Systemic Risk, Banking Regulation and Optimal Monetary Policy*

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Abstract

In this paper, we study the role of the credit channel of monetary policy in a synthesis model of the economy. Specifically, we ask whether the combination of well-described fnancial intermediation channel in the form of banks along with capital regulations induce monetary policy that consistent with observed patterns. Our paper has two primary contributions. One, through the use of a well-specific banking sector and a regulatory capital constraint on lending, we provide an alternate mechanism that can explain the periods of asymmetry in monetary policy without appealing to ad-hoc central bank preferences. Two, we provide the framework via which one can evaluate the relevance of combining monetary policy and bank regulation in a single entity.

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1 Introduction

The recent crisis has highlighted the fact that first-generation new-Keynesian models (e.g. Woodford, 2003) were broad stroke simplifications of the macro economy. They were based a couple of classic imperfections, such as nominal rigidities and monopolistic competition to allow for non-trivial market power and price setting. The goal, of course, was to illustrate how demand shifts could impact output. These constructs permitted an extensive literature that could study the *basic* role of policy. The models, however, omitted details of market imperfections that are central to the study of macroeconomics. This omission is in part responsible for the fact that consensus Taylor rules cannot describe the path of monetary policy (Rudebusch 2006). A new round of (second-generation) new-Keynesian models focus on the implications of these additional imperfections. Because of the current financial crisis, a huge number of new papers, this one included, have turned their attention to the role of a financial and credit market imperfections. This 'financial channel' is now widely believed to play an important role in the conduct of monetary policy. Our question is how. This paper attempts to address the broad question above jointly with a couple of others. One, how can we think about a collapse of financial intermediation on the economy and on the implications for monetary policy? Two, can monetary policy remain effective even if the credit markets are not operating normally?

In this paper, we study the role of the credit channel of monetary policy in a synthesis model of the economy. Specifically, we ask whether the combination of well-described financial intermediation channel in the form of banks along with capital regulations induce monetary policy that consistent with observed patterns. Because monetary policy has empirically been asymmetric and marked by periods of pronounced action, our approach provides an alternative plausible mechanism that is provides the necessary nonlinearity to explain these patterns.

To generate the nonlinearity, we incorporate a well-described banking sector into a specific dynamic stochastic general equilibrium model along the lines of Bernanke et al., (1999). Their mechanism is the well known wedge created by the existence of costly state verification (CSV). The presence of CSV leads to a 'financial accelerator' of monetary policy. Of course, this accelerator is not a lending channel in that banks are completely passive in the model and loans equal deposits at all times. Our banking sector will generate the possibility of additional leverage and thus an increased sensitivity of the financial accelerator. We call this the 'credit channel.' We derive a simple and intuitive characterization that expresses the relationship between the external finance premium, monitoring costs and our credit channel that neatly generalizes Bernanke et al, (1999).

To this still basic setup, we add two very simple features that allow us to capture the world in 'crisis'. First, we include a very simple form of capital regulation, a leverage ratio maximum (i.e. a capital adequacy minimum). Such a ratio is both political salient in that is an integral component of banking regulation and because it generates a mechanism through which the central bank can have a direct credit channel influence. Second, we assume the presence of a systemic shock in returns to capital. This systemic shock gives us the ability to migrate the economy between a good and crisis state and thus to evaluate the role of monetary policy in a world that moves through both.

We show in this paper the salience of the credit channel in this stylized setup and compare our results with the now standard Bernanke et al model. As well, we highlight how the credit channel collapse, and with it the impact of Bernanke et al's costly external finance wedge. Finally, we show that because the channel can collapse monetary policy has irregular patterns, but understandable, patterns.

The intuition behind our results is remarkably simple. Begin with the credit channel. For each dollar of capital, more leverage means more lending. So, monetary interventions have amplified effects. Of course, in the event that bank capital falls below the regulatory minimum, they are by law unable to lend. In this environment, monetary interventions can have no credit channel. What should the monetary authority do in this case? It must pursue the path that leads to the maximum long-run gain. It's straightforward to show that this implies returning to the world with well-capitalized banks, implying a monetary policy that for all intents and purposes mirrors a short-run goal of bank capital accumulation rather than output or inflation considerations.

We think our approach is useful for a few reasons. One, it reconciles the research agendas that look at stability targeting with those that want a pure monetary policy objective function. Two, it provides a simple and tractable mechanism to explain the financial channel that is consistent both with the banking literature that finds a link between monetary policy and real economy and is consistent with the literature on the role of capital regulation on monetary policy.[1] Why does the simple model work? The mechanism is effective because it is a simple version of a commercial bank. Commercial banks lend because it's profitable. Thus, they have an incentive to lend as much as possible up to the regulatory constraint. Change monetary policy and via the Bernanke et al accelerator, the CSV wedge changes and lending can change as well. However, when banks suffer an shock in the form of loan losses, the ability to lend can become curtailed through regulation. Now, a change in monetary policy cannot lead to increased lending.

Importantly, our approach differs from existing work in a few ways. In one sense, it provides a method via which regulation matters in a tractable way. In comparison to models such as Curdia and Woodford (2008), which generate financial channel effects through exogenous spread changes, our model produces an important role for a disruption of intermediation precisely because of the trade-offs present in regulation. In another sense, it differs because it provides a simple way to think about financial intermediation via leverage and constraint.

2 The Benchmark Model

In this section, we describe our stochastic general equilibrium model. The financial system is hampered by asymmetries of information in the borrower-lender relationship and costly state verification, and constrained by regulatory features like capital adequacy and deposit reserve requirements. The economy is populated by a continuum of households and entrepreneurs, each with unit mass. In addition, the economy includes three types of non-financial firms - capital goods producers, wholesale producers, and retailers - and one type of financial institution - the banks -. All firms, whether financial or non-financial, operate under perfect competition, except for the retailers that exploit a monopoly power in their own varieties. Ownership of all the firms is given to the households, except for wholesale producers who are owned and operated by the entrepreneurs.

[1] The literature on this is wide ranging, from Bernanke and Lown (1991) argument that the 1992 Basel 1 deadline contributed to the early 1990s credit crunch to a range of arguments that capital regulation generates magnified business cycles. Some relevant papers include (Berger and Udell (1994), Blum and Hellwig (1995), Brinkmann and Horvitz (1995), Thakor (1996); recent papers include Goodhard *et al.* (2004), Estrella (2004), Kashyap and Stein (2004), Gordy and Howells (2006). As well Borio (2007) provides a comprehensive literature review.

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The financial system is characterized by banks, who originate the loans and channel funds from the households to the entrepreneurs-borrowers, and a central bank with powers to set both banking regulation as well as monetary policy. Monetary policy is characterized by an interest rate feedback rule in the tradition of Taylor (1993). Banking regulation is summarized in a compulsory reserve requirement ratio on deposits and a capital adequacy requirement on bank capital (or bank equity). The fiscal authority plays a passive role purely acting as a device to eliminate the long-run inefficiency due to monopolistic competition in the retail sector.

In the financial accelerator model of Bernanke *et al.* (BGG, 1999) entrepreneurs are inherently different from households, hence borrowing and lending is always possible in equilibrium. Financial frictions arise from asymmetric information between entrepreneurs-borrowers and the lenders (i.e., the banking system). Monitoring costs make external financing costly for entrepreneurs and, therefore, the borrowers' balance sheet conditions play out an important role over the business cycle. Lenders act as a third party inserted between the households and the entrepreneurs-borrowers whose existence and characteristics are all assumed.

Hence, the balance sheet of the lenders that originate the loans becomes passive because ultimately loan supply must be equal to the deposits demanded by the households. Our benchmark extends the BGG (1999) model to enhance the role of the banking balance sheet. In particular, we explore the role that banking regulation has on the 'bank's balance sheet' channel and its relevance for monetary policy. Subsequently, we also explore the interaction between banking regulation and monetary policy. We fit, nonetheless, in the BGG (1999) tradition since the basic structure of banking relationships, intermediation and contract loans is taken as given, rather than arising endogenously, and since we also maintain the illusion of a perfectly competitive banking system.

We depart from BGG (1999) because we note that banking regulation affects the behavior of banks and, therefore, alters the transmission mechanism in the financial accelerator model. We also depart from them because we introduce systemic (or aggregate) risk on capital income to help us analyze the interest rate spreads, the borrower-lender relationship and the business cycle dynamics in response to 'rare events' of large capital income losses.

The Timing Convention. At time t, in the morning, the monetary and productivity shocks are realized and observed. Then, entrepreneurs rent the capital they acquired in the previous period to the wholesale producers. Entrepreneurs and households also supply managerial and unskilled labor, respectively. In turn, wholesale producers manufacture wholesale goods and sell them to the retailers. Households and entrepreneurs get compensated with competitive wages on their labor, and entrepreneurs receive a rental rate on capital as well as the re-sale value of the depreciated capital. However, these capital income is subject to both idiosyncratic and systematic risk shocks which change the disposable capital income available to each individual entrepreneur. The systematic risk is modelled as a potential loss on the average size of the idiosyncratic risk, which can become very large on the left-tail of the distribution. The systemic and idiosyncratic shocks are realized at this point.

In the afternoon, the government raises taxes from households to subsidize the retailers and to allow the central bank flexibility to accommodate changes in deposit reserves from the banks (which occur during the evening). Retailers produce their differentiated varieties, and sell them to households and entrepreneurs for consumption and to capital goods producers for investment purposes. All of them bundle up the retail varieties in the same fashion whether it is then used for investment or consumption. Retailers also distribute their profits back to their shareholders, the households. Capital goods producers combine depreciated capital with investment goods to produce new capital, which will be used at time $t+1$.

In the evening, entrepreneurs repay the outstanding one-period loans made at time $t-1$, and must decide how much to consume and the loans needed to finance the acquisition of new capital in this period. To finance the capital bill in advance, entrepreneurs use internal funds - their disposable income net of consumption - and also external funds - financial loans - from the banking system.[2] Banks do not observe the idiosyncratic shocks on capital income, and therefore originate the loans by taking into account that it is costly to monitor that entrepreneurs report the truth. Simultaneously, the banking system must establish relationships with households to capture enough savings in the form of deposits and bank capital to meet the requirements of banking regulation (on deposit reserves and capital adequacy) and maximize profits,[3] Entrepreneurs subscribe those loans, and then buy the new capital from the capital producers. Entrepreneurs need external financing because they must acquire assets today that will not bear fruits until tomorrow, and they do not dispose of enough funds to finance the expense themselves.

In every period, the same pattern of interactions between the different agents and firms occurs. We now examine the behavior of each one of them more closely.

2.1 The Households

There is a continuum of household of unit mass. Households are infinitely-lived agents with an identical utility function which is additively separable in consumption, C_t, and labor, H_t, i.e.

$$\sum\nolimits_{\tau=0}^{\infty} \beta^{\tau} \mathbb{E}_t \left[\frac{1}{1-\sigma^{-1}} \left(C_{t+\tau} \right)^{1-\sigma^{-1}} - \frac{1}{1+\varphi^{-1}} \left(H_{t+\tau} \right)^{1+\varphi^{-1}} \right], \quad (1)$$

where $0 < \beta < 1$ is the subjective intertemporal discount factor, $\sigma > 0$ $(\sigma \neq 1)$ is the elasticity of intertemporal substitution, and $\varphi > 0$ is the Frisch elasticity of labor supply. Households' income comes from renting unskilled labor to the wholesale producers at competitive nominal wages, W_t, from the ownership of retailers and capital producers which rebate their nominal profits (or losses) to them every period, Π_t^r and Π_t^k respectively, and from interests on their one-period nominal deposits in the banking system, D_t, and yields on their stake on bank capital, B_{t+1}. With this disposable income, households finance their aggregate consumption, C_t, open new deposits, D_{t+1}, buy shares on new banking enterprises[4], B_{t+1}, and pay their nominal (lump-sum) tax bill, T_t. Households, however, do not own physical capital directly.

Accordingly, the households' sequence of budget constraints is described by,

$$P_t C_t + T_t + D_{t+1} + B_{t+1} \leq W_t H_t + I_t D_t + \left(1 - \iota^h\right) R_t^b B_t + \Pi_t^r + \Pi_t^k, \quad (2)$$

[2] Carlstrom and Fuerst (1997) assume that capital goods producers are the ones facing the financing constraints, rather than the producers of final (or wholesale) goods. Alternatively, Carlstrom and Fuerst (2001) focus on the role of financing frictions where the cash-in-advance constraint is placed on the wage bill, rather than on the capital bill. The BGG (1999) framework adopted here is clearly distinct.

[3] We assume that relationships between banks and households do break up after one period. Hence, every period a new relationship has to be initiated. This implies that bank equity, deposits and loans have the same maturity. Therefore, we entirely abstract from the important problem of maturity mismatch in the bank's balance sheet.

[4] The relationships with the banks are always liquidated after one period.

where I_t is the nominal short-term interest rate offered to the depositors, R_t^b is the yield on bank capital, and P_t is the consumption price index (CPI). The nominal tax on bank equity, ι^h, can be viewed as a simplification to capture the differential tax treatment of capital gains from equity holdings and deposits in many tax codes around the world. As a matter of convention, D_{t+1} and B_{t+1}denote nominal deposits and bank equity held from time t to $t+1$. Therefore, the interest rate I_{t+1} paid at $t+1$ is known and determined at time t, but the yield on bank equity R_{t+1}^b could potentially depend on the state of the world at time $t+1$. Household optimization yields the standard first-order conditions for consumption-savings and labor supply,

$$\frac{1}{I_{t+1}} = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma^{-1}} \frac{P_t}{P_{t+1}} \right], \tag{3}$$

$$1 = \beta \mathbb{E}_t \left[\left(1 - \iota^h\right) R_{t+1}^b \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma^{-1}} \frac{P_t}{P_{t+1}} \right], \tag{4}$$

$$\frac{W_t}{P_t} = (C_t)^{\sigma^{-1}} (H_t)^{\varphi^{-1}}, \tag{5}$$

plus the appropriate no-Ponzi, transversality condition. It also implies that each period budget constraint holds with equality.

As we shall see later, the fact that taxes on bank equity are distortionary would have an impact on the optimization problem for the banks. However, the problem of the banks is such that the yield on bank capital is known and determined at time t. Therefore, by simple arbitrage between (3) and (4) it follows that $\left(1 - \iota^h\right) R_{t+1}^b = I_{t+1}$ is necessary for an interior solution (where households hold bank deposits and equity) to exist.

2.2 Retailers

We add a continuum of retailers of unit mass to introduce monopolistic competition in the goods market. The retail sector transforms wholesale output into differentiated goods using a linear technology.[5] Each retail variety is then sold to households, entrepreneurs and capital goods producers, and bundled up for either consumption or investment. Only capital goods producers acquire these varieties for investment purposes. The retailers effectively add a 'brand' name to the wholesale good to introduce differentiation. Variety is valued by all potential costumers, consequently retailers gain monopolistic power to charge a retail mark-up on them.

2.2.1 Aggregation of Final Goods

We denote the differentiated varieties as $Y_t(z)$, where the index $z \in [0, 1]$ identifies each retailer. Final goods used for consumption and investment, Y_t, are bundles of these differentiated varieties, $Y_t(z)$, aggregated by means of a common CES index as follows,

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\theta - 1}{\theta}} dz \right]^{\frac{\theta}{\theta - 1}}. \tag{6}$$

[5] For simplicity, we assume that no capital or labor is needed in the retail sector, so the wholesale good is the only input of production.

The elasticity of substitution across varieties is represented by $\theta > 1$. Under standard results, the corresponding consumption price index (CPI) is given by,

$$P_t = \left[\int_0^1 P_t(z)^{1-\theta}\, dz\right]^{\frac{1}{1-\theta}}, \tag{7}$$

where $P_t(z)$ is the price charged by retailer z for its variety. The optimal allocation of expenditure in each variety, i.e.

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\theta} Y_t, \tag{8}$$

implies that retailers face a downward sloping demand function. The assumption of identical bundling for consumption and investment of final goods rules out the possibility that the relative price of investment in units of consumption is affected by the composition of the bundles. Hence, the relative price of investment (or Tobin's q) varies solely because of the technological constraints on the capital goods producers that we shall introduce later.

2.2.2 The Optimal Pricing Equation

Retailers set prices to maximize profits, but their ability to re-optimize is constrained because they face nominal rigidities à la Calvo (1983). The retailer maintains its previous period price with an exogenous probability $0 < \alpha < 1$ in each period. However, with probability $(1-\alpha)$, the retailer is allowed to optimally reset its price. The law of large numbers implies that a fraction $(1-\alpha)$ of retailers re-optimize their prices in each period. Therefore, whenever re-optimization is possible, a retailer z chooses its price, $\widetilde{P}_t(z)$, to maximize the expected discounted value of its net nominal profits, i.e.

$$\sum\nolimits_{\tau=0}^{\infty} \mathbb{E}_t\left[\alpha^\tau M_{t,t+\tau}\widetilde{Y}_{t,t+\tau}(z)\left(\widetilde{P}_t(z) - (1-\iota^r) P^w_{t+\tau}\right)\right], \tag{9}$$

where $M_{t,t+\tau} \equiv \beta^\tau \left(\frac{C_{t+\tau}}{C_t}\right)^{-\sigma^{-1}} \frac{P_t}{P_{t+\tau}}$ is the household's stochastic discount factor (SDF) for τ-periods ahead nominal payoffs, $P^w_{t+\tau}$ is the nominal price of wholesale goods, and $\widetilde{Y}_{t,t+\tau}(z) = \left(\frac{\widetilde{P}_t(z)}{P_{t+\tau}}\right)^{-\theta} Y_{t+\tau}$ is the demand at time $t+\tau$ given that prices remain fixed at $\widetilde{P}_t(z)$ (see equation (8)). We also include a subsidy on inputs for retailers, ι^r, which is used by the government to eliminate the retail mark-up distortion whenever $\iota^r = \frac{1}{\theta}$.

The solution to the retailer's maximization problem satisfies the following first-order condition,

$$\sum\nolimits_{\tau=0}^{\infty} \mathbb{E}_t\left[(\alpha\beta)^\tau \left(\frac{C_{t+\tau}}{C_t}\right)^{-\sigma^{-1}} \widetilde{Y}_{t,t+\tau}(z)\left(\frac{\widetilde{P}_t(z)}{P_{t+\tau}} - \frac{\theta(1-\iota^r)}{\theta-1}\frac{P^w_{t+\tau}}{P_{t+\tau}}\right)\right] = 0, \tag{10}$$

where $\frac{\theta}{\theta-1}$ denotes the retail mark-up, and $\frac{P^w_t}{P_t}$ denotes the price of wholesale output in units of consumption. The latter provides a measure for the real marginal costs before the government subsidy. In the literature, the first-order condition in (10) is often referred to as the price-setting rule. Given that a fraction α of retailers maintains prices in period t, and that all re-optimizing retailers face a symmetric problem, the aggregate CPI in (7) can be re-written in the following terms,

$$P_t = \left[\alpha P_{t-1}^{1-\theta} + (1-\alpha)\widetilde{P}_t(z)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{11}$$

where $\widetilde{P}_t(z)$ is the (symmetric) optimal price implied by equation (10).

Technically, there is no 'aggregate production function' for the final ouput, Y_t. However, there is a simple way to account for the distribution of resources. By market clearing, the sum of the individual retailers demands of the wholesale good has to be equal to the total production of the wholesale producers, i.e.

$$\int_0^1 Y_t(z)\,dz = Y_t^w. \tag{12}$$

Using the optimal allocation of expenditure in (8), we get that,

$$Y_t = \left(\frac{P_t^*}{P_t}\right)^{\theta} Y_t^w, \tag{13}$$

where $P_t^* \equiv \left[\int_0^1 P_t(z)^{-\theta}\,dz\right]^{-\frac{1}{\theta}} = \left[\alpha\left(P_{t-1}^*\right)^{-\theta} + (1-\alpha)\widetilde{P}_t(z)^{-\theta}\right]^{-\frac{1}{\theta}}$. The term $p_t^* \equiv \left(\frac{P_t^*}{P_t}\right)^{\theta} \leq 1$ characterizes the magnitude of the efficiency distortion due to sticky prices. In other words, it determines the costs of missallocating expenditures whenever relative price dispersion is non-negligible in an environment with sticky prices.

Since households own the retailers, we assume that all profits (or losses) from the retail activity are rebated lump-sum to the households every period. After a bit of algebra, the aggregate nominal profits received by the representative household can be computed as,

$$\begin{aligned}\Pi_t^r &\equiv \int_0^1 \left[Y_t(z)\left(P_t(z) - (1-\iota^r)P_t^w\right)dz\right] \\ &= P_t\left(\frac{P_t^*}{P_t}\right)^{\theta} Y_t^w - (1-\iota^r)P_t^w Y_t^w,\end{aligned} \tag{14}$$

where the second equality follows from the optimal allocation of expenditure in each variety described in (8), the aggregation formulas in (6) – (7), and the relationship between final output and wholesale output implied by (12).

2.3 Capital Goods Producers

There is a continuum of capital goods producers of unit mass, which are perfectly competitive. At time t, these producers combine investment goods, X_t, and depreciated capital, $(1-\delta)K_t$, to manufacture new capital goods, K_{t+1}. The production of new capital is limited by technological constraints. We assume that the aggregate stock of new capital evolves according to the following law of motion,

$$K_{t+1} \leq (1-\delta)K_t + \Phi(X_t, X_{t-1}, K_t)X_t, \tag{15}$$

where X_t is real aggregate investment, K_t stands for real aggregate capital, and $0 < \delta < 1$ is the depreciation rate. The function $\Phi(X_t, X_{t-1}, K_t)$ implicitly characterizes the technology available to the capital goods producers.

We explore three different specifications of the technological constraint. The neoclassical case (NAC) assumes that the transformation of investment goods into new capital can be attained at a one-to-one rate, i.e.

$$\Phi\left(X_t, X_{t-1}, K_t\right) = 1. \tag{16}$$

The so-called capital adjustment (CAC) specification, favored *inter alia* by BGG (1999), takes the following form,

$$\Phi\left(\frac{X_t}{K_t}\right) = 1 - \frac{1}{2}\chi\frac{\left(\frac{X_t}{K_t} - \delta\right)^2}{\frac{X_t}{K_t}}, \tag{17}$$

where $\frac{X_t}{K_t}$ denotes the investment-to-capital ratio. And, finally, the investment adjustment (IAC) specification, prefered by Christiano *et al.* (2005), takes the following form,[6]

$$\Phi\left(\frac{X_t}{X_{t-1}}\right) = 1 - \frac{1}{2}\kappa\frac{\left(\frac{X_t}{X_{t-1}} - 1\right)^2}{\frac{X_t}{X_{t-1}}}, \tag{18}$$

where $\frac{X_t}{X_{t-1}}$ denotes the gross investment growth rate. The parameters $\chi > 0$ and $\kappa > 0$ regulate the degree of concavity of the technological constraint and, therefore, the sensitivity of investment in new capital to the relative price of capital in units of consumption.[7]

Capital goods producer choose their investment demand, X_t, and their supply of new capital, K_{t+1}, to maximize the expected discounted value of their net profits, i.e.

$$\sum\nolimits_{\tau=0}^{\infty} \mathbb{E}_t\left[M_{t,t+\tau}P_{t+\tau}\left(Q_{t+\tau}K_{t+\tau+1} - (1-\delta)\overline{Q}_{t+\tau}K_{t+\tau} - X_{t+\tau}\right)\right], \tag{19}$$

subject to the law of motion for capital described in (15). Here, $M_{t,t+\tau} \equiv \beta^{\tau}\left(\frac{C_{t+\tau}}{C_t}\right)^{-\sigma^{-1}}\frac{P_t}{P_{t+\tau}}$ is the household's stochastic discount factor (SDF) for τ-periods ahead nominal payoffs, since households own the capital goods producers. As a matter of convention, K_{t+1} denotes the real stock of capital built (and determined) at time t for use at time $t+1$.

The investment good is bundled up in the same fashion as the consumption good and is bought at the same price, P_t, as can be inferred from the profit function in (19). The depreciated capital is bought at a re-sale price $\overline{Q}_t$ in units of the consumption good. However, the new capital is then sold to the entrepreneurs at a price Q_t, that determines the relative cost of investment in units of consumption and is often denoted as Tobin's q. In a conceptual departure from BGG (1999), we assume that frictions in the secondary market for used capital prevent arbitrage between the re-sale value of old capital and the sale value of new capital, i.e. $\overline{Q}_t \neq Q_t$. Those frictions are left unmodelled. However, we assume that the parties involved in the secondary market (entrepreneurs and capital goods producers) view these frictions as entirely out of their control and, hence, treat the resulting wedge $o_t \equiv \frac{\overline{Q}_t}{Q_t}$ as an exogenous and random shock. Moreover, each

[6] The IAC specification can be viewed as a reduced form for a richer economic environment. For interesting motivations, see Matsuyama (1984) and Lucca (2006).

[7] In steady state, the CAC function satisfies that $\Phi(\delta) = 1$, $\Phi'(\delta) = 0$, and $\Phi''(\delta) = -\frac{\chi}{\delta} < 0$. Similarly, the IAC function satisfies that $\Phi(1) = 1$, $\Phi'(1) = 0$, and $\Phi''(1) = -\kappa < 0$.

individual entrepreneur and capital producer matched in the secondary market get a different draw of this random wedge.[8]

The optimization of capital producers yields a standard first-order condition that determines the linkage between Tobin's q, Q_t, and investment, X_t, i.e.

$$Q_t\left[\Phi\left(X_t, X_{t-1}, K_t\right)+\frac{\partial\Phi\left(X_t, X_{t-1}, K_t\right)}{\partial X_t}X_t\right]+\beta\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma^{-1}}Q_{t+1}\frac{\partial\Phi\left(X_{t+1}, X_t, K_{t+1}\right)}{\partial X_t}X_{t+1}\right]=1, \tag{20}$$

which does not depend on the wedge o_t. The law of motion for capital is also binding in each period. Given our alternative specifications of the technological constraint, we could re-write the first-order condition in (20) more compactly as,

$$\begin{cases} Q_t=1, \text{ if NAC,} \\ Q_t\left[\Phi\left(\frac{X_t}{K_t}\right)+\Phi'\left(\frac{X_t}{K_t}\right)\frac{X_t}{K_t}\right]=1, \text{ if CAC,} \\ Q_t\left[\Phi\left(\frac{X_t}{X_{t-1}}\right)+\Phi'\left(\frac{X_t}{X_{t-1}}\right)\frac{X_t}{X_{t-1}}\right]=1+\beta\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma^{-1}}Q_{t+1}\Phi'\left(\frac{X_{t+1}}{X_t}\right)\left(\frac{X_{t+1}}{X_t}\right)^2\right], \text{ if IAC.} \end{cases} \tag{21}$$

The first-order conditions in (21) reveal that while Q_t adjusts to shocks, the response of investment varies depending on the underlying technology. Profits (or losses) may arise since X_{t-1} and K_t are pre-determined at time t and cannot be adjusted freely. The profits at each point in time, i.e.

$$\begin{aligned} \Pi_t^k &\equiv P_tQ_tK_{t+1}-(1-\delta)P_t\overline{Q}_tK_t-P_tX_t \\ &= P_tQ_t\Phi\left(X_t, X_{t-1}, K_t\right)X_t-(o_t-1)(1-\delta)P_tQ_tK_t-P_tX_t, \end{aligned} \tag{22}$$

are added to the budget constraint of the households since they are the only shareholders.

2.4 Wholesale Producers

There is a continuum of mass one of wholesale producers, all of which are perfectly competitive. Wholesale producers combine capital rented from the entrepreneurs with the unskilled labor provided by the households and the managerial labor provided by the entrepreneurs to produce wholesale goods according to the following Cobb-Douglas technology, i.e.

$$Y_t^w \leq e^{a_t}\left(K_t\right)^{1-\psi-\varrho}\left(H_t\right)^{\psi}\left(H_t^e\right)^{\varrho}, \tag{23}$$

where Y_t^w is the output of wholesale goods, K_t is the aggregate capital rented by wholesale producers, and H_t and H_t^e are the labor demands for unskilled and managerial labor respectively.

With a constant returns-to-scale technology, the unskilled and managerial labor shares in the production function are determined by the coefficients $0<\psi<1$ and $0<\varrho<1$ respectively. In keeping with BGG (1999), the entrepreneurial share is often assumed to be very small, i.e. ϱ would be close to zero. The

[8] While a conceptual departure from BGG (1999), we are going to map this re-sale shock into the idiosincratic shock to returns on capital that BGG (1999) introduce in their framework. That keeps our departure from the original model to a minimum, but requires a comment. As we shall see later, that mapping entails that Z_t ought to be viewed as a function of a number of endogenous aggregate variables and prices. We must assume that individual capital goods producers and entrepreneurs are small enough to take them as purely exogenous and out of their control. Otherwise, we could not model the wedge Z_t the way we do it here.

productivity shock, a_t, follows an AR(1) process of the following form,

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a, \tag{24}$$

where ε_t^a is a zero mean, uncorrelated and normally-distributed innovation. The parameter $-1 < \rho_a < 1$ determines the persistence of the productivity shock, and $\sigma_a^2 > 0$ the volatility of its innovation.

Wholesale producers maximize their static profit, i.e.

$$\Pi_t^w \equiv P_t^w Y_t^w - R_t^w K_t - W_t H_t - W_t^e H_t^e, \tag{25}$$

subject to the technological constraint implied by (23). Wholesale producers rent labor from households and entrepreneurs at competitive nominal wages, W_t and W_t^e respectively, and pay a nominal price per unit of capital rented from the entrepreneurs, R_t^w. The optimization of the wholesale producers results in the following well-known rules to compensate the factors of production, i.e.

$$R_t^w = (1 - \psi - \varrho) \frac{P_t^w Y_t^w}{K_t}, \tag{26}$$

$$W_t = \psi \frac{P_t^w Y_t^w}{H_t}, \tag{27}$$

$$W_t^e = \varrho \frac{P_t^w Y_t^w}{H_t^e}. \tag{28}$$

The optimization of the wholesale producer can be summarized in these first-order conditions plus the technological constraint in (23) holding with equality. Wholesale producers make zero profits in every period (i.e., $\Pi_t^w = 0$), therefore the entrepreneurs who own them do not receive any dividends. Wholesale producers rent the capital they use, so in every period entrepreneurs get rents from the wholesale producers and also the re-sale value on the depreciated capital from the capital goods producers.

As we shall see shortly, uncertainty about the re-sale value of capital is the underlying risk that distorts the relationship between borrowers (the entrepreneurs) and lenders (the banks). Hence, it distorts the allocation of household and entrepreneurial savings towards productive investments through capital.

2.5 Entrepreneurs

There is a continuum of entrepreneurs of unit mass. Entrepreneurs are infinitely-lived agents with identical preferences which are linear in consumption, C_t^e, i.e.[9]

$$\sum_{\tau=0}^{\infty} (\beta\eta)^\tau \, \mathbb{E}_t \left[C_{t+\tau}^e \right], \tag{29}$$

where $\beta\eta$ is the subjective intertemporal discount factor. Entrepreneurs inelastically supply one unit of managerial labor, i.e.

$$H_t^e = 1, \ \forall t. \tag{30}$$

[9] For our purposes, the ease of aggregation is an important characteristic. The linear utility function gives rise to linear Engel curves (is a Gorman polar function), so this choice satisfies the necessary conditions for exact linear aggregation. However, it should be noted that our aggregation also relies on the presumption that all individual problems for the entrepreneurs would have an interior solution.

The entrepreneurs utility function also differs from that of the households because they are risk-neutral (linear utility), and they discount utility at a higher rate (i.e., $0 < \eta < 1$).[10] The assumption of risk-neutrality implies that entrepreneurs care only about expected returns and, therefore, considerably simplifies the financial contract which we discuss in the next subsection. The assumption that entrepreneurs are more impatient than households is meant to insure that entrepreneurs never save enough resources to overcome their financing constraints.

At the end of period t, the entrepreneur receives a competitive nominal wage, W_t^e, and earns income from the capital rented in the production of wholesale goods at the beginning of the period, $R_t^w K_t$, as well as from the re-sale value on the depreciated capital, $(1-\delta) P_t \overline{Q}_t K_t$.[11] For a given entrepreneur, we define the returns on capital acquired at time $t-1$, $\omega_t R_t^e$, as the total income generated by a unit of capital at time t relative to its acquisition cost, i.e.[12]

$$\omega_t R_t^e \equiv \frac{R_t^w K_t + (1-\delta) P_t \overline{Q}_t K_t}{P_{t-1} Q_{t-1} K_t} = \left(\frac{\frac{R_t^w}{P_t} + (1-\delta) o_t Q_t}{Q_{t-1}} \right) \frac{P_t}{P_{t-1}}, \tag{31}$$

where the rental rate on capital, R_t^w, is determined in equation (26). Returns on capital are subject to idiosyncratic shocks, ω_t, which are derived from the random re-sale distortion, $o_t \equiv O(\omega_t; \cdot)$, which we have already discussed. After repaying the outstanding loans to the banking system based on the terms and conditions of the financial contract, entrepreneurs receive a fraction of the aggregate returns on capital, i.e. a share of $\omega_t R_t^e P_{t-1} Q_{t-1} K_t$. The entrepreneurs own the wholesale producers, but these firms generate zero profits and, therefore, get no dividends.

Using the resources coming from managerial wages and capital income, the entrepreneurs must buy the new capital, K_{t+1}, and decide how much to consume, C_t^e. New capital is needed for the production of wholesale goods at time $t+1$. Net of consumption, the entrepreneurs set aside a portion of their income in the form entrepreneurial net worth (in BGG's (1999) own terminology), N_{t+1}. Entrepreneurial net worth is, anyway, just another word to refer to the savings of the entrepreneur. The entrepreneurs use these savings, N_{t+1}, as well as loans from the banking system, L_{t+1}, to fund the acquisition of new capital, $P_t Q_t K_{t+1}$, i.e.

$$P_t Q_t K_{t+1} = N_{t+1} + L_{t+1}. \tag{32}$$

Equation (32), which holds with equality, can be interpreted as a cash-in-advance constraint on the purchase of new capital, K_{t+1}. It also tells us that new capital is the only asset available to receive the savings of entrepreneurs. As in BGG (1999), we rule out a more complex portfolio setting for entrepreneurs.[13]

[10] The relative impatience of entrepreneurs is for all practical purposes equivalent to the assumption of exponential death emphasized in the presentation of the BGG (1999) model.

[11] Implicitly we assume that capital goods producers are willing to buy back the depreciated capital from the entrepreneurs at some price, but that distortions in the secondary market create a random wedge between the acquisition cost of new capital and the re-sale value of old capital in each period. For further discussion on the distinction between these two prices, see footnote 13 in BGG (1999, p. 1357).

[12] To be more precise, we define the rate of return on capital, R_t^e, as the rate that would prevail if the secondary market for used or depreciated capital led to arbitrage between the re-sale value of capital and the cost of acquiring new capital, i.e. $\overline{Q}_t = Q_t$. The returns of capital are realized under distortions in the secondary market, so the actual rate of return on capital is $\omega_t R_t^e$ as defined in equation (31). For convenience, we implicitly characterize the randomness of the wedge in the re-sale value, o_t, by positing that ω_t is a random variable.

[13] That is likely not without consequence for the risk-taking behavior of entrepreneurs and for the external loan contracts

As we shall see, the costly-state verification theory implies that external funding (loans) is more expensive than internal funding (savings). Therefore, the Modigliani-Miller theorem on the irrelevance of the capital structure no longer holds, and the balance sheet of the entrepreneurs-borrowers becomes rather crucial. In order to understand the budget constraint of the entrepreneurs, we first need to discuss the resulting split of capital income revenues between the entrepreneurs (the borrowers) and the banking system (the lenders) implied by the available financial contracts. We also need to specify the nature of the inefficiency due to monitoring costs. Afterwards, we would discuss the budget constraint fully and the optimization for the entrepreneurs conditional on the implementation of the available financial contracts.

Idiosyncratic and Systematic Risk. At time t, the entrepreneurs-borrowers and the lenders must agree on a contract that facilitates the acquisition of new capital, K_{t+1}. The entrepreneurs operate in a legal environment that ensures their financial commitment is subject to limited liability. Hence, in case of default at time $t+1$, the banks can only appropriate at most the total capital income of the entrepreneurs, i.e. $\omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1}$. The loan is restricted to take the standard form of a one-period risky debt contract as in Townsend (1979), Gale and Hellwig (1985) and BGG (1999).[14] The capital income of an individual entrepreneur at time $t+1$ on capital acquired at time t is greatly affected by the realization of $\omega_{t+1} \in (0, \infty)$, which characterizes an idiosyncratic shock.

We interpret this shock ω_{t+1} as a reduced form representation of exogenous losses on the re-sale value of depreciated capital due to frictions of the secondary market left unmodelled. Those frictions imply a wedge between the re-sale value of capital and the acquisition cost of new capital (or Tobin's q) within the period. It is assumed that the shock is not known at time t when the contract is signed, and can only be observed privately by the entrepreneur itself at time $t+1$. Banks, however, have access to a costly monitoring technology that permits them to uncover the true realization at a cost, i.e. $\mu \omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1}$ where $0 < \mu < 1$.

We denote $\phi(\omega_{t+1} \mid s_{t+1})$ the density and $\Phi(\omega_{t+1} \mid s_{t+1})$ the cumulative distribution of ω_{t+1} conditional on a given realization of the aggregate shock s_{t+1}. The mean return of each entrepreneur on its capital acquisition is a function of the aggregate shock s_{t+1} (e.g., Faia and Monacelli, 2007). The aggregate shock s_{t+1} captures our notion of systemic risk on the re-sale value of depreciated capital, which has the effect of shifting the distribution of the risky capital income. The systemic risk shock, s_t, follows an AR(1) process of the following form,

$$s_t = \rho_s s_{t-1} + \varepsilon^s_t, \tag{33}$$

where ε^s_t is a zero mean, uncorrelated and normally-distributed innovation. The parameter $-1 < \rho_s < 1$ determines the persistence of the productivity shock, and $\sigma^2_s > 0$ the volatility of its innovation. We also assume that the innovations of the systemic risk are potentially correlated with the innovations of the productivity shocks, i.e. $-\infty < \sigma_{a,s} \equiv cov(\varepsilon^a_t, \varepsilon^s_t) < +\infty$.

The expected idiosyncratic shock on capital income, ω_{t+1}, conditional on the realization of the aggregate shock, s_{t+1}, is given by,

$$\mathbb{E}[\omega_{t+1} \mid s_{t+1}] = \frac{1}{\xi + 1} [\xi + \tanh(\lambda\gamma + \gamma s_{t+1})], \tag{34}$$

that the entrepreneurs-borrowers and the lenders rely upon.

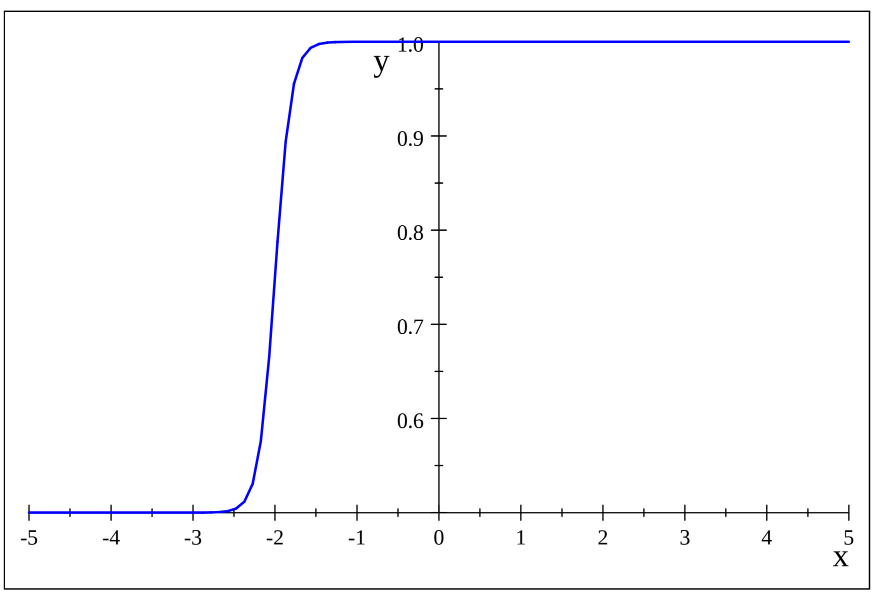
[14] For a discussion of optimal contracts in a dynamic costly state verification framework, see Monnet and Quintin (2005).

where $\lambda > 0$ determines the level of the expected losses, $\gamma > 0$ characterizes the speed (or the slope) on the expected losses and $\xi > 1$ captures the depth of the expected systemic losses in the secondary market. We arbitrarily cap the expected losses to lie somewhere in the interval $\left[\frac{\xi-1}{\xi+1}, 1\right]$. We believe this specification is flexible enough to allow for catastrophic losses in the left-tail of the distribution of the systemic risk shock, s_{t+1}.

By choosing λ sufficiently high, we would ensure that during most of the time the expected idiosyncratic shock is relatively close to one, i.e. $\mathbb{E}\left[\omega_{t+1} \mid s_{t+1}\right] \simeq 1$. This is the assumption in BGG (1999), and means in our interpretation that on average entrepreneurs get a re-sale value on their depreciated capital that is approximately equal to the acquisition cost of new capital. By choosing γ sufficiently high, we ensure that the transition towards catastrophic losses would be abrupt, rather than gradual. That gives us a practical approximation to a world with two regimes: in good times, the economy is close to the BGG (1999) framework; in bad times, the economy confronts catastrophic losses on the secondary capital markets.

In any event, there is a non-negligible probability that markets will fall into disarray due to catastrophic losses in the secondary market. We conjecture that the systemic risk shock is positively correlated with the productivity shock, i.e. $\sigma_{a,s} > 0$, implying that periods of catastrophic losses are more likely whenever productivity is also unusually low. We would speculate that following a different monetary policy rule depending on whether the economy is in bad times and good times could make sense in this context.

We keep the same structure of BGG (1999), which implies that when systemic losses occur in 'bad times' they hit the income generated by the capital acquired by the entrepreneurs, but do not affect the stock of physical capital directly. We leave for future research a fully-fledged theory of frictions in the secondary market for depreciated capital and the exploration of what would happen if these losses would also affect the size of the capital stock.



This figure plots the expected loss function due to systemic risks on the secondary market for the case where $\lambda = 2$, $\gamma = 5$, $\xi = 3$. Notice that in the special case in which $\rho_s = 0$, $\sigma_s = 1$ and $\sigma_{a,s} = 0$, the systemic shock is distributed as a standard normal. Therefore, under this specification of the expected loss function, the separation between the good and bad states occurs around -2. The probability of falling into the bad state is merely 2.3% in every period. This example shows that we can easily model catastrophic losses as "rare" event occurances. Hence, this framework allows us to grasp how rare events affect the behavior of rational agents in an environment where financial frictions are of first-order importance.

2.5.1 Loan Origination

At time t, the entrepreneur-borrower and the lender must agree on the terms of the loan to be repaid at time $t+1$. Default on a loan signed at time t occurs when the income from capital received at time $t+1$ after the idiosyncractic shock ω_{t+1} and the aggregate shock s_{t+1} are realized, i.e. $\omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1}$, falls short of the amount that needs to be repaid. Hence the default space is implicitly characterized by,

$$\omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1} \leq I^l_{t+1} L_{t+1}, \tag{35}$$

where I^l_{t+1} is short-hand notation for the amount of repayment agreed at time at time t per unit of loan, and L_{t+1} defines the loan size. A risky one-period loan contract at time t can be defined in terms of a threshold on the capital losses, $\overline{\omega}_t$,[15] and a measure of the total capital income, $R^e_{t+1} P_t Q_t K_{t+1}$, such that the repayment function is given by,

$$I^l_{t+1} L_{t+1} = \overline{\omega}_{t+1} R^e_{t+1} P_t Q_t K_{t+1}. \tag{36}$$

When default occurs, i.e. when $\omega_t < \overline{\omega}_t$, the entrepreneur cannot repay the amount it owns based on the received capital income. To avoid misreporting on the part of the defaulting entrepreneur, the lender must verify the individual's capital income. That requires the lender to expend in monitoring costs the nominal amount of $\mu \omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1}$. In that case, the entrepreneur gets nothing, while the bank gets $(1-\mu)\,\omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1}$. If the entrepreneur does not default, i.e. if $\omega_t \geq \overline{\omega}_t$, then he pays $\overline{\omega}_{t+1} R^e_{t+1} P_t Q_t K_{t+1}$ back to the lender and keeps the rest for himself, i.e. the entrepreneur gets to keep $\left(\omega_{t+1} - \overline{\omega}_{t+1}\right) R^e_{t+1} P_t Q_t K_{t+1}$.

We take this defaulting rule and the implied distribution of capital income between the entrepreneur-borrower and the lender as given. At time $t+1$, the capital income expected by the entrepreneur after observing all aggregate shocks,[16] but before the realization of its own idiosyncratic shock ω_{t+1}, can be computed as,

$$\begin{aligned}
& \int_{\overline{\omega}_{t+1}}^{+\infty} \left[\omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1} - I^l_{t+1} L_{t+1}\right] \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1} \\
= \quad & R^e_{t+1} P_t Q_t K_{t+1} \left[\int_{\overline{\omega}_{t+1}}^{+\infty} \left(\omega_{t+1} - \overline{\omega}_{t+1}\right) \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1}\right] \\
= \quad & R^e_{t+1} P_t Q_t K_{t+1} f\left(\overline{\omega}_{t+1}, s_{t+1}\right),
\end{aligned} \tag{37}$$

[15] Notice that by choosing $\overline{\omega}_{t+1}$ and I^l_{t+1} it follows that the supply of loans L_{t+1} is also pin down.

[16] Here, aggregate shocks includes the productivity shock, a_{t+1}, the monetary shock and, m_{t+1}, the systemic risk shock, s_{t+1}.

where,

$$f\left(\overline{\omega}_{t+1}, s_{t+1}\right) \equiv \int_{\overline{\omega}_{t+1}}^{+\infty} \omega_{t+1} \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1} - \overline{\omega}_{t+1}\left(1 - \Phi\left(\overline{\omega}_{t+1} \mid s_{t+1}\right)\right) \tag{38}$$

which, by the law of large numbers, can be interpreted also as the fraction of the expected capital income that accrues to the entrepreneurs. With the information available at time t, entrepreneurs expected capital income should therefore be equal to,

$$P_t Q_t K_{t+1} \mathbb{E}_t\left[R_{t+1}^e\right] \mathbb{E}_t\left[U_{t+1}^e f\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right], \tag{39}$$

where $U_{t+1}^e \equiv \frac{R_{t+1}^e}{\mathbb{E}_t\left[R_{t+1}^e\right]}$ determines the ratio of returns between the realized and the expected capital income, and $\mathbb{E}_t\left[U_{t+1}^e\right] = 1$. The amount of physical capital itself, K_{t+1}, is known because it has been determined at time t. The information up to time t includes the realization of the aggregate shock s_t which has a direct impact on the amount of capital income that would be going to the entrepreneurs in the next period.

In a similar fashion, we can also compute the average nominal capital income received by the lenders on this type of loan contracts net of monitoring costs. At time $t+1$, the capital income expected after observing all aggregate shocks, but before the realization of the idiosyncratic shock ω_{t+1}, would be equal to,

$$\begin{aligned}
& (1-\mu) \int_0^{\overline{\omega}_{t+1}} \left[\omega_{t+1} R_{t+1}^e P_t Q_t K_{t+1}\right] \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1} + \int_{\overline{\omega}_{t+1}}^{+\infty} \left[I_{t+1}^l L_{t+1}\right] \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1} \\
= \; & R_{t+1}^e P_t Q_t K_{t+1} \left[(1-\mu) \int_0^{\overline{\omega}_{t+1}} \omega_{t+1} \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1} + \overline{\omega}_{t+1} \int_{\overline{\omega}_{t+1}}^{+\infty} \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1}\right] \\
= \; & R_{t+1}^e P_t Q_t K_{t+1} g\left(\overline{\omega}_{t+1}, s_{t+1}\right), \qquad (40)
\end{aligned}$$

where,

$$g\left(\overline{\omega}_{t+1}, s_{t+1}\right) \equiv (1-\mu) \int_0^{\overline{\omega}_{t+1}} \omega_{t+1} \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1} + \overline{\omega}_{t+1}\left(1 - \Phi\left(\overline{\omega}_{t+1} \mid s_{t+1}\right)\right) \tag{41}$$

which, by the law of large numbers, can be interpreted as the fraction of the expected capital income that accrues to the lenders. With the information available at time t, lenders expected capital income should therefore be equal to,

$$P_t Q_t K_{t+1} \mathbb{E}_t\left[R_{t+1}^e\right] \mathbb{E}_t\left[U_{t+1}^e g\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right], \tag{42}$$

where $U_{t+1}^e \equiv \frac{R_{t+1}^e}{\mathbb{E}_t\left[R_{t+1}^e\right]}$ again determines the ratio of returns between the realized and the expected capital income.

On the Aggregate Sharing of Capital Income. As BGG (1999), we define the following two variables,

$$\begin{aligned}
\Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) & \equiv \int_0^{\overline{\omega}_{t+1}} \omega_{t+1} \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1} + \overline{\omega}_{t+1}\left(1 - \Phi\left(\overline{\omega}_{t+1} \mid s_{t+1}\right)\right), & (43) \\
\mu G\left(\overline{\omega}_{t+1}, s_{t+1}\right) & \equiv \mu \int_0^{\overline{\omega}_{t+1}} \omega_{t+1} \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1}. & (44)
\end{aligned}$$

Then, we can re-write the share on capital income going to the lenders more compactly as,

$$g\left(\overline{\omega}_{t+1}, s_{t+1}\right) = \Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \mu G\left(\overline{\omega}_{t+1}, s_{t+1}\right). \tag{45}$$

Given the definition of the capital income share going to the entrepreneurs, i.e. $f(\overline{\omega}_{t+1}, s_{t+1})$, it also follows that,

$$\begin{aligned} f(\overline{\omega}_{t+1}, s_{t+1}) &= \int_0^{+\infty} \omega_{t+1} \phi(\omega_{t+1} \mid s_{t+1})\, d\omega_{t+1} - \Gamma(\overline{\omega}_{t+1}, s_{t+1}) \\ &= 1 - J(s_{t+1}) - \Gamma(\overline{\omega}_{t+1}, s_{t+1}), \end{aligned} \tag{46}$$

where the second equality follows from our characterization of the expectation of the idiosyncratic shock. Based on these definitions, we can infer that the capital income sharing rule resulting from this financial contract satisfies that,

$$f(\overline{\omega}_{t+1}, s_{t+1}) + g(\overline{\omega}_{t+1}, s_{t+1}) = 1 - J(s_{t+1}) - \mu G(\overline{\omega}_{t+1}, s_{t+1}), \tag{47}$$

where $J(s_{t+1}) \equiv 1 - \frac{1}{\xi+1}\left[\xi + \tanh(\lambda\gamma + \gamma s_{t+1})\right]$ accounts for the expected systemic losses on the re-sale value of capital and $\mu G(\overline{\omega}_{t+1}, s_{t+1})$ characterizes the conventional monitoring costs associated with the costly-state verification framework.

As can be inferred from (47), a fraction of the capital income is transferred to the capital goods producers due to potential systemic losses in the secondary market for capital while another portion is purely and simply lost due to the burden of monitoring after defaulting. These functions represent the sharing rule between entrepreneurs-borrowers and lenders on the capital income implied by the risky one-period loan described before, and all of them depend on the realization of the systemic risk shock, s_{t+1}. It is worth pointing out that only monitoring costs result in a direct loss of capital income, but the fact that resources are siphoned out of the hands of borrowers and lenders due to market imperfections somewhere else has the potential to substantially distort the incentives of both parties and, therefore, to affect the funding of new capital in this economy.

The Optimization Problem. The formal contracting problem reduces to choosing the quantity of physical capital, K_{t+1}, and the state-contingent threshold, $\overline{\omega}_{t+1}$,[17] that maximize the entrepreneurs' expected nominal return on capital income net of the loan costs (see equation (39)), i.e.

$$P_t Q_t K_{t+1} \mathbb{E}_t\left[R^e_{t+1}\right] \mathbb{E}_t\left[U^e_{t+1}\left(1 - J(s_{t+1}) - \Gamma(\overline{\omega}_{t+1}, s_{t+1})\right)\right], \tag{48}$$

subject to the state-contingent participation constraint for the lenders (see equation (42)), i.e.

$$P_t Q_t K_{t+1} U^e_{t+1} \mathbb{E}_t\left[R^e_{t+1}\right]\left(\Gamma(\overline{\omega}_{t+1}, s_{t+1}) - \mu G(\overline{\omega}_{t+1}, s_{t+1})\right) \geq I^b_{t+1} L_{t+1} = I^b_{t+1}\left[P_t Q_t K_{t+1} - N_{t+1}\right], \tag{49}$$

where it is implicitly agreed that if lenders participate in this contract, they always supply enough loans, L_{t+1}, as long as an *ex ante* participation rate, I^b_{t+1}, is guaranteed to them after accounting for all potential losses. In other words, the BGG (1999) framework does not explicitly consider the possibility of credit rationing, and neither do we. All banks share equally on the aggregate size of the loan. For all practical

[17] In other words, $\overline{\omega}_{t+1}$ is not a fixed threshold determined at time t and applicable to each state of nature at time $t+1$. Instead, the contract determines a different threshold for any possible state of nature that can be reached at time $t+1$. That is what is meant by state-contingent. [Alternatively, we can simply argue that $\overline{\omega}_{t+1}$ is chosen as a function of R^k_{t+1} and S_{t+1}. All uncertainty in this problem revolves around how these two variables will behave in the next period.]

purposes, banks are perfectly competitive but the loans they offer are syndicate loans and the allocation to individual entrepreneurs is done according to need.

The participation constraint ensures that the aggregate revenue that the lenders get from this contract ensures them at least a return of I^b_{t+1} in their aggregate portfolio of loans L_{t+1} at each state. The portfolio of loans L_{t+1} is determined at time t, and therefore so is the desired participation return I^b_{t+1}. We assume that the participation rate satisfies that $I^b_{t+1} < R^e_{t+1}$ by consistency with BGG (1999). Otherwise, I^b_{t+1} is a known function determined by the balance sheet of the banks or, to be more precise, by the costs to the banks of raising funds to make good on their loan pledges. We shall discuss this point later. Notice also that, given the definition of U^e_{t+1}, the participation constraint is written to ensure a certain return I^b_{t+1} on the loans for any possible realization of all the aggregate shocks.[18] This implies that all risk arising from either differences between expected and realized capital income or from losses due to systemic valuation risk is taken by the risk-neutral entrepreneur.

The first-order condition with respect to $\overline{\omega}_{t+1}$ defines the function $\lambda_{t+1} \equiv \lambda\left(\overline{\omega}_{t+1}, s_{t+1}\right)$ in the following terms,

$$\Gamma_1\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \lambda\left(\overline{\omega}_{t+1}, s_{t+1}\right)\left[\Gamma_1\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \mu G_1\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right] = 0, \tag{50}$$

where λ_{t+1} is the Lagrange multiplier on the lenders' participation constraint. By virtue of this optimality condition, we say that the shadow cost of enticing the participation of the lenders in this contract is given by,

$$\lambda\left(\overline{\omega}_{t+1}, s_{t+1}\right) = \frac{\Gamma_1\left(\overline{\omega}_{t+1}, s_{t+1}\right)}{\Gamma_1\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \mu G_1\left(\overline{\omega}_{t+1}, s_{t+1}\right)}, \tag{51}$$

which, in turn, implies that the participation constraint must be binding. Efficiency on the participation constraint requires that,

$$\frac{P_t Q_t K_{t+1}}{N_{t+1}}\left(\frac{R^e_{t+1}}{I^b_{t+1}}\right)\left(\Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \mu G\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right) = \left[\frac{P_t Q_t K_{t+1}}{N_{t+1}} - 1\right], \tag{52}$$

or, more compactly,

$$\frac{P_t Q_t K_{t+1}}{N_{t+1}} = \frac{1}{1 - \left(\frac{R^e_{t+1}}{I^b_{t+1}}\right)\left(\frac{\Psi\left(\overline{\omega}_{t+1}, s_{t+1}\right) + J\left(s_{t+1}\right) + \Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) - 1}{\lambda\left(\overline{\omega}_{t+1}, s_{t+1}\right)}\right)}, \tag{53}$$

where we define $\Psi\left(\overline{\omega}_{t+1}, s_{t+1}\right) \equiv 1 - J\left(s_{t+1}\right) - \Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) + \lambda\left(\overline{\omega}_{t+1}, s_{t+1}\right)\left(\Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \mu G\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right)$. The optimization also requires the following first-order condition with respect to capital, K_{t+1}, to hold,

$$\mathbb{E}_t\left\{\frac{R^e_{t+1}}{I^b_{t+1}}\Psi\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \lambda\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right\} = 0. \tag{54}$$

Simply re-arranging gives us the following expression,

$$\mathbb{E}_t\left(\frac{R^e_{t+1}}{I^b_{t+1}}\right) = \frac{\mathbb{E}_t\left[\lambda\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right]}{\mathbb{E}_t\left[U^e_{t+1}\Psi\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right]}, \tag{55}$$

[18] In other words, we can say that I^b_{t+1} would potentially vary as a function of R^e_{t+1} and $\overline{\omega}_{t+1}$. Notice also that $\overline{\omega}_{t+1}$ is itself a function of R^e_{t+1} and s_{t+1}.

which determines the excess returns per unit of capital above the returns on banks loans that would be required to make the financial contract worthwhile to both entrepreneurs-borrowers and lenders.[19]

Given these relationships, it can be argued in the spirit of BGG (1999), that a formulation for the external financing premium arises in the following terms,

$$\mathbb{E}_t\left[\frac{R^e_{t+1}}{I^b_{t+1}}\right] = s\left(\frac{P_tQ_tK_{t+1}}{N_{t+1}}, \mathbb{E}_t\left(s_{t+1}\right)\right), \tag{56}$$

where we ignore the role of U^e_{t+1} since it drops out at the level of a first-order approximation (as can be seen in the appendix). This characterization of the external financing premium expands the BGG (1999) framework by adding the explicit possibility that the spread itself be affected by the impact of an aggregate shock, s_t. The required return on loans is set at the time the contract is signed, therefore I^b_{t+1} is known at time t and can be taken out of the expectation, i.e.

$$\mathbb{E}_t\left[R^e_{t+1}\right] = s\left(\frac{P_tQ_tK_{t+1}}{N_{t+1}}, \mathbb{E}_t\left(s_{t+1}\right)\right) I^b_{t+1}. \tag{57}$$

This relationship is the key feature of the financial accelerator model.

2.5.2 The Optimal Decision of the Entrepreneurs

As we noted before, entrepreneurs obtain income from entrepreneurial labor at a competitive nominal wage, W^e_t, and from renting and re-selling capital, $\omega_{t+1}R^e_{t+1}P_tQ_tK_{t+1}$. With these resources at hand, each entrepreneur must repay the previous period loans at the agreed rate, i.e. it must repay $I^l_tL_t \equiv \overline{\omega}_{t+1}R^e_{t+1}P_tQ_tK_{t+1}$ or choose to default. It must also finance its own consumption, C^e_t, acquire new capital from the capital producers, $P_tQ_tK_{t+1}$, and borrow again, L_{t+1}. In this environment, the budget constraint of a representative entrepreneur can be described in the following terms,

$$P_tC^e_t + P_tQ_tK_{t+1} \leq W^e_tH^e_t + \int_{\overline{\omega}_t}^{+\infty}\left[\omega_tR^e_tP_{t-1}Q_{t-1}K_t - I^l_tL_t\right]\phi\left(\omega_t \mid s_t\right)d\omega_t + L_{t+1}, \tag{58}$$

which accounts for the uses of all those resources. After observing the aggregate shocks at time t, all uncertainty is resolved regarding the aggregate split of capital income between the entrepreneurs-borrowers and the lenders. Using the sharing rule described in equations (37) – (38) and (46) we can re-write the

[19] Notice that we can re-write the participation constraint as follows using the first-order condition on capital,

$$\begin{aligned}\frac{P_tQ_tK_{t+1}}{N_{t+1}} &= \frac{1}{1 - U^e_{t+1}\mathbb{E}_t\left(\frac{R^e_{t+1}}{I^b_{t+1}}\right)\left(\frac{\Psi(\overline{\omega}_{t+1},s_{t+1})+J(s_{t+1})+\Gamma(\overline{\omega}_{t+1},s_{t+1})-1}{\lambda(\overline{\omega}_{t+1},s_{t+1})}\right)} \\ &= \frac{1}{1 - \frac{\mathbb{E}_t\left[\lambda(\overline{\omega}_{t+1},s_{t+1})\right]}{\lambda(\overline{\omega}_{t+1},s_{t+1})}\left(\frac{U^e_{t+1}\Psi(\overline{\omega}_{t+1},s_{t+1})+U^e_{t+1}(-1+J(s_{t+1})+\Gamma(\overline{\omega}_{t+1},s_{t+1}))}{\mathbb{E}_t\left[U^e_{t+1}\Psi(\overline{\omega}_{t+1},s_{t+1})\right]}\right)}.\end{aligned}$$

If we were to take appropriate expectations on this expression, we would get that,

$$\frac{P_tQ_tK_{t+1}}{N_{t+1}} = \frac{\mathbb{E}_t\left[U^e_{t+1}\Psi\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right]}{\mathbb{E}_t\left[U^e_{t+1}\left(1 - J\left(s_{t+1}\right) - \Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right)\right]}.$$

budget constraint as,

$$P_t C_t^e + P_t Q_t K_{t+1} \leq W_t^e H_t^e + \left[1 - J\left(s_t\right) - \Gamma\left(\overline{\omega}_t, s_t\right)\right] R_t^e P_{t-1} Q_{t-1} K_t + L_{t+1}. \tag{59}$$

The objective of a representative entrepreneur that internalizes the default risk would be to maximize (29) subject to the sequence of budget constraints described in (59) and the cash-in-advance constraint on the capital purchase already noted in equation (32).

Since the cash-in-advance constraint in (32) holds with equality, the budget constraint in (59) can be re-written to define an upper bound on entrepreneurial savings as follows,[20]

$$N_{t+1} \leq W_t^e H_t^e + \left[1 - J\left(s_t\right) - \Gamma\left(\overline{\omega}_t, s_t\right)\right] R_t^e P_{t-1} Q_{t-1} K_t - P_t C_t^e. \tag{60}$$

Here, we depart slightly from BGG (1999) because we assume that a representative entrepreneur consumes in every period, while in the original framework only individual entrepreneurs that fail were allowed to consume.[21] However, we still keep the underlying assumption that entrepreneurial labor income ensures that savings would always be positive, i.e. $N_{t+1} \geq 0$, and that consumption and savings are never zero for any individual entrepreneur. As long as individual entrepreneurs do not eat up all their managerial labor income and their share of capital income, the entrepreneurial savings should be positive and this suffices to close the model.[22]

This intertemporal optimization must satisfy the following Euler equation,

$$1 = (\beta\eta)\, \mathbb{E}_t \left[\left(1 - J\left(s_{t+1}\right) - \Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right)\left(R_{t+1}^e \frac{P_t}{P_{t+1}}\right)\right], \tag{61}$$

which determines the consumption-savings margin for the representative entrepreneur in the aggregate. The left-hand side of (61) is the marginal utility of entrepreneurs' consumption. The right-hand side is the expected discounted rate of return of acquiring a unit of capital after taking into account the costs associated with the need for external funding. The latter term has two components. The first is the share on capital income after discounting the possibility of systemic losses and the probability and costs of default on the loans. The second component is the real rate of return on capital income as implied by equation (31).

This first-order condition is also complemented with the cash-in-advance constraint and the budget constraint, which must hold with equality for an interior solution in equilibrium. Hence, this implies that the entrepreneurial savings must be equal to,

$$N_{t+1} = W_t^e H_t^e + \left[1 - J\left(s_t\right) - \Gamma\left(\overline{\omega}_t, s_t\right)\right] R_t^e P_{t-1} Q_{t-1} K_t - P_t C_t^e. \tag{62}$$

[20] A volatile Tobin's q contributes to the fluctuations of entrepreneurial wealth because, *ceteris paribus*, it can change the total capital bill that the entrepreneur has to pay.

[21] Notice that in the standard BGG (1999) consumption only occurs at the end of the life of the entrepreneur. Those entrepreneurs that default, eat their equity and disappear, while for the entrepreneurs that survive defer their consumption, i.e. $C_t^e = 0$.

[22] There is an additional constraint on the problem of the entrepreneurs, i.e. $N_{t+1} \geq 0$. We are searching for an interior solution in which $N_{t+1} > 0$, therefore the lagrange multiplier on the non-negativity of savings would be equal to zero and the first-order condition in (61) is correctly specified. By concentrating on the problem of an aggregate entrepreneur, we might be allowing for some odd behavior at the individual level. We shall make sure that the interior solution is satisfied, at least at the aggregate level.

In order to formally close the model, we would still need to go back and figure out how the consumption of the entrepreneurs is determined. For that we would need to introduce the resource constraint of the economy, which we will do shortly.

2.6 Banking System

There is a continuum of banks of unit mass. In every period, households and banks have to create banking relationships anew.[23] Both households and banks are symmetric and perfectly competitive, so they take all prices as given. The bank offers the households two types of assets for investment purposes: bank equity shares and one-period deposits. The relationship between the banks and the households breaks off in every period. At liquidation, all benefits accrued on capital are rebated to the shareholders, but there is no re-sale value on the bank shares. So in every period new banking relationships have to be created and new banking stakes and deposits have to be offered. Banks, in turn, use the resources they attract to offer one-period loans to the entrepreneurs with the conditions described before.

For convenience, we define the profits generated by these banking partnerships in terms of a yield, R^b_{t+1}, over the value of the banks equity, B_{t+1}. We normalize the supply of banking shares to be equal to one in each period. Ruling out the possibility of long-lasting relationships between the financial intermediaries and the households implies that bank equity and deposits are indistinguishable for the household up to rate of return. Household deposits are perfectly insured, and pay a risk-free rate, I_{t+1}. A pre-condition for an equilibrium to exist in which bank equity and deposits are held simultaneously is that households be indifferent between the bank yield, R^b_{t+1}, and the risk-free rate, I_{t+1}. Otherwise, households would want to save exclusively on either deposits or bank equity, but not both.

At the end of period t, the balance sheet of the banking system can be summarized as follows,

$$L_{t+1} + \varpi D_{t+1} = B_{t+1} + D_{t+1}, \tag{63}$$

where the right-hand side describes the liabilities, which includes deposits taken at time t, D_{t+1}, and equity offered at the same time, B_{t+1}. The left-hand side shows the assets, $L_{t+1} + \varpi D_{t+1}$. Among the assets, we count the reserves on deposits maintained at the central bank, i.e. ϖD_{t+1}, where $0 \leq \varpi < 1$ represents the compulsory reserve requirement on nominal deposits set by the regulator, and the loans offered at time t, L_{t+1}. As a matter of convention, D_{t+1} denotes nominal deposits and L_{t+1} nominal loans held from time t to $t+1$. For the same token, B_{t+1} is the bank capital issued at time t to be liquidated at time $t+1$.

We can re-write more conveniently the balance sheet as,

$$L_{t+1} = \left(\frac{1-\varpi}{1-\upsilon_{t+1}}\right) D_{t+1}, \tag{64}$$

where we define the leverage ratio on bank capital as $\frac{1}{\upsilon_{t+1}} \equiv \frac{B_{t+1}}{L_{t+1}}$. In other words, the rate of transformation from deposits into loans is affected by the compulsory reserve requirement as well as by the bank's capital leverage policy. In BGG (1999), with $\varpi = 0$ and no bank equity, the transformation rate is one-to-one, i.e.

[23] This matches are exogenously broken after one period with probability one. We implicitly assume that the market for bank equity is such that long-lasting relationships are infeasible. We left open for future research the possibility that banking operations survive for more than one period.

$L_{t+1} = D_{t+1}$. Although the model preserves the basic underlying structure of the bank's balance sheet in BGG (1999), equation (64) already points out that regulatory features should play a significant role on loan supply.

The banks profits would be realized at time $t+1$, and afterwards the bank should be liquidated. We can expressed the profits of the banking system as,

$$\Pi^b_{t+1} \equiv I^b_{t+1} L_{t+1} + \varpi \overline{I}_{t+1} D_{t+1} - R^b_{t+1} B_{t+1} - I_{t+1} D_{t+1}. \tag{65}$$

The required nominal returns on loans, I^b_{t+1}, are determined at time t when the loans are signed with the entrepreneurs-borrowers (see the participation constraint in (49)). Deposits held at the central bank in the form of reserves are also returned to the banks. We assume that they earn an interest on reserves fixed at time t, $\overline{I}_{t+1}$, and designed as a two-part tariff, i.e.

$$\overline{I}_{t+1} \equiv (1-c) + \zeta \left(I_{t+1} - 1 \right), \tag{66}$$

whereby banks pay a fee as a management cost per unit of reserve held at the central bank, $0 < c < 1$, and get back the principal (minus the management fee) and a net rate of return that is proportional to the net risk free rate, $0 < \zeta < 1$. Although in most instances the practice is to set this rate of return to zero (i.e., $c = \zeta = 0$), there are precedents for paying interest on reserves.[24] We also make the simplifying assumption that there is full deposit insurance, as a consequence deposits are riskless and the gross interest rate paid on deposits is equal to the risk-free nominal rate, I_{t+1}, which is known at time t.

Bank capital shareholders, the households, have to be compensated with a certain nominal yield determined at time t, R^b_{t+1}. Since everything piece of the profit function is decided at time t and is known by the banks and the households, competitive banks end up offering a yield to the shareholders that is also known at time t.[25] By arbitrage implied by equations (3) and (4), then it must be the case that,

$$\left(1 - \iota^h \right) R^b_{t+1} = I_{t+1}, \tag{67}$$

which insures that households remain indifferent between holding bank capital or deposits. For a competitive banking sector, the profit function in (65) can be re-written as a zero-profit condition (i.e., $\Pi^b_{t+1} = 0$) in the following terms,

$$\Pi^b_{t+1} \equiv \left[I^b_{t+1} - \upsilon_{t+1} R^b_{t+1} - (1 - \upsilon_{t+1}) \left(\frac{I_{t+1} - \varpi \overline{I}_{t+1}}{1 - \varpi} \right) \right] L_{t+1} = 0, \tag{68}$$

using the constraint of the balance sheet in (64). The problem of the banks is to optimize their capital structure, their trade-off between bank equity and deposits, subject to the constraint that banks must offer

[24] Currently, reserve requirements held at the Federal Reserve do not pay interest. In 2006, congress gave the Federal Reserve permission to pay interest on reserves, but mandated that this wait until 2011 to take place. The Federal Reserve announced changes to reserve management after winning the power to pay interest on excess reserves on October 3, 2008. The Federal Reserve has argued that paying interest would deter banks from lending out excess reserves and as such would make it easier for the Fed to attain its target rate. We do not model this feature explicitly.

[25] It is worth pointing out that the return on loans set in the participation constraint can be made independent of the realization of the states tomorrow, insulating the banks and ultimately the households from all uncertainty. Since households are risk-averse, this scheme appears to be the preferred one.

a yield on bank capital that would make households indifferent given the risk-free rate paid on deposits as given by equation (67), subject to the returns on reserves paid by the central bank as given by equation (66), and finally subject to a regulatory constraint on capital adequacy that implies banks must satisfy that,

$$1 \geq \upsilon_{t+1} \equiv \frac{B_{t+1}}{L_{t+1}} \geq \upsilon, \tag{69}$$

where $0 \leq \upsilon < 1$ could be equal to the minimum mandatory capital adequacy requirement set by the regulator,[26] or could be a lower bound that reflects a buffer above the minimum requirement implied by the statutory requirements of the banks.

We shall make two key parametric assumptions to simplify the problem of the banks, and we will leave the exploration of more complex banking cost structures for future research. Our goal, at this stage, is to make only the smallest possible departure from the original BGG (1999). We assume that $\zeta = 1 - c$ and, furthermore, that taxes on bank equity are bounded by $0 < 1 - \iota^h < \frac{1-\varpi}{1-\zeta\varpi}$.[27] Both assumptions put together imply that,

$$R^b_{t+1} > \left(\frac{I_{t+1} - \varpi \overline{I}_{t+1}}{1 - \varpi} \right). \tag{70}$$

In other words, it is costlier for banks to finance themselves with bank equity than with deposits and, therefore, the lower bound on the the leverage ratio must be binding at all times.[28]

In turn, these assumptions imply that the returns on the portfolio loans that the banks require to participate in funding the entrepreneurs are fully determined by the cost structure of the banks as follows,

$$\begin{aligned} I^b_{t+1} &= \upsilon R^b_{t+1} + (1-\upsilon) \left(\frac{I_{t+1} - \varpi \overline{I}_{t+1}}{1 - \varpi} \right) \\ &= \upsilon \left(\frac{1}{1 - \iota^h} \right) I_{t+1} + (1-\upsilon) \left(\frac{1 - \varpi\zeta}{1 - \varpi} \right) I_{t+1}. \end{aligned} \tag{71}$$

This is what we call the balance sheet channel of banking regulation. It can be easily seen that without capital adequacy requirements, i.e. $\upsilon = 0$, and without reserve requirements, i.e. $\varpi = 0$, we would be back in the original world of BGG (1999). Our equation (71) is essentially a heavily parameterized version of the

[26] The current regulatory regime was shaped primarily by the 1988 international Basle Accord and the 1991 Federal Deposit Insurance Corporation Improvement Act (FDICIA). The Basle Accord (Basel 1) established minimum capital requirements as ratios of two aggregates of accounting capital to risk weighted assets (and certain off-balance sheet activities). The risk weights are supposed to reflect credit risk. For example, commercial and industrial loans have weight one, while U.S. government bonds have zero weight, and consequently do not require any regulatory capital. Primary or tier 1 (core) capital (= book value of its stock plus retained earnings) is required to exceed 4% of risk weighted assets, while total (tier 1 plus tier 2) capital must be at least 8%. In calculating the risk weighted capital asset ratio all loans are assumed to be in the highest risk category in the sense of the Basle Accord, with a risk weight of 100%. This category includes all claims to the non-bank private sector, except for mortgages on residential property, which receive a risk weight of 50%. The riskless securities are in the lowest risk category, with weight zero. Typical examples are Treasury bills and short loans to other depository institutions. Revisions to the Basel Accord (Basel II) allows a degree of additional flexibility in the determination of these weights in order to more closely capture the risk profile of banks.

[27] Whenever $\xi = 0$, this bound implies that $\iota^h > \varpi$; whenever $\xi = 1$, it merely requires that $\iota^h > 0$. Given the fact that tax rates are quite often much higher than the minimum reserve ratios, these bounds are likely not excessively restrictive.

[28] One could justify this conclusions by appealing to the fact that consumers are often willing to pay a convinience yield to banks to have their deposits readily available. While this distinction is not available in our model, it helps to justify why the assumption that deposits are cheaper than equity is reasonable.

following expression for returns on the loan of portfolio under constant returns to scale,

$$\frac{I_{t+1}^{b}}{I_{t+1}} \equiv \upsilon_{t+1} \times \frac{cost(bank\ equity_{t+1})}{I_{t+1}} + (1-\upsilon_{t+1}) \times \frac{cost(deposits_{t+1})}{I_{t+1}}, \tag{72}$$

where $\frac{1}{\upsilon_{t+1}}$ represents the leverage ratio as before.

Arguably, our model remains a very naive characterization of the behavior of banks. We are far from having an integrated model of the business cycle where banks operate in multiple periods and confront simultaneously frictions in their lending operations and nontrivial distortions on the way in which they raise capital or attract depositors. However, the emphasis that we are making with this characterization of the economy is on the regulatory power to alter the operational costs of the banking system. Even in this simplified framework, it immediately transpires that the regulator is able to alter the terms of the banks' operating costs. Hence, the regulator has in its hands a tool to either amplify or reduce the loan supply without directly changing the short-term interest rate. It is our goal to explore how the model responds to monetary policy and regulatory features like those.

The relationship in (71) clearly ties down the return on the portfolio of loans to the risk-free rate, which happens to be also the relevant instrument for monetary policy. The regulatory restriction on capital adequacy in (69) does not have the purpose in this model of protecting the financial system from bad outcomes, since that is already taken care off by the contracting problem with the entrepreneurs-borrowers. Instead, this regulatory constraint gives the monetary authority a way to 'regulate' the supply of loans without having to manipulate the interest rate directly. Then, we can visualize the banks' 'balance sheet' channel in the framework of BGG (1999) by combining (57) and (71) as follows,

$$\mathbb{E}_t\left[R_{t+1}^{e}\right] = \underbrace{s\left(\frac{P_t Q_t K_{t+1}}{N_{t+1}}, \mathbb{E}_t\left(s_{t+1}\right)\right)}_{\text{"agency costs" channel as in BGG (1999)}} \underbrace{\left[\upsilon\left(\frac{1}{1-\iota^{h}}\right) + (1-\upsilon)\left(\frac{1-\varpi\zeta}{1-\varpi}\right)\right]}_{\geq 1 \text{ "balance sheet" channel}} I_{t+1}. \tag{73}$$

This equation shows that the balance sheet channel has the potential to amplify the external financing premium spread. However, because this channel is regulated by the central bank, the monetary authority can potentially 'manipulate' the requirements in order to reduce the amplification effect at times when the agency cost component is rising.

A Short Digression. Of course, this type of model embeds a number of simplifications. Among them are the fact that we do not capture the standard maturity mismatch that most banks maintain through holding long maturity assets and short maturity liabilities. As well, bank equity in this class of models arises only as a result of regulatory requirements. They are also forced to liquidate this capital once the banking relationship with households expires after one period. Hence, we do not account explicitly for the existence of bank capital buffers.

It is not obvious how to solve the problem of the banks where they have to both decide on a capital buffer and loan origination strategy. The simple setting presented here affords us the possibility to consider the problems of bank capital funding and loan origination as essentially independent of each other. We cannot discuss issues of counterparty risk, securitization, etc, however we believe this is still a meaningful extension. It shows that the balance sheet channel has the potential to amplify the agency costs. Most importantly, changes in regulation may affect the impact that the balance sheet channel has on the external financing premium.

Regardless, our goal is to strike a balance between parsimony and detail. We believe that this model has sufficient detail to allow us to think consider the relevant issues: especially regarding a central bank with both regulatory and monetary policy powers.

2.7 Monetary and Fiscal Authorities

We close our description of the government with the specification of a consolidated (and balanced) budget constraint and an interest rate rule for monetary policy. We assume that government expenditures and the subsidy on inputs for the retailers are financed through lump-sum taxes, taxes on bank equity dividends and seigniorage, i.e.

$$\begin{aligned} P_t G_t + T_t + \iota^h R_t^b B_t + M_{t+1} &= \iota^r P_t^w \left[\int_0^1 Y_t(z)\, dz \right] + \overline{I}_t M_t \\ &= \iota^r P_t^w Y_t^w + \overline{I}_t M_t, \end{aligned} \tag{74}$$

where G_t denotes the real government expenditure. However, we do assume for simplicity that government consumption is equal to zero in every period, i.e. $G_t = 0$. The characteristics and bounds on the tax subsidy, ι^r, and the tax rate on dividends, ι^h, as well as the nature of the non-distortionary (lump-sum) taxes or transfers, T_t, have already been discussed elsewhere. The government also funds its operations by issuing at time t high-powered money (the monetary base)[29], M_{t+1}, which is used by the banking sector to finance their compulsory reserve requirements at the central bank.

This model ignores issuing securities, e.g. Treasury bills, of possibly different maturities. For the purpose of defining the monetary base, the model also ignores the currency held by households and the relations with other foreign countries. Money consists only of the sum of the reserves of the banking sector on their accounts at the central bank. Therefore, given the compulsory requirement on reserves, the equilibrium in the money market requires that,

$$M_{t+1} = \varpi D_{t+1}. \tag{75}$$

As it was noted before, those reserves deposited at time t accrue a rate of return, $\overline{I}_t$, which is characterized by the formula in (66). For simplicity, money plays exclusively the role of a unit of account and acts as the counterpart for deposit reserves on the balance sheet of the central bank.

The central bank as a policy-maker is modelled by means of an interest rate reaction function. In the spirit of Taylor (1993), the rule targets the short-term nominal interest rate, I_{t+1}, and is linear in the logs of the relevant arguments in the spirit of Taylor (1993),

$$i_{t+1} = \rho_i i_t + (1 - \rho_i) \left[\psi_\pi \ln \left(\frac{P_t}{P_{t-1}} \right) + \psi_q \ln (Q_t) + \psi_y \ln (Y_t) \right] + m_t, \tag{76}$$

where $i_t \equiv \ln(I_t)$ is the logarithm of the risk-free rate. In line with most of the literature, we assume that monetary authorities are willing to smooth changes in the actual short-term nominal interest rate, i.e.

[29] More precisely, the monetary base is the sum of currency in circulation plus reserves held by the banks at the central bank.

$0 \leq \rho_i \leq 1$, where ρ_i is the smoothing parameter. The other parameters of the reaction function satisfy that $\psi_\pi \geq 1$, $-\infty < \psi_q < +\infty$ and $\psi_y \geq 0$. The monetary shock in logs, m_t, follows an AR(1) process of the following form,

$$m_t = \rho_m m_{t-1} + \varepsilon_t^m, \tag{77}$$

where ε_t^m is a zero mean, uncorrelated and normally-distributed innovation. The parameter $-1 < \rho_m < 1$ determines the persistence of the monetary shock, and $\sigma_m^2 > 0$ the volatility of its innovation. The policy rules reflect the assumption that monetary authorities react to a trade-off between inflation and output, but it can also react to asset prices, Q_t.

A few observations on the specification of (76) are in order. First, we model monetary policy in terms of an implementable rule, whereby the central bank sets the short-run nominal interest rate in response to observable variables only. Second, this general specification allows for a reaction of the monetary policy instrument to deviations of the relative price of capital goods Q_t from its long-run value of one. The latter corresponds also to the hypothetical efficient value of Q_t, i.e., its value in the absence of credit frictions or technological constraints on the production of new capital (regardless of whether or not nominal rigidities are built into the model). Hence, this is the channel through which we allow asset price fluctuations to feed into the setting of monetary policy.

2.8 Resource Constraint

Equilibrium in the final good market requires that the production of the final good be allocated to total private consumption by households and entrepreneurs, investment by capital goods producers, and to cover the costs that originate from either systemic losses in the secondary market for used capital or the monitoring technology required to enforce the loan contract, i.e.

$$Y_t = C_t + C_t^e + X_t + \left[\underbrace{J(s_t)}_{\text{loss due to systemic risk}} + \underbrace{\mu G(\overline{\omega}_t, s_t)}_{\text{loss from monitoring costs}} \right] R_t^e \frac{P_{t-1}}{P_t} Q_{t-1} K_t, \tag{78}$$

where $Y_t = \left(\frac{P_t^*}{P_t}\right)^\theta Y_t^w$. In the above equation, there is no government consumption of the final good. In the model of BGG (1999) government consumption evolves exogenously and is assumed to be financed by means of lump-sum taxes, but having an additional exogenous variable at this stage does not really add anything fundamental to our model.

3 The Benchmark Model

This section describes the log-linearized version of our model and its variants to make the presentation more compact. For more details, we refer the reader to the appendix or suggest further readings along the way. Aggregate consumption evolves according to a standard Euler equation,

$$\widehat{c}_t \approx \mathbb{E}_t\left[\widehat{c}_{t+1}\right] - \sigma\left(\widehat{\imath}_{t+1} - \mathbb{E}_t\left[\widehat{\pi}_{t+1}\right]\right), \tag{79}$$

where $\sigma > 0$ ($\sigma \neq 1$) is the elasticity of intertemporal substitution, $\widehat{c}_t$ denotes consumption, $\widehat{i}_{t+1}$ is the nominal interest rate, and $\widehat{\pi}_t \equiv \widehat{p}_t - \widehat{p}_{t-1}$ stands for inflation. The intertemporal elasticity of substitution, σ, regulates the sensitivity of the consumption path to the Fisherian real interest rates, i.e. $\widehat{r}_{t+1} \equiv \widehat{i}_{t+1} - \mathbb{E}_t\left[\widehat{\pi}_{t+1}\right]$. We approximate the labor supply as follows,

$$\widehat{w}_t - \widehat{p}_t \approx \frac{1}{\sigma}\widehat{c}_t + \frac{1}{\varphi}\widehat{h}_t, \tag{80}$$

where $\varphi > 0$ denotes precisely the Frisch elasticity of labor supply, $\widehat{h}_t$ represents labor, $\widehat{w}_t$ are nominal real wages and $\widehat{p}_t$ is the consumption price index (CPI).

Capital accumulation evolves according to a conventional law of motion,

$$\widehat{k}_{t+1} \approx (1-\delta)\,\widehat{k}_t + \delta\widehat{x}_t, \tag{81}$$

where $\widehat{k}_t$ denotes physical capital and $\widehat{x}_t$ stands for investment. Investment dynamics, however, are conditional on our underlying assumptions regarding the technological constraints of the capital goods producers. The first equation that we add to our model, as in BGG (1999), assumes that this technology is a function of the investment-to-capital ratio. Investment dynamics are governed by,

$$\widehat{q}_t \approx \chi\delta\left(\widehat{x}_t - \widehat{k}_t\right), \tag{82}$$

where $\chi > 0$ regulates the degree of concavity of the cost function around the steady state, and δ denotes the depreciation rate for capital. Both parameters affect the sensitivity of investment to fluctuations in the real value of installed capital (or Tobin's q), $\widehat{q}_t$, through the investment equation.

We also consider two different alternative specifications for this adjustment costs. On one hand, we explore the Christiano *et al.* (2005) conjecture that the technology could be a function of investment growth instead (aka, IAC function). Accordingly, the investment equation behaves as follows,

$$\widehat{x}_t \approx \frac{1}{1+\beta}\widehat{x}_{t-1} + \frac{\beta}{1+\beta}\mathbb{E}_t\left[\widehat{x}_{t+1}\right] + \frac{1}{\kappa\left(1+\beta\right)}\widehat{q}_t, \tag{83}$$

where $\kappa > 0$ regulates the degree of concavity of the cost function around the steady state, and where $0 < \beta < 1$ is the subjective intertemporal discount factor of the households. Both parameters affect the sensitivity of investment to fluctuations in Tobin's q, $\widehat{q}_t$, but the investment equation in (83) reveals now that investment is both inertial and forward-looking unlike in BGG (1999)'s setting. On the other hand, we also consider the simpler case in which there are no technological constraints and one unit of investment can be costlessly transformed into one unit of new capital. Hence, that would imply,

$$\widehat{q}_t \approx 0. \tag{84}$$

This case is of particular interest because without the asset price fluctuations captured by Tobin's q, the BGG (1999) framework loses the characteristic that asset price movements serve to reenforce credit market imperfections. For more details on the derivations of the investment equations, see Martínez-García and Søndergaard (2008).

Retailers choose their price to maximize the expected discounted value of their net profits, subject to a demand constraint. Due to Calvo-signals (e.g., Calvo, 1983), in each period only a fraction $0 < 1 - \alpha < 1$ of the retailers gets to re-optimize. The resulting inflation dynamics aggregating over the optimal price-setting rules of all retailers are captured by the following equation,

$$\widehat{\pi}_t \approx \beta \mathbb{E}_t \left[\widehat{\pi}_{t+1} \right] + \left(\frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha} \right) \left(\widehat{p}_t^w - \widehat{p}_t \right). \tag{85}$$

This equation takes the form of a conventional Phillips curve. In an environment with price rigidity, retailers will price taking into account current as well as future marginal costs, giving rise to this type of forward-looking Phillips curve. We can also argue that up to a first-order approximation it follows that,

$$\widehat{y}_t^w \approx \widehat{y}_t, \tag{86}$$

which means that the final goods delivered by the retailers are approximately equal to the wholesale goods produced by the wholesale producers.

The wholesale producers require homogenous labor and capital to produce wholesale output. All factor markets are perfectly competitive, and each producer relies on the same Cobb-Douglas technology. Naturally, wholesale output can be expressed as follows,

$$\widehat{y}_t^w \approx \widehat{a}_t + (1 - \psi - \varrho) \widehat{k}_t + \psi \widehat{h}_t + \varrho \widehat{h}_t^e, \tag{87}$$

where $0 < \psi < 1$ is the unskilled labor share in the production function and $0 < \varrho < 1$, $\widehat{y}_t^w$ denotes the wholesale output, $\widehat{h}_t$ is unskilled labor and $\widehat{h}_t^e$ is the managerial labor. Since managerial labor is in fixed supply as described in (30), then it must be the case that,

$$\widehat{h}_t^e = 0. \tag{88}$$

Then, final output becomes equal to,

$$\widehat{y}_t \approx \widehat{a}_t + (1 - \psi - \varrho) \widehat{k}_t + \psi \widehat{h}_t, \tag{89}$$

where $\widehat{a}_t$ is the aggregate productivity shock. The productivity shock follows an AR(1) process of the following form,

$$\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \varepsilon_t^a, \tag{90}$$

where ε_t^a is a zero mean, uncorrelated and normally-distributed innovation. The parameter $-1 < \rho_a < 1$ determines the persistence of the productivity shock.

Since wholesale producers operate in a competitive labor market, the real wages paid to households and entrepreneurs should be equal to the marginal return on their respective types of labor. That gives us the following pair of equations for the labor demand,

$$\begin{aligned} \widehat{w}_t - \widehat{p}_t &\approx \left(\widehat{p}_t^w - \widehat{p}_t \right) + \left(\widehat{y}_t - \widehat{h}_t \right), & (91) \\ \widehat{w}_t^e - \widehat{p}_t &\approx \left(\widehat{p}_t^w - \widehat{p}_t \right) + \widehat{y}_t, & (92) \end{aligned}$$

where we already account for the fact that $\widehat{y}_t \approx \widehat{y}_t^w$ and $\widehat{h}_t^e = 0$. Combining equations (80) and (91), we can easily derive a labor market equilibrium condition in the following terms,

$$\widehat{y}_t - \widehat{h}_t + (\widehat{p}_t^w - \widehat{p}_t) - \frac{1}{\sigma}\widehat{c}_t \approx \frac{1}{\varphi}\widehat{h}_t. \tag{93}$$

This equilibrium condition allows us to internalize the behavior of real wages, but we still have to account for the cost of capital. We know that the capital income obtained on capital is implied by equations (31) and (26) for the case where arbitrage is possible in the secondary market for used capital. A first order approximation, then, gives us that,

$$\widehat{r}_t^e - \widehat{\pi}_t \approx \left(1 - \frac{(1-\delta)}{R^e}\right)\left((\widehat{p}_t^w - \widehat{p}_t) + \widehat{y}_t - \widehat{k}_t\right) + \frac{(1-\delta)}{R^e}\widehat{q}_t - \widehat{q}_{t-1}, \tag{94}$$

where R^e is precisely the steady state rate of return on capital income, and the inflation rate is defined as follows $\widehat{\pi}_{t+1} \equiv \widehat{p}_{t+1} - \widehat{p}_t$. Once again, this expression is derived making use of the fact that $\widehat{y}_t \approx \widehat{y}_t^w$.

What this equation says is that, free of distortions on the secondary market for used capital, the real returns on capital income, $\widehat{r}_{t+1}^e - \widehat{\pi}_{t+1}$, must be approximately equal to the marginal returns on capital from the production function and the cost of buying and re-selling the stock of capital to the capital producers (as captured by Tobin's q). The marginal return on capital, which is defined as $\left(\widehat{p}_{t+1}^w - \widehat{p}_{t+1}\right) + \widehat{y}_{t+1} - \widehat{k}_{t+1}$, would give us the competitive rental value of capital. Equations $(82) - (84)$ give us an asset pricing characterization of the Tobin's q which is quite instrumental in the model. Thus far, the model is fairly standard and follows BGG (1999), in particular, closely (although we distinguish upfront between nominal and real variables).

Following the costly state verification framework of BGG (1999), wholesale producers cannot borrow at the riskless rate. The cost of external financing differs from the risk-free rate because the returns to capital of the wholesale producers are unobservable from the point of view of the financial intermediaries. In order to infer the realized return of the entrepreneur, the bank has to pay a monitoring cost. The banks monitor the producers that default, pay the verification cost and seize the remaining capital. In equilibrium, entrepreneurs borrow up to the point where the expected return on capital income equals the cost of external financing,

$$\mathbb{E}_t\left[\widehat{r}_{t+1}^e\right] \approx \widehat{i}_{t+1} + \vartheta(s)\left(\widehat{p}_t + \widehat{q}_t + \widehat{k}_{t+1} - \widehat{n}_{t+1}\right) + \overline{\upsilon}\widehat{\upsilon}_{t+1} + \Theta(s)\,\mathbb{E}_t\left[\widehat{s}_{t+1}\right], \tag{95}$$

where $\widehat{k}_{t+1}$ denotes capital, $\widehat{n}_{t+1}$ are entrepreneurial end-of-period savings (or net worth in BGG (1999)'s terminology), $\widehat{q}_t$ is Tobin's q, $\widehat{p}_t$ is the CPI, $\widehat{i}_{t+1}$ is the risk-free rate, $\widehat{\upsilon}_{t+1}$ determines changes in banking regulation (capital adequacy), and $\widehat{s}_{t+1}$ stands for the systemic risk shock which captures distortions in the secondary market for capital. The systematic risk shock follows an AR(1) process of the following form,

$$\widehat{s}_t = \rho_s \widehat{s}_{t-1} + \varepsilon_t^s, \tag{96}$$

where ε_t^s is a zero mean, uncorrelated and normally-distributed innovation. The parameter $-1 < \rho_s < 1$ determines the persistence of the shock, while $\sigma_{a,s} > 0$ defines the covariance with the productivity shock.

The right-hand side of the external financing premium equation in our model can be decomposed in two terms: the nominal risk-free rate itself on one hand, and the external financing premium on the other hand.[30] The parameter $\vartheta(s)$ measures the elasticity of the external financing premium to variations in

[30] The key mechanism involves the link between "external finance premium" (the difference between the cost of funds raised externally and the opportunity cost of funds internal to the firm) and the net worth of potential borrowers (defined as the

wholesale producer internal funds, measured by its capital expenditures relative to end-of-period savings. The closer both are to being equal, the lower the associated moral hazard. In case entrepreneurs have sufficient savings to finance the entire capital stock, agency problems vanish, so the risk-free rate and the expected return to capital income must coincide by arbitrage. So far, this is exactly the same result found in BGG (1999). Our model, however, indicates that changes in the banking regulation on capital adequacy and systemic risk also add a new dimension to external financing premium.

Aggregate savings for the entrepreneurs accumulate according to the following equation[31],

$$\begin{aligned}
&\widehat{n}_{t+1}-\widehat{p}_t \approx \left(1+\tfrac{PC^e}{N}-\tfrac{W^e}{N}\right)\widetilde{\Upsilon}(s)\left(\widehat{n}_t-\widehat{p}_{t-1}\right)+\tfrac{W^e}{N}\left(\left(\widehat{p}^w_t-\widehat{p}_t\right)+\widehat{y}_t\right)+\\
&+\left(1+\tfrac{PC^e}{N}-\tfrac{W^e}{N}\right)\left(1-\widetilde{\Upsilon}(s)\right)\left(\widehat{r}^e_t-\widehat{\pi}_t+\widehat{q}_{t-1}+\widehat{k}_t\right)-\tfrac{PC^e}{N}\widehat{c}^e_t-\left(1+\tfrac{PC^e}{N}-\tfrac{W^e}{N}\right)\widetilde{\Psi}(s)\,\widehat{s}_t+\\
&+2\left(1+\tfrac{PC^e}{N}-\tfrac{W^e}{N}\right)\widetilde{\Upsilon}(s)\left(\tfrac{PQK}{N}-1\right)\widehat{\pi}_t-\left(1+\tfrac{PC^e}{N}-\tfrac{W^e}{N}\right)\widetilde{\Upsilon}(s)\left(\tfrac{PQK}{N}\left(\widehat{r}^e_t-\widehat{\pi}_t\right)-\left(1-\tfrac{PQK}{N}\right)\left(\widehat{i}_t-\widehat{\pi}_t+\overline{\upsilon}\widehat{\upsilon}_t\right)\right),
\end{aligned} \tag{97}$$

which is fundamentally a state variable in our model. Equation (97) simply tells us that the real present discounted value of the end-of-period savings of the entrepreneur, where nominal savings are denoted $\widehat{n}_{t+1}$, must be approximately equal to the previous period savings adjusted by taking out the cost of capital and adding the differential between the returns on capital capital and the risk-free rate. In turn, this equation is complemented with the following condition,

$$\begin{aligned}
&\widehat{p}_t+\widehat{q}_t+\widehat{k}_{t+1}-\widehat{n}_{t+1}\approx\\
&\approx \Xi(s)\,\mathbb{E}_t\left[\widehat{r}^e_{t+1}-\widehat{\pi}_{t+1}\right]-\left(\tfrac{PQK}{N}-1\right)\mathbb{E}_t\left[\widehat{i}_{t+1}-\widehat{\pi}_{t+1}\right]-\Sigma(s)\,\mathbb{E}_t\left[\widehat{s}_{t+1}\right]-\left(\tfrac{PQK}{N}-1\right)\overline{\upsilon}\widehat{\upsilon}_{t+1},
\end{aligned} \tag{98}$$

which determines the willingness of entrepreneurs to save rather than consume given the expected returns on capital income and the real rates. This margin is also sensitive to changes in banking regulation or systemic risk shocks.

The standard goods market equilibrium condition is augmented with a term capturing the costs of variable bankruptcy derived from the costly-state verification framework of BGG (1999) and the costs associated with systemic risk losses,

$$\begin{aligned}
&\widehat{y}_t \approx \gamma_c\widehat{c}_t+\gamma_{c^e}\widehat{c}^e_t+\gamma_x\widehat{x}_t+\left(1-\gamma_c-\gamma_{c^e}-\gamma_x\right)\left(\widehat{r}^e_t-\widehat{\pi}_t+\widehat{q}_{t-1}+\widehat{k}_t\right)+\\
&+\left(1-\gamma_c-\gamma_{c^e}-\gamma_x\right)\Psi(s)\,\widehat{s}_t+\\
&+\left(1-\gamma_c-\gamma_{c^e}-\gamma_x\right)\Upsilon(s)\left[\widehat{p}_{t-1}+\widehat{q}_{t-1}+\widehat{k}_t-\widehat{n}_t-\left(\tfrac{PQK}{N}-1\right)\left(\widehat{r}^e_t-\widehat{i}_t-\overline{\upsilon}\widehat{\upsilon}_t\right)\right],
\end{aligned} \tag{99}$$

where γ_c denotes the household consumption share, γ_{c^e} denotes the entrepreneurs consumption share, γ_x is the investment share, and $\gamma_{csv} = (1-\gamma_c-\gamma_{c^e}-\gamma_x)$ is the share attributed to the bankruptcy costs and the secondary market imperfections in steady state. In this class of models, the consumption share is a function of the elasticity of substitution across varieties, θ, which is a structural parameter, but does not appear anywhere else in the linearization. Therefore, the consumption share can be viewed as a free parameter in itself. The share on the costly state verification costs is taken as a free parameter to ensure that our model

borrowers' savings).

[31] Our approach here is somewhat similar to Meier and Müller (2005). One particular strand of models we have in mind is that of limited enforcement (e.g. Kiyotaki and Moore 1997). Although the underlying microeconomic assumptions are entirely different, these models give rise to similar financial accelerators.

is flexible enough; however, we adopt in most of our simulations the assumption that these costs are very small in steady state. The costs of monitoring are a function of the value of capital at liquidation plus the returns on capital, all of which is appropriated by the bank after the entrepreneurs declare bankruptcy (and the banks pay for the verification).

In line with most of the literature, we assume that monetary authorities are willing to smooth changes in the actual short-term nominal interest rate, $\widehat{i}_t$, but do target inflation and output (the dual mandate). Short-term rates, however, may deviate unexpectedly from their target rates for exogenous reasons (out of the control of the monetary authorities). Thus, the monetary policy is determined by the following Taylor-type interest rate rule,

$$\widehat{i}_{t+1} = \rho_i \widehat{i}_t + (1 - \rho_i) \left[\psi_\pi \widehat{\pi}_t + \psi_q \widehat{q}_t + \psi_y \widehat{y}_t \right] + \widehat{m}_t, \tag{100}$$

where ρ_i is the smoothing parameter, ψ_π, ψ_q and ψ_y are the weights on inflation, asset prices and output for the target rate, and $\widehat{m}_t$ defines the monetary shock in the economy. We assume that bank regulation changes are at best infrequent, so for short-run analysis we concentrate on the spacial case no changes occur, i.e.

$$\widehat{\upsilon}_{t+1} = 0. \tag{101}$$

The monetary policy shock follows an AR(1) process of the following form,

$$\widehat{m}_t = \rho_m \widehat{m}_{t-1} + \varepsilon_t^m, \tag{102}$$

where ε_t^m is a zero mean, uncorrelated, and normally-distributed innovation. The parameter $-1 < \rho_m < 1$ determines the persistence of the monetary shock.

We have followed very closely the derivation of the linearized equilibrium conditions in BGG (1999), and therefore our model shows obvious similarities. The main differences arise because we have introduced frictions on the secondary market for used capital that have the potential to alter the conditions under which borrowers and lenders operate in this economy, and because we have expanded the balance sheet only so slightly to give bank regulation a role on loan pricing decisions. Now, it is our turn to explore the implications that this type of model economy will have.

4 Simulation

As noted above we make the empirical claim that the model here provides sufficient flexibility to explain periods of monetary policy asymmetry. In particular, we believe that it provides the basis to explain why Taylor rules have to date been unable to explain the path of monetary policy in periods of crisis or, at times, recession. The notable unexplained pattern is the sharp decrease in interest rates in times of crisis. These decreases in rates are rejected by consensus Taylor rules. In Figure 3, below, we show the path of US monetary policy and the residuals from a consensus Taylor rule (see Rudebusch 2006 for additional discussion). What we can observe is that monetary poicy often deviates quite far from these rules; this suggests that the research community has not yet managed to capture the full mechanism of monetary policy - particularly in times of relative crisis.

So why does the central bank decrease rates in crisis times? Getting to this point conceptually is straightforward. Monetary policy works through the financial accelerator in the BGG (1999) framework, and this credit channel is further enhanced here with a view on the impact that banking regulation has on the balance sheet of the banks.

When the economy enters a ‘bad’ state, determined by large losses in the inneficient secondary market for used capital, banks can become undercapitalized vis-a-vis the regulatory requirements. This leads to an inability of banks to lend and thus translate into higher external funding premium for the entrepreneurs, to dissuade them from taking more loans than the banking system can actually channel for them. This complicates both the conduct of monetary policy, but also ultimately the decisions on banking regulation. One way to restore the economy to a ‘good’ state quickly with the help of monetary policy would be to lower interest rates.

In the next couple of section we show some of the properties of our model through a basic simulation of the model.

4.1 Parameters and Calibration

As our goal here is to comment on the role of monetary policy in the context of potential systemic stress on the banking relationship with entrepreneurs-borrowers due to systemic losses in the secondary market, we follow the literature in our calibration effort. Some baseline values are taken directly from the relevant literature and a few are calibrated to match existing data. We highlight a few of the key parameters and point readers to Tables 1-4 for full information on parameter values used in simulations.

We set quarterly capital depreciation to 0.025. Our Calvo-price stickiness parameter is set at 0.75, and the Frisch labor supply elasticity is set to 0.5. Each of these parametric choices follows BGG (1999) closely. We also set the discount rate to a quarterly 0.99, which is consistent with a 4% annual real rate of return, and the elasticity of substitution across varieties at 10 in order to match the 11% mark-ups estimated by Basu (1996). We set elasticity of intertemporal substitution is set at 0.5 (Lucas 1990).

We set our capital adjustment parameters such that we capture the intuition of the BGG (1999) framework in that an equivalent parameter in their model would be close to 1; implying relatively small adjustment costs. We parameterize our Taylor rule as follows to include interest rate inertia, ρ_i, of 0.9. The shocks themselves are zero mean, with uncorrelated innovations whose variance is fixed as specified in Table 1. As well, for our baseline parameterization, the weights on inflation and output in our Taylor rule are set at 1.5 and 0.5 respectively.

Perhaps most importantly are choices for the banking sector. We set the leverage maximum to 25, which implies a capital to asset ratio of 0.04. Basle I requirements stipulate that a Tier 1[32] capital to assets ratio of below 0.04 implies that the institution is ‘undercapitalized.’ New Basel II requirement allows a reduction of this capital by up to 15%, thus down to just over 0.03, over a long period of time. Being ‘well capitalized’ requires a Tier 1 capital to assets ratio of above 0.06. We use the 0.04 level for the industry as this reflects the point at which financial institutions can be considered in ‘distress’ from the point of view of the regulator, which in turn menas that monetary policy becomes almost completely ineffective.

Tables 2-4 show the actual and simulated moments for each of three shocks (productivity, monetary and aggregate risk) for a variety of data. In tables 2 and 3 for the productivity and monetary shocks, we also show the results with and without the presence of a systemic risk shock. We believe that we’ve done

[32] Tier 1 capital is defined as common equity, non-cumulative perpetual preferred stock and minority interests in equity accounts of minority shareholders.

a reasonably good job in matching these moments such we can interpret the simulations with a degree of confidence.

4.2 Methodology and Preliminary Results

Thus, we begin with a standard simulation exercise to answer question 1 by looking at two economies separately. One is the standard unconstrained banking system popularized by Bernanke, *et al.* (1999), modified in the ways discussed above and studied also by many others. The other is the constrained system. In the constrained economy, banks cannot expand lending as capital levels lie below the regulatory threshold. In each of the two economies, the central bank follows a Taylor rule.

We show this in two ways. The two panels of Figure 1 show the impulse response functions for our basic economy following a productivity and aggregate risk shock respectively. The first panel shows a set of fairly well known patterns following a productivity shock. Output falls initially. The monetary response is to lower interest rates relatively aggressively. Following this monetary reaction, output responds at the cost of an inflation increase vis-a-vis the post shock level. Our results differ from well known patterns in their persistence, which is relatively high.

The second panel shows the impact from a shock to aggregate risk. Recall that aggregate risk is our method of describing how the economy can enter into a crisis state. For our purposes, the relevant component of the impulse response function for the shock to aggregate risk is the large and persistent drop in interest rates.

We show in Figure 2 our simulated model aligned with simulation of the BGG model. We simulate BGG by shutting down the credit channel completely in our model. This is done by setting deposits equal to loans as done in BGG (1999). In each of the two panels of Figure 2, the BGG model is represented by the dotted blue line. The solid black line shows the paths of these variables for our economy. It is worthwhile to mention a couple of key differences. First, our model produces greater magnitudes as a result of a shock as well as longer persistence. This is consistent with the intuition that leverage has an additional role to play above and beyond the impact of spread differences. Second, we notice the difference in monetary responses in the interest rate impulse response functions. This supports the notion that the presence of a credit channel can contribute to the large pro-inflationary reaction seen in 2007-2008.

5 Discussion and Conclusions

Our paper has offered a model of the economy that generalizes the BGG (1999) to include a compact characterization of both the financial accelerator and the role of the financial sector in propagating monetary policy to the real economy. We've identified the output cost systemic risk as well as its role in determining the external finance premium. Equation (73) neatly summarizes this relationship and makes clear how the financial sector can amplify or dampen the monitoring cost effect discussed in BGG (1999). Such a characterization is important as it provides a parsimonious explanation that can be easily reconciled with existing research on the interaction between monetary policy and bank regulation. This results arises as part and parcel of a model designed to explain the transmission and amplification of monetary actions.

In particular, we believe that a model that includes this type of lending channel can go some length towards explaining some of the monetary policy asymmetries that Taylor rules have been unable to in the last few years. As well, we think that since our model is built around the existence of a regulatory capital constraint, it provides the basis for discussions of the implications for the joining of monetary policy and regulation. Indeed, the presence of differences in monetary policy discussed in this model implies a strong incentive for the joint monetary/regulatory authority to ensure that financial institutions remain above the capital constraint. In times of falling asset values, banks will approach or fall below capital requirements, rendering monetary policy ineffective at stimulating lending. At this point, the monetary/regulatory authority has an incentive to lower capital requirements in order to facilitate monetary intervention. If falling asset values were due to a realization of inaccurate risk measurements, reduced capital levels may simply encourage reckless lending.

With this framework in place, there are potentially more open questions that lie beyond the scope of this paper. For example, while the model appears to do a reasonably good job in describing the stylized patterns of the US monetary authority during the recent crisis, at least in the reduction of interest rates, it is potentially rejected by the European case. The European Central Bank held interest rates constant until late in 2008. Though there are many possible reasons for this, we speculate that this emerges from the differences in mandate. The Federal Reserve has responsibility both for monetary policy and bank regulation of some of the financial system. This produces well-known conflict between counter-cyclical monetary policy and pro-cyclical banking goals. It also produces an incentive to keep banks above regulatory thresholds through the use of monetary policy (see Cechetti and Li, 2008 and neutralization of the capital constraint). Why does this matter here? Two avenues are worth pursuing in future research. One, did the ECB keep rates constant as it saw no direct role within its mandate for financial sector debt deflation? Did the Fed use alternate methods of liquidity provision as a way to provide ad-hoc regulatory tolerance - effectively removing the concern that near-term liquidity problems would decrease asset values sufficiently to lead to a binding capital constraint. By doing so, it attempted to re-open the accelerator for monetary policy?

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Appendix

A The Log-Linearized Model

As a notational convention, all variables identified with lower-case letters and a caret on top represent a transformation of the corresponding variable in upper-case letters. They are variables in logs and expressed in deviations relative to their steady state values.

Aggregate Demand Equations.

$$
\begin{aligned}
&\widehat{y}_t \approx \gamma_c \widehat{c}_t + \gamma_{c^e} \widehat{c}^e_t + \gamma_x \widehat{x}_t + (1-\gamma_c-\gamma_{c^e}-\gamma_x)\left(\widehat{r}^e_t - \widehat{\pi}_t + \widehat{q}_{t-1} + \widehat{k}_t\right) + \\
&\quad + (1-\gamma_c-\gamma_{c^e}-\gamma_x)\Psi(s)\widehat{s}_t + \\
&\quad + (1-\gamma_c-\gamma_{c^e}-\gamma_x)\Upsilon(s)\left[\widehat{p}_{t-1} + \widehat{q}_{t-1} + \widehat{k}_t - \widehat{n}_t - \left(\tfrac{PQK}{N}-1\right)\left(\widehat{r}^e_t - \widehat{i}_t - \overline{\upsilon}\widehat{\upsilon}_t\right)\right], \\
&\widehat{c}_t \approx \mathbb{E}_t\left[\widehat{c}_{t+1}\right] - \sigma\left(\widehat{i}_{t+1} - \mathbb{E}_t\left[\widehat{\pi}_{t+1}\right]\right), \\
&\widehat{r}^e_t - \widehat{\pi}_t \approx \left(1 - \frac{(1-\delta)}{R^e}\right)\left((\widehat{p}^w_t - \widehat{p}_t) + \widehat{y}_t - \widehat{k}_t\right) + \frac{(1-\delta)}{R^e}\widehat{q}_t - \widehat{q}_{t-1}, \\
&\mathbb{E}_t\left[\widehat{r}^e_{t+1}\right] \approx \widehat{i}_{t+1} + \vartheta(s)\left(\widehat{p}_t + \widehat{q}_t + \widehat{k}_{t+1} - \widehat{n}_{t+1}\right) + \Theta(s)\mathbb{E}_t\left[\widehat{s}_{t+1}\right] + \overline{\upsilon}\widehat{\upsilon}_{t+1}, \\
&\widehat{p}_t + \widehat{q}_t + \widehat{k}_{t+1} - \widehat{n}_{t+1} \approx \\
&\quad \approx \Xi(s)\mathbb{E}_t\left[\widehat{r}^e_{t+1} - \widehat{\pi}_{t+1}\right] - \left(\tfrac{PQK}{N}-1\right)\mathbb{E}_t\left[\widehat{i}_{t+1} - \widehat{\pi}_{t+1}\right] - \Sigma(s)\mathbb{E}_t\left[\widehat{s}_{t+1}\right] - \left(\tfrac{PQK}{N}-1\right)\overline{\upsilon}\widehat{\upsilon}_{t+1}, \\
&\widehat{q}_t \approx \begin{cases} 0, \text{ if NAC}, \\ \chi\delta\left(\widehat{x}_t - \widehat{k}_t\right), \text{ if CAC}, \\ \kappa\left(\widehat{x}_t - \widehat{x}_{t-1}\right) - \kappa\beta\mathbb{E}_t\left[\widehat{x}_{t+1} - \widehat{x}_t\right], \text{ if IAC}. \end{cases}
\end{aligned}
$$

Aggregate Supply Equations.

$$
\begin{aligned}
&\widehat{y}_t \approx \widehat{a}_t + (1-\psi-\varrho)\widehat{k}_t + \psi\widehat{h}_t, \\
&\widehat{y}_t - \widehat{h}_t + (\widehat{p}^w_t - \widehat{p}_t) - \frac{1}{\sigma}\widehat{c}_t \approx \frac{1}{\varphi}\widehat{h}_t, \\
&\widehat{\pi}_t \approx \beta\mathbb{E}_t\left[\widehat{\pi}_{t+1}\right] + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(\widehat{p}^w_t - \widehat{p}_t\right),
\end{aligned}
$$

Evolution of the State Variables.

$$
\begin{aligned}
&\widehat{k}_{t+1} \approx (1-\delta)\widehat{k}_t + \delta\widehat{x}_t, \\
&\widehat{n}_{t+1} - \widehat{p}_t \approx \left(1 + \tfrac{PC^e}{N} - \tfrac{W^e}{N}\right)\widetilde{\Upsilon}(s)\left(\widehat{n}_t - \widehat{p}_{t-1}\right) + \tfrac{W^e}{N}\left((\widehat{p}^w_t - \widehat{p}_t) + \widehat{y}_t\right) + \\
&\quad + \left(1 + \tfrac{PC^e}{N} - \tfrac{W^e}{N}\right)\left(1 - \widetilde{\Upsilon}(s)\right)\left(\widehat{r}^e_t - \widehat{\pi}_t + \widehat{q}_{t-1} + \widehat{k}_t\right) - \tfrac{PC^e}{N}\widehat{c}^e_t - \left(1 + \tfrac{PC^e}{N} - \tfrac{W^e}{N}\right)\widetilde{\Psi}(s)\widehat{s}_t + \\
&\quad + 2\left(1 + \tfrac{PC^e}{N} - \tfrac{W^e}{N}\right)\widetilde{\Upsilon}(s)\left(\tfrac{PQK}{N}-1\right)\widehat{\pi}_t - \left(1 + \tfrac{PC^e}{N} - \tfrac{W^e}{N}\right)\widetilde{\Upsilon}(s)\left(\tfrac{PQK}{N}\left(\widehat{r}^e_t - \widehat{\pi}_t\right) - \left(1 - \tfrac{PQK}{N}\right)\left(\widehat{i}_t - \widehat{\pi}_t + \overline{\upsilon}\widehat{\upsilon}_t\right)\right),
\end{aligned}
$$

Monetary Policy Rule and Banking Regulation.

$$\begin{aligned}\widehat{i}_{t+1} &= \rho_i \widehat{i}_t + (1-\rho_i)\left[\psi_\pi \widehat{\pi}_t + \psi_q \widehat{q}_t + \psi_y \widehat{y}_t\right] + \widehat{m}_t, \\ \widehat{v}_{t+1} &= 0,\end{aligned}$$

Exogenous Shock Processes.

$$\begin{aligned}\widehat{a}_t &= \rho_a \widehat{a}_{t-1} + \varepsilon_t^a, \\ \widehat{m}_t &= \rho_m \widehat{m}_{t-1} + \varepsilon_t^m, \\ \widehat{s}_t &= \rho_s \widehat{s}_{t-1} + \varepsilon_t^s,\end{aligned}$$

Definitions.

$$\widehat{\pi}_t \equiv \widehat{p}_t - \widehat{p}_{t-1},$$

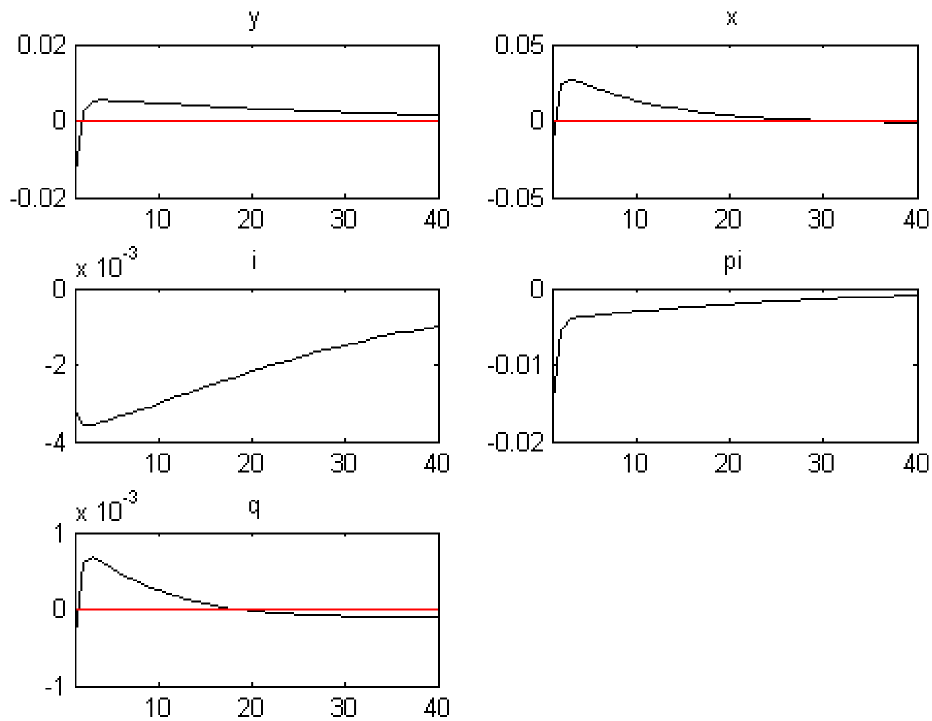
Composite Parameters.

$$
\begin{aligned}
\overline{\upsilon} &\equiv \frac{\left(\frac{1}{1-\iota^{h}}-\frac{1-\varpi\zeta}{1-\varpi}\right)\upsilon}{\upsilon\left(\frac{1}{1-\iota^{h}}\right)+(1-\upsilon)\left(\frac{1-\varpi\zeta}{1-\varpi}\right)}, \\
\vartheta(s) &\equiv \frac{1}{\frac{\Gamma_1(\overline{\omega},s)}{\lambda_1(\overline{\omega},s)}\frac{PQK}{N}\left(\frac{PQK}{N}\frac{R^e}{I^b}\right)+\left(\frac{PQK}{N}-1\right)}, \\
\Theta(s) &\equiv \frac{\frac{PQK}{N}\left(\frac{R^e}{I^b}\right)\left[\left(\frac{\Gamma_1(\overline{\omega},s)}{\lambda_1(\overline{\omega},s)}\right)\left(\frac{PQK}{N}\right)\Omega(s)-\left(\Gamma_2(\overline{\omega},s)-\mu G_2(\overline{\omega},s)\right)s\right]}{\frac{\Gamma_1(\overline{\omega},s)}{\lambda_1(\overline{\omega},s)}\frac{PQK}{N}\left(\frac{PQK}{N}\frac{R^e}{I^b}\right)+\left(\frac{PQK}{N}-1\right)}, \\
\Omega(s) &\equiv \frac{\lambda_2(\overline{\omega},s)-\frac{R^e}{I^b}\left(-J'(s)-\Gamma_2(\overline{\omega},s)+\lambda_2(\overline{\omega},s)\left(\Gamma(\overline{\omega},s)-\mu G(\overline{\omega},s)\right)+\lambda(\overline{\omega},s)\left(\Gamma_2(\overline{\omega},s)-\mu G_2(\overline{\omega},s)\right)\right)}{\lambda(\overline{\omega},s)}s, \\
\Psi(s) &\equiv \frac{\left(J'(s)+\mu G_2(\overline{\omega},s)\right)s}{\left(J(s)+\mu G(\overline{\omega},s)\right)}-\left(\frac{\mu G_1(\overline{\omega},s)}{J(s)+\mu G(\overline{\omega},s)}\right)\left(\frac{\left(\Gamma_2(\overline{\omega},s)-\mu G_2(\overline{\omega},s)\right)s}{\Gamma_1(\overline{\omega},s)-\mu G_1(\overline{\omega},s)}\right), \\
\Upsilon(s) &\equiv \left(\frac{\left(\frac{\mu G_1(\overline{\omega},s)\overline{\omega}}{J(s)+\mu G(\overline{\omega},s)}\right)}{\frac{PQK}{N}\left(\frac{R^e}{I^b}\right)\left(\Gamma_1(\overline{\omega},s)-\mu G_1(\overline{\omega},s)\right)\overline{\omega}}\right), \\
\Xi(s) &\equiv \left[\frac{\frac{PQK}{N}\left(\frac{R^e}{I^b}\right)\left(\Gamma_1(\overline{\omega},s)-\mu G_1(\overline{\omega},s)\right)\overline{\omega}+\left(\frac{PQK}{N}-1\right)\left(\frac{\Gamma_1(\overline{\omega},s)\overline{\omega}}{1-J(s)-\Gamma(\overline{\omega},s)}\right)}{\left(\frac{\Gamma_1(\overline{\omega},s)\overline{\omega}}{1-J(s)-\Gamma(\overline{\omega},s)}\right)}\right], \\
\Sigma(s) &\equiv \frac{PQK}{N}\left(\frac{R^e}{I^b}\right)\left(\Gamma_1(\overline{\omega},s)-\mu G_1(\overline{\omega},s)\right)\left[\left(\frac{\left(J'(s)+\Gamma_2(\overline{\omega},s)\right)s}{\Gamma_1(\overline{\omega},s)}\right)-\left(\frac{\Gamma_2(\overline{\omega},s)-\mu G_2(\overline{\omega},s)}{\Gamma_1(\overline{\omega},s)-\mu G_1(\overline{\omega},s)}s\right)\right], \\
\widetilde{\Psi}(s) &\equiv \left(\frac{\left(J'(s)+\Gamma_2(\overline{\omega},s)\right)s}{1-J(s)-\Gamma(\overline{\omega},s)}\right)-\left(\frac{\Gamma_1(\overline{\omega},s)}{1-J(s)-\Gamma(\overline{\omega},s)}\right)\left(\frac{\left(\Gamma_2(\overline{\omega},s)-\mu G_2(\overline{\omega},s)\right)s}{\Gamma_1(\overline{\omega},s)-\mu G_1(\overline{\omega},s)}\right), \\
\widetilde{\Upsilon}(s) &\equiv \frac{\left(\frac{\Gamma_1(\overline{\omega},s)\overline{\omega}}{1-J(s)-\Gamma(\overline{\omega},s)}\right)}{\frac{PQK}{N}\left(\frac{R^e}{I^b}\right)\left(\Gamma_1(\overline{\omega},s)-\mu G_1(\overline{\omega},s)\right)\overline{\omega}}, \\
\gamma_c &\equiv \frac{C}{Y}, \\
\gamma_{c^e} &\equiv \frac{C^e}{Y}, \\
\gamma_x &\equiv \frac{X}{Y}.
\end{aligned}
$$

Tables and Figures

Figure 1: Simulation of Model Economy

Panel 1: Impact of Productivity Shock in this model



Panel 2: Impact of Aggregate Risk Shock in this model

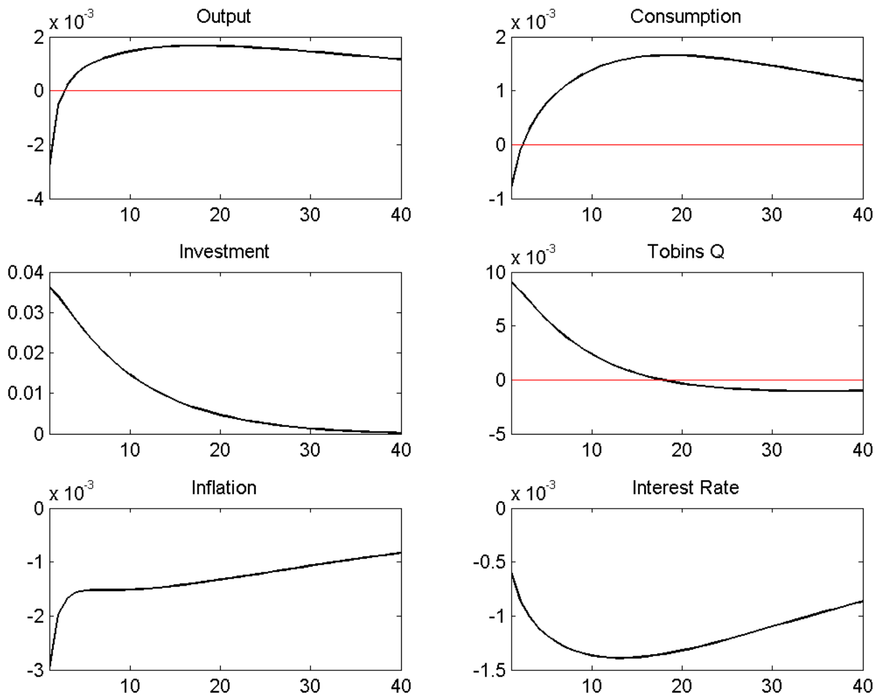
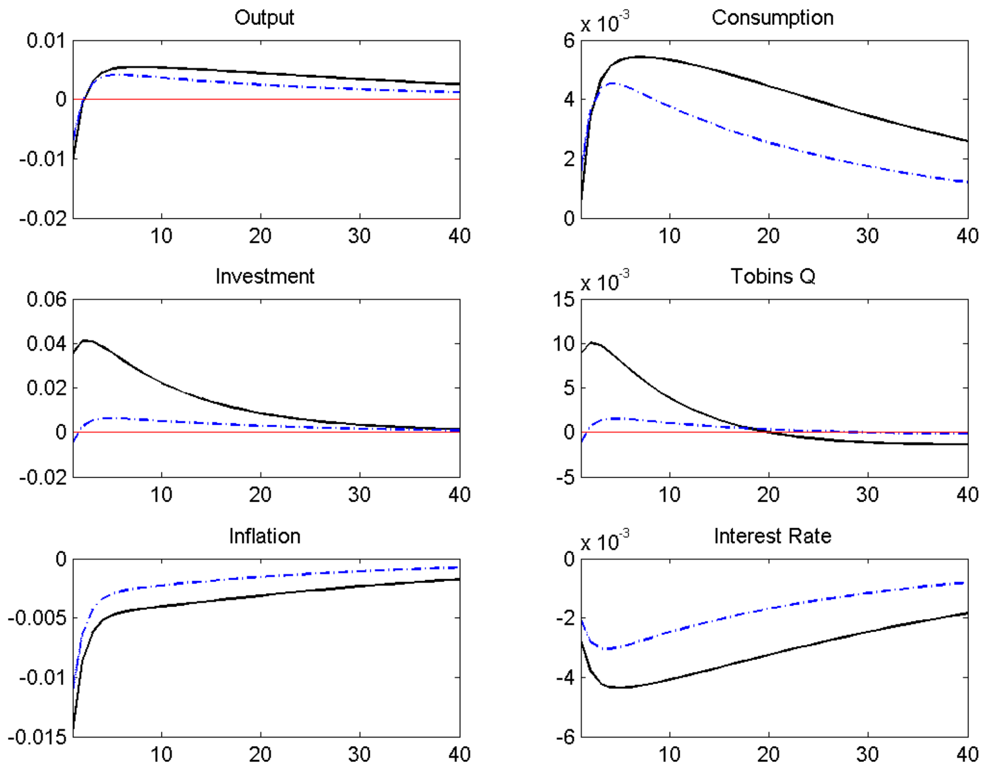


Figure 2: Comparisons to BGG(1999)

Panel 1: Impact of Productivity Shock in BGG (1999) and this model



Panel 2:Impact of Monetary Shock in BGG (1999) and this model

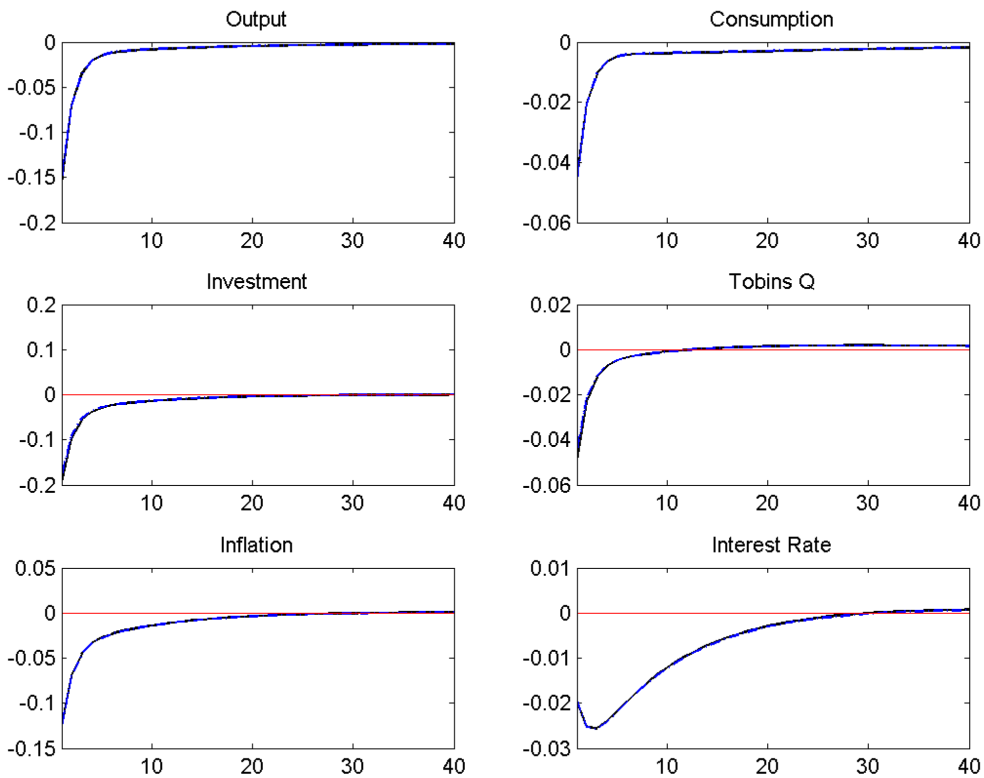
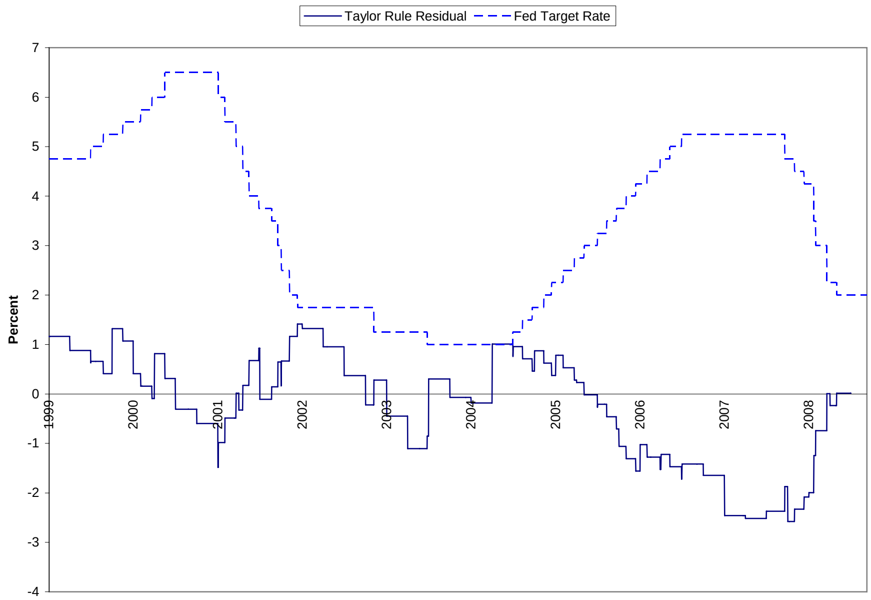


Figure 3: Monetary Policy and Taylor Rules

US Monetary Policy target rates and Taylor Rule residuals



see Rudebusch (2006) for full discussion. These residuals calcuated based on the rule:

$$i_t = 2.04 + 1.39\pi_t + .92y_t.$$

Table 1: Benchmark Calibration

Structural Parameters:			
Discount Factor for Households	$0 < \beta < 1$	0.99	BGG (1999)
Elasticity of Intertemporal Substitution	$\sigma > 0$ $(\sigma \neq 1)$	0.5	Lucas (1990)
Frisch Elasticity of Labor Supply	$\varphi > 0$	0.5	$\sigma = \varphi$
Elasticity of Substitution across Varieties	$\theta > 1$	10	Basu (1996)
Calvo Price Stickiness Parameter	$0 < \alpha < 1$	0.75	BGG (1999)
Depreciation Rate	$0 < \delta < 1$	0.025	BGG (1999)
Capital Adjustment / Investment Adjustment	$\chi > 0$ / $\kappa > 0$	$\chi = 10$ $\kappa = 10$	BGG (1999)
Unskilled Labor Share	$0 < \psi < 1$	0.64	BGG (1999)
Managerial Labor Share	$0 < \varrho < 1$	0.01	BGG (1999)
Relative Impatience of Entrepreneurs	$0 < \eta < 1$	0.8	-
Monitoring Costs	$0 < \mu < 1$	0.12	BGG (1999)
Level of Expected Systemic Losses	$\lambda > 0$	2	-
Slope of Expected Systemic Losses	$\gamma > 0$	5	-
Depth of Expected Systemic Losses	$\xi > 1$	3	-
Monetary Policy and Regulatory Parameters:			
Interest Rate Inertia	$0 \leq \rho_i \leq 1$	0.9	BGG (1999)
Weight on Inflation Target	$\psi_\pi \geq 1$	1.5	-
Weight on Asset Value (Tobin's q) Target	$-\infty < \psi_q < +\infty$	0	-
Weight on Output Target	$\psi_y \geq 0$	0.5	-
Reserve Requirement on Deposits	$0 \leq \varpi < 1$	0.003	FDIC[33]
Capital Adequacy Requirement	$0 \leq \upsilon < 1$	0.04	Bassle II
Reserve Payment Haircut	$0 < \zeta < 1$	0.99	-
Tax Rate on Bank Equity Holdings	$\frac{\varpi - \zeta\varpi}{1-\zeta\varpi} < \iota^h < 1$	0.05	-
Exogenous Shock Parameters:			
Persistence of Productivity Shock	$-1 < \rho_a < 1$	0.95	Cooley & Prescott (1995)
Volatility of Productivity Shock	$\sigma_a^2 > 0$	$(0.007)^2$	Cooley & Prescott (1995)
Persistence of Monetary Shock	$-1 < \rho_m < 1$	0.9	-
Volatility of Monetary Shock	$\sigma_m^2 > 0$	$(0.007)^2$	-
Persistence of Systemic Risk Shock	$-1 < \rho_s < 1$	0.9	-
Volatility of Systemic Risk Shock	$\sigma_s^2 > 0$	$(0.01)^2$	-
Covariance Productivity, Systemic Shocks	$-\infty < \sigma_{a,s} < +\infty$	$\frac{\sigma_{a,s}}{\sigma_s \sigma_a} = 0.75$	-

This table defines the benchmark parameterization of the structural parameters. The results of the sensitivity analysis for a given parameter are discussed in the paper, but not always reported. They can be obtained directly from the authors upon request.

[33] http://www.fdic.gov/deposit/insurance/assessments/risk.html

Table 2: Simulation Results (Monetary Shocks).

		Benchmark	
		- Systemic Risk	+ Systemic Risk
Variable	U.S. Data	Monetary Shocks	
Std. dev. to GDP			
Investment	3.38	1.233	1.313
Consumption	0.81	0.316	0.310
Tobin's q	–	0.299	0.318
Inflation	–	0.934	0.918
Short-Term Rate	–	0.383	0.380
Autocorrelation			
GDP	0.87	0.522	0.487
Investment	0.91	0.575	0.552
Consumption	0.87	0.555	0.554
Tobin's q	–	0.535	0.511
Inflation	–	0.652	0.638
Short-Term Rate	–	0.953	0.953
Correlations			
Cons., GDP	–	0.983	0.980
Cons., Invest.	–	0.966	0.963
Cons., Tobin's q	–	0.870	0.858
GDP, Tobin's q	–	0.945	0.943

Table 3: Simulation Results (Productivity Shocks).

		Benchmark	
		- Systemic Risk	+ Systemic Risk
Variable	U.S. Data	Productivity Shocks	
Std. dev. to GDP			
Investment	3.38	1.219	1.392
Consumption	0.81	0.978	1.006
Tobin's q	–	0.263	0.301
Inflation	–	0.961	0.943
Short-Term Rate	–	0.664	0.697
Autocorrelation			
GDP	0.87	0.811	0.850
Investment	0.91	0.900	0.919
Consumption	0.87	0.987	0.989
Tobin's q	–	0.863	0.890
Inflation	–	0.716	0.775
Short-Term Rate	–	0.981	0.986
Correlations			
Cons., GDP	–	0.836	0.877
Cons., Invest.	–	0.924	0.940
Cons., Tobin's q	–	0.666	0.693
GDP, Tobin's q	–	0.711	0.704

Table 4: Simulation Results (Systemic Risk Shocks).

		Benchmark	
		- Systemic Risk	+ Systemic Risk
Variable	U.S. Data	Systemic Risk Shocks	
Std. dev. to GDP			
Investment	3.38	–	9.440
Consumption	0.81	–	0.931
Tobin's q	–	–	2.246
Inflation	–	–	0.884
Short-Term Rate	–	–	0.767
Autocorrelation			
GDP	0.87	–	0.901
Investment	0.91	–	0.901
Consumption	0.87	–	0.982
Tobin's q	–	–	0.886
Inflation	–	–	0.915
Short-Term Rate	–	–	0.991
Correlations			
Cons., GDP	–	–	0.972
Cons., Invest.	–	–	0.225
Cons., Tobin's q	–	–	−0.134
GDP, Tobin's q	–	–	−0.233