estimation

Sample:	Non-Oil	Intermediate
Observations:	98	75
OLS Estimation		
Dependent Variable: 1985 per capita GDP		
Constant	3.83	4.43
	(1.18)	(1.15)
Popl. + 0.05	−1.74	−1.50
	(0.42)	(0.40)
Savings	0.70	0.70
	(0.13)	(0.15)
School	0.65	0.73
	(0.07)	(0.10)
$\bar{R}^2$	0.78	0.77
s.e.e.	0.51	0.45
NL2SLS Estimation		
Dependent Variable: 1985 per capita GDP		
Constant	5.24	5.76
	(1.42)	(1.34)
Popl. + 0.05	−1.23	−1.01
	(0.50)	(0.48)
Savings	0.73	0.74
	(0.13)	(0.15)
School	0.66	0.75
	(0.07)	(0.10)
$\bar{R}^2$	0.78	0.77
s.e.e.	0.51	0.46
Specification Test	3.26	3.76
p-value	0.51	0.44

Standard errors in parenthesis.

Instruments: Popl. in 50's and Female L.F.

Table 9: Mankiw, Romer and Weil (1992) Data With Human Capital

OLS and NL2SLS Estimation

Sample:	Non-Oil	Intermediate
Observations:	98	75
OLS Estimation		
Dependent Variable: 1985 per capita GDP		
Constant	3.83	4.43
	(1.18)	(1.15)
Popl. + 0.05	-1.74	-1.50
	(0.42)	(0.40)
Savings	0.70	0.70
	(0.13)	(0.15)
School	0.65	0.73
	(0.07)	(0.10)
$\bar{R}^2$	0.78	0.77
s.e.e.	0.51	0.45
NL2SLS Estimation		
Dependent Variable: 1985 per capita GDP		
Constant	5.24	5.76
	(1.42)	(1.34)
Popl. + 0.05	-1.23	-1.01
	(0.50)	(0.48)
Savings	0.73	0.74
	(0.13)	(0.15)
School	0.66	0.75
	(0.07)	(0.10)
$\bar{R}^2$	0.78	0.77
s.e.e.	0.51	0.46
Specification Test	3.26	3.76
p-value	0.51	0.44

Standard errors in parenthesis.

Instruments: Popl. in 50's and Female L.F.

Table 10: Heteroskedasticity Robust Overidentification Restriction Tests

Sample:	Non-Oil	Intermediate
Observations:	98	75
Income Equation Without Human Capital (NL2SLS)		
Over ID Test	3.80	5.16
p-value	0.05	0.02
Income Equation With Human Capital (NL2SLS)		
Over ID Test	0.58	0.88
p-value	0.44	0.35
Population Growth Equation (2SLS)		
Over ID Test	2.49	0.18
p-value	0.12	0.67

Table 11: Exclusion of Human Capital F-Tests

Sample:	Non-Oil	Intermediate
Observations:	98	75
Income Equation (NL2SLS)		
F-Test	3.25	4.18
p-value	0.07	0.04
Population Growth Equation (2SLS)		
F-Test	0.73	0.40
p-value	0.40	0.53