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Resumen
Este trabajo considera si la "trampa de liquidez" es un tema relevante en la evaluación de la conveniencia de adoptar una meta de inflación como estrategia de política monetaria. Desde una perspectiva teórica, se ha sugerido que bajo se corre el peligro de experimentar una "trampa de expectativas" o "indeterminación"; ocurriendo el segundo caso cuando proyecciones sobre la inflación futura entran en la regla de política. Este trabajo argumenta que estos peligros tienen nula importancia práctica. Desde una perspectiva empírica, se desarrolla un modelo cuantitativo de economía abierta y se explora la probabilidad de encontrar una trampa de liquidez para varias reglas de política. También se enfatiza que, si el instrumento usual de tasa de interés es inmovilizado por una trampa de liquidez, existe aún un canal de tipo de cambio por medio del cual la política monetaria puede ejercer efectos estabilizadores. La variable objetivo relevante puede seguir siendo la tasa de inflación.

Abstract
This paper considers whether “liquidity trap” issues have important bearing on the desirability of inflation targeting as a strategy for monetary policy. From a theoretical perspective, it has been suggested that “expectation trap” and “indeterminacy” dangers are created by variants of inflation targeting, the latter when forecasts of future inflation enter the policy rule. This paper argues that these alleged dangers are probably not of practical importance. From an empirical perspective, a quantitative open-economy model is developed and the likelihood of encountering a liquidity trap is explored for several policy rules. Also, it is emphasized that, if a liquidity trap immobilizes the usual interest rate instrument, there is still an exchange-rate channel by means of which monetary policy can exert stabilizing effects. The relevant target variable can still be the inflation rate.

This paper is a chapter of the forthcoming book Inflation Targeting: Design, Performance, Challenges, edited by Norman Loayza and Raimundo Soto, Santiago, Chile.
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Inflation Targeting and the Liquidity Trap

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This paper considers whether issues regarding liquidity trap or zero lower bound phenomena substantially affect the case for inflation targeting, in comparison with other possible strategies for conducting monetary policy. It examines both theoretical and empirical issues and, in the latter case, emphasizes the importance of an economy's openness to foreign trade in goods and securities.

The first theoretical topic to be investigated is prompted by recent papers by Benhabib, Schmitt-Grohé, and Uribe (2001), Dupor (1999), and Schmitt-Grohé and Uribe (2000), among others, which argue that recognition of the existence of a zero lower bound (ZLB) on nominal interest rates leads to the conclusion that inflation targeting rules—as well as more general Taylor-type rules (see Taylor, 1993a)—are likely to fail. The alleged reason is that the existence of a zero lower bound implies that rational expectations solutions to standard optimizing models with Taylor rules are not unique, and one solution that is likely to be attained involves a deflationary liquidity trap. The present paper contends that the alleged danger should not be considered to be of substantial practical importance. This argument is developed in section 1.

Section 2 takes up a closely related topic concerning the danger of solution indeterminacy, which, according to Woodford (1994) and several other analysts, is generated by the practice of basing policy actions on expected future inflation rates rather than on currently observed values. Again, and for similar reasons, I argue that the danger is probably illusory.

I am indebted to Ben Bernanke, Francisco Nadal de Simone, and Edward Nelson for helpful comments.

The foregoing points are of a theoretical and general nature, so they can be discussed within a highly stylized and extremely simplified theoretical framework. When one turns to empirically oriented issues, however, it becomes important to work with a model that more closely reflects the properties of actual economies. Section 3 therefore specifies an open economy model with slow price-level adjustments and inertia in consumption demand. Quantitative calibration is undertaken in section 4, which also presents aspects of the model's properties.

In section 5, the model is used to examine the frequency, under alternative policy rules, with which zero or negative interest rates are encountered in stochastic simulations designed to mimic realistic conditions. This exercise provides some indication of the relative frequency with which liquidity trap situations may arise under inflation targeting, in comparison with other policy rules.

Then in section 6, it is assumed that the economy is in a liquidity trap, such that the usual interest rate instrument is immobilized. The possibility of using monetary policy for stabilizing purposes, nevertheless, is provided by the existence of a transmission channel involving foreign exchange. The section quantitatively examines the relative potency of this channel with an inflation targeting objective. Some authors contend that this exchange rate channel is not available because of uncovered interest parity; the paper analyzes and strongly disputes such contentions. Finally, section 7 provides a brief concluding summary.

Before beginning with these various topics, it is necessary to mention the way in which the term inflation targeting is used in this paper. An inflation targeting regime is taken to be one in which monetary policy is conducted according to a rule that specifies adjustments of an instrument variable in response to deviations of inflation, or expected future inflation, from a policy-specified target value.[1] Given this definition, are responses to other variables, such as the output gap term in Taylor-style rules, permitted? Here no particular position is taken on that terminological issue; I simply refer to such cases as reflecting departures from pure inflation targeting. Also, responses to previous-period values of the instrument variable are permitted so as

1. Of course, it is not supposed that any actual central bank would ever literally follow the instructions of any simple formula. For analytical purposes, however, the systematic aspects of monetary policy can be clearly expressed in terms of a rule. I do not attempt to find an "optimal" rule, for any such finding would be highly model specific, so I do not need to discuss commitment issues. With regard to the question of rules versus discretion, note that it is implausible that any actual central bank would ever literally follow the instructions of an optimal control exercise repeated anew each decision period.

to reflect the type of smoothing behavior that seems to be widely practiced by central banks.

I am, of course, fully aware that Svensson (1997, 1999) has argued for a different terminological convention, one that would use the word target only to refer to variables that appear in explicitly specified loss functions. It is often useful, however, to proceed without adoption of any explicit loss function. Furthermore, I believe that my terminology is more consistent with actual practice, in part because actual central banks have thus far not adopted explicit loss functions. In any event, the issue is of little importance, since it is always possible to write instrument rules that approximate as closely as desired the instrument settings of a policy regime involving targeting in Svensson's sense.

1. An Expectational Liquidity Trap?

As mentioned above, Benhabib, Schmitt-Grohé, and Uribe (2001) suggest that Taylor-style rules, of which inflation targeting rules provide a special case, are perilous in the sense that they may induce the economy to enter a deflationary liquidity trap.[2] In a previous paper (2000), I briefly argue that this outcome is highly unlikely; that the danger is a theoretical curiosity that should not be considered relevant for practical policy analysis. This section develops that argument more fully.

For the purpose of this purely analytical investigation, it is sufficient to use a closed economy model with full price flexibility. An extremely simple but adequate framework is provided by the following two-equation system:[3]

$$y_t = b_0 + b_1 \left(R_t - E_t \Delta p_{t+1} \right) + E_t\, y_{t+1} + v_t \text{, and} \tag{1}$$

$$R_t = \mu_0 - \mu_1 \pi^* + \left(1 + \mu_1 \right) \Delta p_t + \mu_2 y_t \,. \tag{2}$$

Here y_t and p_t denote the logs of output and the price level, so Δp_t is inflation and R_t is the one-period nominal interest rate. Equation 1 represents a log-linearized expectational IS function, which describes aggregate demand behavior in a fashion that can be rationalized by dynamic optimizing analysis, as explained by Woodford (1995, 2000),

2. The trap discussed by Reifschneider and Williams (2000) and Krugman (1998) is similar in some respects, but it involves a different mechanism, as the models used are entirely backward-looking.

3. This is essentially a linearized version of the first model used by Benhabib, Schmitt-Grohé, and Uribe (2001).

McCallum and Nelson (1999), and many others. The term v_t represents a taste shock that is generated by an exogenous stochastic process, which is assumed to be AR(1), that is, autoregressive of order one, with parameter ρ. Equation 2 is a Taylor rule in which the central bank is depicted as setting an interest rate instrument, R_t, each period so as to tighten policy when inflation exceeds its target value π^* and/or when output is high. In equation 2, y_t should be interpreted as the output gap, $y_t - \bar{y}_t$, with $\bar{y}_t$ for simplicity assumed constant at the value zero. For present purposes, furthermore, I am treating prices as fully flexible, so that $y_t = 0$ in each period. The system thus contains only two endogenous variables, R_t and Δp_t. The model also includes the requirement that Δp_t must not approach $-\infty$ as $t \rightarrow \infty$, which represents a transversality condition that obtains in the underlying optimizing model.[4]

To obtain a rational expectations solution, I first substitute out R_t and, using $y_t = 0$, obtain

$$0 = b_0 + b_1 \left[\mu_0 - \mu_1 \pi^* + (1 + \mu_1)\Delta p_t - E_t \Delta p_{t+1}\right] + v_t . \tag{3}$$

The minimum state variable (MSV) solution is of the form

$$\Delta p_t = \phi_0 + \phi_1 v_t , \tag{4}$$

which implies that $E_t \Delta p_{t+1} = \phi_0 + \phi_1 \rho v_t$. Substitution into equation 3 and application of the undetermined coefficient procedure then yields the requirement that

$$0 = b_0 + b_1 \left[\mu_0 - \mu_1 \pi^* + (1 + \mu_1)(\phi_0 + \phi_1 v_t) - \phi_0 - \phi_1 \rho v_t\right] + v \tag{5}$$

holds identically for all realizations of v_t. That implies unique values for ϕ_0 and ϕ_1 and gives the MSV solution

$$\Delta p_t = \pi^* - \frac{b_0 + b_1 \mu_0}{\mu_1} - \left[b_1 (1 - \rho + \mu_1)\right]^{-1} v_t . \tag{6}$$

Of course, Taylor (1993a) and many others prescribe that the central bank set $\mu_0 = r$, the long-run average real rate of interest, which equation 1 defines as $-b_0/b_1$. Adherence to this recommendation there-

4. See, for example, Woodford (2000, chap. 2).

fore implies that the second term on the right-hand side of equation 6 vanishes, yielding

$$\Delta p_t = \pi^* - \left[b_1\left(1-\rho+\mu_1\right)\right]^{-1} \mathrm{v}_t ,$$

as the MSV solution for inflation. Since the unconditional expectation is $E(\mathrm{v}_t) = 0$, it is clear that $E\Delta p_t = \pi^*$, that is, the long-run average rate of inflation is equal to the target value specified by the central bank's policy rule.

There is, however, another solution that satisfies the usually stated conditions for a rational expectations equilibrium. Consider the candidate solution

$$\Delta p_t = \phi_0 + \phi_1 \mathrm{v}_t + \phi_2 \Delta p_{t-1} , \tag{7}$$

which implies that

$$E_t \Delta p_{t+1} = \phi_0 + \phi_1 \rho \mathrm{v}_t + \phi_2\left(\phi_0 + \phi_1 \mathrm{v}_t + \phi_2 \Delta p_{t-1}\right).$$

Then, presuming $\mu_0 = -b_0/b_1$, the undetermined coefficient conditions are

$$b_1\left[-\mu_1\pi^* + (1+\mu_1)\phi_0 - \phi_0\left(1+\phi_2\right)\right] = 0, \tag{8a}$$

$$b_1\left[(1+\mu_1)\phi_1 - \phi_1\rho - \phi_2\phi_1\right] + 1 = 0, \text{ and} \tag{8b}$$

$$\phi_2^2 = \phi_2\left(1+\mu_1\right). \tag{8c}$$

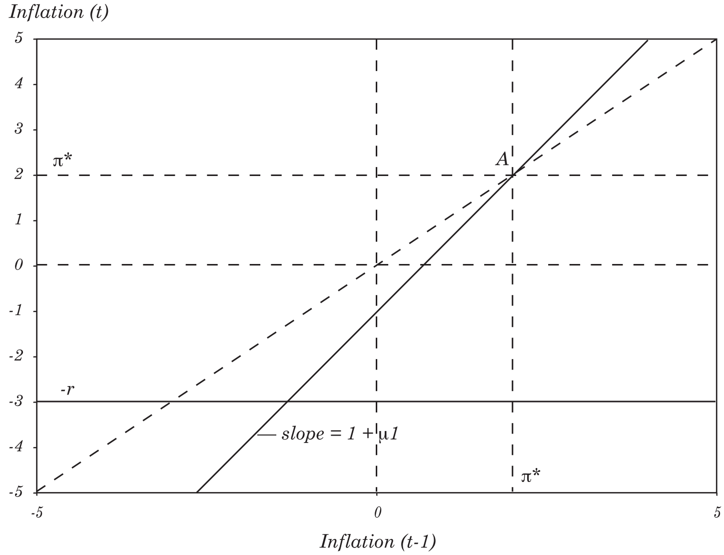
Thus there are two possibilities for ϕ_2, 0 and $1 + \mu_1$. Selecting the former generates the same MSV solution as in equation 6, but if $\phi_2 = 1 + \mu_1$ is designated as relevant, the solution becomes

$$\Delta p_t = -\mu_1\pi^* + \left(1+\mu_1\right)\Delta p_{t-1} + \left(b_1\rho\right)^{-1} \mathrm{v}_t . \tag{9}$$

Clearly, with $\mu_1 > 0$ the latter is explosive. If the system begins with $\Delta p_{t-1} > \pi^*$, then inflation will increase explosively, and if the startup value is below π^*, then Δp_t will tend to approach $-\infty$, according to equation 9 and as illustrated in figure 1.

The last statement, however, ignores the existence of a zero lower bound on the nominal interest rate. In the flexible price system at hand, the latter translates into a lower bound on Δp_t, generating the restriction $\Delta p_t \geq -r$. Thus if the system begins with $\Delta p_{t-1} < \pi^*$, inflation cannot behave as specified by equation 9. Instead, the alleged outcome is that $\Delta p_t \to -r$, which corresponds to $R_t \to 0$. In this case, therefore, the policy rule given by equation 2 fails to stabilize inflation around its target

Figure 1. Inflation Dynamics, System Defined in Equations 1 and 2



value, π*. This is the failure of the Taylor rule proposed and emphasized by the writers mentioned above.

In McCallum (2000), I argue that the foregoing is a pseudo problem, in that the solution just described most likely is not economically relevant. The argument there is that equation 6 provides the MSV or fundamentals solution, whereas equation 9 represents a rational expectations bubble, and that it is doubtful that bubble solutions are of empirical relevance, at least from a macroeconomic perspective. Here I extend that argument with another reason to ignore the non-MSV solution, a reason based on the closely related concepts of E-stability and least squares learnability.

Iterative E-stability was developed in the 1980s, principally by Evans (1985, 1986), and then modified in response to work by Marcet and Sargent (1989). Iterative E-stability involves a thought experiment in which one conceives of expectational behavior with anticipated variables such as Δp^e_{t+1} being described by an expression of a form that would be appropriate under rational expectations, but with parameter values that are initially incorrect.[5] When substituted into the model of the economy, this so-called expectations function implies a law of mo-

5. Here Δp^e_{t+1} denotes the subjective expectation of Δp_{t+1} formed at time t, not necessarily according to rational expectations.

tion that entails systematic expectational errors. One can then conceive of revised values of the parameters of the expectations function that are suggested by the law of motion. These, too, will imply incorrect forecasts, but one can imagine continuing with a series of iterations and consider whether they will converge to a specific rational expectations solution, either the MSV or a non-MSV solution.[6] If such a process converges to a particular solution, then the latter is said to be iteratively E-stable.

By considering ever-smaller time periods for these iterations, one can develop a process that is continuous in notional time (meta-time). Evans and Honkapohja (1999, 2001) emphasize this refined notion of E-stability because it is, under fairly general conditions, equivalent to learnability by means of a least-squares-based adaptive process. For a useful introduction to E-stability and learnability, see Bullard and Mitra (2000).

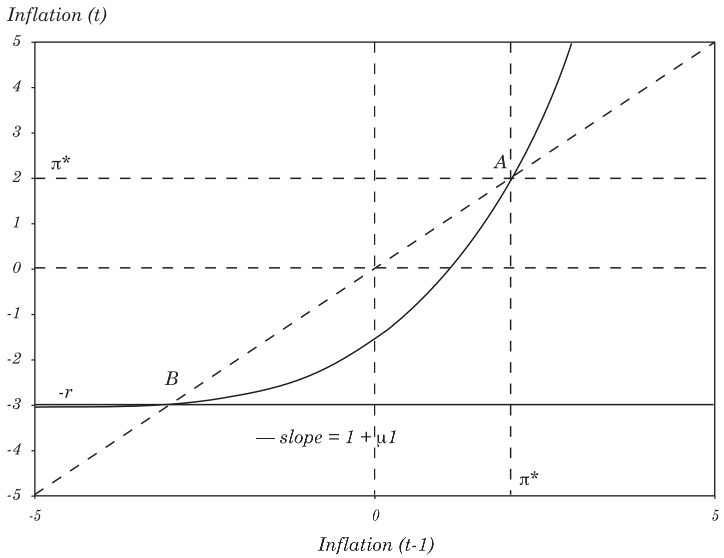
Evans (1986) analyzes the model at hand, as summarized in equation 3, and finds the MSV solution to be E-stable and the bubble solution to be E-unstable. These results extend to the refined definition of E-stability and therefore imply that the MSV solution is least squares learnable and the non-MSV is not (see Evans and Honkapohja, 2001, section 9.7). This statement applies literally to the model without the ZLB constraint, but the constraint does not affect the analysis, which is local in nature, of the MSV solution. For the non-MSV solution, equation 3 must be replaced with the ZLB constraint, which can be done by rewriting equation 3 so as to pass through the point $(-r, -r)$ and inserting a parameter that controls its slope. The constraint would then be imposed by letting the slope approach zero. Thus the analysis would be as before, but with a slope of less than 1.0 at the non-MSV point, which would not yield E-stability.

A more satisfying approach might be to recognize that the lower bound on the nominal interest rate is actually the consequence of a decreasing net marginal benefit, via facilitation of transactions, provided by holdings of money.[7] Figure 2 illustrates the relevant functional form, in which the MSV solution is at point A and the liquidity trap at point B. For this continuous nonlinear case, Evans and Honkapohja (2001, section 11.5) establish that the MSV solution is E-stable and the trap solution is not.

In sum, there are several reasons to believe that MSV solutions generally prevail in actual economies. Thus there is no compelling rea-

6. If there is convergence, it will be to some rational expectations solution.
7. See McCallum (2000).

Figure 2. Inflation Dynamics, Nonlinear System



son to believe that a liquidity trap would be generated, in the manner under discussion, by the adoption of a Taylor rule or the special case of pure inflation targeting.

2. Is Indeterminacy a Problem for Inflation Forecast Targeting?

A closely related issue pertains to policymaking that follows a rule for inflation forecast targeting, that is, a rule of the form

$$R_t = \mu_0 + E_t \Delta p_{t+1} + \mu_1 \left(E_t \Delta p_{t+j} - \pi^* \right) + \mu_2 y_t , \tag{10}$$

with $j \geq 1$. This is evidently how actual inflation targeting regimes have been operated in practice, because of the perceived need for central banks to behave preemptively—that is, adjusting policy instruments to combat inflationary (or deflationary) pressures before measured inflation (or deflation) begins to show up strongly in measured data.[8] However, several analysts, beginning with Woodford (1994), argue that when

8. On the need for preemptive policymaking, see Goodfriend (1997). For descriptions of practices of the Bank of England, the Reserve Bank of New Zealand, and the Bank of Canada, see King (1999); Archer (2000); Freedman (2000).

$j \geq 1$ in equation 10, a danger of indeterminacy is induced, which is not present if the policy rule is of the form given by equation 2.[9] Note that for very large values of μ_1, in a policy rule like equation 10, the implied policy is virtually the same as exact targeting of an expected inflation rate, as promoted by Svensson (1997) and others. The argument thus seems to deserve scrutiny. As in the previous section, however, the danger identified by this line of analysis represents a theoretical curiosity that is probably not of practical relevance.

It is important to keep in mind that the term indeterminacy first became prominent in monetary economics through a series of writings by Patinkin—beginning with Patinkin (1949) and culminating with (1961) and (1965)—that grew out of observations made by Lange (1942) about a putative logical inconsistency in classical monetary theory. Some of Patinkin's conclusions were disputed in a notable book by Gurley and Shaw (1960), and the resulting controversy was prominently reviewed in an influential survey article by Johnson (1962). In all of this early literature, the form of indeterminacy under discussion is price-level indeterminacy, such that the models in question fail to determine the value of any nominal variable, including the money supply. That type of failure occurs basically because of postulated policy behavior that is entirely devoid of any nominal anchor—that is, there is no concern on the part of the central bank for nominal variables.[10] Since rational private households and firms care only about real variables, according to standard neoclassical analysis, the absence of any money illusion by both them and the central bank must imply that no agent (in the model) has any concern for any nominal variable. Thus no nominal variable appears anywhere in the model, so naturally the model cannot determine the value of such variables.

The type of indeterminacy under discussion in the literature cited at the beginning of this section is very different. Instead of a failure to determine any nominal variable, without any implied problematic behavior for real variables, the recent Woodford-warning literature (as termed by Lars Svensson) is concerned with a multiplicity of stable equilibria in terms of real variables.[11] This type of aberrational behavior

9. Other papers that either promote this idea or discuss it with apparent approval include Bernanke and Woodford (1997); Kerr and King (1996); Clarida, Gali, and Gertler (1997); Svensson (1997); Christiano and Gust (1999); Carlstrom and Fuerst (2000); Isard, Laxton, and Eliasson (1999); Bullard and Mitra (2000); King (2000).

10. See Patinkin (1965, p. 309).

11. Dynamically stable equilibria are the relevant issue, because explosive paths of real variables are normally ruled out by transversality conditions that show them to be suboptimal for individual private agents.

stems not from the absence of any nominal anchor (a static concept), but from the essentially dynamic fact that various paths of real money balances can be consistent with rational expectations under some circumstances.[12] As an example of the sort of confusion that can arise if the foregoing distinction is not recognized, consider the analysis of price-level indeterminacy under an interest rate rule developed by Sargent and Wallace (1975). It has long been my belief that this paper is concerned with nominal indeterminacy (see McCallum, 1981, 1986). Woodford (2000, chap. 2), by contrast, interprets this particular Sargent and Wallace discussion as pertaining to solution multiplicity. My position is strengthened by the fact that the only substantive reference cited by Sargent and Wallace is Olivera (1970), which is clearly concerned with nominal indeterminacy. In any event, Sargent and Wallace (1975) and subsequent writings clearly illustrate the importance of observing the distinction.

Consider now the substance of the Woodford warning of multiple solutions when policy is based on rational forecasts of future inflation. It can be illustrated in a model similar to the prototype given by equations 1 and 2 presented above.[13] For convenience, the model is rewritten here, but adding a gradual price adjustment relation and ignoring constant terms that are tedious and for present purposes uninteresting. Finally, suppose that $E_t \Delta p_{t+1}$ is the inflation-forecast variable to which the policy rule pertains. The system can then be written as

$$y_t = b_1 \left(R_t - E_t \Delta p_{t+1} \right) + E_t y_{t+1} + v_t , \tag{11}$$

$$\Delta p_t = \beta E_t \Delta p_{t+1} + \alpha y_t , \text{ and} \tag{12}$$

$$R_t = (1 + \mu_1) E_t \Delta p_{t+1} + \mu_2 y_t + e_t , \tag{13}$$

where e_t in equation 13 is white noise and v_t in equation 11 is, as before, generated by a first-order autoregressive process with parameter ρ.

In this model the unique MSV rational expectations solution is of the form[14]

12. McCallum (1986) proposes that different terms be used for the two types of aberrational behavior to avoid possible semantic confusions, but this proposal has not met with widespread acceptance.

13. I am not disputing the point that central banks need to base policy on their own information and structural models, which is also discussed by Woodford (1994) and Bernanke and Woodford (1997).

14. The minimum state variable (MSV) concept is discussed at length in McCallum (1983, 1999), where it is interpreted as the unique solution that includes no bubble or sunspot components. In those papers, I propose a solution procedure that generates a unique solution by construction in a very wide class of linear rational expectations models.

$$y_t = \phi_{11} v_t + \phi_{12} e_t \text{, and} \tag{14}$$

$$\Delta p_t = \phi_{21} v_t + \phi_{22} e_t . \tag{15}$$

Thus $E_t y_{t+1} = \phi_{11}\rho_1 v_t$ and $E_t \Delta\rho_{t+1} = \phi_{21}\rho_1 v_t$, such that standard undetermined coefficient calculations yield

$$\phi_{11} = \frac{1}{1-\rho_1 - b_1\mu_2 - (\alpha b_1 \mu_1 \rho_1)/(1-\beta\rho_1)}, \tag{16a}$$

$$\phi_{12} = \frac{b_1}{1-b_1\mu_2}, \tag{16b}$$

$$\phi_{21} = \frac{\alpha}{(1-\beta\rho_1)(1-\rho_1 - b_1\mu_2) - \alpha b_1\mu_1\rho_1}, \tag{16c}$$

$$\phi_{22} = \frac{\alpha b_1}{1-b_1\mu_2}. \tag{16d}$$

This implies unique values for $\phi_{11} > 0$, $\phi_{12} < 0$, $\phi_{21} > 0$, and $\phi_{22} < 0$, so the MSV solution suggests that there is no problem with the inflation-forecast targeting rule in equation 13.

Suppose, however, that a researcher looks for non-MSV solutions of the form

$$y_t = \phi_{11} v_t + \phi_{12} e_t + \phi_{13}\Delta p_{t-1} \text{, and} \tag{17}$$

$$\Delta p_t = \phi_{21} v_t + \phi_{22} e_t + \phi_{23}\Delta p_{t-1} , \tag{18}$$

where the extraneous state variable Δp_{t-1} is included. These expressions imply that

$$E_t\, y_{t+1} = \phi_{11}\rho_1 v_t + \phi_{13}(\phi_{21} v_t + \phi_{22} e_t + \phi_{23}\Delta p_{t-1}) \text{, and}$$

$$E_t\, \Delta p_{t+1} = \phi_{21}\rho_1 v_t + \phi_{23}(\phi_{21} v_t + \phi_{22} e_t + \phi_{23}\Delta p_{t-1}).$$

Undetermined coefficient reasoning implies that the values for ϕ_{ij} are given by six relations analogous to equation 16, among which are

$$\phi_{13} = b_1\mu_1\phi_{23}^2 + b_1\mu_2\phi_{13} + \phi_{13}\phi_{23} \text{, and} \tag{19}$$

$$\phi_{23} = \beta\phi_{23}^2 + \alpha\phi_{13} . \tag{20}$$

These equations can be used to solve for ϕ_{23}, yielding the cubic equation

$$\phi_{23} = \beta\phi_{23}^2 + \frac{\alpha b_1 \mu_1 \phi_{23}^2}{1 - b_1 \mu_2 - \phi_{23}}. \tag{21}$$

Inspection of the latter indicates that one solution is provided by $\phi_{23} = 0$, which implies that $\phi_{13} = 0$. This, of course, gives the MSV solution obtained previously, but equation 21 is also satisfied by roots of the quadratic

$$\beta\phi_{23}^2 - \left(1 + \beta + \alpha b_1 \mu_1 - b_1 \mu_2 \beta\right)\phi_{23} + \left(1 - b_1 \mu_2\right) = 0, \tag{22}$$

that is, by

$$\phi_{23} = \frac{d \pm \left[d^2 - 4\beta\left(1 - b_1 \mu_2\right)\right]^{0.5}}{2\beta}, \tag{23}$$

where d is the first bracketed term in equation 22. Therefore, for some values of the parameters α, β, b_1, μ_1, and μ_2, there may be other real solutions in addition to the MSV solution.

To keep matters relatively simple, let $\mu_2 = 0$, so that the policy rule responds only to expected inflation. Then d becomes $1 + \beta + \alpha b_1 \mu_1$, and equation 22 will have two real roots if $\mu_1 < 0$ or if

$$\mu_1 > \mu_1^c \equiv \frac{2\beta^{0.5} + 1 + \beta}{- b_1 \alpha}.$$

Furthermore, while one of the ϕ_{23} values in equation 22 will exceed 1.0 in absolute value when $\mu_1 > \mu_1^c$, the other will not—it will be a negative stable root. Consequently, there will be no transversality condition to rule out that root's implied trajectory as a rational expectations equilibrium. Thus with $\mu_1 > \mu_1^c$, there is an infinite multiplicity of stable rational expectations solutions indexed by the initial start-up value of Δp_{t-1}. In such cases, moreover, so-called sunspot solutions are also possible in the sense of not being ruled out by the conditions of rational expectations equilibria.[15] This is the danger pointed out by the Woodford warning. Furthermore, it is made less likely when values of μ_2 exceed zero, thereby providing an additional reason to avoid pure inflation forecast targeting.[16]

The postulated danger may not be of any practical significance, however, for it is entirely possible that non-MSV solutions—namely, bubbles and sunspots—are empirically irrelevant.[17] This is a cogent and plau-

15. By a sunspot solution I mean one that includes random variables (of a martingale difference variety) that have no connection with other elements of the model.
16. See, for example, Bullard and Mitra (2000).
17. At least in macroeconomic contexts.

sible hypothesis that has not been convincingly contradicted by any empirical tests, despite the enormous amount of interest shown by researchers over the past twenty-five years. The main line of argument in favor of the proposition that only MSV solutions are of empirical relevance again concerns the E-stability and learnability of the alternative solutions. For the model at hand, Bullard and Mitra (2000, figure 3) show that when μ_1 or μ_2 values are large, the MSV solutions are E-stable and, therefore, learnable by a real-time least squares learning procedure.[18] Bullard and Mitra do not analyze the E-stability and learnability properties of the non-MSV solutions, but very closely related cases have been analyzed by Evans (1986, pp. 150–53) and Evans and Honkapohja (1999, pp. 487–506; 2001, chap. 10). Their results indicate that the non-MSV solutions do not possess E-stability in the case at hand.

A second line of argument is developed in McCallum (2001), from which this section is adapted. That paper emphasizes that the unique MSV solution is available in the high-μ_1 cases indicated by the Woodford warning and that this solution is well behaved in the sense of experiencing no discontinuity when μ_1 passes through the critical value that delineates the region of multiple stable solutions. Specifically, impulse response functions for the MSV solution are plotted and shown to be virtually indistinguishable for μ_1 values just above and just below the μ_1^c critical value at which solution multiplicity sets in. Also, the MSV impulse response functions change continuously with μ_1 more generally (McCallum, 2001, figures 3–5). By contrast, the non-MSV solutions are not continuous at μ_1^c, and they feature additional peculiarities. Those results illustrate the well-behaved nature of the MSV solution for the example considered, as well as the erratic nature of the non-MSV (bubble) solutions. Such results also obtain for other parameter values and clearly suggest the desirability of considering the MSV solution as the sole economically relevant solution.

If the MSV solution is taken to represent implied behavior for the model at hand, then there is no compelling reason to believe that strong responses to forecast inflation values will generate undesirable behavior. In that case, preemptive inflation forecast targeting could be an attractive policy regime, despite warnings of the type under discussion.[19]

18. As mentioned above, E-stability pertains to the convergence of meta-time iterations that may or may not drive nonrational expectations functions to their rational expectations values, and it governs least squares learnability.

19. This argument does not apply to the case with $\mu_1 < 0$, in which the Taylor principle does not hold and there is a genuine problem. For analysis and more discussion, see McCallum (2001).

3. A Framework for Quantitative Open Economy Analysis

Whereas the points addressed in the previous two sections could be discussed in the context of extremely simple models with only qualitative specifications, the topics considered below require a more realistic specification of the relations governing the dynamics of both consumption and price adjustment behavior. The foreign trade of goods, services, and financial assets also plays an important role, and postulating alternative monetary policy rules similarly requires a degree of realism. The present section, accordingly, is devoted to describing the open economy model used in sections 4, 5, and 6.

The basic structure of the model is derived from McCallum and Nelson (1999), but with a few adjustments that are intended to improve its match with actual data. The McCallum-Nelson model was designed in the spirit of what has been called the new open economy macroeconomics.[20] In other words, it was intended to be a dynamic open economy macroeconomic model that features rational expectations, optimizing agents, and slowly adjusting prices of goods. It differs from other contributions in the area, however, in the manner in which imported goods are treated. In particular, the McCallum-Nelson model treats imports not as finished goods, as is usual, but rather as raw material inputs to the home economy's production process. This alternative treatment leads to a cleaner and simpler theoretical structure, relative to the standard treatment, and it is empirically attractive in ways that are outlined below. Since the optimizing, general equilibrium analysis (from the perspective of a small economy) is presented in McCallum and Nelson (1999), here I take an informal expository approach designed to facilitate understanding of the model's basic structure.

In a wide variety of infinite-horizon models involving imperfect competition, optimizing analysis leads to a consumption Euler equation that can be expressed or approximated in the form,

$$c_t = \mathrm{E}_t c_{t+1} + \mathrm{b}_0 + \mathrm{b}_1 r_t + \mathrm{v}_t , \tag{24}$$

where c_t is the log of a Dixit-Stiglitz consumption-bundle aggregate of the many distinct goods that a typical household consumes in

20. For references to this line of work, see Lane (1999). Also see Brian Doyle's "New Open Economy Macroeconomics Homepage," at www.geocities.com/brian_m_doyle/open.html.

period t.[21] In equation 24, r_t is the real interest rate on home-country one-period bonds (private or government) and v_t is a stochastic shock term that pertains to household preferences regarding present versus future consumption. In the analysis of a closed economy, relation 24 is often combined with a log-linearized, per-household, overall resource constraint to yield an "expectational IS function," to use the term of Kerr and King (1996). This step presumes that investment and capital are treated as exogenous. The simplest version of that assumption is that the capital stock is fixed; since that assumption is rather common in the new open economy macroeconomics literature, I adopt it here.

For the current open economy application, one might be tempted to write the resource constraint as

$$y_t = \omega_1 c_t + \omega_2 g_t + \omega_3 x_t - \omega_4 im_t , \tag{25}$$

where y_t, g_t, x_t, and im_t are logarithms of real output, government consumption, exports, and imports, respectively, while ω_1, ω_2, ω_3, and ω_4 are steady-state ratios of consumption, government purchases, exports, and imports to output. If imports are exclusively material inputs to the production of home-country goods, however, and if $Y_t = \ln^{-1} y_t$ is interpreted as units of output, then the relevant identity is

$$y_t = \omega_1 c_t + \omega_2 g_t + \omega_3 x_t . \tag{25′}$$

This is, of course, the same as equation 25 with $\omega_4 = 0$. Either of these versions can be thought of as the resource constraint for the model.

It is desirable that import demand be modeled in an optimizing fashion. Toward that end, assume that production of all consumer goods is effected by households that are constrained by a production function of the constant elasticity of substitution (CES) form, with labor and material imports being the two variable inputs. The cost-minimizing demand for imports then equals

$$im_t = y_t - \sigma q_t + \text{const.}, \tag{26}$$

where σ is the elasticity of substitution between materials and labor in production and where "const." denotes some constant.[22] Also, q_t is the

21. Thus $c_t = \ln C_t$, with $C_t = \left[\int C_t(z)^{(\theta-1)/\theta} \mathrm{d}z\right]^{\theta/(\theta-1)}$, where $\theta > 1$, z indexes distinct goods, and the integral is over (0,1), while the corresponding price index is $P_t = \left[\int P_t(z)^{1-\theta} \mathrm{d}z\right]^{1/(1-\theta)}$.

22. That is, the expression "const." in different equations appearing below typically refers to different constant magnitudes.

log price of imports in terms of consumption goods. In other words, $Q_t = \ln^{-1} q_t$ is the real exchange rate. Let P_t and S_t be the home-country money price of goods and foreign exchange, with P_t^* the foreign-money price of home-country imports. If p_t, s_t, and p_t^* are logs of these variables, then

$$q_t = s_t - p_t + p_t^* . \tag{27}$$

Symmetrically, export demand is assumed to be given as

$$x_t = y_t^* + \sigma^* q_t + \text{const.}, \tag{28}$$

where y_t^* denotes production abroad and σ^* is the price elasticity of demand from abroad for home-country goods.

Now consider output determination in a flexible-price version of the model. A log-linear approximation to the home-country production function yields

$$y_t = (1 - \alpha)a_t + (1 - \alpha)n_t + \alpha im_t + \text{const.},$$

where n_t and a_t are logs of labor input and a labor-augmenting technology shock term, respectively. Suppose for simplicity that labor supply is inelastic, with 1.0 units supplied per period by each household. Thus with price flexibility, $n_t = 0$ and the flexible-price, natural-rate (or potential) value of y_t is

$$\bar{y}_t = (1 - \alpha)a_t + \alpha(\bar{y}_t - \sigma q_t) + \text{const., or}$$

$$\bar{y}_t = a_t - \left(\frac{\sigma\alpha}{1 - \alpha}\right)q_t + \text{const.}. \tag{29}$$

But while $\bar{y}_t$ would be the economy's output in period t if prices could adjust promptly in response to any shock, the model assumes that prices adjust only sluggishly. If the economy's demand quantity as determined by the rest of the system (y_t) differs from $\bar{y}_t$, then the former quantity prevails—and workers depart from their inelastic supply schedules so as to provide whatever quantity is needed to produce the demanded output, with im_t given by equation 26.

In such a setting, the precise way in which prices adjust has a direct impact on demand and, consequently, on production. The recent literature uses various models of gradual price adjustment that are intended to represent optimizing behavior. The analysis below explores two candidates, one of which is presented here. Because it is the one used in previous work (McCallum and Nelson, 1999), I begin with the

P-bar model, here expressed in the form

$$p_t - p_{t-1} = (1 - \phi_1)(\bar{p}_{t-1} - p_{t-1}) + \mathrm{E}_{t-1}(\bar{p}_t - \bar{p}_{t-1}). \tag{30}$$

Thus prices adjust in response to prior departures of p_t from its market-clearing value ($\bar{p}_t$) and to expected changes in the latter. In the tabulation of endogenous variables, however, neither p_t nor $\bar{p}_t$ needs to be included in addition to Δp_t, since equation 30 is logically equivalent to

$$\mathrm{E}_{t-1}(p_t - \bar{p}_t) = \phi_1(p_{t-1} - \bar{p}_{t-1}), \text{ and thus to}$$

$$\mathrm{E}_{t-1}(y_t - \bar{y}_t) = \phi_1(y_{t-1} - \bar{y}_{t-1}), \tag{30′}$$

as is shown in McCallum and Nelson (1999). The same conclusion regarding endogenous variables holds in the second model of price adjustment considered below. The adjustment relation in that case is

$$\Delta p_t = 0.5(\mathrm{E}_t \Delta p_{t+1} + \Delta p_{t-1}) + \phi_2(y_t - \bar{y}_t) + \mathrm{u}_t, \tag{30″}$$

where u_t is a behavioral disturbance. This form of equation has been fairly prominent in recent work, primarily because it tends to impart a more realistic degree of persistence to inflation than does the more theoretically attractive Calvo-Rotemberg model.[23]

A standard feature of most current open economy models is a relation implying uncovered interest parity (UIP). One is therefore adopted here, despite the prominent empirical weaknesses of such relations:

$$R_t - R_t^* = \mathrm{E}_t \Delta s_{t+1} + \xi_t. \tag{31}$$

The equation includes a time-varying risk premium term, ξ_t, however, which may have a sizeable variance and could be autocorrelated.

It remains to describe how monetary policy is conducted. In the spirit of most recent research in monetary economics, I presume that the monetary authority conducts policy in a manner suggested by the Taylor (1993a) rule, that is, by adjusting a one-period nominal interest rate in response to prevailing (or forecasted future) values of inflation and the output gap, $\tilde{y}_t = y_t - \bar{y}_t$:

$$R_t = (1 - \mu_3)[\mu_0 + \Delta p_t + \mu_1(\Delta p_t - \pi^*) + \mu_2 \tilde{y}_t] + \mu_3 R_{t-1} + \mathrm{e}_t \tag{32}$$

The quantitative results presented below are based on estimated or calibrated versions of this rule, in most cases with E_{t-1} applied to $\tilde{y}_t$ and Δp_t.

23. See Fuhrer and Moore (1995); Clarida, Gali, and Gertler (1999).

To complete the model, we need only to include the Fisher identity,

$$1 + r_t = \frac{1 + R_t}{1 + \mathrm{E}_t\, \Delta p_{t+1}},$$

which we approximate in the familiar fashion:

$$r_t = R_t - \mathrm{E}_t\, \Delta p_{t+1}. \tag{33}$$

The model is thus a simple log-linear system in which the ten structural relations presented in equations 24 through 33 determine values for the endogenous variables y_t, $\bar{y}_t$, Δp_t, r_t, R_t, q_t, s_t, c_t, x_t, and im_t. Government spending, g_t, and the foreign variables p_t^*, y_t^*, and R_t^* are taken as exogenous—as are the shock processes for v_t, a_t, e_t, and ξ_t. This is probably the simplest and cleanest model extant that includes the essential features of the new open economy macroeconomics literature.

Of course, it is possible to append a money demand function such as

$$m_t - p_t = \gamma_0 + \gamma_1 y_t + \gamma_2 R_t + \eta_t, \tag{34}$$

and one of this general form—perhaps with c_t replacing y_t—would be consistent with optimizing behavior.[24] As many writers have noted, however, that equation would serve only to determine the values of m_t that are needed to implement the R_t policy rule.

With the structure given above, it is possible to calculate the log of the balance on goods and services account—that is, net log exports—as

$$net_t = x_t - \left(im_t + q_t\right), \tag{35}$$

where it is assumed that $\omega_3 = \omega_4$. Also, the log of the GDP deflator can be calculated as

$$p_t^{DEF} = \frac{p_t - \omega_3\left(s_t + p_t^*\right)}{1 - \omega_3}. \tag{36}$$

These represent extra features, however, that need not be included with the basic model (equations 24 through 33).

An advantage of this strategy of modeling imports as material inputs to the production process is that the relevant price index for produced goods is the same as the consumer price index, which implies that the same gradual price adjustment behavior is relevant for all domestic consumption. It also avoids the unattractive assumption,

24. See McCallum and Nelson (1999); Woodford (1995, 2000).

implied by the tradable vs. nontradable goods dichotomization, that export and import goods are perfectly substitutable in production. Theoretical advantages would not constitute a satisfactory justification, of course, if most imports were, in fact, consumption goods. Such is not the case, however, at least for the United States. Instead, the data suggest that under conservative assumptions, productive inputs actually comprise a larger fraction of U.S. imports than do consumer goods (including services).[25]

4. Calibration and Model Properties

There is one way in which the model developed in McCallum and Nelson (1999) differs significantly from the ten-equation formulation just presented. Specifically, it includes a somewhat more complex form of consumption versus saving behavior, featuring habit formation. Thus in place of the time-separable utility function that leads to equation 24, the model assumes that each period-t utility term includes $c_t/(c_{t-1})^h$, with $0 \leq h < 1$, rather than c_t alone. This specification gives rise to the following replacement for equation 24:

$$c_t = \mathrm{h}_0 + \mathrm{h}_1 c_{t-1} + \mathrm{h}_2 \mathrm{E}_t c_{t+1} + \mathrm{h}_3 \mathrm{E}_t c_{t+2} + \mathrm{h}_4 \left(\log \lambda_t\right) + \mathrm{v}_t . \tag{24'}$$

Here λ_t is the Lagrangian multiplier on the household's budget constraint, which obeys

$$\log \lambda_t = \text{const.} + \mathrm{E}_t \lambda_{t+1} + r_t , \tag{37}$$

and there are constraints relating the h_j parameters to others in the system. For details and additional discussion, see McCallum and Nelson (1999) and the recent study by Fuhrer (2000).

Calibration of the model draws on McCallum and Nelson (1999), but it differs in a few ways that, in retrospect, seem appropriate. For the parameters governing spending behavior, I retain here the $h = 0.8$ value taken from an early version of Fuhrer (2000), but for the counterpart of

25. In 1998, imported consumer goods amounted to US$453 billion while imports of business inputs came to US$624 billion, approximately. These figures are based on an examination of categories reported in the August 1999 issue of the *Survey of Current Business*. Several categories are clearly composed predominantly of either consumer or business goods. For other groups, judgmental assignments were required. Those assignments are as follows, with the reported figure being the fraction of the category classified as business inputs: automotive vehicles, engines, and parts, 25 percent; travel, 25 percent; passenger fares, 25 percent; foods, feed, and beverages, 50 percent; and other private services, 75 percent.

b_1 I now use –0.4 rather than –1/6 to reflect the greater responsiveness of investment spending, which is not included explicitly in the model.[26] I again use 1/3 for both σ (the elasticity of substitution in production and, therefore, the elasticity of import demand with respect to Q_t) and the elasticity of export demand with respect to Q_t. In equation 29, the labor-share parameter $1-\alpha$ equals 0.64. The steady-state ratio of imports (and exports) to domestic production is taken to be 0.25, which is a higher value than in McCallum and Nelson (1999) so as to reflect an economy that is more open than the United States. Unlike McCallum and Nelson (1999), I include government consumption, setting $\omega_2 = 0.25$.

In the two price adjustment specifications, the parameter values are $\phi_1 = 0.89$ (estimated by McCallum and Nelson) and $\phi_2 = 0.02$ (based on my reading of a wide variety of studies, plus conversion into nonannualized fractional terms for a quarterly model). Policy rule parameters vary in the experiments, but they should be thought of in relation to realistic values close to $\mu_1 = 0.5$, $\mu_2 = 0.4$, and $\mu_3 = 0.8$, with the latter reflecting considerable interest rate smoothing.[27] In most cases, expectations based on $t-1$ data are used for the Δp_t and $\tilde{y}_t$ variables appearing in the policy rule, to make our version of the rule operational.

The stochastic processes driving the model's shocks must also be calibrated, of course. For both foreign output and the technology shock, I have specified AR(1) processes with AR parameters of 0.95, rather than the 1.0 values used in McCallum and Nelson (1999). The innovation standard deviations are 0.03 and 0.0035, as before. The latter value might appear smaller than is usual, but it is appropriate to generate a realistic degree of variability in $\bar{y}_t$ when the latter is not exogenous but is dependent on q_t. The UIP risk premium term, ξ_t, is generated by an AR(1) process with AR parameter 0.5 and innovation 0.04; these values are based on work reported in Taylor (1993b). Government consumption (ln logs) follows an AR(1) process, with an AR parameter of 0.99 and an innovation standard deviation of 0.02. Finally, the v_t, u_t, and e_t shock processes are taken to be white noise with standard deviation values of 0.011, 0.002, and 0.0017, respectively.

One way to represent a model's properties is in terms of its variances and autocovariances. Unconditional variances for some of the

26. The parameter in question is the intertemporal elasticity of substitution in consumption when h = 0.

27. The coefficient attached to the output gap actually equals 0.1 in the simulations, as they include results based on quarterly fractional units. The literature on Taylor rules usually works with annualized percentages, however, so I here describe the number as 0.4.

Table 1. Standard Deviations of Δp_t, y_t, $\tilde{y}_t$, and R_t

	0.1 trade share		*0.25 trade share*	
Variable	$\mu_3=0.0$	$\mu_3=0.8$	$\mu_3=0.0$	$\mu_3=0.8$
P-bar variant				
Δp_t	4.41	4.42	10.20	7.94
y_t	2.22	1.98	4.41	3.91
$\tilde{y}_t$	2.56	2.25	6.31	4.81
R_t	5.78	2.80	13.12	5.48
Equation 30″ variant				
Δp_t	2.38	2.18	2.58	2.39
y_t	1.19	1.19	1.46	1.59
$\tilde{y}_t$	1.02	1.02	1.91	2.11
R_t	2.97	2.97	3.40	2.22

Source: Author's calculations.

model's crucial variables are shown in table 1 for various specifications. The first segment of the table pertains to the variant with the *P*-bar price adjustment (equation 30), whereas the second is based on the alternative adjustment relation (equation 30″). Two assumptions are considered for the share of exports to total production, namely, that this share is 0.10 or 0.25. The former represents a large economy that is relatively closed to foreign trade, whereas the latter figure is for a more typical economy. Finally, policy rule 32 is used both with and without interest smoothing, that is, with $\mu_3 = 0.8$ (the more realistic case) and with $\mu_3 = 0$ (as in the original version of Taylor's rule). In both cases the other coefficients are given the values mentioned above.

Table 1 clearly demonstrates that the *P*-bar variant of the model generates more variability in all principal variables than does the equation 30″ variant. The simulations do not aim to match the moments of any specific economy, but knowledge of values for the United States gives one the impression that that the equation 30″ values are the more realistic of the two sets, though they are slightly too small. The model also generates much more variability when the economy is more open to foreign trade. This is not surprising, since more trade leads to a bigger effect of exchange rate movements on the natural-rate value of output. The remainder of the paper uses the more open of the two specifications, that is, the one with an export-to-output (or import-to-output) ratio of 0.25. Finally, table 1 also indicates that in most cases interest rate smoothing (that is, $\mu_3 = 0.8$) helps to reduce the variability of inflation and the output gap.

Another way to represent the model's properties is in terms of its impulse response functions. The responses to a unit shock to the policy rule (that is, a 1.0 realization of the shock e_t) are shown in figure 3 for $\mu_3 = 0.8$. This temporary tightening of monetary policy induces temporary but lasting drops in output, inflation, and both the real and nominal exchange rate, together with a temporary increase in net exports. The dynamic patterns are somewhat different for the two price adjustment specifications, with much more inflation persistence apparent in the second case. Since this persistence is more consistent with observed behavior of inflation in most developed economies, this difference in outcomes favors the specification of equation 30″. Consequently, this specification is emphasized in what follows and is henceforth considered the standard price adjustment specification. A questionable feature of both models is that the exchange rates and net exports respond promptly to shocks, rather than with a lagged or gradual pattern. Overall, however, the nature of the models' responses are encouraging. The magnitude of the output response to a policy shock is somewhat larger than in McCallum and Nelson (1999), but this is due to the larger share of foreign trade.

5. Frequency of a Zero Lower Bound with Inflation Targeting

This section now examines the effects of inflation targeting, as compared with other monetary policy regimes, on the frequency of liquidity trap problems. The general strategy is to conduct simulations and determine how often a liquidity trap or zero lower bound (ZLB) constraint is encountered with various policy rules, including inflation targeting. For a given model, the frequency of ZLB constraints being encountered depends on R^*, which is the sum of the target inflation rate, π^*, and the average real interest rate, r. The smaller is π^*, the more frequently will the constraint be encountered. This frequency could be quite low, however, even with a reasonably small value of π^*, say, 2.0 percent per year (that is, 0.005 in quarterly fractional terms).

The simulations reported in this section do not actually impose a ZLB constraint. Instead, they permit negative rates of interest, in order to maintain a linear computational framework. The number of periods with such rates thus overestimates how frequently ZLB constraints would be binding, since in some periods the previous period's rate will have been negative. To more accurately estimate how often ZLB constraints would be encountered, I examine the frequency of periods (quarters)

Figure 3. Responses to Unit Shock to Policy Rule

P-Bar Model

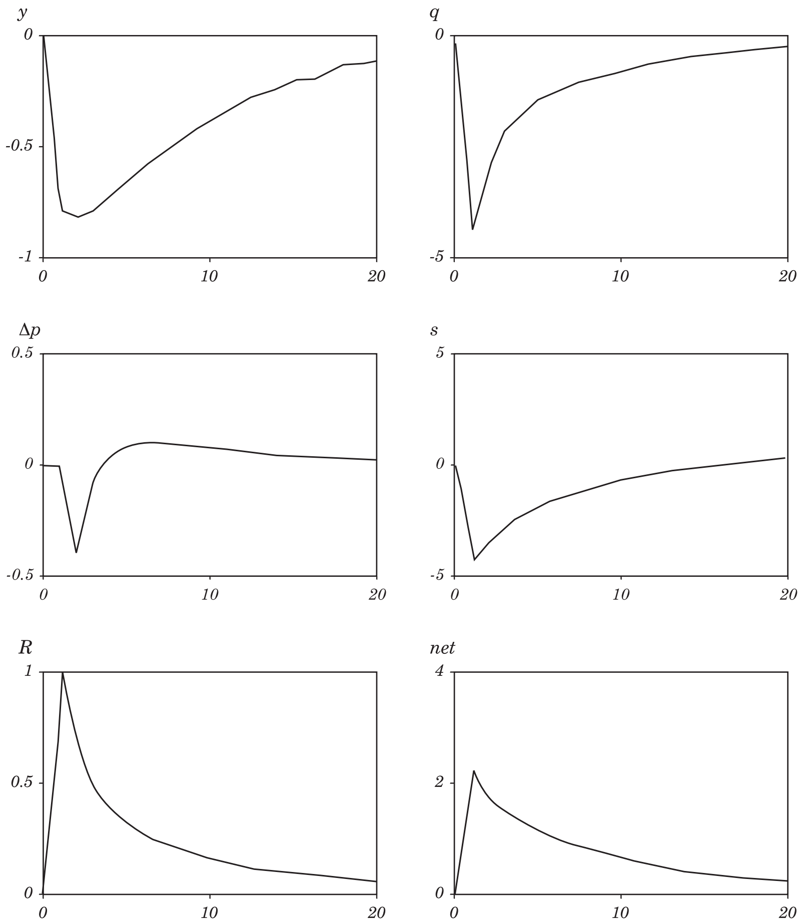


Figure 3. (continued)

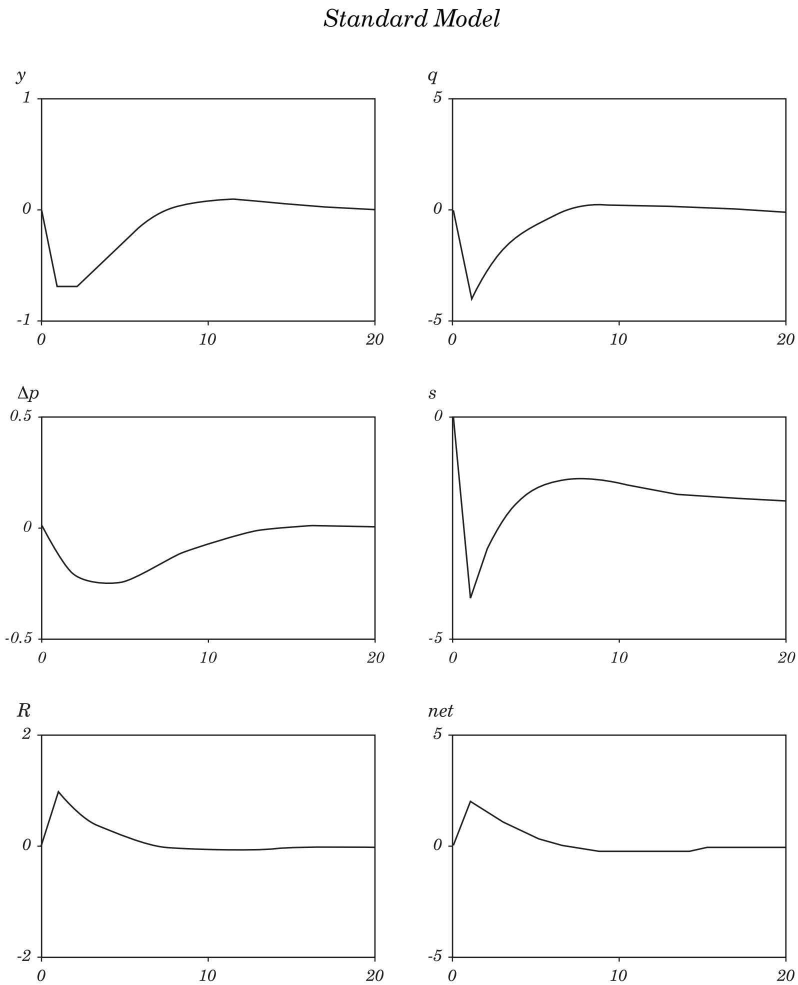


Table 2. Relative Frequency of Zero Lower Bound Statistics

*R** *(percent per year)*	*Statistic 1*	*Statistic 2*	*Statistic 3*
8.0	0.0001	0.0001	0.0001
7.0	0.0006	0.0004	0.0003
6.0	0.0088	0.0060	0.0034
5.0	0.0158	0.0108	0.0061
4.0	0.0408	0.0269	0.0126
3.0	0.0882	0.0565	0.0243
2.0	0.1990	0.1232	0.0423

Source: Author's calculations.

in which the recorded interest rate is both negative and lower than in the previous period. (If a value is negative but higher than in the previous period, the presumption is that R_t movement would also be upward in the model with a ZLB constraint, so the bound would not be encountered.) To illustrate, table 2 reports relative frequencies of three statistics pertaining to the zero lower bound: the fraction of periods in which negative rates are realized;[28] the fraction of periods in which negative rates are realized and the realized value is lower than in the previous period (this is the preferred measure); and the fraction of periods in which negative rates are realized and the value in the previous period was positive. This latter statistic is designed to indicate how many episodes of zero or negative rates occur, with each string of zero or negative values counted only once.

Several assumptions regarding R^* are investigated in table 2, that is, values ranging from 2 to 8 percent per year (0.005 to 0.02 in quarterly fractional units). If one believed that an economy's average real interest rate was about 3 percent and its inflation target was set at 2 percent, then the relevant figure for R^* would be 5 percent. For the calculations in table 2, the standard version of the model is used and the policy rule parameters μ_1, μ_2, and μ_3 are set at 0.5, 0.4, and 0.8, respectively. With $R^* = 5$ percent, negative interest rates are encountered in 1.58 percent of the quarterly time periods, but the preferred measure, for the reasons just explained, is given by the second statistic, which equals 1.08 percent of the time periods. Finally, the third statistic takes on a still smaller value, of 0.61 percent, for the relative frequency of episodes in which interest rate constraints are encountered. Of course, the frequencies

28. The simulations are carried out with all constant terms set equal to zero. Thus the observations described as negative are those in which the simulated value is less than $-R^*$. This way of proceeding is standard and innocuous.

Table 3. Performance Measures with Standard Model and R^* = 5.0 Percent

Standard deviations of inflation and output gap; percent of ZLB periods

μ_1, μ_2	$\mu_3 = 0.0$	$\mu_3 = 0.5$	$\mu_3 = 0.8$	$\mu_3 = 0.9$	$\mu_3 = 0.99$
0.1, 0.4					
Inflation	4.91	4.10	3.32	3.23	20.18
Output gap	2.04	2.13	2.31	2.49	10.88
ZLB	12.18	9.03	3.36	0.71	0.00
0.5, 0.4					
Inflation	2.63	2.52	2.36	2.60	17.33
Output gap	1.94	1.98	2.11	2.31	8.40
ZLB	5.93	4.07	1.23	0.34	0.00
1.0, 0.4					
Inflation	2.16	2.07	2.06	2.26	11.79
Output gap	1.91	1.93	2.04	2.19	6.65
ZLB	6.39	3.30	0.81	0.15	0.00
10.0, 0.4					
Inflation	1.28	1.29	1.36	1.50	2.95
Output gap	2.09	2.09	2.07	2.09	2.55
ZLB	20.31	13.80	6.66	2.96	0.03
0.1, 0.0					
Inflation	4.85	4.11	3.25	3.29	22.31
Output gap	2.12	2.17	2.35	2.59	12.03
ZLB	11.64	8.32	3.49	0.57	0.00
0.5, 0.0					
Inflation	2.60	2.42	2.32	2.55	16.59
Output gap	1.98	2.06	2.16	2.35	9.05
ZLB	6.34	3.32	0.61	0.18	0.00
1.0, 0.0					
Inflation	2.15	2.03	2.05	2.22	12.36
Output gap	1.95	2.00	2.12	2.24	6.98
ZLB	5.89	3.17	0.92	0.16	0.01
10.0, 0.0					
Inflation	1.29	1.26	1.35	1.50	3.01
Output gap	2.14	2.13	2.12	2.09	2.56
ZLB	19.60	13.37	7.30	2.88	0.01

Source: Author's calculations.

are all higher for lower values of R^*, with, for example, the ZLB constraint binding quite rarely at R^* = 7.0 but with a disturbingly high frequency for an R^* of 2.0 or 3.0. The main point here is that the regular and intuitive behavior of the three different statistics gives confidence that the second statistic does indeed provide a reasonable measure of the frequency of periods in which the zero lower bound would be encountered if one were to use nonlinear methods. In what follows, consequently, only that statistic is reported, and it is described simply as the fraction or percentage of periods in which the ZLB constraint is binding (see footnote 30 below).

The first set of basic substantive results is premised on the assumption that a value of 5 percent per year is appropriate for R^*. A hundred simulations were run for each case, and their average results are reported in table 3. The object is to consider alternative values for the policy rule parameters μ_1, μ_2, and μ_3 to determine the relative desirability of different rules. In each cell of table 3, the three numbers represent the standard deviation of inflation, the standard deviation of the output gap, and the frequency of ZLB occurrences. All of these are reported in percent (not fractional units), with the inflation figures annualized. The inflation and output gap figures should be interpreted as root-mean-square deviations from their target values.

Table 3 considers a wide range of values for μ_1 (from 0.1 to 10.0), which represents the strength of reaction to the inflation variable. The degree of interest rate smoothing, measured by μ_3, is also varied over a wide range, from 0 to 0.99. Only two values are reported, however, for μ_2, the response coefficient on the output gap. First, a value of 0.4 is considered, as it is close to the original Taylor-rule value of 0.5. Larger magnitudes are not explored since it is very dangerous, I believe, to respond strongly to perceptions of the output gap, because of the difficulty of measuring or even conceptualizing an operational measure of potential output.[29] Second, a value of μ_2 equal to zero is included to approximate a rule that is representative of pure inflation targeting.

In table 3 it can be seen that ZLB cases appear with excessive frequency for all cases with no interest smoothing or only a small degree (namely, $\mu_3 = 0.5$). At the value 0.8, which is close to those estimated empirically by Clarida, Gali, and Gertler (1997, 1999) and McCallum and Nelson (1999), most of the cases still show ZLB problems arising in over 1 percent of the quarterly time periods. With $\mu_3 = 0.9$, however, the frequency of ZLB periods becomes acceptably small.[30] Next, larger values of the inflation coefficient, μ_1, consistently lead to reduced variability of inflation around its target value.

29. For some discussion and results pertaining to this danger, see McCallum and Nelson (2000).

30. Reifschneider and Williams (2000) use the FRB/US econometric model and actually impose a proper zero lower bound. In their table 1, they report results for a case based on a Taylor rule with coefficients $\mu_1 = 0.5$, $\mu_2 = 0.5$, and $\mu_3 = 0$. They assume $r = 2.5$ percent per year, so their case with an inflation target of 2 percent implies an R^* value of 4.5 percent, which leads to a frequency of ZLB periods of 5 percent. When I use these settings, I get 8 percent, which is close and on the conservative side.

Increasing μ_1 from 0.1 to 1.0, moreover, tends to reduce both the variability of the output gap and the frequency of ZLB occurrences. Higher values, however, seem not to be helpful on balance. Finally, a comparison of the bottom and top halves of the table indicates that there is little difference between the pure inflation targeting case (with $\mu_2 = 0$) and the case with moderate, Taylor-style responses to the output gap (with $\mu_2 = 0.4$).

Next I turn to other, non-Taylor rules that use target variables other than inflation. From the perspective of actual practice, the most important are those that use the exchange rate, or its rate of change, as the principal target variable. I therefore consider policy rules of the form

$$R_t = (1 - \mu_3)\left[\mu_0 + \Delta p_t + \mu_1 \left(z_t - z^*\right)\right] + \mu_3 R_{t-1} + \mathrm{e}_t , \tag{38}$$

where z_t is the target variable. Letting s_t denote the log of the home-country price of foreign exchange, I experiment with s_t and Δs_t as examples of z_t. In addition, since several analysts have promoted nominal income, or its growth rate, as a target variable, I also use $x_t = y_t + p_t$ and Δx_t for z_t. As before, I actually use $\mathrm{E}_{t-1}\Delta p_t$ rather than Δp_t in equation 38 and also use the $t - 1$ expectation of x_t and Δx_t. For the exchange rate, however, it is assumed that the current-period value is observable and so appears in the rule. In addition, I want to consider price level targeting, that is, the use of $\mathrm{E}_{t-1}p_t$ rather than $\mathrm{E}_{t-1}\Delta p_t$ as the rule's target variable. This choice does not necessarily imply that the target for the price level is constant over time, but if it grows at a constant rate then target misses for the price level will subsequently have to be reversed.

Results are shown in table 4. In all cases considerable interest smoothing is assumed, with a realistic value of 0.8 for μ_3. The first column repeats figures from table 3 for reference. The second and third columns give results for s_t and Δs_t. With $\mu_1 = 0.1$, the performance of the s_t target is about as good as for Δp_t with $\mu_1 = 0.5$, but in all other cases both of the exchange rate targets give quite poor results with very high frequencies of ZLB occurrences. The high degree of variability of the exchange rate evidently leads to a great deal of interest variability and thus to a high frequency of ZLB constraints. The nominal income level target performs rather well for values of μ_1 up to 1.0, but it induces many ZLB periods when $\mu_1 = 10$. The nominal income growth target performs less well, although its performance is not too bad when μ_1 equals 0.5 or 1.0. Finally, the price level target yields very good results when μ_1 equals 0.1 or 0.5, but it induces a high frequency of ZLB constraints with stronger feedback parameters.

Table 4. Performance Measures with Alternative Targets and R^* = 5 Percent

Standard deviations of inflation and output gap; percent of ZLB periods[a]

μ_1	$E_{t-1}\Delta p_t$	s_t	Δs_t	$E_{t-1}x_t$	$E_{t-1}\Delta x_t$	$E_{t-1}\pi_t$
0.1						
Inflation	3.25	2.51	3.25	1.98	3.39	1.97
Output gap	2.35	2.05	2.40	2.16	2.44	2.18
ZLB	3.49	0.56	3.25	0.02	3.67	0.03
0.5						
Inflation	2.32	2.66	2.58	1.79	2.47	1.63
Output gap	2.16	1.91	2.34	2.12	2.34	2.21
ZLB	0.61	8.60	7.37	0.10	0.72	0.50
1.0						
Inflation	2.05	2.79	2.73	1.70	2.26	1.50
Output gap	2.12	1.90	2.26	2.12	2.40	2.29
ZLB	0.92	13.99	16.57	0.27	0.63	1.81
10.0						
Inflation	1.35	2.93	2.81	1.60	2.44	1.09
Output gap	2.12	2.01	1.81	2.04	3.10	2.87
ZLB	7.30	24.60	26.60	5.89	2.72	15.21

Source: Author's calculations.

a. Based on the policy rule defined in equation 38 and μ_1 = 0.8

With regard to the basic policy issue at hand, inflation targeting performs somewhat better than other growth rate targets (Δs_t and Δx_t) in certain respects and about the same with regard to the ZLB problem. In comparison with a price level target, inflation targeting appears to be less effective for stabilizing inflation and output, but less open to serious ZLB problems. Exchange rate level targeting is the most sensitive to the ZLB problem of any of the targets considered. Finally, nominal income level targeting seems to perform quite well, but not so well as to dominate inflation targeting.

6. Monetary Stabilization despite a Liquidity Trap

Even if an economy has its interest rate instrument immobilized because of a liquidity trap or zero lower bound, there is nevertheless scope for monetary stabilization policy provided that the economy is open—as all are—to foreign trade. The argument presented here follows that outlined in McCallum (2000), but the model is improved and more open, as described above.[31] Specifically, suppose that the model

31. This proposal does not represent the only way of combating ZLB problems. Other possibilities are promoted by Goodfriend (2000) and Meltzer (2001). For Svensson (2000), see below.

economy's interest rate is fixed rigidly at $R_t = 0$ (or some other constant value), but that the monetary authority adopts a policy rule with an exchange rate instrument—not a target—of the following specification:

$$s_t - s_{t-1} = \nu_0 - \nu_1 \left(\mathrm{E}_{t-1} \Delta p_t - \pi^* \right) - \nu_2 \, \mathrm{E}_{t-1} \left(y_t - \bar{y}_t \right) + \zeta_t , \tag{39}$$

where $\nu_1 > 0$ and $\nu_2 \geq 0$. Here the rate of depreciation of the foreign exchange rate is lowered if inflation or output exceeds its target value. The exchange rate is being used as an instrument or indicator variable in much the same way as is normally the case (in industrialized economies) with a short-term interest rate. Thus the central bank uses open market operations or standing facilities to keep the asset price at the desired value—the value specified by the policy rule—so as to promote the achievement of macroeconomic targets (inflation and output).

To represent such a policy process, equation 39 is included in the model in place of equation 32. Then, since R_t is no longer a variable, one of the model's equations must be deleted or else modified so as to introduce another endogenous variable. For the moment this step can be understood as involving the deletion of uncovered interest parity, as expressed in equation 31. This is only a shorthand method of describing the actual alteration involved, however, which is explained and defended below. My purpose now is to demonstrate that with policy rule 39 in place, stabilizing monetary policy can be conducted even though the nominal interest rate is held fixed at a constant value.

The main simulation results are given in table 5.[32] As ν_1, the coefficient attached to the inflation target, is increased, the variability of inflation drops sharply—that is, inflation is stabilized. Also, larger values of ν_2, the coefficient on the output gap target, lead to reduced variability of the output gap.

Another way of demonstrating the effectiveness of monetary policy stabilization with the policy rule 39 is via impulse response functions. In figure 4, the top panels present the responses of key endogenous variables to a policy rule shock (that is, an upward blip in Δs_t) when the rule parameter values are $\nu_1 = 1.0$ and $\nu_2 = 1.0$. This loosening of policy brings about an increase in both inflation and output, as would be expected. Then in the bottom panels the rule parameter ν_2 is set at the larger value of 10. Thus the rule is designed to exert stronger stabilizing tendencies for inflation. Indeed, the response of inflation and output to the shock are

32. The disturbance ζ_t is assumed to possess the same stochastic properties as ξ_t in equation 31.

Table 5. Performance Measures with Policy Rule 39 and Fixed *R*

Standard deviations of Δp_t, $\tilde{y}_t$, and Δs_t

v_1	$v_2=0$	$v_2=1$	$v_2=10$	$v_2=50$
0				
Δp	11.66	8.86	4.00	4.12
$\tilde{y}_t$	5.73	4.56	2.14	1.52
Δs_t	18.61	17.22	18.47	26.24
1				
Δp	6.46	5.54	3.27	3.55
$\tilde{y}_t$	3.91	3.49	2.02	1.49
Δs_t	17.74	17.46	18.49	25.32
10				
Δp	2.14	2.05	1.93	2.51
$\tilde{y}_t$	2.52	2.40	1.88	1.47
Δs_t	21.23	20.66	20.84	24.34
50				
Δp	1.23	1.23	1.30	1.64
$\tilde{y}_t$	2.63	2.57	2.21	1.68
Δs_t	33.35	32.78	30.11	27.43

Source: Author's calculations.

muted in comparison with the top panels. A similar comparison is provided in figure 5 for the case of a technology shock, which tends to increase output and decrease inflation. Again the bottom panels feature the higher value for ν_2, and again the inflation and output responses are muted by this stronger attempt at stabilization. The cost, of course, is that the nominal and real exchange rates both respond more strongly, since the former is the policy instrument variable. These responses induce larger fluctuations in net exports, as well. From the results shown in table 5 and figures 4 and 5, it seems clear that the policy rule 39 does exert stabilizing influence on the economy despite the liquidity trap immobilization of the nominal interest rate.

Let me now take up some issues regarding this way of modeling this phenomenon. In his comment on McCallum (2000), Christiano (2000) objects to the elimination of the uncovered interest parity (UIP) relation from the model.[33] That this objection would be made is surprising,

33. Christiano's lengthy comment largely consists of variously expressed assertions to the effect that a model should include the UIP relation.

Figure 4. Responses to Unit Shock to Policy Rule in Equation 39

$\nu 1 = 1, \nu 2 = 1$

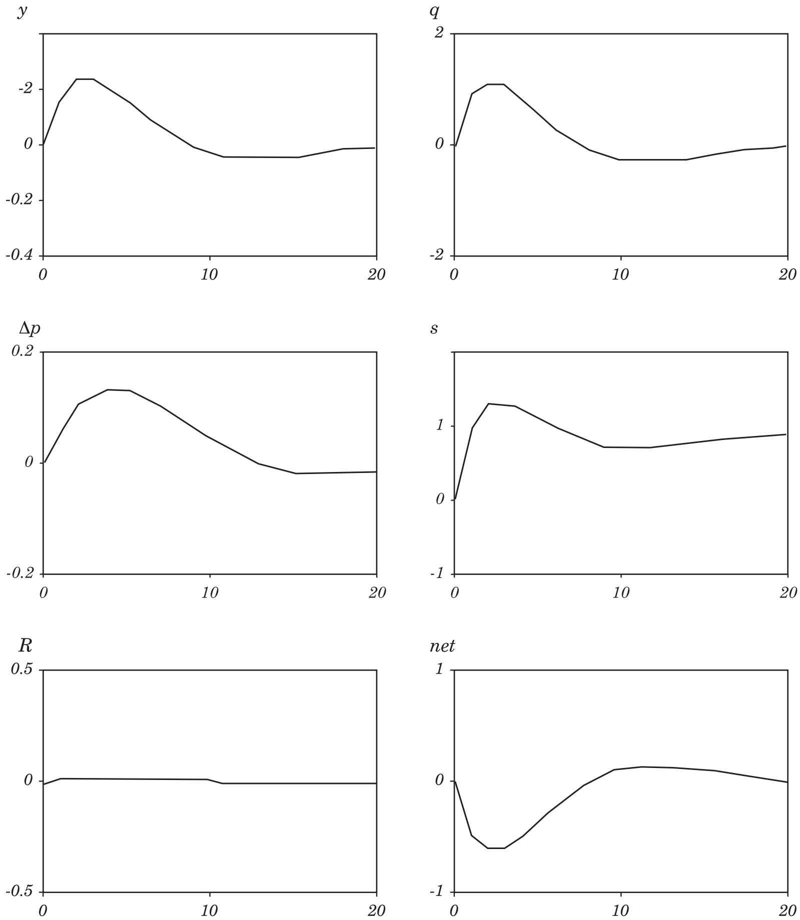


Figure 4. (continued)

$\nu 1 = 10, \nu 2 = 1$

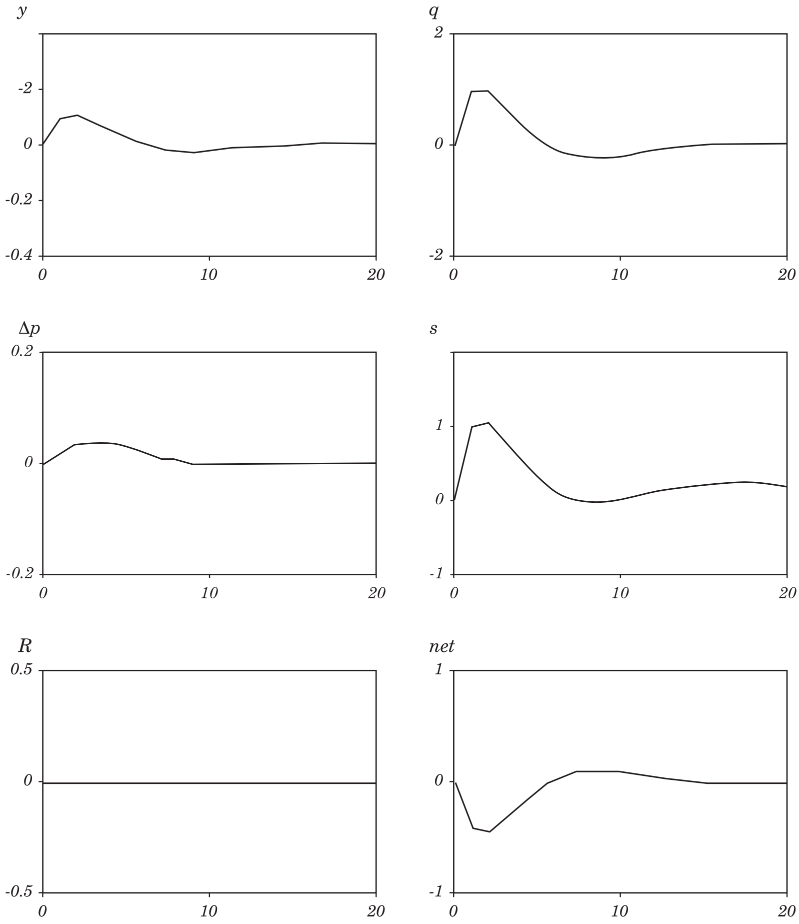


Figure 5. Responses to Unit Technology Shock to Policy Rule in Equation 39

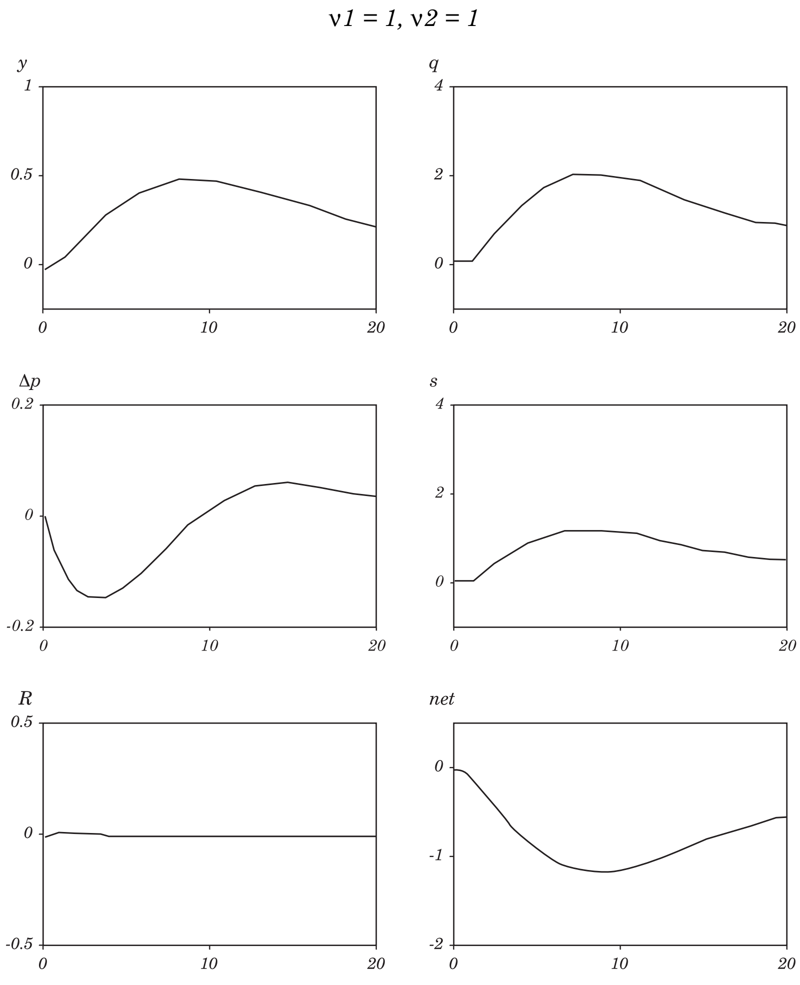
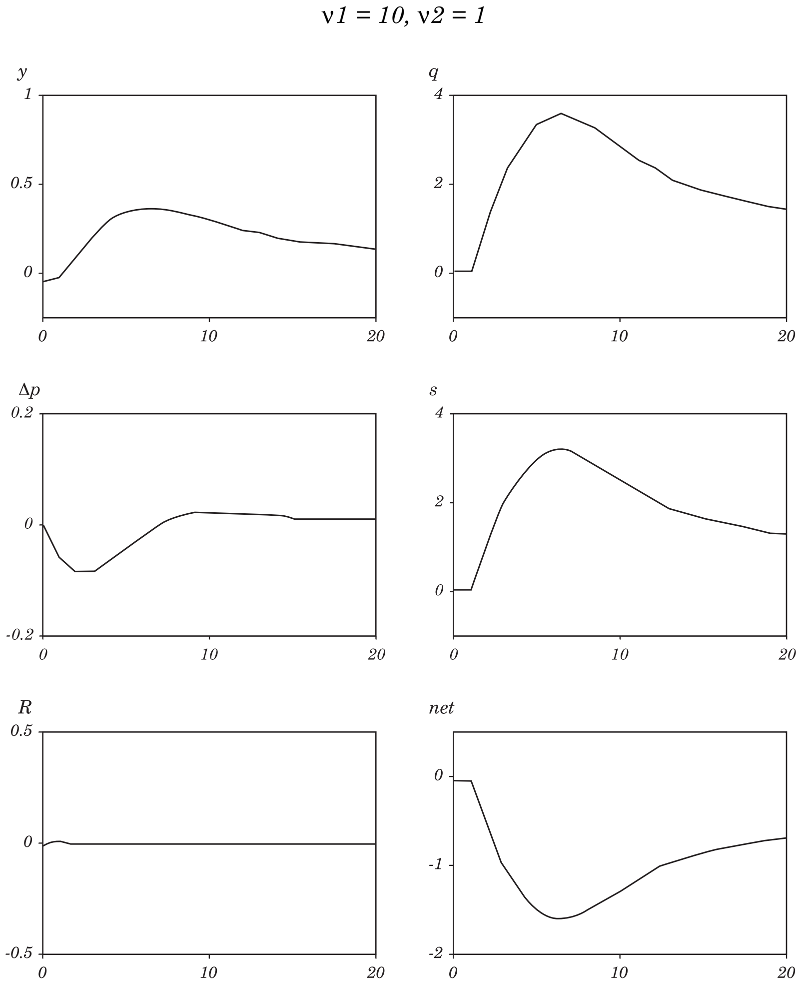


Figure 5. (continued)



given the enormous volume of empirical evidence that finds major departures from UIP. In the most standard empirical test, the slope coefficient that should equal 1.0 if UIP holds usually turns out to be negative—often significantly so.[34] It thus seems peculiar to insist on including the relation, since its drastic empirical failure is well documented. Despite this evidence, UIP is retained in many models, of course, but that is partly because it is unclear how to complete the model in its absence.[35] That is not a problem, however, if the exchange rate is used as the instrument variable; the relation can simply be omitted. This strategy is entirely analogous to omitting a base money demand function from models in which an interest rate is used as the instrument. The point is that in such cases it is not necessary to know how much base money must be supplied to set R_t at its desired value, since its current value is immediately observable in the asset markets. Thus a poor understanding of the demand function for base money does not preclude the use of an interest rate instrument in standard models because the only role of the base money demand function is to specify how much base money must be supplied to implement the interest rate rule. In the case of the exchange rate instrument, it again is not necessary to know the magnitude of the exchange market purchases (increases in base money) needed to implement the rule, because the value of the exchange rate can be immediately observed from the relevant asset market (the foreign exchange market).

In McCallum (2000), it is recognized that the foregoing argument implies that there is some effect on the home country's exchange rate of purchases of foreign exchange with domestic base money. In other words, it is assumed that domestic and foreign currency assets are not perfect substitutes. In that paper, the lack of perfect substitutability is described in terms of the portfolio balance model of exchange rate determination that has been out of favor since the late 1970s.[36] That particular description is not necessary; it was adopted primarily in the belief that it would make the general argument more transparent. The fundamental point is merely that assets denominated in domestic and foreign currencies are not perfect substitutes, so there is scope for departure from exact UIP to be affected by unsterilized purchases of for-

34. Well-known references include Lewis (1995) and Froot and Thayler (1990).

35. For many purposes, using the UIP assumption is entirely sensible. The application under discussion here, however, is an extreme, special case.

36. Specialists in exchange rate analysis have recently shown a renewed attraction to the basic aspects of this approach. See Flood and Marion (2000); Jeanne and Rose (1999).

eign exchange, possibly in very large quantities.[37]

Svensson (2000) puts forth a proposal that, although different in detail, is in essence closely related to the use of a policy rule such as that described by equation 39. Svensson's "foolproof" way of providing monetary stimulus, when a country cannot reduce R_t because of a zero lower bound, is, first, to announce an upward sloping p_t path with the initial value above the current price level; second, to announce that the currency will be devalued immediately and will depreciate henceforth at the rate of increase planned for the price level; third, to announce that the scheme will be converted into a normal price-level or inflation targeting arrangement once the target price path has been achieved; and fourth, to implement the second step by offering to buy and sell foreign exchange at the specified value. The first, second, and fourth parts of this scheme are clearly similar to the adoption of an inflation target and the use of exchange rate depreciation as implied by equation 39. Svensson understandably emphasizes the differences between his scheme and the one presented in McCallum (2000). He exaggerates the differences, however, in stating that his argument "does not depend on any portfolio-balance effect of foreign-exchange interventions, in contrast to the argument of Meltzer [2001] and McCallum [2000], and thus, it is more general.... As long as the central bank supplies an unlimited amount of domestic currency at the target exchange rate,... arbitrage in the foreign-exchange market will ensure that this exchange rate is the equilibrium exchange rate" (Svensson, 2000, p. 24). My point is that exactly the same can be said for equation 39; the central bank is by assumption willing to make whatever unsterilized exchange market purchases (or sales) are needed to make s_t take on the value that the rule specifies. That Svensson's path for s_t is not contingent on other variables does not alter this aspect of the situation. To put the matter differently, if domestic and foreign assets were perfect substitutes, which they are not, then the central bank would not be able to achieve the initial exchange rate specified by his scheme.[38]

37. Notable recent evidence that dollar and deutsche mark assets are not perfect substitutes, based on market-microstructure analysis, is provided by Evans and Lyons (2000).

38. Both Svensson's scheme and mine, incidentally, are feasible only under the proviso that the situation is one in which the central bank must raise the inflation rate and depreciate the currency to escape the liquidity trap. In this case the central bank will not run out of reserves, because it is supplying domestic currency that it can print in unlimited amounts. A major reason why it is widely believed that central banks have no control over exchange rates is that in practice, most have attempted to keep the value of the domestic currency higher than the equilibrium rate, not to lower it to a non-ZLB rate. This requires supplying large amounts of foreign exchange, which cannot be printed by the economy in question.

7. Conclusion

The paper has argued, first, that the danger of a liquidity trap induced solely by self-confirming expectations, due to the existence of two rational expectations equilibria when there is a zero floor on interest rates, is probably minimal. Such a situation implies that the trap equilibrium, which is of a bubble nature, prevails despite the existence of a well-behaved MSV or fundamental equilibrium that yields the target rate of inflation. Crucially, the MSV solution possesses the property of E-stability, which implies that it is achievable by an adaptive least squares learning process, while the trap equilibrium is not. The paper's suggestion is that this form of a liquidity trap represents a theoretical curiosity that is not of practical importance.[39]

Second, a similar analysis applies to the issue of indeterminacy induced by a policy rule that responds strongly to expected future inflation, rather than to currently observed or recent inflation. This situation again appears to be more of a theoretical curiosity than a genuine problem. In considering this issue, it is important to be clear about the nature of two very different concepts of indeterminacy that have been prominent at different times in the monetary policy literature.

Third, the paper quantitatively examines the likelihood of encountering a liquidity trap or zero lower bound, in which the central bank is powerless to combat a recession by reduction of short-term nominal interest rates. This exercise requires a carefully calibrated numerical model of an open economy; the one used here is adapted from McCallum and Nelson (1999). The paper's findings are that the chances of a ZLB constraint are strongly dependent on the sum of the inflation target and the long-run average real interest rate. If that sum is 5 percent per year, the chances of encountering the ZLB constraint can be kept well below 1 percent per quarter by an interest rate policy rule that targets inflation and incorporates a fairly high degree of interest rate smoothing. Adopting the inflation rate as the target variable, instead of other candidate macroeconomic measures, does not exacerbate the difficulty of avoiding the ZLB problem.

Finally, the paper describes a policy rule for escaping a zero lower bound if the economy does fall into a liquidity trap. The proposed rule is one that (temporarily) makes the foreign exchange rate the instrument

39. The recent experience of Japan is quite different. Because the target inflation rate is too low, the economy can fall into a trap of the fundamental type, as in the examples of section 5.

variable, rather than the immobilized interest rate. Macroeconomic stimulus is generated by the purchase (with base money) of foreign exchange so as to satisfy the rule, which includes inflation as a principal target variable. Simulation exercises and impulse response functions indicate that macroeconomic stabilization can in fact be exerted by monetary policy in this manner, despite ZLB immobilization of the usual interest rate instrument.

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