# Monetary Policy through Asset Markets: Lessons from Unconventional Measures and Implications for an Integrated World

Elías Albagli, Diego Saravia, and Michael Woodford
editors

Banco Central de Chile / Central Bank of Chile

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The global financial crisis of 2008 and its aftermath brought many new challenges for the world's central banks. These new challenges have resulted, in turn, in bold experimentation—not just the vigorous application of traditional policy tools, but the use of new ones, or at least ones that were rarely resorted to in the decades leading to the crisis. Now that the most urgent stages of the crisis are in the past, the central banks of many countries need to take stock of the lessons learned during this period of experimentation.

To what extent have we learned that, at least during times of crisis, the central bank's toolkit should be bigger than the one that was regarded as sufficient during the years of the "Great Moderation"?

To what extent have we learned the uses of additional tools that should become routine aspects of the conduct of monetary policy, even when the financial sector is not subject to unusual stress? What do we know about the effects of using these new tools, and what role should they play in the years to come?

The nineteenth annual conference of the Central Bank of Chile conference series addresses these issues, bringing together a distinguished multinational group of scholars to discuss the latest research findings. The structure of the conference consists of three sessions, each addressing a different aspect of the new issues raised by the unconventional monetary policies of recent years.

*Afueras de Chillán*
*Enrique Swinburn (1859–1929)*
*Oil on canvas, 160 x 90 cm*
*Collection of the Central Bank of Chile*

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# Monetary Policy through Asset Markets: Lessons from Unconventional Measures and Implications for an Integrated World

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The global financial crisis of 2008 and its aftermath have brought many new challenges for the world's central banks. These new challenges have, in turn, resulted in bold experimentation—not simply particularly vigorous use of traditional policy tools, but also the use of new tools or, if not entirely new, tools that had seldom been invoked in the decades immediately prior to the crisis. Now that the most urgent period of the crisis is past, central banks are taking stock of the lessons learned from this period of experimentation. Should the central bank's toolkit be larger than what was regarded as sufficient during the years of the Great Moderation, at least during times of crisis? Should the use of additional tools perhaps become routine aspects of the conduct of monetary policy, even when the financial sector is not subject to unusual stresses?

One of the more notable new developments in monetary policy since 2008 has been the greater use of central banks' balance sheets as a tool of policy. Central banks have always engaged in certain kinds of financial transactions to implement monetary policy, but prior to 2008, monetary policy was commonly viewed as involving solely a decision

*Monetary Policy through Asset Markets: Lessons from Unconventional Measures and Implications for an Integrated World*, edited by Elías Albagli, Diego Saravia, and Michael Woodford, Santiago, Chile. 

about a single short-term interest rate—generally an overnight rate at which banks lend to one another, such as the federal funds rate in the case of the U.S. Federal Reserve. In the transactions undertaken to implement changes in the operating target for this overnight rate, the particular assets acquired by the central bank were generally viewed as unimportant (the goal being to vary the supply of bank reserves). Consequently, prudence dictated that the central bank should only hold extremely riskless and very short-maturity securities (sometimes called a "bills only" doctrine in the United States). Moreover, the central bank's balance sheet could be quite small under ordinary circumstances: controlling the overnight interest rate by varying the supply of reserves required only a small volume of reserves to support inter-bank payments, to the extent that significant percentage changes in the reserve supply could be achieved with quite modest transactions in terms of the quantity of assets purchased or sold.

As a result, central banks were not major players in asset markets, even if their policy decisions had important consequences for the market pricing of many assets. Monetary policy decisions affected longer-term bond prices through arbitrage relationships between the prices of longer-term bonds and the expected path of short rates, not through direct purchases or sales of long-term bonds by the central bank with a view to influencing their prices. Similarly, monetary policy decisions affected exchange rates, but again—under the doctrine of a floating exchange rate, which had come to be the standard for inflation-targeting central banks—this was expected to result purely from arbitrage relationships between the exchange rate and the expected path of short-term interest rates at home relative to those abroad, rather than from direct intervention by the central bank to control the exchange rate.

The financial crisis changed this picture dramatically, at least in the short run—with longer-term consequences that are yet to be determined. By the end of 2008, many central banks found that even cutting short-term nominal interest rates to the lowest feasible level (or to the lowest level that they were willing to contemplate) resulted in insufficient monetary stimulus to head off a severely contractionary shock. They were therefore forced to ask what other policy tools were available when further cuts in overnight interest rates would not be possible. This led to a reconsideration of the question of whether the central bank could usefully influence longer-term asset yields and foreign exchange rates through direct asset purchases, even in the absence of any change in the level of the overnight rate (or in the path that it could be expected to follow, at least over the near term). In addition, especially in the period immediately following the onset of the crisis, many central banks faced situations in which the private financial sector could no longer be counted on to efficiently allocate credit in the economy, owing to distress or severe financial constraints in many key institutions. This raised the question of whether the central bank should not itself act as a financial intermediary, channeling credit to particular sectors that would otherwise face funding difficulties, while waiting for private financial institutions to repair their balance sheets and for the climate of panic to be dispelled.

For both of these reasons, the balance sheets of many central banks grew substantially in the years following 2008, and the categories of assets held changed to include many longer-term securities and securities involving risks to which the central banks were not previously exposed. For example, the U.S. Federal Reserve acquired the extensive holdings of mortgage-backed securities in this period. Policies with regard to asset purchases (including, in some cases, significant purchases of foreign exchange with a view to controlling exchange rates) have often been the focus of central bank policy deliberations and communication with the public, given that in many countries, short-term interest rate targets have changed relatively little since late 2008. While asset-purchase policies are not currently being used as actively by central banks like the U.S. Federal Reserve and the Bank of England as in the years immediately following the crisis, they continue to be a central focus of policy at the European Central Bank and the Bank of Japan, among others; and even banks like the Federal Reserve continue to operate with much larger balance sheets than they had prior to the crisis. Thus, the question of the appropriate size of the balance sheet remains an active topic of discussion.

But what do we know about the effects of using these new tools, and what role should they have in the future? The nineteenth annual conference of the Central Bank of Chile addresses these issues, bringing together a distinguished international group of scholars to discuss the latest research findings. The structure of the conference consisted of three sessions, which explored different aspects of the new issues raised by the unconventional monetary policies of recent years.

The first session considered the effects of central bank asset purchases, as well as announcements regarding the intended future path of purchases, on both financial asset prices and the macroeconomy. The second session focused on a specific aspect of the effects of such policies, namely, the extent to which they alter the incentives for risk-taking by financial institutions. This risk-taking channel is found to significantly amplify the effects of policy. The third session explored the scope and magnitude of spillover effects from policies implemented by central banks like the U.S. Federal Reserve on other economies—in particular, emerging economies—with a focus on linkages between the longer-term bond markets of different countries. The conference concluded with a keynote address by Lawrence Summers, former U.S. Treasury Secretary and one of the keenest observers of current economic affairs, on the challenges for stabilization policy going forward, in a global environment in which conventional interest rate policy may have less scope than it had in the past. We now summarize each of these sessions in sequence.

## Session 1: The Impact of Conventional and Unconventional Monetary Policies on Asset Prices

The three papers from the first session study, from both a theoretical and empirical perspective, the impact of the broader set of monetary policies discussed above, the so-called unconventional measures, on interest rates at different horizons and over a wide variety of securities.

In "Forward Guidance in the Yield Curve: Short Rates versus Bond Supply," Robin Greenwood, Samuel G. Hanson, and Dimitri Vayanos characterize and compare the effects of so-called forward guidance policies (that is, pre-announcements about the future path of the federal funds rate) on short-term rates and the supply of bonds, using a model of yield curves and bond rates in which the monetary authority can pre-announce movements in future short-term bond rates or quantitative easing. The results indicate that pre-announcements about short-term bond rates, which operate via expectational hypotheses, have a direct impact on the announced short rates. In particular, if an explicit increase in a rate of specified maturity is pre-announced, this will have an impact of equal magnitude on the referenced short rate. Meanwhile, the pre-announcements of quantitative easing, operating through the expected future risk premium, achieve the maximum rate hikes in the yields of longer-term bonds. Thus, pre-announcements about short rates have direct effects on those rate, and pre-announcements of quantitative easing have a stronger impact on rates at longer maturities.

In the second paper, "Bernanke's No-arbitrage Argument Revisited: Can Open Market Operations in Real Assets Eliminate the Liquidity Trap?," Gauti B. Eggertsson and Kevin B. Proulx show, in a closed-economy context with sticky prices and taxation costs, that open market operations of real asset purchasing by the government can mitigate a deflationary process. This intervention has effects even in a scenario of nominal short-term interest rates near the zero lower bound, since it allows the government to commit to having future inflation that will enable financing the purchase of assets (by either issuing nominal debt or creating money). This commitment prompts a change in private sector inflation expectations (from deflationary to inflationary) and stimulates aggregate demand. The purchase of real assets by the government potentiates other unconventional policies such as a deficit increase (augmenting nominal debt) or a reduced tax burden to boost aggregate demand.

Finally, in "Measuring the Effects of Unconventional Monetary Policy on Asset Prices," Eric T. Swanson adapts the methods used by Gürkaynak, Sack, and Swanson (2005) to estimate the effects of unconventional monetary policies in the United States during the zero lower bound period between 2009 and 2015. In particular, the paper seeks to separately identify the effects of forward guidance and large-scale asset purchases (LSAP) in each Federal Open Market Committee (FOMC) announcement on the U.S. Treasury bill rates, asset prices, parities, and corporate rates. The results show that a one-standard-deviation change in forward guidance or LSAP measures affects equally the medium-term Treasury rates, asset prices, and exchange rates. However, forward guidance policies prove to be relatively more effective on short Treasury rates, while the LSAP policies have greater effects on long Treasury rates and corporate bond rates. Finally, the author stresses that in choosing one policy over the other, it is also necessary to consider the costs that each of them implies.

## Session 2: The Risk-Taking Channel of Monetary Policy: Implications for Financial Fragility

The papers from the second session address a specific issue connected with the effects of monetary policy on asset markets: namely, the consequence of monetary policy decisions for financial stability. This is an important general question for the theory of monetary policy, and it is particularly relevant at present, given the increased concern with reducing the risk of a financial crisis in light of the difficulties created by the recent one. Moreover, some analysts argue that the kind of unconventional policies implemented in response to the crisis distort financial decision-making to an unusual extent, in ways that might pose particular risks to financial stability.

"Risk Premium Shifts and Monetary Policy: A Coordination Approach," by Stephen Morris and Hyun Song Shin, presents a theoretical analysis of a particular type of situation in which small changes in monetary policy can trigger an abrupt shift in portfolios and asset prices. A central bank that fears it may be in this situation may have good reason to tread carefully when even suggesting that it could change its policy. More generally, the paper shows how the effects of monetary policy can, to a large extent, result from its effects on market risk premiums, which change endogenously as a result of the effects of monetary policy expectations on the risk-taking behavior of market participants.

The paper presents a model of risk-neutral investors, who can be interpreted as asset managers, interacting with risk-averse households in the market for a risky long-term bond. Because of the differing degrees of risk aversion of the two types of investors, variation in the share of total issuance of the bond that asset managers are willing to hold results in endogenous variation in the risk premium. This decision by asset managers in turn involves a coordination problem, because asset managers care about their relative performance, making each one's optimal degree of exposure to this type of risk dependent on the degree of exposure that other asset managers are expected to choose. As a result of the coordination problem, it is possible for abrupt changes in the aggregate portfolio decision of asset managers, and hence in the market risk premium, to occur in equilibrium in response to even a very small change in fundamentals, if the fundamental state variables cross a critical threshold that the authors characterize using global game techniques.

Morris and Shin use their model to discuss a possible danger associated with the use of commitments to keep short-term interest rates at an unusually low level for a long time as a tool of monetary stimulus, as practiced by the U.S. Federal Reserve and other central banks in the years immediately following the crisis. In their analysis, such a policy can be a source of stimulus by lowering long-term interest rates. However, an important channel through which this occurs is by encouraging asset managers to increase their holdings of risky longer-term bonds, reducing equilibrium risk premiums. This increase in the share of risky assets held by asset managers who are concerned with their relative performance (and able to shift their positions rapidly) increases the ease with which a signal that interest rates will begin to rise can trigger an abrupt sell-off. Thus, a policy that has desirable effects in the short run can create a sort of trap, in which a central bank finds it difficult to unwind its unusually accommodative policies, even if they are no longer appropriate to current macroeconomic conditions.

In "Quantitative Easing and Financial Stability," Michael Woodford also addresses potential consequences of monetary policy decisions for risks to financial stability. Here, the risks considered stem from financial intermediaries financing purchases of illiquid risky assets by issuing short-term riskless collateralized debt instruments, which creates the possibility of a roll-over crisis in which illiquid assets must be sold in a fire sale. The paper considers the effects of two alternative dimensions of monetary policy—both quantitative easing (that is, central bank asset purchases that result in large increases in the supply of safe central bank liabilities) and conventional interest rate policy (implemented without any large change in the central bank's balance sheet)—on the incentives that banks and shadow banks have to engage in liquidity and maturity transformation of this kind and hence on the degree of risk to financial stability.

The paper embeds a simple model of endogenous intermediary capital structure in an intertemporal general equilibrium monetary model in which short-term safe instruments earn a money premium owing to their special role in facilitating transactions (for example, by being assets that are suitable for money market mutual funds to hold, which create liabilities that can in turn be used as means of payment). The "outside" supply of short-term safe instruments (both short-term bills supplied by the Treasury and safe liabilities of the central bank) then becomes an important determinant of the size of the equilibrium money premium and hence of the incentive for private intermediaries to supply short-term safe instruments such as asset-backed commercial paper or short-term repos (both of which played significant roles in the funding crises of 2007–08).

Woodford shows why conventional interest-rate policy and quantitative easing are logically independent dimensions of policy and how they jointly determine financial conditions, aggregate demand, and the severity of risks to financial stability. While both interest rate cuts and quantitative easing are shown to have similar effects in the sense that either policy will simultaneously stimulate aggregate demand and increase financial risk, the model implies that quantitative easing policies actually increase financial stability risk less than an interest rate cut, relative to the magnitude of aggregate demand stimulus achieved; and a combination of expansion of the central bank's balance sheet with a suitable tightening of macroprudential policy can have a net expansionary effect on aggregate demand with no increased risk to financial stability. This suggests that quantitative easing policies may be useful as an approach to aggregate demand management not only when the zero lower bound precludes further use of conventional interest rate policy, but also when it is not desirable to further reduce interest rates because of financial stability concerns.

Finally, “Short-term Interest Rates and Bank Lending Terms: Evidence from a Survey of U.S. Loans,” by Giovanni Dell’Ariccia, Luc Laeven, and Gustavo Suarez, provides an empirical assessment of the risk-taking channel for the effects of monetary policy—that is, the thesis that loose monetary policy generates expansionary effects largely by inducing banks to relax lending standards, which allows an expansion of credit (and hence more current spending to be financed), but at the cost of increased risks to financial stability. The importance of this channel is an important issue for assessing the degree to which a prolonged period of low nominal interest rates in the United States in the mid-2000s should be considered one of the important causes of the subsequent crisis and for determining the potential dangers of further prolongation of the current period of unusually low nominal rates as well.

The paper uses confidential data from the U.S. Federal Reserve’s Survey of Terms of Business Lending to measure how bank lending terms in the United States are affected by monetary policy. The authors find that, controlling for the ex-ante riskiness of a given loan (as indicated by the internal risk rating of the loan, which banks report to the survey), the lending terms offered by banks are easier when interest rates are lower. Loan spreads are found to be lower, and loans are less likely to be secured, when the federal funds rate target is lower; the authors argue that this provides support for the risk-taking channel. The paper provides novel evidence on this important issue, which nicely complements previous studies that had instead emphasized changes in the composition of lending (that is, increases in the fraction of lending to higher-risk borrowers), rather than on the terms of lending to a given borrower, in response to low interest rates. Taken as a whole, the papers of this session amply demonstrate, on both theoretical and empirical grounds, that effects on risk-taking decisions are among the effects that should be expected from monetary policy changes, and this should be taken into account when making decisions about such actions.

**Session 3: Monetary Policy Interdependence through Long-term Rates**

The third and last session of the conference focused on the effects of the monetary policy followed in the developed world on emerging market asset prices. Securities from emerging markets fit naturally into the category of riskier assets that are expected to be affected by investors' search for yield in an environment of low interest rates, as has been documented by a growing empirical literature. This topic is of particular concern for central bankers in emerging regions, which have reasons to be worried about the consequences of U.S. monetary normalization in an environment where further exchange rate pass-through will put increasing pressure on our inflation targets, while interest rate pass-through puts increased pressure on subpar levels of growth.

In "The Response of Sovereign Bond Yields to U.S. Monetary Policy," Simon Gilchrist, Vivian Z. Yue, and Egon Zakrajšek compare the impact on international sovereign bond rates of the U.S. conventional monetary policy (from 1992 to late 2008) with respect to unconventional measures used after the target policy rate reached the zero lower bound (ZLB), between late 2008 and early 2014. Using the changes in the two- and ten-year U.S. Treasury bills as a policy surprise, the authors find that U.S. monetary policy has a pronounced effect on the short- and long-term interest rates of developed economies. However, the short-term sovereign bond rate does not respond to U.S. monetary policy in emerging economies (with the exception of Mexico); only longer rates are more responsive. The results also show that the expansionary U.S. monetary policy steepens the yield curve during conventional periods and flattens it during unconventional periods (ZLB).

Finally, Elías Albagli, Danilo Leiva-Leon, and Diego Saravia, in "U.S. Monetary Spillovers to Latin America: The Role of Long-term Interest Rates," assess the impact of unexpected hikes in U.S. Treasury bill rates on some important economies in Latin America: namely, Brazil, Chile, Colombia, Mexico, and Peru. Their results indicate that an increase in the longer-maturity Treasury bill rates (ten years) causes an increase in unemployment, inflation, and nominal exchange rates in the economies analyzed, while reducing the returns of domestic capital markets. The one exception is Mexico, whose behavior differs from the rest of Latin America. There, an increase in Treasury bill rates reduces unemployment and the exchange rate; this is mainly explained by Mexico's greater interaction with the United States relative to the rest of Latin America. The authors also find that a rise in the short-term rate (one year) has limited and less statistically significant effects. Finally, increases in U.S. long-term rates triggered an increase in local bond rates during the zero lower bound period, which was transmitted mainly through the rates' risk premium component.

# Forward Guidance in the Yield Curve: Short Rates versus Bond Supply

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Since late 2008, when short-term interest rates reached their zero lower bound, central banks have been conducting monetary policy through two primary instruments: quantitative easing (QE), in which they buy long-term government bonds and other long-term securities, and so-called forward guidance, in which they guide market expectations about the path of future short rates. Because QE alters the maturity structure of the government debt that is available to the public, it changes the amount of duration risk that market participants must bear, thereby affecting bond risk premiums and long-term interest rates. Forward guidance may also affect long rates because it contains information about the central bank's willingness to keep short rates low in the future.

Although the term forward guidance is normally used in reference to central bank policy on future short rates, QE operations typically involve some forward guidance, as well. This is because announcements that the central bank will purchase long-term securities are made well in advance of the actual purchases, which are spread out over a period of months or years. For example, on 18 March 2009, the Federal Open Market Committee (FOMC) announced that to "help

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improve conditions in private credit markets," the U.S. Federal Reserve (the Fed) would increase the scale of its previously announced asset purchase program from US$600 billion to US$1.75 trillion and that these purchases would be carried out over the next six to twelve months. At the same time, the FOMC provided forward guidance on short rates, stating that it "anticipates that economic conditions are likely to warrant exceptionally low levels of the federal funds rate for an extended period." The impact of announcements such as these on the yield curve has been substantial. Following the March 2009 announcement, for example, ten-year zero-coupon Treasury yields fell by 51 basis points over the course of two days.

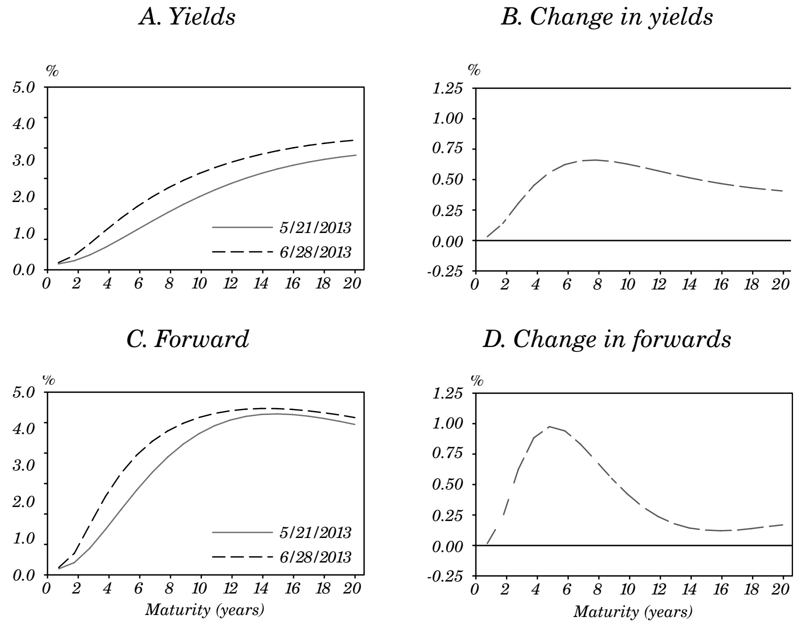
How should forward guidance on short rates and forward guidance on QE be reflected in the yield curve? Policymakers have taken the implicit view that forward guidance on short rates is easy to interpret. If the expectations hypothesis of the yield curve holds, then the expected future path of short rates coincides with the curve of instantaneous forward rates. Forward guidance on QE is inherently more difficult to assess, however, because it depends on how future bond risk premiums change in response to QE and how these changes are incorporated into current bond prices. For example, suppose that market participants believe the central bank plans to acquire large amounts of long-term government bonds, but then plans to sell these bonds in five years. How should these beliefs affect long rates today? What if the market revises its expectations about how long the central bank will maintain its elevated holdings of long-term bonds?

To make these questions concrete, consider the so-called taper tantrum of May–June 2013, a period in which market participants feared that the Fed might reduce the pace of future bond purchases. On 22 May 2013, Federal Reserve Chairman Ben Bernanke testified in front of Congress that the Fed would slow or "taper" its QE program if the economy showed signs of improving. Within a week, yields of ten-year government bonds had increased by 21 basis points. On 19 June 2013, bond yields increased further following a Federal Reserve press conference, as markets feared an end to the Fed's balance-sheet expansion.

Figure 1 shows the evolution of the zero-coupon Treasury yield curve between 21 May and 28 June 2013 (nine days after the Fed's press conference). The peak increase in yields occurred at a maturity of seven years, where the yield to maturity increased by a total of 60 basis points. The peak increase in forward rates occurred at five years to maturity: the one-year yield four years ahead increased by over 100 basis points between the two dates. The change in forward rates was large even as far as ten years into the future.

How should we interpret the yield curve changes in figure 1? Were they mainly driven by market participants' revised expectations about the path of future short rates? If so, then under the expectations hypothesis of the yield curve, expectations were revised the most about short rates five years into the future, and revisions were significant even over a ten-year horizon. Were the changes in the yield curve instead driven by expectations about future purchases of long-term bonds by the Fed? If so, then over what horizon did expectations have to change to generate the observed yield curve changes?

**Figure 1. Changes in U.S. Yields and Forwards during the 2013 Taper Tantrum[a]**




a. Panels A and C plot zero-coupon Treasury yields and one-year forward rates before and after the taper tantrum (21 May to 28 June 2013). Panels B and D plot cumulative changes during the taper tantrum. Yields and forward rates are computed using the continuously compounded yield curve fitted by Gurkaynak, Sack, and Wright (2007).

In this paper, we build a no-arbitrage model of the yield curve that allows us to characterize and compare the effects of forward guidance on short rates and forward guidance on QE. Among other results, we show that forward guidance on QE tends to affect longer maturities than forward guidance on short rates, even when expectations about bond purchases by the central bank concern a shorter horizon than expectations about future short rates. Using our model, we interpret reactions of the U.S. yield curve to policy announcements during the QE period.

Our model builds on Vayanos and Vila (2009) and Greenwood and Vayanos (2014). There is a continuum of default-free, zero-coupon bonds that are available in positive supply. For simplicity, we consolidate the central bank and the fiscal authority, so that the only relevant quantity is the supply of bonds that must be held by the public. The marginal holders of the bonds are risk-averse arbitrageurs with short investment horizons. These arbitrageurs demand a risk premium for holding bonds, because of the possibility that unexpected shocks will cause the bonds to underperform relative to the short rate. In accordance with a long line of research on the portfolio-balance channel (Tobin, 1958, 1969), declines in bond supply lower the amount of duration risk that is borne by arbitrageurs, reducing bond risk premiums and raising bond prices.

Relative to previous work, our key theoretical innovation is that we allow for news about both the future path of short rates and the future supply of bonds. Specifically, the short rate in our model evolves stochastically. However, holding fixed the current level of the short rate, we also allow for shocks to the expected path of future short rates. Similarly, the supply of bonds evolves stochastically, but holding current supply fixed, we also allow for shocks to the expected path of future supply. Shocks to the expected path of future short rates and future supply can be interpreted as policy announcements that provide forward guidance on these variables.

After deriving the equilibrium yield curve, we describe the impact of forward guidance. Forward guidance on short rates in our model works through the expectations hypothesis. Suppose, for example, that arbitrageurs' expectation of the short rate three years from now declines by 100 basis points. This is reflected directly in a 100 basis points decline in the instantaneous forward rate three years from now. The expectations hypothesis describes the effects of shocks to expected future short rates because these shocks do not affect the positions that arbitrageurs hold in equilibrium and hence do not affect bond risk premiums.

Forward guidance on supply works through expected future bond risk premiums. Suppose, for example, that the central bank announces that it will buy ten-year bonds one year from now. After the purchase occurs, arbitrageurs will be holding a smaller position in ten-year bonds and be bearing less duration risk. Hence, the premium associated with that risk will decrease and bond prices will increase. The anticipation of this happening in one year causes an immediate rise in the prices of all bonds with maturity longer than one year. The price increase is not confined to the bonds that the central bank announces it will purchase; in fact, other bonds may be more heavily affected. This is because—as in Vayanos and Vila (2009) and Greenwood and Vayanos (2014)—supply effects operate not locally, but globally through changes in the prices of risk.

Announcements about expected future short rates have a hump-shaped effect on the yield and forward-rate curves, because neither current short rates nor expected short rates far in the future are affected. The location of the hump on the forward-rate curve coincides with that in expected future short rates because of the expectation hypothesis.

Announcements about future supply can also have a hump-shaped effect on the yield and forward-rate curves. The impact of a supply shock on a bond's yield is the average of the shock's effect on the bond's instantaneous expected return over the bond's lifetime. When comparing the effect across bonds of different maturities, there are two opposing forces. On one hand, the supply shock has a larger impact on the current expected return that arbitrageurs require to hold the longer-term bond. On the other, if the shock is expected to revert quickly, required returns are expected to remain elevated over a larger portion of the shorter-term bond's life. The combination of these effects means that a supply shock that is expected to revert quickly has a hump-shaped effect on the yield curve. Moreover, the more quickly the shock is expected to revert, the shorter is the maturity where the hump is located. If the shock is expected to revert slowly, its effect is increasing with maturity (that is, the hump is located at infinity).

A key difference between shocks to future supply and shocks to future short rates is that the former can affect yields and forward rates at maturities much longer than the time by which the shocks are expected to die out. Likewise, the humps on the yield and forward-rate curves associated with supply shocks typically occur at maturities longer than those associated with short-rate shocks, even when the former are expected to revert more quickly. Consider, for example, the impact of a supply shock on the one-year forward rate in nine years. We show that it can be written as the sum of the shock's impact on the difference between expected returns on ten- and nine-year bonds over the next year, plus the impact on the difference between expected returns on nine- and eight-year bonds over the year after, and so on. Even a temporary shock can have a significantly larger effect on the current expected return on ten-year bonds relative to nine-year bonds, thereby affecting the one-year forward rate in nine years.

After developing the theoretical results, we reexamine the empirical evidence on QE announcements in the United States. Existing studies of QE compute changes in bond yields around major policy announcements in the United States and elsewhere. We add to these studies by computing changes in forward rates along the entire curve and considering a large set of announcement dates. We show that the cumulative effect of all expansionary announcements up to 2013 was hump shaped with a maximum effect at the ten-year maturity for the yield curve and the seven-year maturity for the forward-rate curve. Explaining this evidence through changing expectations about short rates would mean that expectations were revised the most drastically for short rates seven years into the future, while revisions one to four years out were much more modest. This seems unlikely. On the other hand, the evidence is more consistent with changing expectations about supply: according to our model, the maximum revision in supply expectations would have to be only one year into the future.

Our findings accord nicely with those of Swanson (2015), who decomposes the effect of FOMC announcements from 2009 to 2015 into a component that reflects news about the future path of short rates (forward guidance) and a component that reflects news about future asset purchases (QE). Consistent with our model, Swanson (2015) finds that both QE-related and forward-guidance-related announcements have hump-shaped effects on the yield curve. Moreover, the hump for the former announcements occurs at a longer maturity than for the latter: QE announcements have their largest impact at around the ten-year maturity, while forward-guidance announcements have their largest impact at two to five years.

Our paper builds on a recent literature that seeks to characterize how shocks to supply and demand affect the yield curve (Vayanos and Vila, 2009; Greenwood and Vayanos, 2014; Hanson, 2014; Malkhozov, and others, 2016). It is also related to a number of event studies that analyze the behavior of the yield curve and prices of other securities around QE-related events. Modigliani and Sutch (1966), Ross (1966), Wallace (1967), and Swanson (2011) study the impact of the 1962–1964 Operation Twist program. More recent event studies of QE in the wake of the Great Recession include Gagnon and others (2011), Krishnamurthy and Vissing-Jorgensen (2011), D'Amico and others (2012), D'Amico and King (2013), Mamaysky (2014), and Swanson (2015) for the United States, and Joyce and others (2011) for the United Kingdom.[1]

The paper proceeds as follows. Section 1 presents the model. Section 2 derives the equilibrium yield curve. Section 3 describes the impact of announcements on the yield and forward-rate curves. Section 4 reexamines the empirical evidence on QE in light of our model. Section 5 concludes.

## 1. Model

The model is set in continuous time. The yield curve at time $t$ consists of a continuum of default-free zero-coupon bonds with maturities in the interval $(0,T]$ and face value one. We denote by $P_t^{(\tau)}$ the price of the bond with maturity $\tau$ at time $t$, and by $y_t^{(\tau)}$ the bond's yield. The yield $y_t^{(\tau)}$ is the spot rate for maturity $\tau$. We denote by $f_t^{(\tau-\Delta\tau,\tau)}$ the forward rate between maturities $\tau - \Delta\tau$ and $\tau$ at time $t$. The spot rate and the forward rate are related to bond prices through

$$y_t^{(\tau)} = -\frac{\log P_t^{(\tau)}}{\tau} \tag{1}$$

$$f_t^{(\tau-\Delta\tau,\tau)} = -\frac{\log\left(\frac{P_t^{(\tau)}}{P_t^{(\tau-\Delta\tau)}}\right)}{\Delta\tau} \tag{2}$$

respectively. The short rate is the limit of $y_t^{(\tau)}$ when $\tau$ goes to zero, and we denote it by $r_t$. The instantaneous forward rate for maturity $\tau$ is the limit of $f_t^{(\tau-\Delta\tau,\tau)}$ when $\Delta\tau$ goes to zero, and we denote it by $f_t^{(\tau)}$. We sometimes refer to $f_t^{(\tau)}$ simply as the forward rate for maturity $\tau$.

1. See also Bernanke, Reinhart, and Sack (2004) for a broader analysis of QE programs, and Joyce and others (2012) for a survey to the theoretical and empirical literature on QE.

We treat the short rate $r_t$ as exogenous, and assume that it follows the process

$$dr_t = \kappa_r(\bar{r}_t - r_t)dt + \sigma_r dB_{r,t}, \tag{3}$$

where

$$d\bar{r}_t = \kappa_{\bar{r}}(\bar{r} - \bar{r}_t)dt + \sigma_{\bar{r}} dB_{\bar{r},t}, \tag{4}$$

$(\kappa_r, \sigma_r, \bar{r}, \kappa_{\bar{r}}, \sigma_{\bar{r}})$ are positive constants, and $(B_{r,t}, B_{\bar{r},t})$ are Brownian motions that are independent of each other. The short rate $r_t$ reverts to a target $\bar{r}_t$, which is itself mean reverting. The assumption that the diffusion coefficients $(\sigma_r, \sigma_{\bar{r}})$ are positive is without loss of generality since we can switch the signs of $(B_{r,t}, B_{\bar{r},t})$. We refer to $\bar{r}_t$ as the target short rate. To emphasize the distinction with $r_t$, we sometimes refer to the latter as the current short rate. Shocks to $\bar{r}_t$ can be interpreted as policy announcements by the central bank that provide forward guidance on the future path of the short rate. The process of equations (3) and (4) for the short rate has been used in the term-structure literature (for example, Chen, 1996; Balduzzi, Das, and Foresi, 1998) and is known as a stochastic-mean process.[2]

Bonds are issued by the government and are traded by arbitrageurs and other investors. We consolidate the central bank and the fiscal authority, so that only the net supply coming out of the two institutions matters. This means, for example, that a QE policy in which the central bank expands the size of its balance sheet, issuing interest-bearing reserves (that is, overnight government debt) to purchase long-term government bonds, is equivalent to a direct reduction in the average maturity of government debt issued by the fiscal authority. For simplicity, we treat the net supply coming out of the government as exogenous and price inelastic. We do the same for the demand of investors other than arbitrageurs, and model explicitly only the arbitrageurs. Hence, the relevant supply in our model is that held by arbitrageurs, and it reflects the combined effects of central bank purchases, issuance by the fiscal authority, and demand by other investors in the economy.

2. Although we refer to $\bar{r}_t$ as the target short rate, this should be interpreted as the central bank's intermediate-term policy target (for example, at a one- to two-year horizon) and not as the current operating target for the short rate (for example, the current target for the federal funds rate set by the FOMC).

We assume that arbitrageurs choose a bond portfolio to trade off the instantaneous mean and variance of changes in wealth. Denoting their time-$t$ wealth by $W_t$ and their dollar investment in the bond with maturity $\tau$ by $x_t^{(\tau)}$, their budget constraint is

$$dW_t = \int_0^T x_t^{(\tau)} \frac{dP_t^{(\tau)}}{P_t^{(\tau)}} d\tau + \left(W_t - \int_0^T x_t^{(\tau)} d\tau\right) r_t dt. \tag{5}$$

The first term in equation (5) is the arbitrageurs' return from investing in bonds; the second term is their return from investing their remaining wealth in the short rate. The arbitrageurs' optimization problem is

$$\max_{\{x_t^{(\tau)}\}_{\tau \in (0,T]}} \left[ E_t(dW_t) - \frac{a}{2} Var_t(dW_t) \right], \tag{6}$$

where $a$ is a risk-aversion coefficient.

We model the supply of bonds in a symmetric fashion to the short rate, so as to be able to capture forward guidance on bond supply. Specifically, we assume that the net supply coming out of the central bank, the fiscal authority, and the other investors is described by a one-factor model: the dollar value of the bond with maturity $\tau$ supplied to arbitrageurs at time $t$ is

$$s_t^{(\tau)} = \zeta(\tau) + \theta(\tau)\beta_t \tag{7}$$

where $\zeta(\tau)$ and $\theta(\tau)$ are deterministic functions of $\tau$, and $\beta_t$ is a stochastic supply factor. Intuitively, it may be useful to think of $\beta_t$ as proportional to the amount of *ten-year bond equivalents*, meaning duration-adjusted dollars of long-term debt. See Greenwood and others (2015) for a calculation along these lines for U.S. government debt.

The factor $\beta_t$ follows the process

$$d\beta_t = \kappa_\beta(\overline{\beta}_t - \beta_t)dt + \sigma_\beta dB_{\beta,t} \tag{8}$$

where

$$d\overline{\beta}_t = -\kappa_{\overline{\beta}}\overline{\beta}_t dt + \sigma_{\overline{\beta}} dB_{\overline{\beta},t} \tag{9}$$

$(\kappa_\beta, \sigma_\beta, \kappa_{\bar{\beta}}, \sigma_{\bar{\beta}})$ are positive constants, and $(B_{\beta,t}, B_{\bar{\beta},t})$ are Brownian motions that are independent of each other and of $(B_{r,t}, B_{\bar{r},t})$. Equations (8) and (9) are a stochastic-mean process, analogous to that followed by the short rate $r_t$. The assumption that the diffusion coefficients $(\sigma_\beta, \sigma_{\bar{\beta}})$ are positive is without loss of generality since we can switch the signs of $(B_{\beta,t}, B_{\bar{\beta},t})$. We refer to $\bar{\beta}_t$ as the target supply. To emphasize the distinction with $\beta_t$, we sometimes refer to the latter as the current supply. Shocks to $\bar{\beta}_t$ can be interpreted as policy announcements by the central bank that provide forward guidance on future purchases or sales of bonds, which in our model affect bond yields.

Since the supply factor $\beta_t$ has mean zero, the function $\zeta(\tau)$ measures the average supply for maturity $\tau$. The function $\theta(\tau)$ measures the sensitivity of that supply to $\beta_t$. We assume that $\theta(\tau)$ has the following properties.

**Assumption 1.** *The function* $\theta(\tau)$ *satisfies*

*(i)* $\int_0^T \theta(\tau) d\tau \geq 0$ *;*

*(ii) There exists* $\tau^* \in [0, T)$ *such that* $\theta(\tau) < 0$ *for* $\tau < \tau^*$ *and* $\theta(\tau) > 0$ *for* $\tau > \tau^*$

Part (*i*) of assumption 1 requires that an increase in $\beta_t$ does not decrease the total dollar value of bonds supplied to arbitrageurs. This is without loss of generality since we can switch the sign of $\beta_t$. Part (*ii*) of assumption 1 allows for the possibility that the supply for some maturities decreases when $\beta_t$ increases, even though the total supply does not decrease. The maturities for which supply can decrease are restricted to be at the short end of the yield curve. As we show in section 2, parts (*i*) and (*ii*) together ensure that an increase in $\beta_t$ makes the overall portfolio that arbitrageurs hold in equilibrium more sensitive to movements in the short rate.

## 2. Equilibrium Yield Curve

Our model has four risk factors: the current short rate $r_t$, the target short rate $\bar{r}_t$, the current supply $\beta_t$, and the target supply $\bar{\beta}_t$. We next examine how shocks to these factors influence the bond prices $P_t^{(\tau)}$ that are endogenously determined in equilibrium. We solve for equilibrium in two steps: first solve the arbitrageurs' optimization problem for equilibrium bond prices of a conjectured form, and second use market clearing to verify the conjectured form of prices. We conjecture that equilibrium spot rates are affine functions of the risk factors. Bond prices thus take the form

$$P_t^{(\tau)} = e^{-\left[A_r(\tau)r_t + A_{\bar{r}}(\tau)\bar{r}_t + A_\beta(\tau)\beta_t + A_{\bar{\beta}}(\tau)\bar{\beta}_t + C(\tau)\right]} \tag{10}$$

for five functions $A_r(\tau)$, $A_{\bar{r}}(\tau)$, $A_\beta(\tau)$, $A_{\bar{\beta}}(\tau)$, and $C(\tau)$ that depend on maturity $\tau$. The functions $A_r(\tau)$, $A_{\bar{r}}(\tau)$, $A_\beta(\tau)$, and $A_{\bar{\beta}}(\tau)$ characterize the sensitivity of bond prices to the current short rate $r_t$, the target short rate $\bar{r}_t$, the current supply $\beta_t$, and the target supply $\bar{\beta}_t$, respectively. Sensitivity to factor $i = r, \bar{r}, \beta, \bar{\beta}$ is defined as the percentage price drop per unit of factor increase.

Substituting equation (10) into equations (1) and (2), we can write spot rates and instantaneous forward rates as

$$y_t^{(\tau)} = \frac{A_r(\tau)r_t + A_{\bar{r}}(\tau)\bar{r}_t + A_\beta(\tau)\beta_t + A_{\bar{\beta}}(\tau)\bar{\beta}_t + C(\tau)}{\tau} \tag{11}$$

$$f_t^{(\tau)} = A'_r(\tau)r_t + A'_{\bar{r}}(\tau)\bar{r}_t + A'_\beta(\tau)\beta_t + A'_{\bar{\beta}}(\tau)\bar{\beta}_t + C'(\tau) \tag{12}$$

respectively. Thus, the sensitivity of spot rates to factor $i = r, \bar{r}, \beta, \bar{\beta}$ is characterized by the function $\frac{A_i(\tau)}{\tau}$, and that of instantaneous forward rates by the function $A'_i(\tau)$.

Applying Ito's Lemma to equation (10) and using the dynamics of $r_t$ in equation (3), $\bar{r}_t$ in equation (4), $\beta_t$ in equation (8), and $\bar{\beta}_t$ in equation (9), we find that the instantaneous return of the bond with maturity $\tau$ is

$$\begin{aligned}\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} = {} & \mu_t^{(\tau)}dt - A_r(\tau)\sigma_r dB_{r,t} - A_{\bar{r}}(\tau)\sigma_{\bar{r}} dB_{\bar{r},t} - A_\beta(\tau)\sigma_\beta dB_{\beta,t} \\ & - A_{\bar{\beta}}(\tau)\sigma_{\bar{\beta}} dB_{\bar{\beta},t}\end{aligned} \tag{13}$$

where

$$\begin{aligned}\mu_t^{(\tau)} \equiv {} & A'_r(\tau)r_t + A'_{\bar{r}}(\tau)\bar{r}_t + A'_\beta(\tau)\beta_t + A'_{\bar{\beta}}(\tau)\bar{\beta}_t + C'(\tau) \\ & + A_r(\tau)\kappa_r(r_t - \bar{r}_t) + A_{\bar{r}}(\tau)\kappa_{\bar{r}}(\bar{r}_t - \bar{r}) + A_\beta(\tau)\kappa_\beta(\beta_t - \bar{\beta}_t) + A_{\bar{\beta}}(\tau)\kappa_{\bar{\beta}}\bar{\beta}_t \\ & + \frac{1}{2}A_r(\tau)^2\sigma_r^2 + \frac{1}{2}A_{\bar{r}}(\tau)^2\sigma_{\bar{r}}^2 + \frac{1}{2}A_\beta(\tau)^2\sigma_\beta^2 + \frac{1}{2}A_{\bar{\beta}}(\tau)^2\sigma_{\bar{\beta}}^2\end{aligned} \tag{14}$$

denotes the instantaneous expected return. Substituting bond returns (equation 13) into the arbitrageurs' budget constraint (equation 5), we can solve the arbitrageurs' optimization problem (equation 6).

**Lemma 1** *The arbitrageurs' first-order condition is*

$$\mu_t^{(\tau)} - r_t = A_r(\tau)\lambda_{r,t} + A_{\bar{r}}(\tau)\lambda_{\bar{r},t} + A_\beta(\tau)\lambda_{\beta,t} + A_{\bar{\beta}}(\tau)\lambda_{\bar{\beta},t} \tag{15}$$

*where for* $i = r, \bar{r}, \beta, \bar{\beta}$,

$$\lambda_{i,t} \equiv a\sigma_i^2 \int_0^T x_t^{(\tau)} A_i(\tau) d\tau \tag{16}$$

According to equation (15), a bond's instantaneous expected return in excess of the short rate, $\mu_t^{(\tau)} - r_t$, is a linear function of the bond's sensitivities $A_i(\tau)$ to the factors $i = r, \bar{r}, \beta, \bar{\beta}$. The coefficients $\lambda_{i,t}$ of the linear function are the prices of risk associated with the factors: they measure the expected excess return per unit of sensitivity to each factor. Although we derive equation (15) from the optimization problem of arbitrageurs with mean-variance preferences, this equation is a more general consequence of the absence of arbitrage: the expected excess return per unit of factor sensitivity must be the same for all bonds (that is, independent of $\tau$); otherwise it would be possible to construct arbitrage portfolios.

Absence of arbitrage imposes essentially no restrictions on the prices of risk or on how they vary over time $t$ and how they depend on bond supply. We determine these prices from market clearing. Equation (16) shows that the price of risk $\lambda_{i,t}$ for factor $i = r, \bar{r}, \beta, \bar{\beta}$ at time $t$ depends on the overall sensitivity $\int_0^T x_t^{(\tau)} A_i(\tau) d\tau$ of arbitrageurs' portfolio to that factor. Intuitively, if arbitrageurs are highly exposed to a factor, they require that any asset they hold yields high expected return per unit of factor sensitivity. The portfolio that arbitrageurs hold in equilibrium is determined from the market-clearing condition

$$x_t^{(\tau)} = s_t^{(\tau)} \tag{17}$$

which equates the arbitrageurs' dollar investment $x_t^{(\tau)}$ in the bond with maturity $\tau$ to the bond's dollar supply $s_t^{(\tau)}$. Substituting $\mu_t^{(\tau)}$ and $x_t^{(\tau)}$ from equations (7), (14), and (17) into equation (15), we find an affine equation in $r_t$, $\bar{r}_t$, $\beta_t$, and $\bar{\beta}_t$. Setting linear terms in $r_t$, $\bar{r}_t$, $\beta_t$, and $\bar{\beta}_t$ to zero yields four ordinary differential equations (ODEs) in $A_r(\tau)$, $A_{\bar{r}}(\tau)$,

$A_\beta(\tau)$, and $A_{\bar{\beta}}(\tau)$, respectively. Setting constant terms to zero yields an additional ODE in $C(\tau)$. We solve the five ODEs in theorem 1.

**Theorem 1.** *The functions* $A_r(\tau)$, $A_{\bar{r}}(\tau)$, $A_\beta(\tau)$, *and* $A_{\bar{\beta}}(\tau)$ *are given by*

$$A_r(\tau) = \frac{1 - e^{-\kappa_r \tau}}{\kappa_r}, \tag{18}$$

$$A_{\bar{r}}(\tau) = \kappa_r \int_0^\tau A_r(\tau') e^{-\kappa_{\bar{r}}(\tau - \tau')} d\tau' = \frac{\kappa_{\bar{r}}(1 - e^{-\kappa_r \tau}) - \kappa_r (1 - e^{-\kappa_{\bar{r}} \tau})}{\kappa_{\bar{r}}(\kappa_{\bar{r}} - \kappa_r)}, \tag{19}$$

$$\begin{aligned} A_\beta(\tau) = {} & Z_1 \left( \frac{\gamma_2}{\gamma_1 - \gamma_2} e^{-\gamma_1 \tau} - \frac{\gamma_1}{\gamma_1 - \gamma_2} e^{-\gamma_2 \tau} + 1 \right) \\ & + Z_2 \left( \frac{\gamma_2 - \kappa_r}{\gamma_1 - \gamma_2} e^{-\gamma_1 \tau} - \frac{\gamma_1 - \kappa_r}{\gamma_1 - \gamma_2} e^{-\gamma_2 \tau} + e^{-\kappa_r \tau} \right) \\ & + Z_3 \left( \frac{\gamma_2 - \kappa_{\bar{r}}}{\gamma_1 - \gamma_2} e^{-\gamma_1 \tau} - \frac{\gamma_1 - \kappa_{\bar{r}}}{\gamma_1 - \gamma_2} e^{-\gamma_2 \tau} + e^{-\kappa_{\bar{r}} \tau} \right) \end{aligned} \tag{20}$$

*and*

$$A_{\bar{\beta}}(\tau) = \kappa_\beta \int_0^\tau A_\beta(\tau') e^{-\kappa_{\bar{\beta}}(\tau - \tau')} d\tau', \tag{21}$$

*respectively, where*

$$Z_1 \equiv \frac{\kappa_{\bar{\beta}} a \left( \frac{\sigma_r^2 I_r}{\kappa_r} + \frac{\sigma_{\bar{r}}^2 I_{\bar{r}}}{\kappa_{\bar{r}}} \right)}{\kappa_{\bar{\beta}} \kappa_\beta - \kappa_{\bar{\beta}} a \sigma_\beta^2 I_\beta - \kappa_\beta a \sigma_{\bar{\beta}}^2 I_{\bar{\beta}}}, \tag{22}$$

$$Z_2 \equiv \frac{(\kappa_r - \kappa_{\bar{\beta}}) a \left( \frac{\sigma_r^2 I_r}{\kappa_r} + \frac{\sigma_{\bar{r}}^2 I_{\bar{r}}}{\kappa_{\bar{r}} - \kappa_r} \right)}{\kappa_r^2 - \kappa_r(\kappa_{\bar{\beta}} + \kappa_\beta - a\sigma_\beta^2 I_\beta) + \kappa_{\bar{\beta}} \kappa_\beta - \kappa_{\bar{\beta}} a \sigma_\beta^2 I_\beta - \kappa_\beta a \sigma_{\bar{\beta}}^2 I_{\bar{\beta}}}, \tag{23}$$

$$Z_3 \equiv \frac{\frac{\kappa_r(\kappa_{\bar{\beta}} - \kappa_{\bar{r}})}{\kappa_{\bar{r}}(\kappa_{\bar{r}} - \kappa_r)} a \sigma_{\bar{r}}^2 I_{\bar{r}}}{\kappa_{\bar{r}}^2 - \kappa_{\bar{r}}(\kappa_{\bar{\beta}} + \kappa_\beta - a\sigma_\beta^2 I_\beta) + \kappa_{\bar{\beta}} \kappa_\beta - \kappa_{\bar{\beta}} a \sigma_\beta^2 I_\beta - \kappa_\beta a \sigma_{\bar{\beta}}^2 I_{\bar{\beta}}}, \tag{24}$$

$$I_r \equiv \int_0^T A_r(\tau) \theta(\tau) d\tau, \tag{25}$$

$$I_{\bar{r}} \equiv \int_0^T A_{\bar{r}}(\tau)\theta(\tau)d\tau \text{ ,} \tag{26}$$

$(\gamma_1, \gamma_2)$ *are the solutions of the quadratic equation*

$$\gamma^2 - \gamma(\kappa_{\bar{\beta}} + \kappa_\beta - a\sigma_\beta^2 I_\beta) + \kappa_{\bar{\beta}}\kappa_\beta - \kappa_{\bar{\beta}} a\sigma_\beta^2 I_\beta - \kappa_\beta a\sigma_{\bar{\beta}}^2 I_{\bar{\beta}} = 0 \ , \tag{27}$$

*and* $(I_\beta, I_{\bar{\beta}})$ *solve the system of equations*

$$I_\beta = \int_0^T A_\beta(\tau)\theta(\tau)d\tau \tag{28}$$

$$I_{\bar{\beta}} = \int_0^T A_{\bar{\beta}}(\tau)\theta(\tau)d\tau \tag{29}$$

*in which the right-hand side is a function of* $(I_\beta, I_{\bar{\beta}})$ *through equations (20) to (27). A solution to the system of equations (28) and (29) exists if a is below a threshold* $\bar{a} > 0$. *The function* $C(\tau)$ *is given by equation (A.16) in appendix A.*

As in Greenwood and Vayanos (2014), an equilibrium with affine spot rates may fail to exist, and when it exists there can be multiplicity. Equilibrium exists if the arbitrageurs' risk-aversion coefficient $a$ is below a threshold $\bar{a} > 0$. We focus on that case and select the equilibrium that corresponds to the smallest value of $I_\beta$. When $a$ converges to zero, that equilibrium converges to the unique equilibrium that exists for $a = 0$.

## 3. Shocks to the Yield Curve

In this section, we examine how shocks to the four risk factors $r, \bar{r}, \beta, \bar{\beta}$ affect the equilibrium yield curve. We start with a numerical example that illustrates the main results. We then return to the analysis of the general model and provide more complete characterizations and intuition.

### 3.1 Numerical Example

Table 1 summarizes the parameters used in our baseline numerical example. While we attempt to choose realistic values for the parameters, the example's main purpose is to illustrate general properties of the effects of the shocks rather than to provide exact quantitative estimates.

**Table 1. Parameters for Baseline Numerical Example**

| *Parameter* | *Value* |
|---|---|
| $\kappa_r$: Rate at which short rate $r_t$ reverts to target short rate $r_t$ | 1.3 |
| $\sigma_r$: Volatility of shocks to short rate $r_t$ | 1.65% |
| $\kappa_{\bar{r}}$: Rate at which target short rate $\bar{r}_t$ reverts to long-run mean | 0.2 |
| $\sigma_{\bar{r}}$: Volatility of shocks to short rate $\bar{r}_t$ | 2.15% |
| $\kappa_\beta$: Rate at which supply factor $\beta_t$ reverts to target supply $\bar{\beta}_t$ | 2.5 |
| $\sigma_\beta$: Volatility of shocks to supply factor $\beta_t$ | 0.18 |
| $\kappa_{\bar{\beta}}$: Rate at which target supply $\bar{\beta}_t$ reverts to long-run mean | 0.25 |
| $\sigma_{\bar{\beta}}$: Volatility of shocks to supply factor $\bar{\beta}_t$ | 0.18 |
| $T$: Maximum bond maturity | 20 |
| $a$: Arbitrageur risk aversion | 1.65 |

We choose values for $\kappa_r$, $\sigma_r$, $\kappa_{\bar{r}}$, and $\sigma_{\bar{r}}$ to match four time-series moments of the short rate. For the purposes of this exercise, we identify the short rate with the one-year nominal yield and use monthly data from June 1961 to September 2015 (from Gurkaynak, Sack, and Wright, 2007). We match the variance ($\sqrt{Var(r_t)} = 3.33\%$), the one-month autocorrelation ($Corr(r_t, r_{t-1/12})$=0.99), the one-year autocorrelation ($Corr(r_t, r_{t-1})$=0.86), and the three-year autocorrelation ($Corr(r_t, r_{t-3})$=0.59). This yields $\kappa_r = 1.3$, $\sigma_r = 1.65\%$, $\kappa_{\bar{r}} = 0.2$, and $\sigma_{\bar{r}} = 2.15\%$. Under these values, 90 percent of the total variance of the short rate is driven by persistent shocks to the target short rate.[3] The half-life of the shocks to the target short rate is 3.46 years (=log(2)/ $\kappa_{\bar{r}}$) whereas the half-life of the shocks to the current short rate is only 0.53 years (=log(2)/$\kappa_r$).

We choose the values of the remaining parameters to capture aspects of the Fed's QE program. We assume that the $\theta(\tau)$ function (which characterizes the sensitivity of the dollar supply of the bond with maturity $\tau$ to the supply factor $\beta_t$) satisfies $\int_0^T \theta(\tau)d\tau = 0$. Under this assumption, changes in $\beta_t$ do not alter the total value of bonds that arbitrageurs hold in equilibrium, but affect only the duration of

3. The variance of the short rate is $Var(r_t) = \frac{\sigma_r^2}{2\kappa_r} + \frac{\kappa_r \sigma_{\bar{r}}^2}{2\kappa_{\bar{r}}(\kappa_r + \kappa_{\bar{r}})}$. The second term in this expression corresponds to the part of the variance that is driven by shocks to the target short rate.

their portfolio. For simplicity, we assume that $\theta(\tau)$ depends linearly on $\tau$. This yields the specification

$$\theta(\tau) = \theta_0 \left(1 - \frac{2\tau}{T}\right).$$

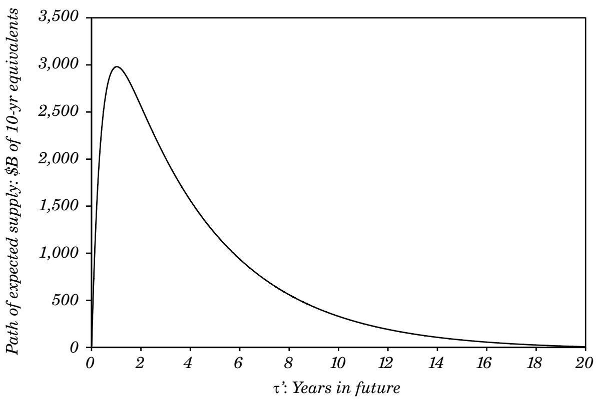
We normalize $\theta_0$ to one, which is without loss of generality because only the product $\theta(\tau)\beta_t$ matters in the definition of the bond supply.

We choose values for $\kappa_\beta$ and $\kappa_{\bar{\beta}}$ to match plausible market expectations about the persistence of the Fed's balance-sheet operations. We assume that the Fed's initial announcement of large-scale asset purchases in 2008 and 2009 led market participants to expect a large reduction in the bond supply over the next twelve months and a gradual increase in supply thereafter. Accordingly, we choose $\kappa_\beta$ and $\kappa_{\bar{\beta}}$ so that the change in the expected supply factor $E_t(\beta_{t+\tau})$ at time $t+\tau$ following a shock to target supply $\bar{\beta}_t$ at time $t$ is maximum after one year ($\tau = 1$) and decays to 50 percent of the maximum after the next three years ($\tau = 4$). This yields $\kappa_\beta = 2.5$ and $\kappa_{\bar{\beta}} = 0.25$. In section 3.5 we examine the sensitivity of our results to a smaller value of $\kappa_{\bar{\beta}}$, under which the effect of a $\bar{\beta}_t$ shock on expected supply is maximum after a period longer than one year.

We assume that a unit shock to $\bar{\beta}_t$ corresponds to the announcement of a QE program that will reduce bond supply by US$ 3 trillion of ten-year bond equivalents. This is without loss of generality because it amounts to a renormalization of the monetary units in which supply is measured. Figure 2 plots the change in the expected supply factor $E_t(\beta_{t+\tau})$ at time $t+\tau$ following a unit shock to $\bar{\beta}_t$ at time $t$. This change, which we denote by $\Delta_{\bar{\beta}} E_t(\beta_{t+\tau})$, is a hump-shaped function of $\tau$ under any parameter values. Indeed, the effect of the $\bar{\beta}_t$ shock on $E_t(\beta_{t+\tau})$, is small for small $\tau$ because the shock does not affect $\beta_t$, increases with $\tau$ as $E_t(\beta_{t+\tau})$ catches up with the new value of $\bar{\beta}_t$, and decreases again to zero because $\bar{\beta}_t$ mean reverts. Under our chosen values for $\kappa_\beta$ and $\kappa_{\bar{\beta}}$, the hump occurs after one year, and the function reaches half of its maximum value after the next three years.

The change $\Delta_{\bar{r}} E_t(r_{t+\tau})$ in the expected short rate $E_t(r_{t+\tau})$ following a unit shock to $\bar{r}_t$ is similarly hump shaped. Under our chosen values for $\kappa_r$ and $\kappa_{\bar{r}}$, the hump occurs after 1.7 years. This is because we assume that supply shocks are less persistent than shocks to the short rate. The mean-reversion parameter for supply shocks is larger than for short-rate shocks both when comparing shocks to current supply $\beta_t$ and the current short rate $r_t$ ($\kappa_\beta > \kappa_r$) and when comparing shocks to the target supply $\bar{\beta}_t$ and the target short rate $\bar{r}_t$ ($\kappa_{\bar{\beta}} > \kappa_{\bar{r}}$).

**Figure 2. Model-Implied Path of QE in Ten-Year Bond Equivalents**




We set $\sigma_\beta = \sigma_{\bar{\beta}} = 0.18$. Under these values, the volatility $\sqrt{Var(\beta_t)}$ of the supply factor is 0.25. We can compare this quantity to the change $\Delta_{\bar{\beta}} E_t(\beta_{t+\tau})$ in the expected supply factor following a unit shock to $\bar{\beta}_t$. This change is 0.75 after one year ($\Delta_{\bar{\beta}} E_t(\beta_{t+1}) = 0.75$), which is three times the standard deviation of $\beta_t$. Thus, a unit shock to $\bar{\beta}_t$ is a rare and large shock to expected future supply, consistent with it being a QE program undertaken in a crisis.

Our final parameter is the arbitrageurs' risk-aversion coefficient $a$, and we choose its value to match the price effects of supply shocks. As noted by Greenwood and others (2015), the Fed's combined QE policies from late 2008 to mid-2014 cumulatively reduced the ten-year bond equivalents available to investors by roughly US$3 trillion. Following the meta-analysis of studies examining the impact of QE announcements in Williams (2014), we assume that an announced purchase of US$500 billion ten-year bond equivalents reduces ten-year yields by 25 basis points. This suggests a total price impact for all QE announcements of 1.50 percent. Therefore, the value of $a$ must be such that $A_{\bar{\beta}}(10)/10=1.50\%$. This yields $a$=1.65.[4]

Figure 3 plots the effects of shocks to the four risk factors $r, \bar{r}, \beta, \bar{\beta}$ on the equilibrium yield curve and the forward-rate curve. There are four plots, each describing the effect that a unit shock to one of the factors has on the yield and forward-rate curves, holding the remaining factors constant. Recall from equations (11) and (12) that the effect of a unit shock to factor $i = r, \bar{r}, \beta, \bar{\beta}$ on the yield for maturity $\tau$ is $\frac{A_i(\tau)}{\tau}$, and the effect on the forward rate for that maturity is $A'_i(\tau)$. Plotting these functions reveals the footprint that shocks to factor $i$ leave on the yield and forward-rate curves.

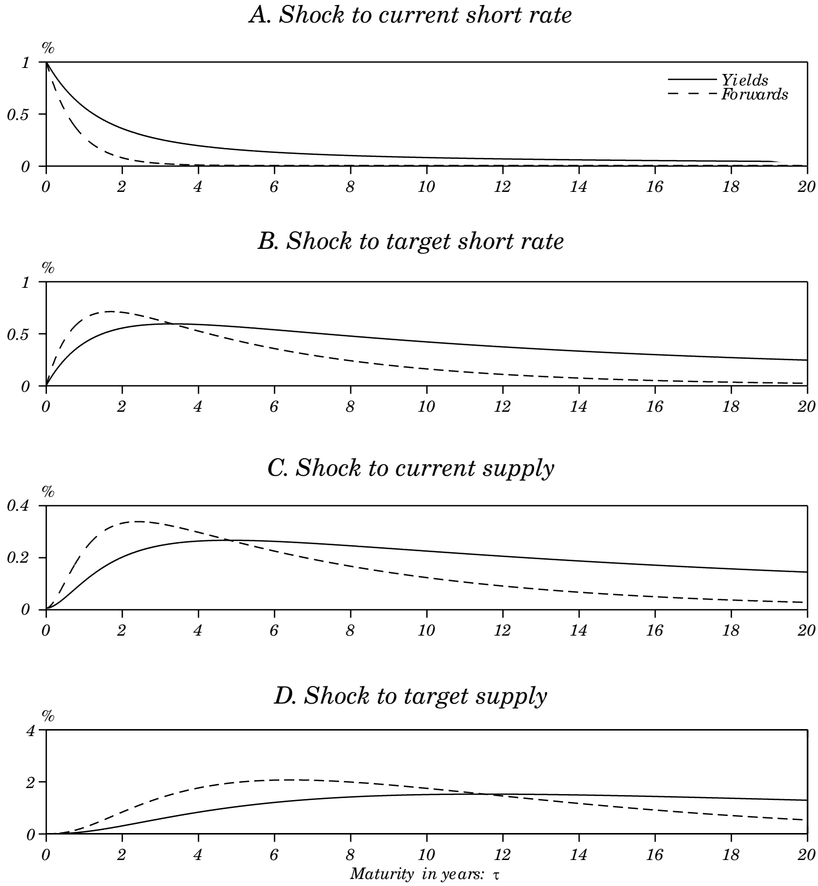
We make three observations regarding figure 3. First, an increase in any of the factors raises all yields and forward rates. Thus, yields and forward rates for any maturity move up in response to increases in the current and the target short rate. They also move up in response to increases in current and target supply.

Second, the effect of shocks to factors other than the current short rate is hump shaped with maturity. Figure 3 thus suggests that policy announcements by the central bank that provide forward guidance on the short rate or on balance-sheet operations should have hump-shaped effects on the yield and forward-rate curves. This is consistent with the evidence on the taper tantrum presented in the introduction.

The third observation suggests a way to differentiate between the two types of forward guidance. The hump for shocks to target supply $\bar{\beta}_t$ occurs at a much longer maturity than for shocks to the target short rate $\bar{r}_t$: 11.5 years versus 3.3 years for the yield curve, and 6.4 years versus 1.7 years for the forward-rate curve. This result cannot be attributed to supply shocks being more persistent than shocks to the short rate: in our baseline numerical example, they are actually less persistent. Figure 3 thus suggests that hump-shaped effects of forward guidance are more likely to concern guidance on supply rather than on the short rate when the hump is located at longer maturities.

4. In principle, one could use the simulated method of moments to estimate the parameters of our model. The parameters that govern the short-rate process ($\kappa_r, \sigma_r, \kappa_{\bar{r}}, \sigma_{\bar{r}}$) (could be identified as above by matching time-series moments of short rates. The parameters that govern the bond supply process ($\kappa_\beta, \sigma_\beta, \kappa_{\bar{\beta}}, \sigma_{\bar{\beta}}$) and arbitrageur risk aversion ($a$) could be identified by matching time-series moments of long-term bond yields of various maturities and the excess returns on long-term bonds. We do not pursue this approach because the supply and demand shocks that have driven bond risk premiums over the past decades may have been of a different nature from the supply shocks generated by the Fed's QE policies since 2008.

**Figure 3. The Effects of a Unit Shock to Each of the Four Risk Factors $r, \bar{r}, \beta, \bar{\beta}$ on the Equilibrium Yield Curve and Forward-Rate Curve**[a]




a. Panel A plots a shock to the current short rate $r_t$; panel B a shock to the target short rate $\bar{r}_t$; panel C a shock to current supply $\beta_t$; and panel D a shock to target supply $\bar{\beta}_t$. For each factor $i = r, \bar{r}, \beta, \bar{\beta}$, the solid line represents the effect $\frac{A_i(\tau)}{\tau}$ on the yield curve, and the dashed line represents the effect $A_i'(\tau)$ on the forward-rate curve.

Figure 3 accords nicely with the empirical findings of Swanson (2015), who decomposes the effect of FOMC announcements from 2009–15 into a component that reflects news about the future path of short rates (forward guidance) and a component that reflects news about future asset purchases (QE). Swanson (2015) finds that both

QE-related and forward-guidance-related announcements have hump-shaped effects on the yield curve. Moreover, QE announcements ( $\bar{\beta}_t$ shocks in our model) have their largest impact at around the ten-year maturity, while forward-guidance announcements ($\bar{r}_t$ shocks) have their largest impact at two to five years.

In the remainder of this section, we show that these three observations hold more generally, and we explain the intuition behind them. Section 3.2 analyzes shocks to the current and the target short rate. Section 3.3 analyzes shocks to current and target supply. Section 3.4 compares the footprints left by shocks to target supply and shocks to the target short rate. Section 3.5 examines how the effects of the shocks depend on various parameters of the model.

## 3.2 Shocks to the Current and the Target Short Rate

Shocks to the current and the target short rate do not affect bond risk premiums in our model. This is because premiums depend only on the positions that arbitrageurs hold in equilibrium, and these depend only on the supply factor $\beta_t$. Since these shocks do not affect risk premiums, their effects on yields and forward rates are only through expected future short rates, and they are fully consistent with the expectations hypothesis. That is, the changes in forward rates caused by these shocks are equal to the changes in expected future short rates.

**Proposition 1.** *The expectations hypothesis holds for shocks to the current and the target short rate.*

- *Consider a unit shock to the current short rate $r_t$ at time t, holding constant the remaining risk factors $(\bar{r}_t, \beta_t, \bar{\beta}_t)$. The change $A'_r(\tau)$ in the forward rate for maturity $\tau$ is equal to the change $\Delta_r E_r(r_{t+\tau})$ in the expected short rate at time $t+\tau$.*
- *Consider a unit shock to the target short rate $\bar{r}_t$ at time t, holding constant the remaining risk factors $(r_t, \beta_t, \bar{\beta}_t)$. The change $A'_{\bar{r}}(\tau)$ in the forward rate for maturity $\tau$ is equal to the change $\Delta_{\bar{r}} E_r(r_{t+\tau})$ in the expected short rate at time $t+\tau$.*

Using proposition 1, we next determine how the effects of shocks to the current and the target short rate depend on maturity. The effect of shocks to the current short rate $r_t$ decreases with maturity and is hence strongest for short maturities. Indeed, because $r_t$ mean reverts, the effect of shocks to $r_t$ on the expected future short rate $E_t(r_{t+\tau})$ is largest in the near future, that is, for small $\tau$. The same applies to the forward rate because of proposition 1. On the other hand, the effect of shocks to the target short rate $\bar{r}_t$ is hump shaped with maturity and is hence strongest for intermediate maturities. Indeed, the effect of shocks to $\bar{r}_t$ on the expected future short rate $E_t(r_{t+\tau})$ is small for short maturities because the shocks do not affect $\bar{r}_t$, increases with maturity as $E_t(r_{t+\tau})$ catches up with the new value of $\bar{r}_t$, and decreases again to zero because $\bar{r}_t$ mean reverts. These results hold both for the yield curve and the forward-rate curve, and are consistent with our baseline numerical example.

**Proposition 2.** *The following results hold for both the yield curve and the forward-rate curve.*

- *An increase in the short rate $r_t$ moves the curve upward. The effect is decreasing with maturity, is equal to one for $\tau = 0$, and to zero for $\tau \to \infty$.*
- *An increase in the target short rate $r_t$ moves the curve upward. The effect is hump shaped with maturity and is equal to zero for $\tau = 0$ and $\tau \to \infty$.*

## 3.3 Shocks to Current and Target Supply

Shocks to current and target supply affect yields and forward rates only through bond risk premiums. Proposition 3 expresses the effects of the shocks on a bond's price as an integral of risk premiums over the life of the bond.

**Proposition 3.** *The effects of supply shocks can be expressed as follows.*

- *Consider a unit shock to current supply $\beta_t$ at time t, holding constant the remaining risk factors $(r_t, \bar{r}_t, \bar{\beta}_t)$. The time-t instantaneous expected return of the bond with maturity $\tau$ changes by*

$$URP(\tau) \equiv a\sigma_r^2 A_r(\tau) I_r + a\sigma_{\bar{r}}^2 A_{\bar{r}}(\tau) I_{\bar{r}} + a\sigma_\beta^2 A_\beta(\tau) I_\beta + a\sigma_{\bar{\beta}}^2 A_{\bar{\beta}}(\tau) I_{\bar{\beta}} \tag{30}$$

The bond's price change in percentage terms is

$$A_\beta(\tau) = \int_0^\tau URP(\tau - \tau')\Delta_\beta E_t(\beta_{t+\tau'})d\tau' \tag{31}$$

*where $\Delta_\beta E_t(\beta_{t+\tau'})$ is the change in the expected supply factor $E_t(\beta_{t+\tau'})$ at time $t + \tau'$.*

- *Consider a unit shock to target supply $\bar{\beta}_t$ at time t, holding constant the remaining risk factors ($r_t$, $\bar{r}_t$, $\beta_t$). The percentage price change of the bond with maturity τ is*

$$A_{\bar{\beta}}(\tau) = \int_0^{\tau} URP(\tau - \tau')\Delta_{\bar{\beta}} E_t(\beta_{t+\tau'})d\tau' \quad (32)$$

where $\Delta_{\bar{\beta}} E_t(\beta_{t+\tau'})$ is the change in the expected supply factor $E_t(\beta_{t+\tau'})$ at time $t + \tau'$.

A unit shock in current supply changes the instantaneous expected return of the bond with maturity τ by a quantity that we denote $URP(\tau)$. This is the unit risk premium that is a required compensation for risk resulting from a unit increase in supply. The unit risk premium for the bond with maturity τ is the product of the arbitrageurs' risk-aversion coefficient $a$ times the change in the bond's instantaneous covariance with the arbitrageurs' portfolio. The covariance changes in response to the supply shock because arbitrageurs change their portfolio in equilibrium. The unit risk premium $URP(\tau)$ is small for bonds with short maturity τ because these bonds have small price sensitivity to the risk factors. As maturity increases, price sensitivity increases and so does $URP(\tau)$.

The impact of a shock to current or target supply on a bond's price derives from its effect on risk premiums over the life of a bond. If, for example, the risk premiums increase, then the price decreases. Equations (31) and (32) make this relationship precise by expressing the effect of a unit supply shock on the percentage price of a bond with maturity τ as an integral of unit risk premiums over the bond's life, that is, from $t$ to $t + \tau$. The risk premium corresponding to time $t + \tau'$, when the bond reaches maturity $\tau - \tau'$, is proportional to the unit risk premium $URP(\tau - \tau')$. Since $URP(\tau - \tau')$ corresponds to a unit increase in the supply factor at $t + \tau'$, we need to multiply it by the actual increase in the expected supply factor. This is $\Delta_{\beta} E_t(\beta_{t+\tau'})$ in the case of a shock to current supply and $\Delta_{\bar{\beta}} E_t(\beta_{t+\tau'})$ in the case of a shock to target supply.

Using proposition 3, we next characterize more fully the effects of shocks to current and target supply: the sign of the effects and how they depend on maturity. As for our analysis on short rates, the results are the same whether we are looking at the yield curve or the forward-rate curve. For the formal propositions that we show in the rest of this section, we assume $\sigma_{\bar{\beta}} = 0$, hence interpreting shocks to $\bar{\beta}_t$ as unanticipated and one-off. However, these formal results are consistent with our baseline numerical example and with other examples that we have explored, all of which assume $\sigma_{\bar{\beta}} > 0$.

As in Greenwood and Vayanos (2014), an increase in current supply $\beta_t$ moves the yield curve upward. Moreover, this occurs even though assumption 1 allows for the possibility that the supply of short-term bonds can decrease. Yields and supply for a given maturity can move in opposite directions because—as in Vayanos and Vila (2009) and Greenwood and Vayanos (2014)—supply effects do operate not locally, but globally through changes in the prices of risk. Equations (16) and (17) show that the prices of risk, $\lambda_{i,t}$ for $i = r, \bar{r}, \beta, \bar{\beta}$, depend on the supply of debt adjusted by measures of duration (the price sensitivities to the factors). An increase in the supply factor raises duration-adjusted supply and hence the prices of risk. Risk premiums also increase, and bond prices decrease from proposition 3. As with $\beta_t$, an increase in target supply $\bar{\beta}_t$ in our model moves the yield curve upward.

We next examine how supply effects depend on maturity. Equation (31) implies that the effect of a unit shock to current supply $\beta_t$ on the yield of a $\tau$-year bond is

$$\frac{A_\beta(\tau)}{\tau} = \frac{\int_0^\tau URP(\tau - \tau') \times \Delta_\beta E_t(\beta_{t+\tau'})d\tau'}{\tau} \tag{33}$$

This is an average of risk premiums over the bond's life. The premium corresponding to time $t + \tau'$, when the bond reaches maturity $\tau - \tau'$, is the product of the unit risk premium $URP(\tau - \tau')$ corresponding to that maturity, times the increase $\Delta_\beta E_t(\beta_{t+\tau'})$ in the expected supply factor at time $t + \tau'$.

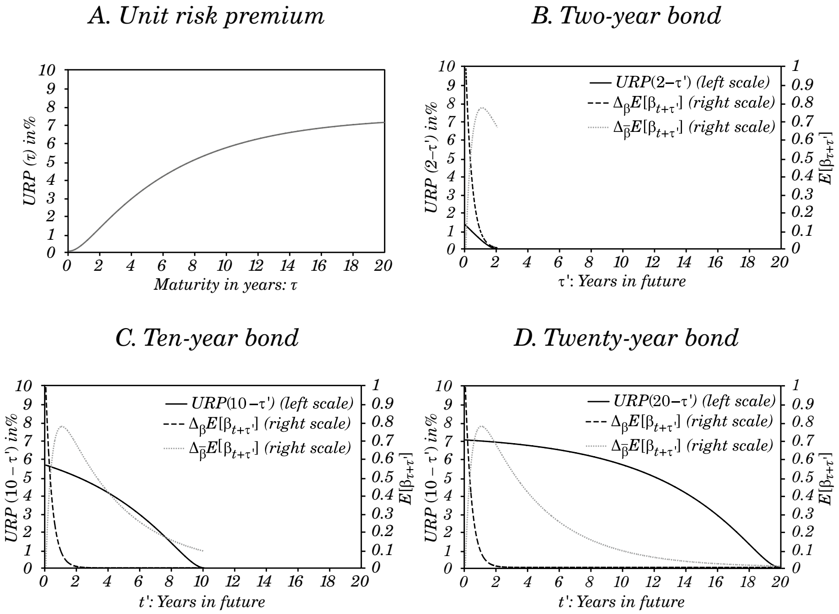
Supply shocks have small effects on short-maturity bonds because these bonds carry small risk premiums. This can be seen formally from equation (33): for small maturity $\tau$, the unit risk premiums $URP(\tau - \tau')$ are small, as is the average in equation (33). As maturity $\tau$ increases, the average in equation (33) increases because unit risk premiums increase. A countervailing effect, however, is that because shocks to $\beta_t$ mean revert, unit risk premiums corresponding to distant times $t + \tau'$ are multiplied by the increasingly smaller quantity $\Delta_\beta E_t(\beta_{t+\tau'})$. This pushes the average down. The countervailing effect is not present in the extreme case where there is no mean reversion ($\kappa_\beta = 0$). In that case, the effect of shocks to $\beta_t$ is increasing with $\tau$, that is, it is strongest at the long end of the term structure. In the other extreme case where mean reversion is high, only the terms for times $t + \tau'$ close to $t$ matter in the average. Because unit risk premiums increase less than linearly with $\tau$ (in particular, changes to $r_t$ or $\bar{r}_t$ have a vanishing effect on spot rates for long maturities), dividing by $\tau$ makes the average converge to zero. The overall effect is hump shaped and hence strongest for intermediate maturities. The same result holds for shocks to $\bar{\beta}_t$. The hump-shaped effects are consistent with our baseline numerical example.

**Proposition 4.** *Suppose that* $\sigma_{\bar{\beta}} > 0$. *An increase in current supply* $\beta_t$ *or target supply* $\bar{\beta}_t$ *moves both the term structure of spot rates and that of instantaneous forward rates upward. The effect is equal to zero for* $\tau = 0$. *For large enough values of* $\kappa_\beta$, *the effect is hump shaped with maturity and is equal to zero for* $\tau \to \infty$. *Otherwise, the effect is increasing with maturity.*

To illustrate the effects of supply, we plot in figure 4 the functions inside the integrals (31) and (32) in the context of our baseline numerical example. Panel A confirms that the unit risk premium $URP(\tau)$ is equal to zero for $\tau = 0$ and increases with $\tau$. Panels B through D plot $URP(\tau - \tau')$, $\Delta_\beta E_t(\beta_{t+\tau'})$, and $\Delta_{\bar{\beta}} E_t(\beta_{t+\tau'})$ as a function of $\tau' \in [0, \tau]$ for three different bonds: a two-year bond ($\tau = 2$), a ten-year bond ($\tau = 10$), and a twenty-year bond ($\tau = 20$). The function $\Delta_\beta E_t(\beta_{t+\tau'})$ is decreasing with $\tau'$: because $\beta_t$ mean reverts, the effect of shocks to $\beta_t$ on the expected future supply factor $E_t(\beta_{t+\tau'})$ is largest in the near future, that is, for small $\tau'$. The function $\Delta_{\bar{\beta}} E_t(\beta_{t+\tau'})$ is hump shaped, as explained in section 3.1.

In the case of the two-year bond, unit risk premiums are small, as are the average values of $URP(\tau - \tau') \times \Delta_\beta E_t(\beta_{t+\tau'})$ and $URP(\tau - \tau') \times \Delta_{\bar{\beta}} E_t(\beta_{t+\tau'})$ over the interval [0,2]. Hence, supply effects are small. In the case of the ten-year bond, unit risk premiums are larger and so are supply effects. In the case of the twenty-year bond, unit risk premiums are even larger, but the average values of $URP(\tau - \tau') \times \Delta_\beta E_t(\beta_{t+\tau'})$ and $URP(\tau - \tau') \times \Delta_{\bar{\beta}} E_t(\beta_{t+\tau'})$ over the interval [0,20] are smaller because of the declines in $\Delta_\beta E_t(\beta_{t+\tau'})$, and $\Delta_{\bar{\beta}} E_t(\beta_{t+\tau'})$. Hence, supply effects are smaller, yielding the hump shape. Note that the smaller supply effect on the yield of the twenty-year bond masks a strong time variation in expected return. The bond's instantaneous expected return is high (and higher than for the other bonds) in the short term, but the effect dies out in the longer term, resulting in a smaller average.

**Figure 4. Decomposition of the Effect of Supply Shocks**




Panel A plots the unit risk premium $URP(\tau)$ . Panels B through D plot $URP(\tau - \tau')$ (solid line), $\Delta_\beta E_t(\beta_{t+\tau'})$ (dashed line), and $\Delta_{\bar\beta} E_t(\beta_{t+\tau'})$ (dotted line) as a function of $\tau' \in [0, \tau]$ for three different bonds: a two-year bond ($\tau = 2$),a ten-year bond ($\tau = 10$), and a twenty-year bond ($\tau = 20$).

## 3.4 Forward Guidance on Supply versus the Short Rate

We next compare the effects of shocks to target supply $\bar\beta_t$ and shocks to the target short rate $\bar r_t$ . Interpreting these shocks as forward guidance by the central bank, we are effectively examining whether different types of forward guidance leave a different footprint on the yield and forward-rate curves. For simplicity, we focus on the forward-rate curve for the rest of this section.

In our baseline numerical example, shocks to target supply $\bar\beta_t$ have their maximum effect at a longer maturity than shocks to the target short rate $\bar r_t$ . While this is the typical outcome in our model, the result is not completely general: if the shocks to current and target supply mean revert very rapidly, the comparison can reverse. Proposition 5 derives sufficient conditions for $\bar\beta_t$ shocks to have their maximum effect at a longer maturity than $\bar r_t$ shocks. The proposition compares the location of the humps associated with two types of shocks, with the convention that if the effect of a shock is monotonically increasing with maturity, then the hump is located at infinity.

**Proposition 5.** *Suppose that* $\sigma_{\bar{\beta}} = 0$. *If* $\kappa_{\bar{r}} \geq \kappa_{\beta}$ *or* $\kappa_{\bar{r}} \geq \kappa_{\bar{\beta}}$, *then the hump on the forward-rate curve associated with shocks to* $\bar{\beta}_t$ *is located at a strictly longer maturity than the hump associated with shocks to* $\bar{r}_t$.

Shocks to $\bar{\beta}_t$ have their largest impact at longer maturities than shocks to $\bar{r}_t$ under the sufficient condition that the latter shocks do not mean revert more slowly than the former shocks ($\kappa_{\bar{r}} \geq \kappa_{\bar{\beta}}$). Alternatively, $\bar{r}_t$ shocks can revert more slowly than $\bar{\beta}_t$ shocks, but then they must not mean revert more slowly than $\beta_t$ shocks ($\kappa_{\bar{r}} \geq \kappa_{\beta}$).Under either sufficient condition, the hump associated with $\bar{\beta}_t$ shocks occurs at a strictly longer maturity than the hump associated with $\bar{r}_t$ shocks, even though the sufficient conditions are weak inequalities. Our baseline numerical example shows that the comparison between the two humps remains the same even when $\kappa_{\beta}$ and $\kappa_{\bar{\beta}}$ are both significantly larger than $\kappa_{\bar{r}}$. (For very large values, however, the comparison can reverse.) Thus, the sufficient conditions in proposition 5 are not tight, and the typical result is that shocks to target supply have their maximum impact at longer maturities than shocks to the target short rate.

The intuition on why shocks to future supply tend to have their largest impact at longer maturities than shocks to the future short rate can be seen from equation (32). The impact of a $\bar{\beta}_t$ shock on the forward rate for maturity $\tau$ is

$$A'_{\bar{\beta}}(\tau) = \int_0^{\tau} \frac{\partial URP(\tau - \tau')}{\partial \tau} \Delta_{\bar{\beta}} E_t(\beta_{t+\tau'}) d\tau' \quad (34)$$

where equation (34) follows from equation (32) by differentiating with respect to $\tau$ and noting that $URP(0) = 0$. The impact on the forward rate can be thought of as the impact on the percentage price of the bond with maturity $\tau$ relative to the same effect for the bond with maturity $\tau - \Delta\tau$. The bond with maturity $\tau$ is affected more heavily because for any given future time $t + \tau'$, the unit risk premium $URP(\tau - \tau')$ associated with that bond is larger than the corresponding premium $URP(\tau - \Delta\tau - \tau')$ associated with the bond with maturity $\tau - \Delta\tau$. The impact on the forward rate hence involves the derivative $\frac{\partial URP(\tau - \tau')}{\partial \tau}$, as equation (34) confirms. This derivative is multiplied by the increase $\Delta_{\bar{\beta}} E_t(\beta_{t+\tau'})$ in the expected supply factor at time $t + \tau'$, and the product is integrated from zero to $\tau$.

Compare next the shock's impact on the forward rate for maturity $\tau$ and for maturity $\hat{\tau}>\tau$. The derivative $\frac{\partial URP(\tau - \tau')}{\partial \tau}$ that is present in the integral (34) for maturity $\tau$ is also present in the integral for maturity $\hat{\tau}$. However, while in the former integral, it corresponds to time $t + \tau'$ and is multiplied by $\Delta_{\bar{\beta}} E_t(\beta_{t+\tau'})$, in the latter integral it corresponds to the more distant time $t + \hat{\tau} - \tau + \tau'$ and is multiplied by $\Delta_{\bar{\beta}} E_t(\beta_{t+\hat{\tau}-\tau+\tau'})$. If $\hat{\tau} \le \tau_{\bar{\beta}}$, where $t + \tau_{\bar{\beta}}$ denotes the location of the hump of $\Delta_{\bar{\beta}} E_t(\beta_{t+\tau'})$, then $\Delta_{\bar{\beta}} E_t(\beta_{t+\hat{\tau}-\tau+\tau'}) > \Delta_{\bar{\beta}} E_t(\beta_{t+\tau'})$ for all $\tau' \in [0, \tau]$. Therefore, the impact of a $\beta_t$ shock on the forward rate for maturity $\hat{\tau}$ is larger than for maturity $\tau$, which means that the shock's maximum impact occurs at a maturity strictly longer than $\tau_{\bar{\beta}}$. On the other hand, proposition 1 implies that the maximum impact of an $\bar{r}_t$ shock occurs exactly at $\tau_{\bar{r}}$, where $t + \tau_{\bar{r}}$ denotes the location of the hump of $\Delta_{\bar{r}} E_t(r_{t+\tau'})$. Therefore, if the shocks to $\bar{r}_t$ and $\bar{\beta}_t$ are symmetric in their persistence, then $\bar{\beta}_t$ shocks have their largest impact at longer maturities than $\bar{r}_t$ shocks.

We can also compare $\bar{r}_t$ and $\bar{\beta}_t$ shocks by focusing on the long end of the term structure rather than on the hump. Proposition 6 derives sufficient conditions for the effect of $\bar{\beta}_t$ shocks to decay more slowly with maturity than that of $\bar{r}_t$ shocks. Under these conditions, $\bar{\beta}_t$ shocks affect the long end of the term structure more than $\bar{r}_t$ shocks do.

**Proposition 6.** *Suppose that* $\sigma_{\bar{\beta}} > 0$. *If* $\min\{\kappa_r, \kappa_{\bar{r}}\} \ge \kappa_{\bar{\beta}}$, *then the effect of shocks to* $\bar{\beta}_t$ *on the forward rate curve decays with maturity at a slower rate than the effect of shocks to* $\bar{r}_t$. *If* $\min\{\kappa_r, \kappa_{\bar{r}}\} \ge \kappa_{\beta}$, *then the same comparison holds and is strict.*

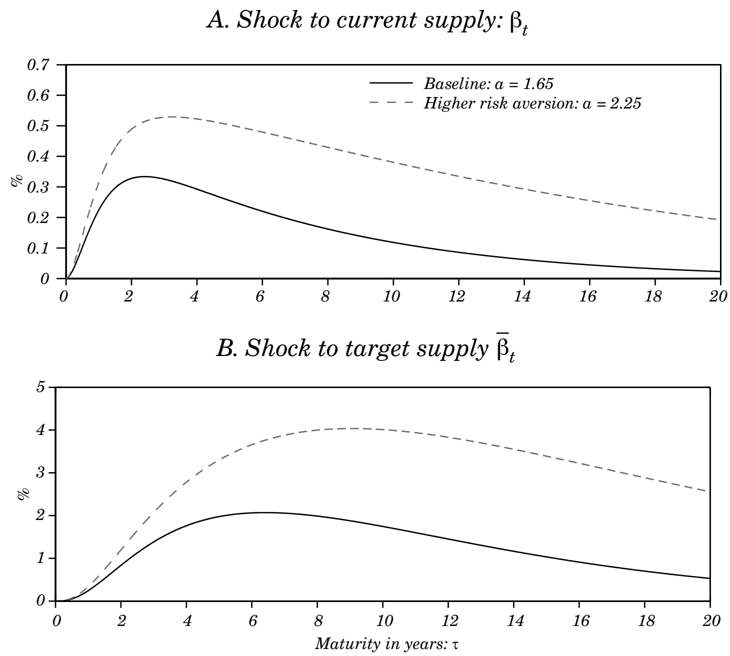
The sufficient conditions in proposition 6 have a similar flavor to those in proposition 5. As with proposition 5, the conditions are not tight. Our baseline numerical example illustrates this.

## 3.5 Comparative Statics

Figure 5 examines how the effect of supply shocks depends on arbitrageur risk aversion. The figure plots the effect of shocks to current supply $\beta_t$ and future supply $\bar{\beta}_t$ on the forward-rate curve in two numerical examples: our baseline example, where the risk-aversion coefficient $a$ is set to 1.65, and an example with $a$ set to 2.25

(and all other parameters remain the same). When arbitrageurs are more risk averse, they require a larger risk premium to accommodate supply shocks, so the shocks have a larger impact on yields and forward rates. Furthermore, the hump for both $\beta_t$ and $\bar{\beta}_t$ shocks occurs at longer maturities. For example, the location of the hump that $\bar{\beta}_t$ shocks generate on the forward-rate curve increases from 6.4 years in the baseline example with $a$ = 1.65 to 9.0 years when $a$ = 2.25. The hump occurs at a longer maturity because when arbitrageurs are more risk averse, the unit risk premium $URP(\tau)$ increases proportionately more for long-term bonds, that is, becomes a more convex function of $\tau$. This is because with more risk-averse arbitrageurs, supply shocks have larger price effects, and the impact of these shocks on long-term bonds relative to short-term bonds is larger than that of short-rate shocks. For example, the impact of $r_t$ shocks is characterized by the increasing function $A_r(\tau)$, while the impact of $\beta_t$ shocks involves an integral of that function.

**Figure 5. Impact of Supply Shocks on the Forward-Rate Curve under Different Values of Arbitrageur Risk Aversion**[a]

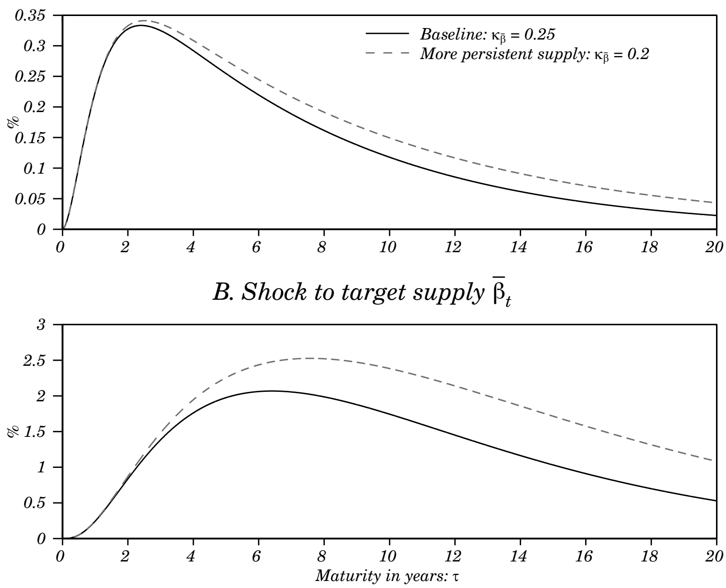



a. The solid lines correspond to our baseline numerical example, where $a$ = 1.65 ; the dashed lines, to an example where $a$ = 2.25 and all other parameters remain the same.

Figure 6 examines how the effect of supply shocks depends on the shocks' persistence. The figure plots the effect of shocks to current and future supply on the forward-rate curve in two numerical examples: our baseline case, where the mean-reversion coefficient $\kappa_{\bar{\beta}}$ of $\bar{\beta}_t$ shocks is set to 0.25, and one where $\kappa_{\bar{\beta}}$ is set to 0.2 and hence shocks are more persistent. When $\kappa_{\bar{\beta}} = 0.2$, the effect of a $\bar{\beta}_t$ shock on the expected supply factor $E_t(\beta_{t+\tau})$ at time $t + \tau$ peaks after 1.1 year ($\tau = 1.1$) and decays to 50 percent of the maximum after the next 3.9 years ($\tau = 5$). When shocks are more persistent, they have a larger impact on the yield and forward-rate curves. Furthermore, the hump for both $\beta_t$ and $\bar{\beta}_t$ shocks occurs at longer maturities. For example, the location of the hump that $\bar{\beta}_t$ shocks generate on the forward-rate curve increases from 6.4 years in the baseline example, where $\kappa_{\bar{\beta}} = 0.25$, to 7.6 years when $\kappa_{\bar{\beta}} = 0.2$. While the shift of the hump to longer maturities may not be surprising in the case of $\bar{\beta}_t$ shocks, whose persistence increases, it may be more surprising in the case of $\beta_t$ shocks, whose persistence does not change. The intuition for $\beta_t$ shocks is that higher persistence means that supply shocks have larger price effects, which makes the unit risk premium $URP(\tau)$ a more convex function of $\tau$.

**Figure 6. Impact of Supply Shocks on the Forward-Rate Curve under Different Values of the Shocks' Persistence[a]**

*A. Shock to current supply:* $\beta_t$




a. The solid lines correspond to our baseline numerical example, where the mean-reversion coefficient of shocks to future supply is $\kappa_{\bar{\beta}} = 0.25$; the dashed lines, to an example where $\kappa_{\bar{\beta}} = 0.2$ and all other parameters remain the same.

## 4. Reassessing QE and the Taper Tantrum

Table 2 summarizes the reaction of the U.S. Treasury yield and forward-rate curves to major QE announcements. The table shows the two-day change in zero-coupon Treasury yields and one-year forward rates around major policy announcements about the Fed's QE operations. We use two-day changes to allow for the possibility that market participants need time to digest news about large-scale asset purchase programs. However, we obtain qualitatively similar results if we restrict attention to one-day changes. We obtain U.S. Treasury yields and forward rates using the fitted nominal Treasury curve estimated by Gurkaynak, Sack, and Wright (2007). We use their zero-coupon yields and compute one-year forward rates from those yields: $f_t^{(\tau-1,\tau)} = \tau y_t^{(\tau)} - (\tau-1) y_t^{(\tau-1)}$. The one-year forward rates are close to the instantaneous forward rates estimated by Gurkaynak, Sack, and Wright (2007). We measure all variables in percentage points.

Our set of QE-related announcement dates is drawn from Fawley and Neely (2013), who provide a comprehensive list of FOMC policy announcements and speeches that contained major news about QE. We classify these events based on whether the announcement contained significant news indicating that the Fed would be expanding or contracting its asset purchases. Many of these events contain a mixture of news about future QE operations and the path of the short rate. For example, the list includes the 18 March 2009 FOMC announcement discussed in the introduction, in which the Fed announced that it was expanding the scale of its long-term asset purchase program from US$600 billion to US$1.75 trillion and that it intended to hold rates at the zero lower bound for "an extended period."

Table 2 shows the change in yields and forward rates around each announcement date. It also shows yield and forward-rate changes aggregated across all expansionary and all contractionary announcements. Figure 7 plots the latter aggregates. As the table and the figure show, both expansionary and contractionary announcements had hump-shaped effects on both the yield and the forward-rate curve. In the case of expansionary announcements, the hump in yields occurred at the ten-year maturity. One-year yields dropped by 33 basis points on aggregate, two-year yields by 52 basis points, three-year yields by 86 basis points, four-year yields by 121 basis points, five-year yields by 153 basis points, seven-year yields by 195 basis points, ten-year yields by 211 basis points, fifteen-year yields by 179 basis points, and twenty-year yields by 144 basis points. The hump in forward rates occurred at the seven-year maturity, with the one-year forward rate seven years into the future dropping by 301 basis points. In the case of contractionary announcements, the hump in yields occurred at the seven-year maturity and that in forward rates at the five-year maturity.

What is the most natural interpretation of these changes? According to our model, the hump-shaped impact on yields and forward rates can be explained either by forward guidance on the path of future short rates or forward guidance on the path of future bond supply. Forward guidance on short rates works through the expectations hypothesis. This means, in particular, that following expansionary announcements, yields and forward rates dropped because market participants revised downward their expectations about future short rates. Moreover, expectations dropped the most for short rates seven years into the future, with the aggregate effect over all announcements being 301 basis points. That market participants revised so drastically their expectations about the short rate seven years into the future, while expectations one to four years out were revised much more modestly, seems unlikely.

Forward guidance on supply works through expected future risk premiums. In contrast to forward guidance on short rates, humps in the yield and forward-rate curves that are consistent with the data could have been the results of changes in supply expectations concerning the near future. Indeed, in our baseline numerical example, shocks to target supply have their largest effect at the 11.5-year maturity in the yield curve and the 6.4-year maturity in the forward-rate curve. Yet, these shocks have their maximum effect on expectations about supply only one year into the future, with the effect four years out being only half of the maximum.

Corroborating evidence on the relative role of supply and short-rate expectations in driving the effects of QE comes from the work by Adrian, Crump, and Moench (2013). These authors construct a methodology for decomposing yields and forward rates into an expectations component and a term-premium component. Drawing on their data for the same announcement dates, figure 8 plots the changes in expected future short rates and in the term premiums.[5] As with the previous figures, we show the results for both yields and forward rates. Adrian, Crump, and Moench's estimates attribute almost all of the impact of QE announcements to changes in term premiums and almost none to changes in expected future short rates.

5. The Adrian-Crump-Moench model does not fit the Gurkaynak-Sack-Wright yields exactly. Thus, the two parts of the Adrian-Crump-Moench curve do not perfectly sum to the Gurkaynak-Sack-Wright curve.

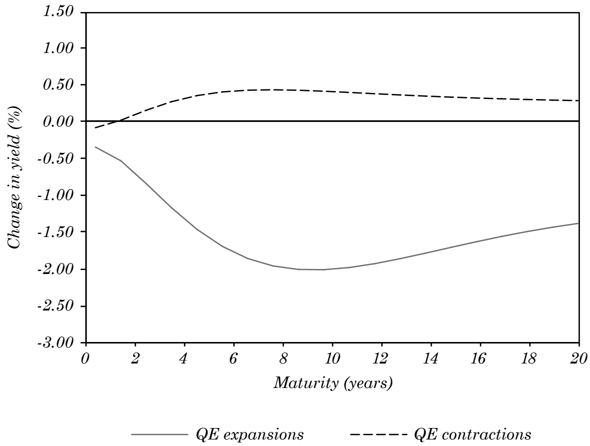
# Table 1. Reaction of U.S. Treasury Yields and Forwards to Major QE Announcements[a]

| | | | *Individual announcement events* | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | *Change in zero coupon yields (%)* | | | | | | | | | *Change in forward rates (%)* | | | | | | | | |
| *Date* | *Type* | *Event description* | *1-yr* | *2-yr* | *3-yr* | *4-yr* | *5-yr* | *7-yr* | *10-yr* | *15-yr* | *20-yr* | *1-yr* | *2-yr* | *3-yr* | *4-yr* | *5-yr* | *7-yr* | *10-yr* | *15-yr* | *20-yr* |
| 11/25/08 | Expand | QE1: Initial announcement: $100B of GSE and $500B MBS | -0.12 | -0.17 | -0.22 | -0.26 | -0.29 | -0.32 | -0.31 | -0.26 | -0.22 | -0.12 | -0.23 | -0.33 | -0.38 | -0.41 | -0.38 | -0.25 | -0.11 | -0.10 |
| 12/1/08 | Expand | QE1: Bernanke QE1 speech mentions extending to USTs | -0.06 | -0.12 | -0.18 | -0.22 | -0.25 | -0.27 | -0.25 | -0.22 | -0.23 | -0.06 | -0.19 | -0.29 | -0.34 | -0.36 | -0.30 | -0.18 | -0.16 | -0.32 |
| 12/16/08 | Expand | QE1: FOMC alludes to possibility of UST purchases | -0.01 | 0.00 | -0.04 | -0.10 | -0.17 | -0.29 | -0.39 | -0.37 | -0.29 | -0.01 | 0.01 | -0.12 | -0.30 | -0.45 | -0.62 | -0.56 | -0.21 | 0.01 |
| 3/18/09 | Expand | QE1: Increased size of QE1 from $600B to $1.75T | -0.12 | -0.20 | -0.28 | -0.35 | -0.40 | -0.48 | -0.51 | -0.43 | -0.30 | -0.12 | -0.28 | -0.43 | -0.55 | -0.63 | -0.67 | -0.53 | -0.11 | 0.21 |
| 8/10/10 | Expand | QE2: Will reinvest coupon/principal from QE1 | -0.01 | -0.03 | -0.05 | -0.08 | -0.10 | -0.14 | -0.15 | -0.13 | -0.10 | -0.01 | -0.05 | -0.11 | -0.16 | -0.20 | -0.21 | -0.16 | -0.05 | -0.01 |
| 9/21/10 | Expand | QE2: Will continue to reinvest coupon/ principal | -0.01 | -0.03 | -0.06 | -0.09 | -0.11 | -0.15 | -0.17 | -0.16 | -0.14 | -0.01 | -0.05 | -0.12 | -0.18 | -0.22 | -0.24 | -0.19 | -0.10 | -0.07 |
| 11/3/10 | Expand | QE2: Will purchase additional $600B of USTs | -0.01 | -0.02 | -0.06 | -0.10 | -0.13 | -0.16 | -0.12 | 0.01 | 0.12 | -0.01 | -0.03 | -0.13 | -0.22 | -0.26 | -0.21 | 0.06 | 0.40 | 0.42 |
| 9/21/11 | Expand | MEP: $400 billion Maturity Extension Program | 0.02 | 0.05 | 0.03 | -0.01 | -0.07 | -0.17 | -0.26 | -0.33 | -0.39 | 0.02 | 0.09 | -0.01 | -0.15 | -0.28 | -0.45 | -0.49 | -0.47 | -0.62 |
| 6/20/12 | Expand | MEP: Continuing MEP through 2012 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.01 | -0.01 | -0.03 | -0.05 | 0.02 | 0.02 | 0.02 | 0.01 | 0.00 | -0.02 | -0.05 | -0.08 | -0.11 |
| 8/22/12 | Expand | QE3: "Additional accommodation may be required fairly soon" | -0.02 | -0.04 | -0.06 | -0.08 | -0.10 | -0.13 | -0.14 | -0.13 | -0.11 | -0.02 | -0.05 | -0.10 | -0.14 | -0.17 | -0.20 | -0.16 | -0.07 | -0.07 |
| 9/13/12 | Expand | QE3: Purchase $40B of MBS per month | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.07 | 0.12 | 0.18 | 0.20 | 0.00 | 0.01 | 0.03 | 0.06 | 0.09 | 0.16 | 0.25 | 0.31 | 0.23 |
| 12/12/12 | Expand | QE3: Purchase $40B of MBS and $45B of USTs per month | 0.00 | 0.01 | 0.03 | 0.04 | 0.06 | 0.08 | 0.09 | 0.08 | 0.07 | 0.00 | 0.03 | 0.06 | 0.09 | 0.11 | 0.13 | 0.11 | 0.05 | 0.04 |
| 11/4/09 | Contract | QE1: Slowing pace of purchases to complete in 2010Q1 | -0.03 | -0.04 | -0.03 | -0.02 | 0.00 | 0.03 | 0.07 | 0.09 | 0.08 | -0.03 | -0.05 | -0.02 | 0.02 | 0.07 | 0.13 | 0.16 | 0.10 | 0.05 |
| 5/22/13 | Contract | QE3: Taper tantrum, may slow pace | -0.01 | 0.02 | 0.04 | 0.06 | 0.07 | 0.09 | 0.10 | 0.07 | 0.05 | -0.01 | 0.04 | 0.08 | 0.11 | 0.13 | 0.14 | 0.08 | 0.00 | 0.00 |
| 6/19/13 | Contract | QE3: Taper tantrum, may slow pace | 0.00 | 0.07 | 0.14 | 0.20 | 0.24 | 0.25 | 0.23 | 0.19 | 0.18 | 0.00 | 0.13 | 0.30 | 0.38 | 0.38 | 0.26 | 0.12 | 0.14 | 0.16 |
| 12/18/13 | Contract | QE3: Announces tapering of QE3 | -0.01 | 0.02 | 0.05 | 0.09 | 0.11 | 0.12 | 0.09 | 0.04 | 0.02 | -0.01 | 0.04 | 0.13 | 0.18 | 0.19 | 0.12 | -0.01 | -0.06 | -0.02 |
| | | | *Total across all announcements* | | | | | | | | | | | | | | | | | |
| | | | *Change in zero coupon yields (%)* | | | | | | | | | *Change in forward rates (%)* | | | | | | | | |
| | | | *1-yr* | *2-yr* | *3-yr* | *4-yr* | *5-yr* | *7-yr* | *10-yr* | *15-yr* | *20-yr* | *1-yr* | *2-yr* | *3-yr* | *4-yr* | *5-yr* | *7-yr* | *10-yr* | *15-yr* | *20-yr* |
| | | Expand | -0.33 | -0.53 | -0.86 | -1.21 | -1.53 | -1.95 | -2.11 | -1.79 | -1.44 | -0.33 | -0.73 | -1.52 | -2.27 | -2.78 | -3.01 | -2.14 | -0.62 | -0.40 |
| | | Contract | -0.05 | 0.06 | 0.20 | 0.33 | 0.42 | 0.50 | 0.48 | 0.40 | 0.34 | -0.05 | 0.17 | 0.49 | 0.70 | 0.77 | 0.65 | 0.35 | 0.19 | 0.19 |

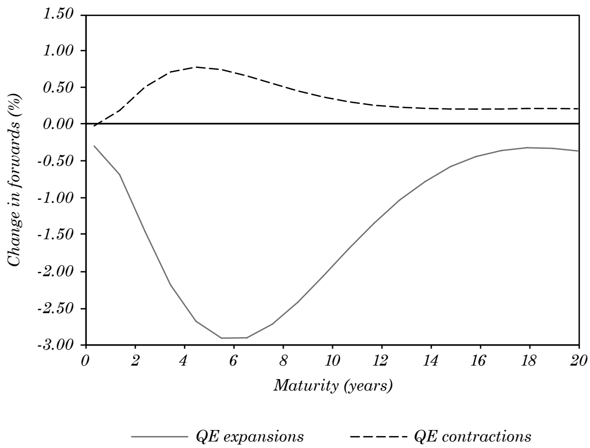
a. This table shows the 2-day change in zero coupon Treasury yields and 1-year forward rates surrounding policy announcements about the Fed's Large Scale Asset Purchase Programs. Yields and forwards are based on the fitted nominal Treasury curve estimated by Gürkaynak, Sack, and Wright (2007). All variables are all measured in percentage points. We classify events based on whether the announcement indicated that the Fed would be expanding or contracting its asset purchases. We compute the totals across all expansionary and contractionary events.

## Figure 7. Changes in Yields and Forward Rates Surrounding QE Announcement Dates

*A. Yields*

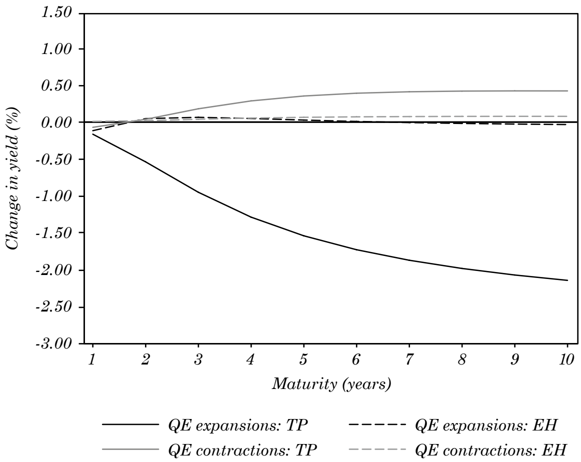



*B. Forward rates*

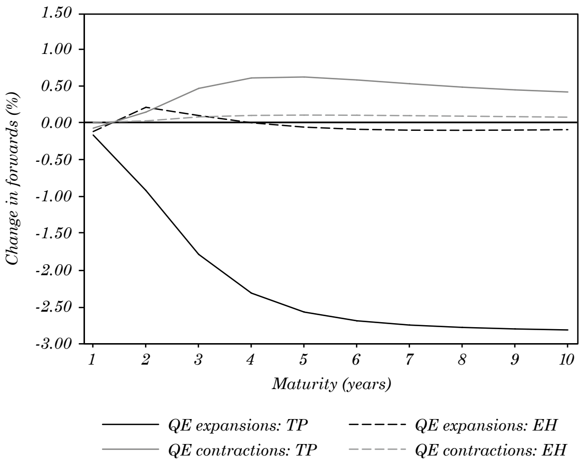


## Figure 8. Changes in Expected Future Short Rates and the Term Premiums Surrounding QE Announcement Dates[a]

*A. Yields*




*B. Forward rates*




a. EH refers to the expectations component and TP to the term-premium component. The decomposition into EH and TP draws on data from Adrian, Crump, and Moench (2013).

## 5. Conclusion

In this paper, we build a model to analyze the impact of forward guidance on the yield curve. Our model recognizes that in recent years, forward guidance pertains not only to the future path of short-term interest rates, but also to the future size of the central bank's balance sheet.

We show that forward guidance on short-term interest rates is easy to interpret because it works through the expectations hypothesis. If, for example, the market expectation of the short rate three years from now declines by 100 basis points, this is reflected directly in a 100 basis points decline in the instantaneous forward rate three years from now. However, when the central bank provides forward guidance on supply, the effects are more subtle. In particular, yields and forward rates are affected at maturities much longer than the time by which supply shocks are expected to die out. Moreover, while the effects of either type of forward guidance on the yield and forward-rate curves can be hump shaped, the humps associated with supply shocks typically occur at maturities longer than those associated with short-rate shocks.

Using our model, we reexamine the empirical evidence on QE announcements in the United States. We show that the cumulative effect of all expansionary announcements up to 2013 was hump shaped with a maximum effect at the ten-year maturity for the yield curve and the seven-year maturity for the forward-rate curve. This evidence is hard to square with changing expectations about short rates, as the maximum change would have to concern short rates seven years into the future. On the other hand, the evidence is more consistent with changing expectations about supply, as the maximum change would have to be only one year into the future.

# Appendix

## Proofs of Theoretical Results

### A.1 Proof of Lemma 1

Using equation (13), we can write equation (5) as

$$dW_t = \left( W_t r_t + \int_0^T x_t^{(\tau)} (\mu_t^{(\tau)} - r_t) d\tau \right) dt - \left( \int_0^T x_t^{(\tau)} A_r(\tau) d\tau \right) \sigma_r dB_{r,t}$$
$$- \left( \int_0^T x_t^{(\tau)} A_{\bar{r}}(\tau) d\tau \right) \sigma_{\bar{r}} dB_{\bar{r},t} - \left( \int_0^T x_t^{(\tau)} A_\beta(\tau) d\tau \right) \sigma_\beta dB_{\beta,t}$$
$$- \left( \int_0^T x_t^{(\tau)} A_{\bar{\beta}}(\tau) d\tau \right) \sigma_{\bar{\beta}} dB_{\bar{\beta},t},$$

and equation (6) as

$$\max_{\{x_t^{(\tau)}\}_{\tau \in (0,T]}} \left[ \int_0^T x_t^{(\tau)} (\mu_t^{(\tau)} - r_t) d\tau - \frac{a\sigma_r^2}{2} \left( \int_0^T x_t^{(\tau)} A_r(\tau) d\tau \right)^2 \right. \tag{A.1}$$
$$\left. - \frac{a\sigma_{\bar{r}}^2}{2} \left( \int_0^T x_t^{(\tau)} A_{\bar{r}}(\tau) d\tau \right)^2 - \frac{a\sigma_\beta^2}{2} \left( \int_0^T x_t^{(\tau)} A_\beta(\tau) d\tau \right)^2 - \frac{a\sigma_{\bar{\beta}}^2}{2} \left( \int_0^T x_t^{(\tau)} A_{\bar{\beta}}(\tau) d\tau \right)^2 \right].$$

Point-wise maximization of equation (A.1) yields equation (15).

### A.2 Proof of Theorem 1

Substituting $x_t^{(\tau)}$ from equations (7) and (17) into equation (16), we find

$$\lambda_{i,t} = a\sigma_i^2 \int_0^T \left[ \zeta(\tau) + \theta(\tau)\beta_t \right] A_i(\tau) d\tau. \tag{A.2}$$

Substituting $\mu_t^{(\tau)}$ and $\lambda_{i,t}$ from equations (14) and (A.2) into equation (15), we find an affine equation in $(r_t, \bar{r}_t, \beta_t, \bar{\beta}_t)$. Identifying terms in $r_t$ yields

$$\kappa_r A_r(\tau) + A_r'(\tau) - 1 = 0, \tag{A.3}$$

identifying terms in $\bar{r}_t$ yields

$$-\kappa_r A_r(\tau) + \kappa_{\bar{r}} A_{\bar{r}}(\tau) + A'_{\bar{r}}(\tau) = 0, \tag{A.4}$$

identifying terms in $\beta_t$ yields

$$\kappa_\beta A_\beta(\tau) + A'_\beta(\tau) = a\sigma_r^2 A_r(\tau)\int_0^T A_r(\tau)\theta(\tau)d\tau$$
$$+a\sigma_{\bar{r}}^2 A_{\bar{r}}(\tau)\int_0^T A_{\bar{r}}(\tau)\theta(\tau)d\tau + a\sigma_\beta^2 A_\beta(\tau) \tag{A.5}$$
$$\int_0^T A_\beta(\tau)\theta(\tau)d\tau + a\sigma_{\bar{\beta}}^2 A_{\bar{\beta}}(\tau)\int_0^T A_{\bar{\beta}}(\tau)\theta(\tau)d\tau,$$

identifying terms in $\bar{\beta}_t$ yields

$$-\kappa_\beta A_\beta(\tau) + \kappa_{\bar{\beta}} A_{\bar{\beta}}(\tau) + A'_{\bar{\beta}}(\tau) = 0, \tag{A.6}$$

and identifying constant terms yields

$$C'(\tau) - \kappa_{\bar{r}}\bar{r}A_{\bar{r}}(\tau) + \frac{\sigma_r^2}{2}A_r(\tau)^2 + \frac{\sigma_{\bar{r}}^2}{2}A_{\bar{r}}(\tau)^2 + \frac{\sigma_\beta^2}{2}A_\beta(\tau)^2 + \frac{\sigma_{\bar{\beta}}^2}{2}A_{\bar{\beta}}(\tau)^2$$
$$= a\sigma_r^2 A_r(\tau)\int_0^T A_r(\tau)\zeta(\tau)d\tau + a\sigma_{\bar{r}}^2 A_{\bar{r}}(\tau)\int_0^T A_{\bar{r}}(\tau)\zeta(\tau)d\tau \tag{A.7}$$
$$+a\sigma_\beta^2 A_\beta(\tau)\int_0^T A_\beta(\tau)\zeta(\tau)d\tau + a\sigma_{\bar{\beta}}^2 A_{\bar{\beta}}(\tau)\int_0^T A_{\bar{\beta}}(\tau)\zeta(\tau)d\tau.$$

The ordinary differential equations (ODEs) (A.3) through (A.7) must be solved with the initial conditions $A_r(0) = A_{\bar{r}}(0) = A_\beta(0) = A_{\bar{\beta}}(0) = C(0) = 0$. The solution to equation (A.3) with the initial condition $A_r(0) = 0$ is equation (18). The solution to equation (A.4) with the initial condition $A_{\bar{r}}(0) = 0$ is equation (19). The solution to equation (A.6) with the initial condition $A_{\bar{\beta}}(0) = 0$ is equation (21). To solve equation (A.5), we write it as

$$\kappa_\beta A_\beta(\tau) + A'_\beta(\tau) = a\sigma_r^2 I_r A_r(\tau) + a\sigma_{\bar{r}}^2 I_{\bar{r}} A_{\bar{r}}(\tau)$$
$$+ a\sigma_\beta^2 I_\beta A_\beta(\tau) + a\sigma_{\bar{\beta}}^2 I_{\bar{\beta}} A_{\bar{\beta}}(\tau), \tag{A.8}$$

using equations (25), (26), (28), and (29). Differentiating with respect to $\tau$, we find

$$\kappa_\beta A'_\beta(\tau) + A''_\beta(\tau) = a\sigma_r^2 I_r A'_r(\tau) + a\sigma_{\bar{r}}^2 I_{\bar{r}} A'_{\bar{r}}(\tau) \tag{A.9}$$
$$+ a\sigma_\beta^2 I_\beta A'_\beta(\tau) + a\sigma_{\bar{\beta}}^2 I_{\bar{\beta}} A'_{\bar{\beta}}(\tau).$$

Multiplying equation (A.8) by $\kappa_{\bar{\beta}}$, adding to equation (A.9), and using equations (18), (19), and (A.6), we find

$$\left(\kappa_{\bar{\beta}}\kappa_{\beta} - \kappa_{\bar{\beta}}a\sigma_{\beta}^2 I_{\beta} - \kappa_{\beta}a\sigma_{\bar{\beta}}^2 I_{\bar{\beta}}\right)A_{\beta}(\tau) + (\kappa_{\bar{\beta}} + \kappa_{\beta} - a\sigma_{\beta}^2 I_{\beta})A'_{\beta}(\tau) + A''_{\beta}(\tau)$$
$$= a\sigma_r^2 I_r\left(\frac{\kappa_{\bar{\beta}}}{\kappa_r} + \frac{\kappa_r - \kappa_{\bar{\beta}}}{\kappa_r}e^{-\kappa_r\tau}\right) \tag{A.10}$$
$$+ a\sigma_{\bar{r}}^2 I_{\bar{r}}\left(\frac{\kappa_{\bar{\beta}}}{\kappa_{\bar{r}}} + \frac{\kappa_r - \kappa_{\bar{\beta}}}{\kappa_{\bar{r}} - \kappa_r}e^{-\kappa_r\tau} + \frac{\kappa_r(\kappa_{\bar{\beta}} - \kappa_{\bar{r}})}{\kappa_{\bar{r}}(\kappa_{\bar{r}} - \kappa_r)}e^{-\kappa_{\bar{r}}\tau}\right).$$

Equation (A.10) is a second-order linear ODE with constant coefficients. Its solution has the form

$$A_{\beta}(\tau) = \Gamma_1 e^{-\gamma_1\tau} + \Gamma_2 e^{-\gamma_2\tau} + \hat{A}_{\beta}(\tau), \tag{A.11}$$

where $(\gamma_1, \gamma_2)$are the solutions of the quadratic equation (27), and $\hat{A}_{\beta}(\tau)$ is one solution to equation (A.10). We look for $\hat{A}_{\beta}(\tau)$ of the form

$$\hat{A}_{\beta}(\tau) = Z_1 + Z_2 e^{-\kappa_r\tau} + Z_3 e^{-\kappa_{\bar{r}}\tau}.$$

Substituting into equation (A.10), we find that $(Z_1, Z_2, Z_3)$ are given by equations (22) through (24), respectively. To determine $(\Gamma_1, \Gamma_2)$ we use the initial conditions. The initial condition $A_{\beta}(0) = 0$ implies

$$\Gamma_1 + \Gamma_2 + Z_1 + Z_2 + Z_3 = 0. \tag{A.12}$$

The initial condition $A'_{\beta}(0) = 0$, which follows from equation (A.5) and $A_r(0) = A_{\beta}(0) = A_{\bar{\beta}}(0)$, implies

$$\gamma_1\Gamma_1 + \gamma_2\Gamma_2 + \kappa_r Z_2 + \kappa_{\bar{r}} Z_3 = 0. \tag{A.13}$$

Solving the linear system of equations (A.12) and (A.13) yields

$$\Gamma_1 = \frac{\gamma_2 Z_1 + (\gamma_2 - \kappa_r)Z_2 + (\gamma_2 - \kappa_{\bar{r}})Z_3}{\gamma_1 - \gamma_2}, \tag{A.14}$$

$$\Gamma_2 = -\frac{\gamma_1 Z_1 + (\gamma_1 - \kappa_r)Z_2 + (\gamma_1 - \kappa_{\bar{r}})Z_3}{\gamma_1 - \gamma_2}. \tag{A.15}$$

Substituting $(\Gamma_1, \Gamma_2)$ from equations (A.14) and (A.15) into equation (A.11), we find equation (20).

The solution to equation (A.7) is

$$C(\tau) = Z_r \int_0^\tau A_r(\tau')d\tau' + Z_{\bar{r}} \int_0^\tau A_{\bar{r}}(\tau')d\tau'$$
$$+ Z_\beta \int_0^\tau A_\beta(\tau')d\tau' + Z_{\bar{\beta}} \int_0^\tau A_{\bar{\beta}}(\tau')d\tau' \tag{A.16}$$
$$-\frac{\sigma_r^2}{2}\int_0^\tau A_r(\tau')^2 d\tau' - \frac{\sigma_{\bar{r}}^2}{2}\int_0^\tau A_{\bar{r}}(\tau')^2 d\tau' - \frac{\sigma_\beta^2}{2}\int_0^\tau A_\beta(\tau')^2 d\tau' - \frac{\sigma_{\bar{\beta}}^2}{2}\int_0^\tau A_{\bar{\beta}}(\tau')^2 d\tau',$$

where

$$Z_r \equiv a\sigma_r^2 \int_0^T A_r(\tau)\zeta(\tau)d\tau,$$

$$Z_r \equiv \kappa_{\bar{r}}\bar{r} + a\sigma_{\bar{r}}^2 \int_0^T A_{\bar{r}}(\tau)\zeta(\tau)d\tau,$$

$$Z_\beta \equiv a\sigma_\beta^2 \int_0^T A_\beta(\tau)\zeta(\tau)d\tau,$$

$$Z_{\bar{\beta}} \equiv a\sigma_{\bar{\beta}}^2 \int_0^T A_{\bar{\beta}}(\tau)\zeta(\tau)d\tau.$$

For $a = 0$, the solutions of equation (27) are $(\gamma_1, \gamma_2) = (\kappa_{\bar{\beta}}, \kappa_{\bar{\beta}})$, and the solution to the system of equations (28) and (29) is $(I_\beta, I_{\bar{\beta}}) = (0,0)$. The existence of a solution to equations (28) and (29) for $a$ close to zero follows from the implicit function theorem.

## A.3 Proof of Proposition 1

Consider first the unit shock to $r_t$. Taking expectations in equation (3), we find that the change $\Delta_r E_t(r_{t+\tau})$ in the expected short rate at time $t + \tau$ follows the dynamics

$$d[\Delta_r E_t(r_{t+\tau})] = -\kappa_r \Delta_r E_t(r_{t+\tau})d\tau.$$

With the initial condition $\Delta_r E_t(r_t) = 1$, these dynamics integrate to

$$\Delta_r E_t(r_{t+\tau}) = e^{-\kappa_r \tau} = A_r'(\tau),$$

where the second step in the first equation follows from equation (18).

Consider next the unit shock to $\bar{r}_t$. Taking expectations in equations (3) and (4), we find that the change $\Delta_{\bar{r}}E_t(r_{t+\tau})$ in the expected short rate and $\Delta_{\bar{r}}E_t(\bar{r}_{t+\tau})$ in the target short rate at time $t+\tau$ follow the dynamics

$$d[\Delta_{\bar{r}}E_t(r_{t+\tau})] = \kappa_r[\Delta_{\bar{r}}E_t(\bar{r}_{t+\tau}) - \Delta_{\bar{r}}E_t(r_{t+\tau})]d\tau,$$

$$d[\Delta_{\bar{r}}E_t(\bar{r}_{t+\tau})] = -\kappa_{\bar{r}}\Delta_{\bar{r}}E_t(\bar{r}_{t+\tau})d\tau.$$

With the initial condition $(\Delta_{\bar{r}}E_t(r_t), \Delta_{\bar{r}}E_t(\bar{r}_t)) = (0,1)$, these dynamics integrate to

$$\Delta_{\bar{r}}E_t(r_{t+\tau}) = \kappa_r \frac{e^{-\kappa_r\tau} - e^{-\kappa_{\bar{r}}\tau}}{\kappa_{\bar{r}} - \kappa_r} = A'_{\bar{r}}(\tau),$$

$$\Delta_{\bar{r}}E_t(\bar{r}_{t+\tau}) = e^{-\kappa_{\bar{r}}\tau},$$

where the second step in the first equation follows from equation (19).

We next show a useful lemma.

**Lemma A.1.** *If a function* $f(\tau)$ *is positive and increasing, then* $\int_0^T f(\tau)\theta(\tau)d\tau > 0$.

**Proof.** We can write the integral $\int_0^T f(\tau)\theta(\tau)d\tau$ as

$$\int_0^T f(\tau)\theta(\tau)d\tau = \int_0^{\tau^*} f(\tau)\theta(\tau)d\tau + \int_{\tau^*}^T f(\tau)\theta(\tau)d\tau$$

$$> f(\tau^*)\int_0^{\tau^*} \theta(\tau)d\tau + f(\tau^*)\int_{\tau^*}^T \theta(\tau)d\tau$$

$$= f(\tau^*)\int_0^T \theta(\tau)d\tau \geq 0,$$

where the second step follows from part (ii) of assumption 1 and because $f(\tau)$ is increasing, and the last step follows from part (i) of assumption 1 and because $f(\tau)$ is positive.

## A.4 Proof of Proposition 2

The effect of an increase in $r_t$ on the term structure of spot rates is described by the function $\frac{A_r(\tau)}{\tau}$, and the effect on the term structure of instantaneous forward rates by the function $A'_r(\tau)$. We will show that these functions have the following properties:

$\frac{A_r(\tau)}{\tau} > 0$ and $A'_r(\tau) > 0$ for $\tau > 0$;

$\lim_{\tau \to 0} \frac{A_r(\tau)}{\tau} = 1$ and $A'_r(\tau) > 0$ for $\tau > 0$;

$\lim_{\tau \to \infty} \frac{A_r(\tau)}{\tau} = 0$ and $\lim_{\tau \to 0} A'_r(\tau) = 0$;

$\frac{A_r(\tau)}{\tau}$ and $A'_r(\tau)$ are decreasing in $\tau$.

Equation (18) implies that the function $A_r(\tau)$ is positive and increasing. Therefore, the functions $\frac{A_r(\tau)}{\tau}$ and $A'_r(\tau)$ are positive, which means that an increase in $r_t$ shifts the term structure upward. Moreover, both $\frac{A_r(\tau)}{\tau} = \frac{1 - e^{-\kappa_r \tau}}{\kappa_r \tau}$ and $A'_r(\tau) = e^{-\kappa_r \tau}$ are equal to one for $\tau = 0$ and to zero for $\tau \to \infty$. Finally, $A'_r(\tau)$ is decreasing in $\tau$, and the same is true for $\frac{A_r(\tau)}{\tau}$ because for a general function $g(\tau)$

$$\frac{d}{d\tau} \frac{g(\tau)}{\tau} = \frac{\tau g'(\tau) - g(\tau)}{\tau^2} = \frac{\int_0^\tau \tau' g''(\tau') d\tau'}{\tau^2}. \tag{A.17}$$

The effect of an increase in $\bar{r}_t$ on the term structure of spot rates is described by the function $\frac{A_{\bar{r}}(\tau)}{\tau}$ and that on the term structure of instantaneous forward rates by the function $A'_{\bar{r}}(\tau)$. We will show that these functions have the following properties:

$\frac{A_{\bar{r}}(\tau)}{\tau}$ and $A'_{\bar{r}}(\tau)$ for $\tau > 0$;

$\lim_{\tau \to 0} \frac{A_{\bar{r}}(\tau)}{\tau} = 0$ and $A'_{\bar{r}}(0) = 0$;

$\lim_{\tau \to \infty} \frac{A_{\bar{r}}(\tau)}{\tau} = 0$ and $\lim_{\tau \to 0} A'_{\bar{r}}(\tau) = 0$;

$\frac{A_{\bar{r}}(\tau)}{\tau}$ and $A'_{\bar{r}}(\tau)$ are hump shaped in $\tau$.

Since the function $A_r(\tau)$ is positive, equation (19) implies that the function $A_{\bar{r}}(\tau)$ is also positive. Moreover, $A_{\bar{r}}(\tau)$ is increasing because

$$\begin{aligned} A'_{\bar{r}}(\tau) &= \kappa_r\left(A_r(\tau) - \kappa_{\bar{r}}\int_0^\tau A_r(\tau')e^{-\kappa_{\bar{r}}(\tau-\tau')}d\tau'\right) \\ &\geq \kappa_r A_r(\tau)\left(1-\kappa_{\bar{r}}\int_0^\tau e^{-\kappa_{\bar{r}}(\tau-\tau')}d\tau'\right) \\ &= \kappa_r A_r(\tau)e^{-\kappa_{\bar{r}}\tau} > 0, \end{aligned} \quad (A.18)$$

where the first step follows by differentiating equation (19) and the second because $A_r(\tau)$ is increasing. Since $A_{\bar{r}}(\tau)$ is positive and increasing, the functions $A_{\bar{r}}(\tau)$ and $A'_{\bar{r}}(\tau)$ are positive, which means that an increase in $\bar{r}_t$ shifts the term structure upward. Since $A_r(0) = 0$, equation (19) implies that $\frac{A_{\bar{r}}(\tau)}{\tau}$ is equal to zero for $\tau = 0$, and equation (A.18) implies the same property for $A'_{\bar{r}}(\tau)$. Since $A_r(\tau)$ converges to the finite limit $\frac{1}{\kappa_r}$ for $\tau \to \infty$, equation (19) implies that $\frac{A_{\bar{r}}(\tau)}{\tau}$ converges to zero for $\tau \to \infty$, and equation (A.18) implies the same property for $A'_{\bar{r}}(\tau)$.

To show that $\frac{A_{\bar{r}}(\tau)}{\tau}$ and $A'_{\bar{r}}(\tau)$ are hump shaped, it suffices to show this property for $A'_{\bar{r}}(\tau)$. Indeed, equation (A.17) would then imply that $\frac{A_{\bar{r}}(\tau)}{\tau}$ can either be increasing or increasing and then decreasing, and the first pattern is ruled out because $\frac{A_{\bar{r}}(\tau)}{\tau}$ is equal to zero for both $\tau = 0$ and $\tau \to \infty$. Differentiating equation (A.18), we find

$$A''_{\bar{r}}(\tau) = \kappa_r\left(A'_r(\tau) - \kappa_{\bar{r}}A_r(\tau) + \kappa_{\bar{r}}^2\int_0^\tau A_r(\tau')e^{-\kappa_{\bar{r}}(\tau-\tau')}d\tau'\right). \quad (A.19)$$

The term in brackets has the same sign as

$$H_{\bar{r}}(\tau) \equiv \left[A'_r(\tau) - \kappa_{\bar{r}}A_r(\tau)\right]e^{\kappa_{\bar{r}}\tau} + \kappa_{\bar{r}}^2\int_0^\tau A_r(\tau')e^{\kappa_{\bar{r}}\tau'}d\tau'.$$

The function $H_{\bar{r}}(\tau)$ is equal to $A'_r(0) = 1$ for $\tau = 0$, and its derivative is

$$H'_{\bar{r}}(\tau) = A''_r(\tau)e^{\kappa_{\bar{r}}\tau} < 0.$$

Therefore, $H_{\bar{r}}(\tau)$ is either positive or positive and then negative. This means that $A'_{\bar{r}}(\tau)$ is either increasing or increasing and then decreasing. The first pattern is ruled out because $A'_{\bar{r}}(\tau)$ is equal to zero for both $\tau = 0$ and $\tau \to \infty$.

### A.5 Proof of Proposition 3

Consider first the unit shock to $\beta_t$. Using the definition (30) of $URP(\tau)$, we can write equation (A.5) as

$$\kappa_\beta A_\beta(\tau) + A'_\beta(\tau) = URP(\tau). \tag{A.20}$$

Integrating equation (A.20) with the initial condition $A_\beta(\tau) = 0$, we find

$$A_\beta(\tau) = \int_0^\tau URP(\tau')e^{-\kappa_\beta(\tau-\tau')}d\tau' \tag{A.21}$$

$$= \int_0^\tau URP(\tau-\tau')e^{-\kappa_\beta\tau'}d\tau'. \tag{A.22}$$

Taking expectations in equation (8), we find that the change $\Delta_\beta E_t(\beta_{t+\tau})$ in the expected future supply factor at time $t+\tau$ follows the dynamics

$$d[\Delta_\beta E_t(\beta_{t+\tau})] = -\kappa_\beta \Delta_\beta E_t(\beta_{t+\tau})d\tau.$$

With the initial condition $\Delta_\beta E_t(\beta_t) = 1$, these dynamics integrate to

$$\Delta_\beta E_t(\beta_{t+\tau}) = e^{-\kappa_\beta\tau}. \tag{A.23}$$

Using equation (A.23), we can write equation (A.22) as equation (31).

Consider next the unit shock to $\bar{\beta}_t$. Taking expectations in equations (8) and (9), we find that the changes $\Delta_{\bar{\beta}} E_t(\beta_{t+\tau})$ in the expected future supply factor and $\Delta_{\bar{\beta}} E_t(\bar{\beta}_{t+\tau})$ in the expected future target supply follow the dynamics

$$d[\Delta_{\bar{\beta}} E_t(\beta_{t+\tau})] = \kappa_\beta[\Delta_{\bar{\beta}} E_t(\bar{\beta}_{t+\tau}) - \Delta_{\bar{\beta}} E_t(\beta_{t+\tau})]d\tau,$$

$$d[\Delta_{\bar{\beta}} E_t(\bar{\beta}_{t+\tau})] = -\kappa_{\bar{\beta}}\Delta_{\bar{\beta}} E_t(\bar{\beta}_{t+\tau})d\tau.$$

With the initial condition $(\Delta_{\bar{\beta}} E_t(\beta_t), \Delta_{\bar{\beta}} E_t(\bar{\beta}_t)) = (0,1)$, these dynamics integrate to

$$\Delta_{\bar{\beta}} E_t(\beta_{t+\tau}) = \kappa_\beta \frac{e^{-\kappa_\beta\tau} - e^{-\kappa_{\bar{\beta}}\tau}}{\kappa_{\bar{\beta}} - \kappa_\beta}, \tag{A.24}$$

$$\Delta_{\bar{\beta}} E_t(\bar{\beta}_{t+\tau}) = e^{-\kappa_{\bar{\beta}}\tau}.$$

Substituting equation (A.21) into equation (21), we find

$$A_{\bar{\beta}}(\tau) = \kappa_\beta \int_0^\tau \left( \int_0^{\tau'} URP(\tau'') e^{-\kappa_\beta(\tau'-\tau'')} d\tau'' \right) e^{-\kappa_{\bar{\beta}}(\tau-\tau')} d\tau' \tag{A.25}$$

$$= \kappa_\beta \int_0^\tau \left( \int_{\tau''}^\tau e^{-\kappa_\beta(\tau'-\tau'')} e^{-\kappa_{\bar{\beta}}(\tau-\tau')} d\tau' \right) URP(\tau'') d\tau''$$

$$= \int_0^\tau URP(\tau') \kappa_\beta \frac{e^{-\kappa_\beta(\tau-\tau')} - e^{-\kappa_{\bar{\beta}}(\tau-\tau')}}{\kappa_{\bar{\beta}} - \kappa_\beta} d\tau'$$

$$= \int_0^\tau URP(\tau-\tau') \kappa_\beta \frac{e^{-\kappa_\beta \tau'} - e^{-\kappa_{\bar{\beta}} \tau'}}{\kappa_{\bar{\beta}} - \kappa_\beta} d\tau'.$$

Using equation (A.24), we can write equation (A.25) as equation (32).

## A.6 Proof of Proposition 4

The effect of an increase in $\beta_t$ on the term structure of spot rates is described by the function $\frac{A_\beta(\tau)}{\tau}$ and that on the term structure of instantaneous forward rates by the function $A'_\beta(\tau)$. For $\sigma_{\bar{\beta}} = 0$, equation (A.8) becomes

$$\kappa_\beta A_\beta(\tau) + A'_\beta(\tau) = a\sigma_r^2 I_r A_r(\tau) + a\sigma_{\bar{r}}^2 I_{\bar{r}} A_{\bar{r}}(\tau) + a\sigma_\beta^2 I_\beta A_\beta(\tau),$$

and integrates to

$$A_\beta(\tau) = a\sigma_r^2 I_r \int_0^\tau A_r(\tau') e^{-\hat{\kappa}_\beta(\tau-\tau')} d\tau' + a\sigma_{\bar{r}}^2 I_{\bar{r}} \int_0^\tau A_{\bar{r}}(\tau') e^{-\hat{\kappa}_\beta(\tau-\tau')} d\tau', \tag{A.26}$$

where $\hat{\kappa}_\beta \equiv \kappa_\beta - a\sigma_\beta^2 I_\beta$.

We will show that the functions $\frac{A_\beta(\tau)}{\tau}$ and $A'_\beta(\tau)$ have the following properties:

$\frac{A_\beta(\tau)}{\tau} > 0$ and $A'_\beta(\tau) > 0$ for $\tau > 0$;

$\lim_{\tau \to 0} \frac{A_\beta(\tau)}{\tau} = 0$ and $A'_\beta(0) = 0$;

For $\hat{\kappa}_\beta > 0$, $\lim_{\tau\to\infty}\frac{A_\beta(\tau)}{\tau} = 0$ and $\lim_{\tau\to 0} A'_\beta(\tau) = 0$;

For $\hat{\kappa}_\beta > 0$, $\frac{A_\beta(\tau)}{\tau}$ and $A'_\beta(\tau)$ are hump shaped in $\tau$. For $\hat{\kappa}_\beta > 0$, $\frac{A_\beta(\tau)}{\tau}$ and $A'_\beta(\tau)$ are increasing in $\tau$.

Since the functions $A_r(\tau)$ and $A_{\bar{r}}(\tau)$ are positive and increasing, lemma 1 implies that $(I_r, I_{\bar{r}})$ are positive. Hence, equation (A.26) implies that the function $A_\beta(\tau)$ is positive. To show that $A_\beta(\tau)$ is increasing, we differentiate equation (A.26):

$$A'_\beta(\tau) = a\sigma_r^2 I_r\left(A_r(\tau) - \hat{\kappa}_\beta\int_0^\tau A_r(\tau')e^{-\hat{\kappa}_\beta(\tau-\tau')}d\tau'\right) \tag{A.27}$$
$$+ a\sigma_{\bar{r}}^2 I_{\bar{r}}\left(A_{\bar{r}}(\tau) - \hat{\kappa}_\beta\int_0^\tau A_{\bar{r}}(\tau')e^{-\hat{\kappa}_\beta(\tau-\tau')}d\tau'\right).$$

If $\hat{\kappa}_\beta \le 0$, then equation (A.27) and the positivity of $(A_r(\tau), A_{\bar{r}}(\tau))$ imply that $A'_\beta(\tau)$ is positive. If $\hat{\kappa}_\beta > 0$, then the same conclusion follows by proceeding as in the proof of the result in proposition 2 that $A_{\bar{r}}(\tau)$ is increasing. Since $A_\beta(\tau)$ is positive and increasing, the functions $\frac{A_\beta(\tau)}{\tau}$ and $A'_\beta(\tau)$ are positive, which means that an increase in $\beta_t$ shifts the term structure upward. Since $(A_r(0), A_{\bar{r}}(0)) = 0$, equation (A.26) implies that $\frac{A_\beta(\tau)}{\tau}$ is equal to zero for $\tau = 0$, and equation (A.27) implies the same property for $A'_\beta(\tau)$. Since $A_r(\tau), A_{\bar{r}}(\tau)$ converge to the finite limit $\left(\frac{1}{\kappa_r}, \frac{1}{\kappa_{\bar{r}}}\right)$ for $\tau \to \infty$, equation (19) implies that when $\hat{\kappa}_\beta > 0$, $A_\beta(\tau)$ converges to a finite limit for $\tau \to \infty$. Therefore, when $\hat{\kappa}_\beta > 0$, $\frac{A_\beta(\tau)}{\tau}$ converges to zero for $\tau \to \infty$, and equation (A.18) implies the same property for $A'_\beta(\tau)$.

We next study the monotonicity of $\frac{A_\beta(\tau)}{\tau}$ and $A'_\beta(\tau)$. Differentiating equation (A.27), we find

$$A''_\beta(\tau) = a\sigma_r^2 I_r\left(A'_r(\tau) - \hat{\kappa}_\beta A_r(\tau) + \hat{\kappa}_\beta^2\int_0^\tau A_r(\tau')e^{-\hat{\kappa}_\beta(\tau-\tau')}d\tau'\right) \tag{A.28}$$
$$+a\sigma_{\bar{r}}^2 I_{\bar{r}}\left(A'_{\bar{r}}(\tau) - \hat{\kappa}_\beta A_{\bar{r}}(\tau) + \hat{\kappa}_\beta^2\int_0^\tau A_{\bar{r}}(\tau')e^{-\hat{\kappa}_\beta(\tau-\tau')}d\tau'\right).$$

If $\hat{\kappa}_\beta \le 0$, then equation (A.28) and the positivity of $(A_r(\tau), A_{\bar{r}}(\tau))$ imply that $A''_\beta(\tau)$ is positive. Therefore, $A'_\beta(\tau)$ is increasing, and equation (A.17) implies that $\frac{A_\beta(\tau)}{\tau}$ is increasing. If $\hat{\kappa}_\beta > 0$, then we will show that $A''_\beta(\tau)$

is positive and then negative, and hence $A'_\beta(\tau)$ is hump shaped. The hump-shape of $\frac{A_\beta(\tau)}{\tau}$ will follow by using equation (A.17) and noting that $\frac{A_\beta(\tau)}{\tau}$ is equal to zero for both $\tau = 0$ and $\tau \to \infty$.

The right-hand side of equation (A.28) has the same sign as

$$H_\beta(\tau) = a\sigma_r^2 I_r \left\{\left[A'_r(\tau) - \hat{\kappa}_\beta A_r(\tau)\right]e^{\hat{\kappa}_\beta \tau} + \hat{\kappa}_\beta^2 \int_0^\tau A_r(\tau')e^{\hat{\kappa}_\beta \tau'} d\tau'\right\}$$

$$+a\sigma_{\bar{r}}^2 I_{\bar{r}} \left\{\left[A'_{\bar{r}}(\tau) - \hat{\kappa}_\beta A_{\bar{r}}(\tau)\right]e^{\hat{\kappa}_\beta \tau} + \hat{\kappa}_\beta^2 \int_0^\tau A_{\bar{r}}(\tau')e^{\hat{\kappa}_\beta \tau'} d\tau'\right\}$$

The function $H_\beta(\tau)$ is equal to $a\sigma_r^2 I_r A'_r(0) = a\sigma_r^2 I_r > 0$ for $\tau = 0$. Its derivative is

$$H'_\beta(\tau) = a\sigma_r^2 I_r A''_r(\tau)e^{\hat{\kappa}_\beta \tau} + a\sigma_{\bar{r}}^2 I_{\bar{r}} A''_{\bar{r}}(\tau)e^{\hat{\kappa}_\beta \tau}.$$

$$= a\sigma_r^2 I_r \left[A''_r(\tau) + \frac{\sigma_{\bar{r}}^2 I_{\bar{r}}}{\sigma_r^2 I_r}\kappa_r H_{\bar{r}}(\tau)e^{-\kappa_{\bar{r}}\tau}\right]e^{\hat{\kappa}_\beta \tau}$$

$$= a\sigma_r^2 I_r \left[A''_r(\tau) + \frac{\sigma_{\bar{r}}^2 I_{\bar{r}}}{\sigma_r^2 I_r}\kappa_r \left(1 + \int_0^\tau A''_r(\tau')e^{\kappa_{\bar{r}}\tau'} d\tau'\right)e^{-\kappa_{\bar{r}}\tau}\right]e^{\hat{\kappa}_\beta \tau}$$

$$= a\sigma_r^2 I_r \left[-\kappa_r e^{-\kappa_r \tau} + \frac{\sigma_{\bar{r}}^2 I_{\bar{r}}}{\sigma_r^2 I_r}\kappa_r \left(1 - \kappa_r \int_0^\tau e^{(\kappa_{\bar{r}} - \kappa_r)\tau'} d\tau'\right)e^{-\kappa_{\bar{r}}\tau}\right]e^{\hat{\kappa}_\beta \tau}$$

$$= a\sigma_r^2 I_r \kappa_r \left[-1 + \frac{\sigma_{\bar{r}}^2 I_{\bar{r}}}{\sigma_r^2 I_r}\left(1 - \frac{\kappa_r\left(e^{(\kappa_{\bar{r}} - \kappa_r)\tau} - 1\right)}{\kappa_{\bar{r}} - \kappa_r}\right)e^{(\kappa_r - \kappa_{\bar{r}})\tau}\right]e^{(\hat{\kappa}_\beta - \kappa_r)\tau}$$

$$= a\sigma_r^2 I_r \kappa_r \left[-1 + \frac{\sigma_{\bar{r}}^2 I_{\bar{r}}}{\sigma_r^2 I_r}\left(e^{(\kappa_r - \kappa_{\bar{r}})\tau} - \frac{\kappa_r\left(1 - e^{(\kappa_r - \kappa_{\bar{r}})\tau}\right)}{\kappa_{\bar{r}} - \kappa_r}\right)\right]e^{(\hat{\kappa}_\beta - \kappa_r)\tau}$$

The term in square brackets is an affine function of $e^{(\kappa_r - \kappa_{\bar{r}})\tau}$ and can hence change sign at most once. Since $A'_r(\tau)$ is decreasing and $A'_{\bar{r}}(\tau)$ is hump shaped, $H'_\beta(\tau)$ is negative for large $\tau$. Since it can change sign at most once, it is either negative or positive and then negative. Therefore, $H_\beta(\tau)$ is either decreasing or increasing and then decreasing. Since $H_\beta(\tau)$ is positive for $\tau = 0$, it is either positive or positive and then negative. The first pattern is ruled out because when $\hat{\kappa}_\beta > 0$, $A'_\beta(\tau)$ is equal to zero for both $\tau = 0$ and $\tau \to \infty$.

The effect of an increase in $\bar{\beta}_t$ on the term structure of spot rates is described by the function $\frac{A_{\bar{\beta}}(\tau)}{\tau}$ and that on the term structure of instantaneous forward rates by the function $A'_{\bar{\beta}}(\tau)$. We will show that these functions have the following properties:

$\frac{A_{\bar{\beta}}(\tau)}{\tau} > 0$ and $A'_{\bar{\beta}}(\tau) > 0$ for $\tau > 0$;

$\lim_{\tau\to 0}\frac{A_{\bar{\beta}}(\tau)}{\tau} = 0$ and $A'_{\bar{\beta}}(\tau)$;

For $\hat{\kappa}_\beta > 0$, $\lim_{\tau\to\infty}\frac{A_{\bar{\beta}}(\tau)}{\tau} = 0$ and $\lim_{\tau\to 0}A'_{\bar{\beta}}(\tau) = 0$;

For $\hat{\kappa}_\beta > 0$, $\frac{A_{\bar{\beta}}(\tau)}{\tau}$ and $A'_{\bar{\beta}}(\tau)$are hump shaped in $\tau$. For $\hat{\kappa}_\beta > 0$, $\frac{A_{\bar{\beta}}(\tau)}{\tau}$ and $A'_{\bar{\beta}}(\tau)$are increasing in $\tau$.

The above properties can be derived from those of $A_\beta(\tau)$ in the same way that the properties of $\frac{A_{\bar{r}}(\tau)}{\tau}$ and $A'_{\bar{\beta}}(\tau)$ are derived from those of $A_r(\tau)$ in the proof of proposition 2. In particular, because $A_\beta(\tau)$ is positive, increasing, equal to zero for $\tau = 0$, and converging to a finite limit for $\tau \to \infty$ when $\hat{\kappa}_\beta > 0$, we can show that $\frac{A_{\bar{\beta}}(\tau)}{\tau}$ and $A'_{\bar{\beta}}(\tau)$ are positive, equal to zero for $\tau = 0$, and converging to zero for $\tau \to \infty$ when $\hat{\kappa}_\beta > 0$. The function $A''_\beta(\tau)$ has the same sign as

$$H_{\bar{\beta}}(\tau) \equiv \left[A'_\beta(\tau) - \kappa_{\bar{\beta}}A_\beta(\tau)\right]e^{\kappa_{\bar{\beta}}\tau} + \kappa_{\bar{\beta}}^2\int_0^\tau A_\beta(\tau')e^{\kappa_{\bar{\beta}}\tau'}d\tau'.$$

The function $H_{\bar{\beta}}(\tau)$ is equal to $A'_\beta(0) = 0$ for $\tau = 0$, and its derivative is

$$H'_{\bar{\beta}}(\tau) = A''_\beta(\tau)e^{\kappa_{\bar{\beta}}\tau}.$$

When $\hat{\kappa}_\beta \leq 0$, $A''_\beta(\tau)$ is positive. Therefore, $A''_{\bar{\beta}}(\tau)$ is also positive and the functions $\frac{A_{\bar{\beta}}(\tau)}{\tau}$ and $A'_{\bar{\beta}}(\tau)$ are increasing. When $\hat{\kappa}_\beta > 0$, $A''_\beta(\tau)$ is positive and then negative. Therefore, $H_{\bar{\beta}}(\tau)$ is increasing and then decreasing. Since $H_{\bar{\beta}}(\tau)$ is equal to zero for $\tau = 0$, it is either positive or positive and then negative. The first pattern is ruled out when $\hat{\kappa}_\beta > 0$ because $A'_{\bar{\beta}}(\tau)$ is equal to zero for both $\tau = 0$ and $\tau \to \infty$.

The final step in the proof is to show that $\hat{\kappa}_\beta$ is a monotone function of $\kappa_\beta$. This will ensure that $\hat{\kappa}_\beta > 0$ corresponds to larger values of $\kappa_\beta$

than $\hat{\kappa}_\beta \le 0$ does. Since the function $A_\beta(\tau)$ is positive and increasing, lemma 1 implies that $I_\beta$ is positive. Since the function

$$G(\hat{\kappa}_\beta) \equiv \hat{\kappa}_\beta - \kappa_\beta + a\sigma_\beta^2 I_\beta$$

is positive for $\hat{\kappa}_\beta \ge \kappa_\beta$, any solution $\hat{\kappa}_\beta$ to $G(\hat{\kappa}_\beta) = 0$ satisfies $\hat{\kappa}_\beta < \kappa_\beta$. Moreover, at the largest solution, which corresponds to our equilibrium selection, the function $G(\hat{\kappa}_\beta)$ crosses the $x$ axis from below. Since $G(\hat{\kappa}_\beta)$ is decreasing in $\kappa_\beta$, the largest solution is increasing in $\kappa_\beta$.

## A.7 Proof of Proposition 5

The humps on the instantaneous-forward-rate term structure associated with shocks to $\bar{r}_t$ and $\bar{\beta}_t$ are located at the solutions to

$$H_{\bar{r}}(\tau) = 1 + \int_0^\tau A_r''(\tau') e^{\kappa_{\bar{r}}\tau'} d\tau' = 0 \tag{A.29}$$

$$H_{\bar{\beta}}(\tau) = \int_0^\tau A_\beta''(\tau') e^{\kappa_{\bar{\beta}}\tau'} d\tau' = 0 \tag{A.30}$$

respectively. We denote these solutions by $(\tau_{\bar{r}}, \tau_{\bar{\beta}})$. Since

$$A_\beta''(\tau) = H_\beta(\tau) e^{-\hat{\kappa}_\beta \tau}$$
$$= \left\{ a\sigma_r^2 I_r + \int_0^\tau \left[ a\sigma_r^2 I_r A_r''(\tau') e^{\hat{\kappa}_\beta \tau'} + a\sigma_{\bar{r}}^2 I_{\bar{r}} A_{\bar{r}}''(\tau') e^{\hat{\kappa}_\beta \tau'} \right] d\tau' \right\} e^{-\hat{\kappa}_\beta \tau}$$

we can write equation (A.30) as

$$\int_0^\tau A_\beta''(\tau') e^{\kappa_{\bar{\beta}}\tau'} d\tau' = 0 \tag{A.31}$$

$$\Leftrightarrow \int_0^\tau \left\{ a\sigma_r^2 I_r + \int_0^{\tau'} \left[ a\sigma_r^2 I_r A_r''(\tau'') e^{\hat{\kappa}_\beta \tau''} + a\sigma_{\bar{r}}^2 I_{\bar{r}} A_{\bar{r}}''(\tau'') e^{\hat{\kappa}_\beta \tau''} \right] d\tau'' \right\} e^{-\hat{\kappa}_\beta \tau'} e^{\kappa_{\bar{\beta}}\tau'} d\tau' = 0$$

$$\Leftrightarrow \int_0^\tau e^{(\kappa_{\bar{\beta}} - \hat{\kappa}_\beta)\tau'} d\tau' + \int_0^\tau \left( \int_{\tau''}^\tau e^{(\kappa_{\bar{\beta}} - \hat{\kappa}_\beta)\tau'} d\tau' \right) \left( A_r''(\tau'') + \frac{\sigma_{\bar{r}}^2 I_{\bar{r}}}{\sigma_r^2 I_r} A_{\bar{r}}''(\tau'') \right) e^{\hat{\kappa}_\beta \tau''} d\tau'' = 0$$

$$\Leftrightarrow 1 + \int_0^\tau \left( A_r''(\tau') + \frac{\sigma_{\bar{r}}^2 I_{\bar{r}}}{\sigma_r^2 I_r} A_{\bar{r}}''(\tau') \right) \frac{e^{(\kappa_{\bar{\beta}} - \kappa_\beta)\tau + \kappa_\beta \tau'} - e^{\kappa_{\bar{\beta}}\tau'}}{e^{(\kappa_{\bar{\beta}} - \hat{\kappa}_\beta)\tau} - 1} d\tau' = 0.$$

A sufficient condition for $\tau_{\bar{\beta}} > \tau_{\bar{r}}$ is that

$$\frac{e^{(\kappa_{\bar{\beta}} - \hat{\kappa}_\beta)\tau_{\bar{r}} + \hat{\kappa}_\beta \tau'} - e^{\kappa_{\bar{\beta}}\tau'}}{e^{(\kappa_{\bar{\beta}} - \hat{\kappa}_\beta)\tau_{\bar{r}}} - 1} < e^{\kappa_{\bar{r}}\tau'} \quad for \quad 0 < \tau' < \tau_{\bar{r}} \tag{A.32}$$

This is because

$$H_{\bar{r}}(\tau_{\bar{r}}) = 1 + \int_0^{\tau_{\bar{r}}} A''_r(\tau') e^{\kappa_{\bar{r}}\tau'} d\tau' = 0 \tag{A.33}$$

$$\Rightarrow 1 + \int_0^{\tau_{\bar{r}}} A''_r(\tau') \frac{e^{(\kappa_{\bar{\beta}} - \hat{\kappa}_\beta)\tau_{\bar{r}} + \hat{\kappa}_\beta \tau'} - e^{\kappa_{\bar{\beta}}\tau'}}{e^{(\kappa_{\bar{\beta}} - \hat{\kappa}_\beta)\tau_{\bar{r}}} - 1} > 0$$

$$\Rightarrow 1 + \int_0^{\tau_{\bar{r}}} \left( A''_r(\tau') + \frac{\sigma_{\bar{r}}^2 I_{\bar{r}}}{\sigma_r^2 I_r} A''_{\bar{r}}(\tau') \right) \frac{e^{(\kappa_{\bar{\beta}} - \hat{\kappa}_\beta)\tau_{\bar{r}} + \hat{\kappa}_\beta \tau'} - e^{\kappa_{\bar{\beta}}\tau'}}{e^{(\kappa_{\bar{\beta}} - \hat{\kappa}_\beta)\tau_{\bar{r}}} - 1} > 0$$

$$\Rightarrow H_{\bar{\beta}}(\tau_{\bar{r}}) > 0$$

where the second step follows from equation (A.32) and $A''_{\bar{r}}(\tau') < 0$, and the third step follows because $A''_{\bar{r}}(\tau') > 0$ for $\tau' < \tau_{\bar{r}}$. Since $H_{\bar{\beta}}(\tau)$ has the same sign of $A_{\bar{\beta}}(\tau)$, and the latter is positive if and only if $\tau < \tau_{\bar{\beta}}$, (A.33) implies that $\tau_{\bar{r}} < \tau_{\bar{\beta}}$.

Equation (A.32) is equivalent to

$$h(\tau') \equiv \frac{e^{(\kappa_{\bar{\beta}} - \hat{\kappa}_\beta)\tau_{\bar{r}} + (\hat{\kappa}_\beta - \kappa_{\bar{r}})\tau'} - e^{(\kappa_{\bar{\beta}} - \kappa_{\bar{r}})\tau'}}{e^{(\kappa_{\bar{\beta}} - \hat{\kappa}_\beta)\tau_{\bar{r}}} - 1} < 1 \quad \textit{for} \quad 0 < \tau' < \tau_{\bar{r}} \cdot$$

The function $h(\tau')$ is equal to one for $\tau' = 0$ and to zero for $\tau' = \tau_{\bar{r}}$. Its derivative is

$$h'(\tau') = \frac{(\hat{\kappa}_\beta - \kappa_{\bar{r}}) e^{(\kappa_{\bar{\beta}} - \hat{\kappa}_\beta)\tau_{\bar{r}} + (\hat{\kappa}_\beta - \kappa_{\bar{r}})\tau'} - (\kappa_{\bar{\beta}} - \kappa_{\bar{r}}) e^{(\kappa_{\bar{\beta}} - \kappa_{\bar{r}})\tau'}}{e^{(\kappa_{\bar{\beta}} - \hat{\kappa}_\beta)\tau_{\bar{r}}} - 1}$$

and has the same sign as

$$h_1(\tau') \equiv \frac{(\hat{\kappa}_\beta - \kappa_{\bar{r}}) e^{(\kappa_{\bar{\beta}} - \hat{\kappa}_\beta)(\tau_{\bar{r}} - \tau')} - (\kappa_{\bar{\beta}} - \kappa_{\bar{r}})}{\kappa_{\bar{\beta}} - \hat{\kappa}_\beta} \cdot$$

If $\kappa_\beta \leq \kappa_{\bar{r}}$, then $\hat{\kappa}_\beta > \kappa_{\bar{r}}$. The function $h_1(\tau')$ is negative, as can be seen by writing it as

$$h_1(\tau') = (\hat{\kappa}_\beta - \kappa_{\bar{r}}) \frac{e^{(\kappa_{\bar{\beta}} - \hat{\kappa}_\beta)(\tau_{\bar{r}} - \tau')} - 1}{\kappa_{\bar{\beta}} - \hat{\kappa}_\beta} - 1 \,.$$

Suppose next that $\kappa_{\bar{\beta}} \leq \kappa_{\bar{r}}$. If $\hat{\kappa}_{\beta} \leq \kappa_{\bar{r}}$, then $h_1(\tau')$ is negative because of the previous argument. If $\hat{\kappa}_{\beta} > \kappa_{\bar{r}}$, then $h_1(\tau')$ is negative, as can be seen by writing it as

$$h_1(\tau') = \frac{\kappa_{\bar{\beta}} - \kappa_{\bar{r}} - (\hat{\kappa}_{\beta} - \kappa_{\bar{r}})e^{(\kappa_{\bar{\beta}} - \hat{\kappa}_{\beta})(\tau_{\bar{r}} - \tau')}}{\hat{\kappa}_{\beta} - \kappa_{\bar{\beta}}}.$$

Since $h_1(\tau')$ is negative, $h(\tau') < 1$ for $0 < \tau' < \tau_{\bar{r}}$, and hence equation (A.32) is satisfied.

## A.8 Proof of Proposition 6

Equation (19) implies that the function $A'_r(\tau)$ decays at rate $e^{-\min\{\kappa_r, \kappa_{\bar{r}}\}\tau}$ for large $\tau$. Equations (20) and (21) imply that the function $A'_{\bar{\beta}}(\tau)$ decays at rate $e^{-\min\{\kappa_r, \kappa_{\bar{r}}, \gamma_1, \gamma_2\}\tau}$ for large $\tau$. Therefore, the effect of $\bar{\beta}_t$ shocks on the instantaneous-forward-rate term structure decays with maturity at a slower rate than the effect of $\bar{r}_t$ shocks if

$$min\{\gamma_1, \gamma_2\} \leq min\{\kappa_r, \kappa_{\bar{r}}\} \qquad (A.34)$$

and at a strictly slower rate if (A.34) is strict. For $\sigma_{\bar{\beta}} = 0$, (27) implies that $(\gamma_1, \gamma_2) = (\hat{\kappa}_{\beta}, \kappa_{\bar{\beta}})$. The proposition follows from this observation, equation (A.34), and $\hat{\kappa}_{\beta} < \kappa_{\beta}$.

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# Bernanke's No-Arbitrage Argument Revisited: Can Open Market Operations in Real Assets Eliminate the Liquidity Trap?

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This paper looks back on the professional consensus about monetary policy at the zero bound prior to the 2008 crisis and proposes a calibrated model that provides one interpretation to explain why it was somewhat off base. The general consensus in the economics profession in the late 1990s, when Japan was experiencing difficulties due to deflation and the zero bound, was that increasing the money supply in one of a variety of ways was a simple and straightforward answer to stimulating aggregate demand.

One example of this point of view is from Kenneth Rogoff (1998), a leading international macroeconomist, in response to Krugman (1998), who launched the modern zero lower bound (ZLB) literature. One of Krugman's key predictions was that increasing the money supply at the ZLB was irrelevant as long as expectations of future money supply were fixed. Rogoff's comment on this summarizes well a commonly held view at the time: "No one should seriously believe that the BOJ [Bank of Japan] would face any significant technical problems in inflating if it puts its mind to the matter, liquidity trap or not. For example, one can feel quite confident that if the BOJ were to issue a 25 percent increase in the current supply and use it to buy back 4 percent of government nominal debt, inflationary expectations would rise."

This basic logic was later spelled out more explicitly in a general equilibrium model by Auerbach and Obstfeld (2005). Their argument

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was that purchasing government debt with money should plausibly lead people to expect a permanent increase in the money supply, in contrast to Krugman's assumption and much as suggested by Rogoff, due to the fact that a permanent increase in the money supply creates seignorage revenues, which reduces tax distortions. In this case, increasing the money supply *should* increase prices and output because people should have no reason to expect the money supply to be contracted to its original level once things normalize and the short-term interest rate is positive, as this would imply higher tax distortions.

Since Rogoff's prediction, the Bank of Japan has increased the monetary base not by 25 percent, but rather by about 550 percent. Furthermore, it has accumulated more than 30 percent of outstanding government debt, as well as several types of real assets, such as stocks, foreign exchange, and mortgage backed securities. A similar story can be told about many other central banks since 2008. Meanwhile, in Japan, government debt as a fraction of gross domestic product (GDP), at 80 percent in 1998, has almost tripled.

The point here is not to single out Kenneth Rogoff for a prediction that in retrospect seems off base as an empirical matter. Instead, it is to illustrate a broad consensus in the profession at the time, a consensus of which the quote from Rogoff is a particularly cogent summary. So as to not seem to be unfairly singling out any particular author, below we provide examples in which one of the authors of this article made statements that had a similar tenor to Rogoff's prediction.

Our suspicion is that the broad consensus at the time had its roots in the classic account of the Great Depression by Friedman and Schwartz (1963), in which the deflation from 1929 to 1933 was explained by a collapse in the money supply. The Great Inflation of the 1970s also appeared to support Friedman's famous dictum that "Inflation is always and everywhere a monetary phenomenon" (Friedman, 1970). It was natural, then, to assume that the same applied to Japan and that simply increasing the money supply would halt the deflation.

Another indication of the consensus of the time was Svensson's (2000) well-known proposal for a "foolproof way" out of a liquidity trap, which, in contrast to Rogoff's proposal, involved printing money to buy up foreign exchange, rather than government debt. The fact that this solution was claimed to be foolproof indicated the general sense among academic economists at the time, especially in the United States, that expansionary monetary policy at the ZLB was only a question of will, rather than posing any technical difficulties for the world's central banks.

To our mind, however, the most pertinent statement about the academic consensus at the turn of the century came up in a personal conversation with Ben Bernanke, then Chairman not of the U.S. Federal Reserve, but the Princeton economics department, and editor of the *American Economic Review*. When the liquidity trap was proposed as a Ph.D. dissertation topic, Bernanke replied, "I have to warn you. I do not believe in the liquidity trap." While the current understanding of the liquidity trap is that it reflects some bound on the short-term nominal interest rate (often referred to as zero, although recent experience suggests it may be somewhat negative), Bernanke was instead referring more broadly to the fact that he believed in the power of the central banks to do something to stimulate demand, in the tradition of Friedman and Schwartz, zero bound or not. This position seemed to have been very much in line with the thinking of Rogoff, Svensson, and Auerbach and Obstfeld, already cited.

In a speech given at the American Economics Association, Bernanke (2000) made a statement that would later become very widely known as he assumed the Chairmanship of the Federal Reserve. Some interpreted the speech as a roadmap for the Fed's subsequent policy actions:

> First, that—despite the apparent liquidity trap—monetary policymakers retain the power to increase nominal aggregate demand and the price level. In my view, one can make what amounts to an arbitrage argument—the most convincing type of argument in an economic context—-that it must be true. The monetary authorities can issue as much money as they like. Hence, if the price level were truly independent of money issuance, then the monetary authorities could use the money they create to acquire indefinite quantities of goods and assets. This is manifestly impossible in equilibrium. Therefore, money issuance *must* ultimately raise the price level, even if nominal interest rates are bounded at zero. This is an *elementary* argument (emphasis added).

In this paper, we revisit this elementary argument on the basis of one particular interpretation of Bernanke's logic. We use it to illuminate why the pre-2008 consensus about the power of monetary policy may have been a bit too optimistic about the ability of central banks to stimulate demand.[1] In making this case, we are not claiming

1. Paul Krugman has often quipped that he should take Svensson and Bernanke to Japan with him on an apology tour for having made it seem too easy at the time. See, for example, Krugman (2014).

that the central bank—or the government as a whole—is unable to stimulate demand at all. Rather, the point is that doing so may require considerably larger intervention than suggested by the precrisis consensus—for example, interventions of the size and scope of the radical regime change implemented by Franklin Delano Roosevelt in 1933. This radical regime change, which is discussed in detail in Eggertsson (2008), involved an explicit commitment to inflate the price level by about 30 percent to the pre-depression level, the abolishment of the gold standard, and a massive increase in government spending and budget deficits. As an indication of how radical it was at the time, the then-director of the budget, Arthur Lewis, declared "this is the end of Western Civilization" and resigned from his post.[2]

To frame the approach of the paper, we raise a basic question: what is an arbitrage opportunity? An arbitrage opportunity refers to a situation in which an agent can acquire profit without taking on any risk. Bernanke (2000) suggests that the liquidity trap can be eliminated as a logical possibility because its existence would imply that the government could generate infinite profits. For the argument to make sense—for example, in the context of a closed economy—one must have in mind an environment in which the government would care about profits and losses in the first place. At first blush, this does not seem obvious, as these profits would necessarily be at the expense of the country's citizens, whose welfare should be a primary concern of the government. Nevertheless, we believe that the proposition that the government cares about profits and losses is entirely reasonable, because the government needs to rely on costly and possibly distortionary taxation to pay for its expenditures. Hence, if there was truly an arbitrage opportunity for the government, any rational government would wish to take it in order to eliminate taxation costs/distortions altogether (not to mention if it could do so at the cost of foreigners via buying up foreign assets).

Framing the question in this way highlights the tight connection between Bernanke's no-arbitrage argument and Auerbach and Obstfeld (2005). As noted, their case for open market operations was made on the basis that open market operations in a liquidity trap *should* imply a permanent increase in the money supply that will last even once the zero bound is no longer binding. This was the most reasonable benchmark to them, since contracting the money supply back to its initial level would imply fiscal costs. Hence, a permanent

2. See references in Eggertsson (2008).

increase in the money supply made sense from the perspective of both macroeconomic stabilization ex ante and fiscal solvency ex post. They made their point explicit by numerically computing a comparative statics that showed the beneficial effect of permanently increasing the money supply (which they coined open market operations). This argument was made slightly differently in an earlier working paper by one of the authors (Eggertsson, 2003). That paper explicitly cites Bernanke's no-arbitrage argument as a motivation, using the same quotation as above. Eggertsson (2003) models Bernanke's argument as a violation of Ricardian equivalence, assuming that the government cannot collect lump-sum taxes but instead needs to pay tax collection costs as in Barro (1979). In this case, the government cares about profits and losses on its balance sheet, as it needs to make up for the losses through costly taxation. By analyzing a Markov perfect equilibrium policy game, which presumes that the government cannot make any credible commitment about future policy apart from paying back the nominal value of debt as in Lucas and Stokey (1983), Eggertsson (2003) formally shows that purchasing "real assets" by printing money (or equivalently bonds, since money and bonds are perfect substitutes at the ZLB) implies a *credible* permanent increase in the money supply in the long run due to the fact that the government has no incentive to revert the supply completely back to its original level on account of the fiscal consequences (leading to costly taxation). This, in turn, provides direct theoretical foundation for Bernanke's no-arbitrage argument to "eliminate" the liquidity trap.

The interpretation suggested in Eggertsson (2003) is that open market operations in real assets provides a straightforward commitment mechanism to lower future interest rates and higher inflation that mitigates the problem of the ZLB.[3] Indeed, the simulations reported in the paper suggest that open market purchases in real asset seem to allow the government to replicate quite closely the ideal state of affairs in which the government can fully commit to future policy, and the problem of the ZLB is trivial in terms of its effect on output and inflation.[4] In retrospect, however, this interpretation

3. In this respect, the intervention in Eggertsson (2003) is different from Auerbach and Obstfeld (2005) in that it increases total government liabilities (money plus bonds) and thus the overall inflation incentive of the government. Since money and bonds are perfect substitutes at the zero bound, it is not obvious that open market operations themselves have any effect on future government objectives.

4. Eggertsson and Woodford (2003) analyze the full commitment equilibrium in a standard New Keynesian model.

was perhaps a little premature. A careful examination of the numerical results illustrates a disturbing feature. The required intervention in real assets needed to generate this outcome in Eggertsson (2003) corresponds to about four times annual GDP. Moreover, the intervention is conducted under ideal circumstances whereby the assets bought have an unlimited supply, their relative returns are not affected by the intervention (but instead are equal to the market interest rate in equilibrium), and the world is deterministic so there are no risks associated with using real asset purchases as a commitment device.

More generally, however, if the government buys real assets corresponding to something like 400 percent of GDP, it seems exceedingly likely that all of these assumptions will be violated in one way or the other. First, an operation of this kind is likely to have a substantial distortionary effect on pricing, which is not modeled. Second, the government is likely to run into physical constraints such as running out of assets to buy. Third, as the scale of the operations increases and uncertainty is taken into account, the risk to the government's balance sheet may be deemed unacceptable, thus lessening the power of this commitment device. Finally, with an intervention of this scale, it is very likely that the central bank will hit some political constraints, due to either public concerns or concerns from trading partners if the assets in question are foreign. Indeed, all the considerations mentioned above have proved to be relevant constraints for banks conducting large asset purchases since 2008. Central banks have faced challenges in finding liquid enough markets to conduct the operation; they have faced strong political backlash for the scale of the operations (for example, because they are viewed as favoring the financial sector and the richest few); and in some cases both the government and the central bank have become exceedingly concerned over the central bank's balance sheet risks. These risks could put central bank independence in question, as they could imply that the treasury must infuse capital into the central bank to prevent unacceptably high levels of inflation, with the associated budgetary implications.[5]

5. Several recent papers evaluate the extent to which these risks have become material for current central banks post crisis (for example, Hall and Reis, 2015; Del Negro and Sims, 2015). Our overall reading of this literature is that these risks are not pertinent for a balance sheet of the size of the U.S. Federal Reserve today, although they would become relevant in some of the numerical examples we provide later in the paper given how extreme some of the numbers in question are.

In this paper, we revisit Bernanke's no-arbitrage argument in the prototypical New Keynesian dynamic stochastic general equilibrium (DSGE) model, in the tradition of Woodford (2003), using conventional calibration parameters. This is in contrast to Eggertsson (2003), who uses a simpler nonconventional modeling approach, which may raise scepticism of the numerical experiments conducted. Inside this model, we ask how large of an intervention in real assets the government needs to undertake to achieve the optimal allocation under discretion, assuming there is no cost of such interventions. As in Eggertsson (2003), we find that the numbers are very large: in our baseline simulation, the corresponding intervention is more than ten times GDP. This suggests that using the government's balance sheet as a commitment device may imply asset positions by the central bank that would be difficult to implement in practice. Thus, while we find that Bernanke's no-arbitrage argument can be correct in theory, it may run into constraints in practice. For this reason, following our baseline experiment, which is conducted in the ideal circumstances of an unlimited supply of the asset and at no cost for the government, we also consider cases in which the assets purchases are costly. In this case, the purchases can lose much of their commitment power.

How does this all relate to recent experience? On 31 October 2014, the Bank of Japan unexpectedly announced an expansion of its comprehensive monetary easing (CME) program from 50 trillion to 80 trillion yen per year. Along with a change in the size of its balance sheet, the announcement included a change in its composition. Beyond long-term government securities, the central bank would purchase additional riskier assets such as exchange traded funds and real estate investment trusts. The expressed goal of the expansion was to meet a 2 percent inflation target within two years. Governor Haruhiko Kuroda described the program as "monetary easing in an entirely new dimension," and in reference to limits in its size relative to GDP said, "We don't have any particular ceiling." As of August 2015, the size of the Bank of Japan's balance sheet stood at approximately 80 percent of GDP. While this seems like a large number, it is much smaller than what is needed according to our calibrated model. Would the Bank of Japan not hit *some* ceiling if it had to buy assets that are more than ten times the current size of its balance sheet? In any event, as of this writing, the Bank of Japan is still unable to hit its inflation target, and most projections paint a pessimistic picture of its prospect of hitting it anytime soon.

As another example, the Swiss National Bank bought foreign currency on the order of 90 percent of GDP in order to fight deflation during the crisis, leading to an 800 percent increase in its money supply. They eventually abandoned this policy since the magnitudes involved had become so high that the central bank faced strong political pressures to halt its purchases. The effect of this policy on the price level was negligible at best, although for a while the Swiss National Bank did manage to prevent an appreciation of the Swiss franc relative to the euro.

The bottom line, then, may be that the irrelevance result of Wallace (1981), which was later extended by Eggertsson and Woodford (2003) to a model with sticky prices and an explicit zero lower bound, may be stronger than the precrisis consensus suggested. Eggertsson and Woodford's (2003) irrelevance result, in turn, is closely related to Krugman's (1998) finding that increasing the money supply has no effect at the ZLB if people expect it to be contracted again to its original level once interest rates turn positive.[6] Those irrelevance results suggested that absent some restrictions in asset trade that prevent arbitrage, equilibrium quantities and assets prices are not affected by a change in the relative supplies of various assets owned by the private sector *if the central bank's policy rule is taken as given*. One way the irrelevance results have been broken in the literature is via changes in expectations about future monetary policy. The results here suggest that at least in a simple calibrated New Keynesian model that imposes a Markov perfect equilibrium as an equilibrium selection device, the asset position of the government needed to achieve the desired commitment, and thus break these irrelevance results, may be extremely high. To be clear—and this is worth reiterating—we do not contend that this implies that nothing can be done at the ZLB, nor even that nothing more could have been done in response to the current crisis. This is clearly illustrated by the impact of Roosevelt's radical reflation program, which coordinated monetary, fiscal, industrial, and exchange rate policy, during the Great Depression. However, it does imply that central bank actions to increase demand may be a bit harder than the precrisis consensus suggested, and the foolproof ways out of the liquidity trap are hard to come by. One policy that we do

6. The difference between Eggertsson and Woodford (2003) and Krugman (1998) is that while Krugman (1998) assumes that the central bank follows a monetary targeting rule, Eggertsson and Woodford (2003) assume a more conventional Taylor-type interest rate reaction function. Moreover, while Krugman (1998) assumes that the money supply is increased via purchases of short-term nominal bonds, Eggertsson and Woodford (2003) assume that the money supply can be increased via purchases of any type of security that is priced in the economy, as in Wallace (1981).

not consider here is to shorten the maturity structure of outstanding government debt. Bhattarai, Eggertsson, and Gafarov (2015) suggest that a policy of that kind may be more potent than the purchases of real assets studied here. Alternatively, if there is a freeze in secondary asset markets, for example, due to a drop in the liquidity of assets, there may also be an important role for asset purchases, as shown by del Negro and others (2016) in the context of the 2008 crisis. Our model abstracts from different degrees of asset liquidity, so this mechanism does not play a role here.

We outline the model in section 1 and summarize the conditions for a Markov perfect equilibrium (MPE) for a coordinated government in section 2. We present and discuss the calibrated model in section 3. With costly taxation and coordinated monetary and fiscal policy, deficit spending and real asset purchases both serve as an additional commitment device for solving the credibility problem created by a liquidity trap. They are effective because they act as an additional device through which a discretionary government can commit future governments to a higher money supply, and thus higher inflation and lower real interest rates. Section 4 presents a brief sensitivity analysis, and section 5 concludes.

## 1. The Model

We start by outlining a standard general equilibrium sticky-price closed-economy model with output cost of taxation, along the lines of Eggertsson (2006). We assume that monetary and fiscal policy are coordinated to maximize social welfare under discretion. The difference in the model from the literature is the introduction of a real asset in the government budget constraint.

### 1.1 Private Sector

A representative household maximizes expected discounted utility over the infinite horizon:

$$E_t \sum_{t=0}^{\infty} \beta^t \left[ u(C_t) + g(G_t) - v(h_t) \right] \xi_t, \tag{1}$$

where $\beta$ is the discount factor; $C_t$ is a Dixit-Stiglitz aggregate of consumption of each of a continuum of differentiated goods,

$$C_t = \left[ \int_0^1 c_t(i)^{\frac{\varepsilon}{\varepsilon-1}} di \right]^{\frac{\varepsilon-1}{\varepsilon}},$$

with elasticity of substitution equal to $\varepsilon > 1$; $G_t$ is a Dixit-Stiglitz aggregate of government consumption defined analogously; $h_t$ is labor supplied; $\xi_t$ is an exogenous shock; and $P_t$ is the Dixit-Stiglitz price index,

$$P_t = \left[ \int_0^1 p_t(i)^{(1-\varepsilon)} di \right]^{\frac{1}{1-\varepsilon}},$$

where $p_t(i)$ is the price of variety $i$. $E_t$ denotes the mathematical expectation conditional on information available in period $t$, $u(\cdot)$ is concave and strictly increasing in $C_t$, $g(\cdot)$ is concave and strictly increasing in $G_t$, and $v(\cdot)$ is increasing and convex in $h_t$.[7]

The household is subject to the following sequence of flow budget constraints:

$$P_t C_t + B_t + E_t \left\{ Q_{t,t+1} D_{t+1} \right\} \leq n_t h_t + \left(1 + i_{t-1}\right) B_{t-1} + D_t - P_t T_t + \int_0^1 Z_t(i) di, \quad (2)$$

where $B_t$ is a one-period risk-free nominal government bond with nominal interest rate $i_t$, $n_t$ is the nominal wage, $Z_t(i)$ is nominal profit of firm $i$, $T_t$ is government taxes, $D_{t+1}$ is the value of the complete set of state-contingent securities at the beginning of period $t + 1$, and $Q_{t,t+1}$ is the stochastic discount factor.

On the firm side, there is a continuum of monopolistically competitive firms indexed by the variety, $i$, that they produce. Each firm has a production function that is linear in labor $y_t(i) = h_t(i)$ and, as in Rotemberg (1982), faces a cost of changing prices given by $d[p_t(i) / p_{t-1}(i)]$.[8] The demand function for variety $i$ is given by

$$\frac{y_t(i)}{Y_t} = \left( \frac{p_t(i)}{P_t} \right)^{-\varepsilon}, \quad (3)$$

7. We abstract from money by considering the cashless limit of Woodford (1998).

8. Our results are not sensitive to assuming instead the Calvo model of price setting so long as we do not assume large resource costs of price changes. See Eggertsson and Singh (2015) for a discussion.

where $Y_t$ is total demand for goods. The firm maximizes expected discounted profits,

$$E_t \sum_{t=0}^{\infty} Q_{t,t+s} Z_{t+s}(i), \tag{4}$$

where the period profits are given by

$$Z_t(i) = \left[ (1+s) Y_t p_t(i)^{1-\varepsilon} P_t^{\varepsilon} - n_t(i) Y_t p_t(i)^{-\varepsilon} P_t^{\varepsilon} - d\left( \frac{p_t(i)}{p_{t-1}(i)} \right) P_t \right].$$

We assume that the production subsidy, $s$, satisfies

$$\frac{\varepsilon - 1}{\varepsilon}(1+s) = 1$$

in order to eliminate steady-state production inefficiencies from monopolistic competition. The household's optimality conditions are given by

$$\frac{v_h(h_t)}{u_c(C_t)} = \frac{n_t}{P_t} \tag{5}$$

and

$$\frac{1}{1+i_t} = E_t \left[ \beta \frac{u_c(C_{t+1})\xi_{t+1}}{u_c(C_t)\xi_t} \Pi_{t+1}^{-1} \right], \tag{6}$$

where $\Pi_t = (P_t / P_{t-1})$ is gross inflation. The firm's optimality condition from price setting is given by

$$\varepsilon Y_t \left[ \frac{\varepsilon - 1}{\varepsilon}(1+s) u_c(C_t, \xi_t) - v_y(Y_t, \xi_t) \right] + u_c(C_t, \xi_t) d'(\Pi_t)\Pi_t, \tag{7}$$
$$= E_t \left[ \beta u_c(C_{t+1}, \xi_{t+1}) d'(\Pi_{t+1}) \Pi_{t+1} \right]$$

where we have replaced $v_h$ with $v_y$ since we focus on a symmetric equilibrium where all firms charge the same price and produce the same amount.

## 1.2 Government

We assume that there is an output cost of taxation $s(T_t)$ as in Barro (1979).[9] Real government spending is then given by

$$F_t = G_t + s(T_t).$$

The government can issue one-period nominal bonds $B_t$ and purchase a real asset $A_t$ with rate of return $q_t$, which we assume satisfies the Fisher no-arbitrage condition in equilibrium. Furthermore, we assume that the government does not internalize the rate of return when optimizing social welfare. That is, the government takes the rate of return on the asset as given when making its policy decision. The consolidated flow budget constraint can be written as

$$B_t + P_t A_t = \left(1 + i_{t-1}\right) B_{t-1} + \left(1 + q_{t-1}\right) P_t A_{t-1} + \psi(A_t) + P_t\left(F_t - T_t\right),$$

where $\psi(A_t)$ is a quadratic cost of asset management. We introduce this quadratic cost as a reduced-form way to capture two phenomena. First, it captures the fact that managing large amounts of assets will involve some administration cost. Second, it is a way to model the relationship that as the scale of the asset purchases increases, the real return of the asset decreases, as this function reflects a loss of real resources. As noted in the introduction, a key conclusion from the numerical experiment we report shortly is that the central bank's intervention is "unreasonably" large. One interesting thought experiment we consider below is to set this cost high enough so as to rationalize the scale of the balance sheet expansion in some central banks observed post crisis. We can then ask if the intervention has a substantial effect in this case.

Next, we define the real value of government debt, inclusive of interest payments to be paid next period, as $b_t = (1 + i_t)(B_t / P_t)$ and the value of the real asset inclusive of returns as $a_t = (1 + q_t) A_t$. We can then write the budget constraint in real terms as

$$\frac{b_t}{1+i_t} + \frac{a_t}{1+q_t} = b_{t-1}\Pi_t^{-1} + a_{t-1} + \psi\left(\frac{a_t}{1+q_t}\right) + \left(F_t - T_t\right). \quad (8)$$

9. The function $s(T)$ is assumed to be twice differentiable with derivatives $s'(T) > 0$ and $s''(T) > 0$.

We define fiscal policy as the choice of $T_t, F_t, b_t$, and $a_t$. For simplicity, we abstract from variations in real government spending, so $F_t = F$ in all that follows. Conventional monetary policy is the choice of the nominal interest rate, $i_t$, which is subject to the zero-bound constraint

$$i_t \geq 0. \tag{9}$$

### 1.3 Private Sector Equilibrium

The goods market clearing condition implies the overall resource constraint

$$Y_t = C_t + F_t + d(\Pi_t) + \psi\left(\frac{a_t}{1+q_t}\right). \tag{10}$$

We define the private sector equilibrium as a collection of stochastic process,

$$\{Y_{t+s}, C_{t+s}, b_{t+s}, a_{t+s}, \Pi_{t+s}, i_{t+s}, T_{t+s}\},$$

for $s \geq 0$ that satisfy equations (5) through (10) for each $s \geq 0$, given $a_{t-1}, b_{t-1}$, and an exogenous stochastic process for $\{\xi_{t+s}\}$. Policy must now be specified to determine the set of possible equilibria in the model.

## 2. Markov-Perfect Equilibrium

We assume that the government policy is implemented under discretion so that the government cannot commit to future policy. To do so, we solve for a Markov perfect equilibrium.[10] However, we also assume that the government is able to commit to paying back the nominal value of its debt as in Lucas and Stokey (1983). The only way the government can influence future governments, then, is through the endogenous state variables that enter the private sector equilibrium conditions.

Define the expectation variables $f_t^E$ and $g_t^E$. The necessary and sufficient condition for a private sector equilibrium is now as twofold. First, the variables $\{Y_t, C_t, b_t, a_t, \Pi_t, i_t, T_t\}$ satisfy the following conditions:

10. See Maskin and Tirole (2001) for a formal definition of the Markov-perfect Equilibrium.

$$\frac{b_t}{1+i_t}+\frac{a_t}{1+q_t}=b_{t-1}\Pi_t^{-1}+a_{t-1}+\psi\left(\frac{a_t}{1+q_t}\right)+\left(F-T_t\right), \tag{11}$$

$$1+i_t=\frac{u_C(C_t,\xi_t)}{\beta f_t^E}, \quad i_t\geq 0, \tag{12}$$

$$\beta g_t^E=\varepsilon Y_t\left[\frac{\varepsilon-1}{\varepsilon}\left(1+s\right)u_c(C_t,\xi_t)-v_y(Y_t,\xi_t)\right]+u_c(C_t,\xi_t)d'(\Pi_t)\Pi_t, \tag{13}$$

and

$$Y_t=C_t+F_t+d(\Pi_t)+\psi\left(\frac{a_t}{1+q_t}\right), \tag{14}$$

given $b_{t-1}$, $a_{t-1}$ and $f_t^E$, and $g_t^E$. Second, expectations are rational, so that

$$f_t^E=E_t\left[u_c(C_{t+1},\xi_{t+1})\Pi_{t+1}^{-1}\right] \tag{15}$$

and

$$g_t^E=E_t\left[u_c(C_{t+1},\xi_{t+1})d'(\Pi_{t+1})\Pi_{t+1}\right]. \tag{16}$$

Since the government cannot commit to future policy apart from its choice of the endogenous state variables $a_{t-1}$ and $b_{t-1}$, the expectations $f_t^E$ and $g_t^E$ are only a function of $a_t$, $b_t$, and $\xi_t$. That is, the expectation functions are defined as

$$f_t^E=\bar{f}^E(a_t,b_t,\xi_t) \tag{17}$$

and

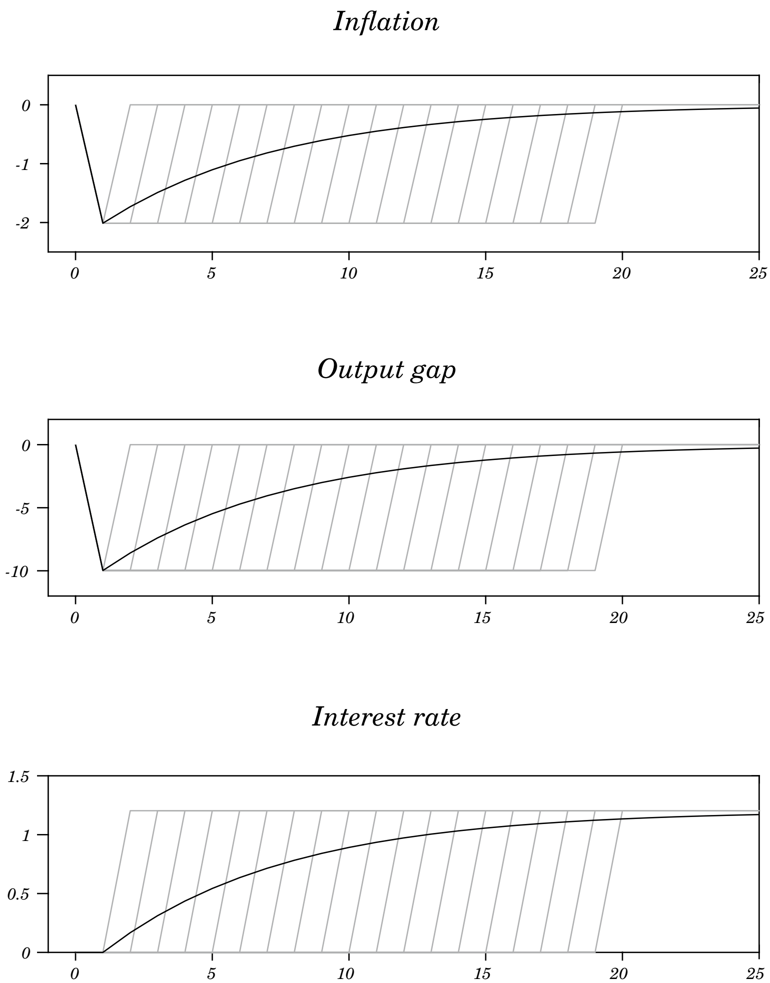
$$g_t^E=\bar{g}^E(a_t,b_t,\xi_t), \tag{18}$$

and we assume that these functions are continuous and differentiable. The discretionary government's dynamic programming problem is

$$V(a_{t-1},b_{t-1},\xi_t)=\max\left[U(\cdot)+\beta E_t V(a_t,b_t,\xi_{t+1})\right], \tag{19}$$

subject to the private sector equilibrium conditions (equations 11–14) and the expectation functions (equations 17–18), which in equilibrium satisfy the rational expectations restrictions (equations 15–16). The period Lagrangian and first-order conditions for this maximization problem are outlined in the appendix, along with their linear approximations.[11] A Markov perfect equilibrium can now be defined as a private sector equilibrium that is a solution to the government problem defined by equation (19).

**Figure 1. Inflation, the Output Gap, and the Short-term Nominal Interest Rate under Discretion when the Government's only Policy Instrument is Open Market Operations**




11. We assume that the government and private sector move simultaneously.

## 3. Results

Following Eggertsson (2006), we model a benchmark deflation scenario as a credibility problem. In particular, we assume that the following three conditions are satisfied: the government's only policy instrument is the short-term nominal interest rate; the economy is subject to a large negative demand shock given by the preference shock $\xi_t$; and the government cannot commit to future policy. We calibrate this benchmark with parameter values from Eggertsson and Singh (2015) that match a 10 percent drop in output and 2 percent drop in inflation.[12]

### 3.1 Optimal Monetary Policy under Commitment

As shown in Eggertsson and Woodford (2003), to increase inflation expectations in a liquidity trap, the central bank commits to keeping the nominal interest rate at zero after the natural interest rate becomes positive again. The consequence of the anticipation of this policy is that the benchmark deflation and large output gap scenario are largely avoided. For the particular calibration that we work with here, deflation and the output gap in the first period of the trap are –0.65 percent and –5.42 percent, respectively. Figure 3 makes this comparison clear.

With the benchmark deflation scenario and optimal monetary commitment in hand, we are now set to conduct numerical experiments to measure how discretionary fiscal policy with real asset purchases and/or deficit spending compare to the worst and best case scenarios, that is, limited discretion and full commitment.

### 3.2 Deficit Spending as an Additional Policy Instrument

To discuss optimal discretion under fiscal policy, we must first calibrate the cost of taxation. We do so by choosing the second derivative of the cost function, $s_1$, so that 5 percent of government spending goes to tax collection costs. With deficit spending as an additional policy instrument, the government can commit to future inflation and a low nominal interest rate by cutting taxes and issuing nominal debt.

12. They parameterize the model using Bayesian methods as in Denes and Eggertsson (2009) and Denes, Eggertsson, and Gilbukh (2013).

**Figure 2. Inflation, the Output Gap, and the Short-term Nominal Interest Rate under Commitment when the Government can Only Use Conventional Monetary Policy**

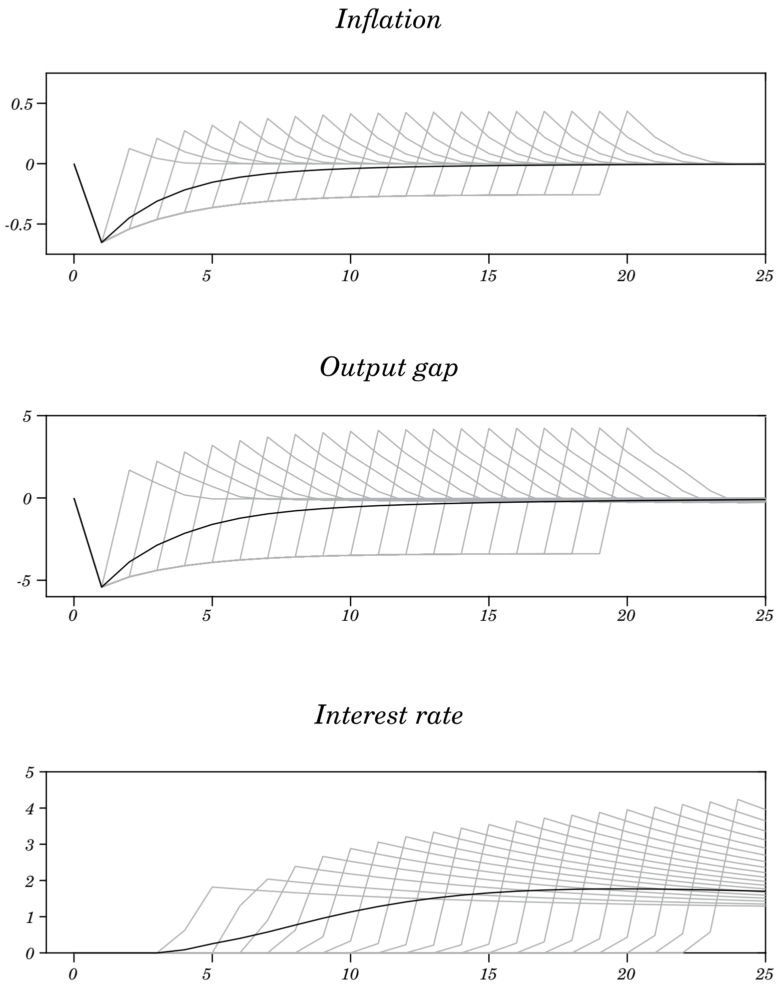


**Figure 3. Inflation, the Output Gap, and the Short-term Nominal Interest Rate under the Benchmark and Commitment when the Aggregate Demand Shock Lasts for 10 Periods**

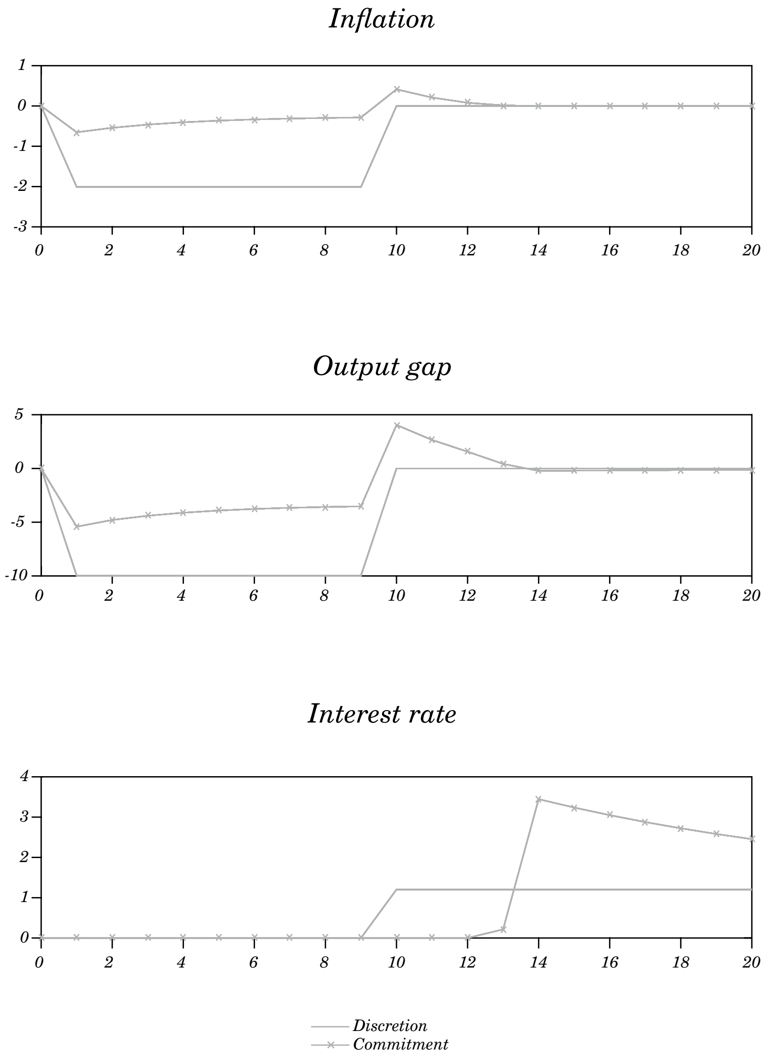


**Figure 4. Inflation, the Output Gap, and the Short-term Nominal Interest Rate under Discretion when the Government can use both Monetary and Fiscal Policies**

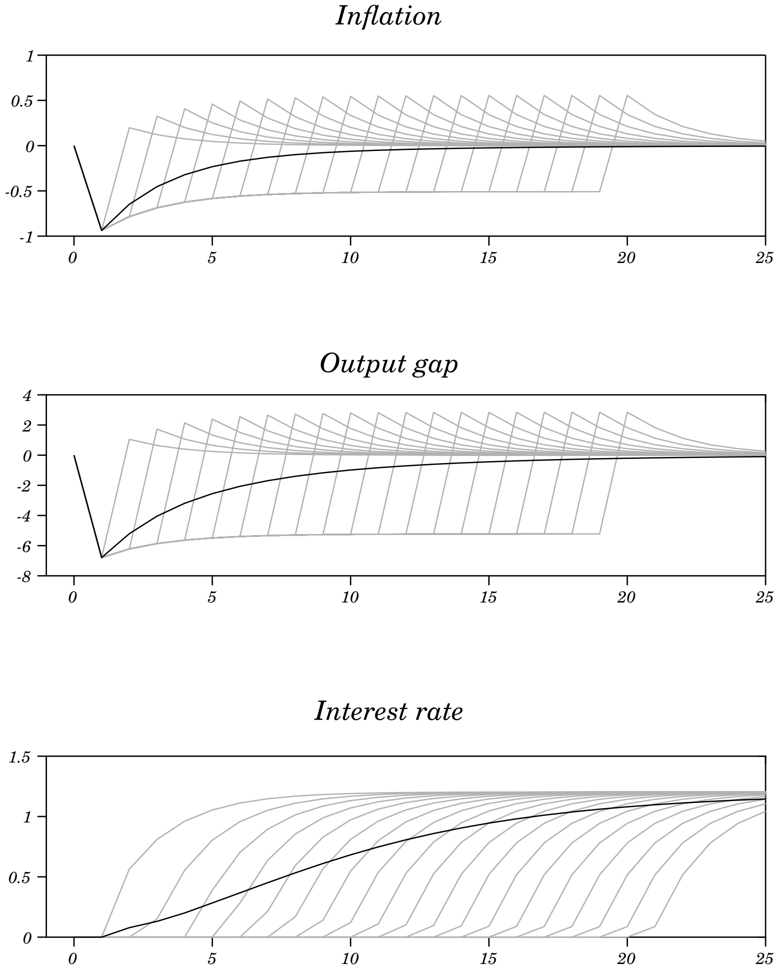
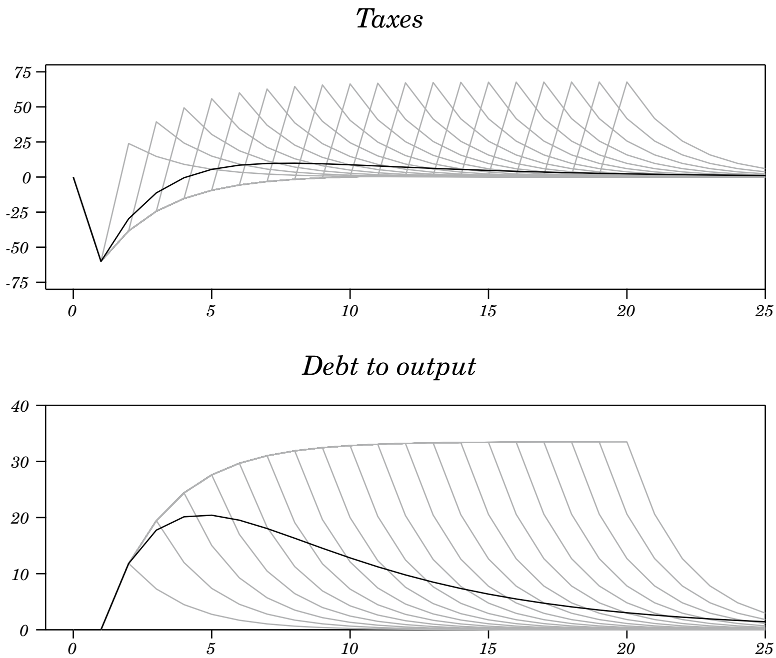


**Figure 5. Taxes and Debt under Discretion when the Government can use Both Monetary and Fiscal Policies**




Nominal debt commits the government to inflation even if it is discretionary because it creates an incentive for the government to reduce the real value of its debt and future interest payments. Since both inflation and taxes are costly, the government will choose a combination of the two in order to achieve this goal. Figures 4 and 5 summarize this result of Eggertsson (2006) for our parameterization.

The intuition is straightforward. Even with the inability to commit, the government can stimulate aggregate demand in a liquidity trap by increasing inflation expectations. To increase inflation expectations, the government can coordinate monetary and fiscal policies in order to run budget deficits. Budget deficits increase nominal debt, which in turn make a higher inflation target credible. Finally, increased inflation expectations lower the real interest rate and thus stimulate aggregate demand.

**Figure 6. Inflation, the Output Gap, and the Short-term Nominal Interest Rate under the Benchmark Discretion, Monetary Commitment, and Fiscal Discretion when the Aggregate Demand Shock Lasts for 10 Periods**

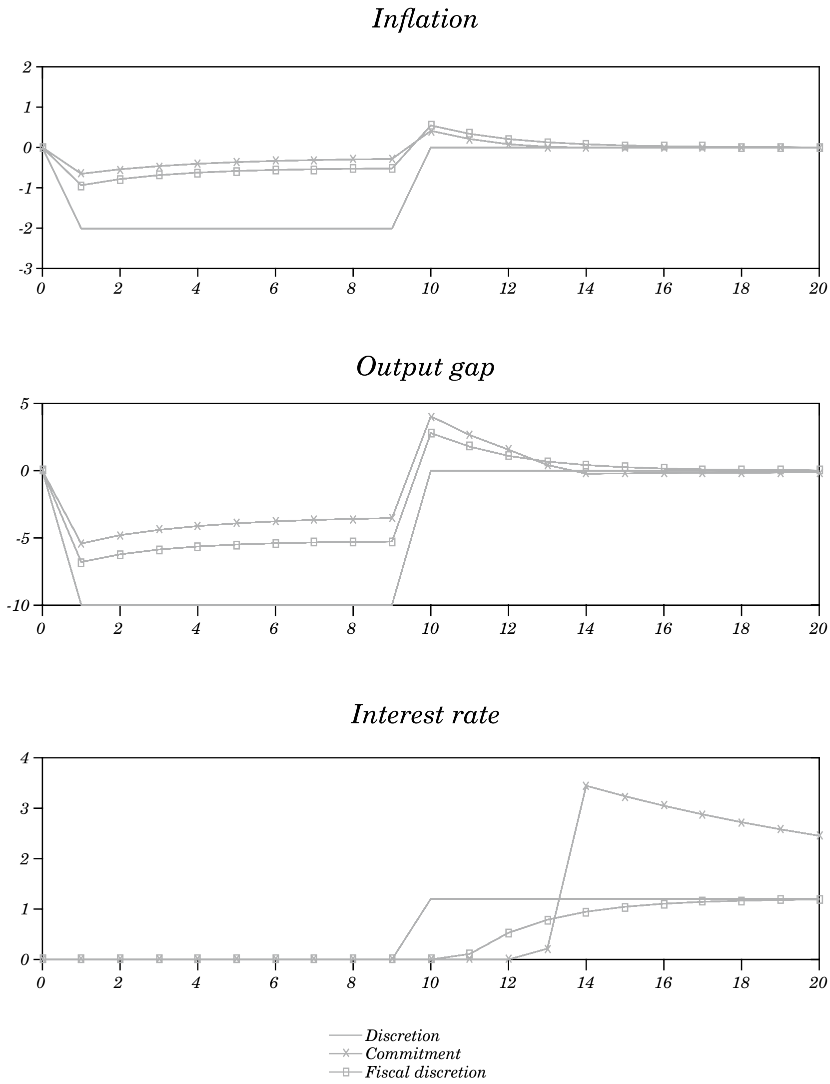


**Figure 7. Taxes and Debt to Output under Fiscal Discretion when the Aggregate Demand Shock Lasts for 10 Periods**

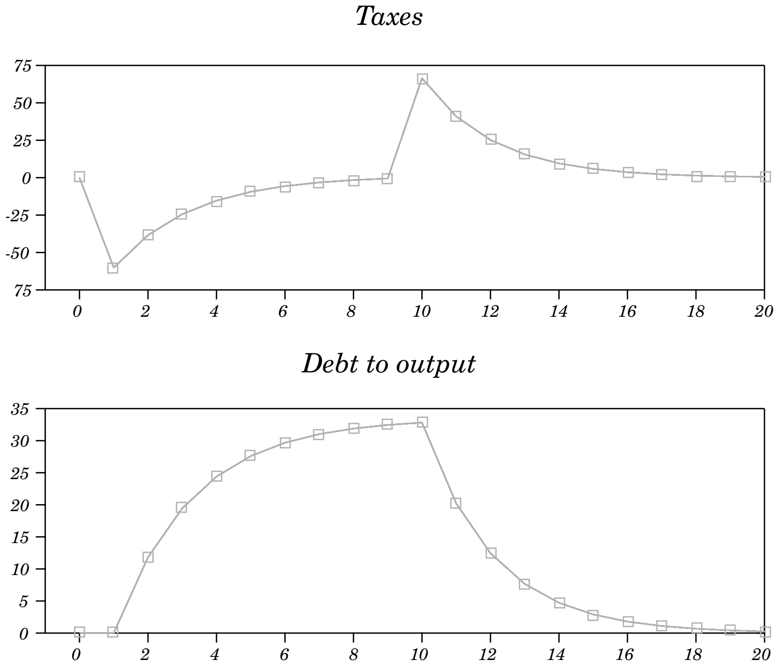



Figure 6 makes the comparison between the benchmark scenario, optimal monetary commitment, and discretionary fiscal policy. In the first period following the shock, the inflation rate and the output gap are –0.93 percent and –6.79 percent under fiscal discretion, quite close to their levels under optimal monetary commitment. Lastly, figure 7 shows taxes and the evolution of debt to output when the shocks lasts for ten periods. Taxes deviate by 60 percent from steady state, while debt peaks at approximately 35 percent of output.

## 3.3 Real Asset Purchases and Deficit Spending

We now turn to how the optimal policy under discretion changes when real asset purchases are used as an additional policy instrument. Figures 8 and 9 show that when asset management is costless and the output cost of taxation is calibrated to 5 percent of government spending, the optimal amount of real asset purchases exceeds 2,000 percent of gross domestic product in all contingencies. Although there is a strong inflation incentive and corresponding output boom due to the large increase in nominal debt, the required amount of asset purchases to obtain this response would clearly be infeasible in practice.[13]

Perhaps a more interesting question, therefore, is what the model predicts for inflation and the output gap if we calibrate the asset management cost so that the optimal amount of real asset purchases is 80 percent of gross domestic product in the first period of the recession. We pick this number as a reference point, as it corresponds approximately to the scale of the Swiss National Bank's foreign exchange intervention before it abandoned its peg. Figures 10 and 11 show that when we perform this thought experiment, the effectiveness of real asset purchases is much more limited. In fact, inflation and the output gap are only reduced to –1.36 percent and –8.23 percent, respectively, which is worse than the case with deficit spending as the only policy instrument. Moreover, as the cost of asset management gets very large, asset purchases approach zero, and we converge to the solution under fiscal discretion.[14]

There are two main takeaways from our results: first, although costless real asset purchases perform the best at reducing inflation and the output gap, the required balance sheet size under this scenario is far too large to be feasible in practice; second, for realistic levels of asset purchases, a combination of deficit spending and asset purchases does not perform much better than the worst-case scenario in the numerical example above. These two points taken together suggest that a combination of fiscal stimulus and central bank balance sheet policies with more weight on the fiscal stimulus may be the most practical. We have abstracted from the ability of the government to increase real government spending in the example above, but the existing literature suggests that this is another way in which the discretionary outcome can be improved.

13. Technically, there still is a negligibly small cost of asset management in this exercise, with $\psi = 1 \times 10^{-7}$. This is the smallest level of $\psi$ that induces stationarity in the equilibrium dynamics. See Schmitt-Grohé and Uribe (2003) for an example of this in closing small open economy models.

14. This numerical result indicates a nonlinearity that is somewhat interesting, in that a discretionary government with intermediate costs of administrating the real assets is better off without the ability to intervene in real assets than with it, as it limits its ability to commit to future inflation. One possible way of getting around this issue, which we do not pursue here, is to impose the constraint that the government cannot have negative asset holdings, in which case the government may still be able to commit to inflation in the intermediate asset management cost range. The key point, however, is that in this case commitment arises due to fiscal commitment as opposed to asset purchases.

**Figure 8. Inflation, the Output Gap, and the Short-term Nominal Interest Rate under Discretion when the Government can Costlessly Conduct Deficit Spending and Open Market Operations in Real Assets**

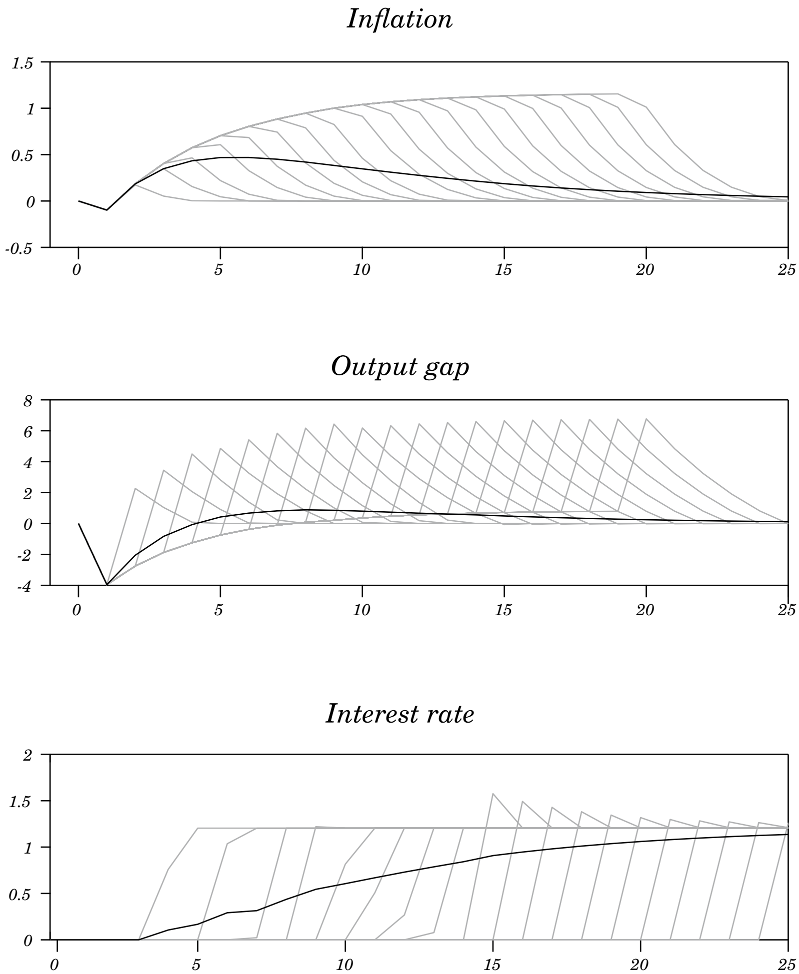


**Figure 9. Taxes, Debt to Output, and Asset Purchases under Discretion when the Government can Costlessly Conduct Deficit Spending and Open Market Operations in Real Assets**

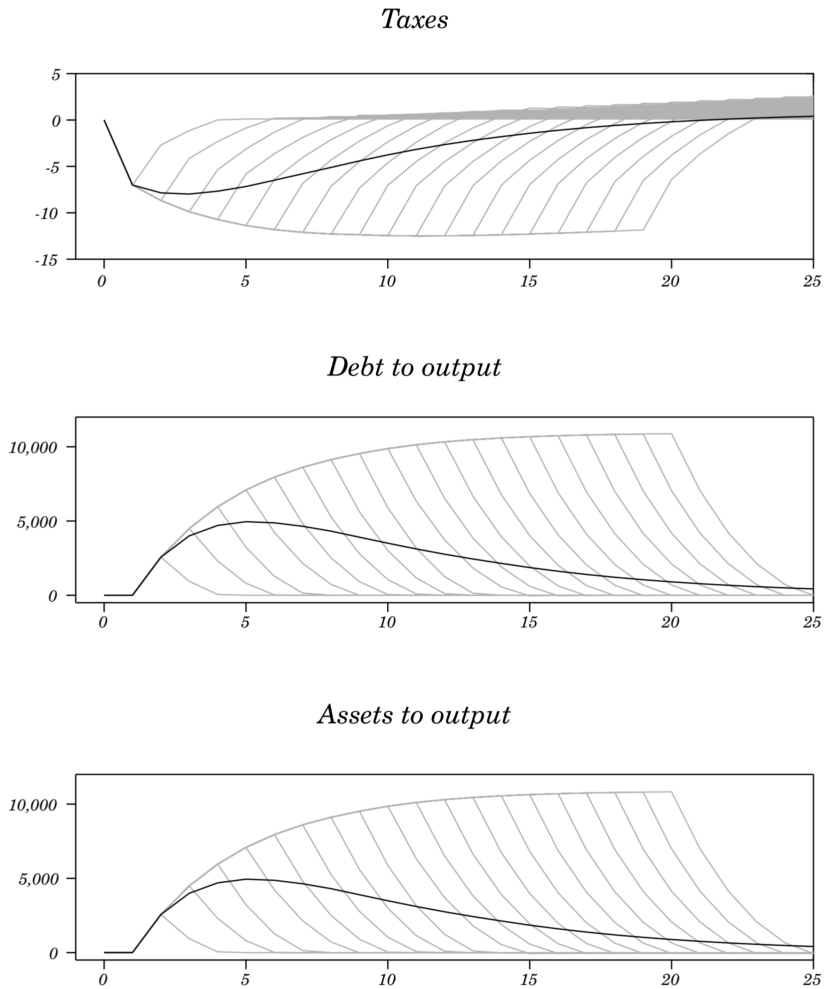


**Figure 10. Inflation, the Output Gap, and the Short-term Nominal Interest Rate under Discretion when the Government can Conduct Deficit Spending and Open Market Operations in Real Assets, with the Cost of Asset Purchases Calibrated to Match Real Asset Purchases of 80% of GDP**

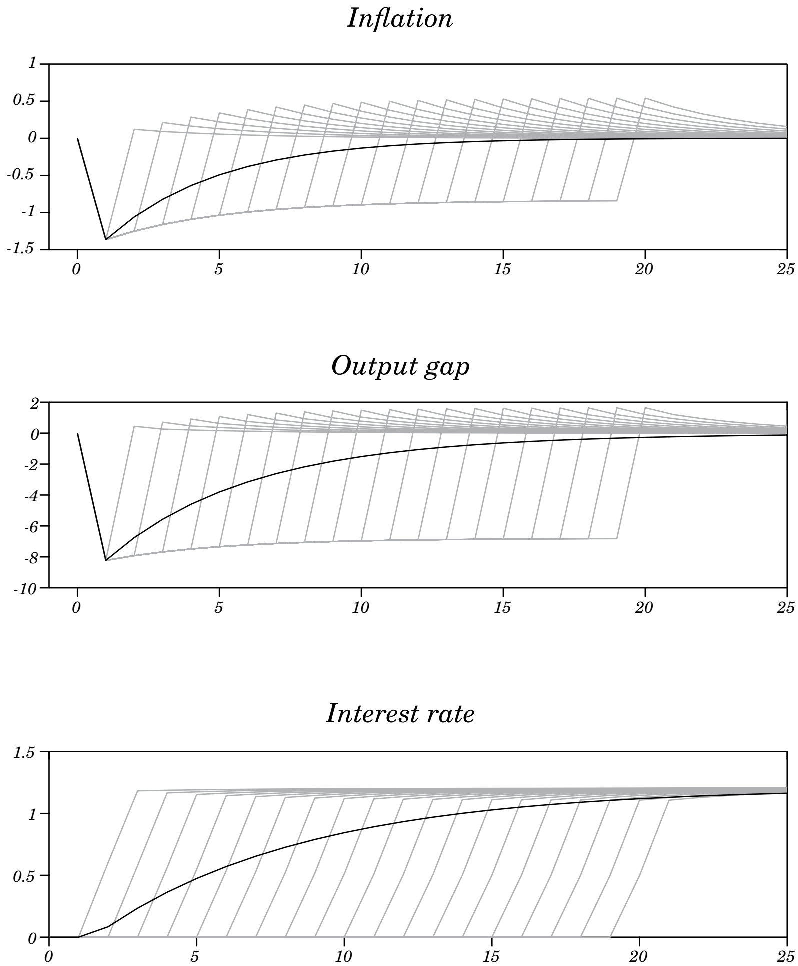


**Figure 11. Taxes, Debt to Output, and Asset Purchases under Discretion when the Government can Conduct Deficit Spending and Open Market Operations in Real Assets, with the Cost of Asset Purchases Calibrated to Match Real Asset Purchases of 80% of GDP**

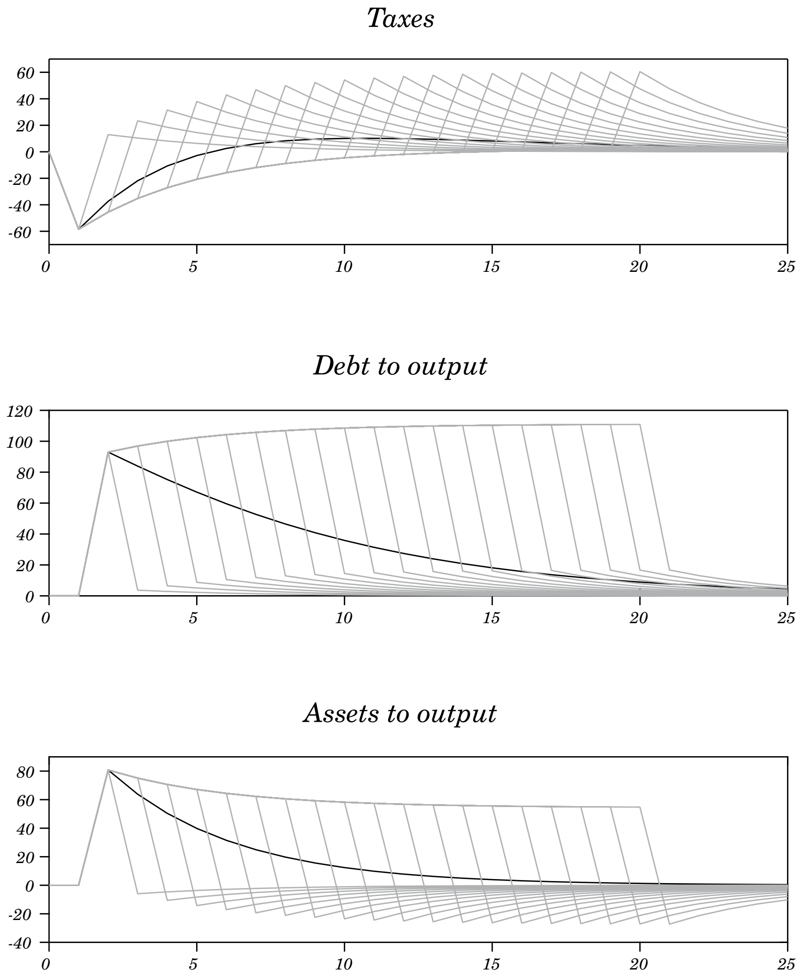


**Figure 12. Inflation, the Output Gap, and the Short-term Nominal Interest Rate under the Benchmark Discretion, Monetary Commitment, Fiscal Discretion, and Real Asset Purchases when the Aggregate Demand Shock Lasts for 10 Periods**

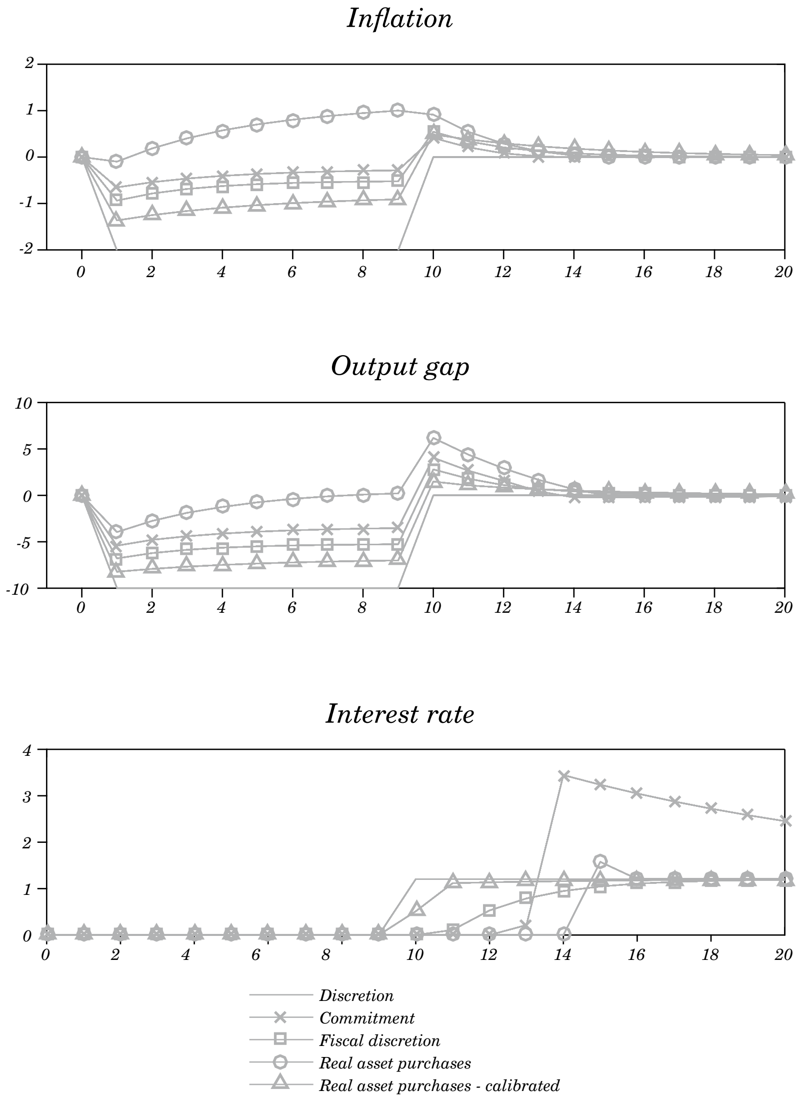
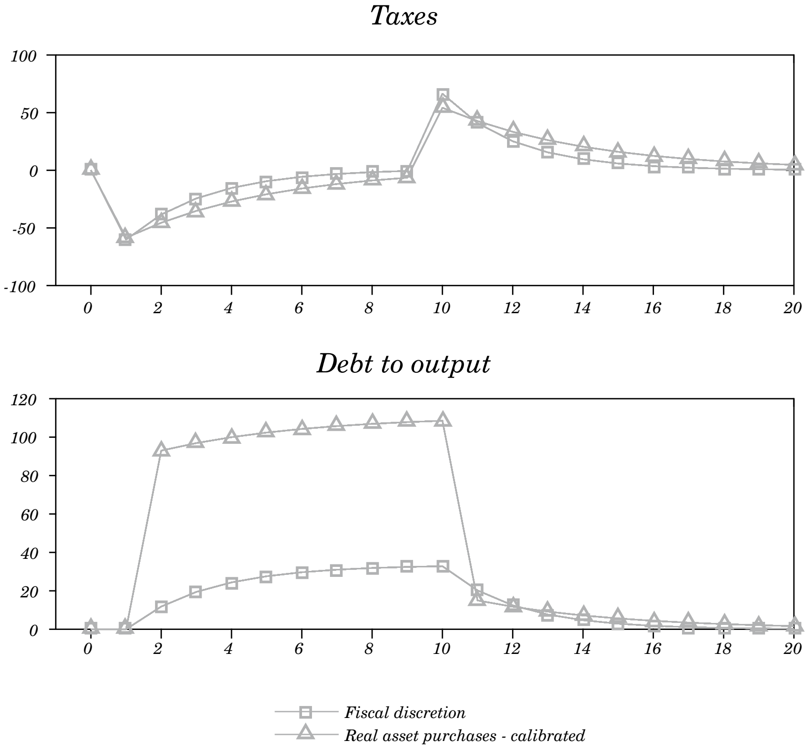


**Figure 13. Taxes and Debt to Output under Fiscal Discretion and Calibrated Real Asset Purchases when the Aggregate Demand Shock Lasts for 10 Periods**




Finally, figures 12 and 13 makes a more precise comparison between all of the policy scenarios that we have considered above (that is, benchmark discretion, commitment, fiscal discretion, costless real asset purchases, and real asset purchases calibrated to match 80 percent of gross domestic product). This confirms that as the cost of asset management gets sufficiently high, the solution converges to the case in which the government only uses deficit spending.

## 4. Sensitivity Analysis

Table 1 shows the sensitivity of our results to the size of taxation costs. The main takeaway is that for any reasonable value of the taxation cost, very large increases in real purchases are needed under full discretion, suggesting a limitation to this policy once more realistic constraints are added.

**Table 1. Varying the Cost of Taxation as a Percentage of Government Spending**

| | *Fiscal discretion* | | | *Discretion with real assets* | | |
|---|---|---|---|---|---|---|
| Taxation cost (%) | $\pi$ (%) | $y$ (%) | $b/gdp$ (%) | $\pi$ (%) | $y$ (%) | $b/gdp$ (%) |
| 0.25 | -1.61 | -9 | 51.8 | -1.78 | -9.41 | 953 |
| 0.5 | -1.47 | -8.58 | 39.44 | -1.85 | -9.46 | 2,414.63 |
| 1 | -1.31 | -8.1 | 28.76 | -1.87 | -9.41 | 5,790.70 |
| 2.5 | -1.09 | -7.37 | 17.75 | -0.99 | -6.21 | 13,074.97 |
| 5 | -0.94 | -6.79 | 11.81 | -0.1 | -3.95 | 2,562.44 |
| 7.5 | -0.87 | -6.52 | 9.33 | -0.05 | -3.86 | 1,390.96 |
| 10 | -0.83 | -6.35 | 7.82 | -0.03 | -3.85 | 979.5 |
| 15 | -0.78 | -6.13 | 6.02 | -0.02 | -3.84 | 609.61 |
| 20 | -0.75 | -6.01 | 4.93 | -0.02 | -3.83 | 461.96 |

## 5. Conclusion

This paper takes Bernanke's no-arbitrage argument to its logical limit and finds that it implies implausibly large asset purchases in a Markov perfect equilibrium. One interpretation of this finding is that open market operations in real assets alone is not sufficient in a liquidity trap, so instead, fiscal policy may be used in one form or another to support a reflation at the zero bound. A key abstraction is that the monetary and fiscal policy objective here corresponds to the utility of the representative household. It may seem more reasonable that the central bank has objectives that are different from social welfare—for example, that it cares greatly about its own balance sheet losses, independently of tax distortions. If one takes that perspective, however, there is no guarantee that real asset purchases provide the magic bullet to escape a liquidity trap, for reasons first articulated by Paul Samuelson in the context of the Great Depression. He argues that during the Great Depression the Fed was a "prisoner of its own independence" and paralyzed from taking any action for fear that they may imply balance sheet losses.[15]

An alternative explanation for the relative ineffectiveness of monetary policy post-2008 in guaranteeing inflation at or above target is that central banks never explicitly committed to an inflationary policy. While one reason central banks refrained from doing so was the high perceived cost of inflation, another was that many of them thought a reflationary program by a central bank would not be credible. The precrisis consensus was that this objection was not relevant because the central bank had the ability to print an unlimited amount of money and buy whatever assets it wanted. The numerical experiments here suggest that governments may face some constraints in practice, due to the scale needed to generate that commitment.

We do not wish to interpret this as suggesting that monetary policy is impotent at the zero bound, however. Rather, central banks need to more explicitly inflate, and they may need some fiscal backing to achieve their objective. This could come from direct government spending, fiscal transfers, and debt accumulation, together with, or perhaps in addition to, some additional institutional reforms that coordinate monetary and fiscal policy. Exploring how this coordination may take place in practice is likely to be a fertile ground for future research (see for example, Turner 2015).

15. See Mayer (1993) p.6.

# APPENDIX

## A.1 Functional forms

We make the following functional form assumptions:

$$u(C,\xi) = \xi \bar{C}^{-\frac{1}{\sigma}} \frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}};$$

$$v(h(i),\xi) = \xi\lambda \frac{h(i)^{1+\varphi}}{1+\phi};$$

$$g(G,\xi) = \xi \bar{G}^{-\frac{1}{\sigma}} \frac{G^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}};$$

$$y(i) = h(i);$$

$$d(\Pi) = d(\Pi - 1)^2;$$

$$\psi(a) = \frac{\psi_1}{2} a^2.$$

The discount factor shock, $\xi$, equals one in steady state, and we scale hours such that $Y = 1$ in steady-state, too. This implies that

$$\tilde{v}(Y,\xi) = \frac{1}{1+\phi} \lambda \xi Y^{\frac{1+\varphi}{\kappa}}.$$

## A.2 Calibration

### Table A1. Model parameters

| *Parameter* | *Value* |
|---|---|
| $\alpha$ | 0.7871 |
| $\beta$ | 0.9970 |
| $\sigma$ | 1 / 1.29 |
| $\varepsilon$ | 13.6012 |
| $\phi$ | 1.7415 |
| $d''$ | 5776.7 |
| $\kappa$ | 0.0072 |
| $F$ | 0.30 |
| $T$ | 0.30 |
| $G$ | 0.25 |
| $s_1$ | 0.3333 |
| $\psi_1$ | 0.0000362 |

### Table A2. ZLB Experiment

| *Parameter* | *Value* |
|---|---|
| $r_l^e$ | −0.0136 |
| $\gamma$ | 0.1393 |

## A.3 Non-Linear Markov-Perfect Equilibrium

Formulate the Lagrangian:

$$
\begin{aligned}
L_t &= u(C_t,\xi_t) + g\left(F - s(T_t)\right) - \tilde{v}(Y_t) + \beta E_t V(a_t, b_t, \xi_{t+1}) \\
&+\phi_{1t}\left[\frac{b_t}{1+i_t} + \frac{a_t}{1+q_t} - b_{t-1}\pi_t^{-1} - a_{t-1} - \psi\left(\frac{a_t}{1+q_t}\right) - \left(F - T_t\right)\right] \\
&+\phi_{2t}\left(\beta f_t^E - \frac{u_c(C_t,\xi_t)}{1+i_t}\right) \\
&+\varphi_{3t}\left\{\beta g_t^E - \varepsilon Y_t\left[\frac{\varepsilon-1}{\varepsilon}\left(1+s\right)u_c(C_t,\xi_t) - \tilde{v}_y(Y_t,\xi_t)\right] - u_c(C_t,\xi_t)d'(\pi_t)\pi_t\right\}
\end{aligned}
$$

$$+\varphi_{4t}\left[Y_t - C_t - F - \psi\left(\frac{a_t}{1+q_t}\right) - d(\pi_t)\right]$$

$$+\eta_{1t}\left(f_t^E - \bar{f}^E(a_t, b_t, \xi_t)\right)$$

$$+\eta_{2t}\left(g_t^E - \bar{g}^E(a_t, b_t, \xi_t)\right)$$

$$+\gamma_{1t}\left(i_t - 0\right).$$

First-order conditions:

$$\pi_t : \phi_{1t}[b_{t-1}\pi_t^{-2}] - \phi_{3t}[u_c d''\pi_t + u_c d'] - \phi_{4t}d';$$

$$Y_t : -\tilde{v}_y - \phi_{3t}\left[\varepsilon\left(\frac{\varepsilon - 1}{\varepsilon}(1+s)u_c\right) - \varepsilon Y_t \tilde{v}_{yy} - \varepsilon\tilde{v}_y\right] - \phi_{4t};$$

$$i_t : -\phi_{1t}\left[b_t\left(1+i_t\right)^{-2}\right] + \phi_{2t}\left[u_c\left(1+i_t\right)^{-2}\right] + \gamma_{1t};$$

$$C_t : u_c - \phi_{2t}\left[u_{cc}\left(1+i_t\right)^{-1}\right] - \phi_{3t}\left[\varepsilon Y_t \frac{\varepsilon - 1}{\varepsilon}\left(1+s\right)u_{cc} + u_{cc}d'\pi_t\right] - \phi_{4t};$$

$$T_t : g_G\left(-s'(T_t)\right) + \phi_{1t};$$

$$a_t : \beta E_t V_a(a_t, b_t, \xi_{t+1}) + \phi_{1t}\left[\left(1+q_t\right)^{-1} - \psi'\left(\frac{a_t}{1+q_t}\right)\right];$$

$$-\phi_{4t}\psi'\left(\frac{a_t}{1+q_t}\right) - \eta_{1t}\bar{f}_a^E - \eta_{2t}\bar{g}_a^E$$

$$b_t : \beta E_t V_b(a_t, b_t, \xi_{t+1}) + \phi_{1t}\left[\left(1+i_t\right)^{-1}\right] - \eta_{1t}\bar{f}_b^E - \eta_{2t}\bar{g}_b^E;$$

$$f_t^E : \beta\phi_{2t} + \eta_{1t};$$

$$g_t^E : \beta\phi_{3t} + \eta_{2t}.$$

Complementary slackness condition:

$$\gamma_{1t} \le 0, \quad i_t \ge 0, \quad \gamma_{1t} i_t = 0.$$

Benveniste-Scheinkman conditions:

$$V_a(a_{t-1}, b_{t-1}, \xi_t) = -\phi_{1t};$$

$$V_b(a_{t-1}, b_{t-1}, \xi_t) = -\phi_{1t}\pi_t^{-1}.$$

### A.4 Steady State

We linearize around an inefficient steady state with positive output cost of taxation, so that

$$\phi_1 = g_G\left(s'(\bar{T})\right).$$

Although we linearize around an inefficient steady-state, to simplify we still assume an appropriate production subsidy, as well as no resource loss from price adjustments, which requires

$$d(\Pi) = 0$$

so that

$$Y = C + F.$$

This requires that we linearize around a zero-inflation steady state,

$$\Pi = 1,$$

which implies

$$d'(\Pi) = 0.$$

Furthermore, we assume that $\bar{a} = \bar{b} = 0$ in steady-state, so that from the first-order condition with respect to $\pi_t$

$$\bar{\varphi}_3 = 0.$$

We assume that the production subsidy satisfies

$$\frac{\varepsilon - 1}{\varepsilon}(1 + s) = 1,$$

so that

$$u_c = \tilde{v}_y.$$

Also, we linearize around a steady state with positive interest rates, so

$$1 + i = \frac{1}{\beta},$$

which implies

$$\bar{\gamma}_1 = 0,$$

and from the first-order condition for $i_i$,

$$\bar{\phi}_2 = 0.$$

Using $d'(\Pi) = 0$ and the first-order condition for $\pi_t$,

$$\bar{\phi}_4 = 0,$$

which implies from the first-order conditions for $Y_t$ and $C_t$

$$\bar{\phi}_5 = \tilde{\upsilon}_y = u_c.$$

The first-order conditions with respect to the expectation variables imply

$$\bar{\eta}_1 = \bar{\eta}_2 = 0,$$

so that we do not need to know the derivatives of the unknown functions.

## A.5 Linear Approximation

### *A.5.1 Private Sector Equilibrium Conditions*

We approximate the equilibrium conditions around an inefficient non-stochastic steady state with zero inflation, $1 + i = 1 + q = \beta^{-1}$, and $\bar{a} = \bar{b} = 0$. We also normalize steady-state output to $\bar{Y} = 1$.

Linearizing the resource constraint $Y_t = C_t + F + d(\Pi_t) + \psi\left(\frac{a_t}{1+q_t}\right)$ gives

$$\hat{Y}_t = \bar{C}\hat{C}_t, \tag{20}$$

where $\hat{C}_t = \frac{C_t - \bar{C}}{\bar{C}}$.

Linearizing the price-setting optimality condition gives

$$\bar{u}_c d''\pi_t + \varepsilon\bar{u}_{cc}\bar{C}\hat{C}_t - \varepsilon\bar{v}_{yy}\hat{Y}_t - \varepsilon\bar{v}_{y\xi}\hat{\xi}_t + \varepsilon\bar{u}_{c\xi}\hat{\xi}_t = \beta\bar{u}_c d''E_t\pi_{t+1},$$

which can be simplified by making use of the linearized resource constraint

$$\pi_t = \beta E_t\pi_{t+1} + \kappa\hat{Y}_t, \tag{21}$$

where $\kappa = \dfrac{\varepsilon\left(\varphi + \sigma^{-1}\right)}{d''}$.

Linearizing the Euler equation gives

$$\bar{u}_{cc}\bar{C}\hat{C}_t + \bar{u}_{c\xi}\hat{\xi}_t = \bar{u}_c\hat{i}_t + \bar{u}_{cc}\bar{C}E_t\hat{C}_{t+1} + \bar{u}_{c\xi}E_t\hat{\xi}_{t+1} - \bar{u}_c E_t\pi_{t+1},$$

which can be simplified by making use of the linearized resource constraint,

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma\left(\hat{i}_t - E_t\pi_{t+1} - \hat{r}_t^e\right), \tag{22}$$

where $\hat{r}_t^e = -\dfrac{\bar{u}_{c\xi}}{\bar{u}_c}\left[E_t\hat{\xi}_{t+1} - \hat{\xi}_t\right]$ and $\sigma = -\dfrac{\bar{u}_c}{\bar{u}_{cc}} = \tilde{\sigma}\bar{C}$.

Imposing the Fisher arbitrage relation as an equilibrium condition and linearizing gives

$$\hat{q}_t = \hat{i}_t - E_t\pi_{t+1}. \tag{23}$$

Linearizing the government budget constraint,

$$b_t + a_t = \beta^{-1}b_{t-1} + \beta^{-1}a_{t-1} - \beta^{-1}\bar{T}\hat{T}_t \tag{24}$$

where $\hat{T}_t = \dfrac{T_t - \bar{T}}{\bar{T}}$.

Lastly, linearizing the expectation functions gives

$$\hat{f}_t^E = -\sigma^{-1}E_t\hat{Y}_{t+1} + E_t\hat{\xi}_{t+1} - E_t\pi_{t+1}; \tag{25}$$

$$\hat{g}_t^E = d''E_t\pi_{t+1}. \tag{26}$$

*A.5.2 Markov-Perfect FOCs*

In steady state, all Lagrange multipliers besides $\bar{\phi}_1 = g_G s'$ and $\bar{\phi}_3 = \tilde{v}_y = u_c$ are equal to zero. Linearizing each FOC in the order given above and using appropriate functional form assumptions,

$$\pi_t : g_G s' b_{t-1} - d''\hat{\phi}_{3t} - d''\pi_t;$$

$$Y_t : \phi \hat{Y}_t - \varepsilon\phi\hat{\phi}_{3t} - \hat{\phi}_{4t};$$

$$i_t : -\beta^2 g_G s' b_t + \beta^2 \hat{\phi}_{2t} + \hat{\gamma}_{1t};$$

$$C_t : \hat{Y}_t - \sigma\hat{\xi}_t - \beta\hat{\phi}_{2t} - \varepsilon\hat{\phi}_{3t} + \sigma\hat{\phi}_{4t};$$

$$T_t : -\left( \frac{s'\bar{T}\bar{C}}{\bar{G}\sigma} + \frac{s''\bar{T}}{s'} \right)\hat{T}_t + \hat{\phi}_{1t};$$

$$a_t : -E_t\hat{\phi}_{1t+1} + \hat{\phi}_{1t} - \hat{q}_t - \psi\beta \frac{g_G s' + 1}{g_G s'} a_t + \frac{\bar{f}_a^E}{g_G s'}\hat{\phi}_{2t} + \frac{\bar{g}_a^E}{g_G s'}\hat{\phi}_{3t};$$

$$b_t : -E_t\hat{\phi}_{1t+1} + \hat{\phi}_{1t} - \hat{i}_t + E_t\pi_{t+1} + \frac{\bar{f}_b^E}{g_G s'}\hat{\phi}_{2t} + \frac{\bar{g}_b^E}{g_G s'}\hat{\phi}_{3t}.$$

Guess solutions for all variables at positive interest rates as a linear function of $a_{t-1}$, $b_{t-1}$, and $\hat{r}_t^e$. Expectations will take the form

$$\hat{f}_t^E = -\sigma^{-1}E_t\hat{Y}_{t+1} + E_t\hat{\xi}_{t+1} - E_t\pi_{t+1} = \bar{f}_a^E a_t + \bar{f}_b^E b_t + \bar{f}_r^E \hat{r}_t^e;$$

$$\hat{g}_t^E = d''E_t\pi_{t+1} = \bar{g}_a^E a_t + \bar{g}_b^E b_t + \bar{g}_r^E \hat{r}_t^e.$$

Under the assumptions about the shock process, $\hat{\xi}_t$, we have

$$E_t\hat{\xi}_{t+1} = (1-\gamma)\hat{\xi}_t$$

and

$$\hat{r}_t^e = \gamma\hat{\xi}_t,$$

where $\gamma$ is the probability of remaining at the *ZLB*. Note that when the *ZLB* no longer binds, $\hat{r}_t^e = 0$.

## A.6 Optimal Policy Commitment

Formulate the Lagrangian:

$$L_t = E_t \sum_{t=0}^{\infty} \beta^t \left\{ u(C_t, \xi_t) + g\left(F - s(T_t)\right) - \tilde{v}(Y_t) \right\}$$

$$+\phi_{1t}\left( \beta u_c(C_{t+1}, \xi_{t+1}) \pi_{t+1}^{-1} - \frac{u_c(C_t, \xi_t)}{1 + i_t} \right)$$

$$+\phi_{2t}\left( \beta u_c(C_{t+1}, \xi_{t+1}) d'(\pi_{t+1}) \pi_{t+1} - \varepsilon Y_t \left[ \frac{\varepsilon - 1}{\varepsilon} \left(1 + s\right) u_c(C_t, \xi_t) - \tilde{v}_y(Y_t, \xi_t) \right] \right.$$

$$\left. - u_c(C_t, \xi_t) d'(\pi_t) \pi_t \right)$$

$$+\phi_{3t}\left(Y_t - C_t - F - d(\pi_t)\right)$$

$$+\gamma_{1t}\left(i_t - 0\right).$$

First-order conditions:

$$\pi_t : -\phi_{1t-1}[u_c \pi_t^{-2}] - \phi_{2t}[u_c d'' \pi_t + u_c d'] - \phi_{2t-1}[u_c d'' \pi_t + u_c d'] - \phi_{3t} d';$$

$$Y_t : -\tilde{v}_y - \phi_{2t}\left[ \varepsilon \left( \frac{\varepsilon - 1}{\varepsilon} \left(1 + s\right) u_c \right) - \varepsilon Y_t \tilde{v}_{yy} - \varepsilon \tilde{v}_y \right] + \phi_{3t};$$

$$i_t : \phi_{1t}\left[ u_c \left(1 + i_t\right)^{-2} \right] + \gamma_{1t};$$

$$C_t : u_c - \phi_{1t}\left[ u_{cc} \left(1 + i_t\right)^{-1} \right] + \phi_{1t-1}[u_{cc} \pi_t]$$

$$- \phi_{2t}\left[ \varepsilon Y_t \frac{\varepsilon - 1}{\varepsilon} \left(1 + s\right) u_{cc} + u_{cc} d' \pi_t \right] + \phi_{2t-1}[u_{cc} d' \pi_t] - \phi_{3t}.$$

Complementary slackness condition:

$$\gamma_{1t} \le 0, \quad i_t \ge 0, \quad \gamma_{1t} i_t = 0.$$

## A.7 Linear Approximation

In steady state, all Lagrange multipliers besides $\bar{\phi}_3 = \tilde{v}_y = u_c$ are equal to zero. Linearizing each FOC in the order given above and using appropriate functional form assumptions,

$$\pi_t : -\hat{\phi}_{1t-1} - d'' \hat{\phi}_{2t} - d'' \hat{\phi}_{2t-1} - d'' \pi_t;$$

$$Y_t : \phi\hat{Y}_t - \varepsilon\phi\hat{\phi}_{2t} - \hat{\phi}_{3t};$$

$$i_t : \beta^2\hat{\phi}_{1t} + \hat{\gamma}_{1t};$$

$$C_t : \hat{Y}_t - \sigma\hat{\xi}_t - \beta\hat{\phi}_{1t} + \hat{\phi}_{1t-1} - \varepsilon\hat{\phi}_{2t} + \sigma\hat{\phi}_{3t}.$$

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# Measuring the Effects of Unconventional Monetary Policy on Asset Prices

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On 16 December 2008, the U.S. Federal Reserve's Federal Open Market Committee (FOMC) lowered the federal funds rate—its traditional monetary policy instrument—to essentially zero in response to the most severe U.S. financial crisis since the Great Depression. Because U.S. currency carries an interest rate of zero, it is essentially impossible for the FOMC to target a value for the federal funds rate that is substantially less than zero. Faced with this zero lower bound (ZLB) constraint, the FOMC subsequently began to pursue alternative, "unconventional" monetary policies, with particular emphasis on forward guidance and large-scale asset purchases (defined below). In this paper, I propose a new method to identify and estimate the effects of these two main types of unconventional monetary policy.

Understanding the effects of unconventional monetary policy is an important topic for both policymakers and researchers. Many central banks around the world have found themselves constrained by the zero lower bound on short-term nominal interest rates. Central banks faced with this constraint must pursue unconventional monetary policy if they wish to affect financial markets and/or the economy. Understanding the effects of different types of unconventional monetary policy, then, allows policymakers and researchers to better understand the efficacy, strengths, and weaknesses of the various alternatives.

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The effectiveness of unconventional monetary policy is also an important determinant of the costs of the zero lower bound constraint. If unconventional monetary policy is relatively ineffective, then the ZLB constraint is more costly, and policymakers should go to greater lengths to prevent hitting the ZLB in the first place—such as by choosing a higher target rate of inflation, as advocated by several authors.[1] On the other hand, if unconventional monetary policy is very effective, then the ZLB constraint is much less costly and policymakers do not need to take such drastic action to avoid hitting it in the future.

In the present paper, I focus on measuring the effects of forward guidance and large-scale asset purchases in particular, since those were the two types of unconventional monetary policy used most extensively by the Federal Reserve during the recent U.S. ZLB period. The term forward guidance refers to communication by the FOMC about the likely future path of the federal funds rate over the next several quarters or years. Large-scale asset purchases (or LSAPs) refers to purchases by the Federal Reserve of hundreds of billions of dollars' worth of longer-term assets, such as long-term U.S. Treasury securities and mortgage-backed securities. The goals of both policies was to lower longer-term U.S. interest rates using methods other than changes in the current federal funds rate. Both types of unconventional monetary policy were used extensively by the Federal Reserve, as can be seen in table 1. In addition to the major unconventional monetary policy announcements listed in table 1, there was incremental news about these policies that was released to financial markets at almost every FOMC meeting, such as updates that a policy was ongoing, was likely to be continued, or might be adjusted.

A major challenge in identifying and estimating the effects of the unconventional monetary policy announcements by the FOMC is determining the size and type of each announcement. For example, many of the statements in table 1 were at least partially anticipated by financial markets prior to their official release. Because financial markets are forward-looking, the anticipated component of each announcement should not have any effect on asset prices; only the unanticipated component should be news to financial markets and have an effect. But determining the size of the unexpected component

1. For example, Summers (1991); Blanchard, Dell'Ariccia, and Mauro (2010); Ball (2014). See also Blanchard, as quoted by Bob Davis, "Q&A: IMF's Blanchard Thinks the Unthinkable," *Wall Street Journal*, 11 February 2010, Real Time Economics blog.

of each announcement in table 1 is very difficult, because there are no good data on what financial markets expected the outcome of each FOMC announcement to be.[2]

**Table 1. Major Unconventional Monetary Policy Announcements by the Federal Reserve, 2009–2015**

| | |
|---|---|
| *March 18, 2009* | FOMC announces it expects to keep the federal funds rate between 0 and 25 basis points (bp) for "an extended period", and that it will purchase \$750B of mortgage-backed securities, \$300B of longer-term Treasuries, and \$100B of agency debt (a.k.a. "QE1") |
| *November 3, 2010* | FOMC announces it will purchase an additional \$600B of longer-term Treasuries (a.k.a. "QE2") |
| *August 9, 2011* | FOMC announces it expects to keep the federal funds rate between 0 and 25 bp "at least through mid-2013" |
| *September 21, 2011* | FOMC announces it will sell \$400B of short-term Treasuries and use the proceeds to buy \$400B of long-term Treasuries (a.k.a. "Operation Twist") |
| *January 25, 2012* | FOMC announces it expects to keep the federal funds rate between 0 and 25 bp "at least through late 2014" |
| *September 13, 2012* | FOMC announces it expects to keep the federal funds rate between 0 and 25 bp "at least through mid-2015", and that it will purchase \$40B of mortgage-backed securities per month for the indefinite future |
| *December 12, 2012* | FOMC announces it will purchase \$45B of longer-term Treasuries per month for the indefinite future, and that it expects to keep the federal funds rate between 0 and 25 bp at least as long as the unemployment rate remains above 6.5 percent and inflation expectations remain subdued |
| *December 18, 2013* | FOMC announces it will start to taper its purchases of longer-term Treasuries and mortgage-backed securities to paces of \$40B and \$35B per month, respectively |
| *December 17, 2014* | FOMC announces that "it can be patient in beginning to normalize the stance of monetary policy" |

2. In contrast, for conventional monetary policy—changes in the federal funds rate—federal funds futures and other short-term financial market instruments provide very good measures of market expectations leading up to each announcement. See Kuttner (2001) and Gürkaynak, Sack, and Swanson (2005, 2007).

A closely related issue is that the FOMC can sometimes surprise markets through its inaction rather than its actions. For example, on 18 September 2013, financial markets widely expected the FOMC to begin tapering its LSAPs, but the FOMC decided not to do so, surprising markets and leading to a large effect on asset prices despite the fact that no action was announced.[3] This implies that even dates not listed in table 1 could have produced a significant surprise in financial markets and led to large effects on asset prices and the economy.

Determining the type of any given announcement—forward guidance versus LSAP—can also be very difficult. For example, many announcements in table 1 clearly contain significant news about both types of policies, which makes disentangling the news on those dates challenging. Even in the case of a seemingly clear-cut announcement, both types of policies may be at work: in particular, several authors argue that LSAPs affect the economy by changing financial market expectations about the future path of the federal funds rate (for example, Woodford, 2012; Bauer and Rudebusch, 2014). To the extent that this channel is operative, even a pure LSAP announcement would have important forward guidance implications. This makes disentangling the two types of policies even more difficult than it might at first seem.

In this paper, I address these problems by adapting the methods of Gürkaynak, Sack, and Swanson (2005, henceforth GSS) to the zero lower bound period in the United States, from 2009 to 2015. The problem GSS faced was similar to the problem I face here, in that they were interested in separately identifying the effects of two dimensions of monetary policy: changes in the current federal funds rate versus changes in FOMC forward guidance. In the zero lower bound environment I consider here, there are also two dimensions of monetary policy: changes in forward guidance and LSAPs. Changes in the current federal funds rate are not a significant component of monetary policy during this period because of the zero lower bound constraint on the funds rate.

Following GSS, I look at how financial markets responded in a thirty-minute window bracketing each FOMC announcement between 2009 and 2015, and compute the first two principal components of those

3. For example, in an article entitled "No Taper Shocks Wall Street," the *Wall Street Journal* reported that "Bernanke had a free pass to begin that tapering process and chose not to follow [through]. . . The Fed had the market precisely where it needed to be. The delay today has the effect of raising the benchmark to tapering" (Steven Rusolillo, "No Taper Shocks Wall Street: Fed 'Running Scared'," *Wall Street Journal*, 18 September 2013, MoneyBeat).

asset price responses. The idea is that forward guidance and LSAPs were by far the two most important components of FOMC announcements for financial markets, so their effects should be well captured by the first two principal components of the asset price responses. I then search over all possible rotations of these two principal components to find the specification in which one of the two factors has the clearest interpretation as a forward guidance factor, using the estimated effect of forward guidance from the pre-ZLB period (computed exactly as in GSS) as the benchmark for what the effects of forward guidance should look like. The remaining, orthogonal factor can then be interpreted as the second main dimension of monetary policy during this period. I interpret this second factor as measuring the FOMC LSAP announcements and present evidence that supports this interpretation. For example, I plot both of these factors—forward guidance and LSAPs—over time and show that they fit identifiable features of major FOMC announcements over the period quite well. In this way, I separately identify the size of the forward guidance and LSAP component of every FOMC announcement between January 2009 and June 2015.

Once the FOMC forward guidance and LSAP announcements are identified, it is then straightforward to estimate the effects of each type of announcement on the high-frequency response of different types of asset prices around those announcements.

The remainder of the paper proceeds as follows. Section 1 reviews the analytical methods of GSS, shows how to adapt them to the recent ZLB period, and describes the data. In section 2, I perform the principal component analysis and rotate the factors as described above. I plot the estimated factors over time and discuss their relationship to identifiable features of major announcements by the FOMC over the ZLB period, showing that my estimates of forward guidance and LSAP announcements seem to be well identified and informative. In section 3, I estimate the effects of these announcements on Treasury yields, stock prices, exchange rates, and corporate bond yields and spreads. In section 5, I discuss the implications of my findings for monetary policy going forward.

## 1. Methods and Data

My methods in the present paper consist of two main steps. First, I extend the analysis of Gürkaynak, Sack, and Swanson (2005) through 16 December 2008, which was the last time the FOMC announced a change in the federal funds rate target. (After that date, the federal funds rate was essentially at a level of zero, and the FOMC was unable or unwilling to lower it any further.) This allows me to identify and estimate the effects of changes in the federal funds rate and changes in forward guidance in normal times, before the ZLB began to bind.[4] Second, I adapt the methods of GSS to the ZLB period from January 2009 through June 2015, during which the FOMC never changed the current federal funds rate target but made multiple unconventional monetary policy announcements involving forward guidance and large-scale asset purchases, as noted in table 1. I thus use the GSS methods, applied to the ZLB sample, to identify and estimate the effects of forward guidance and LSAPs during this later period.

I extend the GSS dataset through June 2015 using data obtained from staff at the Federal Reserve Board. The combined dataset includes the date of each FOMC announcement from July 1991 through June 2015, together with the change in a number of asset prices in a thirty-minute window bracketing each announcement.[5] The asset prices include federal funds futures rates (contracts with expiration at the end of the current month and each of the next five months), Eurodollar futures rates (contracts with expiration near the end of the current quarter and each of the next seven quarters), Treasury bond yields (for the three-month, six-month, and two-, five-, ten-, and thirty- year maturities), the stock market (as measured by the S&P 500), and the U.S. dollar-yen and dollar-euro exchange rates.

To replicate the GSS analysis over the pre-ZLB period, I focus on the responses of the first and third federal funds futures contracts, the second, third, and fourth Eurodollar futures contracts, and the two-, five-, and ten-year Treasury yields to each FOMC announcement from July 1991 through December 2008. The two federal funds futures contracts can be scaled so as to provide good estimates of the market

4. My results are very similar if I end the sample in December 2004, as GSS did, or in December 2007.

5. The window begins 10 minutes before the FOMC announcement and ends 20 minutes after the FOMC announcement. The data set also includes the dates and times of FOMC announcements and some intraday asset price responses going back to January 1990, but the data for Treasury yield responses begin in July 1991, and those data are an important part of my analysis. Also, as is standard in the literature, I exclude the FOMC announcement on 17 September 2001, which took place after financial markets had been closed for several days following the 11 September terrorist attacks. I also include the Federal Reserve Board's announcement on 25 November 2008 that it would begin purchasing mortgage-backed securities and GSE debt (the beginning of "QE1")—although this announcement was not made by the FOMC itself, all subsequent asset purchase announcements were made by the FOMC, so I include it with those others. However, including or excluding this announcement does not noticeably affect any of my results.

expectation of what the federal funds rate will be after the current and next FOMC meetings (see GSS, 2005, for details). The second through fourth Eurodollar futures contracts provide information about the market expectation of the path of the federal funds rate over the horizon from about four months to one year ahead.[6] The two-, five-, and ten-year Treasury yields provide information about interest rate expectations and risk premiums over longer horizons, about one to ten years.

These asset price responses to FOMC announcements can be written as a matrix **X**, with rows of $X$ corresponding to FOMC announcements and columns of $X$ corresponding to different futures rates and Treasury yields. Since there are 159 FOMC announcements from July 1991 through December 2008, and I focus on eight asset price responses, the matrix **X** has dimensions 159 × 8.

As in GSS, I use principal component analysis to estimate the two factors that make the most important contribution to the variation in **X**. The idea is that the asset price responses in **X** are well described by a factor model,

$$\mathbf{X} = \mathbf{F}\boldsymbol{\Lambda} + \boldsymbol{\varepsilon}, \tag{1}$$

where **F** is a 159 × 2 matrix containing two factors, $\boldsymbol{\Lambda}$ is a 2 × 8 matrix of loadings of the asset price responses on the two factors, and $\boldsymbol{\varepsilon}$ is a 159 × 8 matrix of white noise residuals. Letting **F** denote the first two principal components of **X**, the two columns of **F** represent the two components of the FOMC announcements that have had the greatest impact on the assets in **X** over the period from July 1991 to December 2008.

6. The reason for focusing on some rather than all of the possible futures contract rates in the data set is to avoid overlapping contracts as much as possible, since they are highly correlated for technical rather than policy-related reasons. When I conduct the principal components analysis of the data below, futures contracts that are highly correlated will tend to show up as a common factor, which would not be interesting if the correlation was generated by overlapping contracts rather than by the way monetary policy is conducted. For example, FOMC announcements are generally spaced six to eight weeks apart, so there is essentially no gain to including the second federal funds futures contract in addition to the first—the second contract is very highly correlated with the first federal funds futures contract, once the latter contract has been scaled to represent the outcome of the current FOMC meeting. Similarly, including the first Eurodollar futures contract would provide essentially no additional information beyond the first and third federal funds futures contracts. I follow GSS and switch from federal funds futures to Eurodollar futures contracts at a horizon of about two quarters because Eurodollar futures were much more liquid over this sample than longer-maturity federal funds futures, and they are thus likely to provide a better measure of financial market expectations at those longer horizons (see Gürkaynak, Sack, and Swanson, 2007).

Although the first two principal components of **X** explain a maximal fraction of the variation in **X**, they are only a statistical decomposition and typically do not have a structural interpretation. To associate one column of **F** with changes in the federal funds rate and the other column with changes in forward guidance—which is a structural interpretation—it is necessary to transform the factor matrix **F** so that it fits this interpretation.

Given this goal, if **F** and **Λ** characterize the data **X** in equation (1), and **U** is any 2 × 2 orthogonal matrix, then the matrix $\tilde{\mathbf{F}} \equiv \mathbf{FU}$ and loadings $\tilde{\mathbf{\Lambda}} \equiv \mathbf{U'\Lambda}$ represent an alternative factor model that fits the data **X** exactly as well as **F** and **U**, in the sense that it produces exactly the same residuals **ε** in equation (1).[7] Ideally, the two columns of **F** would correspond to changes in the federal funds rate and changes in the FOMC forward guidance, as mentioned above. Although the first two principal components of **X** do not in general have this interpretation, it is possible to choose a rotation matrix **U** such that the rotated factors $\tilde{\mathbf{F}}$ do have such an interpretation. In particular, it is possible to choose **U** such that if $\tilde{f}_1$ and $\tilde{f}_2$ are the two columns of $\tilde{\mathbf{F}}$, then $\tilde{f}_2$ has no effect on the current federal funds rate.[8] This implies that all of the variation in the current federal funds rate (up to the white noise residuals **ε**) in response to FOMC announcements is due to changes in the first factor, $\tilde{f}_1$. The factor $\tilde{f}_1$ can thus be interpreted as the surprise component of the FOMC change in the federal funds rate target. The second factor, $\tilde{f}_2$, then corresponds to all of the other information in the FOMC announcements, above and beyond the surprise change in the funds rate, that changed financial market expectations about the future path of the funds rate. Thus, $\tilde{f}_2$ can be thought of as forward guidance by the FOMC.[9] As GSS show, the second factor $\tilde{f}_2$, identified in this way, corresponds closely to important changes in the FOMC statements about the outlook for the future path of monetary policy, supporting the interpretation of $\tilde{f}_2$ as the change in the FOMC forward guidance.

I next adapt this methodology to the zero lower bound period in the United States, from January 2009 to June 2015. As in GSS and

7. The scale of **F** and **Λ** are also indeterminate: if $k$ is any scalar, then $k\mathbf{F}$ and $\mathbf{\Lambda}/k$ also fit the data **X** exactly as well as **F** and **Λ**. Traditionally, the scale of **F** is normalized so that each column has unit variance.

8. In other words, $\tilde{\lambda}_{21} = 0$, where $\tilde{\lambda}_{ij}$ denotes the $(i,j)$th element of $\tilde{\mathbf{\Lambda}}$, so the current-month federal funds futures contract is not affected by changes in the second factor.

9. GSS called $\tilde{f}_1$ the target factor and $\tilde{f}_2$ the path factor, because it relates to the future path of the federal funds rate, but the latter is now typically referred to as forward guidance.

discussed above, I create a data matrix **X** with rows corresponding to FOMC announcements between January 2009 and June 2015 and columns corresponding to the responses of different futures rates and bond yields in a narrow, thirty-minute window bracketing each announcement. However, I exclude the first and third federal funds futures contracts and the second Eurodollar futures contract from the analysis, because those contracts have such short maturities that they essentially do not respond to news in the ZLB period.[10] The matrix **X** that I construct for the ZLB sample thus has dimensions 52 × 5, corresponding to the 52 FOMC announcements over this period, and five different asset price responses: the third and fourth Eurodollar futures contracts and the two-, five-, and ten-year Treasury yields.

As in GSS and discussed above, I extract the first two principal components from the matrix **X**. These are the two features of FOMC announcements between 2009 and mid-2015 that moved the five yields listed above the most. As before, these two principal components do not have a structural interpretation in general. Let $\mathbf{F}^{zlb}$ denote the 52 × 2 matrix of principal components, let **U** be a 2 × 2 orthogonal matrix, let $\tilde{\mathbf{F}}^{zlb} \equiv \mathbf{F}^{zlb}\mathbf{U}$, and let $\tilde{f}_1^{zlb}$ and $\tilde{f}_2^{zlb}$ denote the first and second columns of $\tilde{\mathbf{F}}^{zlb}$. I search over all possible rotation matrices **U** to find the one where the first rotated factor $\tilde{f}_1^{zlb}$ is as close as possible (in terms of its asset price effects) to the forward guidance factor $\tilde{f}_2$ estimated previously (over the 1991–2008 sample).[11] The identifying assumption is thus that the effect of forward guidance on medium- and longer-term interest rates during the ZLB period is about the same as it was during the pre-ZLB period from 1991–2008. The remaining factor, $\tilde{f}_2^{zlb}$, then corresponds to the component of FOMC announcements, above and beyond changes in forward guidance, that have the biggest effect on medium- and longer-term interest rates. It is natural to interpret this second factor as corresponding to FOMC large-scale asset purchases.

The crucial assumption underlying this identification is that forward guidance has essentially the same effects on medium- and

10. The first and third federal funds futures contracts correspond to federal funds rate expectations over the next one and three months, respectively, and the second Eurodollar futures contract corresponds to funds rate expectations from about three to six months ahead. As shown by Swanson and Williams (2014), interest rates at these short maturities essentially stopped responding systematically to news from 2009 to 2012 (the end of their sample), and this remains true through about mid-2015.

11. In other words, I choose the rotation matrix **U** that matches the factor loadings $\tilde{\lambda}_{11}^{zlb}$, $\tilde{\lambda}_{12}^{zlb}$, $\tilde{\lambda}_{13}^{zlb}$, $\tilde{\lambda}_{14}^{zlb}$, and $\tilde{\lambda}_{15}$ to $\tilde{\lambda}_{24}$, $\tilde{\lambda}_{25}$, $\tilde{\lambda}_{26}$, $\tilde{\lambda}_{27}$, and $\tilde{\lambda}_{28}$ as closely as possible, in the sense of minimum Euclidean distance.

longer-term interest rates before and after the ZLB. This assumption is subject to debate, but it provides a natural starting point for my analysis and in fact seems to work very well, as I show below. Thus, for every FOMC announcement from January 2009 through June 2015, I can separately identify the forward guidance component and the LSAP component of that announcement. Once I have separately identified the two components, it is straightforward to estimate the effects of each component on asset prices using ordinary least squares regressions.

## 2. The FOMC Forward Guidance and LSAP Announcements

I now report the results of these methods applied to the pre-ZLB and ZLB periods.

### 2.1 Federal Funds Rate and Forward Guidance Factors before the ZLB

Table 2 reports the rotated loading matrices $\tilde{\boldsymbol{\Lambda}}$ from the estimation procedure described above. The first two rows report results for the pre-ZLB period, July 1991 to December 2008. Each factor, $\tilde{f}_1$ and $\tilde{f}_2$, is normalized to have a unit standard deviation over this sample, so the coefficients in the table are in units of basis points per standard-deviation change in the monetary policy instrument. A one-standard-deviation increase in the federal funds rate over this period is estimated to cause the current federal funds rate to rise by about 8.6 basis points, the expected federal funds rate at the next FOMC meeting to rise about 6.2 basis points, the second through fourth Eurodollar futures rates to rise by 5.9, 5.6, and 4.8 basis points, respectively, and the two-, five-, and ten-year Treasury yields to increase by 3.8, 1.9, and 0.7 basis points, respectively. The effects of a surprise change in the federal funds rate are thus largest at the short end of the yield curve and die off monotonically as the maturity of the interest rate increases.

**Table 2. Estimated Effects of Conventional and Unconventional Monetary Policy Announcements on Interest Rates before and after Dec. 2008**

| | *MP1* | *MP2* | *ED2* | *ED3* | *ED4* | *2y Tr.* | *5y Tr.* | *10y Tr.* |
|---|---|---|---|---|---|---|---|---|
| *July 1991–Dec. 2008:* | | | | | | | | |
| (1) change in federal funds rate | 8.55 | 6.23 | 5.88 | 5.59 | 4.81 | 3.79 | 1.91 | 0.68 |
| (2) change in forward guidance | 0 | 1.18 | 4.23 | 5.42 | 6.12 | 5.08 | 5.2 | 4.02 |
| *Jan. 2009–June 2015:* | | | | | | | | |
| (3) change in forward guidance | - | - | - | 3.18 | 4.15 | 3.33 | 4.24 | 2.35 |
| (4) change in LSAPs | - | - | - | −0.73 | −0.99 | −1.27 | −4.9 | −7.46 |
| memo: | | | | | | | | |
| (5) row 3, rescaled | - | - | - | 4.68 | 6.11 | 4.89 | 6.24 | 3.45 |

Coefficients in the table correspond to elements of the loading matrix $\Lambda$ from equation (1), in basis points per standard deviation change in the monetary policy instrument (except for row 5, which is rescaled).MP1 and MP2 denote scaled changes in the first and third federal funds futures contracts, respectively; ED2, ED3, and ED4 denote changes in the second through fourth Eurodollar futures contracts; and 2y, 5y, and10y Tr. denote changes in 2-, 5-, and 10-year Treasury yields. See text for details.

The effects of forward guidance, in the second row, are quite different. By construction, a shock to the forward guidance factor has no effect on the current federal funds rate. At longer maturities, however, the forward guidance factor's effects increase, peaking at a horizon of about one year, and then dying off slightly for longer maturities. Thus, changes in forward guidance have a roughly hump-shaped effect on the yield curve. For longer-term yields, such as the five- and ten-year yields, changes in forward guidance are a far more important source of variation than are changes in the federal funds rate, as originally emphasized by GSS.

## 2.2 Forward Guidance and LSAP Factors during the ZLB Period

The third and fourth rows of table 2 report the rotated loadings $\tilde{\Lambda}$ for the ZLB period from January 2009 through June 2015. The third row reports the effects of a one-standard-deviation change in forward guidance on the third and fourth Eurodollar futures contract and the two-, five-, and ten-year Treasury yields, respectively. By construction, these coefficients match those in the second row as closely as possible, up to a constant scale factor, so the effect of forward guidance is hump-shaped with a peak at intermediate horizons of about one year. For reference, the fifth row of table 2 rescales the coefficients in row 3 so that their correspondence to the second row can be seen more easily.

The fourth row reports the effects of a one-standard-deviation increase in FOMC asset purchases. I normalize the sign of this factor so that an increase in purchases causes interest rates to fall. The effect on yields is relatively small at short and medium horizons, but increases steadily with maturity—exactly the opposite of changes in the current federal funds rate. At a horizon of one year, the effect of LSAPs is only about 1.0 basis point, but for the ten-year Treasury yield, the effect is more than seven times larger, about 7.5 basis points.

## 2.3 Correspondence of Factors to Notable FOMC Announcements

In the figure, I plot the time series of estimated values of the forward guidance and LSAP factors for each FOMC announcement from January 2009 to June 2015. The dashed line depicts the forward guidance factor, and the solid line the LSAP factor. To make the interpretation of the LSAP factor more intuitive, I scale it by –1 in the figure, so that an increase in LSAPs appears as a negative value; this sign convention implies that positive values in the figure correspond to monetary policy tightenings and negative values to monetary policy easings. The figure also contains brief annotations that help to explain some of the larger observations in the figure.

The largest and most striking observation in the figure is the negative 5.5-standard-deviation LSAP announcement on 18 March 2009, near the beginning of the ZLB sample. This observation corresponds to the announcement of the first LSAP program, often referred to as QE1 in the press.[12] The key elements of this program are listed in table 1, and the announcement seems to have been a major surprise to financial markets, given the huge estimated size of the factor on that date. My identification procedure for forward guidance versus LSAP announcements described above attributes the effects of this announcement to the LSAP factor.

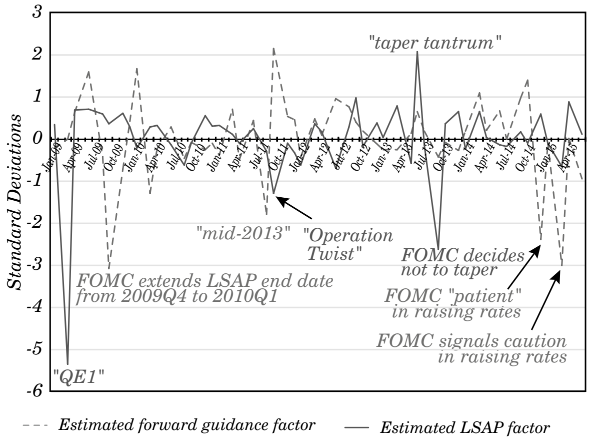
12. The QE1 program began on 25 November 2008, when the Federal Reserve Board announced that it would purchase $600 billion of mortgage-backed securities and $100 billion of debt issued by the mortgage-related government-sponsored enterprises. The term QE1 typically refers to both this earlier program and the huge expansion of that program announced on 18 March 2009.

Given that this FOMC announcement placed such a large emphasis on asset purchases, my identification seems to be working well so far.

The subsequent QE2 program, described in table 1, does not show up as a major event in the figure, perhaps because it was anticipated by financial markets in advance. Looking at the figure around 3 November 2010, the announcement date of the program, there is essentially no estimated effect, because the interest rates included in the estimation responded very little to the announcement. Thus, even though the QE2 announcement was roughly half as large as the earlier QE1 announcement in terms of the quantity of purchases, the surprise component of that announcement appears to have been dramatically smaller.

The next major event in the figure is the negative three-standard-deviation forward guidance announcement on 23 September 2009. On this date, the FOMC stated that it would extend its asset purchase program for an additional three months, through the first quarter of 2010 rather than the fourth quarter of 2009. From the text of the FOMC statement alone, it is unclear whether the announcement should be regarded as forward guidance or LSAPs, or both. However, my identification characterizes this announcement as forward guidance, based on the way financial markets responded (that is, shorter-term interest rates responded more than longer-term interest rates).

**Figure. Estimated Forward Guidance and LSAPF Actors, 2009–2015**




Plot of estimated forward guidance (dashed line) and LSAP (solid line) factors, $\tilde{f}_1^{zlb}$ and $\tilde{f}_2^{zlb}$, overtime. Notable FOMC announcements are labeled in the figure for reference. The LSAP factor is multiplied by −1 in the figure so that positive values in the figure correspond to interest rate increases. See text for details.

By late 2009, the U.S. economy was beginning to recover, and financial markets expected the FOMC to begin raising the federal funds rate in just a few quarters (Swanson and Williams, 2014), but not until a few meetings after completing its asset purchase program. Thus, an extension of the end date of the LSAP program was taken by markets to imply a correspondingly later liftoff date for the federal funds rate.

Another interesting date in the figure is 9 August 2011. That announcement marked the first time the FOMC gave explicit (rather than implicit) forward guidance about the likely path of the federal funds rate over the next several quarters. In that announcement, described in table 1, the FOMC stated that it expected the current (essentially zero) level of the federal funds rate to be appropriate "at least through mid-2013," a date almost two years in the future. Reassuringly, I estimate the announcement on this date as a negative two-standard-deviation surprise in forward guidance, with essentially no LSAP component.

The next FOMC announcement, on 21 September 2011, corresponds to Operation Twist, a program in which the FOMC sold about $400 billion of short-term Treasury securities in its portfolio and used the proceeds to purchase a like quantity of long-term Treasuries. As shown in the figure, this announcement is estimated to have both LSAP and forward guidance components: a negative 1.3-standard-deviation LSAP effect (which is intuitive), and a positive two-standard-deviation forward guidance effect, which is perhaps surprising. This latter effect is due to the fact that shorter-maturity interest rates rose in response to the FOMC announcement—presumably due to a change in risk premiums on those securities resulting from the large increase in expected sales by the Federal Reserve. Although this is probably not an example of forward guidance by the FOMC per se, it nevertheless looks like forward guidance in the data because of the unusual implication of the announcement for short-term Treasury yields. Thus, even though my identification is arguably missing this subtle distinction on this particular date, the estimates coming out of the identification are intuitive and sensible.

For 19 June 2013, I estimate a substantial, two-standard-deviation decrease in the LSAP factor (which is positive in the figure because it represents a monetary policy tightening). There is little change in the FOMC statement on that date, but as reported by the *Wall Street Journal*, the FOMC released economic projections along with the statement that showed a substantial increase in the FOMC economic outlook. Given earlier remarks by then-Chairman Ben Bernanke that the FOMC could begin tapering its asset purchases soon, markets interpreted this as a signal that a tapering was imminent: for example, "Bond prices slumped, sending the yield on the ten-year Treasury note to its highest level in 15 months, as the Federal Reserve upgraded its growth projections for the U.S. economy.… Stronger U.S. growth is widely perceived in the market as heralding an earlier end to the Fed's program of purchasing $85 billion in bonds each month."[13] Thus, this episode fits into the so-called taper tantrum period during the summer of 2013, and it appears to be correctly identified by my procedure as an increase in interest rates due to the LSAP factor.

The flip side of this announcement occurred on 18 September 2013, when the FOMC was widely expected to begin tapering its asset purchases but opted not to do so. The *Wall Street Journal* reported that "The move, coming after Fed officials spent months alerting the public that they might begin to pare their $85 billion-a-month bond-buying program at the September policy meeting, marks the latest in a string of striking turnabouts from Washington policymakers that have whipsawed markets in recent days."[14] The surprise decision by the FOMC not to taper its asset purchases seems to be correctly identified in my estimates as an increase in LSAPs (depicted as a negative value in figure 1 since it is a monetary policy easing).

Near the end of my sample, on 17 December 2014, markets expected the FOMC to remove its statement that it would keep the federal funds rate at essentially zero "for a considerable time." Not only did the FOMC leave that phrase intact, it announced that "the Committee judges it can be patient in beginning to normalize the stance of monetary policy," which was substantially more dovish than financial markets had expected.[15] This announcement thus appears to be correctly identified by my estimation as a large, 2.5-standard deviation decrease in forward guidance by the FOMC.

13. Katy Burne and Mike Chernev, "Bond Markets Sell Off," *Wall Street Journal*, 19 June 2013, Credit Markets.

14. In an article entitled "No Taper Shocks Wall Street," the *Wall Street Journal* reported that "Bernanke had a free pass to begin that tapering process and chose not to follow [through]. . . The Fed had the market precisely where it needed to be. The delay today has the effect of raising the benchmark to tapering" (Steven Rusolillo, "No Taper Shocks Wall Street: Fed 'Running Scared'," *Wall Street Journal*, 18 September 2013, MoneyBeat).

15. For example, "U.S. stocks surged… after the Federal Reserve issued an especially dovish policy statement at the conclusion of the FOMC meetings" (Paul Vigna, "U.S. Stocks Surge after Fed Gets Dovish on Policy," *Wall Street Journal*, 17 December 2014, MoneyBeat).

Finally, on 18 March 2015, the FOMC revised its projections for U.S. output, inflation, and the federal funds rate substantially downward, significantly below what markets had expected. The revised forecast was read by financial markets "as a sign that the central bank would take its time in raising borrowing costs for the economy."[16] Again, my estimation appears to correctly identify this announcement as a substantial, negative three-standard-deviation change in forward guidance.

## 2.4 Scale of Forward Guidance and LSAP Factors

The forward guidance and LSAP factors estimated above and plotted in the figure are normalized to have a unit standard deviation over the sample. Similarly, the loadings in table 2 are for these normalized factors and thus represent an effect measured in basis points per standard deviation. For practical policy applications, however, it is more useful to convert these factors to a scale that is less abstract and more tangible.

For forward guidance, it is natural to think of the factor in terms of a 25 basis points effect on the Eurodollar future rate one year ahead, ED4. A forward guidance announcement of this size would be very large by historical standards, equal to about a six-standard-deviation surprise during the ZLB period or a four-standard-deviation surprise in the pre-ZLB period.[17] To estimate the effects of a forward guidance announcement of this magnitude, the coefficients in the third row of table 2 can be multiplied by a factor of about six, which implies that the effects on the five- and ten-year Treasury yields would be about 25.5 and 14.2 basis points, respectively. The interpretation is that if the FOMC gave forward guidance for the federal funds rate that was about 25 basis points lower one year ahead than financial markets expected, then the five- and ten-year Treasury yields would decline by about 25.5 and 14.2 basis points, on average.

16. Min Zeng, "U.S. Government Bonds Rally after Fed Statement," *Wall Street Journal*, 18 March 2015, Credit Markets. See also *Wall Street Journal*, "U.S. Stocks Surge as Fed Seen Taking Time on Rates," 18 March 2015, Money Beat blog.

17. I estimate that the FOMC forward guidance announcements were larger, on average, before the ZLB than during the ZLB, as presented in table 2. One explanation for why this may be is that, once the FOMC issued its "mid-2013" forward guidance, there were essentially no updates or news about that guidance for many meetings. Similarly, after the FOMC revised the guidance to "late 2014, there were again no updates or news about that guidance for many more meetings, and so on.

For LSAPs, the units would ideally be in billions of dollars of purchases, which is a more difficult transformation than a simple renormalization of the coefficients in table 2. Nevertheless, a number of estimates in the literature suggest that a $600 billion LSAP operation in the United States, distributed across medium- and longer-term Treasury securities, leads to a roughly 15-basis-point decline in the ten-year Treasury yield (see, for example, Swanson, 2011; Williams, 2013, table 1). Using this estimate as a benchmark implies that the coefficients in the fourth row of table 2 correspond to a roughly $300 billion surprise LSAP announcement. Thus, it seems reasonable to interpret the coefficients in that row of table 2 as corresponding to a $300 billion change in purchases. The interpretation is thus that if the FOMC announced a new LSAP program that was about $300 billion larger than markets expected, the effects would be about as large those provided in the fourth row of table 2.

## 3. The Effects of Forward Guidance and LSAPs on Asset Prices

Once the forward guidance and LSAP components of the FOMC announcements from 2009 through 2015 have been identified, it is relatively straightforward to estimate the effects of those announcements on asset prices, using ordinary least squares (OLS) regressions, as follows.

### 3.1 Treasury Yields

Table 3 reports the responses of six-month and two-, five-, ten-, and thirty-year Treasury yields to the forward guidance and LSAP components of the FOMC announcements. As in previous tables and figures, the coefficients here are in units of basis points per standard deviation surprise in the announcement. Each column of the table reports estimates from an OLS regressions of the form

$$\Delta y_t = \alpha + \beta \tilde{\mathbf{F}}_t^{zlb} + \varepsilon_t , \quad (2)$$

where $t$ indices FOMC announcements between January 2009 and June 2015, $y$ denotes the corresponding Treasury yield, $\Delta$ denotes the change in a thirty-minute window bracketing each FOMC announcement, $\tilde{\mathbf{F}}^{zlb}$ denotes the forward guidance and LSAP factors as estimated above, $\varepsilon$ is a regression residual, and $\alpha$ and $\beta$ are parameters.

The point estimates for the two-, five-, and ten-year Treasury yields in table 3 are the same as those in table 2. However, table 3 also reports Huber-White heteroskedasticity-consistent standard errors and $t$ statistics for each coefficient, which indicate that the responses of these yields to both forward guidance and LSAPs are extraordinarily statistically significant, with $t$ statistics ranging from 8.8 to almost 17.0. The regression $R^2$ values are also quite high, over 93 percent, so these two factors explain a very large share of the variation in those yields around FOMC announcements.

Table 3 also reports results for the six-month and thirty-year Treasury yields, which were not included in the estimation of the factors themselves.[18] LSAPs do not have a statistically significant effect on the six-month Treasury yield, and the effect of forward guidance on this yield is statistically significant but small, amounting to only about 0.5 basis points per standard deviation surprise, less than one-sixth the size of the two-year Treasury yield response. This is likely due to the fact that the six-month Treasury yield was very close to zero and largely unresponsive to news over much of this period (Swanson and Williams, 2014). To the extent that the six-month Treasury yield was pinned to zero for a significant part of the sample, I would not expect to see much of a response to any type of announcement.

The effect of forward guidance on the thirty-year Treasury yield is also quantitatively small and, in this case, statistically insignificant. In contrast to the six-month Treasury, the thirty-year Treasury yield was not pinned to zero for any length of time during this period, so the small coefficient reflects the fact that forward guidance apparently had little effect on the longest-maturity Treasuries during the ZLB period. The effect of LSAPs on the thirty-year Treasury yield, however, are large and extraordinarily statistically significant, with a $t$-statistic of almost 12. Interestingly, the effects of LSAPs on the thirty-year yield were not quite as large as their effects on the ten-year yield, presumably because the FOMC LSAP operations were typically concentrated around maturities closer to ten years.

18. Results for the three-month Treasury yield are not reported, since the three-month Treasury yield generally did not respond to news over this period, as shown by Swanson and Williams (2014).

**Table 3. Estimated Effects of Forward Guidance and LSAPs on U.S. Treasury Yields, 2009–2015**

| | *6-month* | *2-year* | *5-year* | *10-year* | *30-year* |
|---|---|---|---|---|---|
| *Change in forward guidance* | 0.53*** | 3.33*** | 4.24*** | 2.35*** | 0.30 |
| (std. err.) | (0.092) | (0.217) | (0.252) | (0.263) | (0.737) |
| [t-stat.] | [5.75] | [15.33] | [16.82] | [8.91] | [0.40] |
| *Change in LSAPs* | −0.08 | −1.27*** | −4.90*** | −7.46*** | −5.78*** |
| (std. err.) | (0.08) | (0.077) | (0.556) | (0.453) | (0.493) |
| [t-stat.] | [−0.99] | [−16.48] | [−8.82] | [−16.47] | [−11.71] |
| *Regression $R^2$* | 0.47 | 0.93 | 0.94 | 0.97 | 0.77 |
| # Observations | 52 | 52 | 52 | 52 | 52 |

Coefficients β from regressions $\Delta yt = \alpha + \beta\tilde{F}_t^{zlb} + \varepsilon_t$, where $t$ indices FOMC announcements between Jan. 2009 and June 2015, $y$ denotes a given Treasury yield, $\tilde{F}$ denotes the forward guidance and LSAP factors estimated previously, and Δ is the intraday change in a 30-minute window bracketing each FOMC announcement.Coefficients are in units of basis points per standard deviation change in the monetary policy instrument. Huber-White heteroskedasticity-consistent standard errors in parentheses; $t$-statistics in square brackets; *** denotes statistical significance at the 1% level. See text for details.

## 3.2 Stock Prices and Exchange Rates

Table 4 reports analogous regression results for the S&P 500 stock index and the dollar-euro and dollar-yen exchange rates. The form of the regressions is the same as in equation (2), except the dependent variable in each regression is now 100 times the log change in the asset price in each column.

As shown in table 4, both forward guidance and LSAPs have statistically significant effects on stock prices and exchange rates. For stocks, a one-standard-deviation increase in forward guidance caused prices to fall by about 0.2 percent, while a one-standard-deviation increase in LSAPs caused stock prices to rise by a similar amount. Both of these coefficients are highly statistically significant, with $t$ statistics of about 2.7 and 3.7, respectively. Both effects are also in the direction one would expect from a standard dividend-discount model, given the interest rate responses reported in the previous table; that is, an increase in interest rates reduces the present value of a stock's dividends (and may reduce the size of the dividends themselves, if the economy contracts), which will tend to cause stock prices to fall. Finally, the $R^2$ for this regression is much lower than those for Treasury yields, due to the high and idiosyncratic volatility of stock prices around FOMC announcements.

The effects of forward guidance and LSAPs on the dollar are more precisely estimated. Both the dollar-euro and dollar-yen exchange rates are expressed as the dollar price per unit of foreign currency.

**Table 4. Estimated Effects of Forward Guidance and LSAPs on Stock Prices and Exchange Rates, 2009–2015**

| | *S&P500* | *$/euro* | *$/yen* |
|---|---|---|---|
| *Change in forward guidance* | −0.19*** | −0.25*** | −0.20*** |
| (std. err.) | (0.07) | (0.037) | (0.04) |
| [t-stat.] | [−2.68] | [−6.66] | [−5.04] |
| *Change in LSAPs* | 0.20*** | 0.33*** | 0.37*** |
| (std. err.) | (0.053) | (0.049) | (0.05) |
| [t-stat.] | [3.66] | [6.65] | [7.32] |
| *Regression* $R^2$ | 0.27 | 0.67 | 0.8 |
| # Observations | 52 | 52 | 52 |

Coefficients β from regressions $\Delta \log xt = \alpha + \beta \tilde{F}_t^{zlb} + \varepsilon_t$, where *t* indices FOMC announcements between Jan. 2009 and June 2015, *x* is the asset price, $\tilde{F}$ denotes the forward guidance and LSAP factors estimated previously, and Δ is the intraday change in a 30-minute window bracketing each FOMC announcement.Coefficients are in units of percentage points per standard deviation change in the monetary policy instrument. Huber-White heteroskedasticity-consistent standard errors in parentheses; *t*-statistics in square brackets; *** denotes statistical significance at the 1% level. See text for details.

In response to a one-standard-deviation increase in forward guidance, the dollar appreciated by about 0.20 to 0.25 percent, and the effect is highly statistically significant, with *t* statistics of about 6.7 for the euro and 5.0 for the yen. A one-standard-deviation increase in LSAPs causes the dollar to depreciate about 0.35 percent, and the effect is again highly statistically significant with *t* statistics of 6.6 and 7.3. These effects have the signs one would expect from uncovered interest parity, given the response of interest rates reported in table 3. That is, an increase in U.S. interest rates makes U.S. dollar investments more attractive relative to foreign investments, which tends to drive the value of the dollar up.

## 3.3 Corporate Bond Yields and Spreads

Table 5 reports results for corporate bond yields and spreads. Corporate bonds are less frequently traded than U.S. Treasuries, stocks, and foreign exchange, so only daily-frequency corporate bond yield data are available. Thus, the regressions in table 5 use the one-day change in corporate bond yields or spreads around each FOMC announcement as the dependent variable. To measure corporate yields, I consider both the Aaa and Baa indices of long-term seasoned corporate bond yields from Moody's.

As shown in the first row of the table, I estimate that changes in FOMC forward guidance had essentially no effect on corporate bond yields during the ZLB period. The point estimates for both Aaa and Baa yields are small (less than one-half of one basis point per standard deviation change in forward guidance) and statistically insignificant. Because ten-year Treasury yields rise modestly in response to a change in forward guidance, the effect on the corporate-Treasury yield spread is thus modestly negative, falling about one to two basis points in response to an increase in guidance, and this effect is moderately statistically significant, with t statistics of 2.2 and 2.5.

The effect of LSAPs on corporate bond yields was much larger and more significant. A one-standard-deviation increase in LSAPs caused both the Aaa and Baa yields to fall about five basis points, and the effect was extraordinarily statistically significant. However, the effect of LSAPs on the ten-year Treasury yield was larger than the effect on corporate bond yields, so the spread between corporate bonds and Treasuries actually increased in response to the LSAP program.[19] This result echoes findings by earlier authors, such as Krishnamurthy and Vissing-Jorgensen (2012) and Swanson (2011), that the Federal Reserve's LSAP programs—which tend to be concentrated in U.S. Treasury securities—push down Treasury yields more than they do private-sector yields. Nevertheless, the effect on corporate bond yields that I estimate here is a bit bigger than those authors find in their studies. For example, Swanson (2011) estimated that corporate bond yields fall by about 4–5 basis points in response to a $600 billion Treasury LSAP, while the estimates in table 5 are closer to 9–10 basis points for the same size operation (assuming this is a roughly two-standard-deviation announcement, as discussed earlier). One reason for the larger estimates here may be that the recent LSAP programs often included a substantial quantity of mortgage-backed securities (MBS) as well as Treasury securities. Those MBS are likely to be closer substitutes for corporate bonds than are Treasuries, so MBS purchases can be expected to have a relatively larger effect on corporate bond yields than purchases of Treasuries alone. The earlier estimates in Krishnamurthy and Vissing-Jorgensen (2012) and Swanson (2011) are for the case of a

19. The ten-year yield response in table 2 is estimated to be about −7.5 basis points, while the effect implied in table 5 is a bit larger, about −8.9 basis points. There are two reasons for this difference. First, the responses in table 2 are thirty-minute responses, while those in table 5 are one-day responses. Second, table 2 uses the on-the-run coupon-bearing ten-year Treasury bond, while table 5 uses the ten-year zero-coupon yield estimate by Gürkaynak, Sack, and Wright (2007). The latter yield has a longer duration than the coupon-bearing ten-year security, which should be a better match to the long-term corporate bonds in the Moody's indices.

Treasury-only LSAP, and they thus could be expected to have smaller effects on private yields than the MBS-and-Treasury LSAPs conducted by the FOMC between 2009 and 2015.

**Table 5. Estimated Effects of Forward Guidance and LSAPs on Corporate Bond Yields and Spreads, 2009–2015**

| | *Corporate yields* | | *Spreads* | |
|---|---|---|---|---|
| | *Aaa* | *Baa* | *Aaa–10-yr.* | *Baa–10-yr.* |
| *Change in forward guidance* | 0.28 | −0.33 | −1.23** | −1.85** |
| (std. err.) | (0.58) | (0.755) | (0.558) | (0.743) |
| [$t$-stat.] | [0.49] | [−0.44] | [−2.21] | [−2.49] |
| *Change in LSAPs* | −4.65*** | −5.17*** | 4.25*** | 3.74*** |
| (std. err.) | (0.373) | (0.577) | (0.546) | (0.911) |
| [$t$-stat.] | [−12.48] | [−8.96] | [7.79] | [4.11] |
| *Regression $R^2$* | 0.44 | 0.49 | 0.56 | 0.55 |
| # Observations | 52 | 52 | 52 | 52 |

Coefficients β from regressions $\Delta yt = \alpha + \beta \tilde{F}_t^{zlb} + \varepsilon_t$, where $t$ indexes FOMC announcements between Jan. 2009 and June 2015, $y$ denotes the corporate bond yield or spread, $\tilde{F}$ denotes the forward guidance and LSAP factors estimated previously, and Δ is the change in a one-day window bracketing each FOMC announcement. Coefficients are in units of basis points per standard deviation change in the monetary policy instrument. Huber-White heteroskedasticity-consistent standard errors in parentheses; $t$-statistics in square brackets; ** and *** denote statistical significance at the 5% and 1% levels, respectively. See text for details.

## 4. Conclusions

In this paper, I show how to identify and estimate the forward guidance and large-scale asset purchase component of every FOMC announcement between 2009 and 2015, the U.S. zero lower bound period. Building on earlier work by Gürkaynak, Sack, and Swanson (2005), I estimate a time series for each type of unconventional monetary policy announcement and show that these series correspond to identifiable characteristics of important FOMC statements during this period.

I use these identified forward guidance and LSAP announcements to estimate the effects of each type of policy on Treasury yields, stock prices, exchange rates, and corporate bond yields and spreads. I find that forward guidance affected Treasury yields at all but the very longest maturities, with a peak effect at a maturity of about one to five years. In contrast, I find that the effects of LSAPs increased with maturity, with LSAPs having their peak effect on the longest maturities (ten and thirty years). LSAPs had essentially no effect on the shortest-maturity Treasuries.

I estimate that forward guidance had no effect on corporate bond yields during the ZLB period. In contrast, LSAPs had substantial and highly significant effects on those yields. Nevertheless, the effects of LSAPs on corporate debt was smaller than their effects on Treasuries, so corporate bond spreads actually increased after an increase in FOMC asset purchases. This finding is consistent with others in the literature and probably reflects the fact that the Federal Reserve's LSAP programs focused largely on purchases of Treasury securities.

Stock prices responded about equally to changes in forward guidance and LSAPs over the zero lower bound period. This is perhaps surprising, given that forward guidance seems to have been relatively unimportant for other long-duration assets, such as the thirty-year Treasury and corporate bonds. Forward guidance certainly had much smaller effects than LSAPs on these other long-duration assets.

Finally, I estimate that forward guidance and LSAPs both had significant effects on exchange rates, with LSAPs being moderately more important. An increase in U.S. interest rates due to either forward guidance or LSAPs caused the U.S. dollar to appreciate, consistent with a standard uncovered interest parity channel.

Looking forward, it is natural to ask which policy is more effective. The answer is that it depends. First, it is difficult to compare the scale of the two different types of policies—for example, is a $100 billion

LSAP operation large or small, and is it larger or smaller than a 25 basis points change in forward guidance about the federal funds rate one year ahead? One natural way of comparing magnitudes across the two types of policies is in terms of their historical importance: over the 2009–2015 period, a one-standard-deviation change in forward guidance by the FOMC corresponded to a change of about six basis points in federal funds rate expectations one year ahead, while a one-standard deviation change in LSAPs corresponded to a roughly $300 billion change in bond purchases. Using these estimates as a basis for comparison, a one-standard-deviation change (six basis points) in forward guidance appears to have been about as effective at changing medium-term Treasury yields, stock prices, and exchange rates as a one-standard-deviation ($300 billion) change in LSAPs. However, LSAPs were much more effective at changing long-term Treasury yields and corporate bond yields, while forward guidance was more effective at moving shorter-maturity Treasury yields.

Finally, the analysis in this paper suggests at least three important avenues for future research. First, it is important to investigate the persistence of the effects estimated above. Wright (2012) does not distinguish between forward guidance and LSAPs, but finds that unconventional monetary policy as a whole had effects that died out with a half-life of just two to three months between November 2008 and September 2011. In ongoing research, I am studying the persistence of the effects of forward guidance and LSAPs on financial markets between 2009 and 2015. Second, the time series of forward guidance and LSAP announcements estimated above can be used to investigate the effects of these announcements on macroeconomic as well as financial variables, which I am also pursuing in ongoing work. Third, the analysis above sheds no light on the relative costs of forward guidance versus LSAPs. Obviously, whether one type of policy should be preferred to the other in practice depends on its costs as well as its effects, which makes this another important avenue for future research.

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# Risk Premium Shifts and Monetary Policy: A Coordination Approach

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Our understanding of crisis propagation and the telling of the crisis narrative have been heavily influenced by the events surrounding the 2008 crisis, which has focused on the leverage of banks and other financial intermediaries. Since then, the focus has shifted from banks to financial market liquidity, in line with the shift in the pattern of financial intermediation as global banks have increasingly given way to long-term investors operating in the bond market. Long-term investors are often portrayed as a stabilizing influence in financial markets, absorbing losses without insolvency and cushioning market shocks caused by leveraged players. However, recent episodes such as the so-called taper tantrum of 2013 have shown that even long-term investors may have limited appetite for losses, and that they will join in a selling spree when one arrives. The issue of evaporating market liquidity and one-sided markets in the face of concerted selling by investors has occupied an important place in recent policy discussions.[1]

The taper tantrum of 2013 is but a recent case of the general phenomenon in which monetary policy shocks are associated with

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1. See, for instance, the BIS report on market-making and market liquidity by the Committee on the Global Financial System (BIS, 2014); the chapters on market liquidity in the *IMF Global Financial Stability Review* (IMF, 2015a, 2015b). Fender and Lewrick (2015) lay out the dimensions of the debate.

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changes in the risk premium inherent in market prices, over and above any change in the actuarially fair long-term interest rate implied by the expectations theory of the yield curve. Shiller, Campbell, and Schoenholtz (1983) document the early evidence. Hanson and Stein (2015) and Gertler and Karadi (2015) add to the accumulated evidence that monetary policy appears to operate through changes in the risk premium inherent in asset prices, in addition to changes in the actuarially fair long-term rate.

The fact that the risk premium fluctuates so much opens up a gap between the theory and practice of monetary policy. Discussions of central bank communication often treat the market as if it were an individual with beliefs. Transparency over the path of future policy rates is seen as a device to guide long-term rates, and crucially, such guidance is seen as something amenable to fine-tuning. The term market expectations is often used in connection with central bank guidance. Although such a term can serve as a shorthand, it creates the temptation to treat the "market" as a person with coherent beliefs. The temptation is to anthropomorphize the market and endow it with attributes that it does not have (Shin, 2013).

However, the market is not a person. Market prices are outcomes of the interaction of many actors, and not the beliefs of any one actor. Even if prices are the average of individual expectations, average expectations fail even the basic property of the law of iterated expectations. In other words, the average expectation today of the average expectation tomorrow of some variable is not the average expectation today of that variable (Allen, Morris, and Shin, 2006).

In this paper, we explore a coordination model of the transmission of monetary policy with heterogeneous market participants. Our model has the feature that monetary policy exerts a direct impact on risk premiums through the risk-taking behavior of market participants. In the model, risk-neutral investors, interpreted as asset managers, interact with risk-averse households in a market for a risky bond. Although the asset managers are motivated by long-term fundamental asset values, there is an element of short-termism generated by the aversion to coming last in short-term performance rankings among asset managers. We interpret the friction as the loss of customer mandates of the asset managers, consistent with the empirical evidence on the sensitivity of fund flows to fund performance. Thus, the friction in the model is that relative performance matters for fund managers.

The importance of relative ranking injects spillover effects across asset managers and an endogenous coordination element in their portfolio choice. The cost of coming last generates behavior that has the outward appearance of shifts in preferences. Just as in a game of musical chairs, when others try harder to grab a chair, more effort must be expended to grab a chair oneself. The ensuing scramble for the relatively safer option of selling the risky bond in favor of the short-term asset leads to a jump in the yield of the risky bond that has the outward appearance of a sudden jump in the risk aversion of the market. The global game approach permits the solution of the trigger level of the floating interest rate when the scramble kicks in. Therefore, when the central bank signals higher future rates, the impact on asset prices is often abrupt, as the risk-taking behavior of market participants undergoes discrete shifts. We could dub this channel of the transmission of monetary policy the risk-taking channel, following Borio and Zhu (2012) who first coined the term.

The key parameter for the strength of the risk-taking channel is the size of the asset management sector. Quantities thus matter. When the sector is large relative to risk-averse households, risk premiums can be driven very low by signaling low future policy rates. In return, however, the central bank must accept a narrower region of fundamentals when risk premiums can be kept low, together with a larger jump in risk premiums when the policy stance changes.

Our main results provide a model of exit of managed funds from key asset markets, generating a jump in the risk premium. We also combine this model with an account of flows into and out of the funds, and the strategic complementarities between the fund managers' investment decisions and decisions of investment managers to invest in or redeem from the funds.

We describe the main model in the next section 1, providing a dynamic context in section 2. Our results hold several implications for the conduct of monetary policy, but we postpone discussion of the implications until section 3. Our paper also bears on investor flows in bond mutual funds. We return in the concluding section to review what incremental lessons our paper can provide to this literature. We first present the model and the solution.

## 1. Model

There are two groups of investors. First, there is a continuum of risk-neutral investors interpreted as asset managers. Asset managers are indexed by the unit interval [0, 1], consume once only at the terminal date, and do not discount the future. Asset managers are evaluated against a benchmark index and rewarded for beating the index (or penalized for lagging behind the index). In other words, the payoff of the asset manager is the difference between the realized return on the portfolio and the realized return on the benchmark index. The benchmark index is fixed exogenously, but its realization is uncertain, as described below. For the purpose of our exercise here, we may interpret the benchmark index as a market interest rate, and the asset managers' performance will be evaluated against this benchmark market interest rate. There is one additional element in the payoffs of the asset managers. Although asset managers care about long-term asset values, they suffer from "last-place aversion" in that they are subject to a penalty (described below) if they are ranked last in the value of their short-term portfolio. We can interpret this penalty as the loss of customers suffered by the asset manager, as reflected in the empirical evidence on the positive relationship between fund flows and fund performance.

The second group of investors are risk-averse household investors. They do not discount the future, they consume once only at the terminal date, and they behave competitively.

All investors form portfolios between two types of assets—a risky asset and a safe asset. The long-term asset is a risky zero-coupon bond that pays only at the terminal date, but the payoff is risky. The expected payoff at the terminal date is $v$ with variance $\sigma^2$. There is an outstanding amount of $S$ units of the risky bond. The safe asset is a storage technology that pays zero.

## 1.1 Three-Period Model

We first examine the benchmark version of our model, which has three dates, 0, 1 and 2. The timeline is depicted in figure 1. At date 1, asset managers choose how much of the risky bond to hold. They all have one unit of wealth, which they can allocate between the risky bond and the floating-rate account. Asset managers cannot borrow and cannot take short positions.

The realized value of the risky bond is uncertain, with expected value $v$. The return on the benchmark index between date 1 and date 2 is denoted by $1 + r$. The price of the risky bond $p$ is determined by market clearing.

Households have mean-variance preferences, and at date 1, they submit a competitive demand curve for the risky bond. Household $h$ has the following utility function:

$$U_h = vy - \frac{1}{2\tau_h} y^2\sigma^2 + \left(e - py\right), \tag{1}$$

where $y$ is the risky bond holding of the household, $e$ is the endowment, and $\tau$ is risk tolerance. We assume that the endowment $e$ is large enough that the first-order condition determines the optimal portfolio. From the first-order condition with respect to $y$ and summing across households, the aggregate demand for the risky bond for the household sector is

$$p = v - \frac{\sigma^2}{\sum_h \tau_h} y \qquad (2)$$

$$= v - cy,$$

where $c$ is the positive constant defined as $c = \sigma^2 / \sum_h \tau_h$, and $\sum_h \tau_h$ is the aggregate risk tolerance for the household sector as a whole.

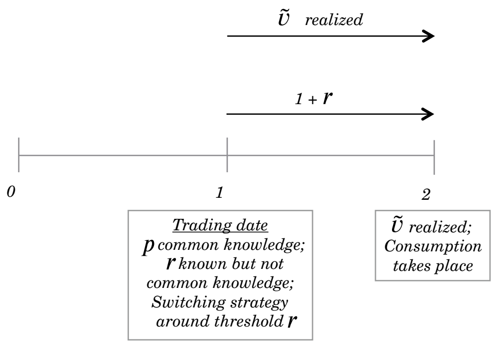
Asset managers hold $A$ units of the bond, which is exogenous for now. Households hold the remainder $S - A$. Thus, prices are determined by the asset market position, with

$$p = v - y(S - A),$$

and the risk premium is

$$\frac{v}{p} = \frac{v}{v - y(S - A)}.$$

**Figure 1. Time Line for Three Period Model**



Asset managers' primary objective is to maximize the return on their investors' funds. The investors in the funds are assumed to be seeking to maximize long-run expected returns. The return to investing in bonds is the risk premium. The alternative investment is the safe asset, with zero return. The excess return relative to the index is given by

$$\frac{v}{v-y(S-A)}-r.$$

However, in our model, asset managers not only care about long-run returns in excess of the benchmark index, but also suffer from last-place aversion.[2] We assume that there is a penalty suffered by any asset manager whose portfolio value is ranked last at date 1. The penalty could be interpreted as a decline in the asset manager's funds under management due to withdrawals by their customers. Below we discuss alternative forms of strategic complementarity that could have generated strategic complementarities in asset managers' incentives.

In particular, if any asset manager is ranked last (or equal last) at date 1, and proportion $x$ of asset managers has a strictly higher portfolio value, then the asset manager suffers a payoff penalty of $\phi x$, where $\phi$ is a positive constant. The asset manager's payoff is

$$\frac{v}{v-y(S-A)}-r-\phi x. \tag{3}$$

## 1.2 Global Game

When viewed as a one-shot game between the asset managers with complete information, there would be an equilibrium where no asset manager sells and everyone gets a payoff

$$\frac{v}{v-y(S-A)}-r,$$

as long as

$$\frac{v}{v-y(S-A)}-r\geq 0,$$

2. The term last-place aversion is taken from Buell and others (2014), who use the concept in the very different context of the welfare economics of social deprivation.

and there will be an equilibrium where all asset managers sell if

$$\frac{v}{v-y(S-A)}-r\leq\phi x.$$

However, asset managers are not certain what other managers will do. We use global games analysis (Morris and Shin, 2003) to capture the idea that there is strategic uncertainty among managers. In particular, suppose that managers are almost sure about the evolution of the benchmark index, but there is a small amount of heterogeneity. Thus, the benchmark index $r$ is uncertain, but investors have good information about it. At date 1, asset manager $i$ observes signal $\rho_i$ of $r$ given by

$$\rho_i = r + s_i, \tag{4}$$

where $s_i$ is a uniformly distributed noise term, with realization in $[-\varepsilon,\varepsilon]$ for small positive constant $\varepsilon$. The noise terms $\{s_i\}$ are independent across asset managers. We further assume that the ex-ante distribution of r is uniform on some interval. The assumption that $r$ and the noise term $s_i$ are uniformly distributed is for expositional simplicity only.

Based on their respective signals, asset managers decide whether to hold the risky bond or sell it. Since asset managers are risk-neutral, it is without loss of generality to consider the binary choice of hold or sell. A strategy for an asset manager is a mapping:

$$\rho_i \mapsto \{\text{Hold, Sell}\} \tag{5}$$

A collection of strategies (one for each asset manager) is an *equilibrium* if the action prescribed by i's strategy maximizes i's expected payoff at every realization of signal $\rho_i$ given others' strategies.

As the first step in the solution, consider switching strategies of the form

$$\begin{cases} \text{Sell} & \text{if } \rho > \rho^* \\ \text{Hold} & \text{if } \rho \leq \rho^* \end{cases} \tag{6}$$

for some threshold value $\rho^*$. We first solve for equilibrium in switching strategies. We search for threshold point $\rho^*$ such that every asset manager uses the same switching strategy around $\rho^*$. We appeal to the following result in global games. Recall that $x$ is our notation for the proportion of investors who sell.

**Lemma 1.** *Suppose that investors follow the switching strategy around* $\rho^*$. *Then, in the limit as* $\varepsilon \to 0$, *the density of* $x$ *conditional on* $\rho^*$ *is uniform over the unit interval* [0, 1].

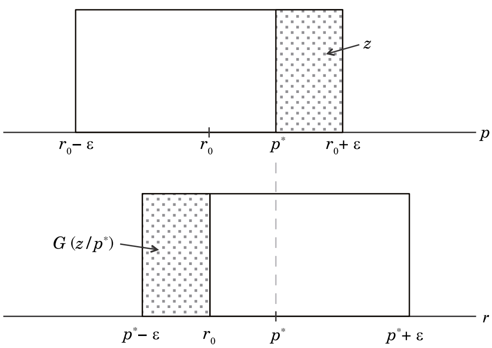
To make the discussion in our paper self-contained, we present the proof of lemma 1. For economy of argument we show the proof only for the case of uniformly distributed $r$ and uniform noise. However, this result is quite general and does not depend on the assumption of uniform density and uniform noise (Morris and Shin, 2003, section 2).

The distribution of $x$ conditional on $\rho^*$ can be derived from the answer to the following question (Q): "My signal is $\rho^*$. What is the probability that $x$ is less than $z$?" The answer to question (Q) gives the cumulative distribution function of $x$ evaluated at $z$, which we denote by $G(z \mid \rho^*)$. The density over $x$ is then obtained by differentiating $G(z \mid \rho^*)$. The steps to answering question (Q) are illustrated in figure 2.

When the true realization of the benchmark index is $r$, the signals $\{\rho_i\}$ are distributed uniformly over the interval $[r-\varepsilon, r+\varepsilon]$. Investors with signals $\rho_i > \rho^*$ are those who sell. Hence,

$$x = \frac{r + \varepsilon - \rho^*}{2\varepsilon}. \tag{7}$$

**Figure 2. Deriving the Subjective Distribution over $x$ at Switching Point $\rho^*$**



When do we have $x < z$? This happens when $r$ is low enough, so that the area under the density to the right of $\rho^*$ is squeezed. There is a value of $r$ at which $x$ is precisely $z$. This is when $r = r_0$, where

$$\frac{r_0 + \varepsilon - \rho^*}{2\varepsilon} = z \tag{8}$$

or

$$r_0 = \rho^* - \varepsilon + 2\varepsilon z. \tag{9}$$

See the top panel of figure 2. We have $x < z$ if and only if $r < r_0$. We need the probability of $r < r_0$ conditional on $\rho^*$.

For this, we must turn to player i's posterior density over $r$ conditional on $\rho^*$. This posterior density is uniform over the interval $[\rho^* - \varepsilon, \rho^* + \varepsilon]$, as in the lower panel of figure 2. This is because the ex ante distribution over $r$ is uniform, and the noise is uniformly distributed around $r$. The probability that $r < r_0$ is then the area under the density to the left of $r_0$, which is

$$\begin{aligned} \frac{r_0 - \left(\rho^* - \varepsilon\right)}{2\varepsilon} &= \frac{\left(\rho^* - \varepsilon + 2\varepsilon z\right) - \left(\rho^* - \varepsilon\right)}{2\varepsilon} \\ &= z, \end{aligned} \tag{10}$$

where the second line follows from substituting in equation (9). Thus, the probability that $x < z$ conditional on $\rho^*$ is exactly $z$. The conditional cumulative distribution function $G(z \mid \rho^*)$ is the following identity function:

$$G\left(z \mid \rho^*\right) = z. \tag{11}$$

The density over $x$ is thus uniform. Finally, the uniform density over $x$ does not depend on the value of $\varepsilon$. For any sequence $(\varepsilon_n)$ where $\varepsilon_n \to 0$, the density over $x$ is uniform. This proves lemma 1.

In the limit as $\varepsilon \to 0$, every investor's signal converges to the true interest rate $r$. Fundamental uncertainty disappears, and it is without loss of generality to write the investor's strategy as being conditional on the true interest rate $r$. Therefore, we search for an equilibrium in switching strategies of the form

$$\begin{cases} \text{Sell} & \text{if } r > r^* \\ \text{Hold} & \text{if } r \le r^* \end{cases} \tag{12}$$

Figure 2 reveals the intuition for lemma 1. As ε shrinks, the dispersion of signals shrinks with it, but so does the support of the posterior density over $r$. The region on the top panel corresponding to $z$ is the mirror image of the region on the bottom panel corresponding to $G(z \mid \rho^*)$. Changing ε stretches or squeezes these regions, but it does not alter the fact that the two regions are equal in size. This identity is the key to the result. The uniform density over $x$, which has been dubbed Laplacian beliefs (Morris and Shin, 2003), implies that the strategic uncertainty faced by players in the global game is at its maximum, even when the fundamental uncertainty faced by players shrinks to zero.

## 1.3 Solution

Given Laplacian beliefs, the switching point $r^*$ is the return that makes each asset manager indifferent between holding and selling. That is, $\rho^*$ satisfies

$$\frac{v}{v - y\left(S - A\right)} - \frac{1}{2}\rho^* . \tag{13}$$

Therefore, the return $r^*$ is given by

$$r^* = \frac{c\left(S - A\right)}{v - c\left(S - A\right)} - \frac{1}{2}\phi. \tag{14}$$

It remains to verify that asset managers strictly prefer to sell when $r > r^*$ and strictly prefer to hold when $r < r^*$. Both propositions follow from the monotonicity of the payoff (equation 3).

The monotonicity of the payoff difference $u(x) - w(x)$ implies that the switching strategy around $r^*$ is the unique dominance-solvable equilibrium in the sense that it is the only equilibrium that survives the iterated deletion of strictly dominated strategies (Morris and Shin, 2003, section 2). Therefore, the solution given by equation (14) is the complete solution in that there is no other equilibrium—whether in switching strategies or in any other strategies. We summarize the solution as follows.

**Proposition 2.** *There is a unique dominance-solvable equilibrium. In this equilibrium, all asset managers use the switching strategy around $r^*$ defined by equation (14), selling the risky bond when $r > r^*$ and holding when $r \leq r^*$.*

We note some properties of the solution. First, the threshold return $r^*$ is decreasing in $\phi$. Therefore, the worse is the last-place aversion of the asset managers, the more jittery they become and the lower is the interest rate at which they jump from holding the risky bond to selling out.

Perhaps more important is the effect of changes in $A$, the size of the asset management sector. When the asset management sector is large relative to the household investors, the price impact of concerted sales is large. The strategic interaction between asset managers is thus heightened. To use our analogy with the musical chairs game, a larger asset management sector means that the musical chairs game becomes more competitive. There is more at stake in coming last in the game, so that asset managers are willing to jump ship at a lower threshold interest rate.

The impact of the asset management sector can be seen in several features of our solution. The larger is $A$ relative to the total stock $S$, the higher is the market price $p$. As $A$ increases, the risk premium of the risky bond becomes more compressed. The risk premium when the size of the asset management sector is $A$ is given by

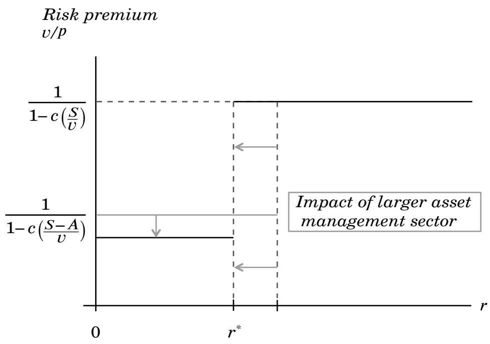
$$\frac{v}{p} = \frac{v}{v - c\left(S - A\right)}, \tag{15}$$

which is decreasing in $A$. Consequently, a large asset management sector can be used by the central bank to keep the risk premium compressed.

However, there is a tradeoff that comes from the larger asset management sector. We see from our solution for the threshold interest rate $r^*$ in equation (14) that the threshold interest rate is also decreasing in $A$. This means that the economy will jump to the high risk premium regime at a lower value of interest rates.

Figure 3 illustrates the effect of a larger asset management sector. Large A entails a lower risk premium in the low risk premium regime, but the jump to the high risk premium regime happens at a lower level of the interest rate. Thus, when the risk premium jumps at the trigger point, the jump will be larger.

**Figure 3. Risk Premium and Critical Threshold $r^*$ as a Function of the Size of Asset Management Sector**




Turning the comparison around, if we interpret the benchmark index realization $r$ as a market interest rate, then there is an upper bound to the size of the asset management sector for any level of the market interest rate that is consistent with the low risk premium regime. From the expression for the critical threshold $r^*$ given by equation (14), for the economy to be in the low risk premium regime, we need

$$r < r^* = \frac{c(S-A)}{v - c(S-A)} - \frac{1}{2}\phi. \tag{16}$$

This gives us an upper bound for $A$ for the low risk premium regime, namely,

$$A < S - \frac{\phi v}{2c + \phi v}. \tag{17}$$

So far, we have assumed that $A$ is exogenous. If instead we suppose that $A$ is growing in the low risk premium regime, then equation (17) represents the relationship between the feasible size of the asset management sector and the interest rate $r$. As $A$ grows, the central bank can maintain low risk premiums by keeping the interest rate low. Once the bound is reached, the central bank must reduce interest rates further to accommodate the growth in $A$. During this process, the risk premium continues to become compressed.

By accommodating further increases in $A$, the central bank is backing itself into a corner, as shown in figure 3. The risk premium gets compressed as $A$ grows, but the threshold point moves down. When, eventually, the central bank has to reverse course and raise interest rates, the jump will happen at a lower interest rate, and the jump in risk premium will be that much larger.

We conclude this section by identifying key features of the model. First, we have assumed that strategic complementarities arose for asset managers because of relative performance concerns—more specifically, last-place aversion. There are many reasons why asset managers might be concerned about the actions of money managers. Short-run concerns (in addition to long-run performance) would immediately give rise to the payoffs above. Following Morris and Shin (2004), we might think that while asset managers would like to perform well in absolute terms, they need to attain some minimum return or they will be fired. Relatedly, following Parlatore (2016), if funds rely on implicit or explicit guarantees from other institutions, then "breaking the buck" will require interventions and thus will give another reason for a performance threshold. Finally, Chen, Goldstein, and Jiang (2010) examine the role of classical bank-run payoffs in the context of equities funds, while Goldstein, Jiang, and Ng (2015) consider an analogous exercise for bond funds. If redemptions reduce investors' returns, then withdrawals by some investors provide incentives for others to withdraw. Our analysis is robust to the exact form of the agency frictions giving rise to strategic complementarities. There is a rich set of results in the literature on mutual fund flows, with the evidence pointing to investor redemptions being reinforced by asset manager sales (see Raddatz and Schmukler, 2012; Shek, Shim, and Shin, 2015). More broadly, our paper adds to the discussion on the procyclicality of the asset management sector (see Bank of England, 2014; Burkart and Dasgupta, 2015).

Second, runs occur in our model when there are changes in the return on short-run assets. We assumed that there was a small degree of heterogeneity in beliefs about those returns. However, all that matters for the global game equilibrium is that there is some heterogeneity in beliefs about some payoff-relevant parameter. As long as this is the case, small changes in returns to short-run assets can give rise to large shifts in funds.

## 2. Dynamics

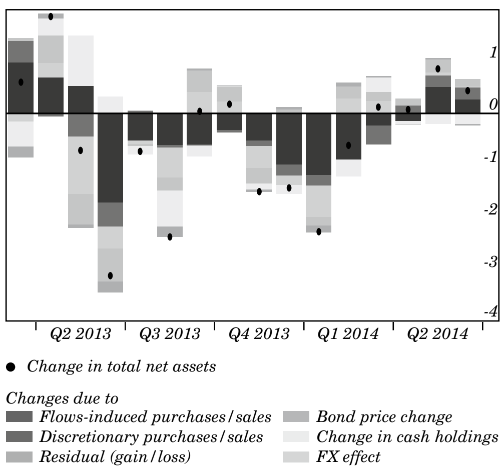
The model described in the previous section focused on the behavior of asset managers, holding fixed the assets $A$ invested in the sector. We now want to complete the model by discussing how investor funds flow into the asset management sector and redemptions from the sector. There are four stylized facts we would like to capture.

First, there is interaction between investor flows and the short-run coordination problem of asset managers. In particular, just as there is an agency friction in how funds are managed within the asset management sector, there is also an agency friction in how investment managers decide how much to invest in managed bond funds, and there are important interactions between these frictions. Figure 4 below from Shek, Shim, and Shin (2015) shows that investor redemptions from emerging market bond funds and discretionary positions of the funds move together.

Second, there is a tendency for the asset management sector to be endogenously at a tipping point, where the size of the asset management sector gives rise to a low but positive risk premium. Under the analysis of the previous section, there is a tendency for a run to occur at this tipping point in response to small changes.

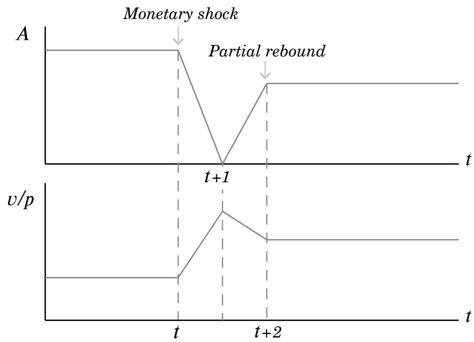
**Figure 4. Breakdown of Monthly Changes in Net Asset Value**

*Sum over 14 global EME local currency bond funds, in billions of US dollars*




Sources: EPFR; authors' calculation.

**Figure 5. Persistent Impact of Increase in Interest Rate above Threshold r***




Third, in a period of low interest rates and thus low expected returns in the short-run asset sector, there is a steady flow into the asset management sector. However, and fourth, the outflow when interest rates reverse jumps with the movement of asset manager's positions, but with "bounce back" where large sales from asset funds are followed by reversals that are not as large as the original outflow (Feroli and others, 2014). See figure 5 for a stylized depiction of such reversals.

How can we explain these four features simultaneously? We assume—consistent with the theory and evidence in Vayanos and Woolley (2013)—that reputational concerns of investment managers give rise to a tendency to allocate funds across sectors based on past performance. This is because investment managers cannot identify whether high or low performance of the sector is sector-specific or reflects overall performance of long-run returns in the economy. This gives rise to momentum in performance and flows. As managers learn, there is a tendency for flows to reverse, giving rise to prices returning to fundamental values and reversal in asset prices. We are now assuming a slow moving friction in fund flows into the management sector which then interacts with the asset managers' behavior. We write $A^*$ for the critical size of the asset management sector—identified in the previous section—where the risk premium is driven down to 0. Thus, we consider a reduced-form description of asset flows where

$$A_{t+1} = \lambda\left(A_t - A_{t-1}\right) + \mu\left(A^* - A_t\right)$$

for some constants λ and μ, where the first term in the equation corresponds to the momentum, with funds moving into the sector, resulting in short-run rising prices and more funds moving into the sector. But there is also a long-run effect—captured by the second term—for funds to move into the sector as long as the risk premium is positive.

This model will give rise to the stylized features above. First, the momentum effect will give rise to comovement of asset managers' positions and investment managers' movements of funds. Second, funds will move into the sector and approach $A^*$, the critical point at which runs will occur. Third, as money flows into the sector, both terms in the above difference equation will act in the same direction, with short-run performance and long-run concerns of investment managers moving in the same direction. Finally, when fund managers all exit, there are dramatic effects on the risk premium. This will create an incentive for asset managers to jump back in to attain good relative performance. However, redemptions by investors in response to the short-run price change will validate the price movement and the bounce back will not equal the initial decrease in prices.

## 3. Implications for Monetary Policy

Monetary policy is a powerful tool for influencing financial conditions. In particular, the commitment to lower interest rates into the future raises the prices of financial assets and compresses risk premiums, with consequences for real economic activity. In this respect, our analysis shares the conclusions from orthodox monetary analyses on the impact of forward guidance, especially the commitment to lower policy rates in the future.[3]

Our analysis parts company with orthodox monetary analysis on whether forward guidance and commitment to future rates is a policy that can be fine-tuned or reversed smoothly when the time comes to change tack. The market is not a person, and market prices need not correspond to the beliefs of that person. In our global game analysis, monetary policy works through the risk-taking channel, that is, through the risk-taking behavior of different sections of the market. Monetary policy affects risk premiums directly, so that the impact on

3. See Woodford (2012) for a forceful statement of this argument.

real economic activity flows through shifts in risk premiums, as well as shifts in the actuarially fair long-term rates.

One lesson from our analysis is that coordination problems can induce jumps in market prices, and quantities matter in the determination of the threshold points. The size of the asset management sector, as encapsulated by the holding of risky bonds $A_t$, determines the risk premium ruling at date $t$, as well as the threshold point for the benchmark index $r_t$ when a sell-off occurs. We can interpret the benchmark index as a market interest rate, and monetary policy will impinge on the coordination problem among asset managers through the determination of the benchmark index $r_t$.

To the extent that quantities matter, the lesson is similar to the one from the 2008 financial crisis. Just as we would be concerned with a build-up of leverage and the size of bank balance sheets, we should similarly be interested in the growth of holdings of fixed-income securities of buy-side investors. The central bank can compress risk premiums further by committing to low future interest rates and accommodating an increase in the size of the asset management sector. Nevertheless, there is a trade-off. By accommodating further growth of the asset management sector, the central bank is trading a lower risk premium today for a more disruptive unwinding at a lower threshold interest rate when, eventually, the central bank has to reverse course.

On the empirical front, our model suggests that observing the joint movements of price changes and quantity changes is informative about the risk-taking of market participants. In particular, the model predicts the joint occurrence of price declines and sales of the risky bond. Thus, rather than cushioning shocks, the demand response tends to amplify shocks.

Feroli and others (2014) conduct a vector autoregressive (VAR) analysis of price and valuation changes for risky fixed income categories, such as mortgage-backed securities, corporate bonds, and emerging market bonds. They find price declines are followed by sales, and sales are followed by further price declines. Consequently, the accumulated impulse responses of price and quantity shocks are large.

An implication for the conduct of monetary policy is that the separation of monetary policy and financial stability policy is much harder to accomplish than is often suggested. Under the risk-taking channel, monetary policy affects the economy through shifts in the risk-taking behavior of market participants. As such, any monetary policy shock is also a shock to risk-taking and hence is inseparable from the concern for financial stability.

Discussions of financial stability after the crisis have been conditioned by the experience of the crisis itself. After neglecting the dangers of excessive leverage and maturity mismatch before the crisis, policymakers have given them central importance since the crisis. As is often the case, accountability exercises usually address known past weaknesses, rather than asking where the new dangers are.

Our analysis suggests that the risk-taking channel may operate through financial institutions that are not leveraged. Asset managers typically have very low effective leverage and therefore do not become insolvent in the way that banks or highly leveraged hedge funds do. However, this does not mean that they do not have an impact on the economy. As the protagonists in financial market dynamics shift from banks to asset managers, researchers need to give more attention to the marketwide impact of institutional investors.

The risk-taking channel of monetary policy affects risk premiums directly, with effects on corporate investment and household consumption. These shocks could have a direct impact on GDP growth through subdued investment and consumption. The potential impact on the real economy is tangible, even though no institutions fail and no financial institutions are bailed out using public funds. Asset managers are not "systemic" in the sense defined in the Dodd-Frank Act as they are not "too big to fail." Nor are there easy regulatory solutions that would substitute for central bank interest rate policy in affecting risk-taking.

Thus, the most important implication of our analysis is that monetary policy and financial stability policy cannot be separated. They are, effectively, the same thing.

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# Quantitative Easing and Financial Stability

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Since the global financial crisis of 2008–09, many of the leading central banks have dramatically increased the size of their balance sheets and have shifted the composition of the assets that they hold toward larger shares of longer-term securities (as well as toward assets that are riskier in other respects). While many have hailed these policies as contributing significantly to containing the degree of damage to both the countries' financial systems and their real economies resulting from the collapse of confidence in certain types of risky assets, the policies have also been and remain quite controversial. One of the concerns raised by skeptics is that such quantitative easing by central banks may have been supporting countries' banking systems and aggregate demand only by encouraging risk-taking by ultimate borrowers and financial intermediaries in areas that increase the risk of precisely the sort of destructive financial crisis that led to the introduction of these policies in the first place.

The most basic argument for suspecting that such policies create risks to financial stability is simply that, according to proponents of these policies in the central banks (for example, Bernanke, 2012), they represent alternative means of achieving the same kind of relaxation of financial conditions that would, under more ordinary circumstances, be achieved by lowering the central bank's operating target for short-term interest rates—but a means that continues to be available even when short-term nominal interest rates have already reached their effective lower bound and so cannot be lowered to provide further

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stimulus. If one believes that a collateral effect of cuts in short-term interest rates—or perhaps even the main channel through which they affect aggregate demand, as argued by Adrian and Shin (2010)—is an increase in the degree to which intermediaries take more highly leveraged positions in risky assets, thereby increasing the likelihood or severity of a potential financial crisis, then one might suppose that to the extent that quantitative easing policies are effective in relaxing financial conditions in order to stimulate aggregate demand, they should similarly increase risks to financial stability.

One might go further and argue that such policies relax financial conditions by increasing the supply of central bank reserves.[1] An increase in the availability of reserves matters for financial conditions precisely because it relaxes a constraint on the extent to which private financial intermediaries can issue money-like liabilities (which are subject to reserve requirements) as a way of financing their acquisition of more risky and less liquid assets, as in the model of Stein (2012). Under this view of the mechanism by which quantitative easing works, one might suppose that it should be even more inevitably linked to an increase in financial stability risk than expansionary interest rate policy (which, after all, might also increase aggregate demand through channels that do not rely on increased risk-taking by banks).

Finally, some may be particularly suspicious of quantitative easing policies on the grounds that these policies, unlike conventional interest rate policy, relax financial conditions primarily by reducing the risk premiums earned by holding longer-term securities, rather than by lowering the expected path of the risk-free rate.[2] Such a departure from the normal historical pattern of risk premiums as a result of massive central bank purchases may seem a cause for alarm. If the premiums that exist when market pricing is not distorted by the central bank's intervention provide an important signal of the degree of risk that exists in the marketplace, then central bank actions that suppress this signal—not by actually reducing the underlying risks, but by preventing them from being fully reflected in market prices—

1. The term quantitative easing, originally introduced by the Bank of Japan to describe the policy it adopted in 2001 in an attempt to stem the deflationary slump that Japan had suffered in the aftermath of the collapse of an asset bubble in the early 1990s, refers precisely to the intention to increase the monetary base (and hence, it was hoped, the money supply more broadly) by increasing the supply of reserves.

2. Again, see Bernanke (2012) for discussion of this view of how the policies work, though he also discusses the possibility of effects of quantitative easing that result from central bank actions being taken to signal different intentions regarding future interest rate policy.

could distort perceptions of risk in a way that will encourage excessive risk-taking.

The present paper considers the extent to which these are valid grounds for concern about the use of this policy tool by central banks, by analyzing the mechanisms just sketched in the context of an explicit model of the way in which quantitative easing policies influence financial conditions and the way in which monetary policies more generally affect the incentives of financial intermediaries to engage in maturity and liquidity transformation of a kind that increases the risk of financial crisis. It argues, in fact, that the concerns just raised are of little merit. However, it does not reach this conclusion by challenging the view that quantitative easing policies can indeed effectively relax financial conditions (and so achieve effects on aggregate demand that are similar to the effects of conventional interest rate policy); nor does it deny that risks to financial stability are an appropriate concern of monetary policy deliberations or that expansionary interest rate policy tends to increase such risks (among other effects). The model developed here is one in which risk-taking by the financial sector can easily be excessive (in the sense that a restriction on banks' ability to engage in liquidity transformation to the same degree as under laissez-faire would raise welfare); in which, when that is true, a reduction in short-term interest rates through central bank action will worsen the problem by making it even more tempting for banks to finance acquisitions of risky, illiquid assets by issuing short-term safe liabilities; and in which the purchase of longer-term and/or risky assets by the central bank, financed by creating additional reserves (or other short-term safe liabilities, such as reverse repos or central bank bills, which would also be useful in facilitating transactions), will indeed loosen financial conditions, with an effect on aggregate demand that is similar, though not identical to, the effect of a reduction in the central bank's operating target for its policy rate. Nonetheless, the paper shows that quantitative easing policies should not increase risks to financial stability, but rather should tend to reduce them.

The reason for this different conclusion hinges on our conception of the sources of the kind of financial fragility that allowed the recent crisis to occur and the way in which monetary policy can affect the incentives to create a more fragile financial structure. In my view, the fragility that led to the crisis was greatly enhanced by the notable increase in maturity and liquidity transformation in the financial sector in the years immediately prior to the crisis (Brunnermeier, 2009; Adrian and Shin, 2010)—in particular, the significant increase in funding of financial intermediaries by issuance of collateralized short-term debt, such as repos (financing investment banks) or asset-backed commercial paper (issued by structured investment vehicles). Such financing is relatively inexpensive, in the sense that investors will hold the instruments even when they promise a relatively low yield, because of the assurance they provide that the investors will receive payment and can withdraw their funds at any time on short notice if desired. Too much of it is dangerous, however, because it exposes the leveraged institution to funding risk, which may require abrupt deleveraging through a fire sale of relatively illiquid assets. The sudden need to sell relatively illiquid assets to cover a shortfall of funding can substantially depress the price of those assets, requiring even more deleveraging and leading to a margin spiral of the kind described by Shleifer and Vishny (1992, 2011) and Brunnermeier and Pederson (2009).

It is important to ask why such fragile financial structures should arise as an equilibrium phenomenon, in order to understand how monetary policy may increase or decrease the likely degree of fragility. According to the perspective adopted here, investors are attracted to the short-term safe liabilities created by banks or other financial intermediaries because assets with a value that is completely certain are more widely accepted as a means of payment.[3] If an insufficient quantity of such safe assets is supplied by the government (through means that discussed below), investors will pay a money premium for privately issued short-term safe instruments with this feature, as documented by Greenwood, Hanson, and Stein (2010), Krishnamurthy and Vissing-Jorgensen (2012), and Carlson and others (2014). This provides banks with an incentive to obtain a larger fraction of their financing in this way. Moreover, they may choose an excessive amount of this kind of financing, despite the funding risk to which it exposes them, because each individual bank fails to internalize the effects of their collective financing decisions on the degree to which asset prices will be depressed in the event of a fire sale. This gives rise to a pecuniary externality, as a result of which excessive risk is taken in equilibrium (Lorenzoni, 2008; Jeanne and Korinek, 2010; Stein, 2012).

Conventional monetary policy, which cuts short-term nominal interest rates in response to an aggregate demand shortfall, can

3. The role of non-state-contingent payoffs in allowing an asset to be widely acceptable as a means of payment is discussed by Gorton and Pennacchi (1990), Gorton (2010), and Gorton, Lewellen, and Metrick (2012).

arguably exacerbate this problem, as low market yields on short-term safe instruments will further increase the incentive for private issuance of similar liabilities (Adrian and Shin, 2010; Giavazzi and Giovannini, 2012). The question of primary concern in this paper is whether quantitative easing policies, pursued as a means of providing economic stimulus when conventional monetary policy is constrained by the lower bound on short-term nominal interest rates, increase financial stability risks for a similar reason.

In the model proposed here, quantitative easing policies lower the equilibrium real yield on longer-term and risky government liabilities, just as a cut in the central bank's target for the short-term riskless rate will, and this relaxation of financial conditions has a similar expansionary effect on aggregate demand in both cases. Nonetheless, the consequences for financial stability are not the same. In the case of conventional monetary policy, a reduction in the riskless rate also lowers the equilibrium yield on risky assets because if it did not, the increased spread between the two yields would provide an increased incentive for maturity and liquidity transformation on the part of banks, which they pursue until the spread has decreased (because of diminishing returns to further investment in risky assets) to where it is again balanced by the risks associated with overly leveraged investment. (This occurs, in equilibrium, partly through a reduction in the degree to which the spread increases—which means that the expected return on risky assets is reduced—and partly through an increase in the risk of a costly fire-sale liquidation of assets.) In the case of quantitative easing, the equilibrium return on risky assets is reduced, but in this case through a reduction—rather than an increase—in the spread between the two yields. The money premium, which results from a scarcity of safe assets, should be reduced if the central bank asset's purchases increase the supply of safe assets to the public, as argued by Caballero and Farhi (2013) and Carlson and others (2014). Hence, the incentives for the creation of a more fragile financial structure are not increased as much by expansionary monetary policy of this kind.

The idea that quantitative easing policies, when pursued as an additional means of stimulus when the risk-free rate is at the zero lower bound, should increase risks to financial stability because they are analogous to an expansionary policy that relaxes reserve requirements on private issuers of money-like liabilities is also based on a flawed analogy. It is true, in the model of endogenous financial stability risk presented here, that a relaxation of a reserve requirement proportional to banks' issuance of short-term safe liabilities will (under a binding constraint) increase the degree to which excessive liquidity transformation occurs. It is also true that in a conventional textbook account of the way in which monetary policy affects financial conditions, an increase in the supply of reserves by the central bank relaxes the constraint on banks' issuance of additional money-like liabilities ("inside money") implied by the reserve requirement, so that the means through which the central bank implements a reduction in the riskless short-term interest rate is essentially equivalent to a reduction in the reserve requirement. However, this is not a channel through which quantitative easing policies can be effective, when the risk-free rate has already fallen to zero (or more generally, to the level of interest paid on reserves). For in such a case, reserves are necessarily already in sufficiently great supply for banks to be satiated in reserves, so that the opportunity cost of holding them must fall to zero in order for the existing supply to be voluntarily held. Under such circumstances (which is to say, those existing in countries like the United States since the end of 2008), banks' reserve requirements have already ceased to constrain their behavior. Hence, to the extent that quantitative easing policies are of any use at the zero lower bound on short-term interest rates, their effects cannot occur through this traditional channel.

In the model presented here, quantitative easing is effective at the zero lower bound (or more generally, even in the absence of reserve requirements or under circumstances where there is already satiation in reserves); this is because an increase in the supply of safe assets (through issuance of additional short-term safe liabilities by the central bank, used to purchase assets that are not equally money-like) reduces the equilibrium money premium. But whereas a relaxation of a binding reserve requirement would increase banks' issuance of short-term safe liabilities (and hence financial stability risk), a reduction in the money premium should reduce their issuance of such liabilities, so that financial stability risk should, if anything, be reduced.

The idea that a reduction in risk premiums as a result of central bank balance sheet policy should imply a greater danger of excessive risk-taking is similarly mistaken. In the model presented here, quantitative easing achieves its effects (both on the equilibrium required return on risky assets and on aggregate demand) by lowering the equilibrium risk premium—that is, the spread between the required return on risky assets and the riskless rate. But this does not imply the creation of conditions under which it should be more tempting for banks to take on greater risk. To the contrary, the existence of a smaller spread between the expected return on risky assets and the risk-free rate makes it less tempting to finance purchases of risky assets by issuing safe, highly liquid short-term liabilities that need pay only the riskless rate. Hence, again, a correct analysis implies that quantitative easing policies should increase financial stability, rather than threatening it.

The remainder of the paper develops these points in the context of an explicit intertemporal monetary equilibrium model, in which it is possible to clearly trace the general equilibrium determinants of risk premiums, the way in which they are affected by both interest rate policy and the central bank's balance sheet, and the consequences for the endogenous capital structure decisions of banks. Section 1 presents the structure of the model, and section 2 then derives the conditions that must link the various endogenous prices and quantities in an intertemporal equilibrium. Section 3 considers the effects of alternative balance sheet policies on equilibrium variables, focusing on the case of a stationary long-run equilibrium with flexible prices. Section 4 compares the ways in which quantitative easing and adjustments of reserve requirements affect banks' financing decisions. Finally, section 5 compares (somewhat more briefly) the short-run effects of both conventional monetary policy, quantitative easing, and macroprudential policy in the presence of nominal rigidities that allow conventional monetary policy to affect the degree of real economic activity. Section 6 concludes.

## 1. A Monetary Equilibrium Model with Fire Sales

This section develops a simple model of monetary equilibrium, in which it is possible simultaneously to consider the effects of the central bank's balance sheet on financial conditions (most notably, the equilibrium spread between the expected rate of return on risky assets and the risk-free rate of interest) and the way in which private banks' financing decisions can increase risks to financial stability. An important goal of the analysis is to present a sufficiently explicit model of the objectives and constraints of individual actors to allow welfare analysis of the equilibria associated with alternative policies that is based on the degree of satisfaction of the individual objectives underlying the behavior assumed in the model, as in the modern theory of public finance, rather than judging alternative equilibria on the basis of a more ad hoc criterion.[4]

Risks to financial stability are modeled using a slightly adapted version of the model proposed by Stein (2012). The Stein model is a three-period model in which banks finance their investments in risky assets in the first period; a crisis may occur in the second period, in which banks are unable to roll over their short-term financing and as a result may have to sell illiquid risky assets in a fire sale; and in the third period, the ultimate value of the risky assets is determined. The present model incorporates this model of financial contracting and occasional fire sales of assets into a fairly standard intertemporal general equilibrium model of the demand for money-like assets, namely, the cash-in-advance model of Lucas and Stokey (1987). In this way, the premium earned by money-like assets, which is treated as an exogenous parameter in Stein (2012), can be endogenized, and the effects of central bank policy on this variable can be analyzed, together with the consequences for financial stability.

## 1.1 Elements of the Model

Like most general equilibrium models of monetary exchange, the Lucas and Stokey (1987) model is an infinite-horizon model, in which the willingness of sellers to accept central bank liabilities as payment for real goods and services in any period depends on the expectation of being able to use those instruments as a means of payment in further transactions in future periods. The state space of the model is kept small (allowing a straightforward characterization of equilibrium, despite random disturbances each period) by assuming a representative household structure; the two sides of each transaction involving payment using cash are assumed to be two members of a household unit with a common objective, which can be thought of as a worker and a shopper. During each period, the worker and shopper from a given household have separate budget constraints (so that cash received by the worker as payment for the sale of produced goods cannot be immediately used by the shopper to purchase goods, in the same market), as is necessary for the cash-in-advance constraint to matter; but at the end of the period, their funds are again pooled in a

4. The proposed framework is further developed in Sergeyev (2016), which considers the interaction between conventional monetary policy and country-specific macroprudential policies in a currency union.

single household budget constraint (so that only the asset positions of households, which are all identical, matter at this point).

I employ a similar device, but increase the number of distinct roles for different household members, in order to introduce additional kinds of financial constraints into the model while retaining the convenience of a representative household. The model assumes that each infinite-lived household is made of four members with different roles during the period: a worker who supplies the inputs used to produce all final goods, and receives the income from the sale of these goods; a shopper who purchases regular goods for consumption by the household and who holds the household's cash balance, for use in such transactions; a banker who buys risky durable goods and issues short-term safe liabilities to finance some of these purchases; and an investor who purchases special final goods and can also bid for the risky durables sold by bankers in the event of a fire sale.[5] As in the Lucas-Stokey model, the different household members have separate budget constraints during the period (which is the significance of referring to them as different people), but pool their budgets at the end of each period in a single household budget constraint.

Four types of final goods are produced each period: durable goods and three types of nondurable goods, called cash goods, credit goods, and special goods. Workers also produce intermediate investment goods that are used as an input in the production of durable goods. Both cash and credit goods are purchased by shoppers; the distinction between the two types of goods is taken from Lucas and Stokey (1987), where the possibility of substitution by consumers between the two types of goods (one subject to the cash-in-advance constraint, the other not) allows the demand for real cash balances to vary with the size of the liquidity premium (opportunity cost of holding cash), for a given level of planned real expenditure. This margin of substitution also results in a distortion in the allocation of resources that depends on the size of the liquidity premium, and I wish to take this distortion into account when considering the welfare effects of changing the size of the central bank's balance sheet.

5. The distinction between bankers, investors, and worker/shopper pairs corresponds to the distinction in the roles of bankers, patient investors, and households in the model of Stein (2012). In the Stein model, these three types of agents are distinct individuals with no sharing of resources among them, rather than members of a single (larger) household; the device of having them pool assets at the end of each period is not needed to simplify the model dynamics, because the model simply ends when the end of the first and only period is reached (in the sense in which the term period is used in this model). In the present model, the representative household device also allows more unambiguous welfare comparisons among equilibria.

The introduction of special goods purchased only by the investor provides an alternative use for the funds available to the investor, so that the amount that investors will spend on risky durables in a fire sale depends on how low the price of the durables falls.[6] The produced durable goods in the model play the role of the risky investment projects in the model of Stein (2012): they require an initial outlay of resources, financed by bankers, in order to allow the production of something that may or may not yield a return later. The device of referring separately to investment goods and to the durable goods produced from them allows investment goods to be treated as perfect substitutes for cash or credit goods on the production side, resulting in a simple specification of workers' disutility of supplying more output, without having to treat durable goods as perfect substitutes for those goods, which would not allow the relative price of durables to rise in a credit boom.

All of the members of a given household are assumed to act so as to maximize a common household objective. Looking forward from the beginning of any period $t$, the household objective is to maximize

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ u(c_{1\tau},c_{2\tau}) + \tilde{u}(c_{3\tau}) + \gamma \underline{s}_\tau - v(Y_\tau) - w(x_\tau) \right]. \tag{1.1}$$

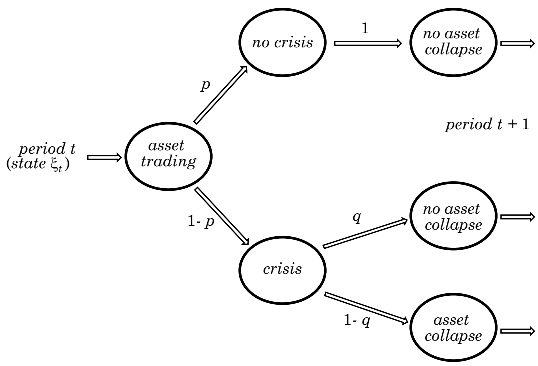
Here $c_{1t}$,$c_{2t}$ and $c_{3t}$ denote the household's consumption of cash goods, credit goods, and special goods, respectively, in period $t$; $\underline{s}_t$ denotes the quantity of durables held by the household at the end of period $t$ that have not proven to be worthless, and hence the flow of services in period $t$ from such intact durables; $Y_t$ denotes the household's supply of normal goods (a term used collectively for cash goods, credit goods, and investment goods, which are all perfect substitutes from the standpoint of a producer) in period $t$; and $x_t$ denotes the household's supply of special goods in period $t$. The functions $u(\cdot,\cdot)$, $\tilde{u}(\cdot)$, $v(\cdot)$ and $w(\cdot)$ are all increasing functions of each of their arguments; the functions $u(\cdot,\cdot)$and $\tilde{u}(\cdot)$ are strictly concave; and the functions $v(\cdot)$ and $w(\cdot)$ are at least weakly convex. The function $u(\cdot,\cdot)$ also implies that both cash and credit goods are normal goods, in the sense that it will be optimal to increase purchases of both types of goods if a household increases its expenditure on these types of goods on aggregate, while the (effective) relative price of the two types of goods remains the

6. The opportunity of spending on purchases of special goods plays the same role in this model as the possibility of investment in late-arriving projects in the model of Stein (2012).

same.[7] In addition, the discount factor satisfies $0 < \beta < 1$ and $\gamma < 0$. The operator $E_t[\cdot]$ indicates the expectation conditional on information at the beginning of period $t$.

Each of the infinite sequence of periods $t = 0, 1, 2, ,\ldots$ is subdivided into three subperiods, corresponding to the three periods in the Stein model. The sequence of events and the set of alternative states that may be reached in each period are indicated in figure 1. In subperiod 1, a financial market is open in which bankers issue short-term safe liabilities and acquire risky durables, and households decide on the cash balances to hold for use by the shopper.[8] In subperiod 2, information is revealed about the possibility that the durable goods purchased by the banks will prove to be valueless. With probability $p$, the no-crisis state is reached, in which it is known with certainty that no collapse in the value of the assets will occur, but with probability $1 - p$, a crisis state is reached, in which it is understood to be possible (though not yet certain) that the assets will prove to be worthless. Finally, in subperiod 3, the value of the risky durables is learned. In both of the no-asset-collapse states, a unit of the durable good produces one unit of services, while in the asset-collapse state (which occurs with probability $1 - q$, conditional on the crisis state being reached), durables provide no service flow.

**Figure 1. The Sequential Resolution of Uncertainty within Period *t***




7. The effective relative price is the relative price taking into account the cost to the household of having to hold cash in order to purchase cash goods.

8. This subperiod corresponds both to the first period of the Stein (2012) model, in which risky projects are financed, and to the securities-trading subperiod of the model in section 5 of Lucas and Stokey (1987), in which bonds are priced and hence the liquidity premium on cash is determined.

The various types of goods are produced and sold in subperiod 2. The markets in which the different goods are sold differ in the means of payment that are accepted. It is assumed, as in Lucas and Stokey (1987), that cash goods are sold only for cash that is transferred from the buyer to the seller at that time; the cash balances used for this purpose must have been acquired in subperiod 1 by the household to which that shopper belongs. (The liquidity premium associated with cash is thus determined in the exchange of cash for other financial claims in subperiod 1.) Credit goods are instead sold to shoppers on credit; this means (as in Lucas and Stokey) that accounts are settled between buyers and sellers only at the end of the period, at which point the various household members have again pooled their resources, so that charges by shoppers during the period can be paid out of the income received by workers for goods sold during that same period. The only constraint on the amount of this kind of credit that a household can draw on is assumed to be determined by a no-Ponzi condition (that is, the requirement that a household be able to pay off its debts eventually out of future income, rather than roll it over indefinitely). Investment goods are sold on credit in the same way. Special goods are also assumed to be sold on credit, but in this case, the amount of credit that investors can draw on is limited by the size of the line of credit arranged for them in subperiod 1. In particular, it is assumed that a given credit limit must be negotiated by the household before it learns whether a crisis will occur in subperiod 2 and thus whether investors will have an opportunity to bid on fire sale assets. The existence of the non-state-contingent credit limit for purchases by investors (both their purchases of special goods and their purchases of risky durables liquidated by the bankers in a fire sale) is important in order to capture the idea that only a limited quantity of funds can be mobilized (by potential buyers with the expertise required to evaluate the assets) to bid on the assets sold in a fire sale.[9]

The nature of the cash that can be used to purchase cash goods requires further comment. Unlike Lucas and Stokey, I do not assume that only monetary liabilities of the government constitute cash that is acceptable as a means of payment in this market. Instead, cash is identified with the class of short-term safe instruments (STSIs)

9. In the model of Stein (2012), this limit is ensured by assuming that the patient investors have a budget that is fixed as a parameter of the model. Here this budget is endogenized by allowing it to be chosen optimally by the household in subperiod 1, but it cannot be changed in subperiod 2.

discussed by Carlson and others (2014) in the case of the United States, which includes U.S. Treasury bills (and not simply monetary liabilities of the U.S. Federal Reserve) and certain types of collateralized short-term debt of private financial institutions. The assumption that only these assets can be used to purchase cash goods is intended to stand in for the convenience provided by these special instruments, which accounts for their lower equilibrium yields relative to the short-period holding returns on other assets.[10] The fact that all assets of this type, whether issued by the government (or central bank) or by bankers, are assumed equally to satisfy the constraint is intended to capture the way in which the demand for privately issued STSIs is observed to vary with the supply of publicly issued STSIs, as shown by Carlson and others (2014).

There are, of course, *also* special uses for base money (currency and reserve balances held at the Fed) as a means of payment, of the kind that Lucas and Stokey sought to model. In particular, when the supply of reserves by the Fed is sufficiently restricted, as was chronically the case prior to the financial crisis of 2008, the special convenience of reserve balances in facilitating payments between financial intermediaries results in a spread between the yield on reserves and that on STSIs such as Treasury bills; and the control of this spread by varying the supply of reserves was the focus of monetary policy prior to the crisis. Nonetheless, the spread between the yield on reserves and the Treasury bill rate (or federal funds rate) is not the one of interest here. Under the circumstances in which the Fed has conducted its experiments with quantitative easing, the supply of reserves has been consistently well beyond the level needed to drive the Treasury bill yield down to (or even below) the yield on reserves. Hence, while certain kinds of payments by banks are constrained by their reserve balances, this has not been a binding constraint in the period in which we wish to consider the effects of further changes in the central bank balance sheet. Granting that reserves have special uses that can result in a liquidity premium specific to them (under

10. One interpretation of the cash-in-advance constraint is that it actually represents a constraint on the type of assets that can be held by money-market mutual funds (MMMFs). Such a constraint gives rise to a money premium only to the extent that there are special advantages to investors of holding wealth in MMMFs, such as the ability to move funds quickly from them to make purchases. Rather than explicitly introducing a demand for cash on the part of MMMFs and assuming that households use their MMMF balances to make certain types of purchases, I obtain the same equilibrium money premium more simply by supposing that the STSIs can directly be used as a means of payment in certain transactions.

circumstances no longer relevant at present) does not in any way imply that STSIs cannot *also* have special uses for which other assets will not serve, giving rise to another sort of money premium—one that need not be zero simply because the premium associated with reserve balances has been eliminated.

The acceptability of a financial claim as cash that can be used to purchase cash goods is assumed to depend on its having a value at maturity that is completely certain, rather than being state-contingent. This requires not only that it be a claim to a fixed nominal quantity at a future date, but that it be viewed as completely safe, for one of two possible reasons: either it is a liability of the government (or central bank),[11] or it is collateralized in a way that allows a holder of the claim to be certain of realizing a definite nominal value from it. Bankers can issue liabilities that will be accepted as cash, but these liabilities will have to be backed by specific risky durables as collateral, and holders of the debt has the right to demand payment of the debt at any time, if they cease to remain confident that the collateral will continue to guarantee the fixed value for it.

When bankers purchase risky durables in the first subperiod, they can finance some portion of the purchase price by issuing safe debt (which can be used by the holder during the second subperiod to purchase cash goods), collateralized by the durables that are acquired. If in the second subperiod, the no-crisis state is reached, the durables can continue to serve as collateral for safe debt, as the value of the asset in the third subperiod can in this case be anticipated with certainty. In this case, bankers are able to roll over their short-term collateralized debt and continue to hold the durables. If instead the crisis state is reached, the durables can no longer collateralize safe debt, as there is now a positive probability that the durables will be worthless in the third subperiod. In this case, holders of the safe debt demand repayment in the second subperiod, and the bankers must sell durables in a fire sale, in the amount required to pay off the short-term debt. It is the right to force this liquidation that makes the debt issued by bankers in the first subperiod safe.

To be more specific, suppose that the sale of goods (and in particular, cash goods) occurs at the beginning of the second subperiod: after it

11. A claim on a government need not be completely safe. If, however, a government borrows in its own fiat currency, and if it is committed to ensure that its nominal liabilities are paid with certainty (by monetizing them if necessary), then it is possible for it to issue debt that is correctly viewed as completely safe (in nominal terms).

has been revealed whether the crisis state will occur, but before the decision whether to demand immediate repayment of the short-term debt is made. Thus, at the time that shoppers seek to purchase cash goods, they may hold liabilities issued by bankers that grant the holder the right to demand repayment at any time; it is the fact that the short-term debt has this feature that allows it to be accepted as cash in the market for cash goods. After the market for cash goods has taken place, the holders of the bankers' short-term debt (who may now include the sellers of cash goods) decide whether to demand immediate repayment of the debt. At this point, these holders (whether shoppers or workers) only care about the contribution that the asset will make to the household's pooled end-of-period budget. In the crisis state, they will choose to demand repayment, since this ensures them the face value of the debt, whereas if they do not demand repayment, they will receive the face value of the debt with probability $q < 1$, but will receive nothing if the asset-collapse state occurs. If they demand repayment, they receive a claim on the investors who purchase the collateral in the fire sale; such a claim is assumed to guarantee payment in the end-of-period settlement, if within the bound of the line of credit arranged for the investor in the first subperiod.

The other source of assets that count as cash is the government. Some very short-term government liabilities (Treasury bills) count as cash. In addition, the central bank can issue liabilities that also count as cash. If the central bank increases its supply of SFSIs by purchasing Treasury bills (which are themselves SFSIs), the overall supply of cash will be unchanged. (This again demonstrates that the concept of cash used here differs importantly from that of Lucas and Stokey.) But if the central bank purchases noncash assets (either longer-term Treasury bonds, which are less able to facilitate transactions than are shorter-term bills, or assets subject to other kinds of risk) and finances these purchases by creating new short-term safe liabilities, it can increase the net supply of SFSIs. We are interested in the effects of this latter kind of policy.

## 1.2 Budget Constraints and Definition of Equilibrium

Each household begins period $t$ with $I_{t-1}$ units of the investment good (purchased in the previous period) and financial wealth $A_t$, which may represent claims on either the government or other households, and is measured in terms of the quantity of cash that would have the same market value in subperiod 1 trading (even though the assets aggregated in $A_t$ need not all count as cash). In the first subperiod, the investment good is used to produce $F(I_{t-1})$ units of the durable good, which can sold on a competitive market at price $Q_t$ per unit.[12] The banker in each household purchases a quantity $s_t$ of these durables, financed partly from funds provided by the household for this purpose and partly by issuing short-term collateralized debt in quantity $D_t$. Here $D_t$ is the face value of the debt, the nominal quantity to which the holder is entitled (with certainty) in the settlement of accounts at the end of period $t$. The price $Q_t$ of the risky asset is quoted in the same (nominal, end-of-period) units; thus, the quantity of funds that the household must provide to the banker is equal to $Q_t s_t - D_t$ in those units.

The household's other uses of its beginning-of-period financial wealth are to acquire cash, in quantity $M_t$, for use by the shopper and to acquire (longer-term) bonds $B_t$, which are government liabilities that do not count as cash. The quantity $M_t$ represents the end-of-period nominal value of these safe assets; thus, if interest is earned on cash (as the model allows), $M_t$ represents the value of the household's cash balances inclusive of the interest earned on them, rather than the nominal value at the time that they are acquired.[13] The quantity of bonds $B_t$ is measured in terms of the number of units of cash that have the same market value in subperiod 1 trading (as with the measurement of $A_t$). Hence, the household's choices of $s_t$, $D_t$, $M_t$, and $B_t$ in the first subperiod are subject to an interim budget constraint,

$$\left(Q_t s_t - D_t\right) + M_t + B_t \leq A_t + Q_t F(I_{t-1}). \tag{1.2}$$

The financing decisions of bankers are also subject to a constraint that safe debt cannot be issued in a quantity beyond that for which

12. We may alternatively suppose that the investment goods are purchased by construction firms that produce the durables and sell them to bankers, and that households simply begin the period owning shares in these construction firms. The explicit introduction of such firms would not change the equilibrium conditions presented below.

13. If cash is equivalent to Treasury bills, $M_t$ represents their face value at maturity, rather than the discounted value at which they are purchased.

they can provide sufficient collateral, given their holdings of the durable $s_t$.[14] This requires that

$$D_t \leq \Gamma_t s_t, \tag{1.3}$$

where $\Gamma_t$ is the market price of the durable good in the fire sale, should one occur in period $t$. (Here $\Gamma_t$ is quoted in terms of the units of nominal value to be delivered by investors in the end-of-period settlement of accounts. Note that while it is not yet known in subperiod 1 whether a crisis will occur, the price $\Gamma_t$ that will be realized in the fire sale *if* one occurs is perfectly forecastable.) Constraint (1.3) indicates the amount of collateral required to ensure that whichever state is reached in subperiod 2, the value of the collateralized debt will equal $D_t$, since sale of the collateral in a fire sale will yield at least that amount.

Regardless of the state reached in subperiod 2, the shopper's cash goods purchases must satisfy the cash-in-advance constraint:

$$P_t c_{1t} \leq M_t, \tag{1.4}$$

where $P_t$ is the price of normal goods in period $t$ (which may depend on the state reached in subperiod 2), quoted in units of the nominal value to be delivered in the end-of-period settlement. It is this constraint that provides a reason for the household to choose to hold cash balances $M_t$. The common price for all normal goods follows from the fact that these goods are perfect substitutes from the point of view of their producers (workers) and that all payments that guarantee the same nominal value in the end-of-period settlement are of equal value to the sellers, once the problem of verifying the soundness of payments made in the cash goods market has been solved.[15]

There is no similar constraint on shopper's purchases of credit goods or investment goods, as these are sold on credit. The investor's

14. We might suppose that bankers can also issue debt that is not collateralized—or not collateralized to this extent. But such liabilities would not be treated as cash by the households that acquire them, so that allowing such debt to be issued by a banker would have no consequences any different from allowing the household itself to issue such debt in the first subperiod, in order to finance a larger equity contribution to its banker. Furthermore, allowing households to trade additional kinds of noncash financial liabilities would make no difference for the equilibrium conditions derived here; it would simply allow us to price the additional types of financial claims. The ability of bankers to issue collateralized short-term debt that counts as cash instead matters; this is not a type of claim that a household can issue other than by having its banker issue it (because it must be collateralized by risky durable goods), and issuing such claims has special value because they can relax the cash-in-advance constraint.

15. Cash goods and credit goods sell for the same price in any given period for the same reason in the model of Lucas and Stokey (1987).

purchases $c_{3t}$ of special goods, and purchases $s_t^{*d}$ of durables in the fire sale[16] must, however, satisfy a state-contingent budget constraint:

$$\tilde{P}_t c_{3t} + \eta_t \Gamma_t s_t^{*d} \leq F_t, \tag{1.5}$$

where $\tilde{P}_t$ is the price of special goods (which are quoted in the same units as $P_t$ and which similarly may depend on the state reached in subperiod 2); $\eta_t$ is an indicator variable for the occurrence of a crisis in period $t$;[17] and $F_t$ is the line of credit arranged for the investor in subperiod 1, quoted in units of the nominal quantity that the investor can promise to deliver in the end-of-period settlement, and with a value that must be independent of the state that is realized in subperiod 2.[18]

If the crisis state is reached in subperiod 2, the banker offers $s_t^{*s}$ units of the durable goods for sale in the fire sale, the quantity of which must satisfy the bounds,

$$D_t \leq \Gamma_t s_t^{*s} \leq \Gamma_t s_t. \tag{1.6}$$

The first inequality indicates that the banker must liquidate sufficient assets to allow repayment of the short-term debt (given that in this state, the holders will necessarily demand immediate repayment); the second inequality follows from the fact that bankers cannot offer to sell more shares of the durable good than they owns. The range of possible quantity offers defined in equation (1.6) is nonempty only because equation (1.3) has been satisfied; thus, a plan that satisfies equation (1.6) necessarily satisfies equation (1.3), making the earlier constraint technically redundant.

Given these decisions, the durables owned by the household in subperiod 3 will equal

$$\underline{s}_t = s_t + \eta_t \left( s_t^{*d} - s_t^{*s} \right) \tag{1.7}$$

16. We use the notation $s_t^*$ for the quantity of durables liquidated in the fire sale, if one occurs in period $t$. An additional superscript $d$ is used for the quantity demanded on this market, and a superscript $s$ for the quantity supplied. Note that $s_t^{*d}$ and $s_t^{*s}$ are two independent choice variables for an individual household, and they need not be chosen to be equal, even though in equilibrium they must be equal (given common choices by all households) in order for the market to clear.

17. That is, $\eta_t$=1 if a crisis occurs, while $\eta_t$=0 if the no-crisis state is reached.

18. Like constraint (1.4), constraint (1.5) is actually two constraints, one for each possible state that may be reached in subperiod 2.

if the durables prove to be valuable, while $\underline{s}_t = 0$ regardless of the household decisions in the asset-collapse state. The household's pooled financial wealth at the end of the period (in nominal units) will be given by

$$W_t = M_t + \left(\frac{R_t^b}{R_t^m}\right) B_t + P_t Y_t - P_t \left(c_{1t} + c_{2t} + I_t\right) + \tilde{P}_t x_t + \eta_t \Gamma_t s_t^{*s} - D_t - F_t + T_t. \tag{1.8}$$

This consists of the household's cash balances at the end of subperiod 1, plus the end-of-period value of the bonds that it holds at the end of subperiod 1, plus additional funds obtained from the sale of both normal goods and special goods in subperiod 2, plus funds raised in the fire sale of assets in the event of a crisis, minus the household's expenditure on normal goods of the various types in subperiod 2, and minus the amounts that it must repay at the end of the period (if not sooner) to pay off the collateralized debt issued by the banker and to pay for the line of credit arranged for the investor, plus the nominal value $T_t$ of net transfers from the government. Because the household must pay $F_t$ regardless of the extent to which the line of credit is used, the investor's expenditure does not need to be subtracted, as it is paid for when $F_t$ is paid.[19] Additionally, bonds that cost the same amount as one unit of cash in subperiod 1 are worth as much as $R_t^b / R_t^m$ units of cash at the end of the period, where $R_t^m$ is the gross nominal yield on cash (assumed to be known when the cash is acquired in subperiod 1, since these assets are riskless in nominal terms), and $R_t^b$ is the gross nominal holding return on bonds (which may depend on the state reached by the end of the period).

Each household is subject to a borrowing limit,

$$W_t \geq \underline{W_t}, \tag{1.9}$$

19. The assumption that $F_t$ must be paid whether or not the full line of credit is used is important because it prevents the household from simply asking for a large line of credit, as much as would be desired in the crisis state, and then not using all of it in the noncrisis state. If that were possible at no cost, the non-state-contingency of the credit available to the investor would have no bite. The assumption that the line of credit must be paid for whether used or not makes this costly and results in the household's wishing ex post in the crisis state that it had provided more funds to the investor—although it also wishes ex post in the noncrisis state that it had provided less credit to the investor. This device implies that the credit available to the investor will be optimal on average, though not optimal in each state because it cannot be state-contingent.

expressed as a lower bound on its net worth after the end-of-period settlement of accounts. I do not further specify the precise value of the borrowing limit, but it can be set tight enough to ensure that any end-of-period net indebtedness can eventually be repaid, while at the same time being loose enough so that the constraint (1.9) never binds in any period. Finally, the household carries into period $t + 1$ the investment goods $I_t$ purchased in subperiod 2 of period $t$, as well as financial wealth in the amount of

$$A_{t+1} = R^m_{t+1} W_t, \tag{1.10}$$

where the multiplicative factor $R^m_{t+1}$ converts the value of the household's financial wealth at the beginning of period $t + 1$ into an equivalent quantity of cash (measured in terms of the face value of the STSIs rather than their cost in subperiod 1 trading).

A feasible plan for a household is then a specification of the quantities $M_t$, $B_t$, $s_t$, $D_t$, $F_t$, $s_t^{*s}$, for each period $t$, as a function of the history $\xi_t$ of shocks up until then, and a specification of the quantities $c_{1t}$, $c_{2t}$, $c_{3t}$, $I_t$, $Y_t$, and $x_t$, for each period $t$, as a function of both $\xi_t$ and $\eta_t$ (that is, whether a crisis occurs in period $t$), that satisfies the constraints (1.2)–(1.3) for each possible history $\xi_t$ and the constraints (1.4)–(1.10) for each possible history $(\xi_t, \eta_t)$, given initial financial wealth $A_0$ and pre-existing investment goods $I_{-1}$ and also given the state-contingent evolution of the prices, net transfers from the government to households, as well as the borrowing limit. An optimal plan is a feasible plan that maximizes equation (1.1).

Equilibrium requires that all markets for goods and assets clear. Thus, it requires that in the first subperiod of period $t$,

$$M_t = \tilde{M}_t + D_t, \tag{1.11}$$

$$B_t = B^s_t, \tag{1.12}$$

and

$$s_t = F(I_{t-1}), \tag{1.13}$$

where $\tilde{M}_t$ is the public supply of cash (short-term safe liabilities of the government or of the central bank) and $B_t^s$ is the supply of longer-term government bonds (not held by the central bank). For simplicity, durables are assumed to fully depreciate after supplying a service flow (in the event that there is no asset collapse) in the period in which they are produced and acquired by bankers; thus, the supply of durables to be acquired by bankers in period $t$ is given simply by the new production $F(I_{t-1})$ and is independent of the quantity $\underline{s}_{t-1}$ of valuable durables in the previous period.

Equilibrium also requires that in the second subperiod, if a crisis occurs,

$$s_t^{*d} = s_t^{*s}, \tag{1.14}$$

and that in either the crisis or in the noncrisis state,

$$c_{1t} + c_{2t} + I_t = Y_t, \tag{1.15}$$

and

$$c_{3t} = x_t. \tag{1.16}$$

A flexible-price equilibrium can then be defined as a specification of prices $Q_t$ and $\Gamma_t$ and cash yield $R_t^m$ for each history $\xi_t$, and prices $P_t$ and $\tilde{P}_t$ and bond yields $R_t^b$ for each history $(\xi_t, \eta_t)$ together with a plan (as described above) for the representative household, such that (i) the plan is optimal for the household, given those prices, and (ii) the market-clearing conditions (1.11)–(1.14) are satisfied for each history $\xi_t$ and conditions (1.15)–(1.16) are satisfied for each history $(\xi_t, \eta_t)$.

## 1.3 Fiscal Policy and Central Bank Policy

The equilibrium conditions above involve several variables that depend on government policy: the supplies of outside financial assets $\tilde{M}_t$ and $B_t^s$ the net transfers $T_t$, and the yields $R_t^m$ and $R_t^b$ on the outside financial assets. Fiscal policy determines the evolution of end-of-period claims on the government,

$$L_t \equiv \tilde{M}_t + \left(\frac{R_t^b}{R_t^m}\right) B_t^s + T_t \tag{1.17}$$

by varying state-contingent net transfers to households appropriately. The Treasury also has a debt management decision: at the beginning of each period $t$, it must decide how much of existing claims on the government will be financed through STSIs (that is, issuance of Treasury bills), as opposed to longer-term debt that cannot be used to satisfy the cash-in-advance constraint. Let $\tilde{M}_t^g$ be Treasury bill issuance by the Treasury in the first subperiod of period $t$; it follows that the total supply of longer-term debt by the Treasury will equal[20]

$$B_t^g = R_t^m L_{t-1} - \tilde{M}_t^g. \tag{1.18}$$

Of these longer-term securities issued by the Treasury, a quantity $B_t^{cb}$ will be held as assets of the central bank, backing central bank liabilities $\tilde{M}_t^{cb}$ of equal value. I assume that all of these central bank liabilities are STSIs that count as cash. The supply of outside assets to the private sector is then given by

$$\tilde{M}_t \equiv \tilde{M}_t^g + \tilde{M}_t^{cb} \tag{1.19}$$

and

$$B_t^s \equiv B_t^g - B_t^{cb}. \tag{1.20}$$

In equilibrium, the net wealth $W_t$ of the representative household at the end of period $t$ must equal net claims $L_t$ on the government.[21] It then follows from equations (1.10) and (1.18) that the beginning-of-period assets $A_t$ of the representative household must equal

$$A_t = \tilde{M}_t^g + B_t^g.$$

Alternatively, since $\tilde{M}_t^{cb} = B_t^{cb}$

$$A_t = \tilde{M}_t + B_t^s, \tag{1.21}$$

in terms of the supplies of outside assets to the private sector.

At the end of period $t$, the central bank's assets are worth $(R_t^b / R_t^m) B_t^{cb}$, while its liabilities are worth $\tilde{M}_t^{cb} = B_t^{cb}$. In general, these quantities will not be equal; I assume, however, that net balance sheet earnings must be rebated to the Treasury at the end of the period, in a transfer of magnitude

20. Note that liabilities with a market value the same as $\tilde{M}_t^g + B_t^g$ units of cash in subperiod 1 will have a market price of $(\tilde{M}_t^g + B_t^g)/ R_t^m$.

21. A comparison of the definition of $W_t$ in equation (1.8) with the definition of $L_t$ in equation (1.17) shows that the market-clearing conditions imply that $W_t = L_t$.

$$T_t^{cb} = \left(\frac{R_t^b}{R_t^m}\right) B_t^{cb} - \tilde{M}_t^{cb}.$$

A transfer from the central bank to the Treasury allows the Treasury to make a larger transfer to the private sector while achieving the same target for end-of-period claims on the government. However, this does not change formula (1.17) for the size of net transfer that is made to the private sector, because that equation was already written in terms of a consolidated budget constraint for the Treasury and central bank. If instead we write

$$T_t^g = L_t - \tilde{M}_t^g - \left(\frac{R_t^b}{R_t^m}\right) B_t^g$$

for the net transfer from the Treasury required to achieve the target $L_t$ neglecting any transfers from the central bank, then

$$T_t = T_t^g + T_t^{cb}.$$

Finally, in addition to choosing the size of its balance sheet, the central bank can choose the nominal interest rate $R_t^m$ paid on its liabilities. In the model, where central bank liabilities (reserves, reverse repos, or central bank bills) are treated as perfect substitutes for all other forms of cash (Treasury bills or STSIs issued by private banks), this policy decision directly determines the equilibrium yield on those other forms of cash, as well.[22] There are thus two independent dimensions of central bank policy each period, each of which can be chosen independently of fiscal policy (that is, of the evolution of both total claims on the government $L_t$ and the supply of short-term safe government liabilities), except to the extent that perhaps $B_t^{cb}$ must be

22. In a more complex model in which reserve balances at the central bank play a special role that other STSIs cannot fulfill and are in sufficiently scarce supply, there will be a spread between the interest rate paid on reserves and the equilibrium yield on other STSIs, although the central bank will still have relatively direct control over the equilibrium yield on STSIs, by varying either the interest rate paid on reserves or the degree of scarcity of reserves. Even before the increased size of central bank balance sheets resulting from the financial crisis, many central banks implemented their interest rate targets largely by varying the interest rate paid on reserve balances, as discussed in Woodford (2003, chap. 1).

no greater than $B_t^g$.[23] These can alternatively be described as either implementation of the central bank's target for the interest rate paid on cash or variation in the size of its balance sheet holding fixed its target for that interest rate.

There is a further potential dimension of central bank policy, which is choice of the composition of its balance sheet. Above I assumed that the central bank holds only longer-term Treasury securities, but it might also hold Treasury bills on its balance sheet (as indeed the U.S. Federal Reserve does). In this model, however, it is easy to see that central bank acquisition of Treasury bills (financed by issuing central bank liabilities that are perfect substitutes for Treasury bills and pay the same rate of interest) will have no effect on any other aspect of equilibrium. To simplify the algebra, this possibility is not introduced in the notation above.

## 2. Determinants of Intertemporal Equilibrium

This section characterizes equilibrium in the model just described, with particular attention to the determinants of the supply of and demand for safe assets and the supply of and demand for risky durables, both when originally produced and in the event of a fire sale.

### 2.1 Conditions for Optimal Behavior

To begin, there are some necessary conditions for optimality of the representative household's behavior. An optimal plan for the household (as defined in the previous section) is one that maximizes a Lagrangian:

$$\begin{aligned}
&\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ u(c_{1t}, c_{2t}) + \tilde{u}(c_{3t}) + \gamma \left[ (1-\eta_t) s_t + \eta_t q \left( s_t + s_t^{*d} - s_t^{*s} \right) \right] - v(Y_t) - w(x_t) \qquad (2.1)\\
&- \phi_{1t} \left[ M_t + B_t + Q_t \left( s_t - F(I_{t-1}) \right) - A_t - D_t \right] - \eta_t \phi_{2t} \left( D_t - \Gamma_t s_t^{*s} \right) \\
&- \eta_t \phi_{3t} \left( \Gamma_t s_t^{*s} - \Gamma_t s_t \right) - \phi_{4t} \left( P_t c_{1t} - M_t \right) - \phi_{5t} \left( \eta_t \Gamma_t s_t^{*d} + \tilde{P}_t c_{3t} - F_t \right) \\
&- \phi_{6t} \left[ \left( \frac{A_{t+1}}{R_{t+1}^m} \right) - M_t - \left( \frac{R_t^b}{R_t^m} \right) B_t - P_t \left( Y_t - c_{1t} - c_{2t} - I_t \right) - \tilde{P}_t x_t - \eta_t \Gamma_t s_t^{*s} + D_t + F_t - T_t \right] \Big\},
\end{aligned}$$

23. In fact, within the logic of the model, there is no problem with allowing $B_t^{cb}$ to exceed $B_t^g$; this would simply require negative holdings of government bonds by the private sector (issuance of "synthetic" bonds by the private sector), which can already be accommodated in the constraints specified above.

where I have substituted equation (1.7) for $\underline{s}_t$ in the utility function, and equation (1.8) for $W_t$ in equation (1.10), in order to eliminate two variables and constraints from the maximization problem (and thus allow simplification of the Lagrangian). There is also no term corresponding to the constraint (1.9), as in the equilibria discussed below I assume that the borrowing constraint is set so as not to bind in any period.[24]

Differentiating the Lagrangian with respect to the choice variables $M_t$, $B_t$, $s_t$, $D_t$, $s_t$, $s_t^{*s}$, $s_t^{*d}$, $F_t$, $c_{1t}$, $c_{2t}$, $c_{3t}$, $I_t$, $Y_t$, $x_t$, and $A_{t+1}$, respectively, yields the first-order conditions:

$$\phi_{1t} = \mathrm{E}_t\left[\phi_{4t} + \phi_{6t}\right] \tag{2.2}$$

$$\phi_{1t} = \mathrm{E}_t\left[\left(\frac{R_t^b}{R_t^m}\right)\phi_{6t}\right], \tag{2.3}$$

$$\phi_{1t} = (1-p)\phi_{2t} + \mathrm{E}_t\phi_{6t}, \tag{2.4}$$

$$\phi_{1t}Q_t = \gamma\left[p + (1-p)q\right] + (1-p)\phi_{3t}\Gamma_t, \tag{2.5}$$

$$\gamma q = (\phi_{2t} - \phi_{3t})\Gamma_t + \phi_{6t}^c\Gamma_t, \tag{2.6}$$

$$\gamma q = \phi_{5t}^c\Gamma_t, \tag{2.7}$$

$$\mathrm{E}_t\phi_{5t} = \mathrm{E}_t\phi_{6t}, \tag{2.8}$$

$$u_1(c_{1t}, c_{2t}) = P_t\left(\phi_{4t} + \phi_{6t}\right), \tag{2.9}$$

$$u_2(c_{1t}, c_{2t}) = P_t\phi_{6t}, \tag{2.10}$$

$$\tilde{u}'(c_{3t}) = \tilde{P}_t\phi_{5t} \tag{2.11}$$

24. We assume a borrowing limit that constrains the asymptotic behavior of the household's net wealth position far in the future, so as to preclude running a "Ponzi scheme," but that does not constrain the household's borrowing over any finite number of periods.

$$\beta\phi_{1,t+1}Q_{t+1}F'(I_t) = P_t\phi_{6t}\,, \tag{2.12}$$

$$v'(Y_t) = P_t\phi_{6t}\,, \tag{2.13}$$

$$w'(x_t) = \tilde{P}_t\phi_{6t}\,, \tag{2.14}$$

and

$$\phi_{6t} = \beta R^m_{t+1}\phi_{1,t+1}\,, \tag{2.15}$$

for each .

In these conditions, the first seven choice variables ($M_t$ through $F_t$) must be chosen only as a function of the history $\xi_t$ (that is, the state at the beginning of period $t$), while the other seven variables ($c_{1t}$ through $A_{t+1}$) may depend on $\eta_t$ (that is, whether a crisis occurs in period $t$) as well as $\xi_t$. This means that while there is only one condition corresponding to each of the equations (2.2)–(2.8) for each history $\xi_t$, each of the equations (2.9)–(2.15) actually corresponds to two conditions for each history $\xi_t$, one for each of the two possible states that may be reached in subperiod 2 (crisis or noncrisis). Similarly, the Lagrange multipliers $\phi_{1t}$, $\phi_{2t}$, and $\phi_{3t}$ will each have a single value for each history $\xi_t$, but the values of the multipliers $\phi_{4t}$, $\phi_{5t}$, and $\phi_{6t}$ may differ depending on the state reached in subperiod 2. The conditional expectation $E[\cdot]$ that appears in conditions such as (2.2) refers to the expected value (as of the first subperiod of period $t$) of variables that may take different values depending which state is reached in subperiod 2.

The superscript c appearing on Lagrange multipliers in equations (2.6)–(2.7) indicates the value of the multiplier in the case that the crisis state occurs in subperiod 2. Thus, condition (2.6) indicates the way in which the values of the multipliers $\phi_{2t}$ and $\phi_{3t}$ (which relate to constraints that apply only in the event that the crisis state is reached) depend on the value of the multiplier $\phi_{6t}$ in the event of a crisis in period $t$; but this value may be different from the value of $\phi_{6t}$ if no crisis occurs.

In writing the first-order conditions in this form, I have assumed for simplicity that any random disturbances (other than learning whether or not an asset-collapse occurs, after a crisis state is reached in subperiod 2) are realized in subperiod 2 of some period. Under this assumption, there is no difference between the information set in the first subperiod of period $t+1$ (denoted $\xi_{t+1}$) and the information set in subperiod 2 of period t.[25] I also assume that while the yield $R^b_{t+1}$ on longer-term government debt may depend on the state reached in subperiod 2 of period $t + 1$, the yield $R^m_{t+1}$ on safe short-term liabilities of the central bank does not; hence, this also must be known as of subperiod 2 of period $t$. Thus, the central bank's decision about the policy rate $R^m_{t+1}$ (which should actually be regarded as the period $t$ interest rate decision[26]) must be announced in subperiod 2 of period $t$.[27] Conditions (2.12) and (2.15) can then be written without conditional expectations, as the variables with subscripts $t + 1$ in these equations are ones with values that are already perfectly predictable in subperiod 2 of period $t$.

In addition to the first-order conditions (2.2)–(2.15), the household's decision variables must satisfy the constraints of the household problem, together with a set of complementary slackness conditions. Condition (2.13), together with the assumption that $v'(Y)>0$ for all possible values of $Y$, implies that $\phi_{6t} > 0$ necessarily; similarly, given nonsatiation in special goods, condition (2.11) implies that $\phi_{5t} > 0$ necessarily. Because it is associated with an inequality constraint—namely, condition (1.4)—the multiplier $\phi_{4t}$ is necessarily nonnegative; condition (2.2) then implies that $\phi_{1t} > 0$ necessarily. The remaining multipliers, $\phi_{2t}$, $\phi_{3t}$, and $\phi_{4t}$, are associated with inequality constraints

25. There is, of course, the difference that by the beginning of period $t + 1$, it will be known whether an asset collapse occurred in period $t$, while this is not yet known in subperiod 2 of period $t$ (in the case that the crisis state is reached). However, because of the assumption of full depreciation of existing durables at the end of each period, while the occurrence of an asset collapse affects household utility, it has no consequences for the assets carried by the household into the following period, the amounts of which are already predictable in subperiod 2 as long as no other random disturbances (such as an unexpected change in the size of net transfers $T_t$) are allowed to occur in subperiod 3. Policy in periods $t + 1$ and later is also assumed to be independent of whether an asset collapse has occurred in period $t$. Therefore, the relevant information set for equilibrium determination in subperiod 1 of period $t + 1$ is independent of whether an asset collapse has occurred.

26. $R^m_{t+1}$ is the nominal yield between the settlement of accounts at the end of period $t$ and the settlement of accounts at the end of period $t + 1$ on wealth that is held in the form of cash. This would often be called the period $t$ riskless rate of interest, as it must be determined before the period for which the safe return is guaranteed. I use the notation $R^m_{t+1}$ rather than $R^m_t$ for consistency with the notation $R^m_{t+1}$ for the one-period holding return on longer-term bonds over the same time period; the latter variable is generally not perfectly predictable in subperiod 2 of period $t$.

27. The model similarly assumes that the Treasury's decision about the Treasury bill supply $\tilde{M}^g_{t+1}$ and the central bank's decision about the size of its balance sheet $\tilde{M}^{cb}_{t+1}$ are announced in subperiod 2 of period $t$. The Treasury's decision about the size of net transfers $T_t$ and hence the value of total claims on the government $L_t$ at the end of period $t$ are also announced in subperiod 2 of period $t$.

and so are necessarily nonnegative, but they may be equal to zero if the constraints in question do not bind (as discussed below). If any of these multipliers has a positive value, the corresponding inequality constraint must hold with equality.

## 2.2 Characterizing Equilibrium

In an equilibrium, all of the necessary conditions for optimality of the household's plan just listed must hold, and in addition, the market-clearing conditions (1.11)–(1.16) must hold. This section draws some further conclusions about relations that must exist among the various endogenous variables in an equilibrium, in order to show how they are affected by central bank policy.

To simplify the discussion, this paper focuses on the case in which any exogenous factors that change over time (apart from the occurrence of crisis states and asset collapses, as depicted in figure 1) are purely deterministic (that is, simply a function of the date $t$). That is, the exploration of the effects of a temporary disturbance of any other type considers only the case of a shock that occurs in the initial period $t = 0$, with consequences that are perfectly predictable after that. The focus is further restricted to the effects of alternative monetary and fiscal policies that are similarly deterministic; this means that while the model can be used to consider the effects of responding in different ways to a one-time disturbance (in section 5), it does not encompass the effects of responding to the occurrence of a crisis that results in a fire sale of bank assets (or to an asset collapse). The reason is that the concern here is with the consequences for the risks to financial stability of alternative central bank policies *prior* to the occurrence of a crisis; the interesting (but more complex) question of what can be achieved by suitable use of these instruments to respond to a crisis *after* it occurs is left for a later study.

Under this assumption, neither the occurrence of a crisis nor an asset collapse in any period $t$ affects equilibrium determination in subsequent periods, and we obtain an equilibrium in which the variables listed above as functions of the history $\xi_t$ depend only on the date $t$, and those listed as functions of the history $(\xi_t, \eta_t)$ depend only on the date $t$ and the value of $\eta_t$. Moreover, because the resolution of uncertainty during the period has no effect on equilibrium in later periods, the Lagrange multiplier $\phi_{6t}$ indicating the shadow value of additional funds in the end-of-period settlement of accounts will be independent of whether a crisis occurs in period $t$. Consequently, the price $P_t$ of normal goods, the quantities purchased of normal goods ($c_{1t}$, $c_{2t}$, $I_t$), and the quantity $Y_t$ that is produced will all be independent of whether a crisis occurs. Similarly, the Lagrange multiplier $\phi_{4t}$ associated with the cash-in-advance constraint will have a value that is independent of whether a crisis occurs.

Thus, an equilibrium can be fully described by sequences $\{A_t, M_t, B_t, D_t, F_t, s_t, s_t^*, c_{1t}, c_{2t}, I_t, Y_t, c_{3t}^c, c_{3t}^n\}$ describing the choices of the representative household;[28] sequences $\{Q_t, \Gamma_t, P_t, \tilde{P}_t^c, \tilde{P}_t^n\}$ of prices and sequences $\{R_t^m, R_t^{bc}, R_t^{bn}\}$ of yields on government securities; and sequences $\{\phi_{1t}, \phi_{2t}, \phi_{3t}, \phi_{4t}, \phi_{5t}^c, \phi_{5t}^n, \phi_{6t}\}$ of Lagrange multipliers. Here the superscripts $c$ and $n$ are used to indicate the values that variables take in a given period conditional on whether the crisis state (superscript $c$) or the noncrisis state (superscript $n$) is reached; variables without superscripts take values that depend only on the date. For these sequences to represent an equilibrium, they must satisfy all of the equilibrium conditions stated above for each date and for each of the possible states in subperiod 2. Conditional expectations are no longer needed in equilibrium relations such as equation (2.2) or (2.4), and the c superscript is no longer needed in equation (2.6).

## 2.3 Prices and Quantities Transacted in a Crisis

We turn now to a more compact description of the conditions that must hold in equilibrium. We begin with a discussion of the relations that determine the equilibrium supply of special goods, the degree to which investors are financially constrained, and the price of durable goods in the event of a fire sale.

Conditions (2.11) and (2.14), together with the requirement that in each state, require that

$$\frac{\tilde{u}'(c_{3t}^s)}{w'(c_{3t}^s)} = \tilde{\phi}_{5t}^s \equiv \frac{\phi_{5t}^s}{\phi_{6t}} \tag{2.16}$$

for each possible state $s$ (equal to either $c$ or $n$) that may be reached in subperiod 2. Since the left-hand side of condition (2.16) is a

28. Here we have reduced the number of separate variables by using a single symbol $s_t^*$ to refer to both $s_t^{*s}$ and $s_t^{*d}$ as these are necessarily equal in any equilibrium, and similarly eliminated separate reference to $x_t$ since it must always be equal to $c_{3t}$ in any equilibrium.

monotonically decreasing function, this equation can be solved uniquely for the demand for special goods in each state,

$$c_{3t}^{s} = c_3(\tilde{\phi}_{5t}^{s}),$$

where $\tilde{\phi}_{kt} \equiv \phi_{kt} / \phi_{6t}$ for any $k \neq 6$, and $c_3(\cdot)$ is the monotonically decreasing function implicitly defined by equation (2.16).

Here $\tilde{\phi}_{5t}^{s}$ measures the degree of financial constraint of investors in state $s$ of subperiod 2. The value $\tilde{\phi}_{5t}^{s} = 1$ would imply no ex post regret in state $s$ about the size of the credit line arranged for the investor, and a demand for special goods that is the same as if there were no constraint separating the funds of the investor from those of the rest of the household; $\tilde{\phi}_{5t}^{s} > 1$ indicates that ex post, the household would wish it had arranged more credit for the investor, while $\tilde{\phi}_{5t}^{s} < 1$ would imply that it would wish it had arranged less. The socially efficient level of production and consumption of special goods in either state is given by the quantity $c_3^*$ such that

$$\frac{\tilde{u}'(c_3^*)}{w'(c_3^*)} = 1.$$

Hence, special goods are underproduced or overproduced in state $s$ according to whether $\tilde{\phi}_{5t}^{s}$ is greater or smaller than 1.

Equation (2.14) can then be used to obtain the implied state-contingent price of special goods (in units of end-of-period marginal utility),

$$\phi_{6t}\tilde{P}_t^{s} = \tilde{p}(\tilde{\phi}_{5t}^{s}) \equiv w'(c_3(\tilde{\phi}_{5t}^{s})),$$

and the implied state-contingent expenditure on special goods (in the same units),

$$\phi_{6t}\tilde{P}_t^{s}c_{3t}^{s} = e_3(\tilde{\phi}_{5t}^{s}) \equiv \tilde{p}(\tilde{\phi}_{5t}^{s})c_3(\tilde{\phi}_{5t}^{s}).$$

Note that $e_3(\tilde{\phi}_5)$ will be a monotonically decreasing function.
Since $\tilde{\phi}_5 > 0$ in each state, budget constraint (1.5) must hold with equality in each state. The fact that $F_t$ must not be state-contingent then implies that the left-hand side of (1.5) must be the same whether a crisis occurs or not, so that in equilibrium,

$$e_3(\tilde{\phi}_{5t}^n) = e_3(\tilde{\phi}_{5t}^c) + \tilde{\Gamma}_t s_t^* \tag{2.17}$$

each period, where $\tilde{\Gamma}_t \equiv \phi_{6t}\Gamma_t$. Moreover, condition (2.8) implies that

$$(1-p)\tilde{\phi}_{5t}^c + p\tilde{\phi}_{5t}^n = 1.$$

This equation can be solved for $\tilde{\phi}_{5t}^n = \tilde{\phi}_5^n(\tilde{\phi}_{5t}^c)$, a monotonically decreasing function with the property that $\tilde{\phi}_5^n(1) = 1$. Substituting this for $\tilde{\phi}_{5t}^n$ in (2.17) yields an equation

$$\tilde{D}(\tilde{\phi}_{5t}^c) = \tilde{\Gamma}_t s_t^*, \tag{2.18}$$

where

$$\tilde{D}(\tilde{\phi}_5^c) \equiv e_3(\tilde{\phi}_5^n(\tilde{\phi}_{5t}^c)) - e_3(\tilde{\phi}_{5t}^c)$$

is a monotonically increasing function with the property that $\tilde{D}(1) = 0$. Finally, equation (2.7) implies that

$$\tilde{\phi}_{5t}^c \tilde{\Gamma}_t = \gamma q. \tag{2.19}$$

This together with (2.18) implies that

$$\tilde{\phi}_{5t}^c \tilde{D}(\tilde{\phi}_{5t}^c) = \gamma q s_t^*.$$

Since the left-hand side of this equation is a monotonically increasing function of $\tilde{\phi}_{5t}^c$, it can be uniquely solved for

$$\tilde{\phi}_{5t}^c = \tilde{\phi}_5^c(s_t^*), \tag{2.20}$$

where $\tilde{\phi}_5^c(s^*)$ is a monotonically increasing function with the property that $\tilde{\phi}_5^c(0) = 1$.

This solution for the equilibrium value of the multiplier $\tilde{\phi}_5^c$ then allows us to solve for the implied values of $\tilde{\Gamma}_t$, $\tilde{\phi}_{5t}^n$, $c_{3t}^c$, $c_{3t}^n$, $\phi_{6t}\tilde{P}_t^c$, and $\phi_{6t}\tilde{P}_t^n$, each as a function of the quantity $s_t^*$ of durable goods that are sold in the fire sale (if one occurs) in period $t$. We observe that $\tilde{\phi}_5^c$ and $c_{3t}^n$ will be increasing functions of $s_t^*$ and $\phi_{6t}\tilde{P}_t^n$ will be nondecreasing, while $\tilde{\Gamma}_t$, $\tilde{\phi}_{5t}^n$, and $c_{3t}^c$ will be decreasing functions of $s_t^*$ and $\phi_{6t}\tilde{P}_t^c$ will be nonincreasing.

In the case that $s_t^* = 0$ (no assets are sold in a fire sale), $c_{3t}^c = c_{3t}^n = c_3^*$ (the efficient quantity of special goods are produced in both states), $\tilde{\phi}_5^c = \tilde{\phi}_{5t}^n = 1$ (no regret about the size of the line of credit arranged for the investor, in either state), and $\tilde{\Gamma}_t = yq$ (the market price of durables in the crisis state is equal to their fundamental value). Instead, if $s_t^* > 0$ (that is, if any assets are sold in a fire sale), $c_{3t}^c < c_3^* < c_{3t}^n$, $\tilde{\phi}_{5t}^n < 1 < \tilde{\phi}_5^c$, and $\tilde{\Gamma}_t = yq$. This means that special goods are underproduced in the crisis state and overproduced in the noncrisis state, and that ex post, the household wishes it had supplied more credit for its investor if the crisis state occurs, while it wishes that it had supplied less credit if the crisis state does not occur. It also means that if the crisis state occurs, the price at which durables are sold in the fire sale is less than their fundamental value, conditional on reaching that state. Moreover, the size of these distortions is greater the larger is the aggregate value of $s_t^*$. The fact that households do not take these equilibrium effects into account when choosing their planned value of $s_t^{*s}$ results in a pecuniary externality.

## 2.4 Implications of the Demand for Safe Assets

We turn next to a discussion of the consequences of the supply of short-term safe instruments for equilibrium purchases of cash and credit goods. We consider first the implications of optimality conditions (2.9)–(2.10), together with the cash-in-advance constraint (1.4) and the associated complementary slackness condition.

Let us first define the demand functions $c_1^*(\lambda)$, $c_2(\lambda)$ as the solution to the problem of choosing $c_1$ and $c_2$ to maximize

$$u(c_1, c_2) - \lambda\left(c_1 + c_2\right)$$

for an arbitrary price $\lambda > 0$. Under the assumption that cash and credit goods are both normal goods, both $c_1^*(\lambda)$ and $c_2^*(\lambda)$ must be monotonically decreasing functions.[29] We can then consider the constrained problem

$$\max_{c_1, c_2} u(c_1, c_2) - \lambda\left(c_1 + c_2\right) \quad \text{s.t. } c_1 \leq m, \tag{2.21}$$

29. The paths followed by the two variables as $\lambda$ is reduced correspond to the "income-expansion path" as a result of increasing the budget available to spend on these two goods, for a fixed relative price (equal prices of the two goods).

where $m > 0$ represents real cash balances available to the household. The solution $c_1(\lambda; m), c_2(\lambda; m)$ to problem (2.21) can be characterized as follows: if $m \leq c_1^*(\lambda)$, then $c_1(\lambda; m) = m$ and $c_2(\lambda; m)$ is implicitly defined by the equation

$$u_2(m, c_2) = \lambda. \tag{2.22}$$

If instead $m \leq c_1^*(\lambda)$ then $c_1(\lambda; m) = c_1^*(\lambda)$ and $c_2(\lambda; m) = c_2^*(\lambda)$.

The Kuhn-Tucker conditions for this latter, constrained problem are easily seen to correspond precisely to conditions (2.9)–(2.10) and constraint (1.4) together with the complementary slackness condition, where the price of normal goods in units of end-of-period marginal utility is given by $\lambda_t \equiv \phi_{6t} P_t$, and available real cash balances are given by $m_t \equiv M_t/P_t$. It follows that the model implies that $c_{1t}, c_{2t}$ must satisfy

$$c_{jt} = c_j(\lambda_t; M_t/P_t)$$

for $j$ = 1,2 where the functions $c_j(\lambda; m)$ are defined in the previous paragraph.

Associated with this solution will be a value for the normalized Lagrange multiplier $\tilde{\phi}_{4t}$, given by

$$\tilde{\phi}_{4t} = \tilde{\phi}_4(\lambda_t; M_t/P_t),$$

where we define

$$\tilde{\phi}_4(\lambda; m) \equiv \frac{u_1(c_1(\lambda; m), c_2(\lambda; m))}{u_2(c_1(\lambda; m), c_2(\lambda; m))} - 1.$$

The Kuhn-Tucker conditions for the problem (2.21) imply that $\tilde{\phi}_{4t}$ $(\lambda; m)$ for all $m \geq c_1^*(\lambda)$, while $\tilde{\phi}_{4t}$ $(\lambda; m) > 0$ for all $m < c_1^*(\lambda)$. Furthermore, in the latter case (where the cash-in-advance constraint binds), the assumption that both cash goods and credit goods are normal goods implies that $\tilde{\phi}_{4t}(\lambda; m)$ is a decreasing function of $\lambda$ for fixed $m$,[30] and a decreasing function of $m$ for fixed $\lambda$.[31]

A comparison of equations (2.2) and (2.4) (and recalling that the conditional expectations have been eliminated from both of these conditions) implies that under any optimal plan, it must be the case that $\tilde{\phi}_{4t} = (1-p)\tilde{\phi}_{2t}$. Hence, in any equilibrium where the cash-in-advance constraint binds in some period, so that $\tilde{\phi}_{4t} > 0$, it must also be the case that $\tilde{\phi}_{2t} > 0$, so that the first inequality in equation (1.6) is also a binding constraint, and $D_t = \Gamma_t s_t^*$ (as much collateralized debt is issued by bankers as can be repaid in the event of a crisis, given the quantity of durables that bankers plan to sell in a fire sale). More generally, we can conclude that the normalized Lagrange multiplier $\tilde{\phi}_{2t}$ will be given by

$$\tilde{\phi}_{2t} = \tilde{\phi}_2(\lambda_t; M_t/P_t),$$

where we define

$$\tilde{\phi}_2(\lambda; m) \equiv \frac{\tilde{\phi}_4(\lambda; m)}{(1-p)}.$$

Condition (2.2) implies that the normalized multiplier $\tilde{\phi}_{1t}$ will similarly be given by a function

$$\tilde{\phi}_{1t} = \tilde{\phi}_1(\lambda_t; M_t/P_t), \tag{2.23}$$

where we define

$$\tilde{\phi}_1(\lambda; m) \equiv 1 + \tilde{\phi}_4(\lambda; m).$$

30. Concavity of the utility function implies that increasing $c_2$ while $c_1$ remains fixed at $m$ implies a decrease in the marginal utility of credit goods consumption, so that increasing $\lambda$ with fixed $m$ must correspond to a reduction in the quantity of $c_2$ that is purchased. In order for the demand $m$ for cash goods to remain the same despite a budget contraction that requires fewer credit goods to be purchased, the relative price of cash goods must decrease (under the assumption of normal goods). This means that $u_1/u_2$ must decrease, and hence that $\tilde{\phi}_4$ must decrease.

31. In the $\lambda - m$ plane, the level curves of the function $\tilde{\phi}_4$ correspond to income-expansion paths, as the budget for cash and credit goods changes with the relative price of the two types of goods fixed. If the two goods are both normal goods, $m$ must increase along such a path as $\lambda$ decreases, as discussed above; hence, the level curves must have a negative slope at all points. It then follows that the sign of this partial derivative follows from the sign of the one discussed in the previous footnote.

It follows that $\tilde{\phi}_{1t} > 1$ if and only if the cash-in-advance constraint binds, while it is equal to 1 otherwise. Additionally, both $\tilde{\phi}_{1t}(\lambda; m)$ and $\tilde{\phi}_{2t}(\lambda; m)$ will be decreasing in both arguments, in the region where the cash-in-advance constraint binds.

A comparison of conditions (2.6) and (2.7) similarly implies that under any optimal plan, it must be the case that

$$\tilde{\phi}^c_{5t} - 1 = \tilde{\phi}_{2t} - \tilde{\phi}_{3t}. \tag{2.24}$$

This allows solving for the implied value of the normalized multiplier $\tilde{\phi}_{3t}$ as

$$\tilde{\phi}_{3t} = \tilde{\phi}_3(\lambda_t; s^*_t, M_t/P_t),$$

where we define

$$\tilde{\phi}_3(\lambda_t; s^*_t, M_t/P_t) \equiv \tilde{\phi}_2(\lambda_t; M_t/P_t) + 1 - \tilde{\phi}^c_5(s^*_t). \tag{2.25}$$

The supply of real cash balances $M_t/P_t$ and the quantity of assets $s^*_t$ sold in the event of a fire sale must be endogenously determined in such a way as to guarantee that in equilibrium, the value of this function is always nonnegative. (The existence of such a solution is shown below.)

Finally, condition (2.5) can be used to determine the equilibrium price of risky durables in the subperiod 1 market. If $\tilde{Q}_t \equiv \phi_{6t} Q_t$ denotes this price in marginal-utility units, then we obtain a solution of the form

$$\tilde{Q}_t = \tilde{Q}(\lambda_t; s^*_t, M_t/P_t),$$

where we define

$$\tilde{Q}(\lambda_t; s^*_t, M_t/P_t) \equiv \frac{\tilde{Q}^* + (1-p)\tilde{\phi}_3(\lambda_t; s^*_t, M_t/P_t)\tilde{\Gamma}(s^*_t)}{\tilde{\phi}_1(\lambda_t; M_t/P_t)}. \tag{2.26}$$

Here, the notation

$$\tilde{Q}^* \equiv \gamma\left[p + (1-p)q\right]$$

is used for the expected marginal utility of the anticipated service flow from a durable purchased in subperiod 1, and

$$\tilde{\Gamma}(s^*) \equiv \frac{\gamma q}{\tilde{\phi}_5^c(s^*)}$$

for the solution for $\tilde{\Gamma}_t$ derived in the previous section.

The fundamental value of a durable purchased in subperiod 1, if the anticipated future service flow were to be valued using the same pricing kernel that is used to price bonds in condition (2.3),[32] would equal[33]

$$\tilde{Q}_t^{fund} \equiv \frac{\tilde{Q}^*}{\tilde{\phi}_{1t}}. \quad (2.27)$$

Thus, equation (2.26) implies that durables will be priced at their fundamental value in subperiod 1 if and only if the second inequality in equation (1.5) is not a binding constraint; that is, the quantity of durables held by bankers (and thus the availability of collateral) does not constrain bankers to issue less collateralized debt than they would otherwise wish. When the constraint binds, so that $\tilde{\phi}_{3t} > 0$ durables are overvalued in subperiod 1. The above discussion of the equilibrium value of $\tilde{\phi}_{3t}$ implies that in order for this to happen, the cash-in-advance constraint must bind (so that $\tilde{\phi}_{2t} > 0$), while the supply of durables (and hence the equilibrium value of $s_t^*$) must not be too large, so that $\tilde{\phi}_5^c(s_t^*)$ is not too much greater than 1.

## 2.5 Determinants of the Supply of Safe Assets

We turn now to the endogenous determination of the cash supply $M_t$, as a result of the financing decisions of bankers. Since $\tilde{\phi}_5^c(s_t^*) > 1$ if $s_t^* > 0$, the left-hand side—and hence also the right-hand side—of equation (2.24) must be positive if any assets will be sold by bankers in the event of a fire sale. But the right-hand side of equation (2.24) can be positive only if $\tilde{\phi}_{2t}$ is positive, which occurs only if the cash-in-

32. That is a general pricing relation for noncash assets, since I make no particular assumption about the nature of the state-contingent return on bonds, only that this asset cannot be used as a means of payment in the cash goods market.

33. Equation (2.3) states that an asset that yields $Y_t$ at the end of period in marginal-utility units should have a price in subperiod 1 of $P_t^Y = E_t[Y_t] / \phi_{1t}$. For the case of longer-term bonds, $Y_t = \phi_{6t} R_t^b$ and the price in the subperiod 1 market is $P_t^Y = R_t^m$.

advance constraint binds. This, in turn, would require that $D_t = \Gamma_t s_t^*$ as argued in the previous paragraph, and hence that, using equation (11),

$$M_t = \tilde{M}_t + \Gamma_t s_t^*. \tag{2.28}$$

On the other hand, if $s_t^*=0$, constraint (1.5) requires that $D_t=0$ as well, so that equation (2.28) must hold in this case, as well. We may thus conclude that in any equilibrium, the total supply of cash will be given by equation (2.28).

It remains to determine the equilibrium value of $s_t^*$. In marginal-utility units, equation (2.28) can be written

$$\hat{M}_t \equiv \phi_{6t} M_t = \lambda_t \tilde{m}_t + \tilde{\Gamma}_t s_t^*, \tag{2.29}$$

using the notation $\tilde{m}_t \equiv \tilde{M}_t / P_t$ for the real supply of safe assets by the government. Then in any equilibrium where

$$\tilde{m}_t + \frac{\tilde{\Gamma}_t s_t^*}{\lambda_t} > c_1^*(\lambda_t),$$

the cash-in-advance constraint will not bind. However, since this implies that $\tilde{\phi}_{2t} = 0$, equation (2.24) implies that $\tilde{\phi}_{5t}^c$ cannot be greater than 1, which requires that $s_t^*=0$.

Hence, such an equilibrium occurs if and only if

$$\tilde{m}_t > \tilde{m}^*(\lambda_t) \equiv c_1^*(\lambda_t), \tag{2.30}$$

and involves $\hat{M}_t = \lambda_t \tilde{m}_t$. In this case, equation (2.25) implies that $\tilde{\phi}_{3t} = 0$ so that $\tilde{Q}_t$ is equal to the fundamental value (2.27). In addition, because $s_t^*=0$, it must be the case that $\tilde{\Gamma}_t = \tilde{\Gamma}(0) = 1$, so that durables are also priced at their fundamental value in subperiod 2, even if the crisis state is reached.

Consider now the possibility of an equilibrium in which the supply of real cash balances is no greater than $c_1^*(\lambda_t)$ (the level required for satiation in cash), but the supply of durables $s_t$ is large enough so that bankers are unconstrained in the amount of collateralized debt that they can issue (so that $\tilde{\phi}_{3t} = 0$). Because of equation (2.24), this requires a value of $s_t^*$ such that

$$\tilde{\phi}_5^c(s_t^*) - 1 = \tilde{\phi}_2(\lambda_t; \tilde{m}_t + \tilde{\Gamma}(s_t^*) s_t^* / \lambda_t). \tag{2.31}$$

It follows from the discussion above that the left-hand side of this equation is an increasing function of $s_t^*$, while the right-hand side is a nonincreasing function of $s_t^*$ (decreasing until the point at which the cash-in-advance constraint ceases to bind, and constant thereafter).[34] Moreover, the right-hand side is at least as large as the left-hand side if $s_t^* = 0$ given the assumption now that $\tilde{m}_t \le c_1^*(\lambda_t)$. Hence, there is a unique value of $0 \le s_t^* < s_t$ that satisfies condition (2.31) if and only if the left-hand side is greater than the right-hand side when $s_t^* = s_t$, which is to say, if and only if

$$\tilde{\phi}_5^c(s_t) - 1 > \tilde{\phi}_2(\lambda_t; \tilde{m}_t + \tilde{\Gamma}(s_t)s_t / \lambda_t). \tag{2.32}$$

Thus, such an equilibrium exists in period $t$ if and only if the outside supply of safe assets $\tilde{m}_t$ fails to satisfy condition (2.30) while the supply of durables $s_t$ does satisfy condition (2.32); in such a case, $s_t^*$ is implicitly defined by condition (2.31), and the total supply of cash is given by condition (2.29). In this case, again $\tilde{\phi}_{3t} = 0$ and hence $\tilde{Q}_t = \tilde{Q}_t^{fund}$. Moreover, if $\tilde{m}_t < c_1^*(\lambda_t)$, the solution must involve $s_t^* > 0$ and hence $\tilde{\Gamma}_t < 1$, so that durables are underpriced in the fire sale in the event of a crisis.

If, instead, $\tilde{m}_t$ does not satisfy condition (2.30) and the supply of durables $s_t$ fails to satisfy condition (2.32), then there can only be an equilibrium in which $s_t^* = s_t$. In this case, the supply of safe assets is given by

$$\hat{M}_t = \lambda_t \tilde{m}_t + \tilde{\Gamma}(s_t)s_t. \tag{2.33}$$

The value of $\tilde{\phi}_{3t}$ is given by equation (2.25), which will be positive in the case of any value of $s_t$ such that the inequality in equation (2.32) is reversed. In any such case, it must be the case that $\tilde{Q}_t > \tilde{Q}_t^{fund}$, so that durables are overvalued in subperiod 1. In addition, the fact that $s_t^* > 0$ implies that $\tilde{\Gamma}_t < 1$ so that durables are *underpriced* in the event of a fire sale, even though they are *overpriced* in subperiod 1. In this case, an asset boom can be followed by a crash.

It is thus possible to completely characterize the equilibrium pricing of risky durables in any period $t$ (both in subperiod 1 and in the event of a crisis) as a function of three quantities: the real supply $\tilde{m}_t$ of safe assets by the government (determined by fiscal policy and central

34. Recall that $\tilde{\Gamma}(s^*)s^* = \tilde{D}(\tilde{\phi}_5^c(s^*))$ is a monotonically increasing function of $s^*$, and that $\tilde{\phi}_2(\lambda; m)$ is a decreasing function of $m$ as long as the cash-in-advance constraint binds, and independent of the value of $m$ for all higher values.

bank asset purchases), the supply of durables $s_t$ (which follows directly from the quantity $I_{t-1}$ of investment goods produced in the previous period), and the marginal utility $\lambda_t$ that the representative household assigns to additional real end-of-period wealth. The latter quantity depends on expectations about subsequent periods, as discussed next.

In particular, the subperiod 1 equilibrium price of durables, expressed in marginal-utility units, can be written as a function

$$\tilde{\phi}_{1t}\tilde{Q}_t = \varphi(\lambda_t; s_t, \tilde{m}_t)$$

derived in the manner just explained. It is useful for the discussion below to consider how this function depends on the supply of durables $s_t$. In the case of an outside cash supply satisfying $\tilde{m}_t > c_1^*(\lambda_t)$, or a supply of durables satisfying equation (2.32), in equilibrium it must be the case that $\tilde{\phi}_{3t} = 0$, so that equation (2.26) implies that $\varphi(\lambda_t; s_t, \tilde{m}_t) = \tilde{Q}^*$. Thus, the value of the function is independent of the value of $s_t$ in either of these cases. If instead there are both an outside cash supply below the satiation level and a supply of durables too small to satisfy equation (2.32), the equilibrium supply of safe assets is given by equation (2.33). The right-hand side of this equation is a monotonically increasing function of $s_t$, so that $M_t / P_t = \hat{M}_t / \lambda_t$ is also an increasing function of $s_t$.

It follows from this that the equilibrium value of $\tilde{\phi}_{3t}$ given by equation (2.25) will be a monotonically decreasing function of $s_t$. It then follows from equation (2.26) that $\tilde{\phi}_{1t}\tilde{Q}_t$ will be a monotonically decreasing function of $s_t$, and hence that the function $\varphi(\lambda_t; s_t, \tilde{m}_t)$ is decreasing in this argument. Thus, in the case that $\tilde{m}_t < c_1^*(\lambda_t)$, the function $\varphi(\lambda_t; s_t, \tilde{m}_t)$ will be a decreasing function of $s_t$ for all supplies of durables too small to satisfy equation (2.32), and will instead be constant at its minimum value of $\tilde{Q}^*$ for all $s_t$ large enough to satisfy equation (2.32). The function is constant (and equal to $\tilde{Q}^*$) whenever $\tilde{m}_t > c_1^*(\lambda_t)$ regardless of the value of $s_t$.

It will also be useful for the discussion below of intertemporal equilibrium to note that the relative value of funds available in subperiod 1 as opposed to the end of the period will be given by a function of the form

$$\tilde{\phi}_{1t} = \hat{\phi}_1(\lambda_t; s_t, \tilde{m}_t). \tag{2.34}$$

This function depends only on the value of $\lambda_t$ in the case that $\tilde{m}_t \geq c_1^*(\lambda_t)$, so that there is satiation in cash. It depends on both $\lambda_t$ and $\tilde{m}_t$ in the case that $\tilde{m}_t < c_1^*(\lambda_t)$ but $s_t$ is large enough to satisfy equation (2.32), but does not depend on $s_t$, since in this case bankers' collateral constraint does not bind, and $s_t^*$ is independent of the size of $s_t$. Finally, in the case that $\tilde{m}_t < c_1^*(\lambda_t)$ and $s_t$ is too small to satisfy equation (2.32), the value of the function depends on all three of its arguments. (In this latter case, $M_t/P_t$ will be an increasing function of $s_t$ for given values of the other two arguments, as just discussed; hence $\tilde{\phi}_{1t}$ will be a decreasing function of $s_t$ for $s_t$ in this range.)

## 2.6 Intertemporal Equilibrium

We now consider the connections between variables in successive periods required for an intertemporal equilibrium. One such connection is given by condition (2.12) for optimal investment demand. Using the solution for the subperiod 1 equilibrium price of durables just derived, condition (2.12) can be written in the alternative form

$$\lambda_t = \beta\varphi(\lambda_{t+1}; F(I_t), \tilde{m}_{t+1})F'(I_t). \tag{2.35}$$

Here I have also used the fact that the supply of durables in period $t+1$ must equal $s_{t+1} = F(I_t)$.

Since the right-hand side of this expression must be a monotonically decreasing function of $I_t$,[35] condition (2.35) has a unique solution for the equilibrium value of $I_t$, which can be written in the form

$$I_t = I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1}). \tag{2.36}$$

Because the right-hand side of equation (2.35) is a decreasing function of $I_t$ the function $I(\lambda; \lambda', \tilde{m})$ implicitly defined by this equation will be a monotonically decreasing function of $\lambda$. Thus, we obtain a demand curve for investment that is a decreasing function of $\lambda_t$, similar to the demands for cash and credit goods as decreasing functions of $\lambda_t$ that can be derived in the way explained above. But whereas the demands for cash and credit goods depend on $s_t$ and $\tilde{m}_t$ along with the value of $\lambda_t$, investment demand depends on expectations regarding the values of $\lambda_{t+1}$ and $\tilde{m}_{t+1}$ along with the value of $\lambda_t$.

35. This relies on the demonstration above that $\varphi(\lambda; s, \tilde{m})$ is a nonincreasing, positive-valued function of $s$, in addition to the assumption that the function $F(I)$ is strictly concave.

If the solution for the sum of the demands for cash and credit goods is written as

$$c_{1t} + c_{2t} = y(\lambda_t; s_t, \tilde{m}_t),$$

then the aggregate demand for normal goods can be written as

$$Y_t = y(\lambda_t; s_t, \tilde{m}_t) + I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1}). \tag{2.37}$$

In a flexible-price equilibrium (the kind assumed thus far), this quantity of normal goods will also have to be voluntarily supplied, which requires that condition (2.3) be satisfied. Hence, the equilibrium value of $\lambda_t$ must satisfy

$$v'\left(y(\lambda_t; F(I_{t-1}), \tilde{m}_t) + I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1})\right) = \lambda_t. \tag{2.38}$$

Since the left-hand side of this equation is a nonincreasing function of $\lambda_t$ (strictly decreasing if $v'' > 0$), there will be a unique solution for $\lambda_t$ corresponding to given values of $I_{t-1}, \tilde{m}_t, \tilde{m}_{t+1}$, and $\lambda_{t+1}$.

In the initial period of the model, the value of $I_{t-1}$ will be given as an initial condition; but in all subsequent periods, the value will be endogenously determined by equation (2.36). Hence, for all periods after the initial period, we obtain an equilibrium relation of the form

$$v'\left(y(\lambda_t; F(I(\lambda_{t-1}; \lambda_t, \tilde{m}_t)), \tilde{m}_t) + I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1})\right) = \lambda_t. \tag{2.39}$$

Given an initial stock of investment goods $I_{-1}$ in period $t = 0$ and a path for $\{\tilde{m}_t\}$ for all $t \geq 0$ (determined by fiscal policy and the central bank's balance sheet policy), an intertemporal equilibrium is then a sequence of anticipated values $\{\lambda_t\}$ for all $t \geq 0$ that satisfy equation (2.38) when $t = 0$ and the second-order nonlinear difference equation (2.39) for all $t \geq 1$.

Given a solution for the path $\{\lambda_t\}$, the associated path for the production of investment goods is given by (2.36) for all $t \geq 0$. This in turn implies a supply of durables $s_t$ for each period $t \geq 0$ using equation (1.13). One then has sequences of values $\{\lambda_t, s_t, \tilde{m}_t\}$ for each of the periods $t \geq 0$. The implied values for the variables $s_t^*$, $M_t/P_t$, and so on, as well as for the various normalized Lagrange multipliers, can then be determined for each of these periods using the results derived in the previous sections.

This yields a solution for the allocation of resources, all relative prices and all real asset prices, that involves no reference to any nominal variables, as long as the central bank's balance sheet policy is specified in real terms (since the real supply of outside safe assets is used in the above calculations). In fact, the only element of policy that matters for the determination of real variables in the flexible-price version of the model is the path of $\{\tilde{m}_t\}$. The path of government debt as a whole does not matter for the determination of any variables in the model: Ricardian equivalence obtains (given the assumption of a representative household and lump-sum taxes and transfers), except for the qualification that changes in the government supply of safe assets are not neutral in this model, owing to the cash-in-advance constraint.[36]

Conventional monetary policy (the central bank's control of the interest rate on cash balances $R_t^m$) is also irrelevant to the determination of real variables, though it can be used to control the general level of prices (the path $\{P_t\}$, and along with it the prices of other goods and assets in monetary units). Condition (2.15) requires that in equilibrium,

$$R_{t+1}^m = \left(1 + r_{t+1}^m\right)\frac{P_{t+1}}{P_t}, \tag{2.40}$$

where

$$1 + r_{t+1}^m \equiv \frac{\lambda_t}{\beta\lambda_{t+1}\hat{\phi}_1(\lambda_{t+1}; s_{t+1}, \tilde{m}_{t+1})}$$

is the equilibrium real return on cash between the end of period $t$ and the end of period $t + 1$. Note that the path of the variable $\{r_{t+1}^m\}$ is determined for all $t \geq 0$ by the path of $\{\tilde{m}_t\}$ in the manner discussed above, as with all other real variables. Equation (2.40) then describes the Fisher relation that must hold between the nominal interest rate on cash and the rate of inflation.

The equilibrium paths of the price level $\{P_t\}$ for $t \geq 0$ and of the nominal interest rate $\{R_{t+1}^m\}$ for $t \geq 0$ are jointly determined by the equilibrium relation (2.40) and the reaction function—which may, for example, be of the form $R_{t+1}^m = \psi(P_t / P_{t-1})$ —that specifies how

36. For the same reason, it does not matter exactly what type of liabilities the government issues other than short-term safe assets; and it similarly does not matter, in this model, what type of noncash assets are held on the balance sheet of the central bank.

the central bank's interest rate target responds to variation in the price level. The discussion of how this occurs follows exactly the lines of the discussion of price-level determination in a flexible-price cashless economy in Woodford (2003, chap. 2). While the present model includes a number of financial frictions and other complications not present in the simple model used in that discussion, what matters is that the variable $r^m_{t+1}$ in equation (2.40) evolves in a way that is completely exogenous with respect to the evolution of the price level and independent of the specification of (conventional) monetary policy.

It will simplify the discussion that follows if conventional monetary policy is specified not by a central bank reaction function, but rather by a target path for the price level $\{P_t\}$ for all $t \geq 0$. Since this target path can be achieved by a suitable rule for setting the interest rate $R^m_{t+1}$—assuming that equation (2.40) does not imply a negative nominal rate at any time,[37] given the target path of prices— the path of the price level is assumed to conform to the target path chosen by the central bank, and equation (2.40) is used to determine the implied equilibrium evolution of the nominal interest rate on cash.

Finally, condition (2.3) requires that the equilibrium expected return on bonds satisfy

$$\mathrm{E}_t[R^b_t] = \hat{\phi}_1(\lambda_t; s_t, \tilde{m}_t)$$

in all periods $t \geq 0$. Given a specification of the character of this alternative form of government debt to determine the relative value of bonds in states $c$ and $n$, this relation then completely determines the state-contingent returns on bonds. The solution for equilibrium bond yields is not necessary to solve for any of the other variables discussed earlier; hence, it is not necessary to discuss further the character of bonds or their equilibrium prices.

37. The model as described above would not preclude a negative nominal interest rate in equilibrium, that is, a value $R^m_{t+1} < 1$. It is more realistic, however, to add an assumption that households can demand currency from the central bank at any time in exchange for interest-earning cash, which would for institutional reasons earn a zero nominal interest rate, and that such currency would be acceptable as payment for cash goods. The possibility of holding currency would then preclude equilibria with $R^m_{t+1} < 1$ in any period.

## 3. The Size of the Central bank Balance Sheet and Stationary Equilibrium

The paper compares the effects of the two dimensions of central bank policy: variation in its target for the interest rate $R_t^m$ paid on cash; and variation in the size of its balance sheet, holding fixed its target for that interest rate. We first compare alternative possible long-run stationary equilibria, in which the inflation rate, the various interest rates, and relative prices are all constant over time, and the real size of the central bank balance sheet and the real supply of Treasury bills by the Treasury are constant over time as well. It can be shown that there exists a two-dimensional family of such stationary equilibria. Moreover, fixing the real supply of Treasury bills, it is still possible to move in both directions within this two-dimensional family of stationary equilibria by varying the two independent dimensions of central bank policy. Thus, even a simple consideration of stationary equilibria allows us to observe the separate effects of the two dimensions of policy.

### 3.1 Alternative Stationary Equilibria

In a stationary equilibrium, the government pursues a constant inflation target

$$\frac{P_t}{P_{t-1}} = \Pi > 0$$

for all $t \geq 0$, starting from some given initial price level $P_{-1}$, and chooses to supply a constant quantity of real outside cash balances $\tilde{m}_t = \tilde{m}$ in all periods $t \geq 0$ as well.[38] We further assume that there are no transitory disturbances to preferences, technological possibilities, or financial constraints (so that the equations derived above apply in all periods, with no modifications), and that the economy starts from an initial stock of investment goods $I_{-1}$ that takes the particular value $I$ with the property that starting with this level of investment goods results in an equilibrium in which $I_t = I$ for all $t \geq 0$ as well. In such a case (and for choices of the targets $\Pi$ and $\tilde{m}$ within suitable ranges),

38. Note that given our assumption of a constantly growing target path for the price level and our assumption that this target is precisely achieved each period, there is no difference between specifying the target path for the supply of outside cash balances as a constant real level or as a nominal target with a constant growth rate equal to the target inflation rate.

there exists an intertemporal equilibrium with the special property that the variables $c_{1t}$, $c_{2t}$, $c^c_{3t}$, $c^n_{3t}$, $s_t$, $s^*_t$, $\lambda_t$, $\tilde{Q}_t$, $\tilde{\Lambda}_t$, $\tilde{p}^c_t$, $\tilde{p}^n_t$, $\hat{M}_t$, $R^m_t$, and the various normalized Lagrange multipliers all have the same constant values for all $t \geq 0$, which are simply denoted $c_1$, $c_2$ and so on.

From equation (2.39) it is evident that such a stationary equilibrium must correspond to a constant value $\lambda$ for the marginal-utility value of end-of-period real income that satisfies

$$v'\left(y(\lambda; F(I(\lambda;\lambda,\tilde{m})),\tilde{m}) + I(\lambda;\lambda,\tilde{m})\right) = \lambda. \tag{3.1}$$

This gives us a single equation to solve for the stationary equilibrium value of $\lambda$ corresponding to a given stationary target $\tilde{m}$. Given the solution for $\lambda$ from this equation, the implied stationary value of $I$ is then given by $I = I(\lambda;\, \lambda, \tilde{m})$ which is the value of $I_{-1}$ that must be assumed for the existence of such an equilibrium. Such an equilibrium will obviously involve a constant supply of durables, equal to $s = \mathrm{F}(I)$. These constant values for $\lambda_t$, $s_t$ and $\tilde{m}_t$ in all periods can then be used to solve for constant values of all of the other variables listed above, using the methods explained in the previous section.

The constant value of the nominal interest rate on cash will be given by $R^m = (1+r^m(\tilde{m}))\Pi$, where

$$1 + r^m \equiv \frac{1}{\beta\tilde{\phi}_1(\tilde{m})}$$

and $\tilde{\phi}_1(\tilde{m})$ is the stationary value of $\tilde{\phi}_{1t}$, which depends on the value chosen for $\tilde{m}$ as discussed above, but is independent of the choice of $\Pi$. Thus, for any choice of $\tilde{m}$, it is possible to choose any value of $\Pi$ such that

$$\Pi \geq \beta\tilde{\phi}_1(\tilde{m}),$$

so that the required stationary nominal interest rate satisfies $R^m \geq 1$.

There is a stationary equilibrium corresponding to any value $\tilde{m} > 0$, but for all $\tilde{m}$ greater than a critical value $m^*$, the stationary equilibrium is the same. Here $m^*$ is the level of outside real cash balances required for satiation in cash balances, which is determined as follows. In a stationary equilibrium with satiation in cash balances, it must be the case that $c_1 = c_1^*(\lambda)$ and $c_1 = c_1^*(\lambda)$. In addition, $\tilde{\phi}_1 = 1$ $\tilde{Q} = \tilde{Q}^*$ so that $\varphi(\lambda; s, \tilde{m}) = \tilde{Q}^*$ regardless of the values of $\lambda$ and $s$. It follows that the stationary level of investment goods production $I$ must equal $I^*(\lambda)$ the quantity implicitly defined by the equation

$$F'(I) = \frac{\lambda}{\beta \tilde{Q}^*}.$$

From this it follows that the stationary value of λ must satisfy

$$v'\left(c_1^*(\lambda) + c_2^*(\lambda) + I^*(\lambda)\right) = \lambda. \tag{3.2}$$

Since $c_1^*(\lambda)$, $c_2^*(\lambda)$ and $I^*(\lambda)$ are all monotonically decreasing functions, it follows that the left-hand side of equation (3.2) is a nonincreasing function of λ, and the equation must have a unique solution for λ. The associated stationary level of cash balances can be any level greater than or equal to $m^* \equiv c_1^*(\lambda)$. Hence, such a stationary equilibrium exists in the case of any value of $\tilde{m}$ that is greater than or equal to $m^*$.

Finally, in any stationary equilibrium, the equilibrium real return on longer-term bonds (and indeed, any asset that can neither be used as cash nor used as collateral to issue liabilities that can be used as cash) will equal

$$\frac{E[R^b]}{\Pi} = \frac{R^m \tilde{\phi}_1}{\Pi} = \beta^{-1}.$$

This is independent of both $\tilde{m}$ and Π. Thus, a higher value of $R^m/\Pi = 1 + r^m(\tilde{m})$ corresponds to a reduced spread between the returns on longer-term bonds and those on holding cash. The value of $\tilde{\phi}_1$ (or, more precisely, the log of $\tilde{\phi}_1$) measures this spread.

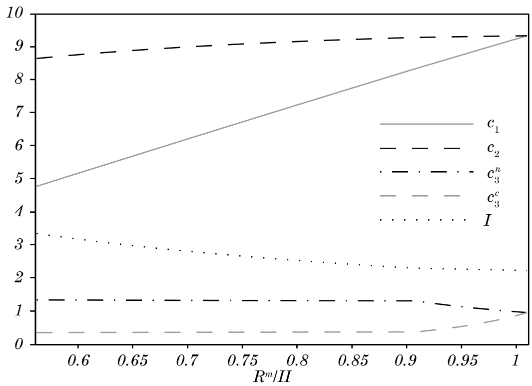
There is thus a two-dimensional family of possible stationary equilibria, which can be indexed by the choice of the two policy variables Π and $\tilde{m}$, which can be independently varied using the two dimensions of central bank policy: conventional monetary policy (interest rate policy) and balance sheet policy (quantitative easing). These two dimensions of monetary policy have quite different effects. In the flexible-price model, interest rate policy has no effect on any real variables, but it can be used (within the limit imposed by the zero lower bound) to control inflation. Balance sheet policy (changing the total supply of outside safe assets by increasing or reducing the quantity of longer-term bonds held by the central bank) can instead affect the steady-state values of all of the real variables in the model, except that further increases in the real supply of outside safe assets beyond the level $\tilde{m} = m^*$ have no further effects.

The possible stationary values of the various real variables that can be achieved by alternative monetary policies can thus be fully characterized by considering the one-parameter family of stationary equilibria corresponding to different values of $\tilde{m}$. These equilibria can be classified into three possible types, according to which of the financial constraints bind. (The three possible cases correspond to the three cases discussed in the treatment of the endogenous determination of the safe asset supply in the previous section.) First, there are equilibria in which the real outside supply of safe assets equals or exceeds the level $m^*$ required for satiation. In these equilibria, the cash-in-advance constraint is slack; bankers finance none of their purchases of durables by issuing collateralized short-term debt (so that the collateral constraint on such issuance is also slack); and as no assets are sold in a fire sale even if the crisis state occurs, there is no ex post regret of the size of investors' credit limit (so that the constraint that this must be fixed in advance also does not bind). Second, there are equilibria in which the real outside supply of safe assets is insufficient. There is some private issuance of safe debt, but the quantity of safe debt issued by bankers is still small enough for the collateral constraint not to bind. Third, there are equilibria in which the incentive for issuance of safe debt by bankers is so strong that their issuance of such liabilities is limited by the availability of suitable collateral. The three cases correspond to different ranges of real outside supply of safe assets: high values of $\tilde{m}$, an intermediate range of values of $\tilde{m}$, and low values of $\tilde{m}$, respectively.

This one-parameter family of stationary equilibria can alternatively be parameterized by the associated value of $R^m/\Pi = 1 + r^m(\tilde{m})$ the stationary gross real rate of return on cash. Values of $\tilde{m}$ increasing from 0 to $m^*$ correspond to values of $R^m/\Pi$ increasing from some minimum value $1 + r^m(0)$ (which may well be positive, though it will generally correspond to a negative real rate of return) to $1 + r^m(m^*) = \beta^{-1} > 1$ (the point at which the spread between the return on bonds and that on cash is completely eliminated). A numerical example may usefully illustrate how systematic variation in this parameter changes the character of the stationary equilibrium.

Figure 2 shows how the stationary equilibrium values of $c_1$, $c_2$, $c_3^c$, $c_3^n$, and $I$ vary with alternative stationary values for $R^m/\Pi$. The figure thus completely displays the allocation of real resources in each possible equilibrium, and it supplies all of the information needed to evaluate the level of expected utility of the representative household in each case and draw conclusions about the welfare effects of alternative possible long-run policy targets. The values of $R^m/\Pi$ considered vary from $1 + r^m(0)$ at the left boundary of the figure to $1 + r^m(m^*) = \beta^{-1} > 1$ at the right boundary.

**Figure 2. The Allocation of Resources in Alternative Stationary Equilibria Corresponding to Different Constant Values of $R^m/\Pi$**




In this example, cash and credit goods enter the household's utility function symmetrically, so that in an efficient allocation, equal quantities of the two goods are produced and consumed; thus, a comparison of the magnitudes of $c_1$ and $c_2$ indicates the size of the distortion created by the cash-in-advance constraint. There is no distortion ($c_1 = c_2$) at the extreme right of the figure, that is, when $R^m/\ \Pi = \beta^{-1}$ so that there is no spread between the return on longer-term bonds and cash. Moving left in the figure, as the real return on cash is reduced (meaning that the spread is made progressively larger), the extent to which $c_1$ is less than $c_2$ grows progressively greater.

The efficiency of the level of production and consumption of special goods can also be seen directly from the figure. Because both the utility from consuming special goods and the disutility of supplying them are independent of which state occurs in subperiod 2, an efficient allocation requires that $c_3^n$ equal $c_3^c$ and for the parameterization used in this example, the common efficient level of special goods production is equal to 1 (regardless of the level of production and consumption of other goods). Thus, the degree to which $c_3^n$ is greater than $c_3^c$ (and to which the former quantity is greater than 1, while the latter quantity is smaller) indicates the degree to which the production and consumption of special goods is distorted by the fact that investors spend some of their resources on acquiring risky durables in the fire sale that occurs in the crisis state. As one moves from right to left in the figure, bankers' incentive to issue collateralized short-term debt increases, but the consequence is an increasing quantity of durables that must be sold to redeem this debt in the event of a fire sale, increasing the wedge between $c_3^n$ and $c_3^c$.

The three different possible types of equilibrium correspond to different regions of the horizontal axis in the figure. The possibility of an equilibrium in which the cash-in-advance constraint is slack is represented by the right boundary ($R^m/\Pi = \beta^{-1}$); while this corresponds to an entire range of possible values of $\tilde{m}$ (any $\tilde{m} \geq m^*$), they all correspond to the same real return on cash and the same allocation of resources. The case in which the cash-in-advance constraint binds but bankers' collateral constraint is slack corresponds to values of $R^m/\Pi$ from around 0.91 to 1.01, while the case in which both constraints bind corresponds to all values of $R^m/\Pi$ from the left boundary to about 0.91.

In the relatively high-cash-return region, because bankers' collateral constraint does not bind, the quantity of short-term debt issuance by bankers increases relatively rapidly as $R^m/\Pi$ is decreased, as a consequence of which the wedge between $c_3^n$ and $c_3^c$ increases relatively sharply. However, because durables are still valued at their fundamental value in subperiod 1, the production of durables does not increase greatly. In the lower-cash-return region, further reductions in $R^m/\Pi$ do not increase debt issuance as rapidly (because now the quantity of debt issued can increase only to the extent that the quantity of durables purchased by bankers also increases enough to provide the required additional collateral), so that the wedge between $c_3^n$ and $c_3^c$ no longer increases so rapidly. Because the ability of durables to allow additional short-term debt issuance increases the price of durables above their fundamental value, the equilibrium production of durables now increases more rapidly with further reductions in $R^m/\Pi$.

Figure 3 shows the stationary values of another set of variables, across the same one-parameter family of stationary equilibria: the supply of short-term collateralized debt $\tilde{D}$ (the stationary value of the variable $\tilde{D}_t \equiv \tilde{\phi}_{6t} D_t$), the resulting total supply of cash $\hat{M}$, the upper bound $\tilde{\Gamma}s$ on issuance of short-term debt by bankers given by the expected market value of their assets in the event of a crisis, and for purposes of comparison, the market value $\tilde{Q}s$ of those same assets in subperiod 1.[39] As the equilibrium return on cash falls and

39. Each of these variables is measured in marginal-utility units, as they have a constant value in marginal-utility units in a stationary equilibrium, regardless of the inflation rate. Also, as shown above, the equilibrium relations determining the values of these variables are in many cases simpler when written in terms of the variables expressed in marginal-utility units.

the money premium correspondingly increases (moving from the right boundary of the figure to the left), the issuance of short-term debt by banks increases from an initial value of zero (when the money premium is zero) to progressively higher values. The rate of increase is sharpest in the high-cash-return region, because the upper bound on debt issuance does not bind; after that constraint begins to bind (around $R^m/\Pi = 0.91$), $\tilde{D}$ increases less sharply with further declines in $R^m/\Pi$ as it can only increase to the extent that $\tilde{\Gamma}s$ also increases. In fact, in the high-cash-return region, $\tilde{\Gamma}s$ decreases as the money premium increases; the reason is that as short-term debt issuance increases, the quantity of assets that must be sold in a fire sale in the event of a crisis increases, depressing the fire-sale value of bankers' assets. Once $R^m/\Pi$ falls to around 0.91, the constraint comes to bind, both because of the increase in desired debt issuance and the reduction in the value of the collateral available to back such debt. Beyond this point, further increases in the size of the money premium cause $\tilde{\Gamma}s$ to increase, rather than continuing to decrease; this is because the value of relaxing the constraint on short-term debt issuance now contributes to a larger market value of durables in subperiod 1,[40] which induces a larger market supply of durables (as can be seen from the $I$ curve in figure 2), so that $\tilde{\Gamma}s$ increases slightly, even though the fire-sale price $\tilde{\Gamma}s$ continues to fall.

The size of the gap between the solid line indicating the value of $\hat{M}$ and the dashed line indicating the value of $\tilde{D}$ shows how the part of the cash supply that comes from outside safe assets (the value of $\lambda\tilde{m}$, in marginal-utility units) varies across the alternative stationary equilibria. This value decreases monotonically as one proceeds from right to left in the figure, both because $\tilde{D}$ increases and because $\hat{M}$ decreases; the latter effect represents the reduction in the demand for cash balances as the opportunity cost of holding them (that is, the money premium) increases. The fact that the equilibrium relationship between the size of the money premium and the quantity of outside safe assets is monotonic indicates how the choice of a stationary level for the supply of outside safe assets (through the combination of the Treasury's debt-

40. Specifically, the value of $\tilde{\phi}_1\tilde{Q}/\lambda$ increases, which is the ratio of the marginal-utility value of the sale price of a unit of the durable good in subperiod 1, given that payment received in subperiod 1 can be used to acquire cash for use by the shopper, to the marginal-utility value of the sale price of a unit of normal goods in subperiod 2. This relative price determines the incentive to produce additional investment goods, as shown by condition (2.12), and hence the supply of durables. The stationary value of $\tilde{Q}$ does not increase, as can be seen from the $\tilde{Q}s$ curve in this figure.

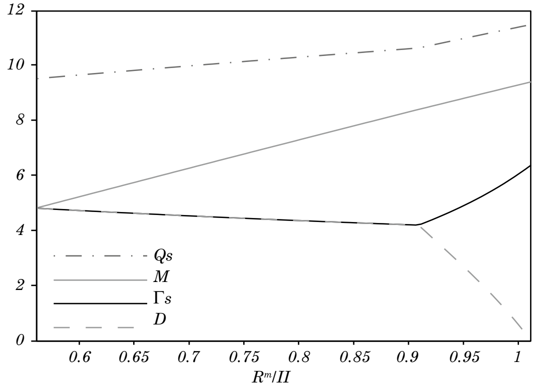
management policy and the central bank's balance sheet policy) can be used to determine the stationary value of $R^m/\Pi$ and thus to select which of the stationary equilibria depicted in these figures should occur.

There is a limit to how far $R^m/\Pi$ can be reduced by shrinking the supply of outside safe assets; at the left edge of the figure, $\tilde{m}$ falls to zero, while $R^m/\Pi$ is still positive. (This is because this lower bound does not correspond to an opportunity cost high enough to reduce the demand for cash balances to zero; it is only necessary that the demand for cash balances fall to a low enough level that it is no greater than the quantity of safe liabilities that bankers wish to supply, which grows the larger the money premium gets.) However, this lower bound for $R^m/\Pi$ can easily be well below 1 (as shown in the figure), corresponding to a negative long-run equilibrium short-term real rate. Thus, in the model it is perfectly possible to have an equilibrium short-term real rate that remains negative forever, as a result of a shortage of safe assets; this results in a safety trap in the sense of Caballero and Farhi (2013), in the case that the inflation target $\Pi$ is too low. An advantage of working with a fully developed monetary equilibrium model, however, is that the existence of a safety trap depends not simply on too low a supply of safe assets (or too great a demand for them), but also on choosing too low an inflation target, just as in the liquidity-trap model of Krugman (1998) and Eggertsson and Woodford (2003).

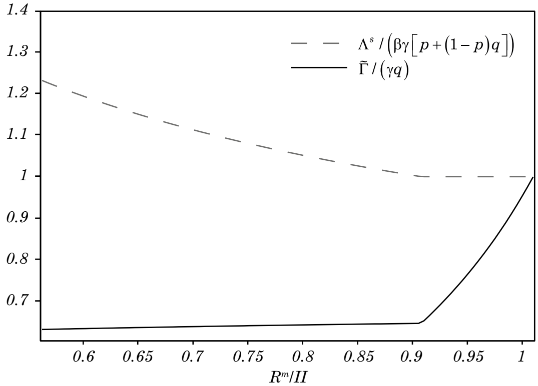
Figure 4 shows how the degree to which durables are both overvalued in subperiod 1 (and at the time that the decision to divert resources into the production of durables is made) and undervalued in the event of a fire sale varies across the alternative stationary equilibria. The dashed line plots the stationary value of $\tilde{\phi}_1\tilde{Q}/\tilde{Q}^*$, which is to say the ratio of the subperiod 1 market price of durables to their fundamental value.[41] Thus, durables are overvalued in subperiod 1 to the extent that this quantity exceeds 1. As the figure shows, it equals 1 (there is no overvaluation) in the high-cash-return region, given that banks do not wish to acquire additional durables for the sake of being able to issue more collateralized short-term debt. However, for all values of $R^m/\Pi$ below 0.91, durables are overvalued, and the degree of overvaluation gets progressively higher the larger is the money premium.

41. Alternatively, the quantity plotted is the ratio of $\Lambda^s$ to its fundamental value $\beta\tilde{Q}^*$, where $\Lambda^s$ is the marginal-utility valuation assigned to an additional quantity of investment goods sufficient to allow production of an additional unit of durables, so that the demand curve for investment goods can be written as $F'(I)=\lambda/\Lambda^s$.

## Figure 3. The Endogenous Supply of Safe Assets in Alternative Stationary Equilibria Corresponding to Different Constant Values of $R^m/\Pi$, with Implications for Bank Capital Structure and the Total Supply of Safe Assets




## Figure 4. The Initial Overvaluation of Durables and Their Subsequent Undervaluation in the Event of a Crisis,[a] in Alternative Stationary Equilibria Corresponding to Different Constant Values of $R^m/\Pi$




a. The dashed line plots the degree of initial overvaluation of durables; the solid line plots the degree of their subsequent undervaluation in the event of a crisis.

The solid line in the same figure plots the stationary value of $\tilde{\Gamma} / \gamma q$, which is the ratio of the fire-sale price of durables to their fundamental value under this contingency (which is smaller than their fundamental value in subperiod 1, since if a crisis occurs the probability that the durables are worthless is higher than previously realized). Thus, durables are undervalued in the fire sale to the extent that this quantity is less than 1. As shown in the figure, durables are undervalued in the fire sale in the case of any $R^m / \Pi = \beta^{-1}$ (corresponding to any $\tilde{m} < m^*$), and the degree of undervaluation increases steadily the larger the money premium. The degree of undervaluation increases especially sharply with increases in the money premium in the high-cash-return region, since in this region $s^*$ (the quantity of assets sold in the fire sale if one occurs) increases relatively sharply with increases in the money premium. Once the constraint that can be no larger than the total quantity $s$ of assets held by bankers becomes binding, $s^*$ increases much less rapidly with further increases in the money premium, and the degree of equilibrium undervaluation correspondingly ceases to increase so rapidly, though it grows somewhat.

Alternatively, the extent to which distortions are created by financial constraints in the alternative stationary equilibria can be measured by looking not at how market valuations differ from fundamental values, but at the extent to which the constraints affect households' decisions, as indicated by the size of the Lagrange multipliers associated with the various constraints. Figure 5 plots the values of the three key (normalized) Lagrange multipliers in the model: $\tilde{\phi}_1$, which indicates a binding cash-in-advance constraint to the extent that it is greater than 1;[42] $\tilde{\phi}_3$, which indicates a binding constraint on the quantity of collateralized short-term debt that bankers can issue to the extent that it is positive; and $\tilde{\phi}_5^c$, which indicates a binding constraint on investors' ability to spend as much in the crisis state as the household would wish ex post, to the extent that it is greater than 1.

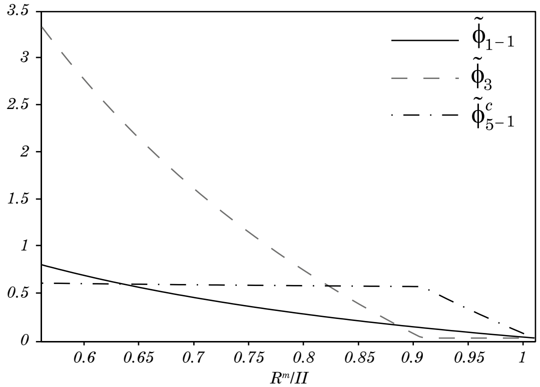
The value of $\tilde{\phi}_1$ is equal to $u_1/u_2$ the marginal rate of substitution between cash and credit goods, and the more this exceeds 1, the greater the inefficiency of the allocation of expenditure between these two types of goods (which have equal disutility of supply). As the figure shows, the magnitude of this distortion increases steadily as $R^m / \Pi$ is reduced (which is to say, as the money premium increases), starting from zero distortion when $R^m / \Pi = \beta^{-1}$, so that there is no

42. The quantity $\tilde{\phi}_1 - 1$ plotted in the figure is also the value of $\tilde{\phi}_4$ as well as $1-p$ times the value of $\tilde{\phi}_2$.

money premium. Moreover, the magnitude of the distortion is a convex function of the size of the money premium, so that the rate at which the distortion increases becomes greater for larger values of the money premium.

As explained in section 2.3, the stationary equilibrium values of $c_3^c$ and $c_3^n$ are both monotonic functions of $\tilde{\phi}_5^c$ (the first an increasing function, the latter a decreasing function), with $\tilde{\phi}_5^c = 1$ corresponding to the efficient level of production $c_3^*$ of special goods in both states. Hence, the extent to which $\tilde{\phi}_5^c$ is greater than 1 indicates the degree of inefficiency in the level of production and consumption of special goods (in both states) owing to the possibility of a fire sale of assets by banks. The figure shows that the magnitude of this distortion also increases as $R^m/\Pi$ is reduced, starting from zero distortion when $R^m/\Pi = \beta^{-1}$. However, the magnitude of this distortion increases sharply with increases in the size of the money premium only in the high-cash-return region; once the availability of collateral becomes a binding constraint on issuance of short-term debt by bankers, the degree of inefficiency in the level of production of special goods increases only gradually with further increases in the size of the money premium.

**Figure 5. Lagrange Multipliers Indicating the Degree to Which the Various Financial Constraints Bind, in Alternative Stationary Equilibria Corresponding to Different Constant Values of $R^m/\Pi$**



Finally, the figure indicates that $\tilde{\phi}_3 > 0$ indicating that the constraint that short-term debt issuance cannot exceed the amount that can be backed by the collateral value of bankers' assets binds, only for values of $R^m/\Pi$ less than 0.91. Below this point, however, the value of the multiplier rises sharply with further increases in the money premium; this accounts for the increase in the subperiod 1 market price of durables, shown in figure 4, over this same region.

## 3.2 Consequences of a Larger Central Bank Balance Sheet

We can now consider how a quantitative easing policy that permanently increases the size of the central bank's balance sheet (in real terms, or relative to the size of the economy)—and more specifically, a policy of purchasing longer-term assets and financing these purchases by issuing short-term safe liabilities—affects the economy's long-run equilibrium. To the extent that the effects of the policy are not undercut by an offsetting shift in the maturity composition of the debt issued by the Treasury,[43] such a policy can increase the steady-state level of $\tilde{m}$. If $\tilde{m} < m^*$, so that there is not already satiation of the demand for safe assets even without any creation of safe assets by the private sector, then increasing $\tilde{m}$ will mean moving to a stationary equilibrium with a higher value of $R^m/\Pi$, corresponding to a movement further to the right in each of the figures just presented.

This has real effects and, in particular, consequences for financial stability. However, a larger supply of outside safe assets as a result of a policy of quantitative easing should *improve* financial stability. Specifically, whether the economy begins in the low-cash-return or high-cash-return region, a higher value of $R^m/\Pi$ (and hence a smaller money premium) reduces private issuance of short-term debt $\tilde{D}$. As a consequence, it reduces the quantity $s^*$ of durables that will have to be sold in a fire sale in the event of a crisis and so reduces the severity of the distortions associated with a crisis.[44] Both the degree to which durables are undervalued in the crisis (as shown in figure 4) and the

43. Such a shift in Treasury policy did offset a significant part of the effect of the Fed's asset purchases in recent years, as shown by Greenwood and others (2014).

44. In the simple model presented here, the probability of a crisis is exogenous and so cannot be affected by policy, but policy can affect the severity of a crisis, conditional on the crisis state being reached.

degree of inefficiency in the level of production of special goods (as shown in figure 2) are smaller, the larger the value of $R^m/\Pi$.

Thus, from the standpoint of financial stability, a larger central bank balance sheet is clearly to be preferred (at least as far as long-run steady states are concerned). In fact, the other real effects of a quantitative easing policy on the long-run steady state are also beneficial. A higher value of $R^m/\Pi$ implies that the cash-in-advance constraint binds less tightly (as shown by the value of $\tilde{\phi}_1$ in figure 5), and this results in a more efficient allocation of household expenditure between cash and credit goods (a ratio of $c_1/c_2$ closer to 1, in figure 2). In the low-cash-return region (where $\tilde{\phi}_3 > 0$), a higher value of $R^m/\Pi$ also results in less overvaluation of durables in subperiod 1, so that there is less inefficient overproduction of durables (as is also seen in figure 2). Each of these considerations points in the same direction: the equilibrium allocation of resources is more efficient (and the welfare of the representative household is increased) if the real supply of outside safe assets is increased.

The conclusion that expansion of the central bank's balance sheet is associated with a more efficient allocation of resources between cash and credit goods might seem surprising in light of the analysis of Lucas and Stokey (1987), who conclude, in the context of a similar model (but without durable goods production or fire sales), that efficiency in this respect is greater the *lower the rate of growth* of the monetary base—with the highest levels of efficiency (and hence of welfare for the representative household) being achieved only in the case of steady contraction of the size of the central bank's balance sheet. The difference in conclusions results from their assumption that the safe liabilities that count as cash must earn a nominal interest rate of zero (so that $R^m = 1$ is assumed). In that case, steady states with different values of $R^m/\Pi$ must correspond to different inflation rates $\Pi$—whereas here the choice of the inflation target $\Pi$ is independent of the aspects of policy that determine $R^m/\Pi$, within the bound required by the lower bound on nominal interest rates.

Lucas and Stokey conclude, as I do, that relaxation of the cash-in-advance constraint, with a more efficient allocation of expenditure between cash and credit goods, requires a higher value of $R^m/\Pi$ but in their analysis this requires a lower inflation rate and hence a lower growth rate of the nominal value of outside safe assets $\tilde{M}_t$. In the model presented here, it is also true that in a long-run stationary equilibrium, the growth rate of $\tilde{M}_t$ must equal the inflation rate. However, it is possible for the central bank to control the value of the currency unit other than through its control of the path of $\tilde{M}_t$ (by appropriate variation in $R_t^m$), so that there is a decision to make about how large $\tilde{M}_t$ should be relative to the level of $P_t$ targeted through interest rate policy, which is separate from the question of the long-run growth rate of the two variables. Thus, it is not correct, more generally, to identify a decision to increase the size of the central bank's balance sheet with a decision to pursue a more *inflationary* policy; in the long run, these are two distinct issues. The short-run consequences of balance sheet expansion are considered in section 5.

## 4. Quantitative Easing Compared with Macroprudential Policy

Another implication of increasing the supply of central bank reserves through a quantitative easing policy, not discussed in the analysis above, is relaxation of the constraint on private banks' ability to issue money-like liabilities that may result from a requirement that they hold reserves in proportion to their issuance of such liabilities. Such reserve requirements apply (at least in some countries, like the United States) to at least some kinds of short-term safe instruments issued by commercial banks—though not, even in the United States, to the kind of privately issued STSIs that were most responsible for the financial fragility exposed by the recent crisis.[45] Under many traditional textbook accounts of the way that monetary policy affects the economy, the key effect of a central bank open-market operation is precisely to relax this constraint on private bank behavior by increasing the quantity of reserves that are available to satisfy the reserve requirement. This might seem to have important implications for financial stability that would cut in the opposite direction to the analysis above; that is, it might seem that expansion of the central bank's balance sheet should have as an effect, or even as its primary effect, an increase in the extent to which private banks acquire risky assets and finance those assets by issuing money-like liabilities. This is a key theme of the analysis by Stein (2012) and the basis for his proposal that monetary policy decisions be considered from the standpoint of financial-stability regulation.

45. The kinds of liability, such as retail deposits at commercial banks, to which such requirements apply were not subject to highly volatile demand. While these funds could, in principle, be withdrawn on short notice, they were not, probably owing to the existence of deposit insurance; so they were not responsible for any appreciable funding risk.

In the analysis here, I have abstracted from reserve requirements, since even in the United States, these have not been binding constraints on banks' behavior during the Fed's experiments with quantitative easing.[46] The framework can, however, be used to discuss the consequences for financial stability of increasing or decreasing the cost to financial institutions of issuing collateralized short-term debt as a source of financing, even when they hold sufficient assets to provide the collateral for such issuance. This as a separate dimension of policy—*macroprudential* policy—that should be distinguished, conceptually, from both conventional monetary policy (interest rate policy) and central bank balance sheet policy.[47] One might well use instruments of macroprudential policy that affect the ability and/or incentives of banks to issue money-like liabilities that are unrelated to the central bank's balance sheet (and that do not depend on the existence of reserve requirements). Even when the tool that is used is a reserve requirement, one can loosen or tighten this constraint independently of the way one changes the size of the central bank's balance sheet— first, because one can vary the required reserve ratio as well as the supply of reserves; second, because the central bank can vary the supply of STSIs without varying the supply of reserves, if it issues central bank bills or engages in reverse repo transactions,[48] or by varying the quantity of Treasury bills on its own balance sheet.

The effects of varying macroprudential policy are quite different from the effects (considered above) of varying the central bank's supply of outside safe assets, when the latter policy is implemented in a way that has no direct effects on financial institutions' cost of short-term debt issuance. Macroprudential policy can be introduced into the model set out above in the following way. Suppose that a banker who issues short-term debt with face value $D_t$ obtains only $\xi_t D_t$ in additional funds with which to acquire assets in subperiod 1, where $0 \leq \xi_t \leq 1$; the quantity $(1-\xi_t)D_t$ represents a proportional tax on issuance of safe debt, collected by the government. The variable $\xi_t$ (or alternatively the

46. They were not relevant, even earlier, for most of the financing decisions modeled in this paper. As noted earlier, the privately supplied "cash" in this model should be identified primarily with repos or asset-backed commercial paper.

47. Macroprudential policy, modeled in a way similar to that used here, is also compared with conventional monetary policy by Sergeyev (2016), who also discusses Ramsey policy when the two distinct types of policy instruments exist. Sergeyev's discussion of optimal policy does not treat the use of balance sheet policies of the kind that are the central focus here.

48. See Carlson and others (2014) on the usefulness of reverse repo transactions, such as the Fed's proposed ON RRP facility, for this purpose.

tax rate) then represents an instrument of macroprudential policy. the value of $\xi_t$ may be varied from period to period, if the degree to which it is desirable to provide a disincentive to safe debt issuance varies over time; and the choice of the path of $\{\xi_t\}$ is independent of the choice of the path of $\{\tilde{m}_t\}$, the real outside supply of safe assets.

One possible way of implementing such a tax on safe debt issuance is through a reserve requirement. Suppose that a bank that issues safe debt with face value $\hat{D}_t$ is required to hold reserves $H_t \geq k_t \hat{D}_t$ where $H_t$ is the value of the reserves in the end-of-period settlement. Suppose, furthermore, that reserves pay a gross nominal interest rate of $R_t^{cb} \geq R_t^m$ which means that $\theta_t R_t^m / R_t^{cb} \equiv 1$ units of cash must be paid in subperiod 1 to acquire a unit of reserves. Finally, suppose that a bank's reserve balance can be used to pay off its safe debt in subperiod 2, if the holders of the bank's short-term debt are not willing to roll it over, with one unit of reserves serving to retire one unit of short-term debt. Then the bank's collateral constraint again takes the form of equation (1.3), and the assets sold in a fire sale must satisfy constraint (1.6), where now $D_t \equiv \hat{D}_t - H_t$ is short-term debt issuance not covered by the bank's reserve balance. The funds obtained by the bank with which to purchase additional assets in subperiod 1 are only $\hat{D}_t - \theta_t H_t$ owing to the need to acquire reserves with some of the proceeds of the debt issuance. This quantity can alternatively be expressed as $\xi_t D_t$ where

$$\xi_t \equiv \frac{1 - k_t \theta_t}{1 - k_t} \leq 1.$$

If we assume that $k_t \theta_t \leq 1$ so that it is possible for the bank to acquire the required reserves out of the proceeds of its short-term debt issuance,[49] then $\xi_t \geq 0$ as assumed above. Thus, reserve requirements are an example of the kind of macroprudential policy that can be modeled in the way proposed above (in the case that the interest rate paid on reserves is less than the rate paid on cash). In this case, $\xi_t$ can be reduced either by reducing the interest rate $R_t^{cb}$ paid on reserves (relative to the central bank's target for the interest rate paid on cash) or by increasing the required reserve ratio $k_t$.

The first-order conditions that characterize optimal household behavior are not changed by the introduction of macroprudential policy, except that equation (2.4) now takes the more general form

49. Tighter reserve requirements than this would have no effect, since when $k_t \theta_t = 1$ banks are already completely precluded from raising any funds by issuing short-term debt.

$$\xi_t \phi_{1t} = (1-p)\phi_{2t} + \mathrm{E}_t \phi_{6t}. \tag{4.1}$$

With this change, the derivation of the conditions for an intertemporal equilibrium proceeds as in section 2. The equilibrium paths of the endogenous variables now depend on the specification of the series $\{P_t, \tilde{m}_t, \xi_t\}$, representing three distinct dimensions of policy: conventional monetary policy; the determination of the outside supply of safe assets by debt management policy and quantitative easing; and macroprudential policy.

This more general version of the model yields a three-parameter family of stationary equilibrium, indexed by stationary values $\Pi$, $\tilde{m}$, and $\xi$. The stationary real allocation of resources depends only the stationary values of $\tilde{m}$ and $\xi$. The previous section showed how variation in $\tilde{m}$ (or alternatively, in $R^m/\Pi$) affects the stationary equilibrium values of real variables and relative prices, for a fixed value of $\xi$. (That discussion used the assumption that $\xi = 1$, but similar qualitative conclusions would obtain in the case of any fixed value of $\xi$). Here we consider instead the consequences of varying the stationary value of $\xi$ and in particular, the extent to which the effects of varying the strength of macroprudential policy (perhaps by relaxing or tightening reserve requirements) are equivalent to the effects of variations in the supply of reserves, discussed in the previous section.

Figure 6 shows again the stationary values of the variables plotted in figure 3 (which compares short-term debt issuance by banks with the total supply of cash and with the available collateral to back such issuance), for alternative constant values of $\xi \leq 1$, holding fixed the target that determines the central bank's balance sheet policy (which is here assumed to be a fixed target for the term premium associated with longer-term bonds, or equivalently a fixed value of $R^m/\Pi$). In the case shown in the figure, the target for $R^m/\Pi$ is low enough that, in the absence of any reserve requirement or other regulation of short-term debt issuance by banks (that is, the case $\xi = 1$), the stationary equilibrium is of the low-cash-return type discussed in the previous section; that is, the incentive for short-term debt issuance by banks is great enough for the collateral constraint to bind, resulting in overvaluation and oversupply of durables in subperiod 1. I consider this case for the numerical illustration because it is the case in which there is the most reason to be interested in whether macroprudential policy can reduce the distortions resulting from banks' excessive incentive to issue short-term debt. The corresponding stationary values for the market valuation of durables are shown in figure 7.

**Figure 6. Short-Term Debt Issuance by Banks in Alternative Stationary Equilibria Corresponding to Different Constant Values of ξ , for a Fixed Value of $R^m/\Pi$**

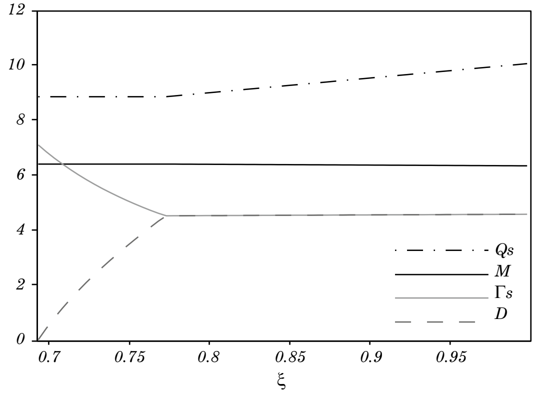
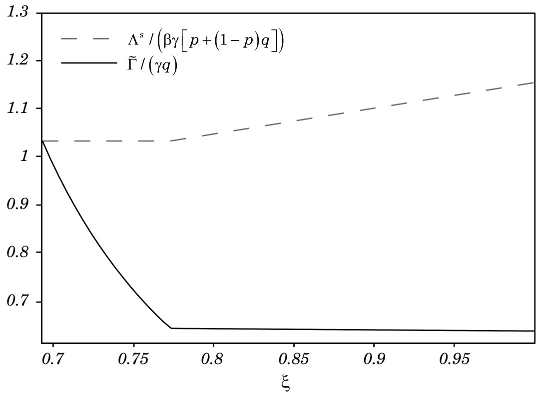



**Figure 7. The Initial Overvaluation of Durables and Their Subsequent Undervaluation in the Event of a Crisis,[a] in Alternative Stationary Equilibria Corresponding to Different Constant Values of ξ**




a. The dashed line plots the degree of initial overvaluation of durables; the solid line plots the degree of their subsequent undervaluation in the event of a crisis. The figure uses the same fixed value of $R^m/\Pi$ as in figure 6.

Figure 6 shows that as the tax rate on short-term debt issuance increases (or the effective tax rate, by increasing the required reserve ratio or reducing the rate of interest paid on reserves), which lowers $\xi$, the stationary value of $\tilde{D}$ falls. For a sufficiently large tax rate (the case of $\xi$ less than 0.77, in the numerical example), the collateral constraint ceases to bind; this implies that durables are no longer overvalued in subperiod 1, as shown in figure 7. In the case of an even larger tax rate (though still less than 100 percent taxation of the proceeds from issuing short-term debt), short-term debt financing of banks is completely driven out ($\tilde{D} = 0$), because the macroprudential tax fully offsets the value of the money premium to issuers of financial claims that can be used as cash. (In the numerical example, this occurs when $\xi = 0.69$, the left boundary of the figures.) When this occurs, bankers no longer have to sell assets in a fire sale, even if the crisis state occurs, and the undervaluation of durables in the crisis state is eliminated, as is also shown in figure 7. Further reductions in $\xi$ below this value are irrelevant, as banks' issuance of short-term debt cannot be further reduced.

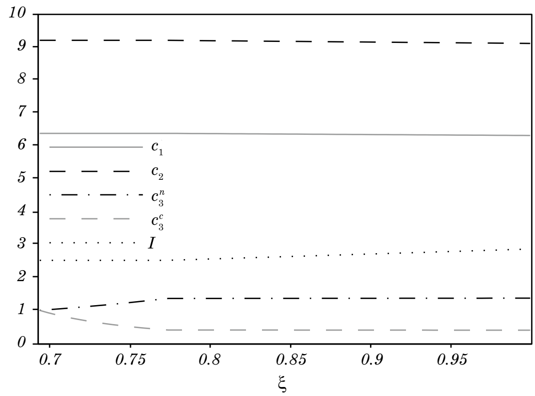
The implications of these alternative equilibria for the allocation of resources are shown in figure 8. Because balance sheet policy is used to fix the value of $R^m/\Pi$, the stationary value of $\tilde{\phi}_1$ and hence the stationary value of $\tilde{\phi}_4$ are unaffected by changing $\xi$. This means that the degree of inefficiency in the allocation of expenditure between cash and credit goods (as measured by the degree to which the marginal rate of substitution $u_1/u_2$ is greater than 1, the relative cost of producing them) is unaffected, so the equilibrium levels of production of cash and credit goods are little affected. However, as $\xi$ is decreased from 1 (while still greater than 0.77), the degree of inefficient overproduction of investment goods is reduced, owing to the decrease in the degree to which banks are willing to pay to relax their collateral constraints. (Once $\xi$ is less than 0.77, the collateral constraint no longer binds, as shown in figure 6, such that further reductions in $\xi$ produce no further reductions in this distortion.) Moreover, because reductions in $\xi$ reduce short-term debt issuance (as long as $\xi$ remains greater than 0.69) and, with that, the value of $s^*$, they reduce the degree of inefficiency in the production and consumption of special goods: both $c_3^c$ and $c_3^n$ move closer to the efficient level of 1, which they reach exactly if $\xi$ is reduced to 0.69.

We can now ask to what extent the effects of expanding the supply of central bank liabilities through quantitative easing are equivalent, or even similar, to the effects of relaxing a reserve requirement that limits banks' ability to issue money-like liabilities. In the context of the model, the former sort of policy corresponds to an increase in $\tilde{m}$

(resulting in an increase in $R^m/\Pi$ if there is not already satiation in cash balances), which can be implemented while keeping $\xi$ fixed; the latter sort of policy corresponds to an increase in $\xi$ (assuming a reserve requirement tight enough to bind), which can be implemented while keeping $\tilde{m}$ fixed or, with an appropriate adjustment of the central bank's balance sheet, while keeping $R^m/\Pi$ fixed.

A comparison of figures 6–8 with figures 2–4 shows that not only are these two policies not equivalent, their effects are in many respects exactly the opposite. An expansion of the central bank's balance sheet while fixing $\xi$ corresponds to a movement from left to right in figures 2–4: short-term debt issuance by private banks falls, both the overvaluation of durables in subperiod 1 and their undervaluation in the event of a crisis are reduced, the overproduction of durables is reduced, and the level of production of special goods in both the $c$ and $n$ states becomes more nearly efficient. A relaxation of a binding reserve requirement while fixing $R^m/\Pi$ corresponds instead to a movement from left to right in figures 6–8, which essentially reverses the effects seen in the earlier figures: short-term debt issuance by private banks increases, both the overvaluation of durables in subperiod 1 and their undervaluation in the crisis state increase, the overproduction of durables in increased, and the level of production of special goods is progressively more severely distorted.

**Figure 8. The Allocation of Resources under Alternative Stationary Equilibria[a]**




a. The figure uses the same set of alternative stationary equilibria shown in figures 6 and 7.

In fact, both an expansion of the outside supply of safe assets and a tightening of reserve requirements (or other forms of macroprudential policy) have similar consequences for financial stability, insofar as both reduce the extent to which banks finance themselves by issuing short-term safe debt. Either of these policies, pursued far enough, will completely eliminate private issuance of money-like claims (the right boundary of figures 2–4 or the left boundary of figures 6–8) and consequently eliminate the distortions resulting from the risk of a fire sale of assets and from the desire of bankers to obtain assets that can be used to collateralize short-term debt issuance. Thus, each of these policies, either of which is welfare-enhancing (when not irrelevant), can serve to some extent as a substitute for the other. However, while a sufficient increase in the outside supply of safe assets would make macroprudential policy unnecessary in the model (since private issuance of money-like claims can be completely eliminated, even if $\xi = 1$), the reverse is not true: even a macroprudential policy of the maximum possible stringency (one that completely prevents private issuance of STSIs) will not eliminate the welfare gains from further expansion of the outside supply of safe assets, since even when $\tilde{D} = 0$ (as in the case with $\xi = 0.69$ in figures 6–8), there will still be inefficient underconsumption of cash goods, owing to the binding cash-in-advance constraint, as long as $\tilde{m} < m^*$.

## 5. Conventional and Unconventional Monetary Policy in the Presence of Nominal Rigidities

In the analysis thus far, all prices have been assumed to be perfectly flexible and to clear markets each period. In such a model, conventional monetary policy has no real effects, but rather only influences the general level of prices in terms of the monetary unit. It follows that conventional monetary policy has no consequences for financial stability. This establishes a sharp distinction between the effects of conventional monetary policy (interest rate policy) and balance sheet policy, since as shown above, the central bank's balance sheet (specifically, the *real* supply of safe assets by the central bank) does have consequences for financial stability.

Such an analysis is adequate for consideration of the possible long-run stationary equilibria achievable under alternative policies, as in the previous two sections. But it does not suffice for an analysis of the considerations at play when alternative dimensions of monetary policy are used to address short-run macroeconomic stabilization objectives, and this is the context in which central banks' recent experiments with quantitative easing have been conducted. To address the issues raised by recent policies, we need to consider the consequences for financial stability of using quantitative easing as a substitute for an interest rate cut that is prevented by the effective lower bound on short-term nominal interest rates, in a situation where such an interest rate cut would otherwise be desired in order to achieve a higher level of output.

The notion that an interest rate cut would be desired in order to increase real activity only makes sense in the presence of nominal rigidities of some kind. Here I discuss a simple extension of the model presented above, which shows how sticky prices allow conventional monetary policy to have real effects in the short run while only affecting the general price level in the long run. This allows the comparison of the effects of quantitative easing and those of an interest rate cut, with respect to both the effects of these policies on aggregate demand and their consequences for financial stability.

## 5.1 Equilibrium with a Sticky Price for Normal Goods

Only the price $P_t$ of normal goods must be set in advance, while the prices of special goods, durable goods, and all financial assets are assumed to be perfectly flexible, as above. (Because all three types of normal goods are perfect substitutes from the standpoint of their suppliers, I assume that a single price $P_t$ is posted, at which goods of any of these types can be purchased, and the buyer determines which type of good will be obtained.) For simplicity, I also consider here the case of a single unexpected aggregate shock (apart from the kind of uncertainty represented in figure 1) at some date $t$, in response to which monetary policy (both interest rate policy and balance sheet policy) may be adjusted; there is no further uncertainty (except for the kind depicted in figure 1) about how the economy will evolve after this shock occurs, and the shock is completely unanticipated prior to its occurrence.

The fact that the shock is completely unexpected means that before it occurs, people expect an equilibrium in which there will never be any random developments except the kind depicted in figure 1. This equilibrium can be assumed to be a stationary equilibrium of the kind described in section 3. In such an equilibrium, the price $P_\tau$ of normal goods in any period $\tau$ is a deterministic function of time; it does not depend on which state is reached in subperiod 2 of period $\tau$, nor does it depend on the history $\xi_\tau$ of states revealed in previous periods. Hence, the same price $P_\tau$ is set for normal goods in all periods $\tau \leq t$ as would clear markets in the flexible-price stationary equilibrium analyzed above, even if the price $P_\tau$ must be set before subperiod 2 of period $\tau$ is reached. For purposes of the present discussion, it is not necessary to define how exactly the predetermined price of normal goods is determined, beyond the assumption that in an environment where the future is perfectly predictable (except for the uncertainty each period depicted in figure 1), the price that is set each period is the one that would clear the market for normal goods.

Let us suppose that period $t$ is one in which no crisis occurs in subperiod 2 (though it is not known up until this time that this would be the case). But let us also suppose that in subperiod 2 of period $t$, an unexpected shock occurs, as a result of which the utility of cash and credit goods consumption is equal to $\chi u(c_{1t}, c_{2t})$, and the disutility of supplying normal goods is equal to $\chi v(Y_t)$, for some factor $\chi > 0$ that need not equal 1; the other components of the utility function are unaffected by the shock. The factor $\chi$ is assumed to take a value different from 1 only in period $t$ (and prior to period $t$, it is assumed to equal 1 with probability 1 in period $t$ as well). The point of assuming a shock of this particular type is that for a given level of production of investment goods, the efficient level of production and consumption of cash and credit goods would not be changed by the shock $\chi$; however, the real interest rate required to sustain that level of demand will change (will be lower if $\chi$ is lower). Hence, the shock $\chi$ represents a demand disturbance to which it would be desirable to respond by lowering interest rates, if this is not precluded by the interest rate lower bound.

Both conventional monetary policy and balance sheet policy are allowed to respond to the occurrence of the shock, though their paths are assumed to be perfectly predictable from then on, as with all other exogenous variables. Both $R_\tau^m$ and $\tilde{m}_\tau$ are determined in subperiod 2 of period $\tau - 1$. Hence, neither $R_\tau^m$ nor $\tilde{m}_t$ can be affected by the value of $\chi$; these variables are both equal to their values in the stationary equilibrium. But $R_\tau^m$ and $\tilde{m}_\tau$ can both differ from their stationary equilibrium values in periods $\tau \geq t + 1$.

For simplicity, I consider here only policy responses to the shock of a special sort. We continue to suppose that from period $t + 1$ onward, conventional monetary policy (that is, the choice of $R_\tau^m + 1$ for all $\tau \geq t + 1$) is used to ensure that the path of normal goods prices $\{P_\tau\}$ grows at the constant rate $\pi^*$ in all periods $\tau \geq t+1$.[50] Moreover, balance sheet policy is used to achieve a real outside supply of cash $m_{\tau+1}$ equal to the stationary equilibrium value $\tilde{m}$ for all $\tau \geq t+1$. The set of alternative monetary policies considered can then be reduced to a two-parameter family, corresponding to different possible choices of $R^m_{t+1}$ and $\tilde{m}_{t+1}$ (both of which must be chosen in subperiod 2 of period $t$, but which may depend on the value of $\chi$).[51]

Because the price $P_t$ has been fixed in advance, it is assumed to be independent of the value of $\chi$ and equal to the price associated with the stationary equilibrium that had previously been expected to continue. Once the shock $\chi$ occurs, there is no further uncertainty about how the economy will evolve from then on (except the uncertainty depicted in figure 1). Hence, the price $P_\tau$ of normal goods in each period $\tau \geq t+1$ is set so as to clear the market for normal goods in that period. (While $P_\tau$ must be set prior to subperiod 2 of period $\tau$, it is not set prior to subperiod 2 of period $\tau-1$.[52]) Thus, in the equilibrium considered in this section, the only period in which the market for normal goods need not clear is period $t$ (the period in which the shock $\chi$ occurs); in that period, $P_t$ is set at the level that would clear the market in the event that $\chi = 1$.

More generally, all variables that are determined in subperiod 1 of period $t$, or earlier, are assumed to be determined as in the equilibrium in which $\chi = 1$ is expected (that is, as in the stationary equilibrium with flexible prices implied by the initial policy). Thus, the values of $A_t$, $M_t$, $B_t$, $D_t$, $F_t$, $s_t$, and $Q_t$ are unaffected by the shock, in addition to $P_t$ and all variables dated $t-1$ or earlier. Instead, the variables $c_{1t}$, $c_{2t}$, $c_{3t}$, $I_t$, $Y_t$, $x_t$, and $\tilde{P}$ as well as all variables dated $t+1$ or later, are determined in a way that takes account of the occurrence of the shock

50. The value of $P_{t+1}$ is set in advance on the basis of expectations about the demand for normal goods in period $t+1$, which will depend on the interest rate $R^m_{t+2}$ because of condition (2.15). Thus, the rule for setting $R^m_{\tau+1}$ in periods $\tau \geq t+1$ can be used to ensure that the market-clearing price for normal goods in all periods $\tau \geq t+1$ is consistent with the inflation target. This desideratum leaves the value of $R^m_{t+1}$ undetermined. The value of $P_t$ reflects expectations about how $R^m_{t+1}$ would be set; but these are expectations about monetary policy in period $t$ that were held prior to the unexpected shock, which may not be confirmed, as a result of the shock.

51. For simplicity, in this section I abstract from the possible use of macroprudential policy, as in sections 1-3; that is, the discussion considers only equilibria in which $\xi_t = 1$ at all times.

52. This means that the length of time for which prices are sticky is limited in the proposed model. A quantitatively realistic model would doubtless need to allow some prices to remain fixed for a longer period, but the simple case considered here suffices to illustrate the qualitative effects of temporary stickiness of prices.

$\chi$.[53] The Lagrange multipliers $\phi_{4t}$, $\phi_{5t}$, and $\phi_{6t}$ are jointly determined with this latter set of variables (as well as the Lagrange multipliers for later periods).

The variables that are affected by the shock $\chi$ are determined by a system of intertemporal equilibrium conditions of the form stated earlier, with the following exceptions. First, the fact that the suppliers of normal goods must supply whatever quantity of such goods is demanded at the predetermined price $P_t$ means that the first-order condition (2.13) need not be satisfied in period $t$ ex post (that is, after the shock $\chi$ occurs). However, the other first-order conditions for optimal household behavior stated above continue to apply, and condition (2.13) also must hold in periods $t + 1$ and later (since normal goods prices in those periods are set in a way that clears the market). Thus, we drop one (but only one) of the conditions that would determine a flexible-price intertemporal equilibrium from subperiod 2 of period $t$ onward, replacing it by the requirement that $P_t$ equal a predetermined value, whether this clears the market for normal goods or not. Second, the partial derivatives $u_i(c_1, c_2)$ in first-order conditions (2.9)–(2.10) are replaced by $\chi u_i(c_1, c_2)$ (for $i = 1,2$) in period $t$ only. All other first-order and market-clearing conditions continue to take the forms stated above.

The demand for cash and credit goods in period $t$ is then given by

$$c_{1t} = c_1(\lambda_t/\chi;\ M_t/P_t)$$

and

$$c_{2t} = c_2(\lambda_t/\chi;\ M_t/P_t),$$

where $M_t/P_t$ is unaffected by the shock. Aggregate demand for normal goods in period $t$ is accordingly

$$Y_t = c_1(\lambda_t/\chi;\ M_t/P_t) + c_2(\lambda_t/\chi;\ M_t/P_t) + I(\lambda_t;\lambda_{t+1},\tilde{m}_{t+1}). \tag{5.1}$$

Since $c_1(\lambda;m)$ and $c_2(\lambda;m)$ are both nonincreasing functions of $\lambda$ and at least $c_2$ must be decreasing, it follows that aggregate demand is a monotonically increasing function of $\chi$, for given values of $\lambda_t$ and $\lambda_{t+1}$.

53. The variables $s_t^*$ and $\Gamma_t$ are undefined, as we have assumed that the crisis state does not occur in period $t$.

Condition (2.40) continues to be a requirement for equilibrium, as a result of which it must be the case that

$$\lambda_t = \beta\lambda_{t+1}\hat{\phi}_1(\lambda_{t+1}; F(I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1})), \tilde{m}_{t+1})R^m_{t+1}\frac{P_t}{P_{t+1}}. \tag{5.2}$$

This equation indicates how the choice of $R^m_{t+1}$ in period $t$ affects the value of $\lambda_t$ —and through it aggregate demand $Y_t$— for given expectations about conditions in period $t+1$. If the price $P_t$ were required to clear the market for normal goods, substitution of equation (5.1) into (2.13) would yield a condition to determine the required value of $\lambda_t$ in equilibrium; equation (5.2) would then indicate the interest rate $R^m_{t+1}$ required to achieve the price-level target $P_t$. Under the assumption that $P_t$ is predetermined and need not clear the market, it is possible for $R^m_{t+1}$ to change in response to the shock, resulting in a value of $Y_t$ that need not satisfy the voluntary supply condition (2.13).

In each period from $t+1$ onward, we effectively have flexible prices, so that condition (2.39) is again required for equilibrium. Thus, for any specification of $R^m_{t+1}$ and of the path $\{\tilde{m}_\tau\}$ for all $\tau \geq t+1$, the equilibrium sequence $\{\lambda_t\}$ for $\tau \geq t$ is determined by condition (5.2) and the sequence of conditions of the form (2.39) for each period from $t+1$ onward. Given a solution for the sequence $\{\lambda_\tau\}$, aggregate demand for normal goods is determined by equation (5.1) in period $t$ and by equation (2.37) in each period from $t+1$ onward. The implied equilibrium values of other variables are then determined as described in section 2.

## 5.2 Real Effects of Conventional and Unconventional Monetary Policy

The effects of quantitative easing can now be compared with those of conventional interest rate policy, as possible responses to a shock $\chi$. If both $R^m_{t+1}$ and the path $\{\tilde{m}_\tau\}$ for $\tau \geq t+1$ remain fixed at the values associated with the stationary equilibrium in which there is no shock, then the values $\lambda_\tau = \bar{\lambda}$ for all $\tau \geq t$ will satisfy condition (2.39) in period $t$ and condition (2.39) for each of the periods $t+1$ and later, where $\bar{\lambda}$ is the constant value of $\lambda_\tau$ in the stationary equilibrium. Aggregate demand for normal goods in period $t$ is then given by equation (5.1). If $\chi = 0$, this implies $Y_t = \bar{Y}$ the constant level of output in the stationary equilibrium. If instead $\chi < 0$, then $Y_t < \bar{Y}$. This reduction in the production of normal goods will be inefficient, since it implies that

$$u_2(c_{1t}, c_{2t}) = \chi\bar{\lambda} < \bar{\lambda} = v'(\bar{Y}) \leq v'(Y_t),$$

so that the marginal utility of additional consumption of normal goods would exceed the marginal disutility of supplying them.

We consider now the extent to which monetary policy can be used to respond to such a shock. In addition to the effects of policy on production and consumption, we are interested in how each of the possible dimensions of central bank policy influence financial conditions. Two measures of financial conditions are especially useful. One is the size of the money premium earned by cash, which can be measured by the extent to which the ratio

$$\frac{E_t[R^b_{t+1}]}{R^m_{t+1}} = \tilde{\phi}_{1,t+1}$$

is greater than one. This is a measure of financial conditions that determines the incentives for short-term debt issuance by banks. Another important measure is the expected one-period real return on longer-term bonds,

$$1 + \bar{r}^b_{t+1} \equiv E_{t+1}\left[\frac{R^b_{t+1}P_t}{P_{t+1}}\right] = \frac{\lambda_t}{\beta\bar{\lambda}}.$$

(This aspect of financial conditions can alternatively be measured by the value of $\lambda_t$.) This is the measure of financial conditions that is relevant for determining the aggregate demand for nondurable normal goods, as a result of equation (5.1). Below we analyze the effects of each of the dimensions of policy on both of these measures of financial conditions.

### 5.2.1 Conventional monetary policy

Conventional monetary policy can be used to mitigate the effects of a $\chi$ shock by lowering $R^m_t$ (if this is not prevented by the lower bound on the nominal interest rate). The effects of such policy are most easily seen in the special case that $v(Y)$ is linear, so that $v'(Y) = \bar{\lambda}$ regardless of the value of $Y$. Then condition (2.39) requires that $\lambda_{t+1} = \bar{\lambda}$ and equation (5.2) reduces to

$$\lambda_t = \beta\bar{\lambda}\hat{\phi}_1(\bar{\lambda}; F(I_t), \tilde{m}_{t+1})R^m_{t+1} / \Pi. \qquad (5.3)$$

Here the target gross inflation rate $\Pi$ has been substituted for $P_{t+1}/P_t$, on the assumption that interest rate policy in period $t+1$ is used to ensure that the target inflation rate is realized, regardless of other conditions.

For simplicity, I only discuss the case of an equilibrium in which bankers' collateral constraint binds in period $t+1$, so that durables are overvalued in subperiod 1. This is the case in which risks to financial stability are of the greatest concern. It follows from equation (2.29) that

$$M_t / P_t = \tilde{m}_t + \lambda_t^{-1}\tilde{\Gamma}(s_t)s_t$$

This, together with the fact that $\tilde{\phi}_1(\lambda;m)$ is decreasing in both arguments (as shown in section 2), can be used to conclude that $\tilde{\phi}_1(\lambda;s,\tilde{m})$ will be a decreasing function of both $s$ and $\tilde{m}$, for any fixed value of $\lambda$.

In addition, equation (2.35) requires that

$$\lambda_t = \beta\varphi(\bar{\lambda}; F(I_t), \tilde{m}_{t+1})F'(I_t). \tag{5.4}$$

This establishes an equilibrium relationship between investment demand $I_t$ and financial conditions as measured by $\lambda_t$ (although the value of $\tilde{m}_{t+1}$ remains of independent relevance). Equating the right-hand sides of equations (5.3) and (5.4) shows that equilibrium investment $I_t$ must satisfy

$$\hat{\phi}_1(\bar{\lambda}; F(I_t), \tilde{m}_{t+1}) = \frac{\Pi}{\bar{\lambda}R^m_{t+1}}\varphi(\bar{\lambda}; F(I_t), \tilde{m}_{t+1})F'(I_t). \tag{5.5}$$

Again restricting attention to the case where bankers' collateral constraint binds in period $t+1$, the function $\varphi(\lambda;s,\tilde{m})$ can alternatively be expressed in terms of the function $\tilde{\phi}_1$. Note that

$$\begin{aligned}
\tilde{\phi}_{1t}\tilde{Q}_t &= \tilde{Q}^* + (1-p)\tilde{\phi}_{3t}\tilde{\Gamma}_t \\
&= \tilde{Q}^* + (1-p)\left[\left(\tilde{\phi}_{2t}+1\right)\tilde{\Gamma}_t - \gamma q\right] \\
&= \gamma p + \left(\tilde{\phi}_{1t} - p\right)\tilde{\Gamma}_t,
\end{aligned}$$

using equations (2.5), (2.6), and (2.4) in succession. It follows that the function $\varphi$ can be expressed as

$$\varphi(\lambda; s, \tilde{m}) \equiv \hat{\varphi}(\hat{\phi}_1(\lambda; s, \tilde{m}); s), \tag{5.6}$$

where

$$\hat{\varphi}(\tilde{\phi}_1; s) \equiv \gamma p + \left(\tilde{\phi}_1 - p\right)\tilde{\Gamma}(s). \tag{5.7}$$

Condition (5.5) can then be written alternatively in the form

$$\tilde{\phi}_{1,t+1} = \frac{\Pi}{\bar{\lambda} R^m_{t+1}} \hat{\varphi}(\tilde{\phi}_{1,t+1}; F(I_t))F'(I_t), \tag{5.8}$$

using equation (5.6). This describes a relationship that must exist between investment demand $I_t$ and the money premium $\tilde{\phi}_{1\,t+1}$ in the case of any given specification of conventional monetary policy. This relationship is unaffected by the value of $\tilde{m}_{t+1}$, which is relevant for the discussion of quantitative easing below.

Because $\tilde{\Gamma}(s) \leq \gamma q < \gamma$, equation (5.7) implies that

$$0 < \frac{\partial \hat{\varphi}}{\partial \tilde{\phi}_1} < \frac{\hat{\varphi}}{\tilde{\phi}_1}, \qquad \frac{\partial \hat{\varphi}}{\partial s} < 0.$$

It follows that equation (5.8) implicitly defines a function

$$\tilde{\phi}_{1,t+1} = \bar{\phi}_1(I_t, R^m_{t+1}) \tag{5.9}$$

which is decreasing in both arguments.

This shows, together with equation (2.34), that the equilibrium level of investment $I_t$ must satisfy

$$\hat{\phi}_1(\bar{\lambda}; F(I_t), \tilde{m}_{t+1}) = \bar{\phi}_1(I_t, R^m_{t+1}). \tag{5.10}$$

This can be solved for the effects of monetary policy on investment. Given that $\hat{\phi}_1(\lambda; s, m)$ is a decreasing function of $s$ (when the collateral constraint binds), both sides of equation (5.10) are decreasing functions of $I_t$.

The comparative statics of $I_t$ in response to a change in either $R^m_{t+1}$ or $\tilde{m}_{t+1}$ then depend on the relative slopes of these two schedules. I assume that in the initial equilibrium, relative to which we wish to consider the effects of a change in monetary policy,

$$\frac{\partial \bar{\phi}_1}{\partial I} < \frac{\partial \hat{\phi}_1}{\partial I} < 0, \tag{5.11}$$

as shown in the upper part of figure 9. In this case, we obtain the conventional signs for the short-run effects of interest rate policy.[54] In particular, because $\bar{\phi}_1(I, R^m)$ is a decreasing function of $R^m$ a reduction of $R^m_{t+1}$ will increase $I_t$, as shown in the figure.

It also follows from equation (5.6) and the fact that $\hat{\phi}_1(\lambda; s, m)$ is a decreasing function of $s$ that $\varphi(\lambda; s, \tilde{m})$ is also a decreasing function of $s$. Thus, the right-hand side of equation (5.4) is a decreasing function of $I_t$, for any fixed value of $\tilde{m}_{t+1}$. Hence, equation (5.4) establishes an inverse relationship between $\lambda_t$ and $I_t$ that must hold regardless of the value chosen for $R^m_{t+1}$. This relationship is graphed in the lower part of figure 9. Since a reduction in $R^m_{t+1}$ increases $I_t$, it must also reduce $\lambda_t$, as shown in the figure. This, in turn, will imply an increase in $Y_t$, because of equation (5.1).[55]

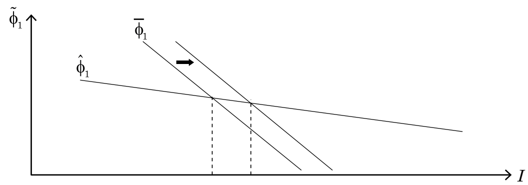
While interest rate policy can be used to stimulate aggregate demand in this way, and so to reduce some of the distortions created by the demand shock $\chi$, it has the side effect of increasing risks to financial stability. Part of the increase in aggregate demand associated with a reduction of $\lambda_t$ will be an increase in the production of investment goods, as a result of which $s_{t+1}$ will be higher. In the case of an equilibrium in which the collateral constraint binds, this will mean a correspondingly higher value of $s^*_{t+1}$, as a consequence of which the degree of undervaluation of durables in the event of a crisis and fire sale will be more severe. Thus, in the sticky-price version of the model, it is indeed the case that reducing short-term nominal interest rates increases risk-taking by banks in a way that makes the distortions associated with a crisis more severe, should one occur.

54. If the inequality (5.11) is reversed, the model would imply that a reduction in the interest rate on cash is associated with a decrease, rather than an increase, in aggregate demand. Condition (5.4) establishes an inverse relationship between $\lambda_t$ and $I_t$, regardless of whether equation (5.11) holds. Therefore, if a reduction of $R^m_{t+1}$ were associated with a reduction of $I_t$, this would have to mean an increase in $\lambda_t$ and hence a decrease in all three terms on the right-hand side of equation (5.1). Such an effect would be contrary to familiar evidence regarding the effects of interest rate policy, and it would also preclude the possibility of a liquidity trap in which the lower bound on nominal interest rates prevents rates from being cut enough to maintain a desired level of real activity.

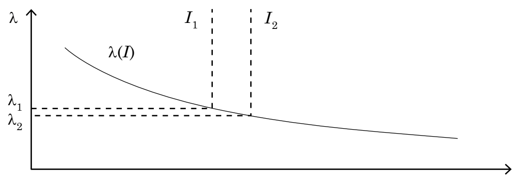
55. The expression $I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1})$ in this equation is just the quantity $I_t$ which must increase as shown earlier.

**Figure 9. The Effects of a Reduction in the Interest Rate Paid on Safe Assets ($R^m_{t+1}$)**

*A. Effect on the Money Premium and Equilibrium Investment in Risky Real Assets*




*B. Effect on Financial Conditions as Measured by $\lambda_t$*




### 5.2.2 Effects of unconventional policies

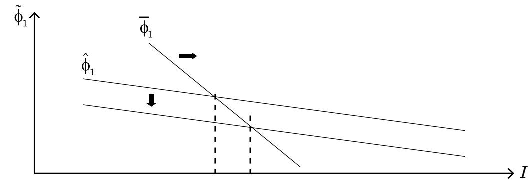
In the event that the lower bound on interest rates prevents $R^m_{t+1}$ from being reduced to the extent that would be necessary to maintain aggregate demand at the desired level, quantitative easing provides an alternative channel through which aggregate demand may be increased. Like conventional interest rate policy, an expansion of the supply of short-term safe assets by the central bank affects aggregate demand by easing financial conditions, as indicated by a reduction in $\lambda_t$ (which can be thought of as the price of a particular very-long-duration indexed bond).

Consider the effects of an increase in $\tilde{m}_{t+1}$, holding $R^m_{t+1}$ fixed. The effect on equilibrium investment demand can again be determined using equation (5.10). The schedule corresponding to the right-hand side of this equation does not shift as a result of an increase in $\tilde{m}_{t+1}$, but the fact that $\hat{\phi}_1(\lambda; s, m)$ is a decreasing function of $\tilde{m}$ means that the schedule corresponding to the left-hand side of the equation shifts down for each possible value of $I_t$, as shown now in panel A of figure 10. Then again, assuming that the relative slopes of the two schedules are given by equation (5.11), it is again possible to conclude that $I_t$ must increase while $\tilde{\phi}_{1\,t+1}$ must decrease.

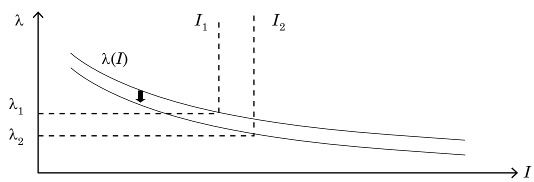
Equation (5.4) can also be used to determine the change in $\lambda_t$ required by a given size increase in $I_t$. As argued above, an increase in $I_t$ reduces the right-hand side of this equation, for any given value of $\tilde{m}_{t+1}$. In addition, equation (5.6), together with the result above that $\phi_1(\lambda; s, m)$ is a decreasing function of $\tilde{m}$, implies that the function $\varphi(\lambda; s, \tilde{m})$ is also a decreasing function of $\tilde{m}$. This means that the curve—the graph of equation (5.4)—plotted in panel B of figure 10 shifts down as a result of an increase in $\tilde{m}_{t+1}$. It then follows that $\lambda_t$ is reduced by an increase in $\tilde{m}_{t+1}$, both because of the decrease in $I_t$ (the shift along the curve) and because of the direct effect of an increase in $\tilde{m}_{t+1}$ (the downward shift of the curve).

**Figure 10. The Effects of an Increase in the Central Bank Supply of Safe Liabilities $\tilde{m}_{t+1}$**

*A. Effect on the Money Premium and Equilibrium Investment in Risky Real Assets*




*B. Effect on Financial Conditions as Measured by $\lambda_t$*



It follows that an increase in $\tilde{m}_{t+1}$ must loosen financial conditions, in the sense that $\lambda_t$ is reduced. This, in turn, means that as in the case of an interest rate cut, $Y_t$ must increase because of equation (5.1). Thus, the effects of quantitative easing are qualitatively similar to those of an interest rate cut: financial conditions are eased, the aggregate demand for normal goods increases (because of an increase in the demand for credit goods and an increase in the demand for investment goods), but at the same time risks to financial stability increase (because of an increase in short-term debt issuance by banks), leading to larger expected distortions in the event that a crisis state occurs in period $t + 1$.

Nonetheless, the two policies do not have quantitatively equivalent effects. A comparison of an interest rate cut (reduction in $R^m_{t+1}$) and an increase in the net supply of safe assets by the central bank (increase in $\tilde{m}_{t+1}$) that increase the equilibrium demand for investment goods $I_t$ by the same amount shows that the increase in $\tilde{m}_{t+1}$ reduces $\lambda_t$ by a greater amount.[56] This can be seen from the fact that equation (5.4) must apply in either case. If, by hypothesis, $I_t$ increases by the same amount in both cases, then the only difference in the implied value for $\lambda_t$ is that $\tilde{m}_{t+1}$ increases in the second case, but remains constant in the first—and this implies a lower value of $\lambda_t$ in the second case. This means that in the case of quantitative easing, a greater share of the total increase in aggregate demand comes from increased demand for credit goods, as opposed to increased demand for investment goods. Thus, a given degree of aggregate demand stimulus can be achieved with less risk to financial stability if it is brought about through an expansion of the central bank's balance sheet, rather than by cutting the interest rate paid on cash.

We can also consider the effects of aggregate demand stimulus through relaxation of macroprudential constraints (that is, an increase in $\xi_{t+1}$). Let us generalize the analysis presented in the earlier part of this section to allow for a macroprudential tax (or reserve requirement), so that $\xi_t$ need not equal 1 (as assumed thus far in this section). Conditions (5.1), (5.3), and (5.4) continue to be required for an equilibrium, and the definition of the function $\hat{\phi}_1(\lambda; s, \tilde{m})$ is unchanged; but in equation (5.4), the expression $\varphi(\bar{\lambda}; F(I_t), \tilde{m}_{t+1})$ must be replaced by $\varphi(\bar{\lambda}; F(I_t), \tilde{m}_{t+1}, \xi_{t+1})$, where we define

56. Compare figures 9 and 10. In figure 10, the amount of quantitative easing is chosen so as to achieve the same increase in investment (from $I_1$ to $I_2$) as the interest rate cut in figure 9. The reduction in $\lambda_t$ is instead larger (from $\lambda_1$ to $\lambda_3 < \lambda_2$).

$$\varphi(\lambda; s, \tilde{m}, \xi) \equiv \hat{\varphi}(\xi\hat{\phi}_1(\lambda; s, \tilde{m}); s),$$

generalizing (5.6).

Condition (5.8) then takes the more general form

$$\tilde{\phi}_{1,t+1} = \frac{\Pi}{\bar{\lambda} R^m_{t+1}} \hat{\varphi}(\xi_{t+1}\tilde{\phi}_{1,t+1}; F(I_t))F'(I_t).$$

As above, this implicitly defines a function

$$\tilde{\phi}_{1,t+1} = \bar{\phi}_1(I_t, R^m_{t+1}, \xi_{t+1}), \tag{5.12}$$

where now $\bar{\phi}_1(I, R^m, \xi)$ is decreasing in $I$ and $R^m$ and increasing in $\xi$. The equilibrium level of investment is again determined by equation (5.10), but now the schedule corresponding to the left-hand side is shifted only by $\tilde{m}_{t+1}$, while the schedule corresponding to the right-hand side is shifted by changes in either $R^m_{t+1}$ or $\xi_{t+1}$. To a linear approximation (which is to say, in the case of small enough policy changes), an increase in $\xi_{t+1}$ (a relaxation of macroprudential policy, as by reducing the required reserve ratio) has the same effects on $I_t$ and $\tilde{\phi}_{1\,t+1}$ as a certain size of cut in $R^m_{t+1}$.

Equation (5.12) can also be used to rewrite equation (5.3) in the form

$$\lambda_t = \beta\bar{\lambda}\bar{\phi}_1(I_t, R^m_{t+1}, \xi_{t+1})R^m_{t+1} / \Pi. \tag{5.13}$$

It follows that since a relaxation of macroprudential policy reduces the value of $\tilde{\phi}_{1\,t+1}$, it must reduce the value of $\lambda_t$. Hence, it increases demand for credit goods and so must increase $Y_t$, like the other two policies just considered. It also follows from equation (5.13), however, that in the case of two policy changes (a cut in $R^m_{t+1}$ or an increase in $\xi_{t+1}$) that reduce $\tilde{\phi}_{1\,t+1}$ to the same extent and that therefore reduce $\bar{\phi}_1(I_t, R^m_{t+1}, \xi_{t+1})$ to the same extent, the interest rate cut must reduce $\lambda_t$ by more, thereby stimulating demand for credit goods to a greater extent. Thus, an even greater share of the increase in aggregate demand achieved by relaxing macroprudential policy comes from an increase in investment demand, as opposed to an increase in the demand for credit goods, than in the case of an increase in aggregate demand achieved by cutting the interest rate on cash.

This leads to an ordering of the three types of expansionary policy as follows: for a given degree of increase in aggregate demand, achieving it by increasing $\tilde{m}_{t+1}$ increases $I_t$ the least, achieving it by reducing $R^m_{t+1}$ increases $I_t$ to an intermediate extent, and achieving it by increasing $\xi_{t+1}$ increases $I_t$ (and hence short-term debt issuance by banks and risks to financial stability) the most. One consequence of this is that increasing aggregate demand through monetary policy *need not* involve any increased risks to financial stability at all. For example, one might combine an increase in $\tilde{m}_{t+1}$ with a tightening of macroprudential policy (a reduction of $\xi_{t+1}$) that exactly offsets the effects of the quantitative easing on desired investment demand, so that there is no net change in $I_t$. Since the former policy change will reduce $\lambda_t$ more than the latter policy change increases it, the net effect will be a loosening of financial conditions, with a corresponding increase in the demand for credit goods. Since there is (by hypothesis) no change in investment demand, aggregate demand $Y_t$ will increase; but there will be no associated increase in $s_{t+1}$, and hence no increase in the severity of the distortions associated with a crisis state in period $t + 1$.

My conclusion is that while quantitative easing *may* increase risks to financial stability in the case that nominal rigidities allow short-run effects of monetary policy on aggregate demand, it *need not* have any such effect. If the increase in the central bank's balance sheet is combined with an increase in the interest rate paid on cash or a tightening of macroprudential policy to a sufficient extent, then it can increase aggregate demand without any adverse consequences for financial stability. It is particularly easy to achieve this outcome by combining the quantitative easing with macroprudential policy, if a suitable macroprudential instrument exists; for in the model, reduction of $\xi_{t+1}$ provides an even greater disincentive to issuance of short-term debt by banks than does raising $R^m_{t+1}$, for a given degree of reduction in aggregate demand.

These results imply that quantitative easing may be a useful addition to a central bank's monetary policy toolkit, even when interest rate policy is not yet constrained by the effective lower bound on short-term nominal interest rates. In the case of a contractionary shock $\chi$, the effects on aggregate demand can be offset purely through a reduction in $R^m_{t+1}$, if the lower bound does not prevent the size of rate cut that is needed; but such a response increases risks to financial stability more than necessary. One could alternatively counter the effects of the $\chi$ shock by increasing $\tilde{m}_{t+1}$, while leaving $R^m_{t+1}$ unchanged; this would have the advantage of posing less of a threat to financial stability. Even better, one could combine a somewhat larger increase in $\tilde{m}_{t+1}$ with a tightening of macroprudential policy, allowing the effects of the $\chi$ shock on aggregate demand to be offset, with even less of an increased risk to financial stability, possibly none at all.

## 6. Conclusions

We can now assess the validity of the concerns about the consequences of quantitative easing for financial stability sketched in the introduction, in the light of the model just presented. The model is one in which monetary policy does indeed influence risks to financial stability; in particular, policies that loosen financial conditions, either by lowering the central bank's operating target for its policy rate (conventional monetary policy) or by relaxing reserve requirements (or other macroprudential constraints), should each increase the attractiveness of private issuance of money-like liabilities, resulting in increased leverage and as a consequence an increased risk of serious resource misallocation in the event of a funding crisis. This means that there can sometimes be a tension between the monetary policy that would be preferable strictly from the standpoint of aggregate demand management and inflation stabilization, on one hand, and the policy that would minimize risks to financial stability, on the other.

The question is whether it is correct to think of quantitative easing as a policy analogous to these, which poses similar risks to financial stability. The model implies that such an analogy is imperfect. A quantitative easing policy (which increases the public supply of safe assets through the issue of additional safe central bank liabilities, used to purchase assets that do not earn a similar safety premium) similarly increases aggregate demand by lowering the equilibrium rate of return on nonsafe assets. Unlike conventional monetary policy, however, it does this by lowering the equilibrium safety premium (by making safe assets less scarce), rather than by lowering the equilibrium return on safe assets; and this does not have the same consequences for financial stability. Lowering the equilibrium return on risky investments (such as the durable goods modeled here, which one may think of as housing) by lowering the return on safe assets works only insofar as the *increased* spread between the two returns that would result if the return on risky investments did not also fall increases the incentive to finance additional risky investment by issuing safe liabilities, thus increasing the leverage of the banks and the degree to which they engage in liquidity transformation. This results in a reduced equilibrium return on risky investment, but not by enough to fully eliminate the increased spread that induces banks to issue additional safe asset-backed liabilities. This mechanism necessarily increases the risk to financial stability at the same time as it increases aggregate demand. Quantitative easing instead *decreases* the spread between these two returns, at least in the absence of any change in the private supply of safe liabilities. This reduction in the spread reduces the incentive for private issuance of such liabilities. Reduced issuance of safe asset-backed liabilities by banks offsets some of the reduction in the spread, but it does not completely eliminate it, as otherwise banks would not have a reason to reduce their issuance. Hence, in this case, the reduction in the equilibrium return on risky investments is associated with a *reduction* of the incentive for liquidity transformation by banks, rather than an increase.

Similarly, quantitative easing increases the total supply of safe assets and so reduces the safety premium. In contrast to a reduction in reserve requirements (or relaxation of macroprudential policy), it achieves this by increasing the public supply of safe assets (and actually reducing the incentive for private issuance), rather than by increasing the incentive that banks have to finance risky investment by issuing safe asset-backed liabilities. Again, the consequences for the degree of liquidity transformation by the banking sector and the risk to financial stability are entirely different.

Likewise, quantitative easing eases financial conditions by reducing the spread between the required return on risky investments and the return on safe assets. This does not mean, however, that risk premiums are artificially reduced in a way that distorts incentives for prudent behavior, leading to excessive risk-taking. In the model presented here, quantitative easing reduces the safety premium, but it does so because the public supply of safe assets for private investors to hold is increased, not because anyone is misled into underestimating the degree of risk involved in undertaking risky investments. Moreover, the reduced spread reduces the incentive for private issuance of safe liabilities and instead favors financing investment through the issuance of nonsafe liabilities, which is desirable on financial stability grounds. Rather than threatening financial stability by encouraging more risk-taking, it favors stability by encouraging forms of financing that reduce the magnitude of the distortions associated with a funding crisis.

The model was used to compare the effects of three alternative policies that can increase aggregate demand by easing financial conditions: reducing the central bank's operating target for the nominal interest rate on safe assets (that is, conventional monetary policy, on the assumption that the zero lower bound does not yet preclude such easing); relaxing reserve requirements or other macroprudential constraints; and quantitative easing. Among these alternative policies, quantitative easing increases risks to financial stability the least, for any given degree of increase in aggregate demand. Not only does quantitative easing make it possible for a central bank to increase aggregate demand even when conventional monetary policy is constrained by the zero lower bound on nominal interest rates, but, at least in principle, the expansion in aggregate demand can be achieved without the collateral effect of greater risk to financial stability, provided that the increased supply of safe liabilities by the central bank is combined with a sufficient tightening of macroprudential measures. The latter measures alone would reduce aggregate demand, but when combined with quantitative easing, the net effect is an increase in aggregate demand, even when the degree of macroprudential tightening is enough to fully offset any increase in risks to financial stability as a result of the balance sheet policy.

This indicates that a concern for the effects of monetary policy on financial stability need not preclude using quantitative easing to stimulate aggregate demand in circumstances where (as in the United States in the aftermath of the recent crisis) conventional monetary policy is constrained by the zero lower bound. The fact that demand stimulus through quantitative easing poses smaller risks to financial stability than demand stimulus through lowering short-term nominal interest rates suggests that balance sheet policy may be a useful tool of monetary stabilization policy even when a central bank is far from the zero lower bound. In the model presented here, the aggregate demand stimulus achieved by lowering nominal interest rates increases the risk to financial stability more than would a quantitative easing policy that is equally effective in increasing aggregate demand. This implies that even if macroprudential policy is unavailable or ineffective, it should be possible to increase aggregate demand without increasing the risk to financial stability by combining expansionary balance sheet policy with an appropriate *increase* in the policy rate. In such a case, conventional monetary policy would essentially be used for macroprudential purposes (to control the risk to financial stability), while balance sheet policy is used for demand stabilization.

Further study of the effects of quantitative easing policies would therefore seem to be warranted, not simply for the sake of having a more effective policy toolkit for use the next time that conventional policy is again constrained by the zero lower bound, but also, arguably, to improve the conduct of stabilization policy under more normal circumstances as well. The availability of this additional dimension of monetary policy is particularly likely to be of use under circumstances where additional monetary stimulus through interest rate reduction is unattractive owing to concerns about financial stability. Such a situation could easily arise even when interest rates are well above their effective lower bound.

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# Short-term Interest Rates and Bank Lending Terms: Evidence from a Survey of U.S. Loans

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The long period of low interest rates that followed the global financial crisis has rekindled interest in how short-term interest rates affect bank behavior. In particular, it has led to a debate on how low policy rates influence bank risk-taking. This risk-taking channel of monetary policy corresponds to the view that interest rate policy affects the quality and not just the quantity of bank credit. From a financial stability perspective, one concern is that a protracted period of low interest rates and monetary stimulus could contribute to an increase in financial risk-taking (Rajan, 2010; Farhi and Tirole, 2012; Acharya, Pagano, and Volpin, 2013; Chodorow-Reich, 2014). Concerns about the risk-taking effects of monetary policy have motivated a lively debate about the extent to which financial stability considerations should be an integral part of the monetary policy framework (Woodford, 2012; Stein, 2014).

Despite the obvious policy interest, the empirical evidence on this topic is scant for the United States. The existing empirical papers on the link between monetary policy and risk-taking are mostly focused

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on Europe (for example, Jimenez and others, 2014; Ioannidou, Ongena, and Peydro, 2015; Altunbas, Gambacorta, and Marques-Ibañez, 2010).

In this paper, we study the link between short-term interest rates and bank risk-taking using confidential data on individual U.S. bank loans from the Federal Reserve's Survey of Terms of Business Lending (STBL). Since 1997, the survey has asked respondents to report their assessed risk rating for each individual loan, which provides a unique ex ante measure of loan riskiness.

We document that banks tend to ease their lending terms during periods of low interest rates. In particular, for a given ex ante internal risk rating of the loan, banks tend to originate new business loans with lower spreads and that are less likely to be collateralized. Our empirical analysis indicates that for the typical new loan, a one-standard-deviation decrease in short-term interest rates is associated with a decrease in loan spreads of roughly 0.1 percentage points. This is a nontrivial effect, although it is somewhat modest when compared with the standard deviation of loan spreads in our sample (1.4 percentage points).

We also show that the negative relationship between short-term interest rates and bank lending terms, as measured by spreads and collateralization, is more pronounced for riskier loans and for banks that are more sensitive to short-term interest rates in their funding needs. Finally, using residuals from Taylor-rule regressions, we show that the less restrictive standards prevailing during periods of low interest rates are explained by the rate component that is orthogonal to cyclical effects captured by the output gap and inflation. We also consider a modified Taylor rule that incorporates financial stability considerations and find similar results. These findings alleviate some concerns that short-term interest rates set by monetary policy are endogenous to bank lending behavior.

Our statistical results are not well suited to answer whether or not the additional risk banks take by easing their standards when facing more accommodative monetary policy is excessive, because we do not model the optimal degree of financial risk-taking. In other words, our results can inform the conduct of monetary policy through an improved understanding of the effects of monetary policy on the financial system, but by themselves they cannot help answer the question of whether a given policy (past or present) is optimal.

The rest of the paper proceeds as follows. Section 1 highlights our contribution to the existing empirical literature. Section 2 presents the methodology used to assess the link between bank lending terms and short-term interest rates and describes the survey of terms of business lending and other data used in our empirical analysis. Section 3 presents and interprets the empirical results, and section 4 concludes.

## 1. Relation with Existing Literature

Different theoretical approaches deliver different predictions on the relationship between the monetary policy rate (or more precisely the interest rate on safe assets) and bank risk-taking.[1] On one hand, most portfolio allocation models will predict that an exogenous decrease in the yield on safe assets will lead to greater risk-taking (for example, Fishburn and Porter, 1976). On the other hand, corporate finance models focusing on the effects of limited liability predict that a decrease in the interest rate that banks have to pay on deposits will reduce risk-taking: this is the classical risk-shifting effect. Due to these offsetting forces, the relationship between short-term interest rates and bank risk-taking is an empirical question (see Dell'Ariccia, Laeven, and Marquez, 2014).

Given the debate on the financial stability concerns of maintaining low interest rates, it is not surprising that the empirical literature on the relationship between interest rates and bank risk-taking has grown notably in recent years. This paper adds to our understanding of this relationship by (1) studying changes in lending along different loan terms, (2) measuring loan risk at origination from an ex ante perspective, and (3) focusing on the United States using a detailed loan-level database.

Exploiting loan-level information from the Federal Reserve's STBL, we study the relationship between monetary policy rates and different terms of business lending, maintaining constant the ex ante risk profile of the loan. In particular, we study the effect of monetary policy on loan spreads and the collateralization of new loans. In this context, we define bank lending terms as easier if, controlling for the riskiness of the loan, banks charge lower spreads or are less likely to require collateral.

One of the contributions of our work consists of controlling for the perceived riskiness of loans at origination. By contrast, most measures of bank risk in the literature are measured ex post, which makes it hard to disentangle whether any realized risk was truly an ex ante decision by the bank or an ex post effect of deterioration in

1. For a discussion, see Altunbas, Gambacorta, and Marques-Ibañez (2010); Chodorow-Reich (2014); Dell'Ariccia and Marquez (2013).

economic activity over the business cycle. Other papers measure bank risk using information on changes in lending standards observed in lending surveys (see Lown and Morgan, 2006, for the United States; and Maddaloni and Peydro, 2011, for the euro area) or rating agency estimates (Altunbas, Gambacorta, and Marques-Ibañez, 2010), but they do not control for loan riskiness. Finally, papers based on credit registries generally use borrower-level measures of risk based on preexisting default history or ex post loan default rates (Jimenez and others, 2014; Ioannidou, Ongena, and Peydro, 2015), rather than what the bank perceived at origination.

Another novelty of the present paper is that it employs U.S. loan-level data. Most recent studies focus on Europe. The few papers focusing on the United States use syndicated loans or aggregate data (Paligorova and Santos, 2012; Delis, Hasan, and Mylonidis, 2011; Buch, Eickmeier, and Prieto, 2011). Syndicated lending mostly reflects borrowing by relatively large corporations and thus may not be representative of broader credit markets. A significant advantage of using U.S. data is that it offers a relatively long time series (contrary to, say, euro area surveys), which helps researchers encompass more monetary policy easing and tightening cycles. In a closely related paper, Dell'Ariccia, Laeven, and Suarez (2016) document how a bank's balance sheet structure (leverage and liquidity) affects the relationship between monetary policy and bank risk-taking.

Our paper is most closely related to Jimenez and others (2014) and Ioannidou, Ongena, and Peydro (2015), who use detailed information on borrower quality from credit registry databases for Spain and Bolivia. Consistent with our results, they find a positive association between low interest rates at loan origination and the probability of extending loans to borrowers with bad credit history or no history at all.

## 2. Methodology and Data

To investigate the relationship between short-term interest rates and the terms on newly issued loans, we employ standard panel regression analysis. Our basic regression model is as follows:

$$y_{kit} = \alpha_i + \lambda_j + \beta r_t + \theta \mathbf{X}_{kit} + \mu \mathbf{W}_{it} + \rho \mathbf{Z}_{jt} + \gamma \mathbf{M}_t + \varepsilon_{kit} \quad (1)$$

where $y_{kit}$ is a characteristic of loan $k$ extended by bank $i$ in quarter $t$, are $\alpha_i$ bank-specific fixed effects, $\lambda_j$ are state-specific fixed effects, $r_t$ is the federal funds rate at the beginning of quarter $t$, $\mathbf{X}_{kit}$ are loan characteristics (loan risk rating and loan amount), $\mathbf{W}_{it}$ is a set of bank-specific control variables measured at the beginning of quarter $t$, $\mathbf{Z}_{jt}$ is a set of time-varying regional (either U.S. state or Census region) control variables, $\mathbf{M}_t$ is a set of macroeconomic controls (GDP growth and an indicator of NBER recessions), and $\varepsilon_{kit}$ is the error term. To control for the potential dependence of observations within banks and within quarters, standard errors are two-way clustered by bank and quarter. Our coefficient of interest in equation (1) is β. Under the hypothesis that lending terms are easier during periods of low interest rates, we expect β to be positive for a regression explaining loan spreads and the probability of collateralization.

To study how the relationship of short-term interest rates on bank lending standards changes with loan or bank characteristics, we expand equation (1) by including interactions between short-term interest rates and those characteristics. In these specifications, we drop the macroeconomic variables in the vector $\mathbf{M}_t$ and the level of short-term interest rates and introduce time-fixed effects instead. More formally, when considering the interaction of the bank-specific variable $v_{it}$ (part of the vector $\mathbf{W}_{it}$) with the short-term interest rate, we estimate the following equation:

$$y_{kit} = \alpha_i + \lambda_j + \tau_t + \delta r_t * v_{it} + \theta \mathbf{X}_{kit} + \mu \mathbf{W}_{it} + \rho \mathbf{Z}_{jt} + \varepsilon_{kit} . \qquad (2)$$

where $\tau_t$ represents a time fixed effect, and all other variables are defined as in equation (1). The coefficients of interest in these specifications are δ.

## 2.1 The Federal Reserve's Survey of Terms of Business Lending

We use loan-level data from the confidential Survey of Terms of Business Lending (STBL) from 1997 to 2011. The STBL is a quarterly survey on lending to businesses originated by a stratified sample of about 400 banks conducted by the Federal Reserve since 1977. The banks surveyed cover a large share of the U.S. banking sector's assets. The survey asks participating banks about the terms of all commercial and industrial loans originated during the first full business week of the middle month in every quarter (February, May, August, and November). Banks report various loan characteristics, including the bank's internal assessment of the risk of the loan using a scale from one (low risk) to five (highest risk). The risk-rating measure roughly maps to the banks' internal loan risk ratings and has been reported in the survey since 1997.

The STBL is the Federal Reserve's main source of data on marginal returns on business loans for a representative set of banking institutions nationwide and a wide range of loan sizes. As a result, the STBL provides valuable insights into shifts in the composition of banks' business loan portfolios and the implications of those shifts for bank profitability (Carpenter, Whitesell, and Zakrajšek, 2001; Black and Rosen, 2007; Black and Hazelwood, 2013).

## 2.2 Variable Definitions

Our analysis combines loan-level data from the STBL with bank-specific data from the Consolidated Reports of Condition and Income for commercial banks, as well as regional and macroeconomic variables.

### 2.2.1 Loan-level variables

For each loan in the sample, the STBL reports the name of the bank extending the loan, the size (in dollars), whether or not the loan is secured by collateral, the effective interest rate charged by the bank for the loan, and the prime rate used by the bank. In addition, banks report their own ex ante assessment of the riskiness of the loan using a risk-rating index designed by the survey, which increases with risk: 1 = Minimal risk; 2 = Low risk; 3 = Moderate risk; 4 = Acceptable risk; and 5 = Special mention or classified asset.

### 2.2.2 Bank variables

We compile information about the balance sheet of the banks responding to the STBL from the quarterly Consolidated Reports of Condition and Income (FFIEC 031 and 041) (call reports) for commercial banks. In particular, in our empirical analysis, *Tier 1 capital* is the ratio of Tier 1 regulatory capital to total risk-weighted assets; *Bank size* is the log of bank total assets; *Net income / assets* is the ratio of net income to total assets; *Liquid assets / assets* is the ratio of liquid assets to total assets; *Deposits / assets* is the ratio of total deposits to total assets; *Short-term deposits / deposits* is the ratio of short-term (that is, up to one year) deposits to total deposits; *Nonretail deposits / deposits* is the ratio of nonretail deposits to total deposits; *Loans / assets* is the ratio of total loans to total assets; and *C&I loans / loans* is the ratio of commercial and industrial loans to total loans.

We locate banks using their headquarters as reported in the National Information Center (NIC) database. We use information on bank location to match bank-specific data with regional (state-specific) data to control for loan demand conditions.

### 2.2.3 Regional variables

Our regressions include state- or region-level factors (where state-level factors are unavailable) to allow for the possibility that local conditions such as employment, inflation, and house prices affect bank risk-taking. At the state level, we consider the growth rate in personal income, taken from the Bureau of Economic Analysis (BEA); the unemployment rate, taken from the Bureau of Labor Statistics (BLS); and the annualized quarter-over-quarter rate of change in the house price index published by the Office of Federal Housing Enterprise Oversight/Federal Housing Finance Agency (OFHEO/FHFA). We consider the annualized quarter-over-quarter rate of change in the consumer price index (CPI) by U.S. Census Bureau region, as reported by the BLS.

### 2.2.4 Nationwide variables

The short-term interest rate is measured using the three-month average of the nominal target federal funds rate. By adjusting reserves, the Federal Reserve controls the market-determined effective federal funds rate to implement monetary policy. At the macroeconomic level, we also control for the U.S. real GDP growth (quarter over quarter, annual rate), reported by the BEA, and for an indicator variable for recessions dated by the National Bureau of Economic Research (NBER).

## 2.3 Descriptive Statistics of Main Variables

Table 1 reports summary statistics on our main regression variables. We restrict our sample to loans that are not made under a commitment established prior to the quarter of the survey. In contrast with the more discretionary loans that constitute our sample, the terms of loans originated under a commitment (for example, a line of credit) do not necessarily reflect the bank's own assessment of the riskiness of the loan at the time the loan was extended.

## Table 1. Summary Statistics

| | *Obs.* | *Average* | *25th pctl.* | *75th pctl.* | *Standard deviation* |
|---|---|---|---|---|---|
| *Loan-level variables* | | | | | |
| Loan spread (in percentage points) | 1,121,510 | 0.754 | 0.074 | 1.425 | 1.444 |
| Dummy for loans secured by collateral | 1,121,508 | 0.807 | 1 | 1 | 0.395 |
| Risk rating | 1,112,510 | 3.306 | 3 | 4 | 0.837 |
| Loan size (dollars) | 1,121,510 | 520,529 | 14,800 | 142,285 | 4,703,035 |
| *Bank-level variables* | | | | | |
| Bank total assets ($million) | 11,854 | 21,072 | 318 | 5,884 | 104,353 |
| Tier 1 capital ratio | 11,854 | 0.122 | 0.095 | 0.135 | 0.049 |
| Net income / assets | 11,854 | 0.006 | 0.003 | 0.010 | 0.009 |
| Liquid assets / assets | 11,854 | 0.027 | 0.014 | 0.035 | 0.019 |
| Deposits / assets | 11,854 | 0.779 | 0.724 | 0.858 | 0.103 |
| Short-term deposits / deposits | 11,854 | 0.018 | 0 | 0 | 0.071 |
| Nonretail deposits / deposits | 11,854 | 0.362 | 0.190 | 0.461 | 0.267 |
| Loans / assets | 11,854 | 0.641 | 0.566 | 0.737 | 0.141 |
| C&I loans / loans | 11,854 | 0.219 | 0.131 | 0.277 | 0.127 |
| *Regional variables* | | | | | |
| State personal income growth (%) | 2,604 | 2.114 | -0.549 | 4.794 | 4.824 |
| Change in regional CPI (%) | 236 | 2.386 | 1.112 | 3.985 | 2.908 |
| State unemployment rate (%) | 2,604 | 5.434 | 4.000 | 6.233 | 2.079 |
| Change in state housing prices (%) | 2,604 | 3.104 | -0.523 | 7.739 | 8.356 |
| *Nationwide variables* | | | | | |
| Target federal funds rate (%) | 59 | 3.012 | 1.000 | 5.250 | 2.203 |
| Real GDP growth (%) | 59 | 2.257 | 1.318 | 3.600 | 2.837 |
| NBER recession | 59 | 0.186 | 0 | 0 | 0.393 |

a. This table reports descriptive statistics of the variables used in our baseline regressions. The sample includes loans reported to the Federal Reserve's STBL from the second quarter of 1997 to the fourth quarter of 2011. Loan spread is the difference between the interest rate on the loan minus the rate the prime rate reported by the bank. Risk rating is the internal risk rating assigned by the bank to a given loan, as reported in STBL, with 1=Minimal Risk, 2=Low Risk, 3=Moderate Risk, 4=Acceptable Risk, and 5=Special Mention or Classified Asset. Loan spread, loan size, and the dummy for loans secured by collateral are all taken from the STBL. Bank location is based on its headquarters, as reported in the NIC database. Bank total assets, capital, profitability, liquidity, deposit, and loan ratios are based on Call Report data. Real GDP growth and state personal income growth are from the BEA, change in region CPI and state unemployment rate are from the BLS, and the change in state housing prices is based on indices published by OFHEO/FHFA. Growth rates are reported as annual rates. Recession dates are from the NBER. We exclude from the sample loans extended under commitment established prior to the current quarter from the sample.

The average loan spread over the bank's prime rate is about 0.75 percentage point, although there is considerable dispersion, with a standard deviation of 1.44 percentage points. The majority of loans in the sample are collateralized. The mean risk rating in the sample is 3.31, with a standard deviation of 0.84, indicating that the average loan over the sample period as reported by banks is between moderate risk (rating 3) and acceptable risk (rating 4). The average loan amount is US$520,529, but the variation is quite large, reflecting the fact that the survey includes business loans to firms of all sizes.

Banks in our sample vary significantly in size, averaging US$21 billion in total assets but with a standard deviation of over US$104 billion, indicating that the sample includes both small and large banks. Loans constitute about two-thirds of the banks' balance sheets, on average, which suggests that our focus on risk-taking through lending is an important part of the risk profile of banks in our sample. On average, about one-fifth of the lending activity of banks in our sample is commercial and industrial (C&I) loans, and the typical bank in our sample is mostly funded by deposits.

The federal funds rate also displays substantial variation over the sample period, averaging about 3 percent in nominal terms with a standard deviation of 2.2 percent. Finally, about one-fifth of quarters in the sample are recession periods.

## 3. Results

In this section, we present our main results concerning the effect of monetary policy conditions on lending terms. We also present some robustness checks that suggest that our baseline results are not likely driven by the response of monetary policy to the economic cycle or financial stability concerns.

We exclude from the sample those loans that banks made under a commitment (for example, drawn from a line of credit) established prior to the quarter of the survey. Instead, we focus on loans originated entirely at the discretion of the lender, which are more likely to capture risk-taking attitudes for the bank.

We study the effect of short-term interest rates on the terms of bank loans to businesses, controlling for the risk of the loan. In particular, we control for the bank's own assessment of the riskiness of the loan as reported to the STBL in the loan risk rating. We also control for other factors that could affect the risk profile of new loans at the bank level (including the originating bank's capitalization, profitability, and liquidity) and the general environment in which the bank operates (including GDP growth, inflation, and unemployment).

**Table 2. Terms of Business Lending and the Federal Funds Rate[a]**

| | *Dependent variable* | |
|---|---|---|
| *Explanatory variable* | *Loan spread* | *Dummy for secured loan* |
| | *(1)* | *(2)* |
| Target federal funds rate | 0.037*** | 0.008*** |
| | (0.012) | (0.002) |
| Loan risk rating | 0.346*** | 0.056*** |
| | (0.010) | (0.004) |
| Loan size | −0.275*** | −0.006*** |
| | (0.007) | (0.001) |
| Bank size | −0.088*** | −0.030*** |
| | (0.032) | (0.006) |
| Bank tier 1 capital ratio | 3.754*** | −0.106 |
| | (0.571) | (0.152) |
| Bank net income / assets | −6.641*** | 0.277 |
| | (1.495) | (0.248) |
| Bank liquid assets / assets | −2.158** | −0.126 |
| | (0.975) | (0.286) |
| Bank deposits / assets | 1.104*** | 0.068 |
| | (0.224) | (0.052) |
| Short-term deposits / deposits | −0.548*** | −0.102* |
| | (0.193) | (0.053) |
| Nonretail deposits / deposits | −0.08 | −0.014 |
| | (0.073) | (0.015) |
| Bank loans / assets | 0.799*** | 0.107** |
| | (0.115) | (0.046) |
| Bank C&I loans / loans | 0.476** | 0.197*** |
| | (0.184) | (0.039) |
| State personal income growth | −0.000*** | 0.000*** |
| | (0.000) | (0.000) |
| Change in regional CPI | −0.002 | 0.000 |
| | (0.006) | (0.001) |
| State unemployment rate | 0.099*** | 0.019*** |
| | (0.011) | (0.002) |
| Change in state housing prices | −0.002 | 0.000 |
| | (0.001) | (0.000) |
| GDP growth | 0.008 | 0.001 |
| | (0.006) | (0.001) |
| NBER recession dummy | 0.012 | −0.003 |
| | (0.039) | (0.005) |

**Table 2. (continued)**

| *Explanatory variable* | *Dependent variable* Loan spread (1) | *Dependent variable* Dummy for secured loan (2) |
|---|---|---|
| Constant | 2.043*** | 0.809*** |
| | (0.690) | (0.113) |
| *Summary statistic* | | |
| Bank fixed effects | Yes | Yes |
| State fixed effects | Yes | Yes |
| Time fixed effects | No | No |
| No. observations | 1,121,510 | 1,121,508 |
| No. banks | 590 | 590 |
| $R^2$ | 0.331 | 0.183 |

* Statistically significant at the 10 percent level.
** Statistically significant at the 5 percent level.
*** Statistically significant at the 1 percent level.
a. This table reports panel regression estimates of terms of individual new business loans originated from the second quarter of 1997 to the fourth quarter of 2011 by banks reporting to the Federal Reserve's STBL, which correspond to equation (1) in the text. The dependent variables in columns (1) and (2) are, respectively, loan spread and an indicator variable for collateralization as reported to the STBL. Bank size (as measured by the log of total assets), Tier 1 capital ratio, net income, liquid assets, deposits, short-term deposits, nonretail deposits, loans, and C&I loans are measured at the bank level and are all taken from call reports. Risk rating is the internal risk rating assigned by the bank to a given loan, as reported in the Federal Reserve's STBL. Real GDP growth and state personal income growth are from the BEA; change in regional CPI and state unemployment rate are from the BLS; and the change in housing prices is based on indices published by OFHEO/FHFA. The sample excludes loans extended under commitment established prior to the current quarter from the sample. All regressions include state and bank fixed effects. Standard errors two-way clustered by quarter and bank are reported in parentheses.

Our results on the relationship between short-term interest rates and the terms of business lending are reported in table 2. The dependent variable in column (1) is the loan spread. The statistically significant positive coefficient on the federal funds rate suggests that, controlling for the riskiness of the loan as assessed by the bank itself at origination, banks tend to charge relatively narrower spreads when short-term interest rates are lower, suggesting some easing of loan terms in low-interest rate environments.

Beyond the pricing of loans, banks appear to adjust risk-taking through some other terms of their lending. In column (2) of table 2, we report the results of estimating equation (1) with an indicator for loans collateralized by real estate as the dependent variable. The positive coefficient on the federal funds rate in the regression suggests that, conditional on their assessment of loan riskiness, banks are less likely to originate business loans secured by collateral in low-interest-rate environments.

**Table 3. Terms of Business Lending and the Federal Funds Rate by Loan Risk Rating**[a]

| *Explanatory variable* | *Dependent variable* | |
|---|---|---|
| | *Loan spread* | *Dummy for secured loan* |
| | *(1)* | *(2)* |
| Target federal funds rate × Loan risk rating | 0.022*** | 0.008*** |
| | (0.003) | (0.001) |
| Loan risk rating | 0.319*** | 0.028*** |
| | (0.014) | (0.003) |
| Loan size | −0.265*** | −0.005*** |
| | (0.007) | (0.001) |
| Bank size | −0.142*** | 0.006 |
| | (0.030) | (0.009) |
| Bank tier 1 capital ratio | 1.888** | −0.008 |
| | (0.712) | (0.156) |
| Bank net income / assets | −8.708*** | 0.465 |
| | (1.586) | (0.367) |
| Bank liquid assets / assets | 0.635 | −0.683** |
| | (0.932) | (0.266) |
| Bank deposits / assets | 1.140*** | 0.117* |
| | (0.263) | (0.058) |
| Short-term deposits / deposits | −1.063*** | −0.088 |
| | (0.188) | (0.055) |
| Nonretail deposits / deposits | 0.166* | −0.005 |
| | (0.091) | (0.017) |
| Bank loans / assets | 0.609*** | 0.075 |
| | (0.151) | (0.048) |
| Bank C&I loans / loans | −0.059 | 0.191*** |
| | (0.180) | (0.042) |
| State personal income growth | −0.000** | 0.000*** |
| | (0.000) | (0.000) |
| Change in regional CPI | 0.006 | −0.002 |
| | (0.015) | (0.003) |
| State unemployment rate | −0.028** | 0.028*** |
| | (0.012) | (0.003) |
| Change in state housing prices | 0.002 | −0.001 |
| | (0.002) | (0.001) |
| Constant | 4.176*** | 0.053 |
| | (0.607) | (0.184) |

**Table 3. (continued)**

| | *Dependent variable* | |
|---|---|---|
| *Explanatory variable* | *Loan spread* *(1)* | *Dummy for secured loan* *(2)* |
| *Summary statistic* | | |
| Bank fixed effects | Yes | Yes |
| State fixed effects | Yes | Yes |
| Time fixed effects | Yes | Yes |
| No. observations | 1,121,510 | 1,121,508 |
| No. banks | 590 | 590 |
| $R^2$ | 0.338 | 0.186 |

* Statistically significant at the 10 percent level.
** Statistically significant at the 5 percent level.
*** Statistically significant at the 1 percent level.
a. This table reports panel regression estimates of terms of individual new business loans originated from the second quarter of 1997 to the fourth quarter of 2011 by banks reporting to the Federal Reserve's STBL, which correspond to equation (2) in the text. The dependent variables in columns (1) and (2) are, respectively, loan spread and an indicator variable for collateralization as reported to the STBL. Explanatory variables are defined as in table 2. The sample excludes loans extended under commitment established prior to the current quarter from the sample. All regressions include time, state, and bank fixed effects. Standard errors two-way clustered by quarter and bank are reported in parentheses.

In table 3, we report the results of expanding the regressions reported in table 2 by interacting the effect of the federal funds rate with the risk rating of the loan. Analogous to table 2, the dependent variable in column (1) is the loan spread. The positive coefficient on the interaction between the federal funds rate and the loan risk rating indicates that in periods with low interest rates, banks lower their spreads relatively more for riskier loans. The results for the regression using an indicator for loans secured by real estate in column (2) suggest that the additional easing of non-pricing loan terms during periods of low interest rates is also more pronounced for riskier loans.

In table 4 we study whether banks that are more interest-rate sensitive change their loan terms more aggressively during periods of lower interest rates. Banks with higher short-term funding needs tend to be more exposed to changes in interest rates. Thus, we proxy reliance on short-term funding using the fraction of short-term deposits (maturing in less than one year). Table 4 reports the results of expanding the regressions in table 2 by including the interaction between short-term interest rates and bank reliance on short-term funding.

## Table 4. Terms of Business Lending and the Federal Funds Rate by Bank Sensitivity to Interest Rates[a]

| | Dependent variable | |
|---|---|---|
| *Explanatory variable* | *Loan spread* | *Dummy for secured loan* |
| | *(1)* | *(2)* |
| Target federal funds rate × Short term deposits / deposits | 0.201***<br>(0.063) | 0.034*<br>(0.020) |
| Loan risk rating | 0.353***<br>(0.009) | 0.056***<br>(0.004) |
| Loan size | −0.279***<br>(0.007) | −0.006***<br>(0.001) |
| Bank size | −0.074**<br>(0.028) | 0.005<br>(0.009) |
| Bank tier 1 capital ratio | 2.984***<br>(0.504) | −0.053<br>(0.157) |
| Bank net income / assets | −6.054***<br>(1.539) | 0.652*<br>(0.356) |
| Bank liquid assets / assets | −0.009<br>(0.798) | −0.719***<br>(0.265) |
| Bank deposits / assets | 1.090***<br>(0.228) | 0.099*<br>(0.057) |
| Short-term deposits / deposits | −1.677***<br>(0.416) | −0.263**<br>(0.113) |
| Nonretail deposits / deposits | 0.096<br>(0.079) | −0.014<br>(0.016) |
| Bank loans / assets | 0.581***<br>(0.102) | 0.085*<br>(0.047) |
| Bank C&I loans / loans | −0.114<br>(0.147) | 0.168***<br>(0.045) |
| State personal income growth | −0.000***<br>(0.000) | 0.000***<br>(0.000) |
| Change in regional CPI | 0.012<br>(0.012) | −0.002<br>(0.003) |
| State unemployment rate | −0.01<br>(0.011) | 0.028***<br>(0.003) |
| Change in state housing prices | 0.002<br>(0.001) | −0.001<br>(0.001) |
| Constant | 2.719***<br>(0.673) | 0.002<br>(0.181) |

**Table 4. (continued)**

| *Explanatory variable* | *Dependent variable* | |
|---|---|---|
| | *Loan spread* | *Dummy for secured loan* |
| | *(1)* | *(2)* |
| *Summary statistic* | | |
| Bank fixed effects | Yes | Yes |
| State fixed effects | Yes | Yes |
| Time fixed effects | No | No |
| No. observations | 1,121,510 | 1,121,508 |
| No. banks | 590 | 590 |
| $R^2$ | 0.338 | 0.185 |

* Statistically significant at the 10 percent level.
** Statistically significant at the 5 percent level.
*** Statistically significant at the 1 percent level.
a. This table reports panel regression estimates of terms of individual new business loans originated from the second quarter of 1997 to the fourth quarter of 2011 by banks reporting to the Federal Reserve's STBL, which correspond to equation (2) in the text. The dependent variables in columns (1) and (2) are, respectively, loan spread and an indicator variable for collateralization as reported to the STBL. Explanatory variables are defined as in table 2. The sample excludes loans extended under commitment established prior to the current quarter from the sample. All regressions include time, state, and bank fixed effects. Standard errors two-way clustered by quarter and bank are reported in parentheses.

The results reported in column (1) suggest that banks that ex ante appear more sensitive to interest rates decrease their spreads by more during periods of low interest rates. Similarly, the results in column (2) are consistent with the hypothesis that rate-sensitive banks are also less likely to collateralize their loans when interest rates are lower compared with banks that are less rate sensitive.

Even though our regressions control for a large set of factors correlated with the risk profile of loans, our ability to identify an exogenous effect of monetary policy on bank lending terms is limited, in part because monetary policy typically responds to macroeconomic conditions. To alleviate this type of endogeneity concern, we explicitly replace the federal funds rate as the dependent variable in our regressions with a Taylor rule residual, which represents the monetary policy surprise. We obtain the Taylor rule residuals from rolling regressions of the target federal funds rate on the deviation of CPI inflation from 2 percent and the difference between actual and potential GDP growth.

**Table 5. Terms of Business Lending and the Taylor Rule Residuals[a]**

| | *Dependent variable* | |
|---|---|---|
| *Explanatory variable* | *Loan spread* | *Dummy for secured loan* |
| | *(1)* | *(2)* |
| Taylor rule residual | 0.034*** | 0.005*** |
| | (0.013) | (0.002) |
| Loan risk rating | 0.346*** | 0.056*** |
| | (0.010) | (0.004) |
| Loan size | −0.275*** | −0.006*** |
| | (0.007) | (0.001) |
| Bank size | −0.098*** | −0.033*** |
| | (0.031) | (0.006) |
| Bank tier 1 capital ratio | 3.818*** | −0.095 |
| | (0.552) | (0.153) |
| Bank net income / assets | −7.305*** | 0.152 |
| | (1.513) | (0.232) |
| Bank liquid assets / assets | −2.214** | −0.136 |
| | (0.970) | (0.285) |
| Bank deposits / assets | 1.060*** | 0.069 |
| | (0.227) | (0.052) |
| Short-term deposits / deposits | −0.549*** | −0.101* |
| | (0.194) | (0.053) |
| Nonretail deposits / deposits | −0.086 | −0.013 |
| | (0.073) | (0.015) |
| Bank loans / assets | 0.815*** | 0.114** |
| | (0.115) | (0.046) |
| Bank C&I loans / loans | 0.508*** | 0.207*** |
| | (0.183) | (0.039) |
| State personal income growth | −0.000*** | 0.000*** |
| | (0.000) | (0.000) |
| Change in regional CPI | 0.013* | 0.002** |
| | (0.008) | (0.001) |
| State unemployment rate | 0.093*** | 0.015*** |
| | (0.010) | (0.002) |
| Change in state housing prices | −0.002 | 0.000 |
| | (0.001) | (0.000) |
| GDP growth | 0.012* | 0.002** |
| | (0.007) | (0.001) |
| NBER recession dummy | 0.015 | −0.003 |
| | (0.041) | (0.005) |

**Table 5. (continued)**

| *Explanatory variable* | *Dependent variable* | |
|---|---|---|
| | *Loan spread* | *Dummy for secured loan* |
| | *(1)* | *(2)* |
| Constant | 2.719*** | –1.407*** |
| | (0.673) | (0.413) |
| *Summary statistic* | | |
| Bank fixed effects | Yes | Yes |
| State fixed effects | Yes | Yes |
| Time fixed effects | No | No |
| No. observations | 1,121,510 | 1,121,508 |
| No. banks | 590 | 590 |
| $R^2$ | 0.330 | 0.183 |

* Statistically significant at the 10 percent level.
** Statistically significant at the 5 percent level.
*** Statistically significant at the 1 percent level.
a. This table reports panel regression estimates of terms of individual new business loans originated from the second quarter of 1997 to the fourth quarter of 2011 by banks reporting to the Federal Reserve's STBL, which correspond to equation (2) in the text. The dependent variables in columns (1) and (2) are, respectively, loan spread and an indicator variable for collateralization as reported to the STBL. Taylor rule residuals are obtained from rolling regressions of the target federal funds rate on deviations of median SPF projections for GDP growth from potential output growth and deviations of CPI inflation from 2 percent. All other explanatory variables are defined as in table 2. The sample excludes loans extended under commitment established prior to the current quarter from the sample. All regressions include state and bank fixed effects. Standard errors two-way clustered by quarter and bank are reported in parentheses.

Table 5 reports the results of reestimating equation (1) replacing the federal funds rate with the Taylor rule residual. We find that the results reported in table 2 are robust to using a measure of monetary policy conditions that is orthogonal to the degree of slack in economic activity and deviations of inflation from target. In other words, we find that the component of interest rates that reflects economic activity is likely not the main driver for our baseline results reported in table 2.

An additional endogeneity concern is that short-term interest rates set by monetary policy could respond directly to financial stability considerations. To alleviate this concern, we also report our results replacing the federal funds rate with the residual from a Taylor rule expanded to include financial risk. In particular, we reestimate the Taylor rule used in table 5 with a rule that also includes the option-implied volatility on the S&P 500 index one month out (that is, the VIX), in addition to measures of the output gap and deviations of inflation from its target. We report the results of this new estimation in table 6.

**Table 6. Terms of Business Lending and Modified Taylor Rule Residuals[a]**

| | *Dependent variable* | |
|---|---|---|
| *Explanatory variable* | *Loan spread* *(1)* | *Dummy for secured loan* *(2)* |
| Modified Taylor rule residual | 0.019* | 0.003** |
| | (0.011) | (0.001) |
| Loan risk rating | 0.333*** | 0.050*** |
| | (0.010) | (0.004) |
| Loan size | –0.261*** | –0.004*** |
| | (0.007) | (0.001) |
| Bank size | –0.105*** | –0.040*** |
| | (0.035) | (0.008) |
| Bank tier 1 capital ratio | 4.241*** | –0.196 |
| | (0.643) | (0.164) |
| Bank net income / assets | –7.458*** | 0.222 |
| | (1.696) | (0.250) |
| Bank liquid assets / assets | –1.181 | –0.286 |
| | (1.115) | (0.358) |
| Bank deposits / assets | 1.391*** | 0.114** |
| | (0.264) | (0.056) |
| Short-term deposits / deposits | –0.767*** | –0.116* |
| | (0.200) | (0.060) |
| Nonretail deposits / deposits | 0.019 | –0.011 |
| | (0.081) | (0.017) |
| Bank loans / assets | 0.940*** | 0.052 |
| | (0.131) | (0.045) |
| Bank C&I loans / loans | 0.790*** | 0.244*** |
| | (0.209) | (0.040) |
| State personal income growth | –0.000*** | 0.000*** |
| | (0.000) | (0.000) |
| Change in regional CPI | 0.002 | 0.001 |
| | (0.007) | (0.001) |
| State unemployment rate | 0.087*** | 0.013*** |
| | (0.012) | (0.002) |
| Change in state housing prices | –0.003** | 0.000 |
| | (0.001) | (0.000) |
| GDP growth | 0.006 | 0.000 |
| | (0.008) | (0.001) |
| NBER recession dummy | 0.012 | –0.005 |
| | (0.048) | (0.004) |

**Table 6. (continued)**

| *Explanatory variable* | *Dependent variable* Loan spread (1) | *Dependent variable* Dummy for secured loan (2) |
|---|---|---|
| *Constant* | 1.815** | 1.116*** |
| | (0.690) | (0.126) |
| *Summary statistic* | | |
| Bank fixed effects | Yes | Yes |
| State fixed effects | Yes | Yes |
| Time fixed effects | No | No |
| Observations | 941,063 | 941,062 |
| Number of banks | 543 | 543 |
| $R^2$ | 0.318 | 0.185 |

* Statistically significant at the 10 percent level.
** Statistically significant at the 5 percent level.
*** Statistically significant at the 1 percent level.
a. This table reports panel regression estimates of terms of individual new business loans originated from the second quarter of 1997 to the fourth quarter of 2011 by banks reporting to the Federal Reserve's STBL, which correspond to equation (2) in the text. The dependent variables in columns (1) and (2) are, respectively, loan spread and an indicator variable for collateralization as reported to the STBL. Modified Taylor rule residuals are obtained from rolling regressions of the target federal funds rate on deviations of median SPF projections for GDP growth from potential output growth, deviations of CPI inflation from 2 percent, and the VIX. All other explanatory variables are defined as in table 2. The sample excludes loans extended under commitment established prior to the current quarter from the sample. All regressions include state and bank fixed effects. Standard errors two-way clustered by quarter and bank are reported in parentheses.

The results in table 6 are fairly similar to those reported in table 5, suggesting that the component of short-term interest rates that reflects financial stability considerations is likely not responsible for explaining the results in table 2, which alleviates some endogeneity concerns[2].

## 4. Conclusions

This paper provides evidence that banks tend to ease lending terms for new loans in an environment of low short-term interest rates, controlling for the ex ante assessment of loan riskiness. For

2. Dell'Ariccia, Laeven, and Suarez (2016), who find that bank loans tend to be assessed as being more risky at origination during periods of low interest rates, document that the interest-rate effects tend to be stronger in times of lower financial stress (for example, periods with few bank failures), when financial stability considerations are less likely to weigh on short-term interest rates.

example, our empirical analysis shows that a one-standard-deviation decrease in short-term interest rates would result in a decrease in loan spreads for new loans of about 0.1 percentage point (compared with its standard deviation of 1.4 percentage points). Moreover, we also find evidence that banks are less likely to require collateral for new loans originated during low-interest rate periods.

We obtain these results using loan-level data on newly issued loans, which is critical for assessing the impact on general credit conditions and on the riskiness of U.S. bank loans. This contrasts with most existing studies, which largely rely on firm-level or aggregate measures of risk in other countries. By restricting our attention to the extension of new loans, we can focus on changes in lending terms, while controlling for ex ante perceptions of loan risk. Most existing studies analyze ex post loan performance, which could be affected by subsequent events.

We also find that the link between lower short-term interest rates and easier loan terms (as measured by spreads and collateral requirements) is more pronounced for banks that are more sensitive to short-term interest rates, as measured by the fraction of short-term deposits in their total deposit base and for riskier loans. These findings suggest that the negative relationship between interest rates and lending terms in our baseline results likely operates through decisions made by the bank in response to changes in interest rates and not through an omitted variable.

We also find similar results when replacing short-term interest rates with Taylor rule residuals that control for the degree of economic slack (as captured by the output gap and deviations of inflation from its target level) and for overall financial risk (as captured by the VIX), suggesting that our results are not explained by endogenous and predictable responses of interest rates to economic and financial conditions.

This paper has focused on a very specific margin of risk-taking: the terms of business lending. The effect on the overall asset portfolio of banks could be different. In fact, Dell'Ariccia, Laeven, and Suarez (2016) find that banks increase their holding of riskier securities during periods of low interest rates. In addition, there are several other channels through which interest rate policy can affect bank stability, including leverage, liquidity, and maturity mismatches (Adrian and Shin, 2009).

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# The Response of Sovereign Bond Yields to U.S. Monetary Policy

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To provide further stimulus to the economy in response to a cascade of shocks that roiled financial markets in the latter part of 2008, the U.S. Federal Reserve started to aggressively employ unconventional monetary policy measures after the Federal Open Market Committee (FOMC) lowered the target for the federal funds rate to its effective lower bound on 16 December 2008. In this paper, we explore whether the Federal Reserve's unconventional monetary policy actions have significantly influenced asset markets beyond U.S. borders. Until recently, the empirical work on this question has been relatively limited. A few prior studies find evidence of cross-country spillovers in the international bond market, but they provide little insight into how the strength and scope of these spillovers compare with those during the conventional monetary policy period. The characteristics of the international spillovers across advanced economies and emerging economies are also an interesting topic with relatively little discussion so far.

Our aim in this paper is to quantify the transmission of U.S. monetary policy shocks to foreign countries, during both the

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conventional monetary policy regime and the unconventional policy period since late 2008. Specifically, we employ the empirical methodology of Gilchrist, López-Salido, and Zakrajšek (2015) to estimate the degree of U.S. monetary policy spillovers—during both the conventional and unconventional policy regimes—on foreign bond yields for a set of advanced foreign and emerging market economies.[1] To compare the efficacy of conventional and unconventional policy measures, we use changes in the two-year nominal U.S. Treasury yield on policy announcement days as a common instrument across the two policy regimes. These movements in the two-year Treasury yield—the "short" surprises—are calculated within a narrow window surrounding FOMC and other policy announcements and thus identify unanticipated changes in the stance of U.S. monetary policy.

To provide a more encompassing stance of monetary policy during the unconventional policy regime, we adopt an identification scheme that allows for an additional unanticipated component of policy—a component that has an independent effect on longer-term interest rates. Specifically, we decompose the observed change in the ten-year nominal U.S. Treasury yield over a narrow window bracketing an FOMC announcement into two components: (1) an anticipated component that reflects the effects of policy-induced changes in the two-year U.S. Treasury yield on longer-term yields within that narrow window; and (2) a surprise component that is orthogonal to the changes in the two-year Treasury yield within the same time interval. The second component—the "long" surprise—is intended to capture the direct effect of U.S. unconventional policy measures on longer-term interest rates.

We focus on the impact of U.S. monetary policy actions on the yields of government bonds denominated in local currency issued by selected advanced and emerging economies.[2] The advanced economies are Australia, Canada, Germany, Italy, Japan, and the United Kingdom, while our panel of emerging market economies consists of Brazil, India, South Korea, Mexico, Singapore, and Thailand. Our results indicate that during the conventional policy regime, an unanticipated easing of monetary policy in the United States has a pronounced effect on both the short- and long-term interest rates for advanced foreign countries. In addition, the

1. This empirical approach is also similar to that used by Hanson and Stein (2015) and Gertler and Karadi (2015).

2. The analysis of the effects of U.S. monetary policy spillovers on yields and spreads on dollar-denominated bonds issued by emerging market economies is part of an ongoing project.

expansionary U.S. policy short surprise steepens the yield curve in those countries. For the emerging economies, except for Mexico, the short-term bond yields do not respond to the U.S. monetary policy action, whereas yields on longer-term bonds are more responsive to such short surprises.

During the U.S. unconventional policy regime, monetary stimulus engineered through the short-end of the yield curve has a mixed effect on short-term interest rates in advanced foreign economies. Yet this short policy shock has a significantly larger effect on the foreign long-term bond yields, implying a flattening of the yield curve in those countries. At the same time, an unconventional stimulus orchestrated vis-à-vis the long-end of the U.S. yield curve also has significant effects on the long-term interest rates in Australia, Canada, Germany, and Japan. Moreover, the impact of U.S. unconventional monetary policy on longer-term bond yields in the emerging market economies is similar. As a result, during the unconventional period, an easing of U.S. monetary policy flattens yield curves in both advanced and emerging market economies.

We also calculate the implied pass-through of the U.S. short monetary policy surprises to the longer-term foreign interest rates across the two policy regimes. Our estimates indicate that during the unconventional period, the degree of the pass-through across countries ranges between 50 and 90 percent of the domestic pass-through to the ten-year U.S Treasury yield. We find that, using such a pass-through metric, the degree of international transmission of U.S. policy shocks to long-term foreign bond yields is very similar across the two policy regimes, at least for advanced countries for which we are able to compute the relevant comparison.

Our analysis of the international effects of unconventional U.S. monetary policy on foreign asset prices contributes to a rapidly growing empirical literature that evaluates the transmission of such measures through financial markets. Much of this research focuses on the question of whether purchases of large quantities of Treasury coupon securities by the U.S. Federal Reserve and various forms of forward guidance have altered the level of longer-term Treasury yields. Employing a variety of approaches, Gagnon and others (2011), Krishnamurthy and Vissing-Jorgensen (2011), Swanson (2011), Christensen and Rudebusch (2012), D'Amico and others (2012), Campbell and others (2012), Hamilton and Wu (2012), Wright (2012), D'Amico and King (2013), Li and Wei (2013), and Bauer and Rudebusch (2014) present compelling evidence that the unconventional policy measures employed by the FOMC since the end of 2008 have significantly lowered longer-term Treasury yields. Our paper is also related to the recent work of Nakamura and Steinsson (2013) and Hanson and Stein (2015), who analyze the effects of U.S. monetary policy on real and nominal Treasury yields over a period that includes both the conventional and unconventional policy regimes.

Regarding the international spillovers of U.S. monetary policy, Neely (2010) finds that the unconventional monetary policy actions by the FOMC substantially reduced international long-term bond yields and the spot value of the dollar. He adopts event-study methods to evaluate the joint effect of unconventional policies on nominal longer-term foreign bond yields denominated in local currencies and the corresponding exchange rates. Bauer and Neely (2014) use dynamic term structure models to uncover the extent to which those declines can be attributed to signaling and portfolio balance channels and find substantial effects of both channels. Bowman, Londoño, and Sapriza (2015) study the transmission of U.S. unconventional monetary policy to emerging market economies. On the study of the broader international effects of unconventional U.S. monetary policies on asset markets, Fratzscher, Lo Duca, and Straub (2013) analyze the global spillovers of the FOMC unconventional monetary policy measures. Rogers, Scotti, and Wright (2014) examine the effects of unconventional monetary policy by the U.S. Federal Reserve, the Bank of England, the European Central Bank, and the Bank of Japan on the corresponding bond yields, stock prices, and exchange rates.

The remainder of the paper is organized as follows. Section 1 outlines our empirical methodology, including a brief discussion of the identification of conventional U.S. monetary policy surprises and a presentation of our framework for estimating the causal effect of U.S. unconventional monetary policy on asset prices. Section 2 contains the estimation results comparing the effects of monetary policy on foreign bond yields across the two policy regimes. Section 3 concludes.

## 1. Empirical Framework

In this section, we present the empirical approach used to estimate the impact of monetary policy on market interest rates during both the conventional and unconventional policy regimes. As noted above, our approach follows Gilchrist, López-Salido, and Zakrajšek (2015). The key aspect of this approach involves the use of intraday data to directly infer monetary policy surprises associated with policy announcements. In combination with the daily data on market interest rates, these high-frequency policy surprises allow us to estimate the causal impact of U.S. monetary policy actions on foreign bond yields.

Before delving into econometric details, we briefly discuss the dating of the two policy regimes. The sample period underlying our analysis runs from 6 February 1992 to 30 April 2014. We divide this period into two distinct monetary policy regimes: a conventional policy regime, a period in which the primary policy instrument was the federal funds rate; and an unconventional policy regime, during which the funds rate has been stuck at the zero lower bound and the FOMC conducted monetary policy primarily by altering the size and composition of the Federal Reserve's balance sheet and by issuing various forms of forward guidance regarding the future trajectory for the federal funds rate.

The dating of these two regimes is relatively straightforward. The key date in our analysis is 25 November 2008, when the FOMC announced—outside its regular schedule—that it would initiate a program to purchase the debt obligations of the government-sponsored enterprises (GSEs) and the mortgage-backed securities (MBS) issued by those agencies in an effort to support housing markets and counteract the massive tightening of financial conditions sparked by the collapse of Lehman Brothers in mid-September. One week later, the FOMC announced—again outside its regular schedule—that in addition to purchasing of agency debt and MBS, it was also considering purchasing longer-term Treasury securities. With the global financial system in severe turmoil and faced with a rapidly deteriorating economic outlook, the FOMC announced at its 16 December meeting that it was lowering the target federal funds rate to a range of 0 to 0.25 percent—its effective lower bound—a decision ushering in the ELB period.

Given this sequence of events, we assume that the unconventional policy regime began on 25 November 2008 and that prior to that point, the conventional policy regime was in effect. Nearly all of the 143 announcements during the conventional policy period followed regularly scheduled FOMC meetings; only four were associated with intermeeting policy moves.[3] According to this chronology, the last

3. The four intermeeting moves occurred on 3 January 2001; 18 April 2001; 22 January 2008; and 8 October 2008. As is customary in this kind of analysis, we excluded the announcement made on 17 September 2001, which was made when trading on major stock exchanges resumed after it was temporarily suspended following the 9/11 terrorist attacks. Most of the FOMC announcements took place at 2:15 pm (Eastern Standard Time); however, announcements for the intermeeting policy moves were made at different times of the day. We obtained all the requisite times from the Office of the Secretary of the Federal Reserve Board.

FOMC meeting during the conventional policy regime took place on 29 October 2008, at which point the FOMC lowered its target for the federal funds rate 50 basis points, to 1.0 percent.

## 1.1 U.S. Conventional Monetary Policy

Changes in the stance of conventional monetary policy have typically been characterized by a single factor—the "target" surprise or the unanticipated component of the change in the current federal funds rate target (see Cook and Hahn, 1989; Kuttner, 2001; Cochrane and Piazzesi, 2002; Bernanke and Kuttner, 2005). As emphasized by Gürkaynak, Sack, and Swanson (2005), however, this characterization of monetary policy is incomplete, and another factor—namely, changes in the future policy rates that are independent of the current target rate—is needed to fully capture the impact of conventional monetary policy on asset prices. This second factor, commonly referred to as a "path" surprise, is closely associated with the FOMC statements that accompany changes in the target rate and represents a communication aspect of monetary policy that assumed even greater importance after the target rate was lowered to its effective lower bound in December 2008.

To facilitate the comparison of the efficacy of conventional and unconventional monetary policy, we follow Hanson and Stein (2015), Gertler and Karadi (2015), and Gilchrist, López-Salido, and Zakrajšek (2015) and reduce this two-dimensional aspect of conventional policy by assuming that the change in the two-year nominal Treasury yield over a narrow window bracketing an FOMC announcement reflects the confluence of the target and path surprises.[4] Under this assumption, the effect of unanticipated changes in the stance of conventional policy on foreign bond yields can be inferred from

$$\hat{\Delta} y_{i,t+1}(n) = \beta_i(n)\tilde{\Delta} y_t^{US}(2) + \varepsilon_{i,t+1}, \qquad (1)$$

where $\hat{\Delta} y_{i,t+1}(n)$ denotes the two-day change in an $n$-year bond yield for country $i$, and $\tilde{\Delta} y_t^{US}(2)$ is the intraday change in the (on-the-run)

4. We examine the robustness of this assumption by decomposing the change in the two-year Treasury yield into the target and path surprises. Our results indicate that the first-order effects of conventional monetary policy actions can be summarized adequately by the intraday changes in the two-year nominal Treasury yield bracketing FOMC announcements.

two-year nominal U.S. Treasury yield over a 30-minute window surrounding an FOMC announcement (10 minutes before to 20 minutes after) on day t. The stochastic disturbance $\varepsilon_{i,t+1}$ captures the information that possibly was released earlier in the day, as well as noise from other financial market developments that took place throughout the next day. Compared with Gilchrist, López-Salido, and Zakrajšek (2015), the only difference is that we use the two-day change in foreign bond yields because markets in Asia and Europe are closed when the FOMC makes its policy announcements; therefore, we need to use the yield on day $t + 1$ to measure the response of these asset markets to the U.S. monetary policy actions.[5]

Using the sample of 143 FOMC announcements during the conventional policy regime, we estimate equation (1) by ordinary least squares (OLS). Underlying this empirical strategy is the assumption that movements in the two-year Treasury yield in a 30-minute window surrounding FOMC announcements are due entirely to the unanticipated changes in the current stance of monetary policy. As discussed by Gilchrist, López-Salido, and Zakrajšek (2015), this is a reasonable assumption because we are virtually certain that no other economic news was released within such a short interval of time.

## 1.2 U.S. Unconventional Monetary Policy

After bringing the target federal funds rate down to its effective lower bound in December 2008, the FOMC has taken numerous steps to provide further monetary accommodation to the U.S. economy. As part of its efforts to stimulate economic activity and ease broad financial conditions, the FOMC has employed different forms of forward guidance regarding the future path of the federal funds rate and has undertaken large-scale purchases of longer-term securities—a policy commonly known as quantitative easing—in order to put further downward pressure on longer-term market interest rates.

As shown in table 1, the provision of guidance about the likely future path of the policy rate has evolved significantly from the Committee's initial statement on 16 December 2008, in which it indicated that economic conditions were "likely to warrant exceptionally low levels of the federal funds rate for some time." Starting with the March

5. For Canada, Mexico, and Brazil, we also calculated the one-day changes in yields and use them as a dependent variable in equation (1). All of our results were robust to this alternative measurement.

2009 meeting, the FOMC communicated its expectation that an exceptionally low funds rate would be in force "for an extended period." This calendar-based approach was clarified in August 2011, when the Committee changed the statement language from "for an extended period" to "at least through mid-2013," and then again in January 2012, when the calendar-dependent forward guidance was changed to "at least through late 2014."

**Table 1. Key Unconventional Monetary Policy Actions[a]**

| *Date* | *Time*[b] | *FOMC*[c] | *Highlights* |
|---|---|---|---|
| **25 Nov 2008** | 08:15 | N | Announcement that starts LSAP-I. |
| **01 Dec 2008** | 08:15 | N | Announcement indicating potential purchases of Treasury securities. |
| **16 Dec 2008** | 14:20 | Y | Target federal funds is lowered to its effective lower bound; statement indicating that the Federal Reserve is considering using its balance sheet to further stimulate the economy; first reference to forward guidance: "economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time." |
| **28 Jan 2009** | 14:15 | Y | "Disappointing"This FOMC statement because of its lacked of concrete language regarding the possibility and timing of purchases of longer-term Treasuries. |
| **18 Mar 2009** | 14:15 | Y | Announcement to purchase Treasuries and increase the size of purchases of agency debt and agency MBS; also, first reference to extended period: "interests rates are likely to remain low for an extended period." |
| **10 Aug 2010** | 14:15 | Y | Announcement that starts LSAP-II. |
| 27 Aug 2010 | 10:00 | N | Chairman's speech at Jackson Hole. |
| **21 Sept 2010** | 14:15 | Y | Announcement reaffirming the existing reinvestment policy. |
| 15 Oct 2010 | 08:15 | N | Chairman's speech at the Federal Reserve Bank of Boston. |
| **03 Nov 2010** | 14:15 | Y | Announcement of additional purchases of Treasury securities. |

**Table 1. Continued**

| Date | Time[b] | *FOMC*[c] | *Highlights* |
|---|---|---|---|
| 09 Aug 2011 | 14:15 | Y | First "calendar-based" forward guidance: "anticipates that economic conditions are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013." |
| 29 Aug 2011 | 10:00 | N | Chairman's speech at Jackson Hole. |
| **21 Sept 2011** | 14:15 | Y | Announcement of the Maturity Extension Program (MEP). |
| 25 Jan 2012 | 12:30 | Y | Second "calendar-based" forward guidance: "keep the federal funds rate exceptionally low at least through late 2014." |
| **20 Jun 2012** | 12:30 | Y | Announcement of continuation of the MEP through end of 2012. |
| 31 Aug 2012 | 10:00 | N | Chairman's speech at Jackson Hole. |
| **13 Sept 2012** | 12:30 | Y | Third "calendar-based" forward guidance: "likely maintain the federal funds rate near zero at least through mid-2015." In addition, first forward guidance regarding the pace of interest rates after lift-off: "likely maintain low rates for a considerable time after the economic recovery strengthens," and announcement of LSAP-III (flow-based; $40 billion per month of agency MBS). |
| **12 Dec 2012** | 12:30 | Y | Announcement of an increase in LSAP-III (from $40 billion to $85 billion per month); first "threshold-based" forward guidance: maintain the funds rate near zero for as long as unemployment is above 6.5%, inflation (1-–2 years ahead) is below 2.5%, and long-term inflation expectations remain well -anchored. |
| 19 Jun 2013 | 14:00 | Y | Forward guidance lays out plans to start tapering asset purchases later that year (unemployment rate below 7.5%); and end LSAP-III by mid-2014, when the unemployment rate is around 7.0%. |
| 17 Jul 2013 | 08:30 | N | Chairman's semiannual Monetary Policy Report to the Congress. |
| 18 Sept 2013 | 14:15 | Y | "Asset purchases are not on a preset course." |

a. Dates in bold correspond to the LSAP-related announcements (see the text for details).
b. All announcements are at Eastern Standard Time.
c. *Y* = an announcement associated with a regularly-schedule FOMC meeting; *N* = an intermeeting policy announcement.

The first round of purchases was completed in March 2010. The next development in the Federal Reserve's balance sheet policy (LSAP-II) was launched with the FOMC's announcement in August 2010 of reinvestment arrangements, under which the Federal Reserve would maintain the elevated level of holdings of longer-term securities brought about by LSAP-I "by redeploying into longer-term Treasury investments the principal payments from agency securities held in the System Open Market Account (SOMA) portfolio." As a result, from November 2010 through the end of June 2011, the Federal Reserve was engaged in the program involving the purchase of $600 billion of longer-term Treasury securities. Subsequently, the FOMC decided to continue to maintain the level of security holdings attained under the first two purchase programs. In September 2011, the Committee made further adjustments to its investment policy, which included an extension of the average maturity of its Treasury securities portfolio (MEP) and reinvesting principal payments from agency securities in MBS rather than in longer-term Treasury securities.

Although these announcements clearly stated the amount of securities that the Federal Reserve anticipated purchasing, they were nevertheless vague about the conditions that might lead the policymakers to change that amount. In an effort to resolve this ambiguity, the FOMC implemented an alternative approach in September 2012 by announcing a monthly rate at which the Federal Reserve would purchase securities. The expectation was that such a flow-based balance sheet policy, if clearly communicated, might lead market participants and the public more generally to expect that the Committee would pursue the program as long as appropriate to achieve its mandated goals.

The rationale underlying LSAPs was predicated on the assumption that the relative prices of financial assets are influenced to an important extent by the quantity of assets available to investors. Economic theory suggests that changes in the central bank's holdings of long-term securities will affect long-term interest rates if private investors have a preference for keeping a portion of their portfolios in the form of such securities, a notion formalized by the so-called preferred habitat models. According to this view, investors are inclined to keep a fraction of their investments in the form of long-term fixed-interest debt such as Treasury securities, on the grounds that these assets have characteristics not shared by alternative longer-term investments—namely, the absence of default risk and a high degree of marketability.

In light of investors' preferences for longer-term government paper, defined broadly to include securities issued or guaranteed by the GSEs, a reduction in the supply of long-term government debt relative to the supplies of other financial assets will, all else equal, lead to a decline in government bond yields in order to induce investors to decrease their holdings of such obligations. In other words, purchases of Treasury securities, agency debt, and agency-guaranteed MBS by the Federal Reserve lower longer-term nominal interest rates, as investors find themselves demanding more government debt than is available on the market at the existing configuration of interest rates; conversely, an increase in the stock of government debt held by the private sector boosts bond yields. This adjustment mechanism hinges importantly on the presumption that the term premiums are sensitive to the volume of long-term debt outstanding, so that longer-term interest rates are affected by purchases even if expectations for the future path of the policy rate remain unchanged.

Because asset purchases were an integral part of the unconventional policy measures employed by the FOMC during the ELB period, changes in the two-year Treasury yield around policy announcements during that period will fail to capture the full impact of unconventional monetary policy on asset prices. Following Gilchrist, López-Salido, and Zakrajšek (2015), we capture this extra dimension of unconventional policy by assuming that

$$\tilde{\Delta} y_t^{US}(10) = \lambda \tilde{\Delta} y_t^{US}(2) + \tilde{\Delta} u_t^{US}, \tag{2}$$

where $\tilde{\Delta} y_t^{US}(10)$ denotes the change in the (on-the-run) ten-year nominal U.S. Treasury yield over a narrow window surrounding a policy announcement on day $t$, $\tilde{\Delta} y_t^{US}(2)$ is the change over the same window in the (on-the-run) two-year U.S. Treasury yield, and $\tilde{\Delta} u_t^{US}$ represents the unanticipated component of the U.S. unconventional policy that potentially has an independent effect on longer-term interest rates.

As above, let $\hat{\Delta} y_{i,t+1}(n)$ denote the two-day change in the $n$-year bond yield for country $i$. Then the full impact of U.S. unconventional monetary policy on this asset can be inferred by estimating

$$\begin{aligned} \hat{\Delta} y_{i,t+1}(n) &= \beta_i(n) \tilde{\Delta} y_t^{US}(2) + \gamma_i(n) \tilde{\Delta} u_t^{US} + \nu_{i,t+1} \\ &= (\beta_i(n) - \gamma_i(n)\lambda) \tilde{\Delta} y_t^{US}(2) + \gamma_i(n) \tilde{\Delta} y_t^{US}(10) + \nu_{i,t+1}, \end{aligned} \tag{3}$$

where $v_{i,t+1}$ captures all nonpolicy shocks that can influence the behavior of asset prices on policy announcement days, and the coefficients $\beta_i(n)$ and $\gamma_i(n)$ determine the relative impact of the short and long U.S. unconventional policy shocks on the $n$-year bond yield for country $i$, respectively. The system implied by equations (2) and (4) can be estimated jointly by nonlinear least squares (NLLS), thereby taking into account the specified cross-equation restrictions.

This empirical approach of quantifying the multi-dimensional aspect of monetary policy is similar to that put forth by Gürkaynak, Sack, and Swanson (2005). Specifically, they use a two-step estimation procedure, where the first step involves the use of the principal components analysis to extract two latent factors from a panel of narrow-window changes in short-term interest rates, which—after a suitable rotation and normalization—are interpreted as the target and path surprises associated with FOMC announcements during the conventional policy regime. Our approach, however, identifies two orthogonal aspects of unconventional monetary policy—a short and a long policy surprise—using two interest rates and, therefore, relies on less information than is embedded in the entire term structure of interest rates. The advantage of our approach lies in the fact that it avoids the two-step estimation procedure and hence the need to adjust standard errors owing to the use of generated regressors in the second step.

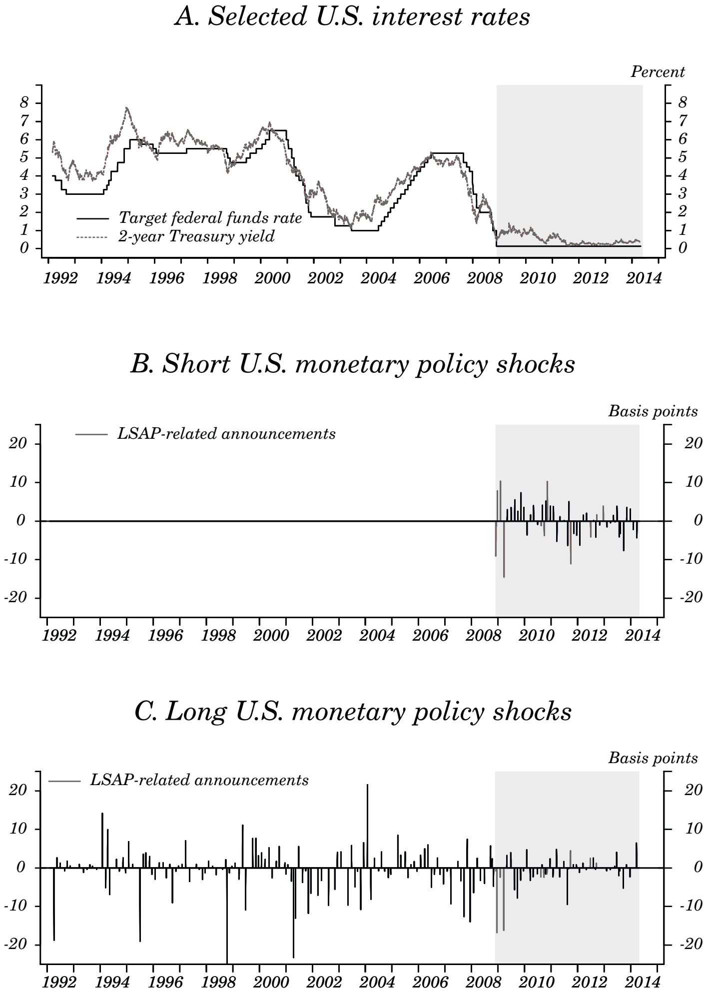
We apply this methodology to a sample of 51 unconventional FOMC policy announcements that took place between 25 November 2008 and 30 April 2014. Our sample includes announcements containing communication about LSAPs, the various forms of forward guidance used during this period, or both. The sample also includes several key speeches and testimonies through which the policymakers elaborated on the various aspects of unconventional policy measures being employed by the FOMC, in an effort to elucidate for market participants the strategic framework guiding their decisions. In many of these instances, the announcements represent the interpretation of statements and speeches—as opposed to conveying information about the numerical value of the target funds rate. Consequently, we use a wider 60-minute window surrounding an announcement (10 minutes before to 50 minutes after) to calculate the intraday changes in the two- and ten-year U.S. Treasury yields.[6]

6. The use of a 60-minute window should allow the market a sufficient amount of time to digest the news contained in announcements associated with unconventional policy measures.

The figure shows the interest rate paths and the identified U.S. monetary policy shocks implied by our approach. Panel A shows the target federal funds rate and the two-year U.S. Treasury yield over the entire sample period. Clearly, our sample period is marked by substantial variation in shorter-term interest rates and contains a number of distinct phases of U.S. monetary policy: the 1994–95 tightening phase that followed the jobless recovery in the early 1990s; the tightening phase that preceded the bursting of the tech bubble in early 2001; the subsequent easing of policy in response to a rapid slowdown in economic activity and the emergence of substantial disinflationary pressures; the 2003–04 period of very low interest rates; the gradual removal of monetary accommodation that commenced in the spring of 2004; the aggressive reduction in the target federal funds rate in the early stages of the 2007–09 financial crisis; and the period when the federal funds rate was stuck at the zero lower bound.

Panels B and C show our U.S. monetary policy surprises. Panel B depicts the sequence of short surprise—that is, the $\tilde{\Delta} y_t^{US}(2)$ —associated with the FOMC actions across both the conventional and unconventional periods. Panel C depicts the sequence of long policy surprises, $\tilde{\Delta} y_t^{US}(10)$, measured during the unconventional period. Under the conventional policy regime, the largest (absolute) short policy surprises are associated with the intermeeting policy actions. As shown by the red spikes, the largest (absolute) short surprises during the unconventional policy regime correspond to the early LSAP announcements. Moreover, the largest short surprises during the unconventional period are associated with monetary policy easings. The volatility of this series is dampened over time, as the two-year U.S. Treasury yield reaches values close to zero. The largest movements in long surprises are also associated with LSAP announcements. In contrast to the short surprises, large long surprises are two-sided. In addition, the volatility of this series shows no evidence of attenuation, as the two-year U.S. Treasury yield approaches the zero lower bound.

**Figure. The Stance of U.S. Monetary Policy**



## 1.3 U.S. Monetary Policy and Domestic Asset Prices

Before we analyze the effects of U.S. monetary policy on foreign bond yields, it is helpful to present the impact of such actions on the asset prices in the United States. Table 2 presents the results using the intraday changes in the ten-year U.S. Treasury yield and the S&P 500 stock price index, as well as the corresponding two-day changes in the two assets. Clearly, the intraday narrow-window changes are much cleaner measures to study the effects of monetary policy surprises. Nevertheless, we also estimate these effects using the two-day changes, in order to compare the results with our benchmark estimation of changes in international bond yields.

**Table 2. The Impact of U.S. Monetary Policy on Selected Domestic Asset Prices[a]**

| | *Conventional[b]* | | *Unconventional[c]* | | |
|---|---|---|---|---|---|
| *Asset (window)* | *Short* | $R^2$ | *Short* | *Long* | $R^2$ |
| Ten-yr Treasury (intraday) | 0.533***<br>(0.058) | 0.646 | 1.407***<br>(0.204) | - | 0.590 |
| Ten-yr Treasury (two-day) | 0.506***<br>(0.122) | 0.091 | 1.770***<br>(0.425) | - | 0.276 |
| S&P 500 (intraday) | –53.126***<br>(13.348) | 0.165 | –70.925***<br>(24.551) | –0.115<br>(18.686) | 0.180 |
| S&P 500 (two-day) | –10.248***<br>(2.145) | 0.134 | –14.454**<br>(6.551) | 12.240**<br>(6.105) | 0.164 |

* Statistically significant at the 10 percent level.
** Statistically significant at the 5 percent level.
*** Statistically significant at the 1 percent level.
a. For the conventional policy regime, the entries under the column heading "Short" denote the OLS estimates of the response coefficients to a U.S. policy-induced surprise in the two-year Treasury yield. For the unconventional policy regime, the entries under the column heading "Short" denote the NLLS estimates of the response coefficients to a U.S. policy-induced surprise in the two-year Treasury yield, while the entries under the column heading "Long" denote the estimates of the response coefficients to a policy-induced surprise in the ten-year Treasury yield that is orthogonal to the surprise in the two-year yield. All specifications include a constant (not reported). Heteroskedasticity-consistent asymptotic standard errors are reported in parentheses.
b. Sample period: 143 FOMC announcements between 06 February 1992 and 24 November 2008. Intraday asset price changes are measured using a 30-minute window bracketing a policy announcement.
c. Sample period: 51 LSAP- and non-LSAP-related FOMC announcements between 25 November 2008 and 30 April 2014. Intraday asset price changes are measured using a 60-minute window bracketing a policy announcement.

According to the entries in the table, an unanticipated easing of monetary policy that lowers the two-year nominal U.S. Treasury yield by 10 basis points induces a 5 basis point decline in the ten-year nominal U.S. Treasury yield during the conventional monetary policy period. During the unconventional policy period, this monetary stimulus leads to a 15 basis point reduction in the ten-year U.S. Treasury yield. These results are very much in line with the estimates of Hanson and Stein (2015) and Gilchrist, López-Salido, and Zakrajšek (2015).

Next, a monetary stimulus of this magnitude significantly boosts the domestic stock market, by a factor of 50 during the conventional policy regime according to the narrow window estimates and by a factor of 10 using the two-day window. The response of the S&P 500 stock price index to the U.S. short shock is even more pronounced in the unconventional policy period. In contrast, the U.S. long monetary policy shock does not seem to have a separate effect on broad equity prices.

## 2. U.S. Monetary Policy and Foreign Bond Yields

This section contains our main analysis regarding the effects of U.S. monetary policy shocks on the yields of foreign government bonds across the two policy regimes. We consider here the yields on local-currency-denominated bonds issued by governments of selected advanced and emerging market economies. The advanced countries are Australia, Canada, Germany, Italy, Japan, and the United Kingdom. The emerging market economies are Brazil, India, South Korea, Mexico, Singapore, and Thailand. The selection of the countries is based on data availability, particularly the coverage of the local-currency-denominated government bond yields during the conventional monetary policy regime.[7]

### 2.1 The Effects of U.S. Conventional Monetary Policy

The responses of foreign bond yields to U.S. monetary policy surprises under the conventional monetary policy regime are presented in tables 3 and 4. Table 3 shows the impact of U.S. monetary policy on government bond yields for the six advanced foreign economies, while

7. For emerging economies, a parallel analysis on dollar-denominated government bond yields is underway.

table 4 shows the results for the six emerging market economies. In both tables, panel A summarizes the estimation results for two-year nominal government bond yields, while those for the ten-year nominal government bond yields are shown in panel B.

As shown in panel A of table 3, a surprise cut in the two-year U.S. Treasury yield of 10 basis points leads to a decline of 4 to 10 basis points in the yields on short-term government bonds issued by advanced foreign economies. The one exception is Japan, which has had very low and stable short-term interest rates since the early 1990s. The strongest international effect of U.S. monetary policy actions is on Canadian bond yields, followed by Australian and U.K. yields. Canadian short-term government bond yields are the most sensitive to U.S. monetary policy moves during the conventional policy regime, a result that underscores the close connection between the two neighboring economies.

**Table 3. U.S. Conventional Monetary Policy and Government Bond Yields: Selected Advanced Economies[a]**

| *U.S.policy shock* | *Australia* | *Canada* | *Germany* | *Italy* | *Japan* | *United Kingdom* |
|---|---|---|---|---|---|---|
| | | | *A. Two-year nominal government bond yields* | | | |
| Short | 0.621*** | 0.972*** | 0.364*** | 0.427*** | 0.104 | 0.518** |
| | (0.182) | (0.144) | (0.089) | (0.094) | (0.067) | (0.238) |
| $R^2$ | 0.108 | 0.225 | 0.121 | 0.1 | 0.023 | 0.1 |
| No. observations | 143 | 143 | 143 | 143 | 143 | 143 |
| | | | *B. Ten-year nominal government bond yields* | | | |
| Short | 0.483*** | 0.435*** | 0.262*** | 0.348*** | 0.106 | 0.407* |
| | (0.162) | (0.127) | (0.099) | (0.117) | (0.066) | (0.218) |
| $R^2$ | 0.084 | 0.088 | 0.069 | 0.074 | 0.012 | 0.069 |
| No. observations | 143 | 143 | 143 | 131 | 122 | 143 |

* Statistically significant at the 10 percent level.
** Statistically significant at the 5 percent level.
*** Statistically significant at the 1 percent level.
a. Sample period: 143 FOMC announcements between 06 Februrary 1992 and 24 November 2008. The dependent variable is a two-day change bracketing an FOMC announcement in two-year the government bond yield (panel A) and the ten-year government bond yield (panel B) for the specified country. The entries labeled "Short" denote the OLS estimates of the response coefficients to a U.S. policy-induced surprise in the two-year Treasury yield. All specifications include a constant (not reported). Heteroskedasticity-consistent asymptotic standard errors are reported in parentheses.

**Table 4. U.S. Conventional Monetary Policy and Government Bond Yields: Selected Emerging Economies[a]**

| U.S. policy shock | Brazil | India | South Korea | Mexico | Singapore | Thailand |
|---|---|---|---|---|---|---|
| *A. Two-year nominal government bond yields* | | | | | | |
| Short | 1.221 | 0.145 | −0.103 | 0.678*** | 0.416*** | 0.161 |
| | (1.279) | (0.156) | (0.103) | (0.186) | (0.119) | (0.127) |
| $R^2$ | 0.025 | 0.008 | 0.007 | 0.085 | 0.213 | 0.022 |
| No. observations | 73 | 59 | 70 | 49 | 92 | 71 |
| *B. Ten-year nominal government bond yields* | | | | | | |
| Short | 3.440*** | 0.230*** | −0.058 | 0.508* | 0.146 | 0.455*** |
| | (1.153) | (0.086) | (0.119) | (0.278) | (0.114) | (0.173) |
| $R^2$ | 0.233 | 0.05 | 0.002 | 0.025 | 0.02 | 0.107 |
| No. observations | 25 | 66 | 64 | 60 | 89 | 69 |

* Statistically significant at the 10 percent level.
** Statistically significant at the 5 percent level.
*** Statistically significant at the 1 percent level.
a. Sample period: 143 FOMC announcements between 06 Februrary 1992 and 24 November 2008. The dependent variable is a two-day change bracketing an FOMC announcement in the two-year government bond yield (panel A) and the ten-year government bond yield (panel B) for the specified country. The entries labeled "Short" denote the OLS estimates of the response coefficients to a U.S. policy-induced surprise in the two-year Treasury yield. All specifications include a constant (not reported). Heteroskedasticity-consistent asymptotic standard errors are reported in parentheses.

The response of the long-term government bond yields in advanced foreign economies to the policy-induced movement in the U.S. two-year Treasury yield is also significant, except for Japan. The estimates indicate that an easing of U.S. monetary policy during the conventional period generates an overall decline in foreign interest rates along the entire term structure. Moreover, the two-year foreign bond yields are more responsive to the U.S. short monetary policy shock than the ten-year bond yields. These results imply that a U.S. monetary policy easing induces a widening of the foreign yield spreads between long- and short-term nominal interest rates. With regard to the domestic impact of the U.S. monetary policy, the standard view is that in periods when the ELB is not binding, U.S. monetary policy influences the short end of the yield curve, and an easing steepens the yield curve. Our results point to a similar effect on the foreign government bond yield curves in major advanced foreign economies.

In comparison, as shown in table 4, the effects of conventional U.S. monetary policy on government bond yields in emerging economies is weaker and less pervasive. For short-term interest rates, a surprise cut in the two-year U.S. Treasury yield leads to a significant reduction in the two-year government bond yields in Mexico and Singapore. For the other emerging economies in our sample, the effect of the U.S. policy short shock is insignificant. For long-term interest rates, the ten-year bond yields for Brazil, India, and Thailand are most responsive to the U.S. monetary policy short shocks. The response coefficients for the Brazilian ten-year bond yield is especially large, which may reflect the short estimation period because the data are available only starting in 2006. Another remark regarding the results in table 4 is that the markets for emerging countries' government bonds denominated in local currency are significantly less developed, especially in the early part of our sample period. As a result, the limited liquidity of these bonds is a potential concern, which could influence our results.

## 2.2 The Effects of U.S. Unconventional Monetary Policy

As discussed above, narrow-window changes in the two-year U.S. Treasury yield bracketing FOMC announcements during the ELB period fail to capture the full impact of unconventional monetary policy on asset prices. The estimation results reported in tables 5 and 6 show the effects of both the short and long unconventional U.S. policy shocks on foreign bond yields.

As shown in panel A of table 5, Australia and the United Kingdom are the only two advanced foreign economies whose short-term interest rates move significantly in response to the short U.S. policy shock, as measured by a policy-induced change in the two-year U.S. Treasury yield during the unconventional period. By contrast, short-term interest rates in other advanced economies do not respond to the short U.S. policy shock. Moreover, the long U.S. policy shock does not affect the yields on short-term government bonds, except for those of Canada, where the estimate of the response coefficient is marginally significant. This result is consistent with the characterization of the two-dimensional U.S. unconventional policy shocks, as the long shock has—by construction—no effect on the U.S. shorter-term interest rates.

According to panel B, the response of the ten-year foreign bond yields to the short U.S. policy shock is significant for all countries in our sample. Overall, the estimated response coefficients are smaller than the estimate of the corresponding response coefficient on the ten-year U.S. Treasury yield (table 2), although there is a significant heterogeneity across countries: in response to an unanticipated decline in the two-year U.S. Treasury yield of 10 basis points, the ten-year government bond yields decline as little as 2 basis points (Japan) and up to 13 basis points (Australia). The long U.S. policy shock also has significant impact on the foreign long-term interest rates, except for those in Italy and the United Kingdom, two countries that investors considered riskier among this group of countries after the global financial crisis.

**Table 5. U.S. Unconventional Monetary Policy and Government Bond Yields: Selected Advanced Economies[a]**

| *U.S. policy shock* | *Australia* | *Canada* | *Germany* | *Italy* | *Japan* | *United Kingdom* |
|---|---|---|---|---|---|---|
| | *A. Two-year nominal government bond yields* | | | | | |
| Short | 0.878*** | 0.29 | 0.555* | 0.43 | 0.126 | 0.768*** |
| | (0.212) | (0.260) | (0.315) | (0.384) | (0.090) | (0.236) |
| Long | 0.224 | 0.329* | −0.024 | 0.032 | −0.055 | −0.1 |
| | (0.184) | (0.181) | (0.200) | (0.296) | (0.052) | (0.170) |
| $R^2$ | 0.238 | 0.13 | 0.12 | 0.023 | 0.136 | 0.148 |
| No. observations | 51 | 51 | 51 | 51 | 51 | 51 |
| | *B. Ten-year nominal government bond yields* | | | | | |
| Short | 1.344*** | 0.872*** | 0.714*** | 1.045*** | 0.223*** | 0.891*** |
| | (0.239) | (0.250) | (0.210) | (0.266) | (0.076) | (0.294) |
| Long | 0.553** | 0.904*** | 0.520** | 0.305 | 0.151** | 0.358 |
| | (0.225) | (0.212) | (0.204) | (0.240) | (0.070) | (0.281) |
| $R^2$ | 0.384 | 0.478 | 0.217 | 0.206 | 0.188 | 0.171 |
| No. observations | 51 | 51 | 51 | 51 | 51 | 51 |

* Statistically significant at the 10 percent level.
** Statistically significant at the 5 percent level.
*** Statistically significant at the 1 percent level.
a. Sample period: 51 LSAP- and non-LSAP-related FOMC announcements between 25 November 2008 and 30 April 2014. The dependent variable is a two-day change bracketing an FOMC announcement in two-year the government bond yield (panel A) and the ten-year government bond yield (panel B) for the specified country. The entries labeled “Short” denote the NLLS estimates of the response coefficients to a U.S. policy-induced surprise in the two-year Treasury yield, while the entries labeled “Long” denote the estimates of the response coefficients to a policy-induced surprise in the ten-year Treasury yield that is orthogonal to the surprise in the two-year yield. All specifications include a constant (not reported). Heteroskedasticity-consistent asymptotic standard errors are reported in parentheses.

**Table 6. U.S. Unconventional Monetary Policy and Government Bond Yields: Selected Emerging Economies[a]**

| *U.S. policy shock* | *Brazil* | *India* | *South Korea* | *Mexico* | *Singapore* | *Thailand* |
|---|---|---|---|---|---|---|
| | *A. Two-year nominal government bond yields* | | | | | |
| Short | 1.733*** | 0.285 | 0.566 | 0.937** | 0.116 | 1.028** |
| | (0.422) | (0.327) | (0.377) | (0.404) | (0.110) | (0.427) |
| Long | 0.886** | 0.838 | 0.177 | –0.125 | 0.126 | –0.136 |
| | (0.453) | (0.581) | (0.301) | (0.289) | (0.177) | (0.236) |
| $R^2$ | 0.179 | 0.042 | 0.113 | 0.16 | 0.04 | 0.287 |
| No. observations | 44 | 43 | 51 | 51 | 51 | 48 |
| | *B. Ten-year nominal government bond yields* | | | | | |
| Short | 2.271*** | 0.918*** | 0.862*** | 1.479** | 0.627*** | 1.792*** |
| | (0.512) | (0.301) | (0.143) | (0.620) | (0.228) | (0.426) |
| Long | 1.380** | 0.399 | 0.456*** | 0.396 | 0.506** | 0.579* |
| | (0.652) | (0.290) | (0.115) | (0.494) | (0.227) | (0.311) |
| $R^2$ | 0.169 | 0.306 | 0.4 | 0.19 | 0.296 | 0.415 |
| No. observations | 44 | 43 | 51 | 51 | 51 | 48 |

* Statistically significant at the 10 percent level.
** Statistically significant at the 5 percent level.
*** Statistically significant at the 1 percent level.
a. Sample period: 51 LSAP- and non-LSAP-related FOMC announcements between 25 November 2008 and 30 April 2014. The dependent variable is a two-day change bracketing an FOMC announcement in the two-year government bond yield (panel A) and the ten-year government bond yield (panel B) for the specified country. The entries labeled "Short" denote the NLLS estimates of the response coefficients to a U.S. policy-induced surprise in the two-year Treasury yield, while the entries labeled "Long" denote the estimates of the response coefficients to a policy-induced surprise in the ten-year Treasury yield that is orthogonal to the surprise in the two-year yield. All specifications include a constant (not reported). Heteroskedasticity-consistent asymptotic standard errors are reported in parentheses.

These results are consistent with the idea that bonds of certain countries provided a safe haven for investors in fixed-income markets during and following the global financial crisis. In particular, the results imply that the U.S. unconventional monetary policy surprises are transmitted to the long-term borrowing costs of advanced foreign economies whose long-term bonds are considered good substitutes for U.S. Treasury securities. Our findings also provide support for the relevance of a portfolio rebalancing channel of international spillovers. Because the impact of the long U.S. policy shock on the ten-year U.S. Treasury yield is normalized to one, the pass-through of this shock to the foreign government bond yields is, on average, about 50 percent.

In sum, when the ELB is binding, policy surprises to both the short- and long-term U.S. interest rates significantly influence the ten-year nominal government bond yields in advanced foreign economies. These findings indicate that the unconventional policy actions used by the FOMC during the current ELB period generate spillovers to the international markets for government bonds. The evidence on international spillovers to foreign yields from the two-dimensional policy surprise measure is consistent with the findings of Hausman and Wongswan (2011) and Bauer and Neely (2014), who measure the target and path U.S. monetary policy surprises and find that path surprises have significant and positive effects on foreign bond yields.

For the emerging market economies (table 6), the two- and ten-year government bond yields for some emerging market economies are also responsive to unanticipated changes in the stance of U.S. monetary policy stance during the unconventional U.S. monetary policy regime. The difference across countries is also evident. Mexico is the only country where movement in short-term interest rates are still in synchronization with the U.S. monetary policy actions, a result that underscores the tight economic linkages between the two economies, as well as the Mexican exchange rate policy. Brazil and Thailand are the other two countries where short-term interest rates respond to the short U.S. policy shock. As evidenced by panel B, our estimates imply that U.S. monetary policy announcements prompt significant movements in the long-term interest rates in emerging market economies. The ten-year bond yields for our sample of countries decline between 6 and 22 basis points in response to a 10 basis point policy-induced cut in the two-year U.S. Treasury yield. This result shows that the FOMC actions generate sizable movements in long-term interest rates for emerging economies. Moreover, a comparison of coefficient estimates in panel A relative to panel B again implies that during the unconventional period, a policy-induced cut in the two-year U.S. Treasury yield flattens the yield curve across the emerging market economies. This effect on the term spread is remarkably uniform across countries, varying from a low of 0.3 in the case of South Korea to a high of 0.76 in the case of Thailand. On average, a 10 basis point increase in the two-year U.S. Treasury implies a 5 basis point increase in the ten-year/two-year yield spread across the emerging market economies.

Although short U.S. policy shocks have significant effects on all emerging market long-term bond yields—and hence on the slope of the yield curve in these countries during the unconventional period—the transmission of the long U.S. policy shock to the ten-year bond yields of emerging market economies during this period is more varied. The long shock has a significant effect on longer-term bond yields in Brazil, South Korea, Singapore, and Thailand, but it has a less pronounced effect on long-term bond yields in India and Mexico.

## 2.3 Comparison of the Implied Pass-through

Finally, we calculate the implied pass-through of the short U.S. monetary policy surprises to long-term foreign interest rates. Table 7 presents the results: panel A shows the estimates of the implied pass-through of U.S. monetary policy short surprises to the foreign ten-year government bond yield of advanced foreign economies during the conventional policy regime; panel B contains the estimates for the advanced during the unconventional regime; and panel C presents the results for emerging economies during the unconventional regime.[8] The implied pass-through is calculated as the ratio of the regression coefficient $\beta_i(10)$ for the two-day change in the foreign ten-year bond yield to the regression coefficient $\lambda$ based on the intraday data as reported in table 2; we also report the *p* value of the test that the implied pass-through coefficient is equal to the response coefficient on the two-year government bond yields for the specific country.

According to our estimates, the level of the pass-through across countries ranges between 50 and 90 percent of the domestic pass-through to the ten-year U.S. Treasury yield for the advanced foreign economies, with the exception of Japan, where the implied pass-through is only about 20 percent. This suggests that the long end of the foreign yield curve is as responsive to the U.S. monetary policy short shock as the U.S. yield curve. Comparing panel A and panel B for the advanced foreign economies, one can see that the degree of international transmission of U.S. policy shocks to long-term foreign bond yields is very similar across the two policy regimes. As for the emerging market economies, the implied pass-through is also significant and ranges between 45 and 160 percent of that for the U.S. long-term interest rate. Lastly, although pass-through coefficient estimates are less than one, the test results indicate that for almost all the countries in our sample, one cannot reject one-for-one pass-through

8. Because the number of observations we have for both the two- and ten-year bond yields around the U.S. monetary policy action dates is very small, we do not compute the implied pass-through for the emerging market economies during the U.S. conventional monetary policy regime.

at the long end of the yield curve, in response to an unanticipated change in the U.S. monetary policy stance engineered vis-à-vis the short end of the yield curve.

**Table 7: Pass-through of Short U.S. Policy Surprise to Ten-Year Foreign Bond Yields[a]**

| *Period* | *Australia* | *Canada* | *Germany* | *Italy* | *Japan* | *United Kingdom* |
|---|---|---|---|---|---|---|
| | | | *A. Conventional monetary policy[b]* | | | |
| | 0.907*** | 0.816*** | 0.492*** | 0.632*** | 0.192* | 0.764* |
| | (0.269) | (0.205) | (0.167) | (0.187) | (0.112) | (0.412) |
| *Pr > W* | 0.729 | 0.371 | 0.002 | 0.049 | 0 | 0.567 |
| | | | *B. Unconventional monetary policy[c]* | | | |
| | 0.955*** | 0.620*** | 0.508*** | 0.743*** | 0.158** | 0.633** |
| | (0.231) | (0.158) | (0.186) | (0.266) | (0.07) | (0.302) |
| *Pr > W* | 0.846 | 0.016 | 0.008 | 0.334 | 0 | 0.225 |
| | *Brazil* | *India* | *South Korea* | *Mexico* | *Singapore* | *Thailand* |
| | 1.599*** | 0.647** | 0.613*** | 1.051* | 0.446*** | 1.249*** |
| | (0.519) | (0.313) | (0.129) | (0.605) | (0.172) | (0.433) |
| *Pr > W* | 0.248 | 0.26 | 0.003 | 0.932 | 0.001 | 0.566 |

* Statistically significant at the 10 percent level.
** Statistically significant at the 5 percent level.
*** Statistically significant at the 1 percent level.
a. The entries in the table denote the estimates of the implied pass-through of short U.S. monetary policy surprises to the ten-year government bond yield for advanced economies during the conventional policy regime (panel A) and the unconventional policy regime (panel B), as well as for emerging economies during the unconventional policy regime (panel C). Heteroskedasticity-consistent asymptotic standard errors are reported in parentheses. Pr > W denotes the p value of the test of the null hypothesis that the implied pass-through coefficient is equal to one.
b. Sample period: 143 FOMC announcements between Feb-06-1992 and Nov-24-2008.
c. Sample period: 51 LSAP- and non-LSAP-related FOMC announcements between Nov-25-2008 and Apr-30-2014.

## 3. Conclusion

This paper compares the impact of U.S. conventional monetary policy on foreign government bond yields with that of the unconventional measures employed after the target federal funds rate hit the effective lower bound. For this latter period, we identify two U.S. monetary policy surprises: changes in the two-year U.S. Treasury yield around FOMC announcements and changes in the ten-year U.S. Treasury yield that are orthogonal to those of the two-year yield. We find that the U.S. monetary policy has a pronounced effect on both the short- and long-term interest rates in advanced foreign countries. An expansionary U.S. monetary policy steepens the foreign yield curve during the conventional period and flattens the foreign yield curve during the unconventional period. While there is a significant degree of heterogeneity across advanced and emerging economies, our estimates of U.S. monetary policy pass-through imply that the average international spillover effect of U.S. unconventional policy is comparable to that of conventional policy.

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# U.S. Monetary Spillovers to Latin America: The Role of Long-term Interest Rates

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The economic situation in emerging markets has deteriorated in recent years. Perhaps the single most important event, especially for Latin America, has been the end of the so called commodity supercycle, which intensified with the collapse in oil prices in late 2014. But the trend of weaker currencies and rising inflation can, in many cases, be traced back at least a year earlier to the taper talk episode in May 2013. This event was a stark reminder of the interdependence of monetary conditions in small countries with core financial centers, the fickle nature of global liquidity, and the consequences of its evaporation for developing economies.

Since that episode, the U.S. Federal Reserve's actual liftoff date has been on the short list of concerns of virtually every central bank in Latin America—and for good reasons. There is now plenty of evidence showing a link between global liquidity push factors and their consequences on key variables in emerging market economies.[1] The central narrative in this literature is that lax monetary conditions in

The views in this paper are those of the authors and do not represent the views of the Central Bank of Chile. We thank Constantino Hevia for helpful comments and Stefano Banfi for excellent research assistance.

1. See Rey (2015), Bruno and Shin (2015), Ahmed and Zlate (2014), and Obstfeld (2015).

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core zones, such as the United States and the euro area, incentivize more risk taking in the form of maturity extension within the risk-free yield curve (indeed, this is the explicit mechanism by which quantitative easing is supposed to work),[2] or through portfolio rebalancing of fixed-income assets with different default risk, such as a substitution from treasuries to investment-grade bonds, mortgages, or even emerging fixed-income and equity markets. The obvious concern is that this mechanism works in reverse, too, and a Fed tightening might produce a further depreciation of exchange rates when most countries in the region are already deviating from their explicit inflation targets.

This paper contributes to the literature by studying spillovers from monetary conditions in the United States to the five largest economies in Latin America (with reliable economic indicators): namely, Brazil, Chile, Colombia, Mexico, and Peru. One way to go about this question is through a country-by-country analysis. However, this approach makes it hard to convey a coherent narrative for the region without falling into idiosyncrasies of each economy. It also misses the potentially rich interactions between different countries in the region—a potentially important omission, given the close trade and financial ties between them.

To tackle these issues we propose a restricted factor-augmented vector autoregressive (FAVAR) approach that includes an exogenous block of information. Under this approach, we first compute a set of factors that are representative of Latin America's performance on different dimensions. We include here an unemployment factor as a measure of real activity, an inflation factor, an exchange rate factor, and a stock market factor. We then model the dynamics of the estimated factors in a vector autoregressive framework including an exogenous block that contains information about U.S. interest rates. This framework allows us to describe the effects of shocks to interest rates in the United States on a particular variable in a specific Latin America country by tracing the dynamics of the associated factor. This is a suitable methodology for our purposes, as it allows us to assess spillovers from U.S. monetary conditions to the region in a parsimonious way, while at the same time controlling the dimensionality of the problem.

We find that for our sample period (2003–15), there are significant spillovers from U.S. monetary conditions into Latin America.

2. See Hanson and Stein (2015), Greenwood and Vayanos (2014), and Krishnamurthy and Vissing-Jorgensen (2011).

Specifically, an unexpected increase in U.S. ten-year yields is associated with higher unemployment, higher inflation, a fall (depreciation) of local currencies vis-à-vis the U.S. dollar, and a drop in stock markets. These effects hold for most Latin American countries. Mexico is an important exception, probably due to its much closer trade ties and proximity to the United States.

One shortcoming of the data is the lack of systematic interest rate information for Latin America in the earlier part of the sample. To assess the spillover effects into monetary conditions in Latin America, we proceed in two parts. First, we estimate an alternative FAVAR using a shorter subsample beginning in January 2009, after which long-term (ten-year) interest rate data are available for all countries. In this exercise, we find significant spillover effects of long-term interest rates in the United States to those in Latin America, but the effects on the other factors are generally insignificant. One possibility is that economic fundamentals in the region—affected by the commodity super-cycle, among other forces—largely diverged from the United States after the global financial crisis, which could explain why interest rate similarities at the long end of the yield curve might not show up elsewhere.[3]

Second, we focus on two specific countries with a long data series on the yield curve: namely, Chile and Mexico. In this exercise, we study U.S. interest rate spillovers using weekly data from financial variables, including exchange rates, stock returns, and bond yields. Moreover, we decompose long-term rates into a risk neutral component that captures the future evolution of short-term interest rates and a term premium component that reflects compensation for risk.[4] We carry out the same exercise using two subsamples, one before the zero lower bound (ZLB) period (2003–08) and during the ZLB (2009–15). The results confirm that U.S. interest rate spillovers after the ZLB—that is, in the period associated with the global financial crisis—are concentrated on bond yields, particularly on the term premium component. The effects on exchange rates are much smaller in the post-crisis period than in the earlier subsample, while the impact on stock returns is not statistically significant.

3. This result is consistent with Gilchrist, Yue, and Zakrajsek (2015), who show that the impact of U.S. monetary policy shocks on exchange rates in emerging market economies largely diminishes after 2009.

4. We perform the term structure decomposition following the methodology described in Adrian, Crump, and Moench (2013).

There is a growing literature exploring the transmission of global liquidity conditions to emerging market economies, including the papers cited above. A number of papers on fixed-income and yield curve modeling quantitatively evaluate the effect of different monetary policy measures in core economies (typically the U.S. Federal Reserve's conventional and unconventional monetary policy measures) on domestic interest rates at different maturities, as well as the international spillover of these measures into interest rates abroad.[5] Our contribution to this literature is twofold. First, we focus specifically on Latin America, which shares similarities with other emerging market economies but also displays important differences in economic structure, in particular the reliance on commodities. Second, we rely on an identification approach (the FAVAR model) that allows us to trace the effects of U.S. interest rate spillovers on several variables (including activity, prices, and financial markets) while keeping the dimensionality of the system in check. One potentially important result is that while U.S. interest rates have generated large spillovers into long-term rates in Latin America, the effects on other financial and real variables are weaker in the post-2009 sample.

The rest of the paper is structured as follows. In section 1, we document a significant degree of comovement within Latin America on a number of dimensions and construct our set of factors. Section 2 describes the proposed restricted FAVAR model, together with its estimation, and reports the main results regarding the spillover effects of U.S. interest rate shocks under different specifications and sample periods. In section 3, we perform the individual case studies of Chile and Mexico with VAR models, at a weekly frequency, based on financial variables only. Section 5 concludes.

## 1. Comovements in Latin American Economies

Latin American countries have strong commercial ties not only with the U.S. economy, but also with each other. Imbs (2004) and Ductor and Leiva-León (2016) show that trade is a key driver of the comovement among the major world economies, and Latin American countries are

5. See Gagnon and others (2011), Christensen and Rudebusch (2012), Hellerstein (2011), Bauer and Rudebusch (2014), Gilchrist, Yue, and Zakrajsek (2015), Hoffman and Takáts (2015), BIS (2015), Miyajima, Mohanty, and Yetman (2014), and Albagli and others (2015).

not the exception. Our analysis focuses on the largest economies of Latin America: namely, Brazil, Chile, Colombia, Mexico, and Peru. We exclude Argentina from the sample due to the unreliability of official figures, particularly for inflation, the exchange rate, and output data.[6]

This section focuses on comovements among countries along four dimensions of economic fundamentals. First, we use the unemployment rate as a measure of the business cycle, which has the advantages of being available at a monthly frequency and displaying less volatility than industrial production. Second, as a measure of inflation dynamics, we use the year-on-year growth rate of the consumer price index (CPI). Third, we use the nominal exchange rate with respect to the U.S. dollar (in levels). Fourth, we include information about the evolution of financial markets by using monthly stock returns. To abstract from hyperinflation and highly volatile periods, we focus on a sample of data that extends from January 2003 until August 2015.

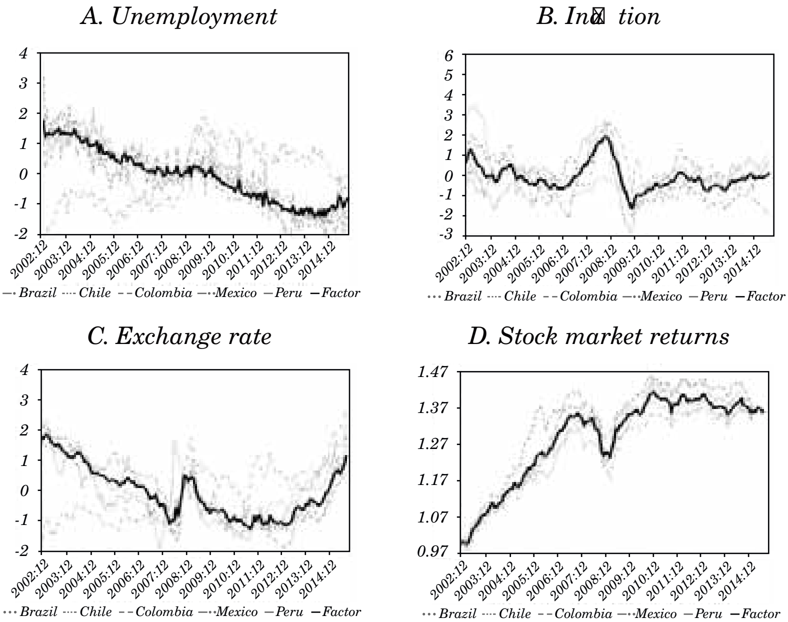
To assess the degree of comovement in unemployment rates, denoted by $u_{i,t}$ , for $i$= Brazil, Chile, Colombia, Mexico, and Peru, we rely on factor analysis and extract the first principal component, $f_t^u$, such that country-specific unemployment rates can be decomposed into common and an idiosyncratic components:

$$u_{i,t} = \lambda_i^u f_t^u + \varepsilon_{i,t}, \tag{1}$$

where $\lambda_i^u$ are the estimated factor loadings, and the common component, $f_t^u$, can be interpreted as a measure of Latin American unemployment. Panel A of figure 1 plots the country-specific unemployment rates, along with the Latin American unemployment. All the data in figure 1 have been standardized to facilitate comparison. The figure shows a strong comovement between Brazil, Chile, Colombia, and Peru, following a decreasing trend from the early 2000s through 2014, with a moderate increase in the last year of the sample. The only significant exception is Mexico, which experienced an important increase in the level of unemployment rate in 2009. This increase is not hard to rationalize due to the close commercial ties with the U.S. economy and the recession that took place at that time.

6. In fact, the IMF took the unprecedented step in February 2013 of censuring this member, encouraging the country to improve its efforts to meet the IMF standards for inflation and GDP data.

**Figure 1. Comovement in Economic Fundamentals in Latin America**




The same procedure is used to extract the common component among inflation rates, $\pi_{i,t}$, to obtain a measure of Latin American inflation, $f_t^{\pi}$ (panel B of figure 1). Inflation across countries has remained relatively stable, with the exception of the 2008–09 period, when inflationary pressures increased temporarily. In the more recent period, inflation has started to pick up, related in most cases to the strong currency depreciation. Again, an important exception is Mexico, where recent inflation has receded even below their expected target.[7]

Panel C shows the country-specific exchange rates with respect to the dollar, $\chi_{i,t}$ , along with a Latin American exchange rate measure, $f_t^{\chi}$. Similar to the case of unemployment, exchange rates demonstrate a

7. According to the Bank of Mexico's latest (June) inflation report (Bank of Mexico, 2015), inflation remains low mostly due to the lack of wage pressures in the context of a relatively weak labor market, despite the exchange rate pass-through pressures due to the recent depreciation of the currency.

significant degree of comovement, with a continuous increase starting in 2013 in response to the U.S. Federal Reserve's taper talk and the global strengthening of the U.S. dollar. Finally, although we extract the common factor of monthly stock market returns across countries, $p_{i,t}$, we recover the level of stock market data to facilitate interpretation and plot it along with the Latin American index of stock markets, $f_t^p$, in in Panel D. The chart shows that the continuous growth of Latin American stock returns until 2007, followed by a significant downturn in 2008–09, associated with the recession in United States. This is followed by a quick recovery up to 2011, after which the stock market factor has been relatively flat, with some deterioration in the las few months of the series.

We performed a variance decomposition analysis to identify the share of each country's economic variables that is explained by the respective economic factor. The results are reported in table 1. For example, the first column of the table reports the share of unemployment rate volatility in each country that is explained by the unemployment factor alone. The unemployment and exchange rate factors explain a largest share of the country-specific series in most cases (with the exception of Mexico). There is somewhat less comovement for inflation, although the common factor is still significant in some cases.

Overall, this section provides evidence of strong comovement in Latin American economic fundamentals. Brazil, Chile, Colombia, and Peru experience similar fluctuations in the unemployment rate, inflation, exchange rates, and stock returns. Mexico, on the other hand, shows some important differences, in particular in unemployment and inflation, which can be attributed to its proximity to and strong integration with the U.S. economy.

**Table 1. Fraction of Country-Specific Variables Explained by Common Factors**

| *Country* | *Unemployment* | *Inflation* | *Exchange rate* | *Stock market returns* |
|---|---|---|---|---|
| Brazil | 0.93 | 0.16 | 0.79 | 0.73 |
| Chile | 0.74 | 0.53 | 0.83 | 0.58 |
| Colombia | 0.82 | 0.64 | 0.96 | 0.52 |
| Mexico | 0.44 | 0.35 | 0.08 | 0.71 |
| Peru | 0.82 | 0.58 | 0.84 | 0.64 |

## 2. Assessing International Spillovers

We now study the spillover effects of monetary conditions in the United States to Latin American countries. As highlighted in the introduction, several recent papers study the implications for emerging market economies of global liquidity conditions more generally and interest rate spillovers specifically. Our central contribution relies on the sample choice (namely, Latin America) and the identification strategy based on the FAVAR approach. Several papers base their identification on event study analysis.[8] However, a VAR-based approach identifies U.S. interest rate shocks from the data by recovering the structural innovations in U.S. interest rates. The FAVAR approach, in particular, allows us to study different Latin American countries jointly, with potentially important interaction effects, while at the same time keeping the dimensionality of the problem under control. Also, as documented in the previous section, the degree of comovement between most Latin American countries is significant along most economic dimensions considered (with the exception of Mexico), which makes the chosen methodology particularly informative.

### 2.1 The Model

We use FAVAR models, initially proposed by Bernanke, Boivin, and Eliasz (2005), to assess the responsiveness of the country-specific economic fundamentals to shocks in U.S. interest rates. This econometric framework allows us to tackle the high dimensionality of the problem and to elegantly relate the two blocks of information: namely, Latin American economic developments and U.S. monetary conditions. In doing so, we also need to deal with the exogeneity of the U.S. block to ensure identification of the spillover effects. We therefore impose some restrictions in the coefficients of the model, in line with Canova (2005).

The proposed FAVAR model with an exogenous block is defined as follows:

$$\begin{bmatrix} X_t \\ r^h_{US,t} \end{bmatrix} = \begin{bmatrix} \Lambda^f & \Lambda^r \\ 0 & I \end{bmatrix} \begin{bmatrix} F_t \\ r^h_{US,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \end{bmatrix}, \ \varepsilon_t : N(0,\Omega) \qquad (2)$$

8. See Hanson and Stein (2015) for U.S. monetary policy spillovers into U.S. long-term yields; and Gilchrist, Yue, and Zakrajsek (2015), and Albagli and others (2015) for the case of emerging market economies.

and

$$\begin{bmatrix} F_t \\ r_{US,t}^h \end{bmatrix} = \begin{bmatrix} \Psi_f(L) & \Psi_{f,US}(L) \\ 0 & \Psi_{US}(L) \end{bmatrix} \begin{bmatrix} F_{t-1} \\ r_{US,t-1}^h \end{bmatrix} + \begin{bmatrix} e_{f,t} \\ e_{US,t} \end{bmatrix}, e_t : N(0,\Sigma), \qquad (3)$$

where the first block of information consists in the economic fundamentals of Latin American economies contained in $\boxtimes_t = (Y_{BRA,t}, Y_{CHI,t}, Y_{COL,t}, Y_{ME\boxtimes,t}, Y_{PER,t})'$. Accordingly, each element of $\boxtimes_t$ is given by $Y_{i,t}\boxtimes(u_{i,t}, \pi_{i,t}, \chi_{i,t}, p_{i,t})'$. The information contained in $\boxtimes_t$ can be appropriately summarized in a small set of factors collected in $F_t=(f_t^u, f_t^\pi, f_t^\chi, f_t^p)'$. We use the factors obtained earlier, which provides two main advantages with respect to unobserved factor models. First, assuming that the factors are observed reduces the estimation uncertainty of the model substantially. Second, this modeling strategy allows us to provide identification and a clear interpretation of what each factor represents. This is usually not well achieved when several factors are extracted from a set of data without imposing any identification restriction.

The second block of information, $r_{US,t}^h$, captures the U.S. monetary conditions, proxied by the U.S. bond yield at horizon $h$. To ensure proper identification of the U.S. interest rate shocks, we impose some constraints on the coefficients of the VAR in equation (3). Specifically, current emerging market dynamics depend on their lagged values and past U.S. developments, while current U.S. dynamics depend only on their lagged values. We also assume a variance-covariance matrix of the VAR disturbances with a block diagonal structure, $\Sigma$ = blockdiag $\{\Sigma_f, \Sigma_{US}\}$, as in Canova (2005).

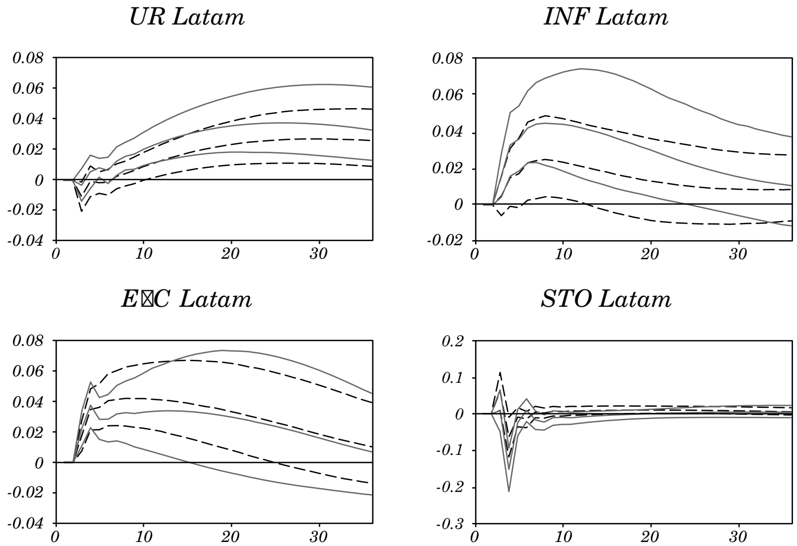
The model is identified with a recursive (Cholesky) structure. We adopt the following ordering for the variables in $F_t$: the unemployment rate factor, followed by the inflation rate factor, the exchange rate factor, and finally the stock market return factor. This ordering is consistent with a criteria of placing slow moving variables first (that is, activity and goods prices) and letting the fast-moving (financial) variables be affected by the previous ones contemporaneously.

The estimation of the model is based on Bayesian methods to provide robust inferences on the parameter estimates. We use the Gibbs sampler to compute draws of the parameter estimates of the FAVAR model and simulate their posterior distribution of parameters and impulse responses. For further details on the estimation method, see Bernanke, Boivin, and Eliasz (2005).

## 2.2 U.S. Monetary Spillovers: The Effects of Interest Rate Shocks

We now quantify and compare the effect of shocks in short- and long-term U.S. bond yields on Latin American economies. We use two FAVAR models with different information on the United States. One model includes only information about the one-year bond yield, $r_{US,t}^{h=1}$, in the U.S. block, while the second model includes only information on the ten-year bond yield, $r_{US,t}^{h=10}$, in the same block. Figure 2 reports the responses of each of the factors to a 25-basis-point shock in the short- and long-term rates. Because we have standardized all the series in the factor analysis, the unit of measure on the *y* axis is the number of standard deviations from the mean.

**Figure 2. Effect of a Shoc⊠ to ⊠bse rved U.S. Short- and Long-term Bond Yields on Latin American Factors**




The dashed (solid) lines plot the responses to a shock in the one-year (ten-year) observed bond yield. In all figures, the central line corresponds to the response according to the median draw of the simulation, while the lower and upper lines correspond to the tenth and ninetieth percentile, respectively.

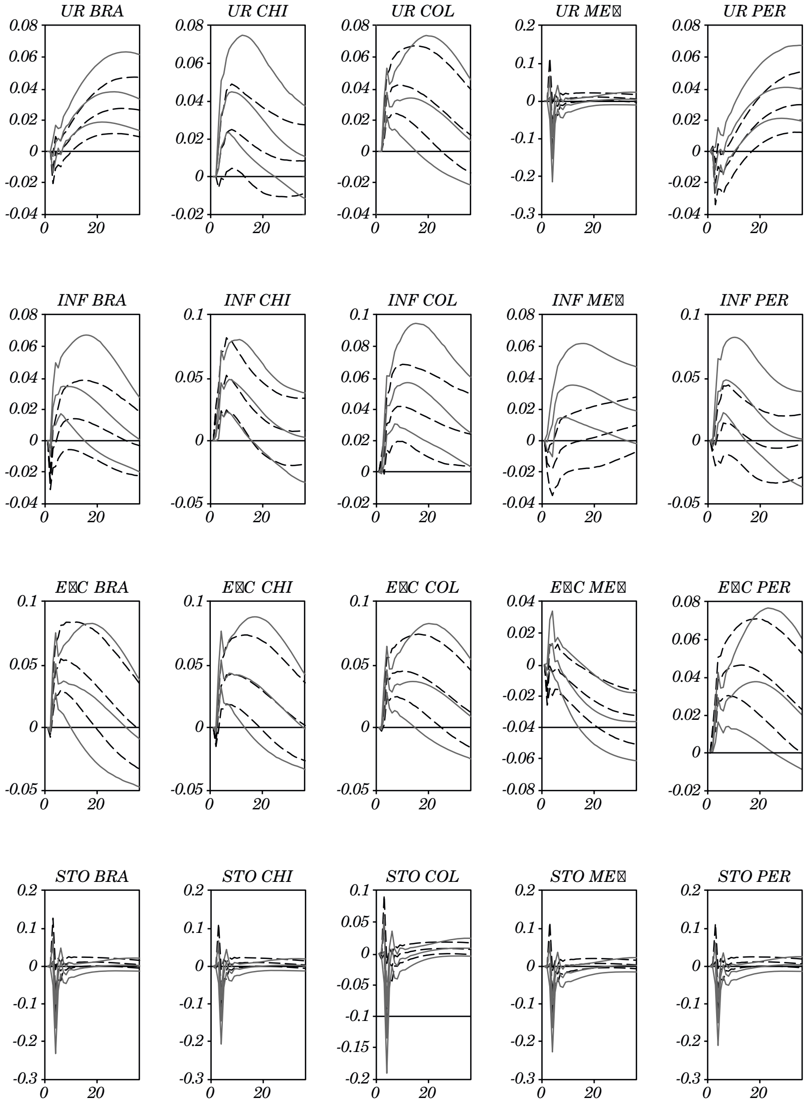
The figure shows that a shock in U.S. short-term rates has a rather small impact on the unemployment factor, the inflation factor, and the stock market factor. The long-term rate, on the other hand, has a significant impact on all factors, in the direction one would expect: higher unemployment, a more depreciated exchange rate, higher inflation, and lower stock market returns.

Figure 3 complements the analysis by showing the response of these variables for each individual country, according to the factor loadings estimated in equation (2). Consistent with figure 2, we find that short-term rate shocks almost always have a negligible effect on Latin American economies. However, these economies are highly affected by long-term rate shocks. Specifically, an increase in the long-term U.S. rate increases unemployment rates, inflation, and exchange rates and decreases stock returns in Brazil, Chile, Colombia, and Peru. Indeed, the direction and magnitude of the responses across countries reflect the high degree of comovement in Latin American economies, as documented above. Mexico exhibits a clearly different pattern of responses in some key variables. As mentioned before, this economy experienced a break that is mainly associated with the global financial crisis and the economy's sensitivity to U.S. economic conditions.

## 2.3 Spillovers at the Zero Lower Bound

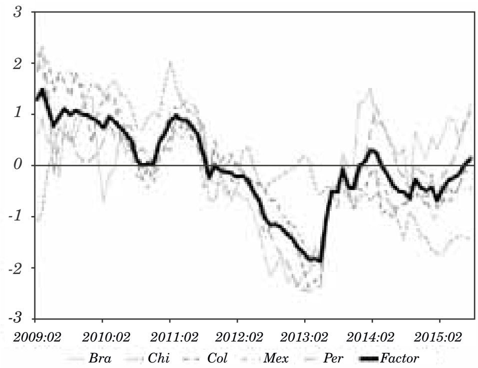
We now present a related exercise by estimating a similar FAVAR model but for a period starting in January 2009. This exercise has two main motivations. First, the conduit of monetary policy and the level of interest rates have truly been exceptional in the period after the global financial crisis, so 2009 seems like a natural break point to test for differential effects. Second, beginning in 2009 we have systematic, reliable data on long-term interest rates for our complete set of Latin American economies, which allows us to enlarge our previous FAVAR model by including information about interest rates and to extract the corresponding factor into the otherwise unchanged specification (equations 1–3). In our baseline specification, the interest rate factor, which is plotted in figure 4, comes in last among the Latin American factors in equation (3). Figure 5 reports the responses of each of the factors to a 25-basis-point shock in the long-term rate for the post-2009 sample, while figure 6 plots the responses on each individual country.

## Figure 3. Effect to a Shock to Observed U.S. Short- and Long-Term Bond Yields on Latin American Factors, by Country

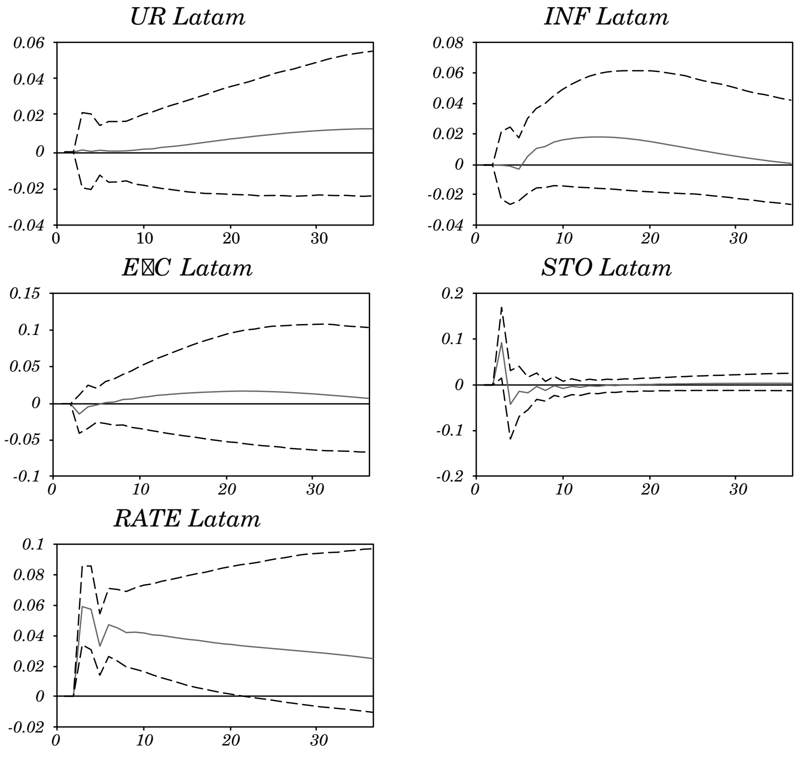



The dashed (solid) lines plot the responses to a shock in observed one-year (ten-year) bond yields. In all figures, the central line corresponds to the response according to the median draw of the simulation, while the lower and upper lines correspond to the tenth and ninetieth percentile, respectively.

## Figure 4. Long-Term Interest Rates in Latin America

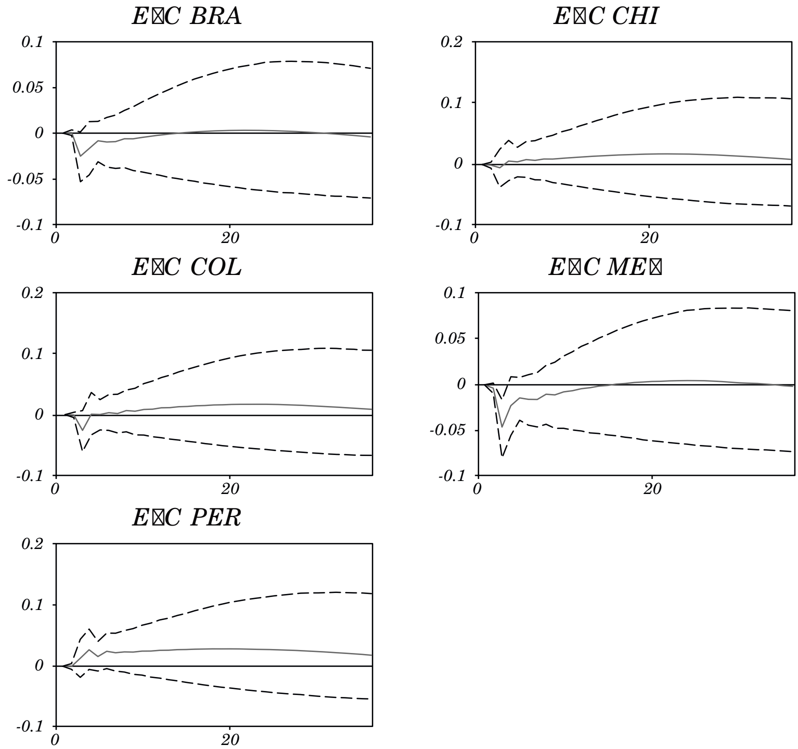



## Figure 5. Effect of a Shock to Observed U.S. Ten-Year Bond Yields on Latin American Factors at the Zero Lower Bound




The lines plot the responses to a shock in the observed ten-year bond yield during the zero lower bound. In all figures, the central lines corresponds to the response according to the median draw of the simulation, while the lower and upper lines correspond to the tenth and ninetieth percentile, respectively.

**Figure 6. Effect of a Shock to Observed U.S. Ten-Year Bond Yields on Latin American Factors at the Zero Lower Bound**




The lines plot the responses to a shock in the observed ten-year bond yield during the zero lower bound. In all figures, the central lines corresponds to the response according to the median draw of the simulation, while the lower and upper lines correspond to the tenth and ninetieth percentile, respectively.

These figures reveal interesting patterns. First and foremost, U.S. interest rate shocks appear to have little to no significant effects on unemployment, inflation, and exchange rates, as compared to the full-sample specification. Based on the median draw of response simulations, the effect has the intuitive sign, but they are generally not significant at the confidence intervals considered. Second, the impact of U.S. long-term interest rates is in general highly significant, both for the Latin American interest rate factor and for individual interest rates in each country. Even Mexico responds in the same direction, breaking the orthogonal behavior displayed on other dimensions with respect to Latin America.

## 3. Spillovers Through Financial Markets

The above analysis suggests that U.S. interest rates have played a role in activity, prices, and financial variables in Latin America over the past 12 years. Moreover, our results indicate that in the post-2009 period, U.S. long-term interest rate shocks have a significant impact on Latin American long-term rates, but a generally insignificant impact on other variables over this shorter subsample. One potential objection to our identification strategy, however, is that it might be too restrictive to assume dynamics associated with lags of one month or more when it comes to financial variables. Moreover, the sample size in monthly frequency after 2009 is rather limited, which could also cast doubt on our results.

To deal with these issues, we perform a country-specific analysis for the cases of Chile and Mexico, where good yield curve information exists for a long sample. In these case studies, we focus on high-frequency data, namely, weekly interest rates, exchange rates, and stock market returns. More specifically, for each country we run a VAR that includes an exogenous bloc for the U.S. interest rate, as before. Among the endogenous domestic variables, we include the level of the nominal exchange rate, the weekly stock return, and a measure of interest rates, in that order. We are thus estimating the restricted VAR specified in equation (3) with new data. The availability of high-frequency data allows us to enlarge the information set in our sample size and also reduces concerns about results being driven by a particular ordering.

We also dig deeper into the specific channels that drive interest rate spillovers from the United States into these countries. Specifically, we perform a decomposition of overall domestic yields into a risk-neutral component, which captures the expected evolution of short-term rates, and a term-premium component. Hence, we run three separate VAR models, using the respective measure of interest rates (one with overall yields, one with the risk-neutral component, and one with the term-premium component). In all cases, the U.S. interest rate variable corresponds to the observed ten-year Treasury yield.

The litter contains a few different approaches for decomposing yields into expected rates and compensation for risk. We follow the methodology advocated by Adrian, Crump, and Moench (2013), which relies only on yield curve data for its estimation, making it suitable for constructing high-frequency variables. Their methodology exploits the log excess holding return predictability showed in empirical studies, such as Cochrane and Piazzesi (2005).[9] Based on that idea, Adrian, Crump, and Moench (2013) propose a simple methodology to construct market prices of risk into an affine model consistent with the predictability of excess bond returns.[10]

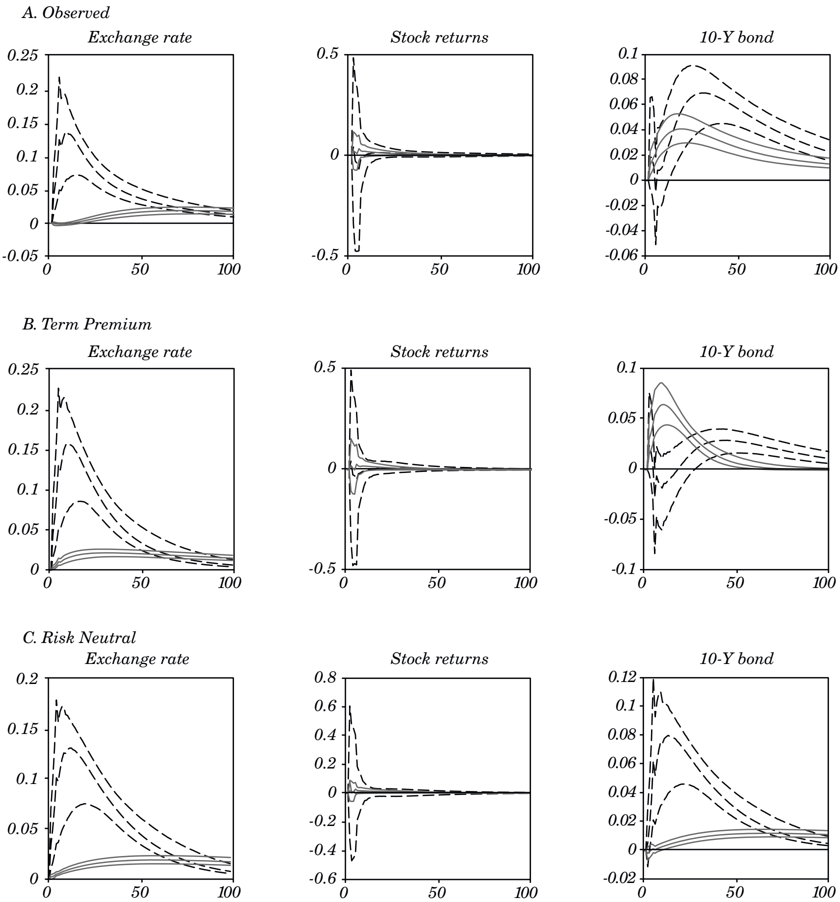
## 3.1 Country-Specific Analysis

Figure 7 plots the impulse response functions to a 25-basis-point increase in U.S. long-term rates for the case of Chile for both the first half of the sample (prior to 2009) and the sample after 2009. The figure includes the results of the VAR for overall yields as a measure of interest rates in Chile, as well as the term-premium component and risk-neutral component. The figure largely confirms our previous results. First, interest-rate shocks have a relatively large impact on exchange rates prior to 2009 (an increase in rates induces a depreciation of the Chilean peso), but a smaller (though still significant) impact during the ZLB period. The impact on stock markets, however, is insignificant in both samples. This result is somewhat consistent with Gilchrist, Yue, and Zakrajsek (2015), who document a significant spillover effect of U.S. interest rates on stock markets before, but not during, the ZLB period.

Second, the impact on Chilean long-term rates is significant in both samples, although in the ZLB period the spillover seems to be acting faster. Moreover, the effect is largely concentrated in the term-premium component after 2009, whereas the bulk of the response is due to the risk-neutral component in the earlier period. This result is consistent with Albagli and others (2015), who use an identification strategy based on event studies to show that spillovers from U.S. monetary policy are concentrated in the term-premium component for a larger sample of emerging countries post-2009. This result probably reflects the fact that activity and monetary policy decisions in Chile have been largely decoupled from the United States after the global financial crisis, so changes in U.S. interest rates are unlikely to affect expectations for future short-term rates (the signaling channel) in Chile. On the other hand, as documented in the papers cited above, there is mounting evidence that U.S. monetary conditions are strongly associated with global liquidity factors, which in turn affect the flow of capital into emerging market economies. The evidence presented here for the case of Chile seems to confirm this notion.

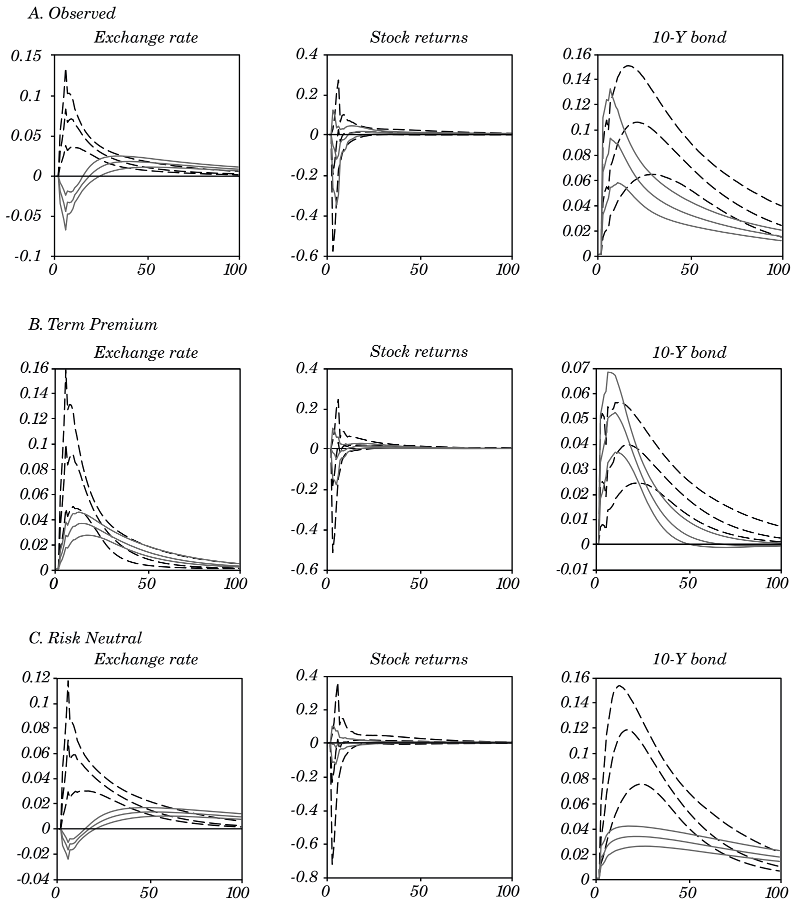
9. See Gürkaynak and Wright (2012) for a comprehensive review of this literature.
10. For more details on the methodology, see Adrian, Crump, and Moench (2013).

## Figure 7. Effect of a Shock to Observed U.S. Ten-Year Bond Yields Before and at the Zero Lower Bound in Chile: Weekly Frequency




The dashed (solid) lines plot the responses to a shock before (at) the zero lower bound.

## Figure 8. Effect of a Shock to Observed U.S. Ten-Year Bond Yields Before and at the Zero Lower Bound in Mexico: Weekly Frequency




The dashed (solid) lines plot the responses to a shock before (at) the zero lower bound.

Figure 8 plots the impulse response functions for the case of Mexico. The effects on the exchange rate change sign between the two subsamples. An increase in U.S. interest rates is associated with a depreciation of the Mexican peso before the ZLB and an appreciation during the ZLB episode. One possible explanation for this phenomenon is that economic activity in Mexico is strongly linked to the United States, particularly after the global financial crisis. It is plausible then that an increase in U.S. yields caused by expectations of stronger economic activity in that country could also reflect good news for the Mexican economy, thus appreciating the currency. On the other hand, as was the case for Chile, there is no significant impact of U.S. interest rates on the Mexican stock market in either period.

With regard to the spillover of U.S. interest rates into Mexican long-term rates, the figure reveals that these spillovers are significantly larger than in the case of Chile, which is intuitive given Mexico's much closer economic interaction with the U.S. economy. Indeed, spillovers are of similar magnitude in both subsamples. However, as was the case with Chile, the spillover effect also seems to be more biased toward the term-premium component during the ZLB, while the risk-neutral component seems to dominate in the earlier part of the sample.

To get a better understanding of the magnitudes of U.S. interest rates spillovers, table 2 computes the pass-through coefficients. These are calculated as the ratio between the cumulative response in domestic bond yields (and the risk-neutral and term-premium components) divided by the cumulative response of the U.S. long-term interest rate. The cumulative pass-through to Chile is close to 0.5 in both samples at a one-year horizon, but at two years the pass-through increases to between 0.8 and 1.0. The component-level analysis shows that, as expected, the pass-through to risk-neutral rates falls drastically during the ZLB subsample, while the pass through to the term premium increases substantially.

In the case of Mexico, the pass-through to overall yields is larger than for Chile at all horizons. Consistent with figure 8, the pass-through to the risk-neutral component is significantly diminished after 2009, while for the term premium it increases somewhat, especially for a one-year horizon.

Overall, the evidence presented in this section corroborates the analysis presented above. We find significant spillover effects of U.S. interest rates to domestic bond yields, which extend for a considerable period of time and which are larger in the case of Mexico. Moreover, we see a more significant role of the term-premium channel during the ZLB episode and a more muted response of risk-neutral rates. This evidence supports the view that spillovers to emerging market economies via the risk-taking channel have been more prevalent in the environment of exceptionally low interest rates that has characterized fixed-income markets after the global financial crisis.

**Table 2. Pass-through of U.S. Interest Rates to Bond Yields, by Component**

| *Type of rate and period* | *Chile* | *Mexico* |
|---|---|---|
| *Observed* | | |
| 1 year before ZLB | 0.56 | 1.05 |
| 2 years before ZLB | 1.01 | 1.55 |
| 1 year at ZLB | 0.56 | 0.96 |
| 2 year at ZLB | 0.81 | 1.34 |
| *Risk neutral* | | |
| 1 year before ZLB | 0.64 | 1.02 |
| 2 years before ZLB | 0.8 | 1.3 |
| 1 year at ZLB | 0.12 | 0.5 |
| 2 year at ZLB | 0.28 | 0.82 |
| *Term premium* | | |
| 1 year before ZLB | 0.13 | 0.36 |
| 2 years before ZLB | 0.36 | 0.45 |
| 1 year at ZLB | 0.57 | 0.46 |
| 2 year at ZLB | 0.58 | 0.46 |

## ⊠. Concluding Remarks

This paper assesses the spillover effects of unexpected increases in U.S. bonds interest rates on the economic fundamentals of Latin American economies. Four main results arise from the analysis. First, we find evidence of strong comovement in the economic fundamentals of Brazil, Chile, Colombia, and Peru. Mexico shows some important differences, which can be attributed to its proximity to and strong integration with the U.S. economy.

Second, we find that short-term rate shocks usually have a negligible effect on Latin American economies. However, these economies are highly affected by long-term rate shocks. Specifically, an unexpected increase in the U.S. long-term rate increases unemployment, inflation, and exchange rates and decreases stock market returns in most of Latin American economies. Mexico exhibits a clearly different pattern of responses in some key variables.

Third, when focusing on the zero lower bound period, we find significant spillover effects of long-term interest rates in the United States to those in Latin America, but also find generally insignificant effects on the other factors. One explanation for this is that after the global financial crisis, economic fundamentals in the region have largely diverged from those in the United States.

Finally, our analysis points to significant spillover effects of U.S. interest rates to domestic bond yields. Moreover, the term premium seems to play a key role during the zero lower bound period, while risk-neutral rates show a more muted response. This evidence supports the view that spillovers to emerging economies via the risk-taking channel have been more prevalent in the environment of exceptionally low interest rates that has characterized fixed-income markets after the global financial crisis.

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