Interest rate policies, banking and the macro-economy[*]

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VERY PRELIMINARY AND INCOMPLETE

Abstract

Low interest rates may stimulate the economy by encouraging spending. However, if the wealth of savers is important for the demand of production inputs (for example labor), low interest rates may also discourage savings with negative effects on the real sector of the economy. I show this channel using a model in which banks play a central role in the intermediation of funds and policy interventions by the monetary/fiscal authority take the form of asset purchases from financial intermediaries. Policies aimed at reducing the interest rate induce a fall in aggregate production and an increase in macroeconomic volatility.

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1   Introduction

The debate over the effectiveness of monetary policy often centers around the benefits of low interest rates as a stimulus for the real economy. The idea is that low interest rates encourage spending, either in the form of consumption or investment, and this promotes employment and production. The potential cost of low interest rates is the possibility of inflation, not only in commodity prices but also in the price of assets (for example, real estate). Therefore, the debate about the desirability of low interest rates centers around the trade-off between economic stimulus and higher inflation.

However, low interest rates have two additional implications that have received less attention in the monetary policy debate. The first implication is that low interest rates reduce the incentive of savers to hold liquid financial assets. The second implication is that low interest rates increase the incentive of financial intermediaries to leverage. In this paper I show that the first implication (lower liquid assets held by savers) discourages economic activity while the second (higher leverage in financial intermediation) increases macroeconomic instability.

I show these results by extending the theoretical framework developed in Quadrini (2017) to include a monetary/fiscal authority that controls interest rates through the supply of funds to the financial intermediation sector. In addition to the monetary/fiscal authority, the model consists of three sectors: a production sector, a household sector, and a financial sector. The equilibrium structure of the model is somewhat special compared to other macroeconomic models with financial intermediation: in equilibrium producers (firms) are net savers while households are net borrowers. By working with this theoretical framework, I am able to capture the fact that US corporations hold high volumes of financial assets (cash) which, in aggregate, are in excess of their financial liabilities. Thus, the corporate sector is no longer a net borrower. On the other hand, household debt has been growing over time and it has reached a very high level in relation to household income.

If firms hold large volumes of cash despite the low rate of return, it must be because they provide some value to firms on top of the earned interests. In the model studied in this paper firms hold low interest bearing assets because they provide insurance against production risks. Because of the insurance service, when firms hold more financial assets they are willing to take more production risks, which translate in higher demand for labor and higher economic activity. But when he interest rate falls, in equilibrium firms

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hold less financial assets. This implies that firms are less insured and, as a result, they cut risky production. This is the mechanism through which lower interest rates have a negative impact on economic activity.

In the model financial intermediaries issue liabilities that are sold to the market. When the interest rate is low, financial intermediaries have a higher incentive to finance investments with more liabilities and less equity, that is, they increase leverage. But higher leverage also implies that a financial crisis has larger macroeconomic consequences. In particular, it generates a larger redistribution of financial wealth from savers (which in the model are producers) to borrowers (which in the model are households). But larger redistribution of financial wealth away from savers-producers implies that they will cut more the demand of labor, generating a stronger macroeconomic contraction. So, ultimately, a policy of low interest rates generates a contraction in economic activity and increases macroeconomic volatility.

The organization of the paper is as follows. Section 2 describes the model, starting with the monetary authority, and characterizes the equilibrium. Section 3 uses the model to study how the action of the monetary authority/fiscal authority affects interest rates and real equilibrium allocations. The final Section 4 provides some concluding remarks.

2 Model

The economy is composed of three sectors: the entrepreneurial sector, the household sector and the financial intermediation sector. The role of financial intermediaries is to facilitate the transfer of resources between entrepreneurs and households. In addition there is a monetary/fiscal authority that purchases bank liabilities with funds raised by taxing households. In strict sense, the funds used by the monetary/fiscal authority to purchase bank liabilities are not fiat money. However, they play a similar role since they increase the funds that banks can use to make loans. I will then refer to the holdings of bank liabilities by the monetary/fiscal authority as 'money' and denote them by M_t.

All variables are in real terms and I abstract from nominal prices. Of course, by doing so, I will not be able to studying the implication of monetary policy for inflation. However, this allows me to simplify the presentation of the central mechanism I would like to emphasize in this paper.

Bank liabilities pay the gross interest R_t^l and the monetary/fiscal author-

ity faces the following budget constraint

$$M_t = \frac{M_{t+1}}{R_t^l} + T_t,$$

where T_t are lump-sum transfers to households (or taxes if negative).

Before continuing it would be useful to stress that the purchase of bank liabilities by the monetary/fiscal authority is similar to open markets operations. In fact, I could assume that there is a stock of government bonds in circulation that pay the gross interest rate R_t^l. Monetary policy interventions would then consist in the purchase of these bonds from banks. By holding government bonds the monetary authority would earn the gross interest rate R_t^l. The difference, however, is that the purchase of government bonds is not made with fiat money but they are fully funded with taxes. As observed above, even if there is no fiat money, the transmission mechanism is similar: purchases of government bonds will generate an injection of funds in the banking system which will then be used by banks to make more loans. I now describe each of the three sectors starting with the financial intermediation sector.

2.1 Financial intermediation sector

There is a continuum of infinitely lived financial intermediaries. Financial intermediaries are profit maximizing firms owned by households. The financial intermediation sector should be interpreted as capturing all financial firms. Thus, it is not limited to commercial banks.

A bank starts the period with investments i_t and liabilities l_t. The difference between investments and liabilities is bank equity $e_t = i_t - l_t$. We will see that in equilibrium the investments of banks are loans made to households and the liabilities are held in part by entrepreneurs and in part by the monetary/fiscal authority. However, this is an equilibrium property of the model and at this stage I do not need to specify which sector holds the liabilities of banks and which sector receives the investments.

Given the beginning of period balance sheet position, the bank could default on its liabilities. In case of default creditors have the right to liquidate the bank investments i_t. However, they may not be able to recover the full value of the investments. In particular, with probability λ creditors recover a fraction $\underline{\xi} < 1$ while with probability $1 - \lambda$ they recover the full value of the

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investments. Denoting by $\xi_t \in \{\underline{\xi}, 1\}$ the fraction of the bank investments recovered by creditors, the recovery value is $\xi_t i_t$.

The stochastic variable ξ_t is the same for all banks (aggregate shock) and its value is unknown when the bank issues liabilities l_t and make investments i_t. In this paper ξ_t follows an exogenous stochastic process. However, this variable can be made endogenous if we interpret ξ_t as the market price of bank investments which depends on the liquidity of the whole banking system (see Quadrini (2017)).

The choice of l_t and i_t are made at the end of period $t-1$. The realization of ξ_t, instead, arises at the beginning of period t. Thus, the bank enters period t with l_t and i_t and, knowing ξ_t, it could use the threat of default to renegotiate its liabilities. Assuming that the bank has the whole bargaining power, the liabilities can be renegotiated to $\xi_t i_t$, that is, to the value that the creditors would recover in case of liquidation. Therefore, after renegotiation, the residual liabilities of the bank are

$$
\tilde{l}_t(l_t, i_t) = \begin{cases} l_t, & \text{if } l_t \leq \xi_t i_t \\ \\ \xi_t i_t & \text{if } l_t > \xi_t i_t \end{cases}. \tag{1}
$$

Financial intermediation implies an operation cost that depends on the leverage chosen by the bank. Denoting the leverage by $\omega_{t+1} = l_{t+1}/i_{t+1}$, the operation cost takes the form

$$
\varphi\left(\omega_{t+1}\right) q_t l_{t+1},
$$

where q_t is the price of the newly issued liabilities and $q_t l_{t+1}$ are the funds raised by the bank.

Assumption 1 *The function $\varphi(\omega_{t+1})$ is twice continuously differentiable. For $\omega_{t+1} \leq \underline{\xi}$ it is constant at τ. For $\omega_{t+1} > \underline{\xi}$ it is strictly increasing and convex, that is, $\varphi'(\omega_{t+1}) > 0$ and $\varphi''(\omega_{t+1}) > 0$.*

The unit cost function is constant and equal to τ if the leverage ω_{t+1} is smaller than $\underline{\xi}$ but it becomes increasing and convex for $\omega_{t+1} > \underline{\xi}$. This assumption captures, in reduced form, the potential agency frictions that become more severe when leverage is higher. From a technical point of view, it insures that the optimal choice of leverage is an interior solution to the bank

problem specified below. Being an interior solution, banks would optimally change the leverage when market conditions change.

Denote by $\overline{R}_t^l$ the expected gross return on the market portfolio of bank liabilities issued in period t and repaid in period $t+1$. This is the expected return on the liabilities of the whole banking sector. Since banks are atomistic and competitive, the expected return on the liabilities issued by an individual bank must be equal to the aggregate expected return $\overline{R}_t^l$. Therefore, the price for the liabilities issued by an individual bank at t must satisfy

$$q_t(l_{t+1}, i_{t+1})l_{t+1} = \frac{1}{\overline{R}_t^l}\mathbb{E}_t\tilde{l}_{t+1}(l_{t+1}, i_{t+1}). \tag{2}$$

The left-hand-side is the payment made by investors (entrepreneurs) to purchase b_{t+1} at price $q_t(b_{t+1}, l_{t+1})$. The right-hand-side is the expected repayment in the next period, discounted by $\overline{R}_t^l$ (the expected market return). The expected repayment and, therefore, the price of the bank liabilities depends on the financial structure chosen by the bank, that is, l_{t+1} and i_{t+1}. Condition (2) guarantees that, whatever the policy chosen by the bank, the holders of its liabilities receive the same expected return $\overline{R}_t^l$.

The budget constraint of the bank, after the renegotiation of its liabilities can be written

$$\tilde{l}_t(l_t, i_t) + \frac{i_{t+1}}{R_t^i} + div_t = i_t + q_t(l_{t+1}, i_{t+1})l_{t+1}\left[1 - \varphi\left(\frac{l_{t+1}}{i_{t+1}}\right)\right]. \tag{3}$$

The left-hand-side contains the residual liabilities after renegotiation, the funds needed to make new investments, and the dividends paid to shareholders (households). The right-hand-side contains the repayment of the loans made to households and the funds raised by issuing new liabilities, net of the operation cost. Using condition (2), the funds raised with the new liabilities are $\mathbb{E}_t\tilde{l}_{t+1}(l_{t+1}, i_{t+1})/\overline{R}_t^l$.

The optimization problem solved by the bank is

$$V_t(l_t, i_t) = \max_{div_t, l_{t+1}, i_{t+1}} \left\{ div_t + \beta\mathbb{E}_t V_{t+1}(l_{t+1}, i_{t+1}) \right\} \tag{4}$$

subject to (1), (2), (3).

Notice that the problem takes into account the renegotiation of the debt through the function $\tilde{l}_t(l_t, i_t)$ in the budget constraint. The leverage cannot

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exceed 1 since in this case the bank would renegotiate with certainty. Once the probability of renegotiation is 1, a further increase in l_{t+1} does not increase the borrowed funds $\mathbb{E}_t \tilde{l}_{t+1}(l_{t+1}, i_{t+1})/\overline{R}_t$ but raises the operation cost. Therefore, Problem (4) is also subject to the constraint $l_{t+1} \leq i_{t+1}$.

The first order conditions with respect to l_{t+1} and i_{t+1}, derived in Appendix A, are

$$\frac{1}{\overline{R}_t^l} \geq \beta\Big[1 + \Phi(\omega_{t+1})\Big] \tag{5}$$

$$\frac{1}{R_t^i} \geq \beta\Big[1 + \Psi(\omega_{t+1})\Big], \tag{6}$$

with $\Phi(\omega_{t+1})$ and $\Psi(\omega_{t+1})$ increasing in leverage $\omega_{t+1} = l_{t+1}/i_{t+1}$. These conditions are satisfied with equality if $\omega_{t+1} < 1$ and inequality if $\omega_{t+1} = 1$.

From condition (5) we can see that leverage $\omega_{t+1} = l_{t+1}/i_{t+1}$ is the relevant variable, not the scale of operation l_{t+1} or i_{t+1}. This follows from the linearity of the intermediation technology and the risk neutrality of banks. Bank leverage matters because it affects the operation cost. These properties imply that in equilibrium all banks choose the same leverage ω_{t+1} (although they could chose different scales of operation).

Because the first order conditions (5) and (6) depend only on ω_{t+1}, there is no guarantee that these conditions are both satisfied for arbitrary values of $\overline{R}_t^l$ and R_t^i. In the general equilibrium, however, these rates adjust to clear the markets for bank liabilities and investments so both conditions will be satisfied.

Lemma 2.1 *If $\omega_{t+1} > \underline{\xi}$, then $\overline{R}_l^l < R_t^i < \frac{1}{\beta}$ and $R_t^i/\overline{R}_t^l$ increases with ω_{t+1}.*

Proof 2.1 *Appendix B.*

Since leverage increases the operation cost, the bank chooses to do so only if there is a differential between the cost of funds and the return on investments. As the spread increases, banks are willing to pay the higher cost induced by higher leverage. When the leverage exceeds $\underline{\xi}$, banks could default with positive probability. Default implies losses for the holders of bank liabilities.

In the next section we will see that the holders of bank liabilities are entrepreneurs. Thus, bank default implies a wealth loss for entrepreneurs and this affects adversely their willingness to undertake production risks with negative macroeconomic consequences.

2.2 Productioin sector

Production is carried out by a unit mass of entrepreneurs with lifetime utility $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t)$. Entrepreneurs are individual owners of firms, each operating the production function $y_t = z_t h_t$, where h_t is the input of labor supplied by households at the wage rate w_t, and z_t is an idiosyncratic productivity shock. The productivity shock is independently and identically distributed among firms and over time, with probability distribution $\Gamma(z)$.

A key assumption is that the input of labor h_t is chosen before observing the idiosyncratic productivity z_t. Since entrepreneurs are risk-averse, this assumption implies that labor is risky.

To facilitate consumption smoothing, entrepreneurs can hold bank liabilities, denoted by b_t. However, since banks could default on their liabilities, what matters for entrepreneurs is the actual value of these liabilities, that is, the value after renegotiation which I denote by $\tilde{b}_t$. The budget constraint faced by an entrepreneur in period t is

$$c_t + q_t b_{t+1} = (z_t - w_t)h_t + \tilde{b}_t. \tag{7}$$

An entrepreneur enters the period with financial wealth b_t (bank liabilities). After banks renegotiate, the wealth of the entrepreneur becomes $\tilde{b}_t$. Given $\tilde{b}_t$, the entrepreneur chooses the labor input h_t and, after the realization of the idiosyncratic shock z_t, he/she chooses consumption c_t and next period financial assets b_{t+1}.

Because labor h_t is chosen before the realization of z_t, while the saving decision is made after the observation of z_t, it will be convenient to define $n_t = \tilde{b}_t + (z_t - w_t)h_t$ as the entrepreneur's net worth after production. Given the timing structure, the input of labor h_t depends on $\tilde{b}_t$ while the saving decision $q_t b_{t+1}$ depends on n_t.

Lemma 2.2 *Let ϕ_t satisfy $\mathbb{E}_z \left\{ \frac{z - w_t}{1 + (z - w_t)\phi_t} \right\} = 0$. The optimal entrepreneur's policies are*

$$
\begin{aligned}
h_t &= \phi_t \tilde{b}_t, \\
c_t &= (1 - \beta)n_t, \\
q_t b_{t+1} &= \beta n_t.
\end{aligned}
$$

Proof 2.2 *Appendix C.*

The demand for labor is linear in the financial wealth of the entrepreneur $\tilde{b}_t$. The term of proportionality ϕ_t is defined by the condition $\mathbb{E}_z \left\{ \frac{z-w_t}{1+(z-w_t)\phi_t} \right\} = 0$, where the expectation is over the idiosyncratic productivity z. Since the only endogenous variable that affects ϕ_t is the wage rate, I denote this term by $\phi(w_t)$. It is easy to verify that this function is strictly decreasing in w_t.

Because $\phi(w_t)$ is the same for all entrepreneurs, the aggregate demand for labor is

$$H_t = \phi(w_t) \int b_t = \phi(w_t)\tilde{B}_t,$$

where I have used capital letters to denote average (per-capita) variables.

We can see from the above expression that the demand for labor depends negatively on the wage rate and positively on the financial wealth of entrepreneurs $\tilde{B}_t$. When banks default, the renegotiated wealth of entrepreneurs $\tilde{B}_t$ drops, that is, $\tilde{B}_t < B_t$. The immediate impact of this is a reduction in the demand for labor.

Although the dependence of the production scale from the wealth of entrepreneurs is a features of many models with financial market frictions, the mechanism that in this model generates this property is different. It does not derive from the need to finance working capital or investments with binding borrowing constraints. Instead, it derives from the assumption that production is risky and the entrepreneur is willing to hire more labor only if he/she holds a larger wealth buffer that allows for smoother consumption. Thanks to this feature, financial market frictions play an important role for the real sector of the economy even if producers are not borrowing constrained.

2.3 Household sector

There is a unit mass of households with utility $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(c_t - \alpha \frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right)$, where c_t is consumption and h_t is labor. Households do not face idiosyncratic risks and the assumption of risk neutrality is not important for the key properties of the model.

Each household holds a non-reproducible asset available in fixed supply $\overline{K}$, each producing χ units of consumption goods. I think of the non-reproducible asset as housing and χ as housing services. Houses are divisible and can be traded at market price p_t. Households can borrow from banks at

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the gross interest rate R_t^i and face the budget constraint

$$c_t + d_t + (k_{t+1} - k_t)p_t = \frac{d_{t+1}}{R_t^i} + w_t h_t + \chi k_t + T_t,$$

where d_t is the loan (household debt) contracted in period $t-1$ (due in period t), and d_{t+1} is the new loan that will be repaid in the next period $t+1$. The variable T_t denotes the transfers received from the monetary/fiscal authority.

Household debt is constrained by the following borrowing limit

$$d_{t+1} \leq \kappa + \eta \mathbb{E}_t p_{t+1} k_{t+1}, \tag{8}$$

where κ and η are constant parameters.

I will consider two special cases. To obtain analytical intuitions, I will first consider the case with $\eta = 0$ so that the borrowing limit is constant. Then, in the quantitative section, I will consider the more general case with $\eta > 0$.

The first order conditions can be written as

$$\alpha h_t^{\frac{1}{\nu}} = w_t, \tag{9}$$

$$1 = \beta R_t^i (1 + \mu_t), \tag{10}$$

$$p_t = \beta \mathbb{E}_t \Big[\chi + (1 + \eta \mu_t) p_{t+1} \Big]. \tag{11}$$

The term $\beta \mu_t$ is the Lagrange multiplier for the borrowing constraint. From the third equation we can see that, if $\eta = 0$, the real estate price p_t must be constant. Instead, when $\eta > 0$, p_t depends on the tightness of the borrowing constraint captured by the multiplier μ_t.

2.4 Equilibrium with direct borrowing and lending

Before characterizing the full equilibrium, it would be convenient to characterize a simplified version of the model without financial intermediaries. In this case the loans taken by households, D_t, are equal to the financial wealth of entrepreneurs plus the financial assets held by the monetary authority, that is, $B_t + M_t$. In this simplified version of the economy the monetary/fiscal authority lends directly to households and there is no default. This implies that $\tilde{B}_t = B_t$. The equilibrium prices satisfy $1/q_t = R_t^l = R_t^i = R_t$. To further simplify the model, I assume that $\eta = 0$ in the borrowing constraint of households, equation (8). Thus, the borrowing capacity of households is fixed.

Proposition 2.1 *In absence of aggregate shocks, the economy converges to a steady state in which households borrow from entrepreneurs and monetary/fiscal authority, and $\beta R < 1$.*

Proof 2.1 *Appendix D.*

The fact that the steady state interest rate is lower than the intertemporal discount rate is a consequence of the uninsurable risk faced by entrepreneurs. If $\beta R = 1$, entrepreneurs would continue to accumulate bonds without limit in order to insure the idiosyncratic risk. The supply of bonds from households, however, is limited by the borrowing constraint of households. To insure that entrepreneurs do not accumulate an infinite amount of bonds, the interest rate has to fall below the intertemporal discount rate.

The equilibrium in the labor market can be characterized as the intersection of aggregate demand and supply as depicted in Figure 1. The aggregate demand was derived in the previous subsection and takes the form $H_t^D = \phi(w_t)B_t$. The supply is derived from the households' first order condition (9) and takes the form $H_t^S = \left(\frac{w_t}{\alpha}\right)^\nu$.
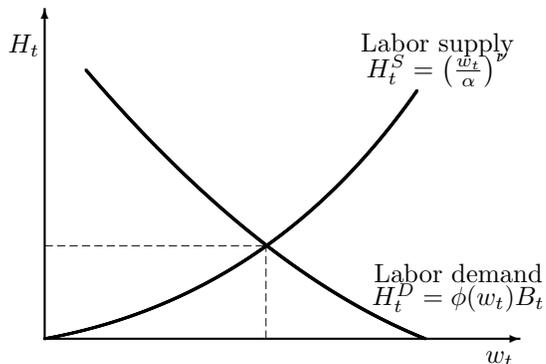

Figure 1: Labor market equilibrium.

The dependence of the demand of labor from the financial wealth of entrepreneurs is a key property of this model. When entrepreneurs hold a lower value of B_t, the demand for labor declines and in equilibrium there is lower employment and production.

In this simplified environment is easy to see the effect of an increase in liquidity. Since $B_t + M_t = D_t$, but $D_t = \kappa$, an increase in M_t has to be

followed by a reduction in B_t. In other words, the financial wealth held by entrepreneurs is crowded out by the financial assets held by the monetary authority. Of course, to induce entrepreneurs to hold less financial assets, the interest rate R_t has to fall. But when the interest rate is low, entrepreneurs hold less financial wealth which in turn reduces the demand of labor. This is the channel through which lower interest rates could have negative macroeconomic effects.

This channel is also present in the general model with financial intermediaries. The intermediation of banks also introduces an additional mechanism in which the fall in the financial wealth of entrepreneurs could be the result of a banking crisis. I will then be able to show that lower is the interest rate and bigger are the macroeconomic consequence of a banking crisis.

2.5 General equilibrium

To characterize the general equilibrium with financial intermediation, I first derive the aggregate demand for bank liabilities from the optimal saving of entrepreneurs, B_{t+1}, and the demand from the monetary authority M_{t+1}. I then derive the supply by consolidating the demand of loans from households with the optimal policy of banks. In this section I continue to assume that $\eta = 0$ so that the borrowing limit specified in equation (8) reduces to $d_{t+1} \leq \kappa$. This allows me to characterize the equilibrium analytically.

Demand for bank liabilities As shown in Lemma 2.2, entrepreneurs' saving takes the form $q_t b_{t+1} = \beta n_t$, where n_t is the end-of-period wealth $n_t = \tilde{b}_t + (z_t - w_t) h_t$.

Since $h_t = \phi(w_t) \tilde{b}_t$ (see Lemma 2.2), the end-of-period net worth can be rewritten as $n_t = [1 + (z_t - w_t)\phi(w_t)]\tilde{b}_t$. Substituting into the optimal saving and aggregating over all entrepreneurs we obtain

$$B_{t+1} = \beta \left[1 + (\bar{z} - w_t)\phi(w_t) \right] \frac{\tilde{B}_t}{q_t}. \tag{12}$$

This equation defines the aggregate demand for bank liabilities from entrepreneurs as a function of its price q_t, the wage rate w_t, and the aggregate wealth $\tilde{B}_t$. Remember that the tilde sign denotes the financial wealth of entrepreneurs after the renegotiation of banks. Also notice that $1/q_t$ is not the 'expected' return from bank liabilities which we previously denoted by $\overline{R}_t^l$ since banks will repay B_{t+1} in full only with some probability.

Using the equilibrium condition in the labor market, we can express the wage rate as a function of $\tilde{B}_t$. In particular, equalizing the demand for labor, $H_t^D = \phi(w_t)\tilde{B}_t$, to the supply from households, $H_t^S = (w_t/\alpha)^\nu$, the wage w_t becomes a function of only $\tilde{B}_t$. We can then use this function to replace w_t in (12) and express the demand for bank liabilities as a function of only $\tilde{B}_t$ and q_t. This takes the form

$$B_{t+1} = \frac{s(\tilde{B}_t)}{q_t}, \tag{13}$$

where $s(\tilde{B}_t)$ is strictly increasing in the financial wealth of entrepreneurs $\tilde{B}_t$.

The total demand of bank liabilities is the sum of the demand coming from entrepreneurs, B_{t+1}, and the demand coming from the monetary authority M_{t+1}. Figure 2 plots the total demand for a given value of $\tilde{B}_t$. As we change $\tilde{B}_t$, the slope of the demand function changes. More specifically, keeping the price q_t constant, higher initial wealth $\tilde{B}_t$ implies higher demand coming from entrepreneurs and, therefore, higher total demand $B_{t+1} + M_{t+1}$.

Supply of bank liabilities The supply of bank liabilities is derived from consolidating the borrowing decisions of households with the investment and funding decisions of banks.

According to Lemma 2.1, when banks are highly leveraged, that is, $\omega_{t+1} > \underline{\xi}$, the interest rate on bank investments must be smaller than the intertemporal discount rate ($R_t^i < 1/\beta$). From the households' first order condition (10) we can see that $\mu_t > 0$ if $R_t^i < 1/\beta$. Therefore, the borrowing constraint for households is binding, that is, $D_{t+1} = \kappa$. Since $L_{t+1} = \omega_{t+1}I_{t+1}$, and $I_{t+1} = D_{t+1}$, the supply of bank liabilities is then $L_{t+1} = \kappa\omega_{t+1}$.

When the lending rate is equal to the intertemporal discount rate, instead, the demand of loans from households is undetermined, which in turn implies indeterminacy in the supply of bank liabilities. In this case the equilibrium liabilities are only determined by the demand. In summary, the supply of bank liabilities is

$$L(\omega_{t+1}) = \begin{cases} \text{Undetermined}, & \text{if} \quad \omega_{t+1} < \underline{\xi} \\[2mm] \kappa\omega_{t+1}, & \text{if} \quad \omega_{t+1} \geq \underline{\xi} \end{cases} \tag{14}$$

So far I have derived the supply of bank liabilities as a function of bank leverage ω_{t+1}. However, the leverage of banks also depends on the cost of

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borrowing $\overline{R}_t^i$ through condition (5). The expected return on bank liabilities for the holder of these liabilities, $\overline{R}_t^i$, is in turn related to the price q_t by the condition

$$\overline{R}_t^i = \left[1 - \theta(\omega_{t+1}) + \theta(\omega_{t+1})\left(\frac{\underline{\xi}}{\omega_{t+1}}\right)\right]\frac{1}{q_t}. \tag{15}$$

With probability $1 - \theta(\omega_{t+1})$ banks do not renegotiate and the ex-post return is $1/q_t$. With probability $\theta(\omega_{t+1})$ banks renegotiate and investors recover only a fraction $\underline{\xi}/\omega_{t+1}$ of the initial investment. Therefore, when banks renegotiate, the ex-post return is $(\underline{\xi}/\omega_{t+1})/q_t$.

Using (15) to replace $\overline{R}_t^i$ in equation (5) I obtain a function that relates the price q_t to the leverage ω_{t+1}. Finally, I combine this function with $L_{t+1} = \kappa\omega_{t+1}$ to obtain the supply of bank liabilities as a function of q_t. This is plotted in Figure 2.
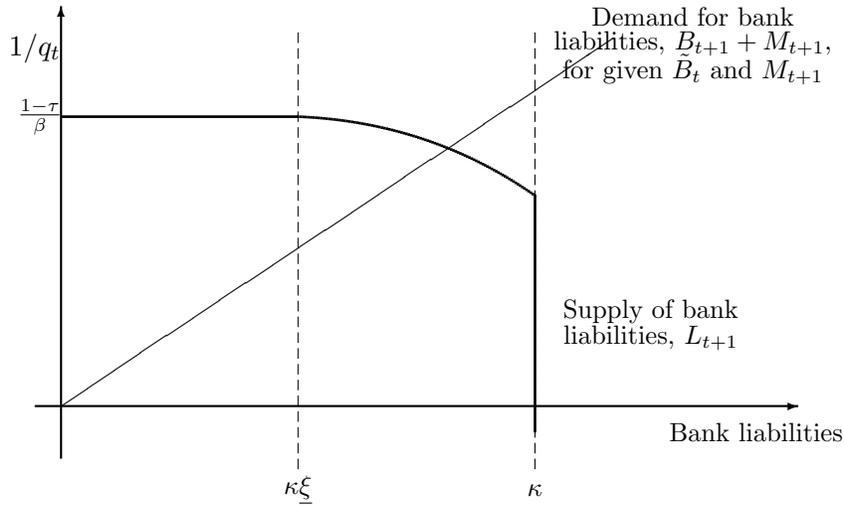

Figure 2: Demand and supply of bank liabilities.

The figure shows that the supply is undetermined when the price q_t is equal to $\beta/(1 - \tau)$ and strictly decreasing for higher values of q_t until it reaches κ. Remember that, according to Assumption 1, $\tau \geq 0$ is the unitary operation cost when $\omega_{t+1} \leq \underline{\omega}$.

Equilibrium The intersection of demand and supply of bank liabilities plotted in Figure 2 defines the general equilibrium. The supply (from banks)

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is increasing in the price q_t while the demand is decreasing in q_t. The demand is plotted for a particular value of outstanding post-renegotiation liabilities held by entrepreneurs, $\tilde{B}_t$, and demand from the monetary authority, M_{t+1}. By changing the outstanding post-renegotiation liabilities, the slope of the demand function changes.

The figure indicates three regions. When the price is $q_t = \beta/(1 - \tau)$, banks are indifferent in the choice of leverage $\omega_{t+1} \leq \underline{\xi}$. If the equilibrium is in this region (that is, the intersection of demand and supply arises at $B_{t+1} + M_{t+1} = L_{t+1} = D_{t+1} \leq \underline{\xi}\kappa$), then a realization of the $\xi_{t+1} = \underline{\xi}$ is inconsequential since banks always repay their liabilities in full. When $q_t > \beta/(1 - \tau)$, however, the optimal leverage starts to increase above $\underline{\xi}$. In this region banks repay only a fraction of their liabilities after a realization of $\xi_{t+1} = \underline{\xi}$. Once $\omega_{t+1} = 1$, a further increase in the price q_t does not lead to higher leverages since the choice of $\omega_{t+1} > 1$ would cause renegotiation with probability 1.

The equilibrium illustrated in Figure 2 is for a particular value of financial wealth held by entrepreneurs, $\tilde{B}_t$, and a given demand from the monetary authority, M_{t+1}. Given the equilibrium value of L_{t+1} and the random draw of ξ_{t+1}, we determine the next period financial wealth of entrepreneurs $\tilde{B}_{t+1}$. The new $\tilde{B}_{t+1}$ will determine a new slope for the demand of bank liabilities which, together with M_{t+2}, will determine the new equilibrium value of L_{t+2}. Depending on parameters, the economy may or may not reach a steady state. It would reach a steady state if, starting with $L_t < \underline{\xi}\kappa$, bank liabilities will never increase above $\underline{\xi}\kappa$. An important factor determining this is the operation cost $\varphi(\omega_t)$.

According to Assumption 1, the unit operation cost is constant and equal to τ for values of $\omega_t \leq \underline{\xi}$. This parameter plays an important role in determining the existence of a steady state as stated in the following proposition.

Proposition 2.2 *Suppose that M_{t+1} is constant and equal to $\bar{M}$. There exists $\hat{\tau} > 0$ such that: If $\tau \geq \hat{\tau}$, the economy converges to a steady state without renegotiation. If $\tau < \hat{\tau}$, the economy never converges to a steady state but switches stochastically between equilibria with and without renegotiation in response to ξ_t.*

Proof 2.2 *Appendix E.*

In a steady state, the price at which banks sell their liabilities must be equal to $q_t = \beta/(1 - \tau)$. At this price banks do not have incentive to leverage

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because the funding cost is equal to the return on loans. In order to have $q_t = \beta/(1-\tau)$, the demand for bank liabilities must be sufficiently low. This cannot be the case when $\tau = 0$. With $\tau = 0$, in fact, the steady state price of bank liabilities must be equal to $q = \beta$. But then, because of precautionary savings, entrepreneurs continue to accumulate bank liabilities without a bound. The demand for bank liabilities will eventually become bigger than the supply (which is bounded by the borrowing constraint of households), driving the price q_t above $\beta/(1-\tau)$. As the interest rate falls, equilibria with renegotiation become possible. But then the economy fluctuates stochastically and a steady state is never reached.

The above proposition is stated for a constant value of $M_{t+1} = \bar{M}$. But how does an increase in $\bar{M}$ affects the properties of the equilibrium? The next proposition states that if $\bar{M}$ increases sufficiently, the economy will move away from the steady state and becomes stochastic.

Proposition 2.3 *Suppose that for $M_{t+1} = \bar{M}$, $\tau \geq \hat{\tau}$ so that the economy converges to a steady state without renegotiation. Then a sufficiently high increase in $\bar{M}$, will induce a transition away from the steady state and, at some point, the economy becomes stochastic.*

Proof 2.3 *Appendix F.*

The proposition has a simple intuition. A higher value of $\bar{M}$ induces a decrease in the interest rate on bank liabilities, that is, an increase in q_t. The lower interest rate then increases the incentive of banks to leverage. But higher leverage implies that banks renegotiate their liabilities when $\xi_t = \underline{\xi}$. More specifically, bank liabilities are renegotiated to $\kappa\underline{\xi}$. Therefore, bigger are the liabilities issued by banks and larger are the losses incurred by entrepreneurs holding these liabilities. Larger financial losses incurred by entrepreneurs then imply bigger declines in the demand for labor, which in turn cause bigger macroeconomic contractions (financial crises).

Proposition 2.3 thus shows that low interest rates could lead to greater macroeconomic instability. I will now show this property with the calibrated model.

3   Quantitative analysis

In this section I study the impact of interest rate policies using a calibrated version of the model. In the baseline calibration I set $M_t = 0$ for all t. I will

then consider an increase M_t (leading to lower interest rates) and show as the economy responds to this change.

The period in the model is a quarter. I set the discount factor β so that the expected return on equity for banks is 7% annually. Since the expected quarterly return for banks is $1/\beta - 1$, I set $\beta = 1/1.07^{\frac{1}{4}}$.

The parameter ν in the utility function of households is the elasticity of labor supply. To mimic an environment with rigid wages while keeping the model simple, I set $\nu = 50$. With this calibration wages are almost constant while equilibrium labor is almost completely demand determined. The parameter α in the dis-utility from working is chosen to have an average labor supply is 0.3.

The average productivity is normalized to $\bar{z} = 1$. Since the average input of labor is 0.3, the average production in the entrepreneurial sector is also 0.3. The supply of houses is normalized to $\bar{k} = 1$ and its services are set to $\chi = 0.05$. Total production is the sum of entrepreneurial production (0.3) plus housing services (0.05). Therefore, total output is 0.35 per quarter (about 1.4 per year).

The borrowing constraint (8) has two parameters: κ and η. The parameter κ is the constant limit which I set to zero. The parameter η determines the fraction of the value of houses that can be used as a collateral. I calibrate η to 0.6 so that the leverage of the household sector is similar to the data. The productivity shock follows a truncated[1] normal distribution with standard deviation of 0.3. This implies that the standard deviation of entrepreneurial wealth is about 7%. This is within the range of estimates for rich households reported by Fagereng, Guiso, Malacrino, and Pistaferri (2016) based on data from Norway.

The last set of parameters pertain to the banking sector. The probability that the liquidation value of bank assets is $\underline{\xi}$ (which could lead to a bank crisis) is set to 1 percent ($\lambda = 0.01$). Therefore, provided that banks choose sufficiently high leverage, a crisis is a low probability event that arises, on average, every 25 years. This number is close to calibrations of crisis probabilities used in the literature. See for example Bianchi and Mendoza (2013).

For $\underline{\xi}$, I do not have direct evidence to calibrate this parameter. I set it to 0.75 which implies that, if a crisis arises, creditors recover at least 75 percent of the bank investment. A loss of 25% for the investments of the

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[1]The truncation is motivated by that assumption that the idiosyncratic shock is be bounded.

whole banking sector (excluding safe financial investments like government bonds) seems plausible. Notice that the actual losses for the creditors of banks are lower than 25% since they depend on bank leverage. For example, if the leverage of banks is 80%, creditors would lose 5% of their assets (held in bank liabilities).

The operation cost takes the quadratic form $\varphi(\omega) = \tau + \bar{\varphi} \cdot \max\{\omega - \underline{\xi}, 0\}^2$. The convex part of the cost is scaled by the parameter $\bar{\varphi}$. This parameter determines the response sensitivity of bank leverage and interest rates to a change in market conditions. Higher is the value of $\bar{\varphi}$, and higher is the sensitivity of interest rates to shocks but lower is the response of bank leverage. I set $\bar{\varphi} = 0.05$ which allows for reasonable responses of the interest rate and leverage.

The linear component of the operation cost, τ, is chosen so that the long-run leverage in absence of crises is higher than $\underline{\xi} = 0.75$. This guarantees that the economy experiences stochastic dynamics in response to realizations of ξ_t. The calibrated value is $\tau = 0.0025$, which implies an average leverage of 0.76. Therefore, abstracting from the convex component of the cost, the operation cost is 0.25% per quarter or about 1% per year. Since the calibration of $\underline{\xi}$, τ and $\bar{\varphi}$ are not based on direct empirical targets, the quantitative results should be interpreted with caution.

3.1 Interest rate policies and macroeconomic stability

The only aggregate shock in the model is ξ_t, that is, the liquidation value of bank investments. The distribution of this variable is iid: in each period, with 1% probability $\xi_t = \underline{\xi}$ and with 99% probability $\xi_t = 1$. If in equilibrium banks choose to be sufficiently leveraged (which is the case for the chosen calibration), the economy displays stochastic dynamics.

To illustrate the stochastic properties of the economy and the importance of asset purchases by the monetary/fiscal authority, I simulate the model with a random sequence of ξ_t over a period of 2,200 quarters. During the first 2,100 quarters $M_{t+1} = 0$ (baseline calibration). In the remaining 100 quarters I set $M_{t+1} = 0.129$, which is about 10% the average value of bank liabilities before the policy change.

The simulation is repeated 1,000 times (with each simulation performed over 2,200 periods based of new random draws of ξ_t). I use only the last 200 quarters of each simulation to illustrate the stochastic properties of the economy. By discarding the first 2,000 quarters I eliminate the possible

impact of initial conditions. The numerical procedure used to solve the model is described in Appendix G.

Figure 3 plots the average as well as the 5th and 95th percentiles of the 1,000 repeated simulations for each of the last 200 quarters of simulated data (the first 100 quarters with $M_{t+1} = 0$ and the subsequent 100 quarters with $M_{t+1} = 0.129$). The range of variation between the 5th and 95th percentiles captures the volatility of the economy at any point in time.
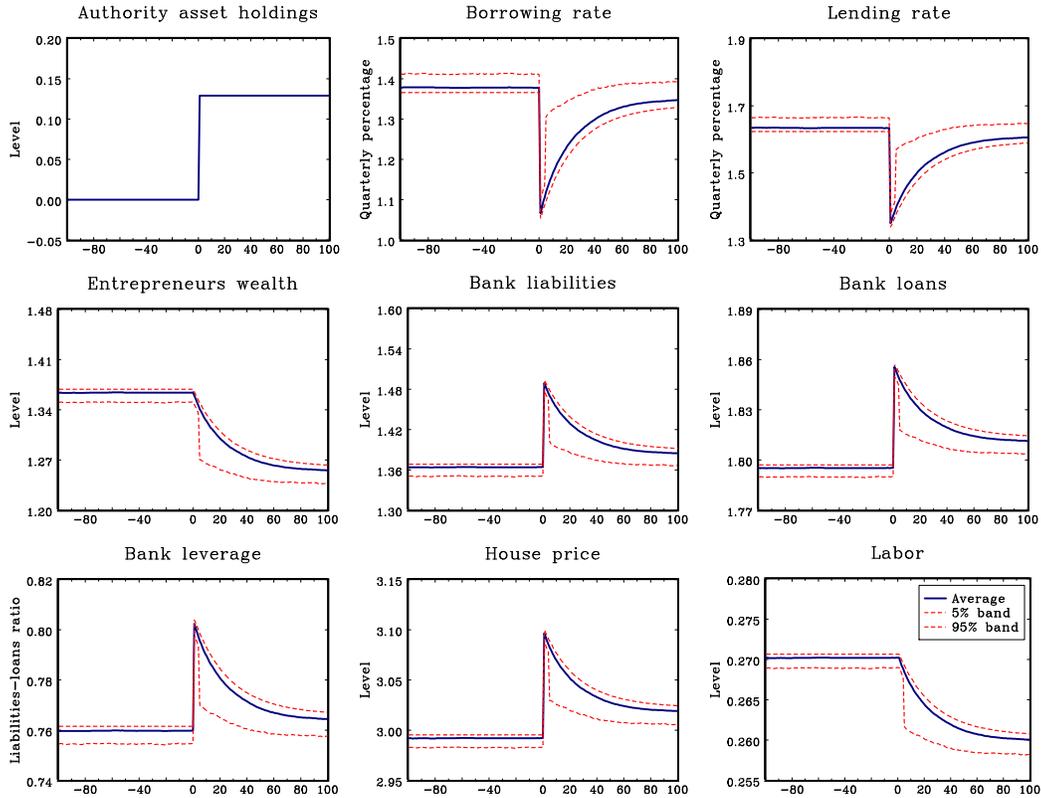

Figure 3: The effects of asset purchases by the monetary/fiscal authority, M_{t+1}. Mean and percentiles over 1,000 repeated simulations.

The first panel plots the bank liabilities held by the monetary/fiscal authority, M_{t+1} (asset purchases). This variable is exogenous in the model and the permanent change that takes place in quarter 1 is what drives the changes in key statistics for the endogenous variables plotted in the other panels.

Let's focus first on the mean of the endogenous variables (continuous

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line). This is the average, for each quarter, calculated over the 1,000 repeated simulations. Following the increase in M_{t+1}, we observe a decline in the average interest rates (the rate paid by banks on its liabilities and the rate charged to their loans to households). Notice that, even if these rates recover somewhat over the transition periods, they remain lower than the pre-intervention values also in the long-run. Thus, the second part of the simulation is characterized by lower interest rates induced by the higher supply of funds (asset purchases) from the monetary/fiscal authority.

The panels in the middle and bottom rows illustrate the impact of lower interest rates. First, since banks pay lower interests on their liabilities, they increase leverage and supply more loans. This induces a fall in the lending rate paid by households which in turn induces, on average, an increase in the price of houses. Even thought there is more liquidity in the economy and the interest rates fall, the input of labor (and therefore production) falls on average.

The mechanism that generates the macroeconomic contraction (lower employment and production) can be described as follows. The lower interest rate implies that savers (entrepreneurs in the model) have less incentive to hold financial assets. We can then see from the first panel in the middle of Figure 3 that the financial wealth of entrepreneurs falls. Even if bank liabilities L_{t+1} rise, the increase is smaller than the increase in M_{t+1} and, therefore, entrepreneurs' wealth B_{t+1} has to decline. Remember that in equilibrium $L_{t+1} = B_{t+1} + M_{t+1}$. Thus, if L_{t+1} rises less than M_{t+1}, B_{t+1} has to decline. Effectively, the asset purchased by the monetary/fiscal authority crowd out the purchases from savers (entrepreneurs). But lower financial wealth held by savers implies that they are less willing to hire labor. Thus, the unintended consequences of asset purchase policies are the reduction in the demand for labor.

Let's now look at the percentiles of the repeated simulations. The dashed lines in Figure 3 show the 5th and 95th percentiles for the 1,000 repeated simulations.[2] The intervals between the two percentiles widen after the policy intervention. This shows that the asset purchase by the monetary/fiscal

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[2]The percentiles are calculated as follows. For each quarter, the values of each variable in the 1,000 repeated simulations are sorted from the lowest to the highest values. The 5th percentile is then the value that is located in position 950 of the 1,000 sorted realizations in the particular quarter (and, therefore, only 5% of all realizations have higher values). The 95th percentile is the value that is located in position 51 (and, therefore, only 5% of all realizations have lower values).

authority increases financial and macroeconomic volatility. The probability of a bank crisis is always positive even before the structural break associated with the higher M_t. However, after the structural break, the consequence of a crisis could be much bigger since banks become more leveraged. Because they are more leveraged, when the economy experiences a negative shock, entrepreneurs face higher losses due to larger renegotiation from banks.

Counter-cyclical asset purchases. Suppose that the monetary/fiscal authority increases M_{t+1} in response to a crisis. To make the analysis simple, suppose that M_{t+1} becomes positive when a financial crisis arrives (that is, $\xi_t = \underline{\xi}$) and stays at this high level for N periods (unless another crisis hits). Formally,

$$M_{t+1} = \begin{cases} \bar{M}, & \text{if } \xi_{t-j} = \underline{\xi} \text{ for any } j = 0, ..., N-1; \\ \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

With this policy rule, I simulate the model for a particular sequence of shocks. In this particular sequence, a crisis ($\xi_t = \underline{\xi}$) arises after a long period without crises, that is, $\xi_t = 1$. The crisis is only for one period and then the economy does not experience any more crises. More specifically, I simulate the economy for the same number of periods as before, 2,200 quarters. The exogenous variable ξ_t takes the value of 1 in all simulation periods with the exception of quarter 2101. I then discard the first 2,000 quarters and show the statistics for the remaining 200. Thus, the negative shock arises in the middle of the last 200 quarters. Since the simulation is for a particular sequence of shocks, I do not need to repeat the simulation as I did before.

Figure 4 plots the simulated variables with two policy regimes. In the first regime the monetary authority behaves passively and keeps $M_t = 0$ in all periods, and therefore, it does not respond to the negative shock. In the second regime the monetary authority follows the rule described in (16), with $N = 8$. Thus, in response to a crisis, the monetary authority increases liquidity for 8 quarters.

Let's look first at the case in which the monetary/fiscal authority does not respond to the crisis. The realization of $\xi_t = \underline{\xi}$ generates a wealth loss for entrepreneurs (due to the renegotiation from banks) which in turn reduces the demand of labor (macroeconomic contraction). Even if the negative shock is only for one period and there are no crises afterwards, the recovery in the
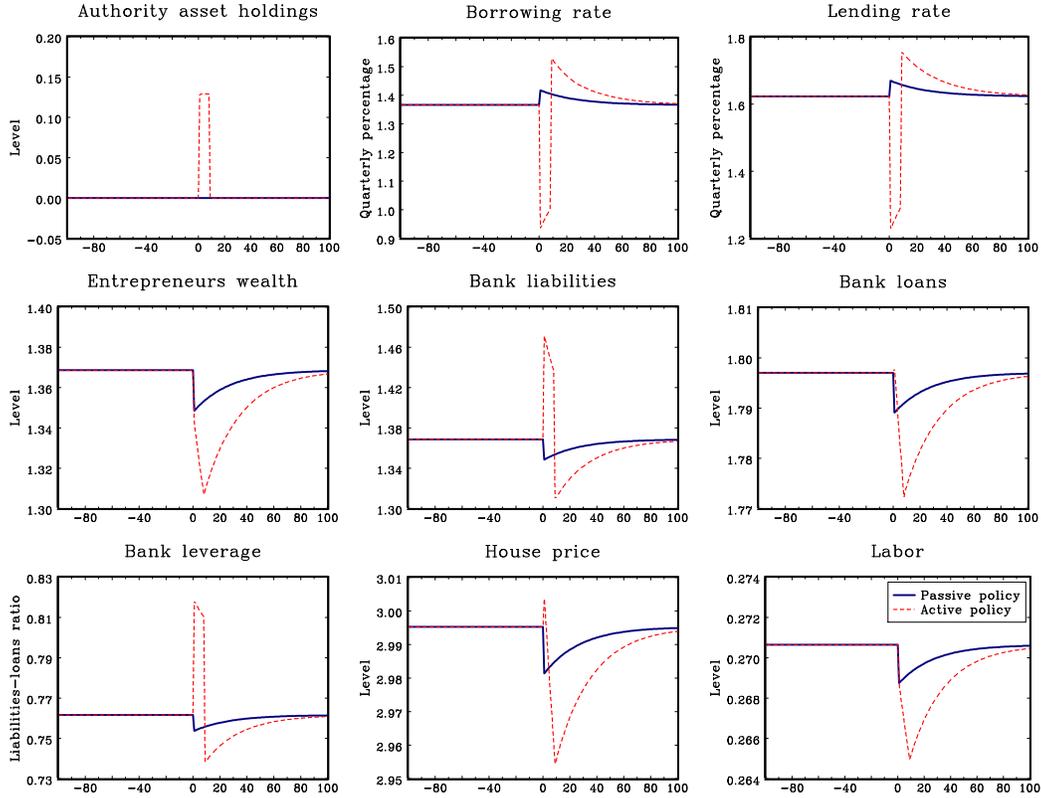
Figure 4: Countercyclical asset purchases by the monetary/fiscal authority. Increase in M_t for 2 years in response to a financial crisis hits $(\xi_t = \underline{\xi})$ at quarter 1.

labor market is very slow. This is because it takes a while for employers to rebuilt the lost wealth with savings.

When the monetary/fiscal authority reacts to the crisis with asset purchases, the negative macroeconomic impact of the shock gets amplified. The intervention has the effect of reducing the interest rates but lower interest rates discourage savings. As a result, entrepreneurs take longer to rebuilt their financial wealth. The lower interest rates have an immediate positive effect on house prices. But the positive effect is only temporary. This is because the reversal of the policy after 8 quarters is anticipated by the market and, therefore, there is the anticipation of higher future interest rates. The positive effect of the policy on asset prices would be long-lasting if the policy intervention was permanent (as in the simulations shown in Figure 3).

But a permanent policy intervention would also make the negative impact on employment permanent.

4 Conclusion

Monetary policy interventions that reduce interest rates encourage spending, either in the form of consumption or in the form of investment, and stimulate the real sector of the economy. In this paper I show that low interest rates could also have a negative macroeconomic impact if they discourage savings. I illustrated the idea in a model in which the financial wealth of producers has a positive impact on production. Since low interest rates reduce the incentive of producers to hold financial wealth, they have negative consequences on production.

Low interest rates may also increase macroeconomic volatility if they encourage financial intermediaries to become more leveraged.

The goal of this paper is not to prove that low interest rates are necessarily counter-productive for the performance of the economy, both in level and volatility. In fact, to show these effects I have used a model where the hypothesized positive channels of low interest rates are absent. For example, even if low interest rates increase the price of houses, it does not increase the production of new houses (which is zero by assumption). The purpose of the paper is to emphasize that there could be alternative channels that have not be fully explored by academic researchers and practitioners. But there is very weak evidence that low interest rates are associated with economic growth and macroeconomic stability. Although in emerging countries the correlation between interest rates and macroeconomic indicators is negative, the opposite seems to be true for industrialized countries.

Appendix

A First order conditions for problem (4)

The probability of renegotiation, denoted by θ_{t+1}, is defined as

$$\theta_{t+1} = \begin{cases} 0, & \text{if} \quad \omega_{t+1} < \underline{\xi} \\ \bar{\lambda}, & \text{if} \quad \underline{\xi} \le \omega_{t+1} \le 1 \\ 1, & \text{if} \quad \omega_{t+1} > 1 \end{cases}$$

Define $\beta(1-\theta_{t+1})\gamma_t$ the Lagrange multiplier associated to the constraint $l_{t+1} \le i_{t+1}$. The first order conditions for problem (4) with respect to l_{t+1} and i_{t+1} are

$$\frac{1-\varphi_t}{\overline{R}_t^l} \mathbb{E}_t \frac{\partial \tilde{l}_{t+1}}{\partial l_{t+1}} - \frac{\partial \varphi_t}{\partial l_{t+1}} \frac{\mathbb{E}_t \tilde{l}_{t+1}}{\overline{R}_t^l} - \beta \mathbb{E}_t \frac{\partial \tilde{l}_{t+1}}{\partial l_{t+1}} - \beta(1-\theta_{t+1})\gamma_t = 0, \tag{17}$$

$$-\frac{1}{R_t^i} + \frac{1-\varphi_t}{\overline{R}_t^l} \mathbb{E}_t \frac{\partial \tilde{l}_{t+1}}{\partial i_{t+1}} - \frac{\partial \varphi_t}{\partial i_{t+1}} \frac{\mathbb{E}_t \tilde{l}_{t+1}}{\overline{R}_t^l} + \beta \mathbb{E}_t \left(1 - \frac{\partial \tilde{l}_{t+1}}{\partial i_{t+1}}\right) + \beta(1-\theta_{t+1})\gamma_t = 0. \tag{18}$$

I now use the definition $\tilde{b}_{t+1}$ provided in (1) to derive the following terms

$$\begin{aligned}
\frac{\partial \varphi_t}{\partial l_{t+1}} &= \varphi'_{t+1} \frac{1}{i_{t+1}}, \\
\frac{\partial \varphi_t}{\partial i_{t+1}} &= -\varphi'_{t+1}\omega_{t+1} \frac{1}{i_{t+1}}, \\
\mathbb{E}_t \frac{\partial \tilde{l}_{t+1}}{\partial l_{t+1}} &= 1 - \theta_{t+1}, \\
\mathbb{E}_t \frac{\partial \tilde{l}_{t+1}}{\partial i_{t+1}} &= \theta_{t+1}\underline{\xi}, \\
\mathbb{E}_t \tilde{l}_{t+1} &= (1-\theta_{t+1})l_{t+1} + \theta_{t+1}\underline{\xi} i_{t+1}.
\end{aligned}$$

Substituting in (17) and (18) and re-arranging we obtain

$$\frac{1}{\overline{R}_t^l} = \beta \left[1 + \frac{\varphi_{t+1} + \varphi'_{t+1}\hat{\omega}_{t+1} + \gamma_t}{1 - \varphi_{t+1} - \varphi'_{t+1}\hat{\omega}_{t+1}}\right], \tag{19}$$

$$\frac{1}{R_t^i} = \beta \left[1 + \frac{\varphi'_{t+1}\hat{\omega}_{t+1}^2(1 - \theta_{t+1})(1 + \gamma_t)}{1 - \varphi_{t+1} - \varphi'_{t+1}\hat{\omega}_{t+1}} + \left(1 - \theta_{t+1} + \theta_{t+1}\underline{\xi}\right)\gamma_t\right], \tag{20}$$

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where $\hat{\omega}_{t+1} = \omega_{t+1} + \frac{\theta_{t+1}\underline{\xi}}{1-\theta_{t+1}}$.

The multiplier γ_t is zero if $\omega_{t+1} < 1$ and positive if $\omega_{t+1} = 1$. Therefore, the first order conditions can be written as

$$\frac{1}{\overline{R}_t^l} \geq \beta \left[1 + \frac{\varphi_{t+1} + \varphi_{t+1}'\hat{\omega}_{t+1}}{1 - \varphi_{t+1} - \varphi_{t+1}'\hat{\omega}_{t+1}} \right],$$

$$\frac{1}{R_t^i} \geq \beta \left[1 + \frac{\varphi_{t+1}'\hat{\omega}_{t+1}^2(1 - \theta_{t+1})}{1 - \varphi_{t+1} - \varphi_{t+1}'\hat{\omega}_{t+1}} \right],$$

which are satisfied with the inequality sign if $\gamma_t > 0$. Since they are all functions of ω_{t+1}, the first order conditions can be written as in (5) and (6).

B Proof of Lemma 2.1

Let's consider the first order conditions (19) and (20) when $\omega_{t+1} < 1$. In this case the lagrange multiplier γ_t is zero. Provided that $\varphi_{t+1} + \varphi_{t+1}'\hat{\omega}_{t+1} < 1$—which will be satisfied for plausible calibrations—conditions (19) and (20) imply that $\overline{R}_t^l$ and R_t^i are smaller than $1/\beta$.

The return spread can be computed from (19) and (20) as

$$\frac{R_t^i}{\overline{R}_t^l} = \frac{1}{1 - \varphi_{t+1} - \varphi_{t+1}'\hat{\omega}_{t+1}[1 - (1 - \theta_{t+1})\hat{\omega}_{t+1}]}. \tag{21}$$

Given the properties of the cost function (Assumption 1), to show that the spread is bigger than 1 I only need to show that $(1 - \theta_{t+1})\hat{\omega}_{t+1} < 1$. Using $\hat{\omega}_{t+1} = \omega_{t+1} + \frac{\theta_{t+1}\underline{\xi}}{1-\theta_{t+1}}$ and taking into account that $\omega_{t+1} < 1$ and $\theta_{t+1} < 1$, we can verify that $(1 - \theta_{t+1})\hat{\omega}_{t+1} < 1$. Therefore, the spread is bigger than 1.

To show that the spread is increasing in the leverage, I differentiate (21) with respect to ω_{t+1} to obtain

$$\frac{R_t^i}{\overline{R}_t^l} = \frac{(\varphi_{t+1}''\hat{\omega}_{t+1} + 2\varphi_{t+1}')[1 - (1 - \theta_{t+1})\hat{\omega}_{t+1}]}{\left[1 - \varphi_{t+1} - \varphi_{t+1}'\hat{\omega}_{t+1}(1 - (1 - \theta_{t+1})\hat{\omega}_{t+1}\right]^2}$$

Given the properties of the cost function (Assumption 1), the derivative is zero for $\omega_{t+1} \leq \underline{\xi}$. To prove that the derivative is positive for $\omega_{t+1} > \underline{\xi}$, I only need to show that $(1 - \theta_{t+1})\hat{\omega}_{t+1} < 1$, which has already been shown above. Therefore, the return spread is strictly increasing for $\omega_{t+1} > \underline{\xi}$. *Q.E.D.*

C   Proof of Lemma 2.2

The optimization problem of an entrepreneur can be written recursively as

$$V_t(\tilde{b}_t) \quad = \quad \max_{h_t} \mathbb{E}_t \tilde{V}_t(n_t) \tag{22}$$

subject to

$$n_t = \tilde{b}_t + (z_t - w_t)h_t$$

$$\tilde{V}_t(n_t) \quad = \quad \max_{b_{t+1}} \left\{ \ln(c_t) + \beta \mathbb{E}_t V_{t+1}(\tilde{b}_{t+1}) \right\} \tag{23}$$

subject to

$$c_t = n_t - q_t b_{t+1}$$

Since the information set changes from the beginning of the period to the end of the period, the optimization problem has been separated according to the available information. In sub-problem (22) the entrepreneur chooses the input of labor without knowing the productivity z_t. In sub-problem (23) the entrepreneur allocates the end of period wealth in consumption and savings after observing z_t. Notice that the expectation in sub-problem (23) takes into account the dependence of $\tilde{b}_{t+1}$—the renegotiated value of bank liabilities—from the pre-renegotiated value b_{t+1}.

The first order condition for sub-problem (22) is

$$\mathbb{E}_t \frac{\partial \tilde{V}_t}{\partial n_t}(z_t - w_t) = 0.$$

The envelope condition from sub-problem (23) gives

$$\frac{\partial \tilde{V}_t}{\partial n_t} = \frac{1}{c_t}.$$

Substituting in the first order condition we obtain

$$\mathbb{E}_t \left(\frac{z_t - w_t}{c_t} \right) = 0. \tag{24}$$

At this point we proceed by guessing and verifying the optimal policies for employment and savings. The guessed policies take the form:

$$h_t \quad = \quad \phi_t \tilde{b}_t \tag{25}$$

$$c_t \quad = \quad (1 - \beta)n_t \tag{26}$$

Since $n_t = \tilde{b}_t + (z_t - w_t)h_t$ and the employment policy is $h_t = \phi_t \tilde{b}_t$, the end of period wealth can be written as $n_t = [1 + (z_t - w_t)\phi_t]\tilde{b}_t$. Substituting in the guessed consumption policy we obtain

$$c_t = (1 - \beta)\Big[1 + (z_t - w_t)\phi_t\Big]\tilde{b}_t. \tag{27}$$

This expression is used to replace c_t in the first order condition (24) to obtain

$$\mathbb{E}_t\left[\frac{z_t - w_t}{1 + (z_t - w_t)\phi_t}\right] = 0, \tag{28}$$

which is the condition stated in Lemma 2.2.

To complete the proof, we need to show that the guessed policies (25) and (26) satisfy the optimality condition for the choice of consumption and saving. This is characterized by the first order condition of sub-problem (23), which is equal to

$$-\frac{q_t}{c_t} + \beta \mathbb{E}_t \frac{\partial V_{t+1}}{\partial \tilde{b}_{t+1}} \frac{\partial \tilde{b}_{t+1}}{\partial b_{t+1}} = 0.$$

From sub-problem (22) we derive the envelope condition $\partial V_t / \partial \tilde{b}_t = 1/c_t$ which can be used in the first order condition to obtain

$$\frac{q_t}{c_t} = \beta \mathbb{E}_t \frac{1}{c_{t+1}} \frac{\partial \tilde{b}_{t+1}}{\partial b_{t+1}}.$$

We have to verify that the guessed policies satisfy this condition. Using the guessed policy (26) and equation (27) updated one period, the first order condition can be rewritten as

$$\frac{q_t}{n_t} = \beta \mathbb{E}_t \frac{1}{[1 + (z_{t+1} - w_{t+1})\phi_{t+1}]\tilde{b}_{t+1}} \frac{\partial \tilde{b}_{t+1}}{\partial b_{t+1}}.$$

Multiplying both sides by b_{t+1}/β the first order condition can be further rewritten as

$$\frac{q_t b_{t+1}}{\beta n_t} = \mathbb{E}_t \frac{1}{[1 + (z_{t+1} - w_{t+1})\phi_{t+1}]} \frac{b_{t+1}}{\tilde{b}_{t+1}} \frac{\partial \tilde{b}_{t+1}}{\partial b_{t+1}}.$$

Notice that, since in case of bank renegotiation the entrepreneur recovers a fraction of his financial wealth $(\tilde{b}_{t+1}/b_{t+1} < 1)$ and this fraction does not depend on the size of the financial wealth held by an individual entrepreneur b_{t+1}, the term $\frac{b_{t+1}}{\tilde{b}_{t+1}} \frac{\partial \tilde{b}_{t+1}}{\partial b_{t+1}} = 1$. Furthermore, using the guessed policy (26) we have that

$q_t b_{t+1} = \beta n_t$. Substituting in the last expression for the first order condition and rearranging we obtain

$$1 = \mathbb{E}_t \left[\frac{1}{1 + (z_{t+1} - w_{t+1})\phi_{t+1}} \right]. \tag{29}$$

The final step is to show that, if condition (28) is satisfied, then condition (29) is also satisfied. Let's start with condition (28), updated by one period. Multiplying both sides by ϕ_{t+1} and then subtracting 1 in both sides we obtain

$$\mathbb{E}_{t+1} \left[\frac{(z_{t+1} - w_{t+1})\phi_{t+1}}{1 + (z_{t+1} - w_{t+1})\phi_{t+1}} - 1 \right] = -1.$$

Multiplying both sides by -1 and taking expectations at time t we obtain (29). Q.E.D.

D Proof of Proposition 2.1

As shown in Lemma 2.2, the optimal saving of entrepreneurs takes the form $q_t b_{t+1} = \beta n_t$, where n_t is the end-of-period wealth $n_t = \tilde{b}_t + (z_t - w_t)h_t$. Since $h_t = \phi(w_t)\tilde{b}_t$ (see Lemma 2.2), the end-of-period wealth can be rewritten as $n_t = [1 + (z_t - w_t)\phi(w_t)]\tilde{b}_t$. In the environment with direct borrowing and lending there is not default and, therefore, $\tilde{b}_t = b_t$. Substituting into the optimal saving and aggregating over all entrepreneurs we obtain

$$B_{t+1} = \beta \left[1 + (\bar{z} - w_t)\phi(w_t) \right] \frac{B_t}{q_t}. \tag{30}$$

This equation defines the aggregate demand for bonds as a function of the price q_t, the wage rate w_t, and the beginning-of-period aggregate wealth of entrepreneurs $\tilde{B}_t$. Notice that the term in square brackets is bigger than 1. Therefore, in a steady state equilibrium where $B_{t+1} = B_t = \tilde{B}_t$, the condition $\beta < q$ must be satisfied.

Using the equilibrium condition in the labor market, I can express the wage rate as a function of B_t. In particular, equalizing the demand for labor, $H_t^D = \phi(w_t)B_t$, to the supply from households, $H_t^S = (w_t/\alpha)^\nu$, the wage w_t can be expressed as a function of only B_t. We can then use this function to replace w_t in (30) and express the demand for bank liabilities as a function of only B_t and q_t as follows

$$B_{t+1} = \frac{s(B_t)}{q_t}. \tag{31}$$

The function $s(B_t)$ is strictly increasing in the wealth of entrepreneurs, B_t.

Consider now the supply of bonds from households. For simplicity I assume that $\eta = 0$ in the borrowing constraint (8). Therefore, the constraint takes the

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form $d_{t+1} \leq \kappa$. Using this limit together with the first order condition (10), we have that, either the price satisfies $q = \beta$ or households are financially constrained, that is, $d_{t+1} = \kappa$. Notice that in equilibrium $B_{t+1} = D_{t+1} - \bar{M}$. Therefore, if the borrowing constraint is binding for households, we must have $B_{t+1} = \kappa - \bar{M}$.

When the price is equal to the inter-temporal discount factor (first case), we can see from (30) that $B_{t+1} > B_t$. So eventually, the borrowing constraint of households becomes binding and the equilibrium condition is $B_{t+1} = \kappa - \bar{M}$ (second case). When the borrowing constraint is binding, the multiplier μ_t is positive and condition (10) implies that the price is bigger than the inter-temporal discount factor. So the economy has reached a steady state. The steady state price is determined by condition (31) after setting $B_t = B_{t+1} = \kappa - \bar{M}$. This is the only steady state equilibrium.

When $\eta > 0$ in the borrowing constraint (8), the proof is more involved but the economy also reaches a steady state with $\beta < q$. $\hspace{2cm}$ Q.E.D.

E Proof of Proposition 2.2

Given a fixed price q, the aggregate demand for bank liabilities, equation (13), has a converging fix point $B^*(q)$. The fixed point is decreasing in q and converges to infinity as q converges to β. This implies that, if $\tau = 0$, the leverage of banks is always bigger than $\underline{\xi}$.

To show this, suppose that banks choose a leverage of $\omega < \underline{\xi}$. According to conditions (5) and (6), we have that $q = 1/R^i = \beta$. But when $q = \beta$ the demand for bank liabilities is unbounded in the limit. This implies that to reach a stable equilibrium without renegotiation (that is, $\omega < \underline{\xi}$), q must be bigger than β. This requires τ to be sufficiently large. In fact, when $\tau > 0$ and $\omega < \underline{\xi}$, we have $(1 - \tau)q = 1/R^i = \beta$. Since the demand for bank liabilities is decreasing in q, there must be some $\hat{\tau} > 0$ such that, for $\tau > \hat{\tau}$, the equilibrium is characterized by $\omega < \underline{\xi}$. This implies that the economy is not subject to crises and converges to a steady state.

For $\tau < \hat{\tau}$, instead, the equilibrium is characterized by $\omega > \underline{\xi}$. In this case the economy displays stochastic dynamics and never converges to a steady state.

F Proof of Proposition 2.3

TO BE DONE. $\hspace{8cm}$ Q.E.D.

G Numerical solution

I describe first the numerical procedure when M_t is constant. I will then describe the procedure when M_t changes to a new value.

G.1 Computation of equilibrium with constant M_t

The states of the economy are given by the bank liabilities L_t, the bank loans I_t and the realization of the stochastic variable ξ_t. Since M_t is constant, the financial wealth of entrepreneurs is $B_t = L_t - \bar{M}$. The three states are important in determining the renegotiated liabilities $\tilde{L}_t$. However, once we know the renegotiated liabilities, $\tilde{L}_t$ becomes the sufficient state for solving the model. Therefore, in the computation I will solve for the recursive equilibrium using $\tilde{L}_t$ as a state variable.

I will use the following equilibrium conditions:

$$H_t = \phi(w_t)\tilde{B}_t, \tag{32}$$

$$q_t B_{t+1} = \beta N_t, \tag{33}$$

$$N_t = \tilde{B}_t + (1 - w_t)H_t \tag{34}$$

$$\alpha H_t^{\frac{1}{\nu}} = w_t, \tag{35}$$

$$1 = \beta R_t^i (1 + \mu_t), \tag{36}$$

$$p_t = \beta \mathbb{E}_t \Big[\chi + (1 + \eta\mu_t)p_{t+1} \Big], \tag{37}$$

$$I_{t+1} = \eta \mathbb{E}_t p_{t+1}, \tag{38}$$

$$\frac{1}{\overline{R}_t^l} = \beta \left[1 + \frac{\varphi_{t+1} + \varphi'_{t+1}\hat{\omega}_{t+1}}{1 - \varphi_{t+1} - \varphi'_{t+1}\hat{\omega}_{t+1}} \right], \tag{39}$$

$$\frac{1}{R_t^l} = \beta \left[1 + \frac{\varphi'_{t+1}\hat{\omega}_{t+1}^2(1 - \theta_{t+1})}{1 - \varphi_{t+1} - \varphi'_{t+1}\hat{\omega}_{t+1}} \right], \tag{40}$$

$$\overline{R}_t^i = \left[1 - \theta(\omega_{t+1}) + \theta(\omega_{t+1})\left(\frac{\xi}{\omega_{t+1}}\right) \right] \frac{1}{q_t}, \tag{41}$$

$$\omega_{t+1} = \frac{I_{t+1}}{L_{t+1}} \tag{42}$$

$$L_{t+1} = B_{t+1} + \bar{M} \tag{43}$$

Equations (32)-(34) come from the aggregation of the optimal policies of entrepreneurs (labor demand, savings and end of periods wealth). Equations (35)-(38) come from the optimization problem of households (labor supply, optimal

borrowing, optimal holding of the fixed asset, and borrowing constraint). Notice that the borrowing constraint of households (equation (38) is not always binding. However, when it is not binding and the multiplier is $\mu_t = 0$, households' borrowing is not determined. Therefore, without loss of generality I assume that in this case households borrow up to the limit.Equations (39)-(40) are the first order conditions of banks. These conditions are satisfied with equality if $\omega_{t+1} < 1$ and with inequality if $\omega_{t+1} = 1$. Equation (41) defines the expected return on bank liabilities given their price q_t. Equation (42) defines leverage and (42) is the market clearing condition for bank liabilities.

One complication in solving this system of equations is that the expectation of the next period price of the fixed asset, $\mathbb{E}_t p_{t+1}$, is unknown. All we know is that the price is a function of $\tilde{B}_{t+1}$, that is, $p_{t+1} = P(\tilde{B}_{t+1})$. If I knew the function $P(\tilde{B}_{t+1})$, for any given state $\tilde{B}_t$, the above conditions would be a system of 12 equations in 12 variables: H_t, N_t, μ_t, w_t, p_t, q_t, R_t^i, $\overline{R}_t^l$, B_{t+1}, L_{t+1}, I_{t+1}, ω_{t+1}. Notice that $\tilde{B}_{t+1}$ is a known function of B_{t+1}, L_{t+1}, I_t and the realization of ξ_t. Therefore, knowing the function $P(\tilde{B}_{t+1})$, I can compute the expectation of the next period price p_{t+1}. We can then solve the 12 equations for the 12 variables and this would provide a solution for any given state $\tilde{B}_t$.

However, the function $P(\tilde{B}_{t+1})$ is unknown. Therefore, the numerical procedure will parameterize an approximation of this function. In particular, I approximate $P(\tilde{B}_{t+1})$ with a piece-wise linear function over a grid for the state variable $\tilde{B}_t$. I then solve the above system of equations at each grid point for $\tilde{B}_t$. As part of the solution I obtain the current price p_t. I then use the solution for the current price to update the approximated function $P(\tilde{B}_{t+1})$ at the grid point. I repeat the iteration until convergence, that is, the values guessed for $P(\tilde{B}_{t+1})$ at each grid point must be equal (up to a small rounding number) to the values of p_t obtained by solving the model (given the guess for $P(\tilde{B}_{t+1})$).

G.2 Computation of equilibrium with changing M_t

When M_t changes, the economy transits from a stochastic equilibrium to a new stochastic equilibrium. This requires to solve for the transition and the solution method is based on the following steps.

1. I first compute the stochastic equilibrium under the regime with the initial constant M_t.

2. I then compute the stochastic equilibrium under the new and constant M_t.

3. At this point I solve the model for the transition period in which M_t changes. I solve for the transition backward starting at the terminal period when M_t

becomes constant at the new level. At each t I solve the system (32)-(43) using the approximated function $P_{t+1}(\tilde{B}_{t+1})$ found at time $t+1$. In the first backward step (last period of the transition), $P_{t+1}(\tilde{B}_{t+1})$ is the approximated price function found in the stochastic stationary equilibrium after the break (see previous computational step).

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