

Exchange Rate Policies at the Zero Lower Bound*

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Abstract

This paper studies how the Central Bank of a small open economy achieves an exchange rate objective in an environment that features a zero lower bound (ZLB) constraint on nominal interest rates and limits to arbitrage in international capital markets. If the nominal interest rate that is consistent with interest parity is positive, the Central Bank can achieve its exchange rate objective while giving up its monetary independence, a well known result in international finance. However, if the nominal interest rate consistent with interest rate parity is negative, the pursue of an exchange rate objective necessarily results in zero nominal interest rates, deviations from interest rate parity, capital inflows, and welfare costs associated with the accumulation of foreign reserves by the Central Bank. We characterize how these costs vary with the economic environment, and discuss situations in which these interventions are optimal from the point of view of a small open economy. We finally show that the recent break-downs in covered interest rate parity documented in the literature are associated to large foreign exchange interventions carried out by Central Banks operating at the ZLB.

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1 Introduction

In the aftermath of the global financial crisis of 2008, many central banks in advanced economies have faced sustained appreciation pressures on their exchange rates. The Swiss franc, for example, went from trading at roughly 1.6 Francs per Euro prior to 2008, to 1.05 at the beginning of 2011. For a variety of reasons, Central Banks in these countries have adopted policies geared to resist or delay appreciation, i.e. they have tried to keep their currencies weak for some time. The Swiss Central Bank, for example, has maintained a peg of the Swiss Franc with the Euro for the period 2011-2014, before letting the Swiss Franc appreciate further in 2015. However, facing an environment with interest very close to their lower bound, central banks cannot simply keep their currency depreciated using conventional reductions in the interest rates, and have resorted to large interventions in currency markets, resulting in large accumulation of foreign reserves. Indeed, the ratio of foreign reserves held by the Swiss National Bank to gross domestic product went from being 10% at the beginning of 2010 to over 100% in 2015. Many commentators have referred to this large accumulation of foreign reserves as potential risk to central banks, but the nature of these risks are not yet, to the best of our knowledge, well understood.

The goal of this paper is to study the effects of pursuing these exchange rate exchange policies, and their associated reserves accumulation in a standard monetary environment which potentially features nominal interest rates that are the zero lower bound (ZLB). Towards this goal, we study a set up where the Central Bank of a small open economy (SOE) wants to implement a temporary depreciation of its exchange rate. A key assumption is the presence of limited international arbitrage which we model, in the simplest possible fashion, as an upper bound on the amount of foreign wealth that can be invested in the SOE.

Our first set of results extends the well known trilemma of international finance for an economy that operates at the ZLB.¹ We show that pursuing an exchange rate objective in such an environment necessarily entails violations of the interest parity condition. These deviations from arbitrage create an incentive for foreign investors to accumulate assets of the SOE. In absence of an intervention by the monetary authority, these capital inflows would put pressure on the exchange rate. In order to sustain its exchange rate target, the Central Bank needs to accumulate foreign assets, reversing the trades made by the foreigners. Therefore, foreign exchange interventions are a necessary instrument for a Central Bank that wishes to implement an exchange rate policy when the ZLB constraint binds. Importantly, as identified by [Cavallino \(2016\)](#) and [Fanelli and Straub \(2015\)](#), these interventions in the

¹There is a large literature that explores the trilemma, including the role of capital controls in escaping the trilemma. Recent contributions include, among others, [Rey \(2013\)](#), [Farhi and Werning \(2014\)](#), and [Devereux and Yetman \(2014a\)](#).

presence of deviations from interest rate parity are *costly* because the Central Bank takes the opposite side of the arbitrage profits made by the foreign investors, leading to a loss for the SOE as a whole.

This result stands in contrast with what happens when nominal interest rates are positive. In this scenario, the Central Bank can always fight appreciation pressures by decreasing its policy rate, eliminating the possibility of arbitrage opportunities in international capital markets. Thus, when nominal interest rates are positive, our model replicates the textbook result that an exchange rate objective can be implemented by giving up monetary independence. When the ZLB constraint binds, instead, the Central Bank needs to expand its balance sheet and carry out costly interventions in foreign exchange markets.

While a zero nominal interest rate makes foreign exchange rate interventions necessary, it also makes it easier to implement them. In our model, the Central Bank can, in principle, sustain an exchange rate policy and maintain monetary independence when nominal interest rates are positive. In order to do so, it must expand its balance sheet and accumulate foreign assets while generating the required deviations from interest rate parity. However, a Central Bank that is constrained in its ability to issue interest paying liabilities and does not receive transfers from the fiscal authority cannot expand its balance sheet without limits. When nominal interest rates are positive, money is a dominated asset, and the size of the Central Bank balance sheet is constrained by the utility services provided by the money it issues. We show that, when foreign wealth is sufficiently large, in the unique monetary equilibrium, such a constrained Central Bank cannot sustain a deviation from interest rate parity. This is not the case when operating at the ZLB, as in this case, the Central Bank can expand its balance sheet without a coordination with the fiscal authority as money and domestic bonds are perfect substitutes.

We next turn to analyze in more details the costs of defending an exchange rate target at the ZLB. Changes in the economic environment, that would have otherwise been beneficial, increase these costs. For example, a deepening in international capital market integration (i.e., an increase on the upper bound on foreign wealth) is always beneficial when the zero lower bound constraint does not bind. However, such an increase is always detrimental at the ZLB. In other words, deeper financial integration with the outside world switches from being a virtue to being a curse when the economy moves from positive to zero nominal rates. The zero lower bound environment also changes the way the economy reacts to reductions in the international interest rate. When the domestic nominal interest rate is positive, a reduction in the foreign interest rate is generally beneficial (or at least irrelevant) for a borrowing country. At the zero lower bound, instead, such a reduction makes the country strictly worse off. The key reason behind these two reversals is the behavior of the Central Bank

when off and on the ZLB constraint. Away from the ZLB, the Central Bank does not need to engage in costly interventions to sustain an exchange rate path. At the ZLB, instead, the Central Bank needs to intervene, and the size of these costly interventions increase with a reduction in either the limits of international arbitrage or in foreign interest rates, magnifying the losses.

The ZLB also has implications for the ability of the Central Bank to exploit expectational mistakes by private agents. We show that, away from the ZLB, if private agents expect a higher appreciation of the currency than what the Central Bank will in effect implement, the Central Bank can always exploit this mistake and increase domestic welfare. The result again hinges on the ability of the Central Bank to lower the nominal interest rate, accumulate foreign assets, and take advantage of the foreign investor’s mistaken beliefs. At the ZLB, the opposite holds. The Central Bank cannot take advantage of the mistakes, and, because it cannot lower the nominal interest rate, it is forced to intervene and accumulate foreign reserves to maintain its exchange rate policy. This intervention at the ZLB unambiguously decreases domestic welfare. Although we do not pursue this in the present paper, this result could have implications for the possibility of self-fulfilling appreciation “runs” at the ZLB.

In the final theoretical section of the paper we turn to the choice of the exchange rate (which in the initial part of the theory we take as given), and ask whether, given the costs we have highlighted, a central bank would ever want pursue a policy of exchange rate depreciation. To this purpose, we embed our mechanism in a simple two-period New-Keynesian model with wage rigidities (see e.g. Rendahl 2016), where the exchange rate can affect real economic activity. We show that in this context, if wage rigidities are severe enough, the Central Bank will still want to pursue a policy of exchange rate depreciation and suffer the associated cost. This model, by allowing an evaluation of the costs and benefits of a given exchange rate policy, can also shed light on when and why a monetary authority might want to abandon that policy.

In the second part of the paper we check whether some key implications of our framework are confirmed in the data. We collect data for a panel of 11 advanced economies over the 2000-2015 period on foreign reserves held by Central Banks, nominal interest rates, and deviations from covered interest rate parity (CIP), which are our measure of limits to arbitrage. First we uncover a strong relation, both in the time series and in the cross-section, between the size of the foreign reserves held by Central Banks and the deviations from CIP. Moreover, we replicate the finding of [Du et al. \(2016\)](#) that countries with nominal interest rates close to zero experienced positive cross-currency bases during this period. When taken together, these two facts are consistent with the key mechanisms at play in our model: Central Banks can sustain deviations in the interest rate parity condition by accumulating foreign assets,

and these interventions become a necessity when trying to pursue an implicit or an explicit exchange rate target at the ZLB.

One remaining question is how large the costs of following an exchange rate policy could be in practice. Our model suggests that the losses equal the product of foreign reserves and deviations from covered interest rate parity. We apply this formula to the experience of the Swiss National Bank during the 2010-2015 period. We document that the foreign exchange interventions by the Swiss National Bank conducted over this period resulted in substantial losses, reaching around 0.8-1% of GDP in January of 2015.

Our work is related to recent work on segmented international markets and exchange rates, such as [Alvarez et al. \(2009\)](#) and [Gabaix and Maggiori \(2015\)](#). In particular [Gabaix and Maggiori \(2015\)](#) introduce frictions in international financial intermediaries, motivated by the portfolio balance approach, and study how foreign exchange rate intervention affects exchange rates by altering the balance sheet of intermediaries. Following their work, [Cavallino \(2016\)](#) studies optimal monetary policy and foreign exchange rate intervention with a focus on the desirability of interventions in response to currency misalignments arising in financial markets. A particularly relevant contribution is [Fanelli and Straub \(2015\)](#). They show how a deviation from interest parity generates a cost in the inter-temporal resource constraint of the economy, which is proportional to size of the deviation. This is an insight that we exploit in our analysis, although in a slightly different model. A very related literature makes a similar point. [Calvo \(1991\)](#) first raised the warning about the potential costs of sterilizations by Central Banks in emerging markets.² Subsequent papers have discussed and estimated the “quasi-fiscal” costs of these operations, and similarly identified the costs of sterilization as a loss in the inter-temporal budget constraint of the government, proportional to the interest parity deviations and the size of the accumulated reserves (see [Kletzer and Spiegel 2004](#), [Devereux and Yetman 2014b](#), [Liu and Spiegel 2015](#), and references therein). In their analysis of speculative runs on interest rate pegs, [Bassetto and Phelan \(2015\)](#) have explored the implications of limits to arbitrage in a monetary (but closed) economy. Differently from the above papers, our main objective is to study the effects that the zero lower bound constraint imposes for exchange rate management in the presence of limited arbitrage, as well as using the CIP deviations to quantify the potential losses. Other related recent work has also explored the international implications of the zero lower bound on interest rates, in particular see [Acharya and Bengui \(2015\)](#), [Caballero et al. \(2015\)](#) and [Eggertsson et al. \(2016\)](#).

²[Backus and Kehoe \(1989\)](#) is an earlier paper showing general conditions under which sterilization (i.e., a change in the composition of the currency of denomination of the government debt) is irrelevant; a situation that happens in our framework when the economy operates away from the zero bound and the foreign wealth is large enough.

The structure of the paper is as follows. Section 2 introduces the basic monetary setup. Section 3 discusses the implementation of an exchange rate path when the zero lower bound constraint on nominal interest rate is slack, and when it binds. Section 4 presents a comparative static analysis of the costs of foreign exchange interventions, while Section 5 explains the role of expectational mistakes of private agents. Section 6 discusses optimal exchange rate interventions in an extension of the model with wage rigidities. Section 7 presents empirical evidence consistent with the mechanisms discussed in this paper and it measures the costs of foreign exchange rate interventions by the Swiss National Bank during the 2010-2015 period. Section 8 concludes.

2 The model

We consider a two-periods ($t = 1, 2$), two currencies (domestic and foreign), one-good, deterministic small open economy, inhabited by a continuum of domestic households, a monetary and a fiscal authority. The small open economy trades with a continuum of foreign investors and can potentially also access an international financial market. We now proceed to describe the economy in detail.

2.1 Exchange rates, and interest rates

We denote by s_t the exchange rate in period t , i.e. the amount of domestic currency needed to purchase one unit of foreign currency in period t . We normalize the foreign price level (i.e. the amount of foreign currency needed to buy one unit of the good) to 1 in each period, and we assume that the law of one price holds. As a result, s_t is the domestic price level, i.e. the units of domestic currency needed to purchase one unit of the consumption good.

There are three assets available. First, there is a domestic nominal bond, which is traded within the domestic economy. This bond is denominated in domestic currency and has an interest rate which we denote by i . Domestic agents are also able to access the international financial markets and save in a foreign bond, denominated in foreign currency, with an interest rate denoted by i^* . In addition, there is domestic currency circulating in the domestic economy.

While the domestic interest rate will be determined endogenously on the domestic credit market, the foreign rate is exogenously given, in accord with the small open economy assumption.

2.2 Domestic households

Domestic households value consumption of the final good as well as from holding real money balances according to the following utility function:

$$U(c_1, c_2, m) = u(c_1) + h\left(\frac{m}{s_1}\right) + \beta u(c_2) \quad (1)$$

where $u(\cdot)$ is a standard utility function, c_i is household consumption in period i , m is the nominal stock of money held by the household at the end of period 1 and $h(\cdot)$ is an increasing and concave function, also displaying a satiation level \bar{x} (i.e. there exists an \bar{x} s.t. $h(x) = h(\bar{x})$, for all $x \geq \bar{x}$).

Domestic households are endowed with y_1 and y_2 units of the good in the two periods. The domestic households' budget constraints in periods 1 and 2 are

$$y_1 + T_1 = c_1 + \frac{m + a}{s_1} + f \quad (2)$$

$$y_2 + T_2 = c_2 - \frac{m + (1 + i)a}{s_2} - (1 + i^*)f \quad (3)$$

where a and f represent the domestic holdings of domestic and foreign bonds and T_i represent the (real) transfer from the fiscal authority to the households in period i .

We assume that households cannot borrow directly in international financial markets, $f \geq 0$. This assumption will guarantee later on that domestic households cannot fully arbitrage the difference between domestic and foreign real interest rates.

The domestic households' problem is thus:

$$\max_{m, a, f, c_1, c_2} U(c_1, c_2, m)$$

s.t.:

$$\text{equations (2), (3)}$$

$$f \geq 0; m \geq 0$$

2.3 Monetary authority

We impose for now that the monetary authority has a given nominal exchange rate objective, which we denote by the pair (s_1, s_2) . In general, an exchange rate objective would arise from the desire of achieving a particular inflation target or from the presence of nominal rigidities. In Section 6 we will study optimal exchange rate policies in a model with wage rigidities. For the moment, however, we simply assume that the monetary authority follows this objective,

and define equilibrium for the economy given (s_1, s_2) .

In period 1, the monetary authority issues monetary liabilities M . It uses these resources to purchase foreign and domestic bonds by amounts F and A , respectively, as well as to make a transfer, τ_1 , to the fiscal authority.

In the second period, the monetary authority uses the proceeds from these investments to redeem the outstanding monetary liabilities at the exchange rate s_2 , and to make a final transfer to the fiscal authority, τ_2 .

Just as the domestic agents, we assume that the monetary authority cannot borrow in foreign bonds. As a result, the monetary authority faces the following constraints:

$$\begin{aligned} \frac{M}{s_1} &= F + \frac{A}{s_1} + \tau_1 \\ (1 + i^*)F + (1 + i)\frac{A}{s_2} &= \frac{M}{s_2} + \tau_2 \\ M &\geq 0; F \geq 0 \end{aligned}$$

We will sometimes find it useful to analyze the case where the Central Bank cannot receive transfers from the fiscal authority in the first period, and cannot issue government bonds:

Assumption 1 (Lack of Fiscal Support). *The monetary authority does not receive a positive transfer from the fiscal authority in the first period, and cannot issue interest paying liabilities: $\tau_1 \geq 0$ and $A \geq 0$.*

2.4 Fiscal authority

The fiscal authority makes transfers (T_1, T_2) to households in each period. It also receives transfers from the monetary authority, (τ_1, τ_2) in each period. The fiscal authority issues domestic nominal bonds B in period 1 and redeems them in period 2. The associated budget constraints are:

$$\frac{B}{s_1} + \tau_1 = T_1 \tag{4}$$

$$\tau_2 = T_2 + (1 + i)\frac{B}{s_2} \tag{5}$$

Note that we assume that the fiscal authority does not borrow, or invest, in foreign markets.

Because public debt does not affect equilibrium outcomes due to Ricardian equivalence, we will treat the amount of bonds issued by the fiscal authority, B , as a fixed parameter.

2.5 Foreign investors and the international financial markets

A key assumption is that domestic and foreign markets are not fully integrated. In particular, there is a limit to the resources that foreign investors can channel to the domestic economy.³ We assume that the only foreign capital that can be invested in the domestic economy is in the hands of a continuum of foreign investors, and is limited by a total amount \bar{w} , denominated in foreign currency.⁴

We assume that the foreign investors only value consumption in the second period. The investors cannot borrow in any of the financial markets, but can purchase both domestic and foreign assets.⁵ In period 1, they decide how to allocate their wealth between foreign assets f^* , domestic assets a^* , and domestic currency m^* ; while in the second period they use the proceeds from their investments to finance their second period consumption, c^* . The foreign investor's problem is

$$\max_{f^*, a^*, m^*} c^* \tag{6}$$

$$\text{subject to:} \tag{7}$$

$$\bar{w} = f^* + \frac{a^* + m^*}{s_1} \tag{8}$$

$$c^* = (1 + i^*)f^* + (1 + i)\frac{a^*}{s_2} + \frac{m^*}{s_2} \tag{9}$$

$$f^* \geq 0, a^* \geq 0 \text{ and } m^* \geq 0. \tag{10}$$

Notice that unlike domestic investors, foreign investors do not enjoy a utility flow from holding domestic currency, so as expected, they will choose not hold domestic currency when the domestic interest rate i is strictly positive.

2.6 Market clearing and the monetary equilibrium

Recall that our objective is to study whether a particular exchange rate policy can be attained as an equilibrium by the monetary authority, and to compute the costs of pursuing such a policy. Towards this goal, we will define equilibrium *for a given exchange rate policy* (s_1, s_2) :

³There is a recent literature on segmented international asset markets, see for example [Alvarez et al. \(2009\)](#) and [Gabaix and Maggiori \(2015\)](#).

⁴This way of modeling foreign investors is different from [Fanelli and Straub \(2015\)](#). In that paper, foreign demand for domestic assets ends up been a linear function of the arbitrage return, that crosses the origin. In our model, instead, the foreign demand will be a step function of the arbitrage return. That is, there is always a strictly positive amount of foreign wealth ready to arbitrage away any profits from investing in the SOE.

⁵An alternative interpretation is that \bar{w} already represents the total wealth available for investing in period 0, inclusive of any amount that could be borrowed.

Definition 1. A monetary equilibrium, given an exchange rate policy (s_1, s_2) , is a consumption profile for households, (c_1, c_2) , and asset positions, (a, f, m) ; a consumption for investors, c^* , and their asset positions (a^*, f^*, m^*) ; money supply, M ; transfers from the fiscal to the monetary authority, (τ_1, τ_2) ; investments by the monetary authority, (A, F) ; transfers from the fiscal authority to the households, (T_1, T_2) ; and a domestic interest rate i , such that:

- (i) the domestic households make consumption and portfolio choices to maximize utility, subject to their budget and borrowing constraints;
- (ii) foreign investors make consumption and portfolio choices to maximize their utility, subject to their budget and borrowing constraints;
- (iii) the purchases of assets by the monetary authority, its decision about the money supply and its transfers to the fiscal authority satisfy its budget constraints, as well as $F \geq 0$;
- (iv) the fiscal authority satisfies its budget constraints;
- (v) and the domestic asset market clears for both money and bond

$$\begin{aligned} m + m^* &= M \\ a + a^* + A &= B \end{aligned}$$

3 Implementing an exchange rate policy

We now study how the small open economy achieves an equilibrium given a policy for the exchange rate (s_1, s_2) . We start in Section 3.1 by analyzing how foreign reserves affect the equilibrium in a real version of the model. We will see that the monetary authority can, by accumulating foreign reserves, generate a wedge between domestic and foreign real interest rates, and that such interventions will be costly from the point of view of the small open economy. We next turn in Section 3.2 to study the monetary equilibria given the exchange rate policy (s_1, s_2) . The main result will be that a monetary authority that wishes to sustain a given exchange rate policy will *have to* engage in these costly interventions when the domestic nominal interest rate hits the zero lower bound constraint.

3.1 The accumulation of foreign reserves

In order to explain in the most transparent way how the accumulation of foreign reserves affects the equilibrium, we consider a version of the model without money. In this subsection,

we assume that the Central Bank and the fiscal authority are just one single government agency.

We denote by r the rate of return on a real domestic bond. Because we have assumed that the foreign price level is constant, the return on the real foreign bond equals i^* . In this environment, the only action of the monetary authority consists in choosing the amount of foreign reserves F in the first period. Because foreign reserves (plus interests) are rebated back to the representative household in the second period, an increase in F is equivalent to a shift of the domestic endowment from the first to the second period. It is convenient to define the households' endowment after the monetary authority sets the level of foreign reserves,

$$\begin{aligned}\tilde{y}_1 &= y_1 - F, \\ \tilde{y}_2 &= y_2 + (1 + i^*)F.\end{aligned}$$

The domestic households maximize utility $u(c_1) + \beta u(c_2)$ subject to the following budget constraints:

$$\begin{aligned}c_1 &= \tilde{y}_1 - f - a \\ c_2 &= \tilde{y}_2 + (1 + i^*)f + (1 + r)a\end{aligned}$$

where f and a represent their purchases of foreign and domestic assets, respectively. As in the monetary economy, we impose that they cannot borrow abroad, so $f \geq 0$.

The foreign investors are willing to invest up to the maximum of their wealth, \bar{w} , to maximize their returns. That is, their demand of domestic assets a^* satisfies,

$$\max_{0 \leq a^* \leq \bar{w}} a^*(r - i^*) = \bar{w}(r - i^*) \quad (11)$$

where the last equality follows from the maximization.

We will assume that $B = 0$, and market clearing in the domestic financial market implies that $a^* + a = 0$.

We can then define an equilibrium for a given policy of the monetary authority as follows

Definition 2. A *non-monetary equilibrium* given F for $F \geq 0$ is a consumption pair (c_1, c_2) and a domestic real interest rate, r , such that there exists a demand for domestic assets by foreign investors, a^* , and bond holdings by domestic households, (a, f) , with the properties that (i) (c_1, c_2) and (a, f) maximize the households' utility subject to the budget and borrowing constraints, (ii) a^* maximize the foreign investor's utility, and (iii) the domestic asset

market clears.

To characterize an equilibrium, note that the first order conditions of the household imply

$$u'(c_1) = (1 + r)\beta u'(c_2) \quad (12)$$

$$r \geq i^* \quad (13)$$

with $f = 0$ if the last inequality holds strictly.

The first condition is the standard Euler equation, while the second condition imposes that the real interest rate at home cannot be below the one abroad. If that was the case, the demand for domestic asset by households will be unbounded. Importantly, the converse is not true because we have assumed that households cannot borrow in foreign currency, $f \geq 0$, and because of the foreign investors' limited wealth.

We can eliminate a in the household's budget constraints, and obtain an inter-temporal resource constraint for the small open economy:

$$\tilde{y}_1 - c_1 + \frac{\tilde{y}_2 - c_2}{1 + r} - f \left[\frac{r - i^*}{1 + r} \right] = 0.$$

From the household optimality condition stated above, we know that $f = 0$ if $r > i^*$, so it follows then that the inter-temporal budget constraint simplifies to

$$\tilde{y}_1 - c_1 + \frac{\tilde{y}_2 - c_2}{1 + r} = 0. \quad (14)$$

There is an additional equilibrium condition, constraining the trade deficit that the small open economy can run in the first period. Indeed, because $-a = a^* \leq \bar{w}$, one must have that

$$c_1 = \tilde{y}_1 - f - a \leq \tilde{y}_1 + \bar{w}, \quad (15)$$

where the last inequality follows from the fact that $f \geq 0$. This expression tells us that the first period consumption of the households and the foreign reserves of the monetary authority cannot exceed the endowment of the small open economy and the wealth of foreigners \bar{w} . The non-monetary equilibrium is then fully characterized by conditions (12)-(15).

Before turning to the characterization of the equilibrium, it is useful to define the "first

best" consumption allocation,

$$(c_1^{fb}, c_2^{fb}) \equiv \arg \max_{(c_1, c_2)} \{u(c_1) + \beta u(c_2)\}$$

subject to:

$$c_1 + \frac{c_2}{1 + i^*} = y_1 + \frac{y_2}{1 + i^*}.$$

That is, (c_1^{fb}, c_2^{fb}) represents the equilibrium consumption allocation when the constraint on the first period trade balance does not bind.

We then have the following proposition.

Proposition 1. *Non-monetary equilibria given F are characterized as follows:*

(i) *If $F \in [0, y_1 + \bar{w} - c_1^{fb}]$, there is a unique non-monetary equilibrium, and it features $r = i^*$, $c_1 = c_1^{fb}$, and $c_2 = c_2^{fb}$.*

(ii) *If $F \in (y_1 + \bar{w} - c_1^{fb}, y_1 + \bar{w})$, there is a unique non-monetary equilibrium, and it features $c_1 = y_1 - F + \bar{w} < c_1^{fb}$, and c_2 solves*

$$c_2 = y_2 - (1 + r)\bar{w} + (1 + i^*)F, \tag{16}$$

$$\text{with } r = \frac{u'(c_1)}{\beta u'(c_2)} - 1 > i^*.$$

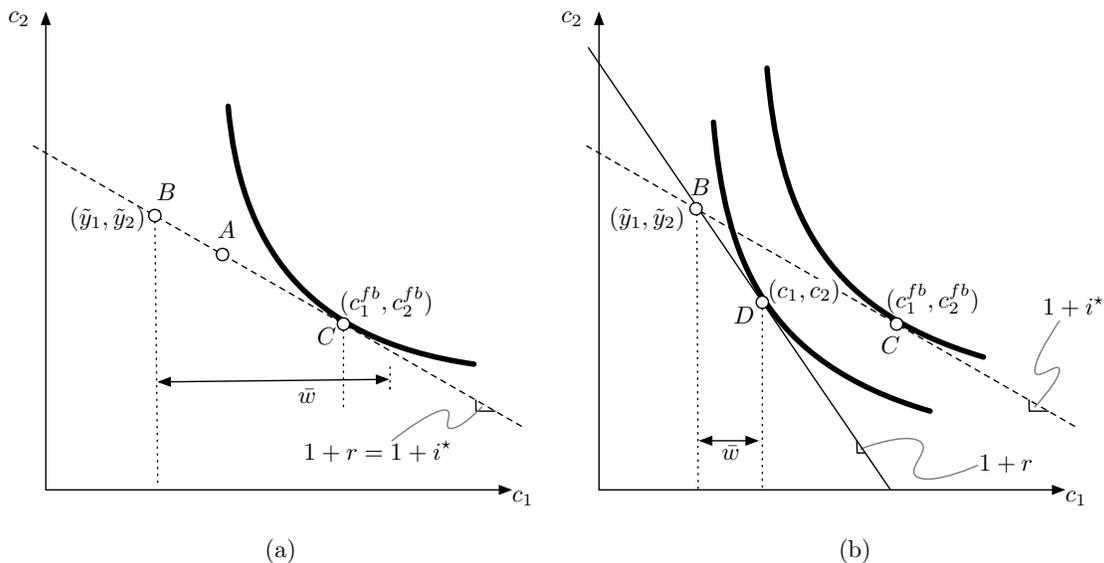
(iii) *If $F > y_1 + \bar{w}$, then there is no non-monetary equilibrium.*

Proposition 1 tells us that there are only two possible equilibrium outcomes in the real economy, depending on the accumulation of foreign reserves by the Central Bank. We illustrate these two cases in Figure 1.

Panel (a) in the figure illustrates the first case. Point A represents the original endowment of the representative household, while point B is the households endowment after taking into account the foreign reserves accumulated by the Central Bank, F . Point C in the figure represents the first best consumption allocation, the one that would arise if the household could freely borrow and lend at the world interest rate i^* . Importantly, point C is feasible for the small open economy only if there is sufficient foreign wealth to cover the first period trade balance, that is, if $y_1 - F - c_1^{fb} < \bar{w}$. This is precisely what happens in case (i) of proposition 1.

Panel (b) in the figure illustrates the second case. The accumulation of foreign reserves by the Central Bank is now so large that there is not enough foreign wealth to finance the trade deficit that would arise with the first best consumption allocation. Therefore, the

Figure 1: Non Monetary Equilibria given F



constraint (15) binds, and consumption allocation is now in point D . Competition for these limited external resources results in a higher domestic real interest rate, which induces the household to consume less in the first period than what they would under the first best. In period 2, the household's consumption equals the endowment minus payments to foreigners, net of the proceeds from the accumulation of foreign reserves by the Central Bank.

We can now characterize the effects that foreign reserves have on the non-monetary equilibrium.

Corollary 1. *In the non-monetary equilibrium given F , for $F \in (y_1 + \bar{w} - c_1^{fb}, y_1 + \bar{w})$, the domestic real interest rate r is strictly increasing in F while the welfare of the domestic households is strictly decreasing in F . Foreign reserves have no impact on the domestic interest rate ($r = i^*$), nor on domestic welfare, when $F \leq y_1 + \bar{w} - c_1^{fb}$.*

The increase in F reduces \tilde{y}_1 and increases \tilde{y}_2 . When F is small (that is $F < y_1 + \bar{w} - c_1^{fb}$) these interventions have no effects on the equilibrium because the private sector is able to undue the external position taken by the Central Bank: enough foreign wealth flows in from the rest of the world to equilibrate the domestic and foreign real rates. When F is large enough (that is, $F > y_1 + \bar{w} - c_1^{fb}$), however, the private sector cannot undue these interventions because the available foreign wealth is not large enough. In this case, the Central Bank interventions effectively make the small open economy “credit constrained”, and induces an increase in the domestic real interest rate.

To understand the adverse consequences of this policy, let us rewrite the inter-temporal

resource constraint for the small open economy, equation (14), as follows

$$BC \equiv (1+r)(y_1 - c_1) + y_2 - c_2 - F(r - i^*) = 0. \quad (17)$$

The term $F(r - i^*)$ captures the losses associated to foreign reserve accumulation by the Central Bank.⁶ These losses appear because the Central Bank strategy consists in saving abroad, at a low return, while the economy is in effect borrowing at a higher one.⁷

The welfare of the domestic household is given by the maximization of their utility subject to just (17), so we can read the effects on domestic welfare by understanding the effects of F on the budget constraint. Taking first order conditions (assuming that the equilibrium r is differentiable), we obtain that the marginal effect of F , for $F \in (y_1 + \bar{w} - c_1^{fb}, y_1 + \bar{w})$, is

$$\frac{dBC}{dF} = -(r - i^*) - \underbrace{(c_1 + F - y_1)}_{\bar{w}} \frac{dr}{dF} < 0$$

From the above, we can see that there are two effects generated by an increase in F . First, one additional unit of reserves directly increases the budget constraint losses by the interest rate differential, $(r - i^*) > 0$. But in addition, an increase in F also increases the equilibrium domestic real rate in this region, $dr/dF > 0$; and given that domestic households are net borrowers with respect to endowment point \tilde{y}_1, \tilde{y}_2 , this induces a negative effect on the budget constraint.⁸

⁶These losses correspond to the “quasi-fiscal” losses of Central Bank interventions highlighted in the sterilization literature.

⁷This represents a loss to the entire small open economy. In [Bassetto and Phelan \(2015\)](#), an arbitrage gain also appears in the private agents budget constraint. But in their closed economy environment, the private agents’ gains equal the government’s losses; and as a result, the gains/losses do not affect the resource constraint of the economy.

⁸As done in [Fanelli and Straub \(2015\)](#), another way of representing the losses faced by domestic households is to rewrite the inter-temporal budget constraint solving out for foreign reserve holdings, using that $a^*(r - i^*) = \bar{w}(r - i^*)$ together with the market clearing condition, which leads to:

$$y_1 - c_1 + \frac{y_2 - c_2}{1 + i^*} - \bar{w} \left[\frac{1 + r}{1 + i^*} - 1 \right] = 0 \quad (18)$$

The first two terms represent the standard inter-temporal resource constraint for an economy that could borrow and save freely at rate i^* . But there is an additional term, which captures the reason why the equilibrium consumption outcome lies strictly within the feasibility frontier. As stressed by [Fanelli and Straub \(2015\)](#), this term represents a loss: as the foreigners invest when the domestic interest is above the foreign one, they gain a profit which is a loss to the country. As can be seen, the losses are proportional to the amount of wealth invested by the foreign investors, a^* , and the differential interest rate, $r - i^*$. Note that differently from [Fanelli and Straub \(2015\)](#), in our environment, these losses may arise even absent a Central Bank intervention, if the foreign wealth is not large enough to take the economy to the first best allocation. When studying the zero lower bound environment, we will find it more useful to work with a version of equation (17), rather than with (18).

Figure 2: The welfare costs of Central Bank interventions

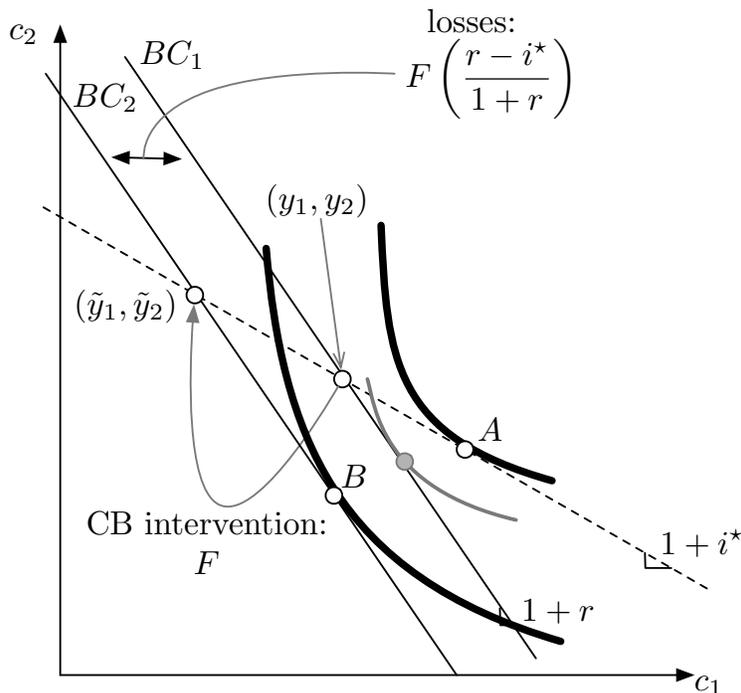


Figure 2 illustrates these welfare losses graphically. Without intervention, the equilibrium is denoted by point A, which in this case corresponds to the first best allocation. With a sufficiently large accumulation of foreign reserves, the Central Bank moves the economy from the income profile (y_1, y_2) to $(\tilde{y}_1, \tilde{y}_2)$. In this example, the first best allocation cannot be attained because foreign wealth is not large enough. The intervention leads to an increase in the equilibrium domestic real rate, which now exceeds i^* , and a new consumption allocation that is now at point B.

We can see from Figure 2 the two effects associated with this intervention of the Central Bank. The movement from point A to the gray dot in the figure isolates the effect that operates through an increase in the domestic interest rate (which negatively affects the country, given that it is originally a borrower). The movement of the budget set from BC_1 to BC_2 captures the resource costs associated with the Central Bank interventions.

In this non-monetary world, Central Bank interventions would not be desirable (at best they have no effect). As a result, it would be optimal in this environment for the Central Bank to always set $F = 0$. We show below how, in a monetary environment, the Central Bank may be forced (because of its exchange rate objective and the zero lower bound) to engage on this type of costly interventions.

3.2 The implementation of an exchange rate policy

So far, we have seen that the Central Bank can generate a wedge between the domestic and the foreign interest rate by accumulating foreign reserves. In a non-monetary world, these interventions are never desirable because they entail welfare losses for the domestic household. The question we ask now is when, and under what conditions, the Central Bank will need to engage in these costly interventions in order to sustain a given exchange rate objective (s_1, s_2) .

To explore this issue, we return to the monetary economy. From the household's optimization problem we know that in any monetary equilibrium given (s_1, s_2) , the following conditions must hold

$$u'(c_1) = \beta(1+i)\frac{s_1}{s_2}u'(c_2) \quad (19)$$

$$(1+i)\frac{s_1}{s_2} \geq (1+i^*) \quad (20)$$

$$h'\left(\frac{m}{s_1}\right) = \frac{i}{1+i}\frac{u'(c_1)}{s_1}, \quad (21)$$

and $f = 0$ if $(1+i)\frac{s_1}{s_2} > (1+i^*)$.

Using the budget constraints of the households, together with market clearing condition in the money market, we get the following equation:

$$y_1 - c_1 + \frac{y_2 - c_2}{\frac{s_1}{s_2}(1+i)} - (f + F) \left[1 - \frac{s_2(1+i^*)}{s_1(1+i)} \right] + \frac{i}{\frac{s_1}{s_2}(1+i)} \frac{m^*}{s_2} = 0.$$

Note however that $f = 0$ if $1 - \frac{s_2(1+i^*)}{s_1(1+i)} > 0$. Therefore, the above expression simplifies to

$$y_1 - c_1 + \frac{y_2 - c_2}{\frac{s_1}{s_2}(1+i)} - F \left[1 - \frac{s_2(1+i^*)}{s_1(1+i)} \right] + \frac{i}{\frac{s_1}{s_2}(1+i)} \frac{m^*}{s_2} = 0.$$

The first three terms in the above expression correspond to the inter-temporal resource constraint for the non-monetary economy, equation (17), as the domestic real interest rate in this monetary economy equals $(1+i)\frac{s_1}{s_2}$. The last term, which is peculiar to the monetary economy, captures the potential seigniorage collected from foreigners. Because foreigners do not receive liquidity services from holding money balances, they set $m^* = 0$, unless the domestic nominal interest rate is 0, implying that $im^* = 0$. As a result, the inter-temporal

resource constraint further simplifies to

$$y_1 - c_1 + \frac{y_2 - c_2}{\frac{s_1}{s_2}(1+i)} - F \left[1 - \frac{s_2(1+i^*)}{s_1(1+i)} \right] = 0 \quad (22)$$

The final equilibrium condition revolves around the Central Bank asset position. Recall from equation (??) that

$$c_1 - y_1 + F = \frac{m^* + a^*}{s_1} - f \leq \bar{w},$$

where the last inequality follows from $f \geq 0$ and $m^* + a^* \leq s_1 \bar{w}$. In addition, if $\frac{1+i}{1+i^*} \frac{s_1}{s_2} - 1 > 0$, then we know that $m^* + a^* = s_1 \bar{w}$ and $f = 0$ (i.e., foreigners invest everything in the domestic assets, and households do not invest in the foreign asset). Therefore, in any monetary equilibrium we must have

$$c_1 \leq y_1 - F + \bar{w}; \text{ with equality if } \frac{1+i}{1+i^*} \frac{s_1}{s_2} - 1 > 0 \quad (23)$$

In other words, the foreign wealth must finance the trade deficit plus the reserve accumulation of the Central Bank.

Note that equations (19), (20), (22), and (23) are the same equations that characterize a non-monetary equilibrium, equations (12), (13), (14), and (15), with $r = (1+i) \frac{s_1}{s_2} - 1$, $\tilde{y}_1 = y_1 - F$, and $\tilde{y}_2 = y_1 + (1+i^*)F$. Thus, any monetary equilibrium must deliver an allocation consistent with a non-monetary equilibrium outcome. In addition, however, a monetary equilibrium imposes the restriction that the nominal interest rate must be non-negative (i.e., the zero lower bound), a key restriction that will play an important role in what follows.

As a result, there is potentially a continuum of monetary equilibria given the exchange rate objective (s_1, s_2) . Each equilibrium differs for the level of foreign reserve F accumulated by the Central Bank and potentially for the level of the nominal interest rate i and for the consumption allocation.

For future reference, we denote by \underline{r} the domestic real interest rate in the non-monetary equilibrium associated with $F = 0$. From Proposition 1 we know that $\underline{r} \geq i^*$.

We can now study how the Central Bank can implement a given policy for the exchange rate (s_1, s_2) in the monetary economy. We will distinguish between two cases, depending on whether the zero lower bound constraint under the exchange rate policy binds or not.

3.2.1 Implementation when the zero lower bound constraint does not bind

We first consider the case in which $(1 + \underline{r}) \frac{s_2}{s_1} \geq 1$. We have the following result.

Proposition 2. *Suppose that $(1 + \underline{r}) \frac{s_2}{s_1} \geq 1$. Then, for all $F \in [0, y_1 + \bar{w})$, the non-monetary equilibrium given F constitutes a monetary equilibrium outcome. Household's welfare is maximized in the equilibrium with $F = 0$.*

The intuition behind this proposition is as follows. When the Central Bank does not accumulate foreign reserves, the real interest rate in the non-monetary economy will be equal to \underline{r} . This real rate, along with the exchange rate policy (s_1, s_2) , does not violate the zero lower bound constraint because, by assumption, the domestic nominal interest rate would be such $i = (1 + \underline{r}) \frac{s_2}{s_1} - 1 \geq 0$. Therefore, the allocation $(c_1, c_2, \underline{r})$ for $F = 0$ constitutes a monetary equilibrium outcome. From Corollary 1, we know that the real interest rate is weakly increasing in F . Thus, all non-monetary equilibria given F , for $F > 0$, will not violate the zero lower bound constraint on nominal interest rates, and will also constitute a monetary equilibrium outcome.

Combining Proposition 1 and 2, we can see that the Central Bank can implement an exchange rate objective (s_1, s_2) in two distinct ways. First, the Central Bank could implement (s_1, s_2) by adjusting the nominal interest rate in order to guarantee that foreign investors are indifferent between holding domestic or foreign currency assets, i.e. that the interest rate parity condition in (20) holds with equality. This is case (i) in Proposition 1. In this first scenario, the accumulation of foreign reserves does not impact the equilibrium outcomes (locally), this mirroring the classic irrelevance result of Backus and Kehoe (1989).

There is, however, a second way to implement the exchange rate objective (s_1, s_2) . This is described in case (ii) of Proposition 1: the Central Bank could achieve its desired exchange rate policy (s_1, s_2) by accumulating foreign reserves while setting a higher domestic interest rate than the one consistent with interest rate parity.

These results specialize the classic trilemma of international finance to an environment with limits to international arbitrage. The Central Bank can implement an exchange rate policy by adjusting the nominal interest rate and eliminate arbitrage opportunities in capital markets. In our environment, however, this is not the only option, and the Central Bank could follow an exchange rate policy (s_1, s_2) while maintaining some degrees of monetary independence. To do so, it will need to engage in the costly interventions described in Section 3.1.

In the model described here, though, this trade-off is not operating: given an exchange rate policy (s_1, s_2) , the optimal Central Bank policy would be not to accumulate foreign reserves (a result that follows directly from Proposition 3.1).⁹ However, there is a sense in

⁹There is an issue, related to the value of money balances, a consideration that, of course, does not appear in the non-monetary equilibria analysis. However, the equilibrium with $F = 0$ is the monetary equilibrium with the lowest possible nominal interest rate, given the exchange rate policy. And thus, it ends

which this is a stronger result. If a Central Bank has no fiscal support in the first period, then it may not be feasible for the Central Bank to engineer a deviation from interest parity:

Proposition 3. *Suppose that $(1 + \underline{r})\frac{s_2}{s_1} \geq 1$ and that assumption 1 holds. In addition, suppose that $c_1^{fb} - y_1 + \bar{x} \leq \bar{w}$. Then all monetary equilibria attain the first best consumption allocation, the same domestic welfare, and the interest rate parity condition (20) holds with equality.*

Proposition 3 tells us that a Central Bank that cannot issue interest rate paying liabilities and does not receive transfers from the fiscal authority is constrained in its ability to raise the domestic real rate above the foreign one. In order to understand why, suppose that the Central Bank tries to do so. This leads to an immediate inflow of foreign capital of size \bar{w} , which puts downward pressure on the domestic interest rate. To keep the interest rate from falling, the Central Bank must purchase a large amount of the inflow and accumulate foreign reserves. But the purchasing power of the Central Bank is limited by its balance sheet because, by assumption 1, the Central Bank's liabilities are bounded by the satiation point of money \bar{x} . If the external wealth is sufficiently high, the Central Bank will not be able to sustain a deviation from interest rate parity, and the domestic interest rate will need to adjust. Therefore, it could be challenging for the Central Bank to gain monetary independence while committing to an exchange rate policy when nominal interest rates are positive.

3.2.2 Implementation when the zero lower bound constraint binds

The second case we analyze is when $(1 + \underline{r})\frac{s_2}{s_1} < 1$. In this case, the non-monetary equilibrium with $F = 0$ cannot arise as a monetary equilibrium outcome because it would lead to a domestic nominal interest rate that violates the zero lower bound constraint. As a result, the monetary equilibrium will necessarily feature a deviation from interest rate parity, and the domestic real interest rate will need to lie strictly above the foreign one.¹⁰

So, for there to be a monetary equilibrium, the Central Bank will need to intervene and accumulate reserves of a magnitude sufficient to increase the real interest rate above the

up maximizing total households' utility, inclusive of money balances. Indeed, there is no additional value of raising the domestic interest rate beyond what's necessary to support the exchange rate policy under no reserve accumulation.

¹⁰This follows immediately from the following set of inequalities:

$$(1 + i)\frac{s_1}{s_2} - 1 \geq \frac{s_1}{s_2} - 1 > \underline{r} \geq i^*$$

where the first term is the domestic real rate, the first inequality follows from the zero lower bound constraint, the second defines the case of interest, and the last one is the restriction that appears in any non-monetary equilibrium.

level consistent with interest parity. Let \bar{r} to be the highest possible real interest rate in the non-monetary economy (that is, the interest rate associated with the maximum possible intervention). We then have the following result:

Proposition 4. *Suppose that $1 + \underline{r} < \frac{s_1}{s_2} < 1 + \bar{r}$, then there exists an $\underline{F} > 0$ such that for all $F \in [\underline{F}, y_1 + \bar{w})$, the non-monetary equilibrium given $F, (c_1, c_2, r)$, constitutes a monetary equilibrium outcome. In all these monetary equilibria, the interest rate parity condition (20) holds as a strict inequality. Household's welfare is maximized in the equilibrium with $F = \underline{F}$.*

Proposition 4 tells us that the Central Bank is able to sustain the exchange rate policy. However, because of the zero lower bound, it *has to* engage in the costly interventions described in Section 3.1.

It follows however that, given an exchange rate policy (s_1, s_2) , the optimal Central Bank policy is to accumulate the minimum amount of foreign reserves necessary to deliver a monetary equilibrium. As a result, the best monetary equilibrium in this case will feature $i = 0$ and a violation of the interest parity condition. Differently from the situation in which the zero lower bound constraint is slack, the Central Bank can always sustain these exchange rate policies, even without the support of the fiscal authority:

Proposition 5. *Suppose that $(1 + \underline{r})\frac{s_2}{s_1} < 1$ and that assumption 1 holds. In addition, suppose that $c_1^{fb} - y_1 + \bar{x} \leq \bar{w}$. Then the unique monetary equilibrium outcome is the one where $F = \underline{F}$ and $i = 0$.*

Proposition 5 tells us that a Central Bank without fiscal support is able to raise the domestic real rate above the foreign one, as long as the nominal interest rate remains at zero. In this case, by sustaining the exchange rate path, the Central Bank is forced to issue currency to purchase the foreign assets necessary to maintain the domestic rate above the foreign one. The main difference from the case analyzed previously is that now, because of the zero nominal rate, bonds and money are perfect substitutes. Thus, the Central Bank can expand its balance sheet without limits.

The mechanism at play here is related to the one highlighted in closed-economy New Keynesian models, such as in Eggertsson and Woodford (2003), Christiano et al. (2011) and Werning (2011). In both setups the problem is that there is "too much" desired saving in domestic asset markets. In New Keynesian models usually the excess saving is driven by shocks to patience of households, while in our set-up the excess saving is induced by the exchange rate policy, which makes domestic assets attractive. In both set-ups restoring equilibrium in credit markets at the zero lower bound entails a reduction in the desired savings by domestic agents, and in both setups this adjustment is costly. In new Keynesian closed economy models the reduction in saving arises because of declines in current output, caused by nominal

rigidities, and the cost is the output loss itself. In our setup the reduction in desired savings is generated through the Central Bank intervention, which transfers resources from the present to the future, and the loss is driven by the fact that this intervention entails transfers of resources from domestic to foreign agents.

4 The costs of foreign reserve accumulation

In the previous section we have seen that a Central Bank that wishes to implement an exchange rate path while its nominal interest rates are at zero needs to accumulate foreign reserves. We have also seen that these interventions are costly from the perspective of the small open economy. In this section we study in more details those costs, and discuss how they are affected by changes in the underlying economic environment.

We consider the effects of increases in foreign wealth, \bar{w} , and of reductions in the foreign interest rate i^* (when the country is a net borrower). Before moving to the ZLB environment, let us first argue that both of these changes will unambiguously improve welfare when the zero lower bound constraint does not bind, that is, when $(1 + \underline{r}) \frac{s_2}{s_1} \geq 1$.

To see this, note that, away from the zero lower bound, the best monetary equilibrium given an exchange rate policy (s_1, s_2) sets $F = 0$. As a result, the welfare effects can be read by studying the effects of such changes in the budget constraint of domestic households,

$$y_1 - c_1 + \frac{y_2 - c_2}{1 + r} \geq 0,$$

where r is the domestic equilibrium real rate. So, whether increases in \bar{w} or decreases in i^* are welfare improving or not, depend on the effect of these changes on the equilibrium domestic real interest rate. The following helps in clarifying the effects:

Lemma 1. *Consider the non-monetary equilibrium given $F = 0$. Then,*

- (i) *if $c_1^{fb} > y_1 + \bar{w}$, a marginal increase in \bar{w} strictly decreases the domestic real interest rate, while a marginal decrease in i^* has no effect.*
- (ii) *if $c_1^{fb} < y_1 + \bar{w}$, a marginal increase in \bar{w} has no effect on the domestic interest rate, while a marginal decrease in i^* strictly decreases it.*

The results of this lemma follow from our characterization of the non-monetary equilibrium. When $F = 0$, if $c_1^{fb} < y_1 + \bar{w}$, then the economy achieves the first best consumption outcome, and the domestic real interest rate will equal i^* . As a result, an increase in \bar{w} would have no effect on the real interest rate in this region, but a reduction in i^* will reduce

the domestic rate one to one, explaining part (ii) of the lemma. However, if $c_1^{fb} > y_1 + \bar{w}$, then the economy is constrained, and the domestic interest rate is the unique value r that solves the following equation

$$(1 + r) = \frac{u'(y_1 + \bar{w})}{\beta u'(y_2 - (1 + r)\bar{w})}$$

In this case, changes in the foreign interest rate have no effects on the equilibrium r . An increase in \bar{w} , however, strictly reduces r , a natural outcome of the increase in competition from foreign investors.¹¹

It follows then an increase in \bar{w} either has no effect on the domestic real rate, or reduces it when $c_1 = y_1 + \bar{w}$, that is, when the country is net borrower. From the households budget constraint, an increase in \bar{w} then weakly increases welfare.

A reduction in i^* has no effect on the domestic real rate when the economy is constrained, and reduces the real rate when the economy is at its first best allocation, $c_1 = c_1^{fb}$. If $c_1^{fb} > y_1$, that is, the economy is a net borrower at the first best consumption allocation, then the reduction in i^* will increase welfare.

We now proceed to show how these beneficial changes become welfare-reducing when the economy follows an exchange rate policy at the zero lower bound.

4.1 Changes in foreign wealth

We start analyzing how a change in foreign wealth \bar{w} affects the costs of foreign reserve accumulation when the Central Bank is committed to the exchange rate path (s_1, s_2) and $(1 + \underline{r}) \frac{s_2}{s_1} < 1$. In this case, the best monetary equilibrium will set the nominal interest rate to zero.

We can characterize domestic welfare as follows

$$W \equiv \max_{(c_1, c_2)} u(c_1) + \beta u(c_2) + h(\bar{x}) \tag{24}$$

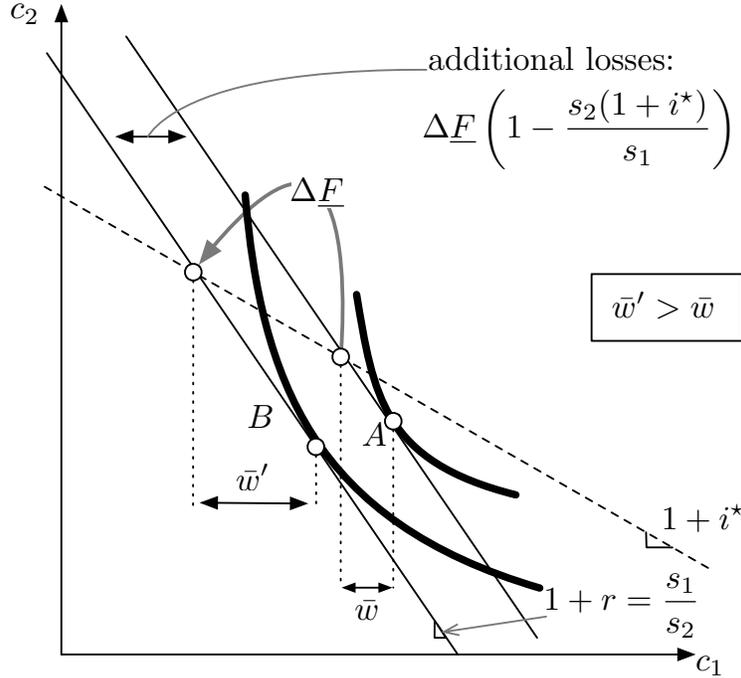
subject to:

$$y_1 - c_1 + \frac{y_2 - c_2}{s_1/s_2} - \underline{F} \left[1 - \frac{s_2(1 + i^*)}{s_1} \right] = 0,$$

¹¹To see this, we can differentiate the equation with respect to \bar{w} and obtain that

$$\frac{dr}{d\bar{w}} = \frac{(1 + r)^2 \beta u''(c_2) + u''(c_1)}{\beta u'(c_2) - (1 + r) \beta u''(c_2) \bar{w}} < 0.$$

Figure 3: Changes in \bar{w} at the ZLB



where \underline{F} is the minimum level of foreign reserves necessary for $i = 0$ given (s_2/s_1) .¹²

Note that at the zero lower bound, the domestic real interest rate equals the rate of appreciation of the currency, which is fixed under the Central Bank policy. Therefore, a change in the welfare of domestic household purely reflects changes in the arbitrage losses that the Central Bank sustain when accumulating foreign reserves, the term $\underline{F} \left[1 - \frac{s_2(1+i^*)}{s_1} \right]$ in the resource constraint.

Let's now consider how an increase in the wealth of foreign investors affects this term. The term in the square bracket is independent on \bar{w} , so a change in \bar{w} affects welfare only through its effect on \underline{F} : if higher \bar{w} leads to higher \underline{F} , then welfare unambiguously declines. This is indeed what happens. When foreign wealth increases, the Central Bank is forced to accumulate more foreign reserves in order to sustain the exchange rate path (s_1, s_2) . Because for every penny of foreign reserve accumulated the Central Bank makes a loss, the overall costs of the intervention increases with \bar{w} .

Figure 3 illustrates this graphically. Point A depicts the equilibrium consumption under the exchange rate policy for a given level of foreign wealth \bar{w} . Suppose now that foreign wealth increases to \bar{w}' . Because the interest rate parity condition is violated under the policy,

¹²The presence of $h(\bar{x})$ arises from the utility value of money balances. At $i = 0$, domestic households are satiated with respect to money balances, so that $m/s_1 \geq \bar{x}$ and $h(m/s_1) = h(\bar{x})$.

more foreign capital will fly toward the small open economy, putting downward pressure on the domestic real interest rate. In order to sustain the path for the exchange rate, the Central Bank will have to lean against these capital flows and purchase foreign assets, so \underline{F} must increase. Point B in the figure represents the equilibrium that prevails when foreign wealth moves to \bar{w}' . The domestic real interest rate at equilibrium B is the same as at A, as the economy is at the ZLB in both points and domestic real rate is pinned down by $\frac{s_1}{s_2}$. Despite the fact the real rate has not changed, welfare at B is unambiguously lower than at A, as the higher \bar{w} forces the Central Bank to intervene more ($\Delta \underline{F} > 0$). The losses generated by this bigger intervention are represented by the parallel shift in the budget lines from point A to point B.

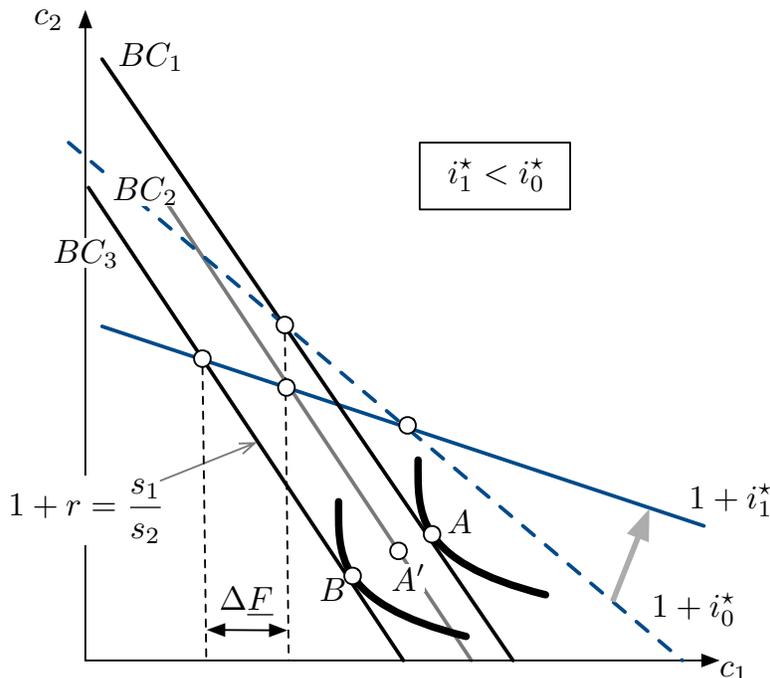
This result shows that a higher degree of capital market integration makes more costly the pursue of an exchange rate objective when the economy is at the zero lower bound: the Central Bank has to accumulate more foreign reserves to sustain the path (s_1, s_2) , and this accumulation leads to resource costs for the small open economy.

4.2 Changes in the foreign interest rate

A similar result occurs when i^* declines. Suppose, again, that the Central Bank is sustaining the path (s_1, s_2) at the zero lower bound, and the foreign interest rate decline. As it was for a change in foreign wealth, the impact of a decline in i^* on welfare depends on its impact on $\underline{F} \left[1 - \frac{s_2(1+i^*)}{s_1} \right]$. However, there are now two effects to consider. First, for given a (s_1, s_2) , the decline in the foreign interest rate implies a larger deviation from interest rate parity, and it leads to an increase in the arbitrage losses made by the Central Bank for a given \underline{F} . Second, the decline in i^* forces the Central Bank to accumulate more reserves, and \underline{F} must increase. Both of these forces increase the costs of sustaining the exchange rate path for the Central Bank.

We illustrate this result in Figure 4. We are considering a situation where the SOE is already at the zero lower bound, and its consumption lies at point A. The dashed budget line represents the resource constraint using an initial foreign rate equal to i_0^* . We then consider a reduction in the international rate to $i_1^* < i_0^*$. The first effect is isolated by the shift of the inter-temporal resource constraint from BC_1 to BC_2 : the Central Bank intervention generates bigger resource costs for the small open economy because the interest parity deviations, for a given exchange rate path, are larger if the foreign interest rate is i_1^* . However, A' is not an equilibrium. The domestic household now would like to save because his endowment in the second period is not as high as it used to be before the decrease in the foreign rate, which implies that the domestic asset market is not in equilibrium. As a result,

Figure 4: Changes in i^* at the ZLB



the Central Bank must increase its foreign reserves, driving the economy to its equilibrium in point B , with an even higher reduction in welfare.¹³

5 Expectational mistakes

So far, we have assumed rational expectations of private agents. We now relax this assumption and ask what happens when private agents are mistaken in their expectations regarding the path of the exchange rate set by the Central Bank.

We show that when the Central Bank follows an exchange rate path that conflicts with the zero lower bound, incorrect beliefs of a future appreciation of the domestic currency necessarily induce an increase in the foreign reserve holdings of the Central Bank, and they end up increasing the welfare costs of sustaining the exchange rate path (s_1, s_2) . This stands in contrast with the case when the zero lower bound constraint does not bind, as in this latter scenario a Central Bank can always exploit these expectational mistakes and strictly

¹³There is potentially another effect that we do not consider here. Suppose that the reduction in i^* allows foreigners to borrow more from the international financial markets. This is equivalent to a larger amount foreign wealth \bar{w} available for investment in the SOE in the first period. The additional effects generated by this will be similar to the already discussed exogenous increase in \bar{w} : it will increase the foreign reserve holdings by the Central Bank, magnifying the welfare losses.

increase the welfare of the domestic household.

We introduce the possibility of expectational mistakes as follows. We continue to let (s_1, s_2) denote the actual exchange rate policy, and we maintain the assumption that the Central Bank will pursue it. Market participants (i.e., domestic households and foreign investors) see the value of s_1 in the first period, and believe that the exchange rate in the second period will be \hat{s}_2 . Expectational mistakes arise when $\hat{s}_2 \neq s_2$. Keeping with our desire to maintain simplicity, we assume that the private agents do not learn or infer any information from the actions of the Central Bank. We define an equilibrium under potentially mistaken market beliefs as follows:

Definition 3. An *equilibrium* given (s_1, s_2) and market beliefs \hat{s}_2 consists of a domestic interest rate i , a consumption profile (c_1, c_2, \hat{c}_2) , asset positions for foreign investors (a^*, f^*, m^*) , money M , investments by the monetary authority (A, F) ; transfers from the monetary authority to the fiscal, $(\tau_1, \tau_2, \hat{\tau}_2)$, and transfers from the fiscal authority to the households, (T_1, T_2, \hat{T}_2) such that

(i) the allocation $(c_1, \hat{c}_2, a, f, m, a^*, f^*, m^*, \tau_1, \hat{\tau}_2, A, F, T_1, \hat{T}_2)$ with nominal interest rate i constitutes a monetary equilibrium given the exchange rate s_1, \hat{s}_2 .

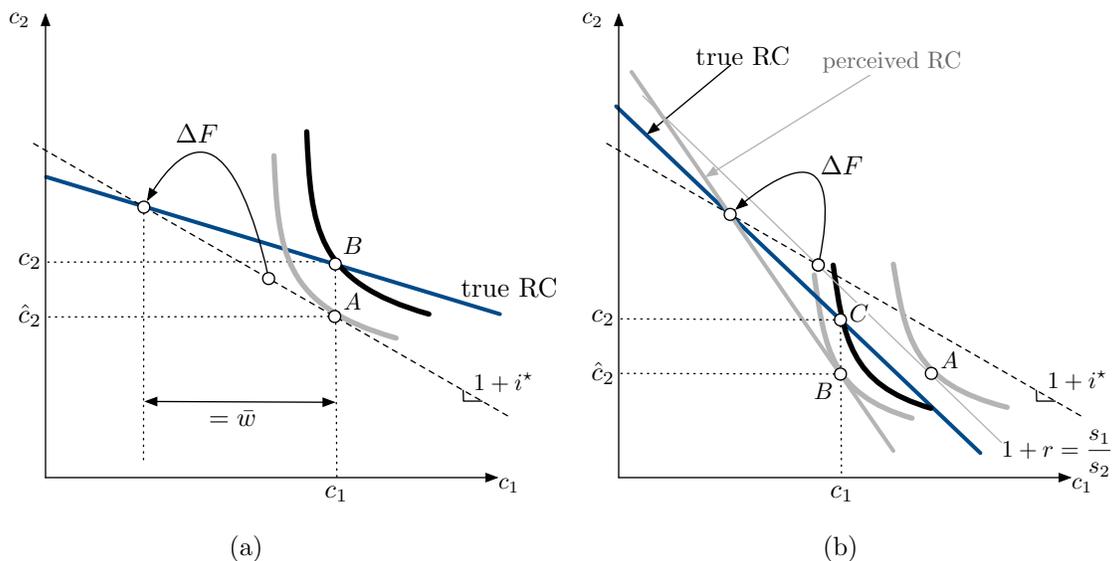
(ii) The second-period consumption and transfers, (c_2, τ_2, T_2) satisfy

$$\begin{aligned} c_2 &= y_2 + T_2 + \frac{(1+i)a + m}{s_2} + (1+i^*)f \\ \tau_2 &= (1+i^*)F + (1+i)\frac{A}{s_2} - \frac{M}{s_2} \\ T_2 &= \tau_2 - (1+i)\frac{B}{s_2} \end{aligned}$$

Note that in part (i) we use the beliefs to define the monetary equilibrium. However, in period 2, the realization of the exchange rate will be s_2 , and the second period “true” allocations (c_2, τ_2, T_2) are calculated with respect to the true exchange rate (part (ii) of the definition). We will call (c_1, \hat{c}_2) the *perceived* consumption allocation, and (c_1, c_2) the *true* consumption allocation. We will also say that $(1+i)\frac{s_1}{\hat{s}_2}$ is the *perceived* real interest rate, and we call $(1+i)\frac{s_1}{s_2}$ the *true* real interest rate. Clearly if $s_2 = \hat{s}_2$, the definition of equilibrium above is identical to our definition of a monetary equilibrium given the exchange rate policy (s_1, s_2) .

As explained earlier, we will consider the case where $s_2 > \hat{s}_2$, that is, when private agents expect the currency to be more appreciated next period relative to the policy that is actually chosen by the Central Bank. We evaluate welfare by considering the household’s utility under the true consumption allocation:

Figure 5: **Expectational mistakes**



Proposition 6. *Suppose that $s_2 > \hat{s}_2$. Consider the equilibrium with market beliefs \hat{s}_2 that maximizes household's welfare. Then*

- (i) *if $(1 + \underline{r})_{s_1}^{\hat{s}_2} \geq 1$, household's welfare strictly decreases with \hat{s}_2 ;*
- (ii) *if $(1 + \underline{r})_{s_1}^{\hat{s}_2} < 1$, household's welfare strictly increases in \hat{s}_2 .*

In case (i) of the proposition, the Central Bank maximizes welfare by reducing the interest rate to that private agents perceive the interest rate parity condition to hold, and by accumulating foreign assets. Ex post, these foreign assets are worth more under the realized exchange rate in period 2 because $s_2 > \hat{s}_2$. These profits are rebated back to the households, and increase their effective second period consumption. As a result, household's welfare, as evaluated by the Central Bank, increases. An example of this is illustrated in Figure 5 panel (a). Point A represents the rational expectation case, $\hat{s}_2 = s_2$. In our example, the Central Bank is sustaining the path for the exchange rate with no interventions ($F = 0$). We then consider how the equilibrium changes when $\hat{s}_2 < s_2$. In this case, the Central Bank reduces the nominal interest rate, such that the “perceived” real rate of return remains the same. Moreover, it accumulates foreign reserves to the point in which all foreign wealth enters the small open economy. These two changes do not affect the behavior of private agents, which believe the equilibrium will be at point A. However, ex-post, the exchange rate equals s_2 , and the consumption of the domestic households will be at point B, rather than A, generating a strictly positive welfare gain.

The key reason why the Central Bank can exploit the mistaken beliefs in case (i) of Proposition 6 is based on its ability to lower the domestic nominal interest rate when the beliefs deviate from the true ones. At the ZLB, this is not possible. As a result, the expectation mistakes cannot be exploited and welfare cannot be increased. We show now an even more negative result: the mistaken beliefs at the ZLB unambiguously generate a reduction in welfare.

Figure 5 panel (b) illustrates case (ii) of the proposition. Point A represents the original equilibrium, where the ZLB binds, and $\hat{s}_2 = s_2$. We consider then how the equilibrium changes if $\hat{s}_2 < s_2$. In this case, the Central Bank cannot further reduce i , because the interest rate is at zero. As a result, the perceived domestic rate of return is necessarily above the foreign one. Thus, the Central Bank has to accumulate foreign reserves in order to maintain its desired path for the exchange rate. The perceived equilibrium consumption allocation shifts to point B. However, ex-post, the exchange rate remains at s_2 , and the realized rate of return is identical to the original. The realized consumption allocation that is attained in equilibrium is given by point C, which is dominated by A. Hence, welfare has been reduced.

The results of this section highlight that even if the Central Bank is committed to its exchange rate policy, if the markets do not believe it, then there will be costs associated to defending the policy when the economy operates at the zero bound. In addition, the larger the expectational mistake, the larger the required foreign exchange interventions by the Central Bank, and the larger the welfare losses. That is, at the ZLB, expectational mistakes are accompanied with costly balance sheet expansions by the Central Bank. Those expansions could, by themselves, trigger an abandonment of the exchange rate policy if the Central Bank finds it costly to maintain a large balance sheet.¹⁴

The results in this section open the door to the possibility of self-fulfilling “appreciation” runs at the ZLB. In particular, if the private agents erroneously anticipate an appreciation, when the Central Bank is at the ZLB, such mistaken expectations will be costly. This may force the Central Bank to abandon the defense of the current exchange rate regime, allowing the currency to appreciate, somewhat validating the mistaken expectations of the markets. The analysis in this section calls for a more detailed study of the game between the Central Bank and the private agents, something that we leave for future work.

¹⁴This is something that we do not analyze here, but is a point we studied in Amador et al. (2016).

6 Optimal Exchange Rate Policy

Until now we have studied how the economy achieves an equilibrium for a given exchange rate policy (s_1, s_2) . We now study optimal exchange rate interventions in a model with nominal rigidities. To simplify the analysis, we still assume that s_2 is given. (In the Appendix we address the joint determination of (s_1, s_2) and the essentially the same results apply [need to add this]).¹⁵

We consider a model where first-period output is produced with labor, $y = h^\alpha$, while in the second period we continue to assume that there is an endowment. Wages, are sticky in domestic currency, $\omega = \bar{\omega}$. Taking wages as given, firms solve

$$\Pi_1 \equiv \max_{h_1} s_1 f(h) - \bar{\omega} h_1,$$

which leads to a labor demand condition

$$s_1 f'(h) = \omega_1 \tag{25}$$

Households lifetime utility is now given by

$$u(c_1) + v(n_1) + \beta u(c_2)$$

where $v(n_1)$ represents the disutility from working. Households face basically the same problem as in the previous version of the model with the difference households' income is composed of labor income and firms' profits. We assume that households are off their labor supply, and essentially supply as many hours as firms demand at the sticky wage (i.e., $n_1 = h_1$). Hence, their problem is exactly as in (), except that the first-period budget constraint () is replaced by

$$s_1 c_1 + s_1 a^* + a = \bar{\omega} n_1 + \Pi_1 + T_1$$

In this economy, the efficient level of employment solves $\max_{h_1} u(f(h)) - v(h)$ and implies a zero labor wedge $u_c(c_1^*) f'(h_1^*) = v_n(h_1^*)$. The flexible wage economy would achieve this level of employment with associated real wage, $\omega^* = f'(h_1^*)$. When ω is above (below) ω^* , we have unemployment (overemployment). Notice from (25), that the central bank can offset

¹⁵The only difference when s_2 is also endogenous is that the government can relax the zero lower bound either by appreciating today or by increasing the depreciation tomorrow. As long, as there is a cost from depreciating in the second period, because of overheating or higher prices, the same trade-offs apply in that version of the model.

the nominal rigidity in wages by adjusting the level of the exchange rate. Following the standard prescription, the central bank can appreciate whenever there is overemployment, and depreciate whenever there is unemployment, to achieve the efficient level of employment. In the former case, the central bank can raise sufficiently the interest rate to achieve the efficient level of employment. The presence of the zero lower bound on Monica interest rates, however, imposes limits on the ability of the Central Bank to reduce unemployment when real wages are too high. We study next the optimal central bank problem of jointly choosing exchange rate policy and foreign exchange rate intervention.

6.1 Central Bank's Problem

government problem consists of choosing $\{s_1, i, F, T_1, T_2\}$ to maximize household lifetime utility. When choosing $\{s_1, F, T_1, T_2\}$, the government is subject to implementability constraints given by firms' employment decisions, household optimization, foreign lenders' optimal demand for domestic assets, as well as the zero lower bound. Combining these conditions, the problem can be written as

$$\max_{\{c_1, c_2, s_1, s_2, i\}} u(c_1) - v\left(g\left(\frac{\bar{\omega}_1}{s_1}\right)\right) + \beta u(c_2)$$

subject to:

$$(1 + i^*)c_1 + c_2 \leq (1 + i^*) \left[f\left(g\left(\frac{\bar{\omega}_1}{s_1}\right)\right) \right] + y_2 - \left(\frac{s_1}{s_2} - (1 + i^*)\right) \bar{x} \quad (\text{Resource C.})$$

$$1 + i \geq 0 \quad (\text{ZLB})$$

$$1 + i \geq (1 + i^*) \frac{s_2}{s_1} \quad (\text{IP})$$

$$1 + i = \frac{u'(c_1)}{\beta u'(c_2)} \frac{s_2}{s_1} \quad (\text{Euler Domestic})$$

$$g\left(\frac{\bar{\omega}_1}{s_1}\right) - c_1 + \bar{x} \geq 0 \quad (\text{Foreign Wealth})$$

where we have denoted $g(x) = f'^{-1}(x)$, replaced the equilibrium level of employment using (25) and we have used that $\frac{u'(c_1)}{\beta u'(c_2)} = 1 + i$ in (ZLB).

Consider a relaxed problem where we ignore the ZLB constraint, $\frac{u'(c_1)}{\beta u'(c_2)} \geq \frac{s_1}{s_2}$. Let c_1^*, c_2^*, s_1^* , denote its solution. Then, we have that if $\frac{u'(c_1^*)}{\beta u'(c_2^*)} \geq \frac{s_1^*}{s_2}$, then the solution to the planning problem coincides with the relaxed problem. The ZLB is not binding and the economy achieves the efficient allocations by setting $1 + i = (1 + i^*) \frac{s_2^*}{s_1^*}$. Instead if $\frac{u'(c_1^*)}{\beta u'(c_2^*)} < \frac{s_1^*}{s_2}$, the solution to the relaxed problem cannot be attained in the planning problem and the

economy is at the ZLB. The exchange rate is too appreciated relative to the efficient level (i.e., $s_1 < s_1^*$).

At the ZLB, the central bank faces a trade-off. It can reduce the level of unemployment by raising s_1 above the interest parity condition (i.e., $s_1^* > (1 + i^*)s_2^*$), but this exposes the CB to losses on the balance sheet. Our key result is when the ZLB is sufficiently binding, the CB induces a deviation from IP. Starting at the efficient allocations, an increase in the real wage induces second order losses, whereas a deviation from IP generates a first-order loss proportional to \bar{x} .

To inspect this trade-off, consider a state with deviation from IP condition and binding ZLB constraint. Let λ, η, ξ be respectively the Lagrange multiplier of the the ZLB constraint, the intertemporal resource constraint and the foreign wealth constraint. We have the following optimality condition with respect to s_1

$$-g' \left(\frac{\bar{\omega}_1}{s_1} \right) \frac{\bar{\omega}_1}{s_1^2} \left(\underbrace{(\lambda(1 + i^*) + \eta)f'(h) - v'(h_1)}_{\text{Labor Wedge}} \right) = \underbrace{\frac{\lambda}{s_2} \bar{w}}_{\uparrow \text{Sterilization Loss}} + \underbrace{\xi \beta u'(c_2)}_{\text{Interest Rate Distortion}} \quad (26)$$

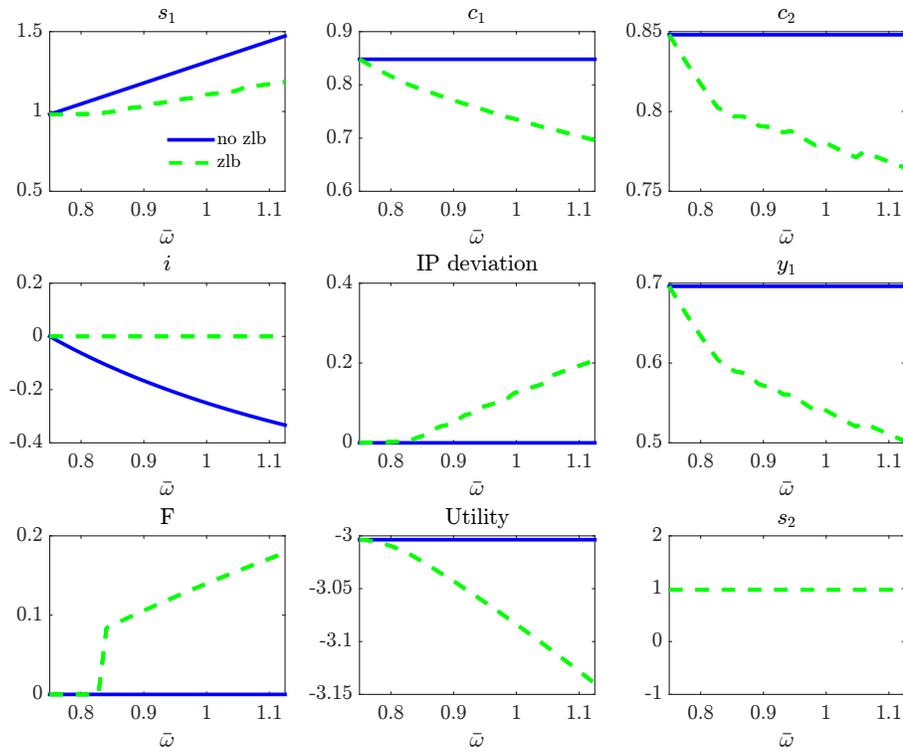
Equation 26 illustrates the key trade-off in the model. The left-hand side indicates the benefits of depreciating the exchange rate. Since $g' < 0$, this shows that there are positive benefits from depreciating when the labor wedge is positive. The right-hand side indicates the marginal costs from depreciating the exchange rate, which is composed of two objects we analyzed before. The first term is the sterilization loss, expressed in period 2 consumption goods, that arise when there is a strict violation from interest parity. As we have discussed before, these losses are proportional to the foreign wealth of investors. The second term is the loss due to the distortion in the consumption-saving decisions of domestic households. A rise in s_1 increases the real rate, and distorts consumption towards the second period.

Figure 6 shows how the optimal policy and allocations vary with the level of the sticky wage. For low levels of ω_1 , the ZLB is not binding and the economy is at the efficient level of employment. As ω_1 increases, the ZLB becomes binding and the economy experiences a steep decline in output. As ω_1 increases further, the CB intervenes by accumulating assets, opening a deviation from interest parity. Due to this intervention, the CB is able to moderate the recession.

7 Empirical Evidence

We now verify whether the main predictions of our theory find support in the data. In Section 7.1 we explore the links between foreign reserves, deviations from interest rate parity and

Figure 6: Optimal Exchange Rate Interventions



nominal interest rate for a group of advanced economies. We document two main facts that are consistent with the basic mechanisms of our theory. First, foreign reserves held by monetary authorities are positively related, both across countries and over time, to deviations from the covered interest rate parity. Second, deviations from the covered interest rate parity arise mostly for currencies whose nominal interest rates gravitate around zero. In Section 7.2 we use our simple formula of equation (XX) to provide an estimate of the costs of the recent foreign exchange interventions by the Swiss National Bank (SNB).

7.1 Foreign reserves, nominal interest rates and CIP gaps

In our model the monetary authority can increase the domestic real interest rate relative to the world real interest rate by accumulating foreign assets. While these interventions are costly from the point of view of a small open economy, we have seen that a Central Bank may optimally implement them in order to temporarily depreciate its currency while at the zero lower bound. We now check whether these model predictions are consistent with basic facts about the relation between foreign reserves, nominal interest rates, and deviations between domestic and the world real interest rates.

To this end, we construct proxies for these three variables for a group of advanced economies over the 2000-2015 period.¹⁶ We obtain yearly data on foreign reserves holdings from the IMF *International Financial Statistics*,¹⁷ and we scale it by annual gross domestic product obtained from the OECD *National Accounts*. Both foreign reserves and gross domestic product are expressed in U.S. dollars at current prices. The ratio between foreign reserves and gross domestic product is our proxy for the size of foreign reserves.

The measurement of deviations between the domestic and the world real interest rates is more involved. In our model, these gaps generate arbitrage opportunities for foreign investors, and they have drastic effects on capital flows. Because our model is deterministic, we could proxy these gaps either with deviations from the covered interest rate parity (CIP) conditions, or using deviations from the uncovered one (UIP). However, it is well known that deviations from UIP may reflect compensation that risk averse lenders require for holding currency risk, and the literature has hardly interpreted their presence in the data as an indication of arbitrage opportunities (Engel, 2014). Because of that, we map the gap between

¹⁶As we detail below, we proxy deviations between the domestic and the world real interest rates using gaps in the covered interest rate parity condition. Because of that, we restrict the analysis to a set of countries for which data on currency forwards are of sufficiently good quality. Our sample borrows mostly from Du et al. (2016) and it includes Switzerland, Japan, Denmark, Sweden, Canada, the U.K., Australia, New Zealand, Norway and Israel.

¹⁷Total reserves comprise holdings of monetary gold, special drawing rights, reserves of IMF members held by the IMF, and holdings of foreign exchange under the control of monetary authorities. The gold component of these reserves is valued at year-end (December 31) London prices.

a country’s real interest rate and the U.S. real interest rate using deviations from the CIP condition. Specifically, letting $i_{t,t+n}^{\$}$ denote the nominal interest rate in US dollars between time t and time $t+n$, $i_{t,t+n}^j$ the corresponding interest rate in currency j , $s_t^{j,\$}$ the spot exchange rate of currency j per U.S. dollar, and $f_{t,t+n}^{j,\$}$ the n -periods ahead associated forward contract, we can express deviations from the CIP condition as

$$\text{cip gaps}_{t,t+n}^{j,\$} = i_{t,t+n}^j - i_{t,t+n}^{\$} + \frac{1}{n}[\log(s_t^{j,\$}) - \log(f_{t,t+n}^{j,\$})].$$

A positive value for this indicator is equivalent, in our model, to a positive gap between the real interest rate in country j and the world real interest rate.

We calculate deviations from the CIP condition at a three months horizon between major currencies and the U.S. dollar for the period 2000-2015. We map $i_{t,t+n}^j$ to the interest rate on an overnight indexed swap (OIS) of three-month duration in currency j , while $i_{t,t+n}^{\$}$ is the respective OIS rate in U.S. dollars with the same duration.¹⁸ The variable $f_{t,t+n}^{j,\$}$ is the three-months forward rate between currency j and the U.S. dollar. All these data are obtained at daily frequency from Bloomberg, and we use the mid-point between the bid and the ask quotes.

Figure x in the Appendix plots these three time series for each country in our group. The figure shows interesting patterns, both over time and across countries. First, there has been a sizable increase in the foreign reserve to GDP ratio for advanced economies over this period, which on average went from 9% in 2001 to 25.4% in 2015. This trend was more pronounced for certain economies than for others: Central Banks in Switzerland, Japan, and Denmark have substantially increased their foreign asset position during the sample, while Central Banks in Australia, Canada and the UK didn’t. Second, nominal interest rates have been declining over time: by the end of the period, we have a group of countries with zero or even negative nominal interest rates (Denmark, Switzerland, Japan, and Sweden), and countries with clearly positive nominal rates (Australia, and New Zealand). The third panel in the figure reports yearly averages CIP gaps in our sample. Prior to 2007, CIP deviations were on average small and close to zero for all the advanced economies considered here, a fact that is well establish in the literature. During the global financial crisis of 2007-2009, we have observed major deviations from covered interest rate parity for all the currencies in our sample.¹⁹ Interestingly, these deviations have persisted even after the financial crisis

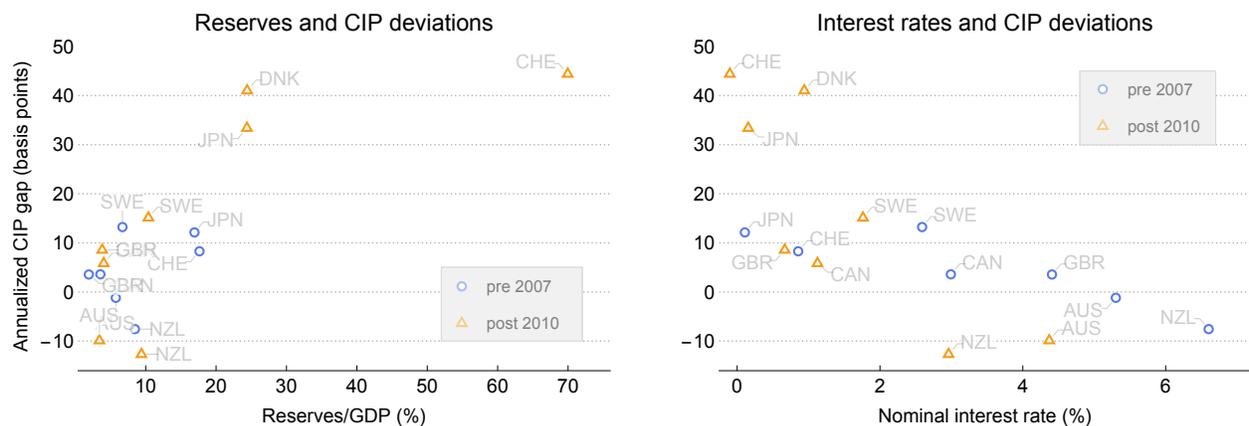
¹⁸We also do the computations using Libor rates. See Appendix X for details.

¹⁹Baba and Packer (2009) and Ivashina et al. (2015) explain the deviations during the financial crisis through a combination of dollar funding shortages in foreign exchange markets and the inability of financial intermediaries to take advantage of these arbitrage opportunities because of funding constraints during the financial crisis. Our focus however, is in the period after the financial crisis.

for a group of countries, most notably Switzerland, Denmark, Japan, and Sweden. [Du et al. \(2016\)](#) and [Borio et al. \(2016\)](#) discuss in details these failures of the CIP condition in the post 2008 period.

We exploit the variation in these three series, both across countries and over time, to verify two main predictions of the model. Because the deviations from CIP during 2007, 2008 and 2009 were extreme and probably due to the effects of the crisis, we exclude this period and split the time dimension of our sample in two subsets: before (2002-2006) and after (2010-2015) the financial crisis. The left panel of [Figure 7](#) plots the average foreign reserve holdings to GDP ratio against the average CIP deviations in these two sub-samples for each country in our group. The plot shows a positive relationship, both across countries and over time, between the level of foreign reserves and the deviations from the CIP. This empirical finding, which to best of our knowledge has not been previously noted in the literature, is consistent with the mechanism at the heart of our model, whereby the monetary authority increase the domestic real interest rate relative to the world real interest rate by accumulating a sufficiently large position in foreign assets.

Figure 7: **Foreign reserves, nominal interest rates and CIP deviations**



The right panel of [Figure 7](#) plots the nominal interest rate against the average CIP deviations in these two subsamples. We can observe that the CIP gaps are positive for countries-time periods characterized by low nominal interest rates, while they tend to be small when nominal interest rates are positive. This negative relation between CIP gaps and nominal interest rates, originally uncovered by [Du et al. \(2016\)](#), lends support to our result

that Central Banks find it optimal to engage in large foreign exchange interventions when the zero lower bound constraint on nominal interest rates binds.

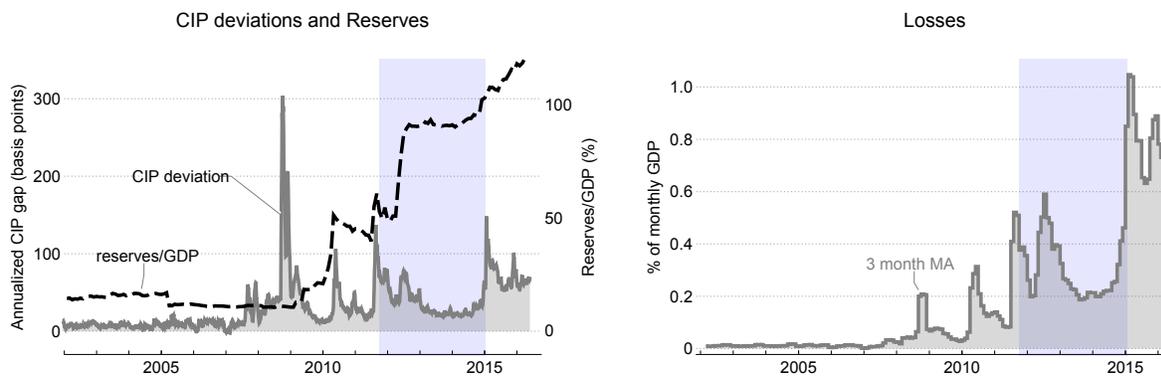
7.2 The costs of the SNB foreign exchange interventions

In this subsection, we use the Swiss experience to obtain guidance on the size of the potential losses faced by Central Banks. Starting from 2010, the Swiss National Bank has intervened massively in foreign exchange markets, either to defend an explicit target for the exchange rate,²⁰ or, more informally, to fight appreciation pressures on the Swiss franc. Our theory provides a simple expression to measure the costs associated to these interventions,

$$\text{losses}_t = \left[\frac{(1 + i_t) s_t}{(1 + i_t^*) s_{t+1}} - 1 \right] \times F_t. \quad (27)$$

We can use our data on CIP deviations (on a horizon of three months) and on foreign reserves to provide an approximation for the costs of these foreign exchange interventions.²¹

Figure 8: Foreign reserves, CIP deviations and losses



In the left panel of Figure 8, we report the monthly three-month CIP deviations between the Swiss franc and the U.S. dollar along with a monthly series for the stock foreign reserves as a fraction of Swiss GDP. This plot confirms that the positive relation between foreign reserves and CIP deviations that we have uncovered earlier in our panel holds at a much higher frequency: after the U.S. financial crisis, spikes in the CIP gaps are associated to massive interventions of the SNB.

²⁰Between 2011 and 2015, the Swiss National Bank successfully kept a floor of 1.2 Swiss franc per euro.

²¹See Appendix A.

In the right panel of Figure 8, we report our corresponding measure of the monthly losses as a fraction of monthly Swiss GDP. The lightly shaded area represents the period where the SNB maintained an official floor on the franc. As can be seen, throughout this period, the losses were significant. They reach their highest point around January 2015, the month when the SNB decided to abandon the currency floor vis a vis the euro.

8 Conclusions

[TO BE COMPLETED]

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A Calculating the losses

Our formula in (27) is an approximation because the Swiss National Bank holds several assets in the form of foreign reserves that differ by maturity, currency of denomination, and underlying riskiness. The appropriate way to measure the losses would be that of computing CIP deviations for different currencies and at different horizons, and appropriately matching these gaps with the different asset purchases made by SNB.

The way we proceed is different. We will assume that all assets in the SNB balance sheet are three month zero coupon bonds, denominated in US dollars. We observe the monthly market value of the reserves portfolio from the SNB. Let us denote that series by S_t . Let n_t denote the amount of foreign denominated zero coupon bonds purchased by the SNB in period t .

The market value of the foreign reserve portfolio at the end of period t is:

$$S_t = q_t^3 n_t + q_t^2 n_{t-1} + q_t^1 n_{t-2},$$

where q_t^i is the international price at time t of a zero-coupon bonds that matures in i periods. The amount of foreign reserves purchased in period t is then

$$F_t = q_t^3 n_t.$$

And thus, we have that the market value of the stock can be written as

$$S_t = F_t + \frac{q_t^2}{q_{t-1}^3} F_{t-1} + \frac{q_t^1}{q_{t-2}^3} F_{t-2}$$

We know that $q_t^3 = (1 + i_t^*)^{-1}$, and we approximate the one period and the 2 period ahead interest rate to be $q_t^2 = (1 + i_t^*)^{-2/3}$ and $q_t^1 = (1 + i_t^*)^{-1/3}$.

Using the observed series for S_t , and the three month international interest rate, i_t^* , we can then solve for the series F_t that solves

$$S_t = F_t + \frac{1 + i_{t-1}^*}{(1 + i_t^*)^{2/3}} F_{t-1} + \frac{1 + i_{t-2}^*}{(1 + i_t^*)^{-1/3}} F_{t-2},$$

for some starting points with $F_{t_0} = F_{t_0+1} = 0$.

Having computed F_t , we then apply the formula (27) to calculate the monthly losses of the Central Bank.