REAL MONEY HOLDINGS, MONEY GROWTH AND INFLATION: BRAZIL

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Resumen

La inflación ha sido una constante preocupación en los países latinoamericanos por mucho tiempo. En el caso brasileño, las frecuentes y persistentes explosiones de precios fueron resultado de malos manejos en políticas macroeconómicas. Este artículo aplica técnicas econométricas modernas para analizar la existencia de una relación estable entre saldos monetarios reales e inflación. La modelación ecónométrica tradicional utiliza funciones simples, lineales y homogéneas para representar esta relación (un ejemplo clásico de esta práctica viene dado por el modelo de Cagan). Este artículo utiliza técnicas de estimación Semi-Noparametricas que demuestran que el proceso bivariado entre los saldos monetarios reales e inflación presenta importantes desviaciones de normalidad y homogeneidad. Estas características proveen información útil al momento de analizar el efecto de innovaciones no anticipadas en ambas variables. Análisis no lineales de impulso-respuesta resaltan la presencia de asimetrías que no podrían ser capturadas en modelos VAR tradicionales. Finalmente, técnicas paramétricas y no paramétricas son utilizadas para evaluar la hipótesis de la existencia de una relación estable de largo plazo entre inflación y saldos monetarios reales. Esta hipótesis es fuertemente rechazada por los datos. El único periodo en que una relación del tipo propuesto por Cagan puede existir es para un rango muy reducido de niveles de inflación.

Abstract

Inflation has been a major concern in Latin American countries for a long time. In the case of Brazil, frequent and persistent price explosions have been the result of important macroeconomic mismanagement. This paper applies modern econometric techniques to analyze the existence of a stable relation between real money holdings and inflation. Traditional econometric models use simple (linear and homogeneous) functions to represent this relation. One classical example is constituted by the Cagan model. We use Semi-Non-Parametric estimation techniques that show that the bivariate process between real money holdings and inflation has important departures from gaussianity and homogeneity. This feature provides insights regarding the effects of unanticipated innovations of inflation and real money holdings on their trajectory in time. Non linear impulse response analysis highlights the presence of asymmetries that would not be detected by traditional VAR models. Finally, parametric and non parametric techniques are used to evaluate the hypothesis of a stable long run relation between inflation and real money holdings. This hypothesis is strongly rejected by the data. The only stage in which a Cagan type relation may hold is under a reduced range of inflation levels.

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I. Introduction

Inflation has been a major concern in Latin American countries for ages. In most cases price explosions have been the result of macroeconomic mismanagement; that is, growing fiscal deficit and a passive monetary policy. In certain countries such as Chile in the late 70's; Bolivia in the earliest 80's and Argentina in the late 80's; the process resulted in an explicit hyperinflation which lead to economic stagnation and in many cases, serious political crises. In the early 80's, many Latin American countries began to give great emphasis to price stabilization policies. Some successful programs to combat inflation were put in place by Chile, Mexico, Bolivia and Argentina. They have combined sustainable fiscal adjustment, tight monetary policy and eventually a monetary anchor such as exchange rate or nominal wage. As a result, in those countries inflation has declined substantially and is reaching low historical levels.

Brazil has been an important exception until recently. The country presents a classical example of high chronic inflation. From 1944 to 1984, only in the short period of 5 years has the inflation rate remained below 10% per year.[1] In a long run view, the tendency was for the inflation to keep rising. This was certainly the case from 1950 to 1962, when there was a sharp acceleration of price and a serious recession that lead to the overthrow of the president at that time, Jango Gular; in the "1964 Coup".

The new military government initiated a serious effort to stabilize the economy which combined classical elements such as fiscal reform, real rate of interests together with some heterodox components like wage and price controls. Therefore, inflation decreased to a level of roughly 16% in 1972; "The Brazilian Miracle".[2] However, the unexpected impact of external exogenous factors such as the two oil shocks and the external debt crises created the conditions for a reacceleration of the inflation process.

In the 80's annual inflation rates were already above 100% per year. In the second half of the decade, there were several attempts to stabilize through the so called "Heterodox

[1] According to the Getulio Vargas Foundation (FGV) IGP index.

[2] Source FGV.

Plans". These plans emphasized price and wage freeze controls rather than attacking the fundamental cause of inflation which was still a large imbalance in the public sector. In 1993, the Brazilian inflation reached the astonishing level of 210.99 per year.[3] Despite all the distortions, the economy was still working and as a matter of fact, the real GDP growth was of 5%.

The main reason why in the Brazilian case it has been possible to live with such high chronic inflation was the existence of a very sophisticated system of indexation. This process started in 1984 with the launching of indexed government bonds in order to make it viable to finance a proportion of the still existing public deficit. It was subsequently extended to different forms of savings which assured a positive real rate of return for investments. Later on, indexation was also applied to wage and exchange rate (the "Crawling Peg" mechanism). Indexation was also present in the tax system to avoid the taxation of pure nominal profits. With such a widespread system of indexation it was possible to minimize some but not all of the allocative and distributive costs of inflation. At the same time indexation makes it more difficult to reduce inflation because of the rigidity imposed upon nominal prices such as the wage rate.

Another problem in the Brazilian economy up to the consolidation of the Real Plan was the presence of the inertia effect, by which past inflation affects the current level of prices through indexation. Therefore, establishing a floor below which it is very hard to bring inflation down. The Brazilian experience has however shown that indexation works relatively well if the rate of inflation is relatively stable around a low level (by Brazilian standards), that is below 20% per year. Therefore, it does not work well when there is an unexpected rise in prices or when inflation reaches monthly levels above 10%. In such circumstances, the timing of the realization of indexation became shorter and shorter; moreover, it could not avoid substantial redistribution of income and less incentive for long run productive investments.

From 1986 until 1994 there were frustrated attempts to combat inflation, but the results were short lived. Inflation did come down in the first months basically because of severe price and wage controls. However, after a period of time the results of an

[3] Source FGV.

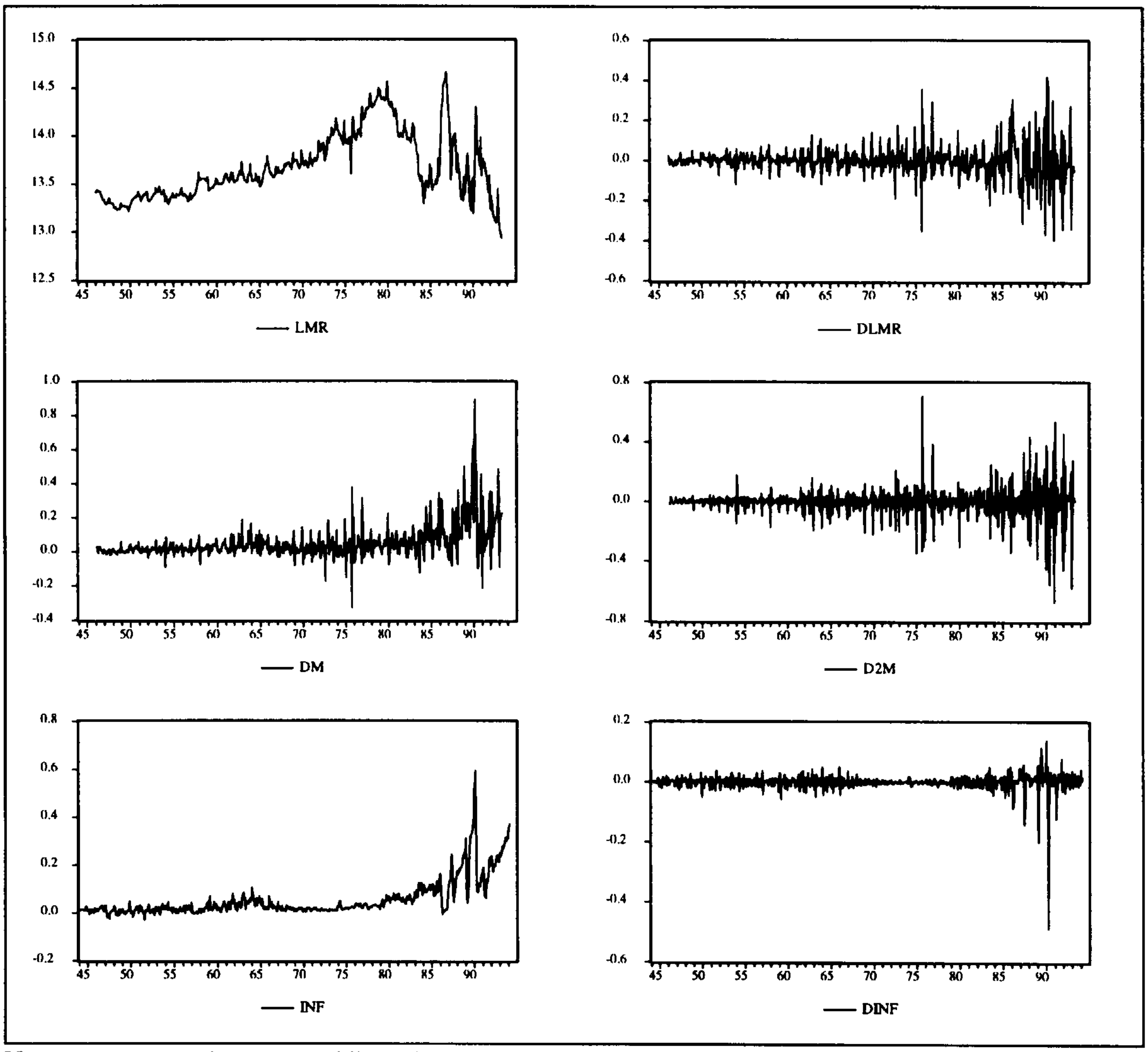
inappropriate fiscal and monetary policy generated an excess aggregate demand and thus a shortage of goods, specifically food. This was followed by an explosion of prices with inflation going back either to the same levels where it was before the implementation of the policies, or in some extreme cases the acceleration of prices brought a higher inflation than the original levels. A good example is the "Cruzado Plan". The fundamental mistake was not having balanced the public budget and having in practice negative real rates of interests which lead to a reallocation of funds from savings to consumption.

The "Collor Plan I" in 1989, reduced the monthly inflation from 80% to approximate 2%, but this low level was made possible through the seizure of financial assets which affected drastically the incentives of the private sector to save. Again, the result of forceful measures on the fiscal side lead in six months to a sharp acceleration of inflation which reached the level of 80% just one year after. As a consequence, there was a tremendous amount of instability in the Brazilian economy caused by the mismanagement of macroeconomic policy. This is reflected in the data by the cyclical behavior of the rate of inflation.

Brazil is undergoing a process of implementing another stabilization plan, "The REAL Plan", which for the first time tries to eliminate a chronic public sector deficit that has been the historical cause of the Brazilian inflation. At the same time, it had a transition period where all prices in the economy were being translated into a stable unit of account, "unidade real de valor" (URV), pegged to the dollar. This was a clever way to eliminate in practice the inertia effect of the Brazilian inflation. Once all prices and wages had been realigned with this unit, a new currency, named REAL, with a fixed parity to the dollar, was introduced. A tight monetary policy with very high real interest rates will be maintained until there is a favorable change in inflationary expectation.

The objective of this study is to apply modern econometric techniques and the traditional Cagan model to Brazilian monthly data from 1944 until 1994.

Figure 1. Real Money Holdings, Rate of Growth of Monetary Base and Inflation (1944:01-1994:12)



Notes: LMR = Real Money Holdings (in logs). DM = Rate of Growth of Monetary Base. INF = Inflation. First differences on the right (DLMR, D2M, and DINF respectively).

The evolution of the times series make clear that there exist at least three different regimes for the inflation and the rate of growth of the monetary base. It is of interest then to evaluate the well known proposition that there exists a stable relation between real money holdings and inflation.

Table 1 and 2 below show some stylized facts of the monthly time series data from 1944 to 1994.

Table 1. Summary Statistics.			
Variable[a]	LMR	DM	INF
Mean	13.673	0.052	0.055
Std.Dev.	0.342	0.102	0.080
Var. Coeff.	0.025	1.961	1.452
Skewness	0.742	2.784	2.832
Kurtosis	2.803	17.030	12.938
J-B[b]	53.071	5392.379	3282.177
Coc (13)[c]	0.665	0.080	0.288
Coc (25)	0.567	0.047	0.193
AR[d]	14	13	1

Notes. [a] See Figure 1 for definitions.
[b] Jarque-Bera Normality Test. Distributed as Chi² (2) with critical value (5%) 5.992.
[c] Cochrane (1988) persistence index. The number in parenthesis corresponds to the number of period taken into account in the construction of the index.
[d] Autoregressive part. Computed using the Schwartz Criterion.

The coefficient of variation is a scale free indicator of volatility, which in these cases show that DM and INF are much more volatile than LMR. Another important fact, is the presence of significantly positive skewness in the three cases, while, at least for the case of DM and INF there exist leptokurtosis which is a sign of fat tails departing from gaussianity. Thus, the Jarque Bera test rejects the assumption of normality in the three series. The COC test is a test for persistence, the closer to one the more persistent the series is. As discussed in Christiano and Eichenbaum (1990) this indicator may be better than the Dickey-Fuller type tests for unit roots. From Table 1 we notice that LMR has the most significant value since COC 13 is relatively close to 1 and when we look at the series at COC 25 the value is not so different from the first 13 observations. This means that even after 2 years, an innovation in real money holdings continues to have an important effect. The other two series seem not to be very persistent according to the COC values. Finally, according to the Schwarz criterion the preferred univariate autorregresive process is an AR (14) for LMR, an AR (13) for DM and an AR (1) for INF.

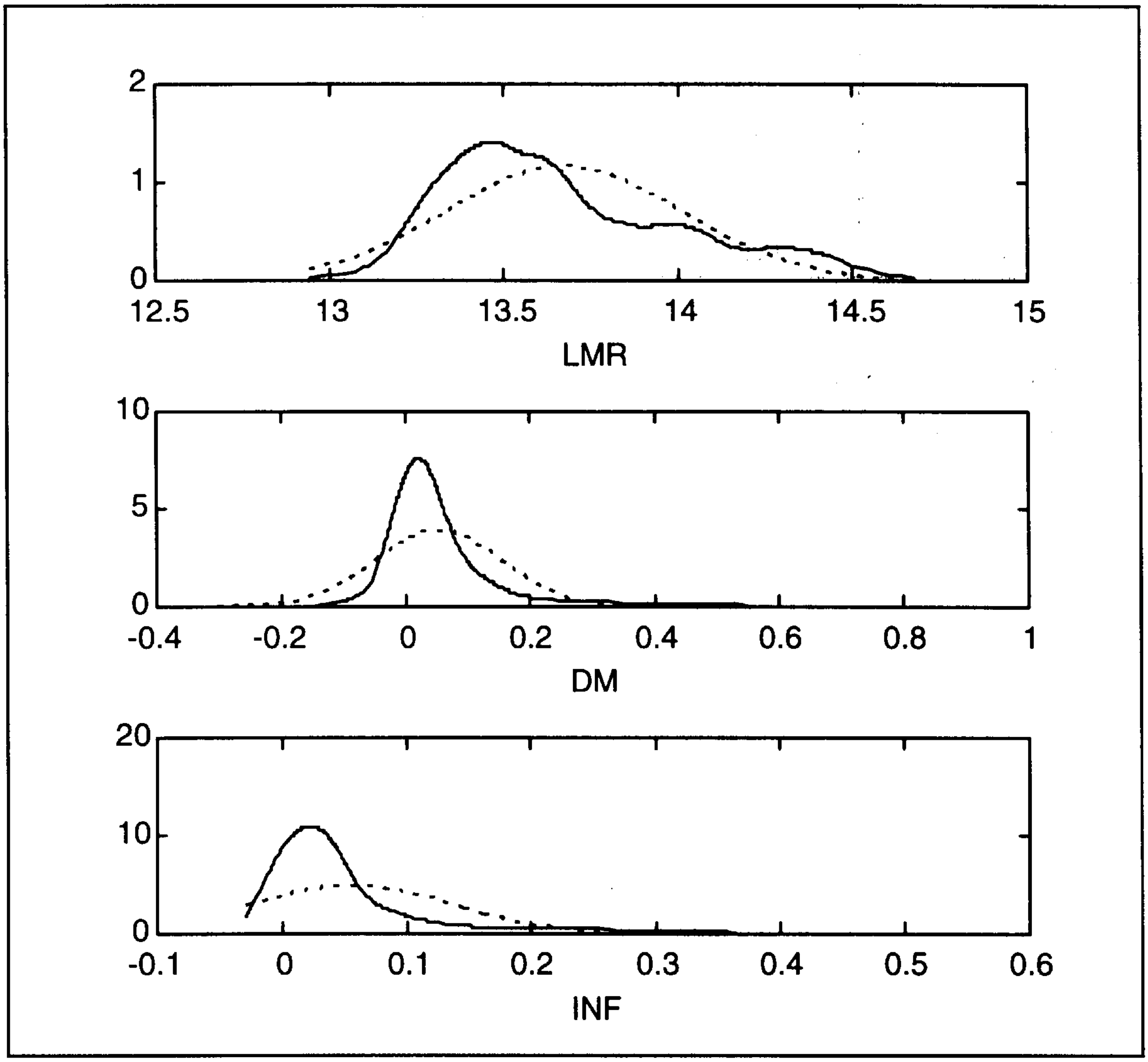
Table 2. Summary Statistics.			
Variable	DLMR	D2M	DINF
Mean	-0.001	0.0003	0.001
Std.Dev.	0.084	0.113	0.030
Skewness	0.119	-0.297	-7.410
Kurtosis	9.404	12.833	118.911
J-B	971.881	2292.447	341946.200
Coc (13)	0.069	0.041	0.062
Coc (25)	0.039	0.023	0.037

Notes. See Figure 1 and Table 1 for definitions.

The skewness of DLMR is positive and far from zero. D2M and DINF present negative skewness indicating the presence of important asymmetries. The Jarque Bera test rejects the null of normality for all variables. The three series present a low value of COC indicating no evidence of persistence. That is, as expected these series are stationary in difference. It is possible that DM and INF are already stationary. Figure 2 presents a "crude" estimate of the unconditional PDF of each series obtained by non parametric methods with automatic bandwidth selection. It is clear that LMR is not unimodal, while DM and INF are asymmetric and have fat tails.[4]

[4] It may be suspected that the leptokurtosis that is evident in all series may be due to the presence of outliers in specific subperiods, however, the stylized facts reported in Tables 1 and 2 as well as the non parametric density estimate does not change fundamentally when subperiods are analyzed instead of computing statistics of the whole sample. It may also be argued that LMR may not be unimodal due to structural changes on its PDF. This point will be discussed later.

Figure 2. Non Parametric Estimates of the Unconditional PDF of LMR, DM, and INF



Notes: The continuous line corresponds to the estimated of the unconditional PDF using Epanechnikov kernel and cross validation (Silverman, 1986). The dotted line corresponds to a normal pdf with the same mean and variance. See Figure 1 for definitions.

Table 3. Granger Causality / Correlation Matrix			
—	LMR	DM	INF
LMR	_ 1	yes 0.022	yes -0.123
DM	yes 0.022	_ 1	yes 0.589
INF	yes -0.123	yes 0.589	_ 1

Even when LMR and INF are negatively correlated, the magnitude is relatively small, while as expected DM and INF have positive correlation. All variables Granger cause each other, so that it would be difficult to try to construct counterfactual scenarios from the estimated money demands in which inflation is on the right hand side. See Engle, et.al. (1983).

Table 4. Unit Root Test Augmented D.F.		
	Const.	Time Trend
LMR	-1.919	-1.435
DM	-2.413	-3.871
INF	-3.796	-5.699
Notes: The critical value with constant term is 2.867 and the critical value with time trend is 3.419; using ordinary least squares.		

Table 4 confirms the results presented in Table 1. LMR seems to be non stationary while DM and INF reject the unit root hypothesis, once a deterministic trend is incorporated. As known, the power of this test is low, but joined with the evidence from Table 1, it is reasonable to conclude that DM and INF seem to be stationary.

Table 5. Best Univariate Fits			
	LMR	DM	INF
Q (30)[a]	39.451	49.630	30.802
J-B	797.145	3358.644	134172.097
ARCH[b]	(1) 134.771	(3) 105.753	(3) 22.221
White[c]	26.443	246.592	269.426

Notes: [a] Ljung and Box Q-statistic test for white noise with critical value Chi²(30)=43.773.
[b] Chi²(1)=3.841, Chi²(3)=7.815.
[c] White test for heteroskedasticity.

Table 5 shows the results from univariate fits constructed according to the AR criteria exposed in Table 1. As can be seen most of the residuals from the fits can be characterized as non gaussian, heteroskedastic innovations.

The remainder of the paper is organized as follows: Section II presents a brief review of the Cagan model of hyperinflation. Section III applies the SNP technique and Johansen Procedure for cointegration using Brazilian monthly data of inflation, money growth and real money holdings from 1944 to 1994. Finally the last section concludes and summarizes the most important findings.

II. The Cagan Model

The traditional Cagan Model of hyperinflation implies the existence of a stable relation between real money holdings (log) and the expected inflation. Furthermore under the assumptions of rational expectations the empirical implication of this model is that LMR and INF should be cointegrated. Formally:

$$(m-p)_t = -\alpha \Delta p^e_{t+1} + \psi_t \tag{1}$$

where m and p denote the logarithm of nominal money balances and prices respectively, superscript e denotes expectations formed at time t and ψ is a random variable with mean zero.[5]

[5] As Phylaktis and Taylor (1993) stress, if the omitted variables captured in ψ_t are stationary, then this random variable would be stationary also. Cagan's insight is that under severe inflationary periods, real money

Assuming rational expectations **(1)** can be expressed as:

$$(m - p)_t = -\alpha\Delta p_{t+1} + \varepsilon_{t+1} \quad \textbf{(2)}$$

where $\varepsilon_{t+1} = \left[\psi_t + \alpha\left(\Delta p_{t+1} - \Delta p^e_{t+1}\right)\right]$.

According to the previous notation LMR=*m-p* and INF=Δp, hence a simple test of the applicability of the hyperinflation model lies in testing whether or not real money holdings and inflation are cointegrated. In that case a consistent estimator of α is the OLS estimator. Another estimator for α can be found applying Johansen's (1988) procedure. In Section 3b we present both.

There are a number of short comings from the model described in equation **(2)**. Some of them are:

- The existence of a stable inverse relation between LMR and INF does not necessarily mean that it should be linear.
- The random variable ε is composed by the combination of two variables, the first of which may have a systematic component and maybe susceptible to nonstationarities due to changes in regime; this would weaken the conclusion of the existence of cointegration.
- In accordance to the last observation, ψ_t may vary in importance for the demand for real money holdings, being more important in periods of low inflation.

III. Empirical Work

a. SNP

This section presents the results of estimates of univariate and bivariate conditional distributions using the SNP method, which as the name suggest, lies halfway between parametric and nonparametric procedures.[6]

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holdings would be basically determined by the expected inflation with ψ_t playing a relatively minor role in their determination.

[6] See Gallant and Tauchen (1993) or Gallant, Rossi and Tauchen (1992).

Following Gallant and Tauchen (1993) closely, a brief description of the method is presented. The method is based on the notion that a Hermite expansion can be used as a general-purpose nonparametric estimator of a conditional density function. Estimation entails using maximum likelihood procedures on a truncated expansion together with a model selection strategy that determines the truncation point. Under reasonable regularity conditions, the estimator is consistent for the true density under a norm that is strong enough to imply consistency of evaluation functionals and conditionals moments.

Letting z denote an M-vector, the particular Hermite expansion employed has the form $\hbar(z) \propto [\mathrm{P}(z)]^2 \phi(z)$, where P(z) denotes a multivariate polynomial of degree K_z and $\phi(z)$ denotes the density function of the Gaussian distribution with mean zero and the identity matrix as its variance-covariance matrix. The constant of proportionality is the divisor $\int [\mathrm{P}(z)]^2 \phi(z)dz$, which makes h(z) integrate to unity. Because of this division, the density is a homogeneous function of the coefficients of the polynomial P(z), and these coefficients can only be determined to within a scalar multiple. To achieve a unique representation, the constant term of the polynomial part is put to unity.

The location scale shift $y = Rz + \mu$, where R is an upper triangular matrix and μ is an M-vector, followed by a change of variables, leads to a parameterization that is easy to interpret: $f(y/\theta) \propto \{P[R^{-1}(y-\mu)]\}^2 \{\phi[R^{-1}(y-\mu)]/|\det(R)|\}$.

Because $\{\phi[R^{-1}(y-\mu)]/|\det(R)|\}$ is the density function of the M-dimensional, multivariate Gaussian distribution with mean μ and variance-covariance matrix $\Sigma = RR'$, and since the leading term of the polynomial part equals unity, the leading term of the entire expansion is the multivariate Gaussian density function; denote it by $\eta_m(y/\mu,\Sigma)$. When K_z is zero one gets $\eta_m(y/\mu,\Sigma)$ exactly. When K_z is positive, one gets a Gaussian density whose shape is modified because of multiplication by a polynomial. The shape modifications thus achieved are rich enough to accurately approximate densities from a large class that includes multimodal densities, densities with fat t-like tails, densities with tails that are thinner than Gaussian, and skewed densities.

The parameters θ of $f(y/\theta)$ are made up of the coefficients of the polynomial P(z) plus μ and R and are estimated by maximum likelihood. A procedure that is equivalent to maximum likelihood, but more stable numerically, is to estimate θ in a sample of size n by minimizing $S_n(\theta) = (-1/n)\sum_{t=1}^{n} \ln[f(y_t/\theta)]$. If the number of parameters p_θ grows with the sample size n, then the true density, and various features of it such as derivatives and moments, are estimated consistently. Because the method is parametric yet has nonparametric properties, it is termed seminonparametric.

The basic approach is adapted to estimate the conditional density of a multiple time series that has a Markovian structure, that is, the conditional density of the M-vector y, given the entire past y_{t-1}, y_{t-2}, ...depends only on L lags from the past. Let x_{t-1} be the vector of lags, which has length M*L. A density is obtained by the location-scale shift $y_t = Rz_t + \mu_x$ off a sequence of normalized errors $\{z_t\}$. Where μ_x is a linear function of x_{t-1} and the leading term of the expansion $\eta_M(y/\mu_x,\Sigma)$, which is a Gaussian vector autoregression (Gaussian VAR). In time series z_t are usually refereed as linear innovations. To allow the innovations to be conditionally heterogeneous the coefficients of the polynomial P(z) are, themselves, polynomials of degree K_x in x_{t-1}; which is denoted P(z, x). Therefore, when K_x is zero the $\{z_t\}$ are homogeneous, since they do not depend on x_{t-1}. When K_x is positive, $\{z_t\}$ are conditionally heterogeneous. The tuning parameter K_z controls the extent to which the model deviates from normality, while K_x controls the extent to which these deviations vary with the past history.

A way of keeping K_x small is to put the leading term of the expansion to a Gaussian ARCH rather than a Gaussian VAR. This can be done by letting R be a linear function of the absolute values of L_r of the lagged y_t, that have been centered and scaled to have mean zero and identity variance-covariance matrix.

Let $\Sigma_x = R_x R_x$ with vech(R_x) = P_0+P_1abs(x_{t-1}^*), where P_0 and P_1 are coefficient matrices of dimension (M*(M+1)/2) x 1 and (M*(M+1)/2) x M*L, respectively. The conditional density becomes $f(y/x,\theta) \propto [P(z,x)]^2 \eta_M(y/\mu_x,\Sigma_x)$, where $z = R_x^{-1}(y-\mu_x)$ and θ denotes the coefficients of the polynomial P(z,x) and the Gaussian ARCH $\eta_M(y/\mu_x, \Sigma_x)$

collected together. The parameters are estimated by minimizing $S_n(\theta) = (-1/n)\sum_{t=1}^{n} \ln[f(y_t / x_{t-1}, \theta)]$.

Table 6, 7 and 8 below show the empirical findings associated with the univariate models. The tuning parameters of the SNP method for a univariate model are the following: L_u corresponds to the number of lags of the VAR part; L_r the number of lags of the ARCH part; L_p the number of lags corresponding to the x part of the polynomial; K_z and K_x corresponds to the degree of the polynomial P(z,x) and hence the nature of the innovation process $\{z_t\}$. Finally I_z and I_x reflect the number of interactions.

The model selection strategy suggested by Gallant, Hsieh and Tauchen (1991), was applied. The Schwartz or BIC criterion was used to select a model, that is the one with the smallest value is preferred. The Schwartz preferred model is then submitted to a battery of specification tests; these tests indicate if further expansion of the model is necessary.

The Schwartz criterion is computed as:

$$BIC = S_n + \tfrac{1}{2}(p_o / n)\log(n) \qquad \textbf{(3)}$$

The Hannan-Quinn criterion is another way to find the most adequate model and is computed as

$$HQ = S_n + (p_o / n)\log[\log(n)] \qquad \textbf{(4)}$$

The diagnostic tests for predictability are conducted in both residuals and square of the residuals. Therefore, they are called the mean and variance tests. For both mean and variance two type of tests can be done: one sensitive to the short term misspecification and the other to long term.

Unfortunately, the long term tests show systematic modifications in the pattern of the mean and variance of the residuals. The results reported in Table 6 show the short term test. In each univariate regression the F-statistic is applied to test the null hypothesis that all coefficients other than the intercept term are zero. All results are in the next three tables below.

Table 6. Univariate Real Money Holdings Series:
Optimized Likelihood and Residuals Diagnostics

L_u	L_τ	L_p	K_z	I_z	K_x	I_x	S_n	P	HQ	BIC	Mean	Var.
14	0	1	0	0	0	0	-0.096	16	-0.045	-0.007	65.34	64.17
14	2	1	0	0	0	0	-0.113	18	-0.055	-0.013	56.97	724.61
14	4	1	0	0	0	0	-0.117	20	-0.052	-0.005	74.02	721.71
14	6	1	0	0	0	0	-0.119	22	-0.048	-0.004	71.29	714.97
14	2	1	2	0	0	0	-0.155	20	-0.091	-0.044	76.03	704.36
14	2	1	4	0	0	0	-0.305	22	-0.234	-0.183	30.60	672.34
14	2	1	6	0	0	0	-0.339	24	-0.261	-0.205	74.43	691.15
14	2	1	8	0	0	0	-0.347	26	-0.263	-0.202	87.22	696.40
14	2	1	10	0	0	0	-0.345	28	-0.254	-0.189	119.60	723.94
14	2	2	8	0	1	0	-0.406	44	-0.263	-0.161	26.49	380.78
14	2	4	8	0	1	0	-0.431	62	-0.230	-0.085	81.65	187.34
14	2	2	8	0	2	0	-0.545	71	-0.314	-0.149	34.50	76.35
14	2	4	8	0	2	0	-0.675	152	-0.181	-0.172	54.20	145.24
4	2	1	6	0	0	0	-0.158	14	-0.113	-0.080	60.87	647.12
10	2	1	6	0	0	0	-0.210	20	-0.146	-0.099	61.73	657.63
16	2	1	6	0	0	0	-0.341	26	-0.256	-0.196	73.92	696.42

From Table 6 above, the Schwarz preferred model has L_u=14, L_τ=2, L_p=1, K_z=6, I_z=0, K_x=0 and I_x=0 with P=24. This result is consistent with the findings presented above; that is, LMR presents non-gaussianity and heterogeneity.

The diagnostics test rejects the null of innovation in the residuals. These results come in the case of the mean by regressing the residuals on up to three lags of this series and its squares. For the case of the variance, given that the mean of the residuals is zero, a regression of the squared residuals with up to three lags of the squares and fourth power of the series was run.

Table 7. Univariate Money Growth Series:
Optimized Likelihood and Residuals Diagnostics

L_u	L_T	L_p	K_z	I_z	K_x	I_x	S_n	P	HQ	BIC	Mean	Var.
13	0	1	0	0	0	0	1.155	15	1.203	1.238	132.61	736.16
13	2	1	0	0	0	0	0.892	17	0.947	0.987	19.04	100.90
13	4	1	0	0	0	0	0.869	19	0.931	0.975	30.15	143.67
13	6	1	0	0	0	0	0.859	21	0.927	0.976	26.42	160.38
13	4	1	2	0	0	0	0.820	21	0.888	0.937	71.73	194.42
13	4	1	4	0	0	0	0.774	23	0.849	0.902	55.07	69.25
13	4	1	6	0	0	0	0.724	25	0.805	0.863	48.12	168.59
13	4	1	8	0	0	0	0.731	27	0.819	0.882	132.92	301.59
13	4	2	6	0	1	0	0.684	39	0.811	0.902	58.10	89.10
13	4	4	6	0	1	0	0.603	53	0.775	0.899	89.49	130.16
13	4	6	6	0	1	0	0.585	67	0.803	0.959	99.04	138.02
13	4	2	6	0	2	0	0.606	60	0.801	0.941	62.96	5.67
13	4	4	6	0	2	0	0.503	123	0.903	1.189	64.93	192.94
3	4	1	6	0	0	0	0.923	15	0.972	1.007	78.28	85.15
8	4	1	6	0	0	0	0.882	20	0.947	0.994	23.85	247.60
15	4	1	6	0	0	0	0.727	27	0.815	0.878	46.83	165.27

In this case, the Schwarz preferred model has L_u=13, L_T=4, L_p=1, K_z=6, I_z=0, K_x=0 and I_x=0 with P=25. This series also shows important departures from gaussianity and homogeneity. The short term diagnostics tests for mean and variance show the failure of the estimation to produce innovations.

Table 8. Univariate Monthly Inflation Rate Series: Optimized Likelihood and Residuals Diagnostics												
L_u	L_r	L_p	K_z	I_z	K_x	I_x	S_n	P	HQ	BIC	Mean	Var.
1	0	1	0	0	0	0	0.403	3	0.412	0.419	150.46	861.87
1	2	1	0	0	0	0	0.105	5	0.120	0.131	49.76	592.21
1	4	1	0	0	0	0	0.091	7	0.113	0.128	37.26	458.88
1	6	1	0	0	0	0	0.072	9	0.100	0.120	64.46	437.27
1	8	1	0	0	0	0	0.069	11	0.103	0.128	65.37	446.21
1	6	1	2	0	0	0	0.006	11	0.040	0.065	144.41	386.59
1	6	1	4	0	0	0	0.027	13	0.068	0.097	142.51	489.53
1	6	1	6	0	0	0	-0.048	15	-0.002	0.032	98.97	380.05
1	6	1	8	0	0	0	0.037	17	0.089	0.127	132.72	607.19
1	6	2	6	0	1	0	-0.193	29	-0.104	-0.039	74.56	111.07
1	6	4	6	0	1	0	-0.194	43	-0.061	0.035	12.28	110.57
1	6	2	6	0	2	0	-0.198	50	-0.044	0.068	96.54	57.53
1	6	4	6	0	2	0	-0.303	113	0.045	0.297	4.81	40.06
2	6	2	6	0	1	0	-0.186	30	-0.093	-0.027	69.18	103.82
3	6	2	6	0	1	0	-0.186	30	-0.103	-0.033	37.37	72.27
5	6	2	6	0	1	0	-0.186	30	-0.103	-0.030	26.42	57.52

From Table 8 above the Schwarz preferred model has L_u=1, L_r=6, L_p=2, K_z=6, I_z=0, K_x=1 and I_x=0 with P=29. In this case, the evidence also shows that gaussianity and homogeneity are not present. In this particular case however, K_x is different from zero, thus enhancing the departure from homogeneous innovations and linearity.

Tables 6-8 are consistent with the findings of previous tables, particularly Table 1; that is, the univariate conditional distribution of this series departs significantly from normality and it is necessary to incorporate important conditional heteroskedasticity to resemble the tempestuous behavior of these series. For the case of LMR and DM large values of autocorrelation are necessary, but this does not assure the presence of innovations, given that the diagnostics tests reject this hypothesis for all the series.

The following table presents the results corresponding to the bivariate fit of LMR and INF. As motivated in Section II, the conditional distribution of real money holdings is assumed to have a stable relationship with (expected) inflation. Under Rational Expectations, the difference between the actual and expected levels of inflation must be an innovation. Thus the results reported here and in the following section use this assumption to replace the actual for the expected inflation in the estimation. A stronger assumption may be that agents have perfect foresight about inflation, but that other components of **(1)** are not known.

Table 9. Bivariate Model: Monthly Real Money Holdings and Inflation Rate Series: Optimized Likelihood

L_u	L_r	L_p	K_z	I_z	K_x	I_x	S_n	P	HQ	BIC
2	0	1	0	0	0	0	0.470	13	0.513	0.543
4	0	1	0	0	0	0	0.453	21	0.521	0.570
6	0	1	0	0	0	0	0.442	29	0.536	0.603
8	0	1	0	0	0	0	0.434	37	0.554	0.640
1	0	1	0	0	0	0	0.503	9	0.532	0.553
3	0	1	0	0	0	0	0.464	17	0.519	0.559
2	2	1	0	0	0	0	0.114	17	0.169	0.209
2	4	1	0	0	0	0	0.090	21	0.158	0.207
2	6	1	0	0	0	0	0.062	25	0.143	0.202
2	8	1	0	0	0	0	0.056	29	0.150	0.218
2	6	1	2	0	0	0	-0.064	30	0.034	0.103
2	6	1	4	0	0	0	-0.177	39	-0.051	0.040
2	6	1	6	0	0	0	-0.298	52	-0.129	-0.008
2	6	1	8	0	0	0	-0.274	69	-0.050	0.111
2	6	1	6	0	1	0	-0.515	108	-0.164	-0.325
2	6	1	6	0	2	0	-0.693	192	-0.069	0.378

The preferred Bivariate model according to the Schwarz criterion is L_u=2, L_r=6, L_p=1, K_z=6, I_z=0, K_x=0, I_x=0. These results show that the bivariate case is consistent with an important departure from gaussianity and homogeneity. However a lower autoregressive representation (AR) is needed.[7]

[7] It may be argued that both in the univariate fits and in the bivariate representation the important departures from linearity and homogeneity that are present come from unstable linear representations and that the estimation procedure should take into account the possibility of structural breaks. It should be pointed out that testing one view against the other requires a rather involved procedure. A preferable way to deal with this problem may be to introduce a nested model in which switching regimes are allowed. This is nevertheless a computationally intensive procedure that (to my knowledge) has not been undertaken yet. The reason is rather simple, numerical optimization of SNP and switching regime models requires a sample size that is not currently available in order to provide sensible answers to this (possibly) competing views. Nevertheless, stability tests performed to the bivariate model show that the stochastic properties of the model are not significantly altered with changes in sample size.

Figures 3 and 4, show the results from impulse response exercises or "error shock" performed with the Schwarz preferred bivariate model following the methodology outlined in Gallant and Tauchen (1993). The basic idea is to trace trough the system the effects of small movements in the innovations, or linear combinations of the innovations. All the various techniques that exist to catch the effects of innovations, provide a means for exploring the characteristics of the conditional density, which can be rather complicated even for linear processes.

Nonlinear impulse-response analysis, involves a comparison of a conditional moment profile to a baseline profile. A conditional moment profile is the forecast made at time t of the time t+j value of a time-invariant function regarded as a function of j. Equivalently, is the conditional expectation evaluated at time t of a time-invariant function evaluated at time t+j regarded as function of j.

This procedure requires an efficient algorithm to simulate a sample path. A conditional moment profile can be obtained by running a time-invariant function out over many simulated sample paths and then averaging. Bootstrap estimates, which are used to compute sup-norm bounds on profiles, are obtained by simulating the sample path on the entire data and re-estimating the density.

Formally; let $\{y_t\}_{t=-\infty}^{\infty}$ with $y_t \in R^M$ be a strictly stationary process with a conditional density function that depends upon at most L lags. Denote the L lags of y_{t+1} by $x_t = (y_{t-l+1}', ..., y_t')' \in R^{ML}$ and write $f(y_{t+1}/x_t)$ for the (one-step ahead) conditional density. Due to the assumption of strict stationarity, $f(y/x)$ does not depend on t; that is, the density is time-invariant.

Define the conditional mean profile $\{\hat{y}_j(x)\}_{j=0}^{\infty}$ corresponding to initial condition x by $\hat{y}_j(x) = \varepsilon(y_{t+j}/x_t = x) = \int y f^j(y/x)dy$ where $f^j(y/x)$ denotes the j-step ahead conditional density:

$$f^j(y/x) = \int \dots \int \left[\prod_{i=0}^{j-1} f(y_{i+1}/y_{i-L+1}, \dots, y_i) \right] dy_1 \dots dy_{j-1} \quad \text{with } x = (y_{-L+1}', \dots, y_0)' .$$

In empirical work, $f^{j}(y/x)$ is approximated by a nonparametric estimate $\hat{f}(y/x)$ in place of $f(y/x)$. Given an efficient algorithm for sampling $\hat{f}(y/x)$, $\hat{y}_j(x)$ is easily computed by Monte Carlo integration.

Let $\delta_y^+, \delta_y^- \in R^M$ be small perturbations to y_0, where:

δ_y^+ is positive and δ_y^- is negative.

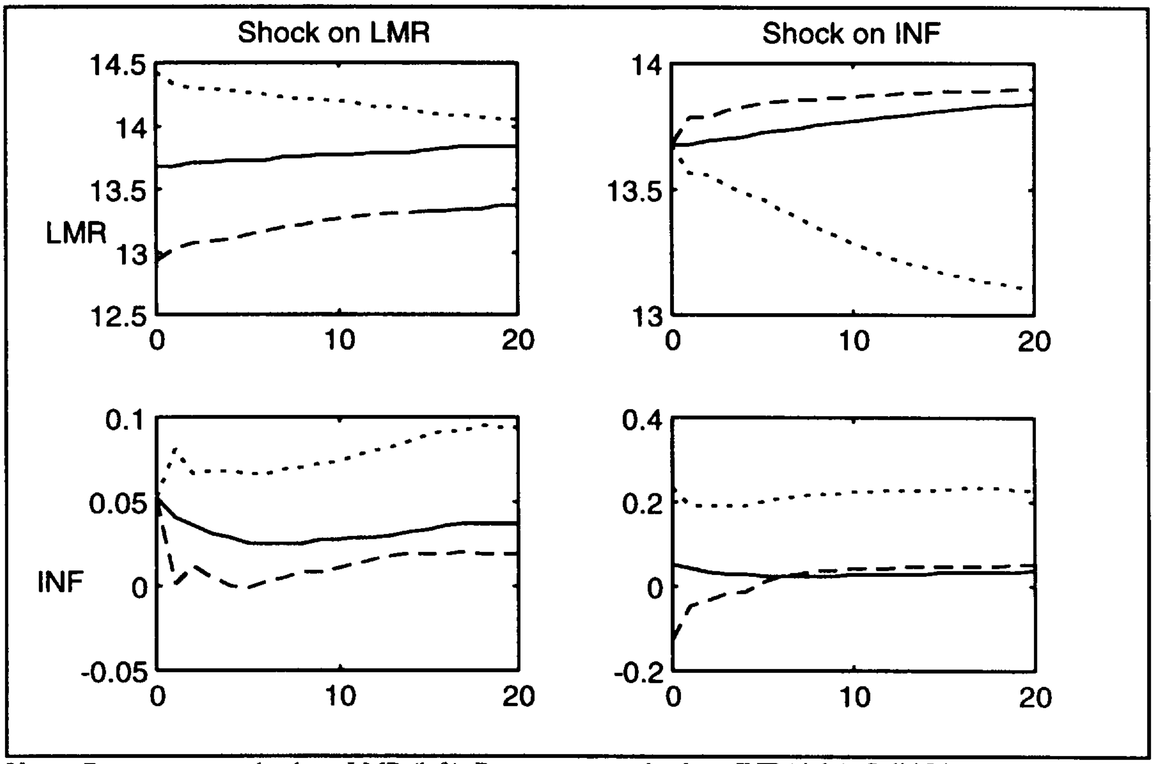
Put $x^+ = \left(y'_{-L+1}, y'_{-L+2}, \ldots, y'_0\right)' + \left(0,0,\ldots,\delta^+\right)'$,

$x^0 = \left(y_{-L+1}', y_{-L+2}', \ldots, y_0\right)'$ and $x^- = \left(y_{-L+1}', y_{-L+2}', \ldots, y_0\right)' + \left(0,0,\ldots,\delta y^-\right)'$.

Thus, x^+ is an initial condition corresponding to a positive impulse or shock δy^+ added to contemporaneous y_0, x^- corresponds to a negative impulse, while x^0 represents the base case with no impulse. A natural definition of the nonlinear impulse response is the net effect of the impulse δy^+ (or δy^-). The net effect is obtained by comparing the profile for δy^+ (or δy^-) to the baseline. Specifically the sequence, $\left\{\hat{y}_j^+ - \hat{y}_j^0\right\}_{j=0}^{\infty}$ represents the net response to the positive impulse while $\left\{\hat{y}_j^- - \hat{y}_j^0\right\}_{j=0}^{\infty}$ represents the net response to a negative one. The impulse response depends upon the initial x, which reflects the nonlinearities of the system.

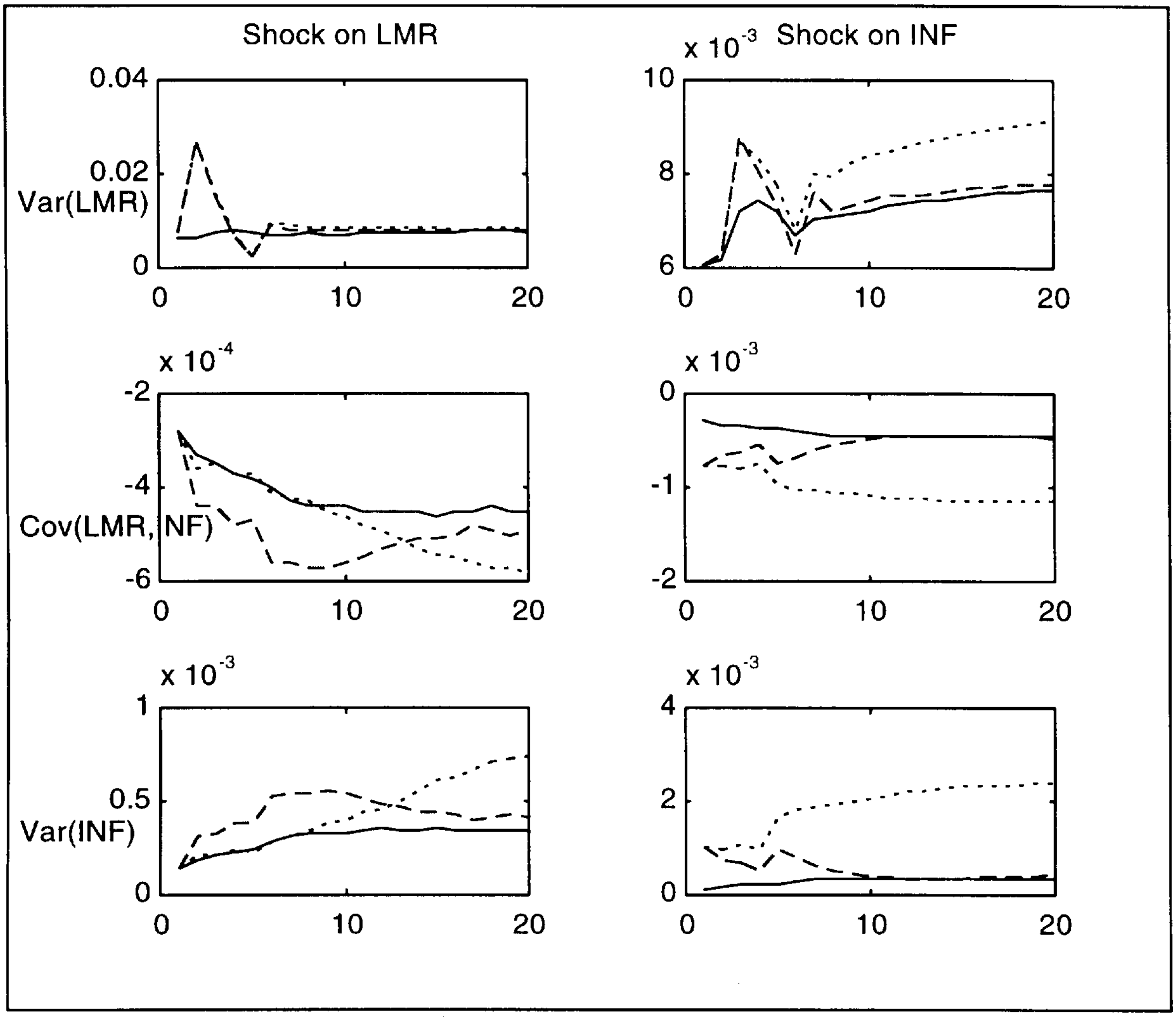
The conditional moment profile can be extended to higher other moments, as is shown in Figure 4 below.

Figure 3. Impulse-Response of LMR and INF in the Bivariate Model.
(First Moments)



Notes: Responses to a shock on LMR (left). Responses to a shock on INF (right). Solid Line = Baseline. Dashed Line = Negative Shock. Dotted Line = Positive Shock.

Figure 4. Impulse-Response of LMR and INF in the Bivariate Model. (Second Moments)



Notes: Responses to a shock on LMR (left). Responses to a shock on INF (right). Solid Line = Baseline. Dashed Line = Negative Shock. Dotted Line = Positive Shock.

The top panel of Figure 3 shows the effect of innovations of 2 times the standard deviation of LMR and INF. In the first case, it can be noticed that even when the effect of this shock seems to dampen and converge to the baseline, after 20 months the effects are still notorious; both the positive and negative have rather symmetric effects on LMR, while the effect on INF is heavily asymmetric. A negative shock converges faster than a positive shock which effect seems to exarcebate the inflationary process even after 20 months.

On the other hand, the effects of an innovation on INF are drastically different; the trajectories of LMR and INF associated with a negative shock tend to converge to the baseline, while a positive shock has more profound and rather permanent effects, the inflationary process stabilizes in a new level roughly equivalent to the magnitude of the innovation and the demand for money is drastically reduced.

Figure 4 shows the effect of shocks to LMR and INF on the variance of LMR, the covariance between LMR and INF and the variance of INF. In the case of an innovation to LMR, the top panel shows that after 6 months both shocks converge to the baseline. Both shocks increase the volatility of LMR at the beginning and then tend to stabilize rapidly. The covariance between LMR and INF is negative, but the effects of the two shocks are very different; while the positive shock does not present important modifications of the comovements of both variables with respect to the baseline (at least in the first ten months). In the case of the bottom panel, we can appreciate that the volatility of INF is not affected in the first 8 months when LMR is hitted with a positive shock, but its volatility is severely increased when the shock is negative. It seems clear that the effect of this shock is transitory.

On the other hand, a shock to INF has more interesting and asymmetric effects. In all the cases a negative shock converge after 8 to 10 months to the baseline while the positive shock on INF is permanently associated with more volatility on INF and LMR and greater covariance between them. This findings are very interesting because they show the asymmetrical effects that this innovation has. In particular, deflationary shocks seem to have almost no effect after a short period of time, while inflationary pressures are very difficult to deal with, because they seem to have permanent effects not only on levels but also on the associated volatilities. Not surprisingly, this evidence shows the fragility of price stabilization measures in an economy like the Brazilian..[8]

b. Cointegration Test

In theory, the Cagan model specifies a stable (negative) relationship between real money holdings and inflation (expected) at least in periods of high inflation. It is obvious that the traditional Cagan specification presents several flows.[9] It is particularly relevant to notice that even when the relation may exist, empirically it has been found important unexplained

[8] As discussed earlier, one advantage of using this type of methodology for constructing the impulse-response functions is that we can obtain confidence intervals for both the positive and negative shock to asses if they are statistically different. Not surprisingly given the non linear nature of the bivariate process these responses are in most cases statistically different.

[9] See McCallum (1989) for an introductory exposition.

autocorrelation. As Figures 1 and 2 make clear, non linearities have to be included in order to be able to represent realistically this relation.

It is known that any regression can be viewed as the expectation of a conditional distribution. There are some cases in which even when series are individually non stationary, the combination of two or more may lead to a stationary series. This is the basic idea of the term cointegration.[10]

It may be the case that even when the residual from the projection of one variable to other presents autocorrelation it is stationary. This part of the paper pretends to see if it is possible to find cointegration between inflation and LMR for the Brazilian economy from 1944 to 1994. That is, we will see if we can find an stable long run relationship between inflation and real money holdings, which is the logical implication of the Cagan model.

In order to do so we use 2 types of tests, namely the Johansen Procedure and Phillips-Oulliaris-Hansen Test.

Johansen Procedure

As described in Johansen (1988, 1991) and Hamilton (1994) and given that we are considering two series (namely INF and LMR) we are interested in testing three hypothesis. The results are:

(1) H_0: zero cointegration relations
H_1: two cointegration relations

The computed value is 101.370 and the critical 15.197
Reject the null hypothesis

(2) H_0: zero cointegration relation
H_1: one cointegration relation

The computed value is 93.305 and the critical 3.962
Reject the null hypothesis

(3) H_0: one cointegration relation
H_1: two cointegration relations

[10] See Engle and Granger (1987) or Hamilton (1994).

The computed value is 8.064 and the critical 3.962
Accept the null hypothesis

The Johansen procedure seems to support the presence of one cointegration relation, thus computing the cointegration vector we obtain:

Cointegration Vector : [1 0.4063]
[LMR INF]

There are several reason why one may consider that this test is suspect, some of them are:

- INF and LMR do not share the same order of integration.
- The assumption of normality used to compute the Maximum Likelihood estimates is unrealistic.

Given these reasons we decided to test the cointegration hypothesis using a different Procedure.

Phillips-Ouliaris-Hansen Procedure

This procedure which is summarized in Hamilton (1994)[11] presents the following results:

$Z_p = -11.280 > -21.5$ Accept no cointegration
$Z_t = -2.268 > -3.42$ Accept no cointegration

Cointegration Vector [1 0.5601]
[LMR INF]

The presence of cointegration is thus inconclusive, Johansen tends to say yes, while Phillips-Ouliaris-Hansen says no.

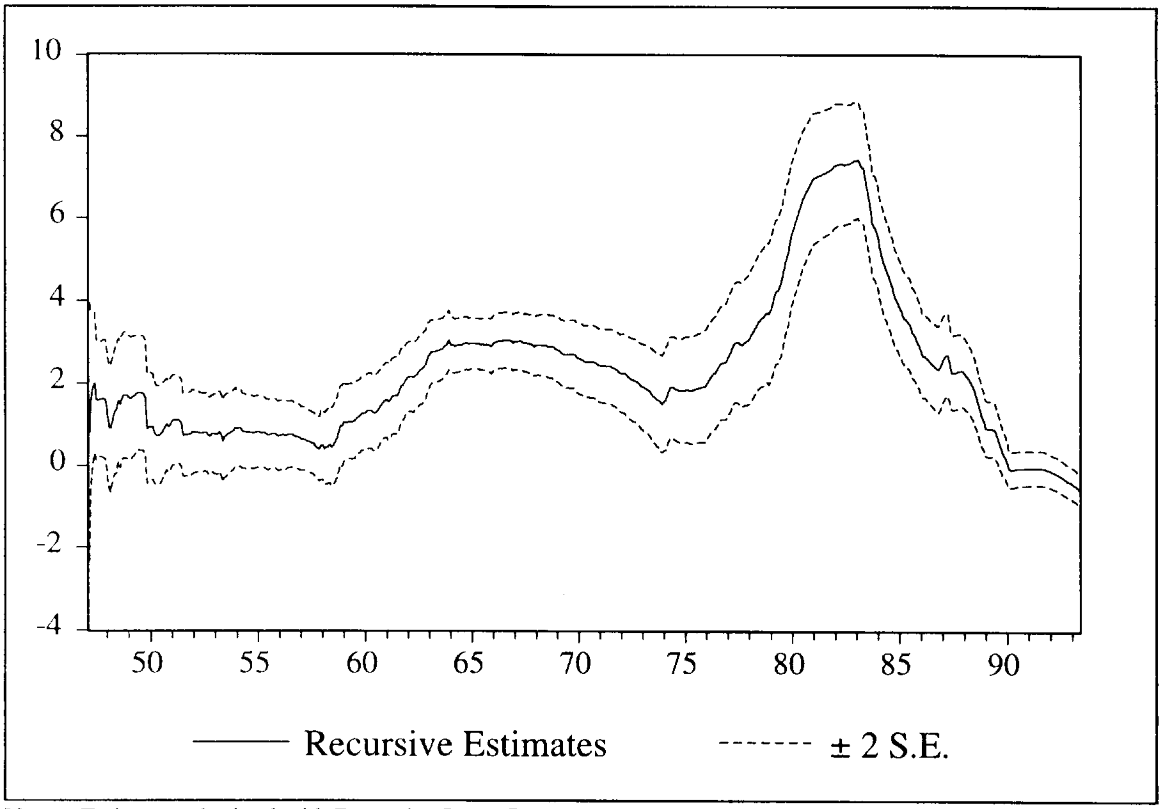
The problems with Johansen procedure were already discussed, while the most important problem with Phillips-Oulliaris-Hansen is that, as the augmented Dickey-Fuller test, it has a low power. It may also be sensible to the choice of lags and trend polynomial.

Given these basic difficulties we decided in the first place to obtain the Recursive Least Squares estimate of the cointegrating vector (in case it exists). This would allow us to

[11] See particularly pages 598-600 for a useful example and the presentation of formulas.

see if given the assumption of the existence of a long run (linear) relation between INF and LMR, this is stable. The results are reported in following Figure.

Figure 5. Evolution of the Parameter Corresponding to INF

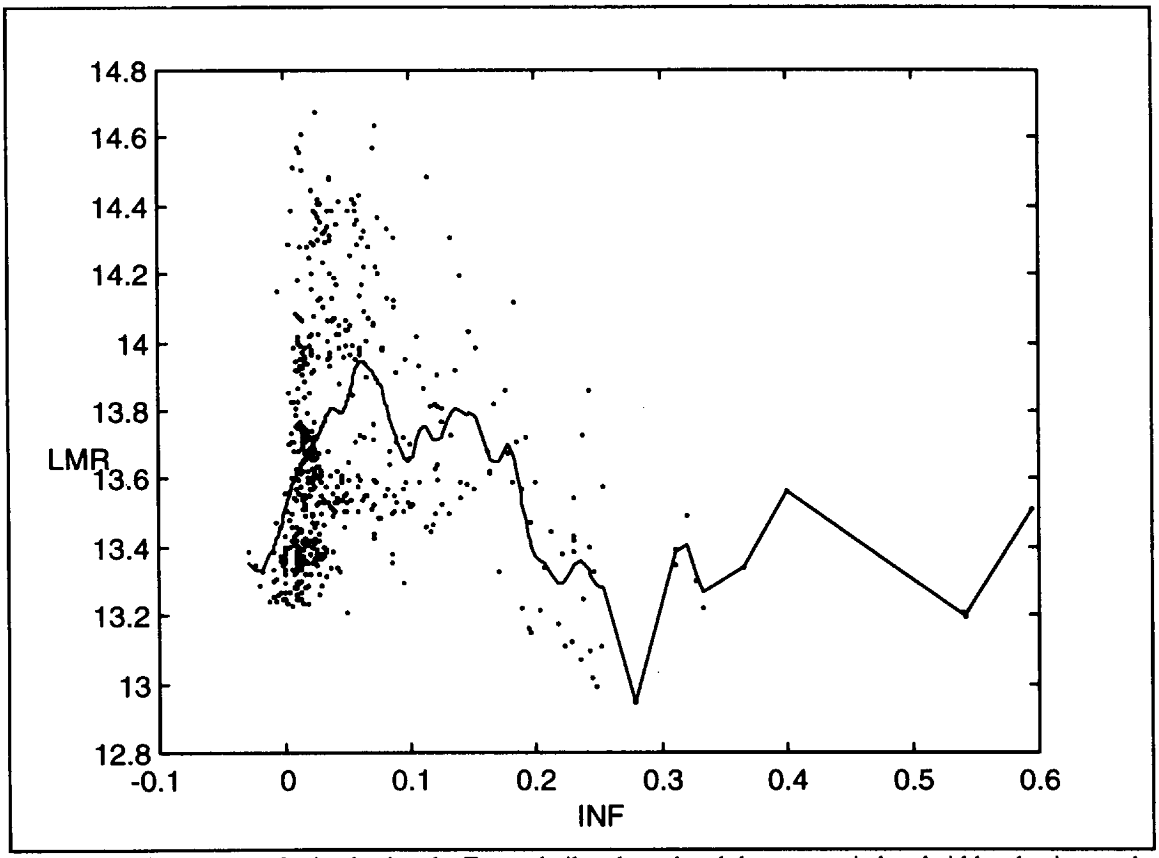


Notes: Estimates obtained with Recursive Least Squares. Dependent Variable: LMR.

It is clear that with the assumption of linearity between LMR and INF we are not able to find an stable (and reasonable) estimate of the semi-elasticity of real money holdings with respect to inflation. Notice that it is only by the end of the sample that the coefficient is significantly negative. For the most part of the sample, the coefficient is positive but highly unstable.

In order to circumvent the problems that normality and linearity impose, an estimation of the distribution of LMR conditional on INF using non-parametric techniques was done, after that it was possible to compute the conditional mean and variance that will (hopefully) help to understand better the contradictions encountered.

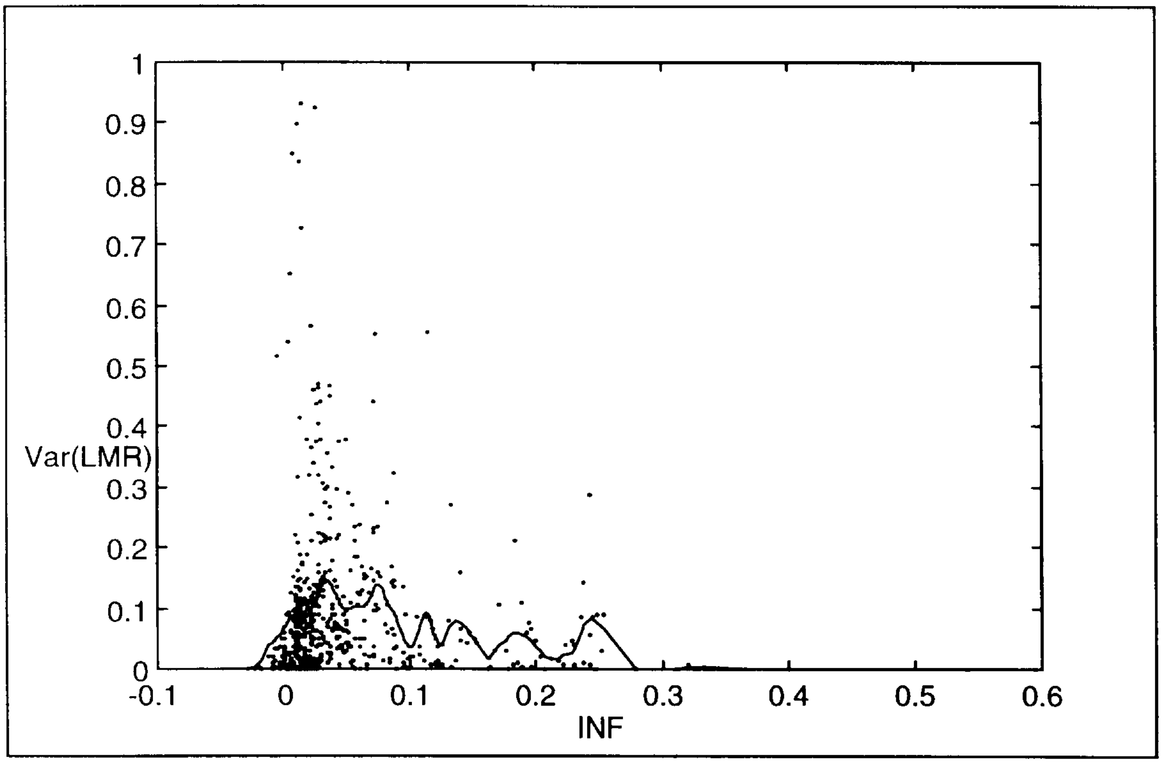
Figure 6. Kernel Estimate of the Conditional Mean of LMR



Notes: The estimator was obtained using the Epanechnikov kernel and the automatic bandwidth selection method proposed by Silverman (1986).

It is clear that there is no stable inverse relation between LMR and INF once a flexible representation for the distribution of LMR is selected. Figure 6 makes evident that for relatively small inflationary episodes, the demand for real holdings tend to increase instead of decreasing. The "expected" Cagan relation seems to begin to hold only when the inflation reaches very high levels. This may be due in part to the fact the some pecuniary and non pecuniary transactions costs are implicitly present in the Brazilian economy and sometimes they seem to raise with inflation. Of course, after a certain level, the cost of maintaining local currency is so high that substitution is performed. Finally, Figure 7 shows the results of the estimation of the conditional variance of LMR.

Figure 7. Kernel Estimate of the Conditional Variance of LMR



Notes: The estimator was obtained using the Epanechnikov kernel and the automatic bandwidth selection method proposed by Silverman (1986).

It is interesting to realize that the conditional variance of LMR is non linear in INF; that is, it is increasing in early stages, seems to stabilize and decrease rapidly for high levels of inflation. Intuitively, as soon as the cost of holding national currency is sufficiently large, the agents seem to recognize that the stable maintenance of a minimal amount of currency is their best policy.

This results are intuitively appealing, but seem to contradict (for moderate inflation rates) the Cagan hypothesis of a stable (inverse) long run relationship between real money holdings and inflation.

IV. Conclusions

The usage of the SNP technique and more traditional approaches to describe the statistical properties of three Brazilian time series proved to be useful to show important departures from normality and homogeneity. In particular, LMR seems to be multimodal and nonstationary, pointing out evidence of possible structural breaks. On the other hand, for a large sample such as the one analyzed here, INF and DM are unimodal, leptokurtic, with positive skewness and stationary.

No evidence of unidirectional Granger causality between series was found. This observation may show the difficulty of exercises that intend to find revenue maximizing inflation taxes; as in Dornbusch, et.al. (1990).

For the period under analysis, there seems to be contradictory evidence regarding the presence of a stable inverse long run relation between LMR and INF. Even when the cointegration vector obtained using OLS is similar to the one estimated by Johansen procedure, INF appears to be stationary thus weakening the cointegration proposition. Furthermore, recursive least squares estimation of a in (2) shows alarming instability through time and (usually) the "wrong" sign.

Specification tests for (2) can be performed using nonparametric techniques; but its application shows a nonmonotone relation between LMR and INF. The only stage in which the Cagan specification may hold is under a reduce range of inflation levels.

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Table 1. Summary Statistics.			
Variable[a]	LMR	DM	INF
Mean	13.673	0.052	0.055
Std.Dev.	0.342	0.102	0.080
Var. Coeff.	0.025	1.961	1.452
Skewness	0.742	2.784	2.832
Kurtosis	2.803	17.030	12.938
J-B[b]	53.071	5392.379	3282.177
Coc (13)[c]	0.665	0.080	0.288
Coc (25)	0.567	0.047	0.193
AR[d]	14	13	1

Notes. [a] See Figure 1 for definitions.

[b] Jarque-Bera Normality Test. Distributed as Chi² (2) with critical value (5%) 5.992.

[c] Cochrane (1988) persistence index. The number in parenthesis corresponds to the number of period taken into account in the construction of the index.

[d] Autoregressive part. Computed using the Schwartz Criterion.

Table 2. Summary Statistics.			
Variable	DLMR	D2M	DINF
Mean	-0.001	0.0003	0.001
Std.Dev.	0.084	0.113	0.030
Skewness	0.119	-0.297	-7.410
Kurtosis	9.404	12.833	118.911
J-B	971.881	2292.447	341946.200
Coc (13)	0.069	0.041	0.062
Coc (25)	0.039	0.023	0.037

Notes. See Figure 1 and Table 1 for definitions.

Table 3. Granger Causality / Correlation Matrix			
—	LMR	DM	INF
LMR	– 1	yes 0.022	yes -0.123
DM	yes 0.022	– 1	yes 0.589
INF	yes -0.123	yes 0.589	– 1

Table 4. Unit Root Test Augmented D.F.		
	Const.	Time Trend
LMR	-1.919	-1.435
DM	-2.413	-3.871
INF	-3.796	-5.699

Notes: The critical value with constant term is 2.867 and the critical value with time trend is 3.419; using ordinary least squares.

Table 5. Best Univariate Fits			
	LMR	DM	INF
Q (30)[a]	39.451	49.630	30.802
J-B	797.145	3358.644	134172.097
ARCH[b]	(1) 134.771	(3) 105.753	(3) 22.221
White[c]	26.443	246.592	269.426

Notes: [a] Ljung and Box Q-statistic test for white noise with critical value $Chi^2(30)=43.773$.

[b] $Chi^2(1)=3.841$, $Chi^2(3)=7.815$.

[c] White test for heteroskedasticity.

Table 6. Univariate Real Money Holdings Series:
Optimized Likelihood and Residuals Diagnostics

L_u	L_r	L_p	K_z	I_z	K_x	I_x	S_n	P	HQ	BIC	Mean	Var.
14	0	1	0	0	0	0	-0.096	16	-0.045	-0.007	65.34	64.17
14	2	1	0	0	0	0	-0.113	18	-0.055	-0.013	56.97	724.61
14	4	1	0	0	0	0	-0.117	20	-0.052	-0.005	74.02	721.71
14	6	1	0	0	0	0	-0.119	22	-0.048	-0.004	71.29	714.97
14	2	1	2	0	0	0	-0.155	20	-0.091	-0.044	76.03	704.36
14	2	1	4	0	0	0	-0.305	22	-0.234	-0.183	30.60	672.34
14	2	1	6	0	0	0	-0.339	24	-0.261	-0.205	74.43	691.15
14	2	1	8	0	0	0	-0.347	26	-0.263	-0.202	87.22	696.40
14	2	1	10	0	0	0	-0.345	28	-0.254	-0.189	119.60	723.94
14	2	2	8	0	1	0	-0.406	44	-0.263	-0.161	26.49	380.78
14	2	4	8	0	1	0	-0.431	62	-0.230	-0.085	81.65	187.34
14	2	2	8	0	2	0	-0.545	71	-0.314	-0.149	34.50	76.35
14	2	4	8	0	2	0	-0.675	152	-0.181	-0.172	54.20	145.24
4	2	1	6	0	0	0	-0.158	14	-0.113	-0.080	60.87	647.12
10	2	1	6	0	0	0	-0.210	20	-0.146	-0.099	61.73	657.63
16	2	1	6	0	0	0	-0.341	26	-0.256	-0.196	73.92	696.42

Table 7. Univariate Money Growth Series:
Optimized Likelihood and Residuals Diagnostics

L_u	L_r	L_p	K_z	I_z	K_x	I_x	S_n	P	HQ	BIC	Mean	Var.
13	0	1	0	0	0	0	1.155	15	1.203	1.238	132.61	736.16
13	2	1	0	0	0	0	0.892	17	0.947	0.987	19.04	100.90
13	4	1	0	0	0	0	0.869	19	0.931	0.975	30.15	143.67
13	6	1	0	0	0	0	0.859	21	0.927	0.976	26.42	160.38
13	4	1	2	0	0	0	0.820	21	0.888	0.937	71.73	194.42
13	4	1	4	0	0	0	0.774	23	0.849	0.902	55.07	69.25
13	4	1	6	0	0	0	0.724	25	0.805	0.863	48.12	168.59
13	4	1	8	0	0	0	0.731	27	0.819	0.882	132.92	301.59
13	4	2	6	0	1	0	0.684	39	0.811	0.902	58.10	89.10
13	4	4	6	0	1	0	0.603	53	0.775	0.899	89.49	130.16
13	4	6	6	0	1	0	0.585	67	0.803	0.959	99.04	138.02
13	4	2	6	0	2	0	0.606	60	0.801	0.941	62.96	5.67
13	4	4	6	0	2	0	0.503	123	0.903	1.189	64.93	192.94
3	4	1	6	0	0	0	0.923	15	0.972	1.007	78.28	85.15
8	4	1	6	0	0	0	0.882	20	0.947	0.994	23.85	247.60
15	4	1	6	0	0	0	0.727	27	0.815	0.878	46.83	165.27

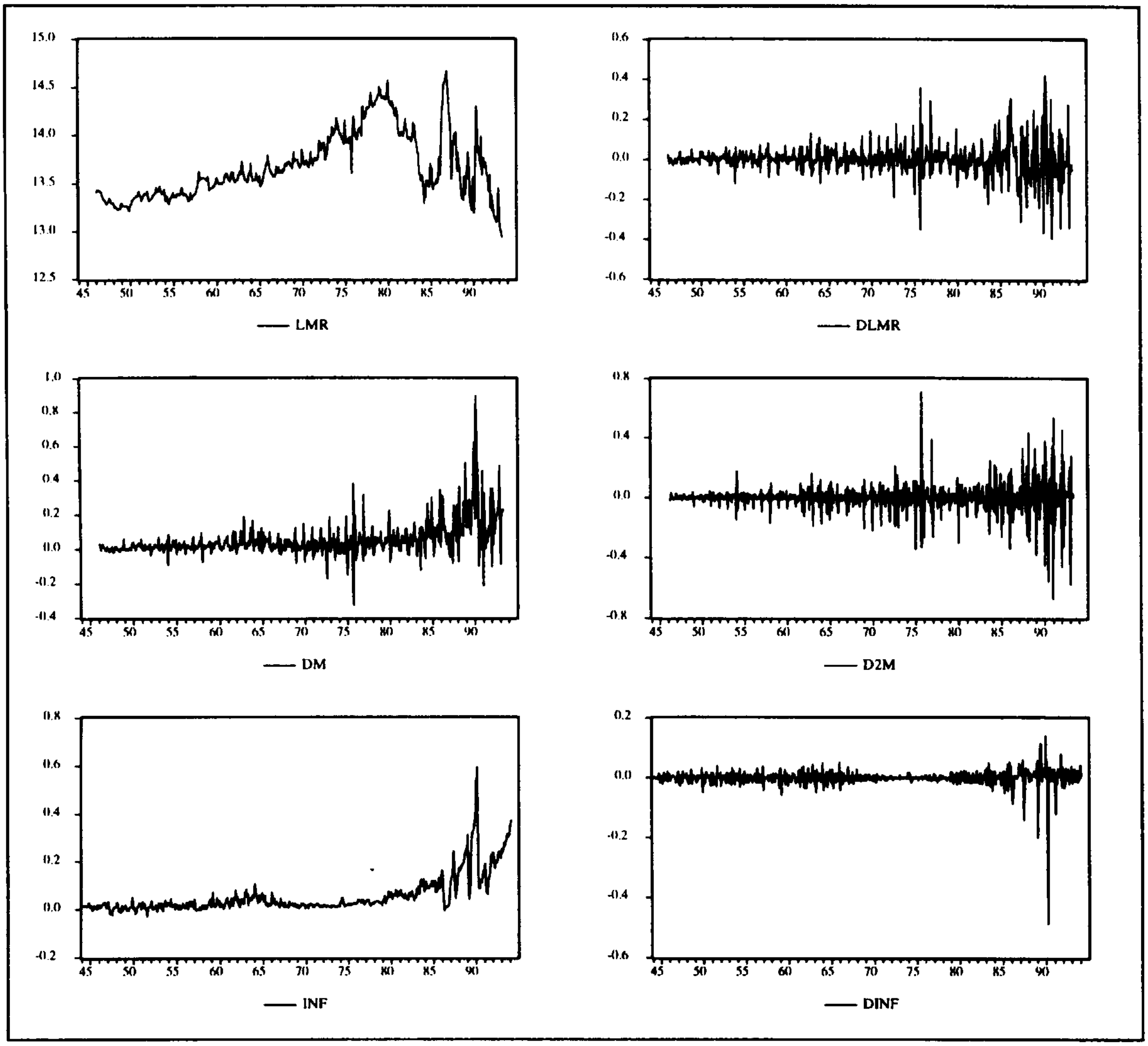
Table 8. Univariate Monthly Inflation Rate Series:
Optimized Likelihood and Residuals Diagnostics

L_u	L_T	L_p	K_z	I_z	K_x	I_x	S_n	P	HQ	BIC	Mean	Var.
1	0	1	0	0	0	0	0.403	3	0.412	0.419	150.46	861.87
1	2	1	0	0	0	0	0.105	5	0.120	0.131	49.76	592.21
1	4	1	0	0	0	0	0.091	7	0.113	0.128	37.26	458.88
1	6	1	0	0	0	0	0.072	9	0.100	0.120	64.46	437.27
1	8	1	0	0	0	0	0.069	11	0.103	0.128	65.37	446.21
1	6	1	2	0	0	0	0.006	11	0.040	0.065	144.41	386.59
1	6	1	4	0	0	0	0.027	13	0.068	0.097	142.51	489.53
1	6	1	6	0	0	0	-0.048	15	-0.002	0.032	98.97	380.05
1	6	1	8	0	0	0	0.037	17	0.089	0.127	132.72	607.19
1	6	2	6	0	1	0	-0.193	29	-0.104	-0.039	74.56	111.07
1	6	4	6	0	1	0	-0.194	43	-0.061	0.035	12.28	110.57
1	6	2	6	0	2	0	-0.198	50	-0.044	0.068	96.54	57.53
1	6	4	6	0	2	0	-0.303	113	0.045	0.297	4.81	40.06
2	6	2	6	0	1	0	-0.186	30	-0.093	-0.027	69.18	103.82
3	6	2	6	0	1	0	-0.186	30	-0.103	-0.033	37.37	72.27
5	6	2	6	0	1	0	-0.186	30	-0.103	-0.030	26.42	57.52

Table 9. Bivariate Model: Monthly Real Money Holdings and Inflation Rate Series:
Optimized Likelihood

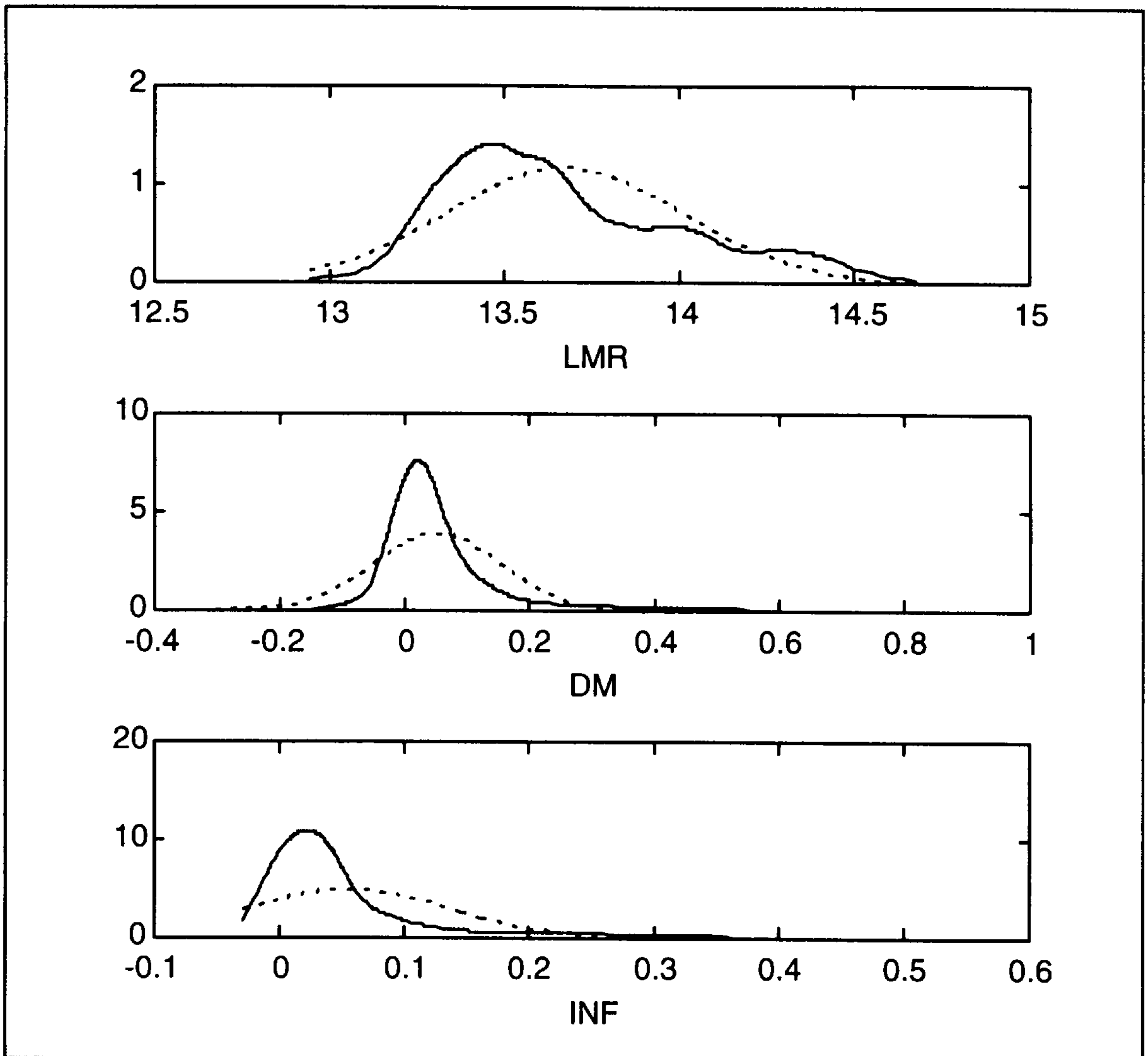
L_u	L_T	L_p	K_z	I_z	K_x	I_x	S_n	P	HQ	BIC
2	0	1	0	0	0	0	0.470	13	0.513	0.543
4	0	1	0	0	0	0	0.453	21	0.521	0.570
6	0	1	0	0	0	0	0.442	29	0.536	0.603
8	0	1	0	0	0	0	0.434	37	0.554	0.640
1	0	1	0	0	0	0	0.503	9	0.532	0.553
3	0	1	0	0	0	0	0.464	17	0.519	0.559
2	2	1	0	0	0	0	0.114	17	0.169	0.209
2	4	1	0	0	0	0	0.090	21	0.158	0.207
2	6	1	0	0	0	0	0.062	25	0.143	0.202
2	8	1	0	0	0	0	0.056	29	0.150	0.218
2	6	1	2	0	0	0	-0.064	30	0.034	0.103
2	6	1	4	0	0	0	-0.177	39	-0.051	0.040
2	6	1	6	0	0	0	-0.298	52	-0.129	-0.008
2	6	1	8	0	0	0	-0.274	69	-0.050	0.111
2	6	1	6	0	1	0	-0.515	108	-0.164	-0.325
2	6	1	6	0	2	0	-0.693	192	-0.069	0.378

Figure 1. Real Money Holdings, Rate of Growth of Monetary Base and Inflation (1944:01-1994:12)



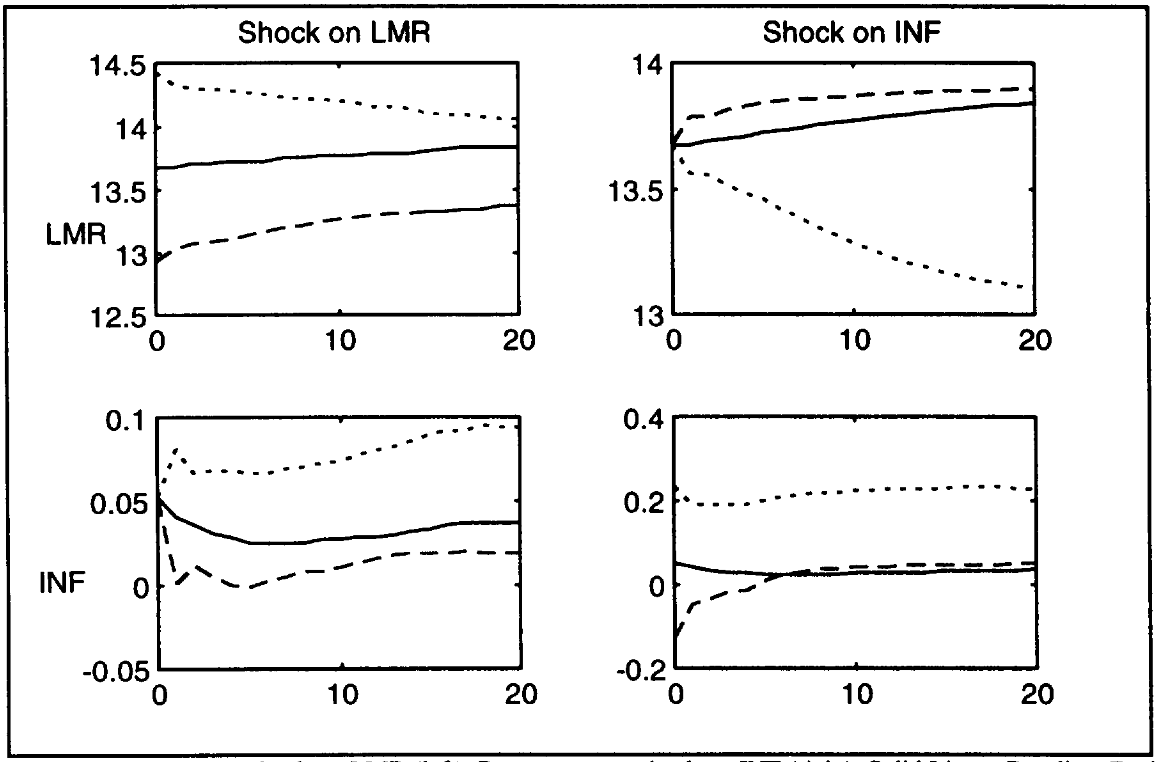
Notes: LMR = Real Money Holdings (in logs). DM = Rate of Growth of Monetary Base. INF = Inflation. First differences on the right (DLMR, D2M, and DINF respectively).

Figure 2. Non Parametric Estimates of the Unconditional PDF of LMR, DM, and INF



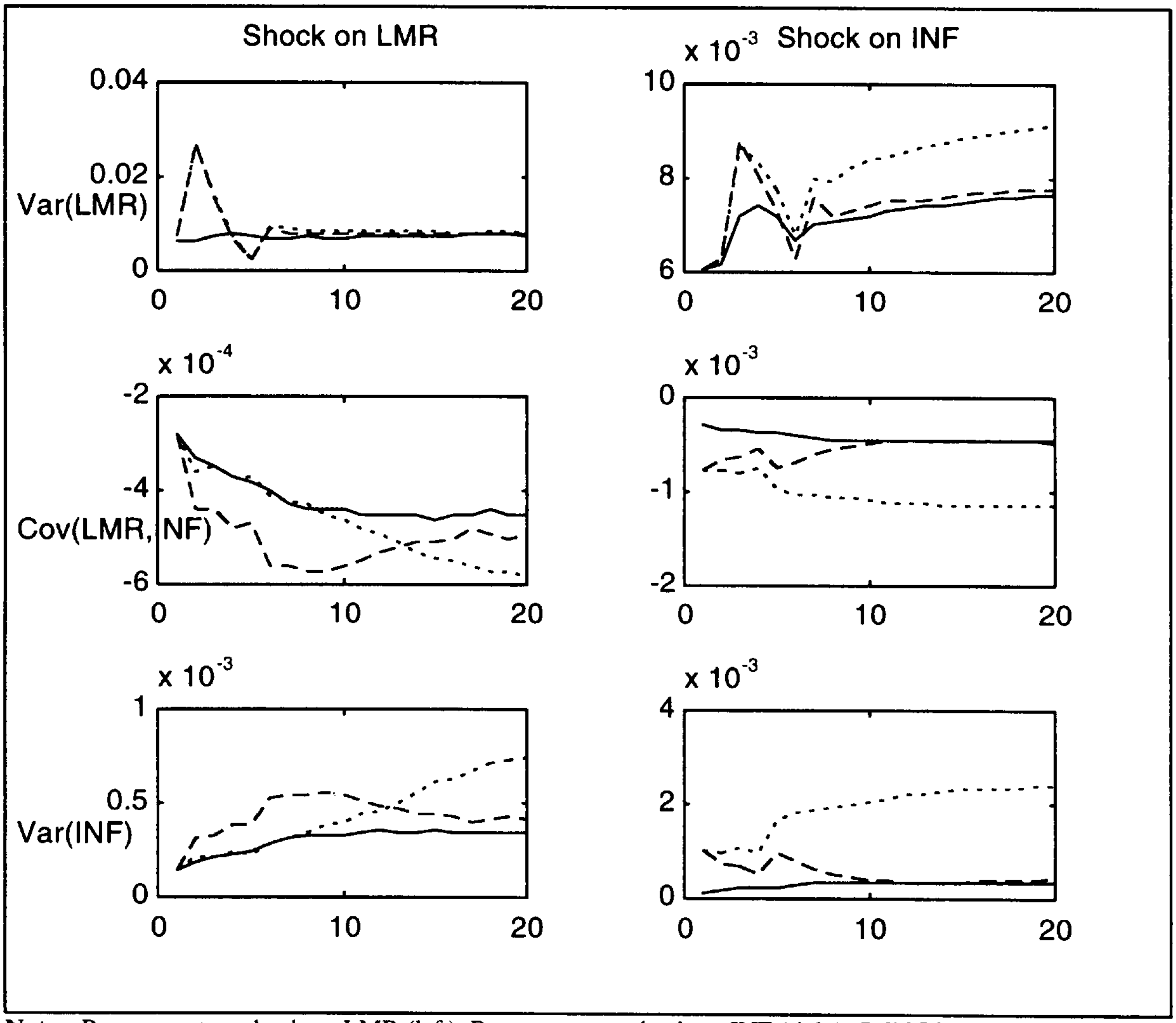
Notes: The continuous line corresponds to the estimated of the unconditional PDF using Epanechnikov kernel and cross validation (Silverman, 1986). The dotted line corresponds to a normal pdf with the same mean and variance. See Figure 1 for definitions.

Figure 3. Impulse-Response of LMR and INF in the Bivariate Model. (First Moments)



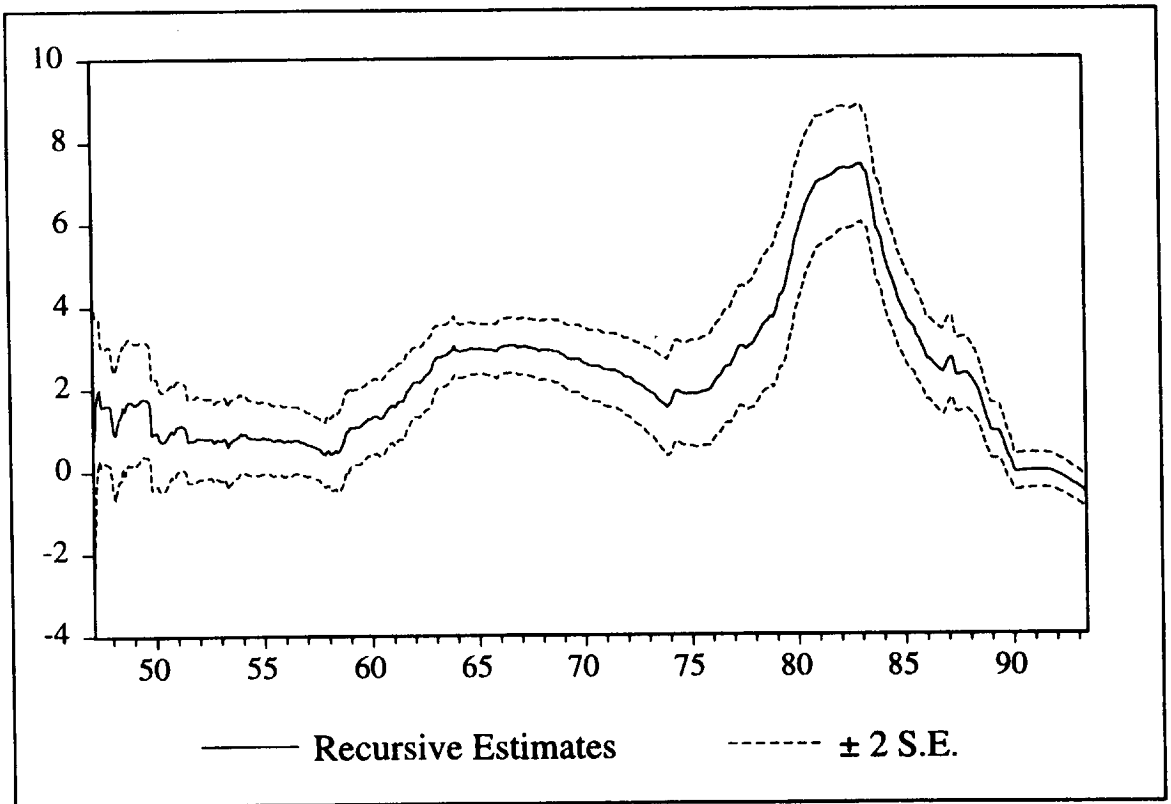
Notes: Responses to a shock on LMR (left). Responses to a shock on INF (right). Solid Line = Baseline. Dashed Line = Negative Shock. Dotted Line = Positive Shock.

Figure 4. Impulse-Response of LMR and INF in the Bivariate Model.
(Second Moments)



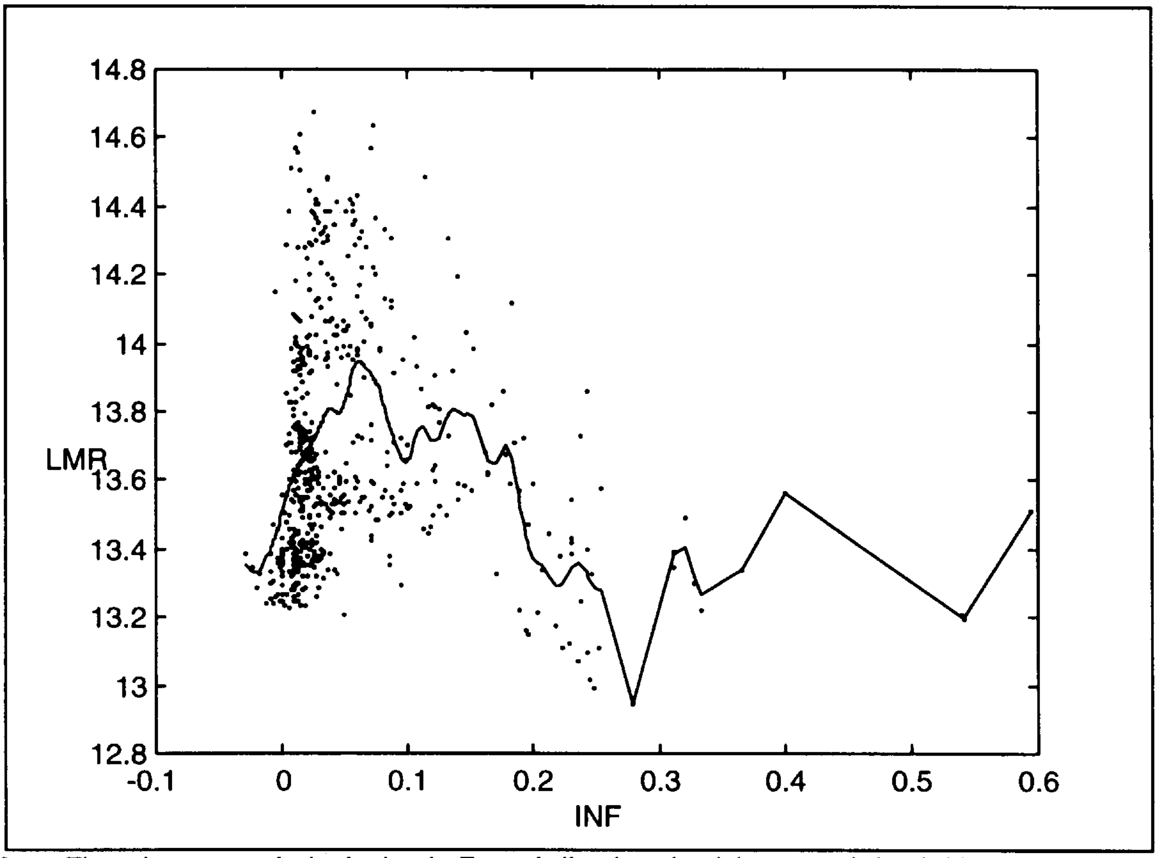
Notes: Responses to a shock on LMR (left). Responses to a shock on INF (right). Solid Line = Baseline. Dashed Line = Negative Shock. Dotted Line = Positive Shock.

Figure 5. Evolution of the Parameter Corresponding to INF



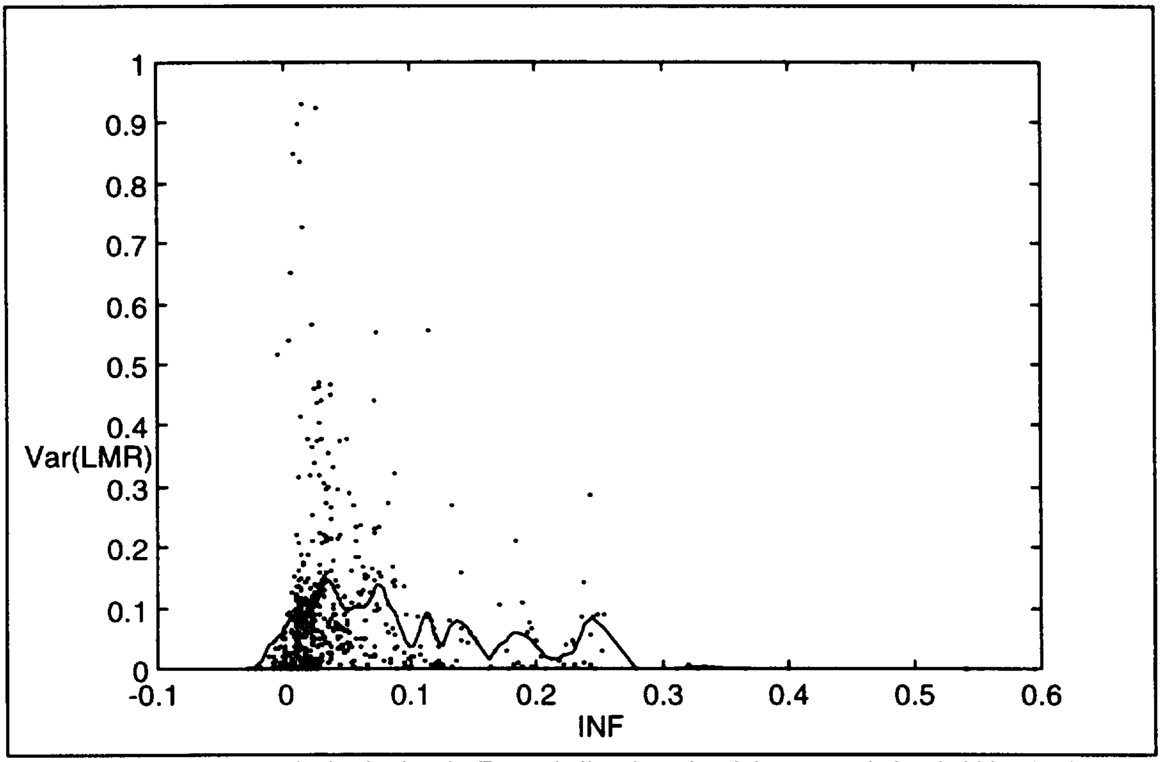
Notes: Estimates obtained with Recursive Least Squares. Dependent Variable: LMR.

Figure 6. Kernel Estimate of the Conditional Mean of LMR



Notes: The estimator was obtained using the Epanechnikov kernel and the automatic bandwidth selection method proposed by Silverman (1986).

Figure 7. Kernel Estimate of the Conditional Variance of LMR



Notes: The estimator was obtained using the Epanechnikov kernel and the automatic bandwidth selection method proposed by Silverman (1986).