SOURCES OF UNCERTAINTY IN CHILEAN MONETARY POLICY*

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Abstract

This paper analyzes the quantitative relevance of additive, multiplicative and data uncertainty in the implementation of Chile's monetary policy. For the analysis of data uncertainty we focus on the uncertainty associated with the estimation of the output gap using real-time data and various well-known methods to estimate the output trend. We found that the revisions of the output gap are important and persistent and that the unobserved components method shows a better performance with real-time data than other more usual ones, like the HP filter. In the case of additive and multiplicative uncertainties we estimate the equations that govern the behavior of the economy with time-varying parameters and with state-dependent variances in the shocks of the model. This allows us to analyze the contribution of these two types of uncertainties on the total uncertainty. We found that additive uncertainty is the most relevant to explain total uncertainty and that shocks to the model are state-dependent.

JEL Classification: E32, E58, E59, C32.

Key words: real-time data, output gap, time-varying parameter models, models with regime changes, forward-looking models.

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1. Introduction

It is widely accepted that monetary policy is inevitably made in an environment of substantial uncertainty. This has led to a considerable increase in the interest of academic researchers to demonstrate formally the implications of uncertainty and the ways in which central banks can deal with uncertainty. The theoretical literature on uncertainty distinguishes between three types of uncertainty: (1) *Additive uncertainty*, which refers to the lack of knowledge of the central banks regarding the future shocks faced by the economy; (2) *Multiplicative uncertainty*, which represents the lack of knowledge, or the erroneous knowledge, of one or more parameters of the model that explains the behavior of the economy; and (3) *Data uncertainty*, which is associated to the fact that the information used by the central bank at the time policy decisions are made could either be incorrect or could show in an incomplete manner the actual state of the economy. The objective of this paper is to check the quantitative relevance of these three types of uncertainty in the case of the monetary policy of Chile's Central Bank. To this end, the paper is divided into two parts: the first one covers the problem of data uncertainty and focuses on the output gap estimates for the full-fledged inflation targeting period, from 1999 onward; and the second focuses on additive and multiplicative uncertainty for the period of 1990 - 2006 but places a special emphasis on the period subsequent to 1999.

In the analysis of data uncertainty we focus on the output gap because it is an important variable in projecting inflation and, simultaneously, what is available at the time monetary decisions are made are preliminary figures for real output (real-time data) which are revised several times afterwards. Also, the estimation of the output trend (part of the output gap) depends on statistical filters applied to a series of output which contains these preliminary figures. In our exercise, we use various well-known filters, among them the Hodrick-Prescott (HP) filter, the Baxter-King (BK) filter, the Christiano-Fitzgerald (CF) filter, the quadratic trend and the Clark method based on the unobserved components model. To analyze the reliability and the statistical accuracy of these methods with real-time data we follow closely the methodology proposed by Orphanides and van Norden (1999). We found that the revisions of the output gap in the case of Chile seem to be important and persistent, and that the correlations between the final data output gap and the real-time data output gap are relatively low. Nonetheless, of the five filters utilized, the Clark method produces the best results. These results imply that caution and judgment should be taken when evaluating the business cycle with real-time data, but also suggest that using popular filters, like HP, could be misleading.

On the other hand, to evaluate the empirical importance of additive and multiplicative uncertainty we use the methodology proposed by Zhang and Semmler (2005). In particular, we estimate the behavioral equations for the Chilean economy with time-varying parameters and shocks with state-dependent variance (two states), which follow a first order Markov process. To estimate the behavioral equations we adopt the *forward-looking* specification of Svensson (2000) and Al-Eyd and Karasulu (2006) for the equations that govern the behavior of a small open economy – an aggregate demand, a Phillips curve, and the real uncovered interest parity condition. Additionally, we use a technique in Kim (1993) to decompose total uncertainty, measured using the conditional variance of the forecast error, into two components: that associated to multiplicative uncertainty and that associated to additive uncertainty. The results of the analysis suggest that for all the behavioral equations of the economy, the uncertainty of shocks, or additive uncertainty, has been the most important in explaining total uncertainty. Moreover, the estimations support the hypotheses of state-dependent variances and that these states could be considered as periods of high and low volatility in the shocks. Another interesting finding is that total uncertainty of both the output gap and the inflation rate have declined over time and the period of greater stability coincides with the establishment of the *full-fledge inflation targeting* framework for the conduct of monetary policy.[1]

Finally, given that the estimated models in the analysis of the additive and multiplicative uncertainty require an output gap estimate, data uncertainty, at least as it refers to the estimation methods of the output gap, will be part of the additive and multiplicative uncertainty without any possibility of discrimination. It is for this reason that a robustness analysis is done through the estimation of the behavioral equations of the economy utilizing the output gap calculated with each of the five methods proposed. The finding is that the previous results concerning the contributions of additive and multiplicative uncertainty are robust and do not change.

The paper is organized as follows. In section 2 we present a literature review on the types of uncertainty faced by central banks, its implications for the conduct of monetary policy and the way in which they have been typically modeled empirically. In section 3 we analyze the

[1] It is important to mention that this period also coincides with the establishment of the structural surplus rule for the conduct of fiscal policy and with a generally highly stable international context.

quantitative relevance of data uncertainty, particularly focusing on the output gap estimates. In section 4 we present the analysis of the importance of additive and multiplicative uncertainty on the models typically used to analyze the effect of monetary policy. Finally, in section 5 we present the conclusions.

2. Monetary policy and uncertainty

In the last few years, there has been a considerable increase in the interest of academic researchers to demonstrate formally the ways in which central banks can deal with uncertainty (Schellekens, 2002, Feldstein, 2003). In particular, some papers have studied the distinct types of uncertainty faced by central banks, which have introduced important challenges in the modeling of monetary policy, and its implications on the behavior of the monetary authority. Some of these papers include Isard et.al. (1999), Martin and Salmon (1999), Svensson (1999), Wieland (2000), Meyer et.al. (2001), Tetlow and von zur Muehlen (2001a), Gianoni (2002), Orphanides and Williams (2002) and Soderstrom (2002). Other papers have proposed different strategies that can be used to deal with uncertainty, namely robust monetary policy rules and learning mechanisms, among others; see, for example, Craine (1979), Sargent (1998) and Onatski and Stock (2000) and Wieland (2002).

Feldstein (2003) argues that central banks typically face four types of uncertainty: uncertainty about the current and future state of the economy, uncertainty about how the economy operates, uncertainty of individuals about their personal futures, and uncertainty about the impact of potential future monetary policies. However, the most common classification speaks of three types of uncertainty: additive uncertainty, multiplicative uncertainty and data uncertainty.[2] *Additive uncertainty* represents the component of a forecast error associated to the outcome of an exogenous variable in the system (the error in a regression model). This type of uncertainty captures the lack of knowledge of central banks regarding the future shocks faced by the economy (Zhang and Semmler, 2005; Grauwe, 2006). *Multiplicative (or parameter) uncertainty*, on the other hand, represents the lack of knowledge, or the erroneous knowledge of

[2] Another type of uncertainty also considered in the literature, but not analyzed in this paper, is uncertainty about the probability distributions over possible events known as Knightian uncertainty.

one or more parameters of the model that explains the behavior of the economy (and its agents).[3] Hall et.al. (1999) claims that this type of uncertainty can occur for several reasons such as the stochastic nature of the parameters and the measurement errors in the data utilized to estimate the model. Finally, *data uncertainty* is associated to the fact that the information used by the central bank at the time policy decisions are made could either be incorrect or could show in an incomplete manner the actual state of the economy (Orphanides and van Norden, 1999). According to Rudebush (2001), when these different types of uncertainty are combined, they weigh heavily on the policy decision-makers.

Since the seminal papers of Phillips (1954) and Theil (1964),[4] additive uncertainty has been widely studied in the literature. In particular, Theil (1964) derived the famous *certainty-equivalence* result, which establishes that in the presence of additive uncertainty the central bank could act as if it were certain about the possible outcomes of the economy.[5] The actions of the monetary authority depend only on its expectations about the future of the variables and not of the uncertainty associated with those expectations (Walsh, 2003). Phillips (1954) initiated this idea by suggesting that the monetary policy, based on the principles of automatic regulating systems, would be adequate to deal with all but the most severe disturbances that could affect the economic system. In this context, there was a certain degree of confidence in econometric modeling, such that in the estimation of the structural models any error could be eliminated, except the error associated with additive uncertainty. It is important to mention, however, that the principle of certainty-equivalence is valid only under certain conditions, particularly in a linear quadratic world. This could be too restrictive in practice. More generally, there are several models that according to their construction can imply either small or large variations in the monetary policy instrument when the central bank is faced with additive uncertainty; see Walsh (2003b). For example, Sack (2000) shows that under this type of uncertainty, the optimal policy rule implies a more aggressive reaction by the central bank.[6]

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[3] Note that the model parameters representing the behavior of the economy (reduced-form model) depend on the parameters that control the behavior of the individuals.

[4] Cited in Hall et.al. (1999).

[5] This implies that for monetary policy what is important is the average of the variables or the parameters; hence the uncertainty should be ignored.

[6] This could occur when the economy is far from full employment, when inflation is low and the expectations are not aligned with the target of the central bank (Feldstein, 2003).

Multiplicative or parameter uncertainty was first analyzed by Holt (1962),[7] who demonstrated that policy performance could be seriously affected if the model parameters used for the evaluation of such policy are uncertain. In this context, the certainty-equivalence principle is not valid and, hence, the central bank should consider this type of uncertainty when making policy decisions. Brainard (1967), in his classic analysis regarding multiplicative uncertainty, showed that it would be optimal for the central bank to respond more cautiously when the impact of its policy on the economy is unknown (that is, the model feedback parameters).[8] This result has important practical implications in the conduct of monetary policy, since it indicates that it could be optimal for policy makers not to expect to completely eliminate the gap between the observed objective variable and its target value, in a particular period. This could be interpreted as a justification for a gradual monetary policy. Although Brainard's (1967) result has been widely discussed in the literature (see Blinder, 1998) and is quite intuitive, it cannot be generalized. For example, Soderstrom (2002) shows that in situations where the coefficients of the lagged variables in the model are subject to uncertainty, the optimal policy for the central bank is to react more aggressively. Other examples in support of the argument that the multiplicative uncertainty does not necessarily lead the central bank to behave more cautiously can be found in Gianoni (2002) and González and Rodríguez (2003).

The study of uncertainty associated with the data is relatively new in the literature on monetary policy and uncertainty. As a matter of fact, only recently, academics and policy makers have cautiously invested resources in this area and, as a result, there has been significant growth in the literature that study the properties of real-time data and its implications on policy decisions (Bernhardsen et.al., 2005). The pioneering work of Croushore and Stark (1999, 2001) set the framework in this area and led to various applications, which have focused primarily on developed countries. Examples of such applications can be found in Orphanides (2001), Croushore and Stark (2002) and Orphanides and van Norden (2002).[9] This literature highlights that the moment at which the data are obtained, their availability and reliability for empirical evaluation of policy rules, is crucial for monetary policy performance since they condition the decisions of the policy makers (Ghlysels, 2002). In this regard, Rudebush (2001) and

[7] Cited in Hall et.al. (1999)

[8] This result was derived from a linear quadratic model with a known probability distribution for the uncertainty model parameters.

[9] For an excellent literature review on the issue for the case of the United States see Kozicki (2004).

Bernhardsen et.al. (2005), argue that the new information that the central banks obtains from one policy meeting to the next does not justify drastic changes in its instrument, which can lead to very slow responses to particular economic events.

One of the variables that summarize the actual state of the economy and is, therefore, fundamental for the monetary policy decisions is the output gap. Naturally, if potential output measures are not reliable, policy decisions may fail to react to the true economic conditions and may instead reflect measurement error. Along these lines, Orphanides and van Norden (2002) argue that the output gap is associated with important components of uncertainty since there are at least three types of problems typically faced by the central banks when evaluating the business cycle with real-time data. First, output data are revised continually. Second, the methods to estimate potential or trend output, in general, provide different results, and this problem is particularly critical with the end of sample estimates that are, precisely, those relevant for policy decisions.[10] Third, a future evaluation of output data can indicate that the economy has experienced a structural change and such a change could have been revealed by something else other than real-time data.

Following Zhang and Semmler (2005) and to provide an example of the concepts previously mentioned, we consider the following economic model that is standard in the literature of optimal rules of monetary policy:

$$\min_{\{u_t\}_0^\infty} E_0 \sum_{t=0}^{\infty} \rho^t L(x_t, u_t) \tag{1}$$

subject to:

$$x_{t+1} = f(x_t, u_t, \varepsilon_t) \tag{2}$$

where ρ is the discount factor bounded between 0 and 1, $L(x_t, u_t)$ is a loss function of an economic agent, in this case the central bank, x_t is the vector of state variables, u_t is the vector of control variables (the policy instrument), ε_t is the vector of shocks and E_0 is the mathematical expectation operator based on the initial values of the state variables. As mentioned before, this kind of model represents the basic framework of monetary policy analysis

[10] Kuttner (1994) and St-Amant and van Norden (1998), using final data of the output and using different methods to estimate its trend, found that there were substantial differences in the estimations of these trends using final data.

and control used by Clarida et.al. (1999), Svensson (1997, 1999) and Beck and Wieland (2002), where the constraints in equation (2) are the Phillips curve and the IS curve plus the interest rate parity condition. (Svensson, 2000).

Given the state equations in (2), the central bank's problem consists in deriving a path for its instrument (the control variable u_t) that satisfies (1). The question that arises, however, is whether the state equations can be correctly specified with time series estimates. Given the previous discussion it is possible to affirm that the response to this question is negative, since these equations can be subject to a high degree of uncertainty caused by shocks ε_t, by parameter uncertainty and by data uncertainty used in the estimations. This is particularly important since the optimal monetary policy rules[11] are derived from the solution of the previous problem and, hence, these rules depend on the parameters of the state equations. Thus, if the parameters in the model are uncertain, the estimated "optimal" monetary policy rule could be unreliable.

However, no matter the effects of uncertainty central banks should not be paralyzed. Following Feldstein (2003), "*..although (this) uncertainty affects the ability of a central bank to limit the variability of inflation and output, it does not limit the ability of central banks to avoid a high and rising level of inflation. There is no uncertainty about the key facts that can guide central banks to low log-term inflation. So, uncertainty is not an excuse for allowing inflation to go back to the bad old days.*"

3. Data uncertainty: the output gap

In this section, we analyze the quantitative relevance of data uncertainty in the case of Chile, focusing as said before on the output gap, and for the sample period 2000-2006. This period was chosen for two particular reasons: (1) the availability of historical information of the publications of the output series at each moment in time; and (2) it is the period in which the central bank adopted the *full-fledged inflation targeting* scheme to conduct its monetary policy. To fulfill this objective, we use real-time data (i.e., data used by the central bank when making policy decisions) and various well-known methods for the estimation of the output trends. For each method we analyze both the behavior of the end-of-sample output gap estimates, which are relevant for policy decisions, and the revisions of these estimates across time. In particular, we

[11] See for example Svensson (1999) and Semmler et.al. (2004).

present the statistical properties of the revisions and verify the reliability of the estimates for each possible method. We divide this section into two subsections: in the first one we describe the methodological issues related to the construction of the output gap with real time-data and the detrending methods; and in the second part we present the results of the estimates and their implications.

3.1. Methodological issues

As mentioned by Bernhardsen et.al. (2005), monetary policy decisions are typically based on real-time data, classified as preliminary data. This also holds, to a lesser degree, for long-past historical data. The preliminary nature of the data calls for it to be in constant revision and the reasons for these revisions can be, among others, of an informative nature and of a methodological nature. In effect, and as suggested by the Central Bank of Chile in its Monetary Policy Report (IPoM) of September 2004, the revision of data is motivated by: (1) the inclusion of new basic information (new sources of information or the improvement of these sources); (2) the recalculation of the estimates (revisions attributed to new estimates);[12] (3) methodological improvements, due to changes in statistical methods, concepts, definitions or classification; and finally, (4) error correction, either in the basic sources or in the calculations. As mentioned in the previous section, one of the variables that encompasses the actual state of the economy and is, therefore, fundamental for the monetary policy decisions is the output gap. Given that at the moment when policy decisions are made this variable is estimated using preliminary output data, according to the previous discussion it is necessary to make an assessment of the degree of reliability of these estimates.[13] For this assessment, we use real-time data to replicate the available information for the policy makers at every point in time. Thus, we simulate the real-time environment of the monetary policy setting process (Ghlysels, 2002).

To analyze the reliability and the statistical accuracy of the output gap estimates commonly used in the literature we follow the methodology proposed by Orphanides and van Norden (1999). This consists of measuring, at any point in time, the degree at which the output gap

[12] This refers to the updating of seasonal factors or of the base period used in the constant price estimates. In Chile, the last change of base year used for the national accounts updated the estimates to 2003 prices.
[13] As a matter of fact, if the output gap measures are not reliable it could be advantageous for the central bank, in some situations, to base their monetary policy decisions on information regarding output growth (Orphanides et.al., 2000; Bernhardsen et.al., 2005).

estimates vary when the data are revised and with the different output gap estimation methodologies. This allows us to capture the effects caused by data revisions and the misspecification of statistical models used to estimate the output trend. The advantage of this methodological approach is that it does not require a priori assumptions on the true structure of the economy or on the process that generated the observed output time series. However, and as stated by Orphanides and van Norden (1999), this approach also has certain limitations, such that data revisions are being analyzed comparing each level of output observed at the end of the sample with the "final output", which could still have measurement errors.

In Orphanides and van Norden (1999) approach there are two key definitions: the "final" and the "real-time" estimates of the output gap. The final estimate of the output gap is simply the difference between the last available vintage of output data and its trend (obtained via a detrending method). The real-time estimate of the output gap, on the other hand, is a time series consisting of the last observed estimate of the output gap constructed as the difference between the observed output for each point in time (each vintage) and its trend. This latter estimate represents the estimate that the central bank may have calculated at the time when policy decisions were made. Formally, if we assume that we have access to the observed output series published at each point in time during N periods we would have a matrix of the form $\left(y^1, y^2,..., y^N\right)$, where each y^i (with $i = 1,..., N$) is a column vector that contains the time series of the output and each column is an observation (row) shorter than the one that follows it.[14] If $f^{dt}(\cdot)$ is a function that detrends the time series y, the final estimate of the output gap is given by:

$$gap^{final} = \ln(y^N) - \ln(f^{dt}(y^N)) \tag{3}$$

On the other hand, if we define the function $\ell(\cdot)$ as a function that extracts the last real observation of the column vector y^i we have that the real-time estimate of the output gap is given by:

[14] In the matrix $\left(y^1, y^2,..., y^N\right)$ we consider the missing observations as non-real numbers.

$$gap^{real-time} = \ln(\ell(y^1), \ell(y^2), ..., \ell(y^N)) - \ln(\ell(f^{dt}(y^1)), \ell(f^{dt}(y^2)), ..., \ell(f^{dt}(y^N)))' \qquad (4)$$

The difference between the final output gaps and the real-time output gaps represents the *total revision* of the estimates at each point in time. The statistical properties of these series of revisions will be a guide to evaluate the reliability and accuracy of the output gap estimates. For the estimates defined in equations (3) and (4) it is necessary to define the function $f^{dt}(\cdot)$, that is the detrending method, given that in practice neither the true potential output of the economy nor its data generating process are known. This selection becomes important, as mentioned previously, since these methods in general provide quite different results. In the case of Chile, Gallego and Johnson (2001) find that the set of methods used to estimate the trend component of the output provide a wide range of estimates. Therefore, besides the revisions in the data, the method chosen also constitutes a source of uncertainty.

A detrending method decomposes the real output (measured in logarithms) y_t into two components: the trend y_t^T and the cycle y_t^C such that $y_t = y_t^T + y_t^C$. In this paper we consider five alternative univariate methods that have been widely used in the literature:[15] (1) the Hodrick-Prescott filter, (2) the Baxter-King filter, (3) the Christiano-Fitzgerald filter, (4) the quadratic trend and (5) Clark's method based on the unobservable components model.[16] It is important to mention that as in the case of Orphanides and van Norden (1999), we focus only on univariate techniques of detrending, since the use of multivariate techniques requires the compilation of information on the data that is not revised (in real time) for each possible regressor in the model. Table 1 summarizes these methods and the models they are based on.

[15] See Orphanides and van Norden (1999) for an extensive revision of the detrending methods and its principal advantages and disadvantages.
[16] See Gallego and Johnson (2001) for an interesting compilation of the use of these methods in different central banks of the world.

Table 1: Alternative Methods to Calculate the Output Trend

HP	Hodrick-Prescott ($\lambda = 1600$)	$y_t^T = \arg\min \sum_{t=1}^{T} \left\{ \left(y_t - y_t^T \right)^2 + \lambda \left(\Delta^2 y_{t+1}^T \right) \right\}$
BK	Baxter-King (6,32)[17]	$y_t^T = \sum_{c=1}^{q+1} \omega^{BK}(1,c) y_{t+1-c} + \sum_{c=2}^{q+1} \omega^{BK}(1,c) y_{t+c-1}$ $t = q+1,...,n-q$
CF	Christiano-Fitzgerald (6,32,1,0,0)[18]	$y_t^T = \sum_{c=1}^{q+1} \omega^{CF}(1,c) y_{t+1-c} + \sum_{c=2}^{q+1} \omega^{CF}(1,c) y_{t+c-1}$ $t = q+1,...,n-q$
QT	Quadratic Trend	$y_t = \alpha + \beta t + \gamma t^2 + y_t^C$
Clark	Unobserved Components	$y_t = y_t^T + y_t^C$ $y_t^T = g_{t-1} + y_{t-1}^T + v_t$ $g_t = g_{t-1} + \omega_t$ $y_t^C = \delta_1 y_{t-1}^C + \delta_2 y_{t-2}^C + e_t$

The Hodrick-Prescott filter is perhaps one of the most popular methods used for detrending and it is based on the choice of the trend that minimizes the variance of the cyclical component of the series, subject to penalization for variations in the second difference of the cyclical growth component (Hodrick and Prescott, 1997). On the other hand, both the Baxter-King filter and the Christiano-Fitzgerald filter are based on the smoothing of the series using weighted moving average. The fundamental difference between both, for the case of symmetric filters as considered in this paper, lies in the choice of the objective function that defines the weights (Baxter and King, 1999; Christiano and Fitzgerald, 2003). Moreover, the Christiano-Fitzgerald filter imposes the restriction that the filter weights add up to zero, when unit roots are considered. On the other hand, the quadratic trend is a method of deterministic components that assumes that the trend series show a behavior triggered by a second order polynomial. Hence, this method is flexible at the moment of detecting slow changes in the trend. It is important to mention that its simplicity has made it quite valuable for empirical applications related to monetary policy (for

[17] The series of numbers 6 and 23 represent the minimum and maximum of the desired oscillation period, respectively, for quarterly data.
[18] The series of numbers 6 and 32 have the same interpretation as in the Baxter-King filter. On the other hand, the series of numbers 1,0,0 represent the existence of unit roots, without drift and symmetric filter, respectively.

example, Clarida et.al, 1998). However, its use has generated much controversy due to the argument that better modeling of the output requires statistical components in the model. Finally, the unobserved components model allows us to specify the data generating processes for the output time series and use these to identify the trend and cyclical components. In the particular case of the model proposed by Clark (1987), it is assumed that the trend component follows a random walk process with drift and the cyclical component follows an AR(2) process. The main advantage of this type of model is that it allows a richer short-term dynamic specification for the model.

3.2. Results

The output data observed at each point in time were constructed using data compiled from the monthly publications (bulletins) of the Central Bank of Chile. For each new statistical entry in which a new output record was published an output series was constructed, which included the revisions of the data published before.[19] As previously mentioned, for the quantitative evaluation of uncertainty in the output gap estimates, we consider the period between the first quarter of 2000 and the last quarter of 2006. Nonetheless, the output gap estimates were calculated based on information since 1986.[20] In this context, the first time series we work with covers the period between the first quarter of 1986 and the first quarter of 2000. The series that follows contains an additional quarter not included in the previous series and this occurs successively up until the last series, which is comprised of the complete period, that is, from the first quarter of 1986 to the last quarter of 2006. Each output series was seasonally adjusted using the X-12 ARIMA procedure employed by the Central Bank of Chile. Hence, the series reflect both the revisions and the re-estimation of seasonal factors. Finally, the series published in the last quarter of 2006 is that which we consider as the final series of output, although we are aware that this series could still contain unrevised data.

The compilation of the information described in the previous paragraph produced a total of 28 output series for each point in time. We apply the five detrending methods described in the

[19] In some cases the revisions were observed for one or two quarters back and in others, such as the periods in which there are base changes, the revisions were performed on the complete series.

[20] Note that for a statistical filter to produce reasonable results we need at least a complete cycle in the series, which implies that long time series are necessary.

previous subsection to each of these estimates to calculate the output gap. Following the methodology applied by Orphanides and van Norden (1999), our final estimates are the output gap series for the last available series and our real-time estimates are the series constructed with the last observation of each of the output gaps estimated with the 28 series. Figures 1 and 2 show these estimates using the five filters, as well as final and real-time data.

Figure 1: Output Gap Estimates for the Chilean Economy with Final Data

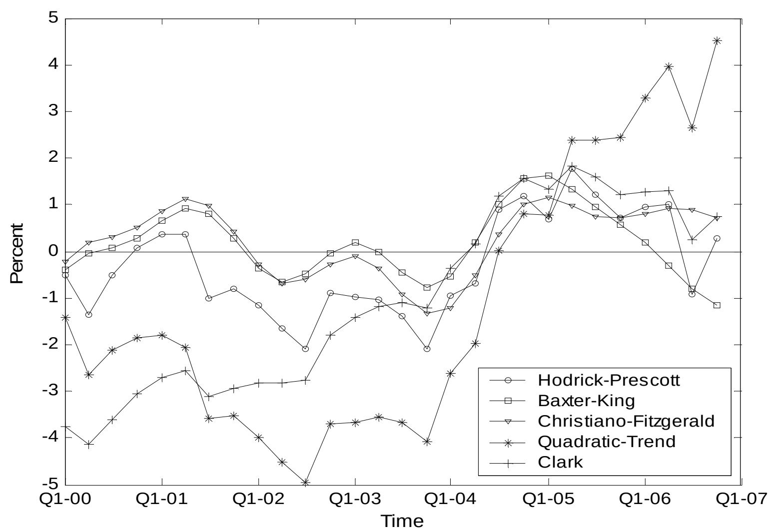
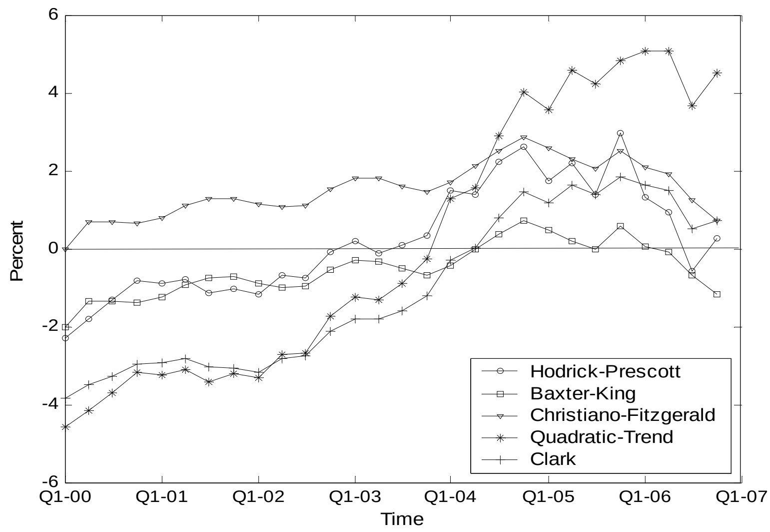
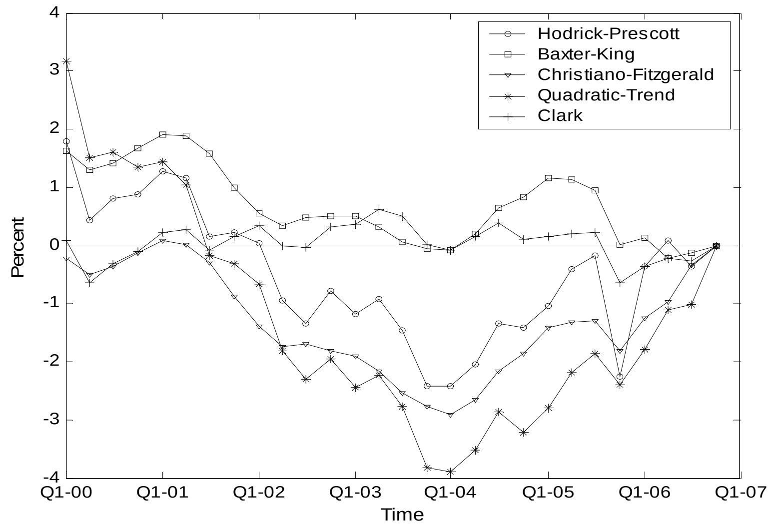


Figure 2: Output Gap Estimates for the Chilean Economy with Real-Time Data



Note from figures 1 and 2 that most of the estimations generated by the different detrending methods reveal similar behavioral patterns as it relates to the increasing or decreasing trends across the entire period. This is true for both the estimations using final data and those using real-time data. The only exception to this rule is observed in the estimation of the output gap based on the quadratic trend. It is important to highlight, however, that despite the comovements observed in the different series, the magnitude of the changes vary considerably from one method to the other. In the same way, the different methods produce a wide range of output gap estimates. The average difference between the highest and lowest estimates is 6% when final data are used and 12% when real-time data are used. The order of magnitude of these differences is considerable since they are quite superior to the difference between the highest and the lowest points of the business cycle within the period considered (approximately 5% for both types of data and for the majority of the filters). On the other hand, the average dispersion that exists between the methods is also important and reaches 2.3% when using final data and 4.3% in the case of real-time data. Another important characteristic of the estimations using final data is that these tend to be clustered between the fourth quarter of 2004 and the third quarter of 2005. In addition, these estimates remain relatively close to the end of the analysis period with the exception, once again, of the output gap based on the quadratic trend. This latter characteristic is not observed with the real-time estimates. To have a qualitative idea of the importance of data revision, figure 3 shows the difference between the estimates with final data and those with real-time data for the five detrending methods. This difference represents the total revision in the output gap.

Figure 3: Total Revisions in the Output Gap for the Chilean Economy



As observed in figure 3, the magnitude of the revisions is also important and differs substantially between the distinct filters used (the average dispersion of the revisions between the different measures is 2.8%). The most extreme cases are observed in early 2004, where the revisions of the HP, CF and quadratic trend methods were the most important in the entire sample. This is due to the fact that these filters do not adequately capture the change in the signs of the output gap in that period (see figures 1 and 2) and, therefore, suggests that real-time estimates were imprecise. Also, note that this is not satisfied for the HP and Clark methods and as a matter of fact, in that same period the revisions were practically null. The most important revisions for these last two filters were observed, on the contrary, at the beginning of the sample. For a better understanding of the differences between the estimates with final data and those with real-time data, we present descriptive statistics of the output gap estimates and of the revisions, respectively, for the five filters considered in tables 2 and 3. Figure 3 shows the time behavior of all these estimates.

Table 2: Descriptive Statistics of the Output Gap Measures calculated with Final and Real-Time Data

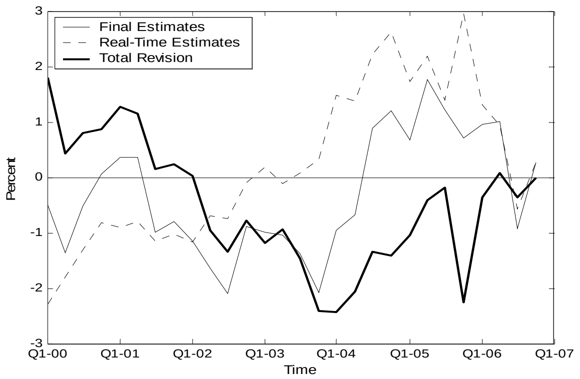
	Mean	Std	Min	Max	Corr
			Hodrick-Prescott		
Final Estimates	-0.0030	0.0108	-0.0209	0.0177	1.0000
Real-Time Estimates	0.0020	0.0141	-0.0230	0.0295	0.6109
			Baxter-King		
Final Estimates	0.0016	0.0074	-0.0117	0.0162	1.0000
Real-Time Estimates	-0.0054	0.0068	-0.0203	0.0071	0.5605
			Christiano-Fitzgerald		
Final Estimates	0.0022	0.0075	-0.0133	0.0117	1.0000
Real-Time Estimates	0.0152	0.0071	-0.0001	0.0287	0.2027
			Quadratic-Trend		
Final Estimates	-0.0116	0.0289	-0.0496	0.0452	1.0000
Real-Time Estimates	0.0009	0.0347	-0.0459	0.0508	0.8408
			Clark		
Final Estimates	-0.0103	0.0198	-0.0414	0.0183	1.0000
Real-Time Estimates	-0.0108	0.0199	-0.0385	0.0185	0.9880

Table 3: Descriptive Statistics of the Total Revisions in the Output Gap

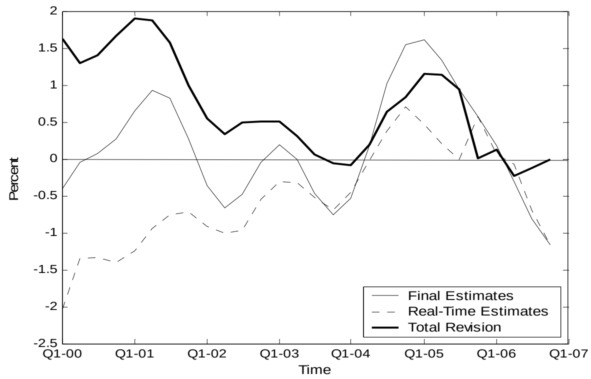
	Mean	Std	Min	Max	AR
Hodrick-Prescott	-0.005	0.011	-0.024	0.018	0.700
Baxter-King	0.007	0.007	-0.002	0.019	0.875
Christiano-Fitzgerald	-0.013	0.009	-0.029	0.001	0.939
Quadratic-Trend	-0.013	0.019	-0.039	0.032	0.842
Clark	0.000	0.003	-0.006	0.006	0.473

Figure 3: Estimation of the Output Gap and the Total Revisions using Final and Real-Time Data for the Five Alternative Filters

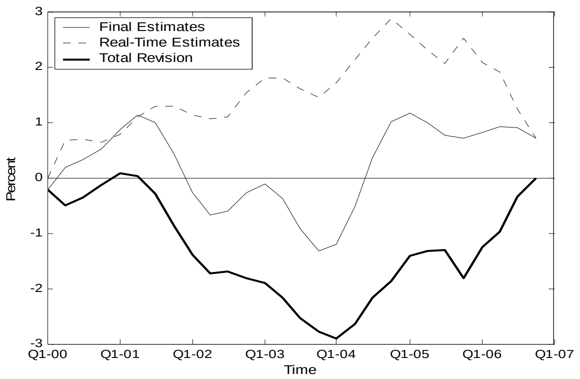
(a) Hodrick-Prescott



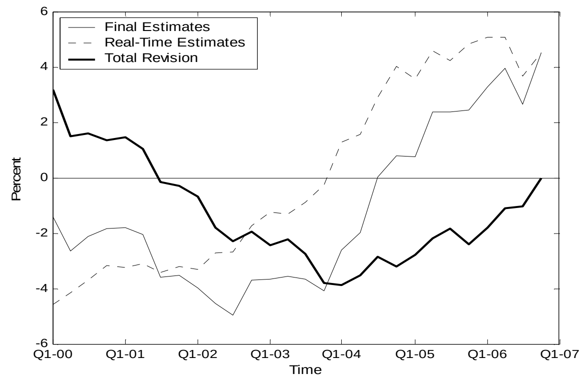
(b) Baxter-King



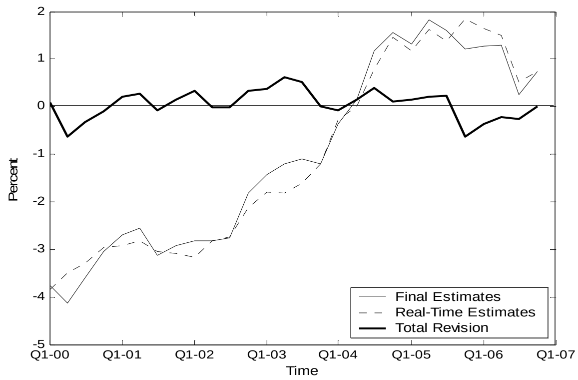
(c) Christiano-Fitzgerald



(d) Quadratic-Trend



(e) Clark



Comparing the results presented in tables 2 and 3 we observe that, on average, the total revisions are of the same or of greater magnitude as the output gap estimates for all filters used. This result is qualitatively similar to that found in Orphanides and van Norden (1999) for the US economy. Something similar occurs in the cases of the HP, BK and CF filters if we compare the variability of the output gap estimates and that of the revisions across the sample. As it relates to the minimum and maximum points of the business cycle in the period considered, it is important to highlight that the estimates with final data and those of real-time data tend to show the lowest values in the same period only in the case of the Clark method (see figure 3 panel e). At the other extreme, the maximum values of the output estimates with final data and those with real-time data coincide in the same period for the BK, the quadratic trend and Clark filters (see panels b, d and e of figure 3). This suggests that the majority of the applied filters fail to identify the magnitude of the recessive periods.

On the other hand, the last column of table 2 shows the correlation coefficients between the final data estimates and the real-time data estimates for each filter employed. Note that the highest correlations are observed for the Clark and the quadratic trend methods, whose correlation coefficients are 0.98 and 0.84, respectively, while the filters that produce the lowest correlations are those of CF and BK (0.5 and 0.2, respectively). This is consistent with the comovements observed in the final data series and the real-time data series of figure 3, since the Clark filter, besides showing the lowest values in the revisions (see table 3), has quite similar movements in both estimates. On the other extreme, the BK filter shows important revisions and opposed movements in its estimations using final and real-time data.

Another important element that needs to be considered is the degree of persistence that the revisions of the output gap estimates could reveal. This is due to the fact that as the revisions persist over time, the discrepancies between the final and real-time estimates would tend to be maintained or take time to disappear. The last column of table 3 reports the estimated first order autocorrelation coefficients for total revisions which indicate that these revisions are highly persistent. Indeed, and with the exception of the Clark model, such persistence is found within the range of 0.7 for the HP filter and 0.94 for the CF filter. The question yet to be responded is whether the distinct measures of the output gap constructed with real-time data are reliable. Since the different methods vary substantially with respect to the size of the cyclical component, it is more convenient to seek to compare the reliability of the real-time estimates using independent scale measures. It is important to mention that these indicators provide a measure of reliability for the distinct filters as it relates to quantifying the difference between the final estimates and the real-time estimates. Hence, it does not indicate anything regarding the reliability, produced by each of the methods, as tools for the estimation of the "true" output gap (Bernhardsen et.al., 2005). Table 4 presents the reliability measures used by Orphanides and van Norden (1999).

Table 4: Descriptive Statistics of the Reliability Measures for the Alternative Distinct Filters

	Corr	**N/S**	**Opsign**	**Xsize**
Hodrick-Prescott	0.611	1.055	0.286	0.500
Baxter-King	0.560	0.902	0.321	0.536
Christiano-Fitzgerald	0.203	1.229	0.393	0.750
Quadratic-Trend	0.841	0.650	0.071	0.214
Clark	0.988	0.156	0.000	0.036

In the first column of Table 4 we present the correlation between the final and the real-time series for each method, previously discussed. The remaining three indicators in table 4 measure in different ways the relative importance of the revisions. It is important to mention that the ideal value for these three indicators is zero. The first indicator, known as N/S, is the ratio of the standard deviation of the revision to that of the final estimate of the output gap and seeks to approximate the noise-to-signal ratio. As can be observed the methods with greater noise levels are the HP and the CF methods, while that with the least noise is the Clark method. On the other hand, the OPSING indicator shows the frequency with which the real-time estimates of the output gap reveals a different sign when compared to the final estimates. In the case of the HP, BK and CF filters, the average error frequency in the sign of the estimation is relatively high (approximately 30%), while the quadratic trend and the Clark methods show frequencies that are considerably lower, and in some cases even show no error in signs. Finally, the XSIZE indicator shows the frequency with which the absolute value of the revision exceeds the absolute value of the final estimates of the output gap. As it occurred in the case of the OPSING indicator, the HP, BK and CF filters produce output gap estimates with a large quantity of observations in which the revision is larger than the gap itself (frequencies between 50 and 75%), while with the quadratic trend and the Clark methods there are much smaller frequencies (between 21 and 3%, respectively). It is important to highlight that, once again, the Clark method reveals to be a better indicator.

According to the analysis presented in this section, we can mention that, in general terms, the revisions of the output gap in the case of Chile seem to be important and persistent for the period considered, and that the correlations between the final estimates and real-time estimates of the output gap are relatively low. Nonetheless, the methods that show more favorable statistics are those of quadratic trend and Clark's. In particular, this last method produces the best results overall. Reliability indicators also indicate that the most reliable filters that will be used with real-time data are the quadratic and Clark methods. Comparing the results obtained with those of Orphanides and van Norden (1999) for the US economy, we find that in general the different reliability measures produce similar values. However, this comparison should be taken with caution since, on one hand, these authors use a much larger study sample period to evaluate the reliability and, on the other hand, the set of detrending methods used where not exactly the same. In general, these results imply that caution and judgment should be taken when assessing the level of the real-time estimates of the output gap. Additionally, our results should be considered a lower bound to the measurement errors that could be present in the output gap estimates because comparisons are made with respect to a measure of the final output gap that could contain unrevised data.

4. Additive and multiplicative uncertainty

Once the problem of data uncertainty has been dealt with, in this section we focus on the empirical importance of the additive and multiplicative uncertainty in the case of Chile. This analysis is done for the period 1990 to 2006 with some emphasis in the sub sample 1999-2006, the *full-fledged inflation targeting* period. We adopt the *forward-looking* specification of Svensson (2000) and Al-Eyd and Karasulu (2006) for the equations that govern the behavior of a small open economy, as is the case of Chile (i.e, the aggregate demand, the Phillips curve and the real uncovered interest parity condition). To capture the sources of uncertainty, we estimate the model with time-varying parameters and assume that shocks have state-dependent variances (two states) and that their behavior follows a first order Markov process. In the end, this strategy allows us to decompose the conditional variance of the forecast error into two components: one component associated with parameter or multiplicative uncertainty and a second component attributed to uncertainty dealing with shocks in the model or additive uncertainty.

4.1. Methodological issues

The empirical literature that has tried to model additive and multiplicative uncertainty have typically used models that explicitly consider stochastic volatility potentially present in the errors (heteroscedasticity) and time-varying parameters (Zhang and Semmler, 2005). Among studies that have explicitly dealt with parameter uncertainty we can cite Cogley and Sargent (2001), who studied the inflation dynamics of the United States in the post World War II period using a Bayesian VAR with time-varying parameters (TVP), and Semmler et.al. (2005), who estimated the Phillips curve and a monetary policy Taylor rule for the Euro Zone also with time-varying parameters. In both cases, the authors found evidence of substantial changes in the model parameters. It is important to mention, however, that even though the evidence found when using models with time-varying parameters suggests the existence of important degrees of uncertainty, in the modeling process this analysis cannot be separated from the additive uncertainty. This is so because when additive uncertainty is not considered, volatility in the parameters could be exaggerated when it is indeed captured (Sims, 2001). An example can be found in Sims and Zha (2006), who study regime changes in the US economy dynamics and find, contrary to Cogley and Sargent (2001), much evidence in favor of stable model dynamics but unstable variance of the disturbances. Thus, Cogley and Sargent (2005) modify their original model considering both the time-varying parameters and the stochastic volatility and also find the existence of regime changes. More recent examples of the estimation of Taylor rules with time-varying parameters and stochastic volatility can be found in Kim and Nelson (2006) and Zampolli (2006). The evidence found for countries such as the United States illustrates the important uncertainty associated with the use of the models for policy evaluation.

According to the previous discussion, to incorporate both types of uncertainty, additive and multiplicative, we follow the Zhang and Semmler (2005) approach. In particular, we use a model with time-varying parameters and shocks that have a state-dependent variance. Contrary to Cogley and Sargent (2005), who assume that the variance of the shocks change with each period, we assume that the variance has only two states (high volatility state and low volatility state) which follow a Markov process, as in the works of Sims and Zha (2006).[21] This specification, besides having the advantage of dealing with both types of uncertainty in the same model, allows the decomposition of the variance of the forecast error into two components: that associated with additive uncertainty and that associated with multiplicative uncertainty (Kim, 1993). In this context, and assuming that we are dealing with Gaussian errors, we can specify the following general model for the state equations contained in equation (2) (Kim and Nelson, 1999):

$$\begin{aligned} x_t &= v_t'\beta_t + \varepsilon_t \qquad \varepsilon_t \sim N(0,\sigma^2_{\varepsilon,S_t}) \\ \beta_t &= \beta_{t-1} + \eta_t \qquad \eta_t \sim N(0,Q) \\ \sigma^2_{\varepsilon,S_t} &= \sigma^2_{\varepsilon,0} + (\sigma^2_{\varepsilon,1} - \sigma^2_{\varepsilon,0})S_t \qquad \sigma^2_{\varepsilon,1} > \sigma^2_{\varepsilon,0} \end{aligned} \tag{5}$$

where x_t, as before, represents a vector of state variables, v_t is a vector of explanatory variables in the model,[22] β_t is a vector of parameters that follow a random walk process,[23] ε_t and η_t are regression errors (ε_t is interpreted as the shocks of the system), $\sigma^2_{\varepsilon,S_t}$ is the state-dependent variance of the shocks, Q is the variance-covariance matrix of the parameter-generating process model errors and S_t is an indicator variable (unobserved) that takes the value of 1 when the state is of high volatility, $\sigma^2_{\varepsilon,1}$, and 0 otherwise (i.e., the variance of the shocks is of low volatility $\sigma^2_{\varepsilon,0}$). The transition probabilities of one state to another under the Markov process can be written as:

$$\begin{aligned} \Pr[S_t = 1 \mid S_{t-1} = 1] &= p \\ \Pr[S_t = 0 \mid S_{t-1} = 0] &= q \end{aligned} \tag{6}$$

[21] These authors assume that the variance of the regression errors follow a Markov process with three states.
[22] This vector can contain both the state variable lags and other variables (endogenous and exogenous) that affect the behavioral equations of the economy.
[23] This assumption is common in the literature concerning time-varying parameter models. Nonetheless, in the analysis of the following section the validity of this assumption was verified with the data.

For the estimation of the model presented in equations (5) and (6) we use maximum likelihood techniques that combine the use of the Kalman filter and the EM algorithm proposed by Hamilton (1989,1990); for a detailed description of the algorithm see Kim and Nelson (1999).

Since Chile is a small open economy, we use a version of the Svensson (2000) and Al-Eyd and Karasulu (2006) specification that can be estimated for the behavioral equations of the economy. Such a specification is a version of the Neokeynesian model for a small and open economy with perfect capital mobility and, as mentioned above, is comprised of the IS curve or the aggregate demand curve, the short-term aggregate supply (the Phillips curve) and the real uncovered interest parity condition. These equations can be written as:

$$y_t = \theta_1 y_{t-1} + \theta_2 E_t[y_{t+1}] + \theta_3 r_{t-1} + \theta_4 q_{t-1} + \varepsilon_t^d \tag{7}$$

$$\pi_t = \phi_1 \pi_{t-1} + \phi_2 E_t[\pi_{t+1}] + \phi_3 y_{t-1} + \phi_4 q_t + \varepsilon_t^s \tag{8}$$

$$q_t = \gamma_1 E_t[q_{t+1}] + \gamma_2 (r_t - r_t^f) + \upsilon_t \tag{9}$$

where y_t represents the real output gap, π_t is the inflation rate, r_t is the short-term real interest rate, q_t is the real exchange rate and r_r^f is the foreign real interest rate, observed in period t. On the other hand, $E_t[y_{t+1}]$, $E_t[\pi_{t+1}]$ and $E_t[q_{t+1}]$ represent the expectations for period $t+1$ of the output gap, the inflation rate, and the real exchange rate, respectively, conditional on the available information at period t (E_t is the expectations operator). The shocks of the economy are represented by ε_t^d, ε_t^s and υ_t. The first two are aggregate demand and supply shocks, respectively, and the third one is associated with the exchange market. In the words of Al-Eyd and Karasulu (2006), this last disturbance term could be interpreted as a risk premium that captures the effects of the unobservables, such as the exchange market sentiments. Finally, θ_i (with $i = 1,2,3,4$), ϕ_i (with $i = 1,2,3,4$) and γ_i (with $i = 1,2$) are the parameters to be estimated.

It is important to mention two interesting issues of the previous specification. First, the explicit inclusion of the exchange rate in the modeling process is relevant for an economy such as Chile that utilizes inflation targeting as a monetary policy framework. Indeed, an important additional transmission channel of monetary policy compared to the closed economy models is introduced and the external shock effect on the domestic economy is incorporated. Following Svensson (2000), the exchange rate affects the inflation rate directly through its effect on the domestic prices of the imported goods. Additionally, as it affects relative prices, the exchange rate also contributes to the aggregate demand transmission mechanism. Second, the specification incorporates both *forward-looking* and *backward-looking* terms (hybrid model), a feature for which there is some empirical evidence in Chile, at least for the Phillips curve (see Caputo et.al., 2006, and Céspedes et.al., 2005). The fact that *forward-looking* terms are introduced in the model also contributes to explicitly consider the price and wage-setting rules in the modeling process of the sticky price models. In the case of the exchange rate, it allows the incorporation of the expectations component, which is inherently *forward-looking*, on the asset prices,[24] which plays a key role in monetary policy.

In spite of the theoretical advantages of the specification in equations (7), (8) and (9), at the practical level this presents some potential problems. In particular, the way in which the *forward-looking* components are measured or approximated can have important implications in the estimation properties (consistency). The literature has proposed various ways to deal with these variables, as well as the most appropriate estimation techniques in each case. A first option suggested by Roberts (1995), is to use data from expectation surveys, for example those prepared on a monthly basis by the Central Bank of Chile, to construct a proxy variable of the expectations. This alternative, however, has two potential problems: the first one is associated to the availability of long period time series for the estimation; and the second one, acknowledged by the same Roberts (1995), is that in general surveys are measured with error. Another option is to utilize ex-post data, that is, approximate the expectation variables with their respective observed future values. Even though this option is operationally simple, it has an important problem since it generates an endogeneity bias in the estimation of the model parameters, which leads to inconsistent estimates (Kim and Nelson, 2006).[25] To illustrate the problem of endogeneity bias note that the *forward-looking* component of the model, under the assumption of

[24] Note that the exchange rate is the price of an asset.
[25] A fact that is relevant if it is considered that one of the objectives of the article is to study precisely parameter uncertainty.

rational expectations, can be written as: $E_t[y_{t+1}] = y_{t+1} + \mu^y_{t+1}$, $E_t[\pi_{t+1}] = \pi_{t+1} + \mu^\pi_{t+1}$ and $E_t[q_{t+1}] = q_{t+1} + \mu^q_{t+1}$, where μ^j_{t+1} with $j = y, \pi, q$ representing the error committed by the agents in their forecasts. These forecast errors are highly correlated with the shocks of the economy (i.e., the regression errors in the previous specification) and, therefore, upon using the ex-post data an endogenous variable is being implicitly introduced in the model.

To deal with the problem of endogeneity bias, Rudd and Whelan (2005) and Lindé (2005) propose to estimate a model using full information maximum likelihood (FIML), for which they first transform the *forward-looking* model into a *backward-looking* model.[26] Even though the methodological process of these authors generates more robust estimates compared to other estimation methods, it requires a priori precise knowledge of the true model that governs the behavior of the economy. In effect, Gali et.al. (2005) using Montecarlo simulations show that the FIML estimation, when there are specification errors, present biases that could be quite significant in magnitude. This point is highly relevant in the context of the present paper since one of the sources of uncertainty is precisely the lack of knowledge or imprecise knowledge of the true model of the behavior of the economy. Additionally, the transformation into a *backward-looking* model generates a composed error that depends on the model parameters (see footnote 26) and if this is estimated with time-varying parameters, it would not be possible to separate the effect of the shocks (additive uncertainty) from the instability of the parameters (multiplicative uncertainty).

Galí and Gertler (1999), Roberts (2001) and Galí et.al (2005) have proposed an alternative methodology to deal with the endogeneity problem, which is based on the use of ex-post data for

[26] For the Rudd and Whelan (2005) and Lindé (2005) transformation, for example for the Phillips curve presented in equation (8), it is assumed that the agents are rational, which implies that this equation can be expressed as:

$$\pi_t = \phi_1 \pi_{t-1} + \phi_2(\pi_{t+1} + \mu^\pi_{t+1}) + \phi_3 y_{t-1} + \phi_4 q_t + \varepsilon^s_t$$

Then, solving for π_{t+1} the following is obtained:

$$\pi_{t+1} = \frac{1}{\phi_2}\pi_t - \frac{\phi_1}{\phi_2}\pi_{t-1} - \frac{\phi_3}{\phi_2} y_{t-1} - \frac{\phi_4}{\phi_2} q_t - \mu^\pi_{t+1} - \frac{1}{\phi_2}\varepsilon^s_t$$

Applying lags to the this equation and renaming the parameters and the error of the model we have:

$$\pi_t = \varphi_1 \pi_{t-1} + \varphi_2 \pi_{t-2} + \varphi_3 y_{t-2} + \varphi_4 q_{t-1} + \omega_t$$

$$\omega_t = -\left(\mu^\pi_{t+1} + \frac{1}{\phi_2}\varepsilon^s_t\right)$$

Note that the transformed Phillips curve is in essence *backward-looking*. Hence, it requires the use of ex-post data and therefore it does not present the endogeneity bias problem.

the *forward-looking* component of the model and estimation by the Generalized Moments Method (GMM) to instrumentalize the expectations. The use of the GMM techniques to estimate the Phillips curve, as well as the *forward-looking* Taylor rules has been very common in the literature. For the case of Chile, there are various examples and to cite only two of them we can mention Céspedes et.al. (2005), who estimated a hybrid Phillips curve, and Corbo (2002), who estimated a reaction function for the Central Bank. It is important to highlight that this method is more robust than the one proposed by Rudd and Whelan (2005) and Lindé (2005) when there are specification errors (Galí et.al, 2005), which makes it desirable for the objectives of our paper. Kim (2004, 2006), along these lines, recently proposed the application of instrumental variables for the estimation with endogenous regressors, in the context of time-varying parameter models and with regime changes that follow a Markov process. More specifically, this methodological proposal solves the endogeneity problem applying the Kalman filter in a two-stage Heckman (1976) estimation. A recent application of this methodology used to estimate a *forward-looking* Taylor rule with ex-post data for the United States can be found in Kim and Nelson (2006). Contrary to this latter study, that uses conditional heteroscedasticity models to capture the volatility in the variance of the errors, we assume that such variance follows a first order Markov process.

For presentation purposes, we put forward the methodology of Kim (2004, 2006) summarized only for the general version of the model described in equation (5). Nonetheless, the procedure is easily applied to the specification of the behavioral equations of the economy; equations (7), (8), and (9). In this context, we have the following model in its state-space representation with time-varying parameters and endogenous variables:

$$\begin{aligned}
&x_t = v_t'\beta_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma^2_{\varepsilon,S_t}) \\
&\beta_t = \beta_{t-1} + \eta_t \quad \eta_t \sim N(0, Q_\eta) \\
&v_t = Z_t'\delta_t + \xi_t \quad \xi_t \sim N(0, Q_\xi) \\
&\delta_t = \delta_{t-1} + \kappa_t \quad \kappa_t \sim N(0, Q_\kappa) \\
&\sigma^2_{\varepsilon,S_t} = \sigma^2_{\varepsilon,0} + (\sigma^2_{\varepsilon,1} - \sigma^2_{\varepsilon,0})S_t \quad \sigma^2_{\varepsilon,1} > \sigma^2_{\varepsilon,0}
\end{aligned} \tag{10}$$

where, as before, x_t represents a vector of state variables, v_t is a vector of explanatory variables in the model, which are correlated with the errors of the model ε_t, Z_t is a vector of instrumental variables, β_t and δ_t are time-varying parameters and η_t, ξ_t and κ_t are Gaussian errors with a matrix of variances Q_i with $i=\eta,\xi,\kappa$. Similarly as before, we assume that the variance of errors ε_t present two states with transition probabilities that follow a Markov process. Kim (2006) proposes to specify the endogeneity present in the model assuming that the correlation that exists between the error term ε_t and the standardized forecast error associated with the endogenous variables ξ_t^* (that is, the prediction error associated with the rational expectations of the agents) is constant and equal to ρ. On the other hand, and considering that the variance of the errors is state-dependent, Kim (2004) suggests that such correlation will also be state-dependent. With this, the error of the model can be rewritten as $\varepsilon_t = \xi_t^{*\prime}\rho_{S_t}\sigma_{\varepsilon,S_t} + \sqrt{1-\rho_{S_t}{}'\rho_{S_t}}\,\sigma_{\varepsilon,S_t}\omega_t$ with $\omega_t \sim N(0,1)$. Using this last expression we can write the first equation of the model (10) as:

$$x_t = v_t{}'\beta_t + \xi_t^{*\prime}\rho_{S_t}\sigma_{\varepsilon,S_t} + \sqrt{1-\rho_{S_t}{}'\rho_{S_t}}\,\sigma_{\varepsilon,S_t}\omega_t \qquad \omega_t \sim N(0,1) \tag{11}$$

where $\rho_{S_t} = \rho_0 + (\rho_1 - \rho_0)S_t$ and S_t is the same indicator variable (unobserved) defined previously. Note in this last equation that the error of the model is independent of v_t and of ξ_t^*, hence, the estimation generates parameters that are consistent. For the estimation, Kim (2004, 2006) proposes the following two-stage procedure. The first stage consists in estimating the model that instrumentalizes the endogenous variables using the maximum log-likelihood method based on the forecast of the error and the conventional Kalman filter, that is:

$$\begin{aligned} v_t &= Z_t{}'\delta_t + \xi_t \qquad & \xi_t &\sim N(0,Q_\xi) \\ \delta_t &= \delta_{t-1} + \kappa_t \qquad & \kappa_t &\sim N(0,Q_\kappa) \end{aligned} \tag{12}$$

With this we calculate the standardized forecast error of v_t as $\xi_t^* = Q_{\xi,t|t-1}^{-1/2}(v_t - Z_t{}'\delta_{t|t-1})$ for all $t=1,2,...,T$. The second stage consists in using the forecast error calculated previously to estimate the following model by applying maximum log-likelihood techniques that combine the use of the Kalman filter and the EM algorithm proposed by Hamilton (1989,1990):

$$\begin{aligned}
&x_t = v_t'\beta_t + \xi_t^{*}{}'\rho_{S_t}\sigma_{\varepsilon,S_t} + \sqrt{1-\rho_{S_t}'\rho_{S_t}}\,\sigma_{\varepsilon,S_t}\omega_t && \omega_t \sim N(0,1)\\
&\beta_t = \beta_{t-1} + \eta_t && \eta_t \sim N(0,Q_\eta)\\
&\sigma^2_{\varepsilon,S_t} = \sigma^2_{\varepsilon,0} + (\sigma^2_{\varepsilon,1} - \sigma^2_{\varepsilon,0})S_t \quad \sigma^2_{\varepsilon,1} > \sigma^2_{\varepsilon,0} &&\\
&\rho_{S_t} = \rho_0 + (\rho_1 - \rho_0)S_t &&
\end{aligned} \tag{13}$$

As before, we assume that the transition probabilities from one state to another follow a first order Markov process and are: $\Pr[S_t = 1 | S_{t-1} = 1] = p$ and $\Pr[S_t = 0 | S_{t-1} = 0] = q$. The estimation algorithm is presented in the appendix. So, what we estimate is the following set of equations:

Aggregate Demand:

$$\begin{aligned}
&y_t = \theta_{1,t}y_{t-1} + \theta_{2,t}y_{t+1} + \theta_{3,t}r_{t-1} + \theta_{4,t}q_{t-1} + \varepsilon_t^d && \varepsilon_t^d \sim N(0,\sigma^2_{\varepsilon^d,S_t^d})\\
&\varepsilon_t^d = v_t^{*}\rho_{S_t^d}\sigma_{\varepsilon^d,S_t^d} + \sqrt{1-\rho^2_{S_t^d}}\,\sigma_{\varepsilon^d,S_t^d}\omega_t && \omega_t \sim N(0,1)\\
&\theta_{i,t} = \theta_{i,t-1} + \eta^\theta_{i,t} && \eta^\theta_{i,t} \sim N(0,\sigma^2_{\eta_i^\theta}) \ \forall i = 1,\dots,4\\
&y_{t+1} = Z_t'\delta_t + \nu_t && \nu_t \sim N(0,\sigma^2_\nu)\\
&\delta_t = \delta_{t-1} + \kappa_t && \kappa_t \sim N(0,\sigma^2_\kappa)\\
&\sigma^2_{\varepsilon^d,S_t^d} = \sigma^2_{\varepsilon^d,0} + (\sigma^2_{\varepsilon^d,1} - \sigma^2_{\varepsilon^d,0})S_t^d && \sigma^2_{\varepsilon^d,1} > \sigma^2_{\varepsilon^d,0}\\
&\rho_{S_t^d} = \rho_0 + (\rho_1 - \rho_0)S_t^d &&
\end{aligned} \tag{14}$$

Phillips Curve:

$$
\begin{array}{ll}
\pi_t = \phi_{1,t}\pi_{t-1} + \phi_{2,t}\pi_{t+1} + \phi_{3,t}y_{t-1} + \phi_{4,t}q_t + \varepsilon_t^s & \varepsilon_t^s \sim N(0,\sigma^2_{\varepsilon^s,S_t^s}) \\
\varepsilon_t^s = \nu_t^* \rho_{S_t^s}\sigma_{\varepsilon^s,S_t^s} + \sqrt{1-\rho^2_{S_t^s}}\,\sigma_{\varepsilon^s,S_t^s}\omega_t & \omega_t \sim N(0,1) \\
\phi_{i,t} = \phi_{i,t-1} + \eta^{\phi}_{i,t} & \eta^{\phi}_{i,t} \sim N(0,\sigma^2_{\eta_i^{\phi}}) \ \ \forall i = 1,\ldots,4 \\
\pi_{t+1} = Z_t{}'\delta_t + \nu_t & \nu_t \sim N(0,\sigma^2_{\nu}) \\
\delta_t = \delta_{t-1} + \kappa_t & \kappa_t \sim N(0,\sigma^2_{\kappa}) \\
\sigma^2_{\varepsilon^s,S_t^s} = \sigma^2_{\varepsilon^s,0} + (\sigma^2_{\varepsilon^s,1} - \sigma^2_{\varepsilon^s,0})S_t^s & \sigma^2_{\varepsilon^s,1} > \sigma^2_{\varepsilon^s,0} \\
\rho_{S_t^s} = \rho_0 + (\rho_1 - \rho_0)S_t^s &
\end{array}
\tag{15}
$$

Real Uncovered Interest Parity:

$$
\begin{array}{ll}
q_t = \gamma_{1,t}q_{t+1} + \gamma_{2,t}(r_t - r_t^f) + \upsilon_t & \upsilon_t \sim N(0,\sigma^2_{\upsilon,S_t^{\upsilon}}) \\
\upsilon_t = \nu_t^* \rho_{S_t^{\upsilon}}\sigma_{\upsilon,S_t^{\upsilon}} + \sqrt{1-\rho^2_{S_t^{\upsilon}}}\,\sigma_{\upsilon,S_t^{\upsilon}}\omega_t & \omega_t \sim N(0,1) \\
\gamma_{i,t} = \gamma_{i,t-1} + \eta^{\gamma}_{i,t} & \eta^{\gamma}_{i,t} \sim N(0,\sigma^2_{\eta_i^{\gamma}}) \ \ \forall i = 1,2 \\
q_{t+1} = Z_t{}'\delta_t + \nu_t & \nu_t \sim N(0,\sigma^2_{\nu}) \\
\delta_t = \delta_{t-1} + \kappa_t & \kappa_t \sim N(0,\sigma^2_{\kappa}) \\
\sigma^2_{\upsilon,S_t^{\upsilon}} = \sigma^2_{\upsilon,0} + (\sigma^2_{\upsilon,1} - \sigma^2_{\upsilon,0})S_t^{\upsilon} & \sigma^2_{\upsilon,1} > \sigma^2_{\upsilon,0} \\
\rho_{S_t^{\upsilon}} = \rho_0 + (\rho_1 - \rho_0)S_t^{\upsilon} &
\end{array}
\tag{16}
$$

Kim (1993)[27] proposes a procedure to decompose the conditional variance of the forecast error (f), calculated from the estimation of the specifications (14), (15), and (16), into two components: (1) f^1 or the conditional variance due to changes (or lack of knowledge) in the model parameters, or multiplicative uncertainty, and (2) f^2 or the conditional variance given the heteroscedasticity in the error term, or additive uncertainty. For this, the author exploits the informational structure of the model related with the probability distributions in the different states. In effect, the conditional variance due to the multiplicative uncertainty depends on the state in a previous period, while the conditional variance due to additive uncertainty depends on

[27] In his paper, Kim (1993) seeks to identify the sources of uncertainty and its importance associated to the process of monetary creation in the United States.

the state in the current period. This decomposition is quite useful for this paper objectives, since it allows us to know what percentage of the total variance of the forecast error is caused as a result of each of the sources of uncertainty considered. Formally, and using the same notation of the general model presented in equation (10) we have:[28]

$$f_t = f_t^1 + f_t^2 \rightarrow \begin{cases} f_t^1 = v_{t-1}\left\{\sum_{i=0}^{1} \Pr[S_t = i \mid \psi_{t-1}]\left[P_{t|t-1}^i + \left(\tilde{\beta}_{t|t-1} - \beta_{t|t-1}^i\right)\left(\tilde{\beta}_{t|t-1} - \beta_{t|t-1}^i\right)'\right]\right\} v_{t-1}' \\ f_t^2 = \sigma_{\varepsilon,S_t}^2 = \sigma_{\varepsilon,0}^2 + (\sigma_{\varepsilon,1}^2 - \sigma_{\varepsilon,0}^2)\Pr[S_t = 1 \mid \psi_{t-1}] \end{cases} \tag{17}$$

where $\tilde{\beta}_{t|t-1} = \sum_{i=0}^{1} \Pr[S_t = i \mid \psi_{t-1}]\beta_{t|t-1}^i$ y $P_{t|t-1}^i$ is the variance-covariance matrix of $\beta_{t|t-1}^i$ at state i

4.2. Results

To estimate (14), (15), and (16) we use quarterly data for the period going from the first quarter of 1990 to the last quarter of 2006. For estimation purposes we define the output gap y_t as the difference between the observed GDP and its trend, which is calculated using the HP filter.[29] Given that the output series ends in 2006, our measure of the output gap, according to the discussion in section 3 above, would be that which we consider there as "final". Thus, the uncertainty associated with data revisions does not form part of the types of uncertainty analyzed in this section. However, the calculation methods of the output trend can have an important effect on the estimations. This is studied in the robustness analysis presented in the end. On the other hand, the quarterly inflation rate π_t is measured as the quarterly variation of the underlying consumer price index (CPIX).[30] As in the work of Céspedes et.al. (2005), we use the CPI variation instead of the implicit deflator variation of the GDP since the latter, for the case of Chile, is measured with considerable noise and is strongly influenced by the variations in the terms of trade. Additionally, the target of the central bank is expressed in terms of CPI

[28] For details on the formal derivation of the decomposition of the conditional variance of the forecast error see Kim and Nelson (1999).

[29] Later in the paper, we replicate the estimations using other filters.

[30] We chose the underlying index to avoid the influence of the regulated prices and of those that show significant variations.

variations. In the case of the real exchange rate q_t we chose the bilateral exchange rate index with the United States. Finally, the foreign and domestic short-term interest rates r_t and r_t^f were defined as the monetary policy rates of Chile and the United States, respectively. All the previous data were obtained from the Central Bank of Chile database. Table 5 shows the parameters estimated using Heckman's two-stage procedure detailed in Kim (2004, 2006) and described in the previous subsection.[31]

There are two interesting elements of these estimations that we can highlighted. The first one is that variances of shocks confirm the existence of two states in the three behavioral equations: one of high volatility and one of low volatility. For the case of the estimations for the aggregate demand, note that the variance of shocks in the high volatility state is substantially greater than the low volatility state (0.48 vs. 0.05). The difference between these variances for the case of the Phillips curve is just as significant (0.54 and 0.03 in the high and low volatility states, respectively). In the case of the real uncovered interest parity condition something similar occurs (3.27 vs. 6.76), even though the magnitude of the difference is not as huge as in the previous two cases. Additionally, all the variances, except that associated with the high volatility state of the Phillips curve, are statistically significant. It is important to highlight, however, that even though the difference between the variances of shocks for the parity condition is not significant, the size of these differences is considerable if we compare them with those found for the aggregate demand and the Phillips curve. This is intuitively correct if we consider that the exchange rate is in essence an asset price.

The second element that can be highlighted refers to the existing correlation between the shocks of the behavioral equations and the errors in the expectations of the economic agents that also vary substantially with the states. In particular, the results suggest that in high volatility states in the shocks, agents tend to commit crucial errors in their forecasts. This fact is particularly true for the Phillips curve, where such correlation varies between 0.001 and 0.47 for both states, and for the real uncovered interest parity condition (0.78 vs. 1). In the case of the aggregate demand there is also an important correlation in the high volatility state. Nonetheless, the difference between the correlations of both states is less evident than in the previous two

[31] It is important to mention that in the application of the Kalman filter for the evaluation of the likelihood function we eliminated observations at the beginning of the sample due to the presence of non-stationary time series in the model; see Kim and Nelson (1999).

cases. Results also indicate that correlation coefficients are highly significant for all cases except for the one associated to the low volatility state in the shocks on the aggregate demand.

Table 5: Estimation of the Behavioral Equations

Aggregate Demand			Phillips Curve			Real Uncovered Interest Parity		
Param.	Estimated	S.D.	Param.	Estimated	S.D.	Param.	Estimated	S.D.
p	0.6571	0.5267	p	0.6639	1.5101	p	0.9992	0.0033
q	0.6586	0.0644	q	0.8475	0.0501	q	0.9453	0.1156
$\sigma_{\eta_1^\theta}$	0.0697	0.2565	$\sigma_{\eta_1^\phi}$	2.4407	1.0338	$\sigma_{\eta_1^\gamma}$	0.0036	0.0008
$\sigma_{\eta_2^\theta}$	0.0797	0.2441	$\sigma_{\eta_2^\phi}$	1.2700	0.8449	$\sigma_{\eta_2^\gamma}$	0.0000	0.1383
$\sigma_{\eta_3^\theta}$	0.2942	0.2540	$\sigma_{\eta_3^\phi}$	0.0000	0.0001			
$\sigma_{\eta_4^\theta}$	0.0002	0.0002	$\sigma_{\eta_4^\phi}$	1.6518	0.9554			
$\sigma_{\varepsilon,0}$	0.0570	0.0098	$\sigma_{\varepsilon,0}$	0.0329	0.0084	$\sigma_{\upsilon,0}$	3.2785	0.2739
$\sigma_{\varepsilon,1}$	0.4806	0.2347	$\sigma_{\varepsilon,1}$	0.5497	1.2718	$\sigma_{\upsilon,1}$	6.7694	0.1583
ρ_0	0.5123	0.1594	ρ_0	0.0010	0.2473	ρ_0	0.7854	0.0866
ρ_1	0.6324	0.1892	ρ_1	0.4705	0.1446	ρ_1	1.0000	0.0475
Loglike	-64.026		Loglike	-80.389		Loglike	-114.74	

Figures 4 to 6 present the time behavior of the estimated parameters for the three set of equations in table 5. In the case of the aggregate demand parameters (figure 4) it can be observed that there are three clearly defined periods. The first one, observed up until the mid 90s, is characterized by high instability and substantial differences between the parameters of the two states associated to the demand shocks. It is important to mention that during this period the Chilean economy experienced a substantial fall in its annual GDP growth rate (from rates of 15% to rates below 6%) and yet maintained moderate to high inflation rates. In the second period, that covered the years of 1998 and 1999, we observed a substantial reduction in the instability of the parameters, as well as in the differences of these reductions with respect to the state that characterizes the shocks. However, such parameters still present certain volatility when compared with those of the years that followed 1999. The Asian crisis, which occurred during this period, seemed to explain, at least in part, the instability that was still present in the aggregate demand. The third period, on the contrary, shows much more stability than the previous two periods and the parameters, with the exception of the expectations parameter regarding output, are quite similar in the two states of the shocks. These results suggest that the multiplicative uncertainty associated with the aggregate demand tends to decline over time and the period of greater stability coincides with the establishment of the *full-fledged inflation targeting* framework to conduct monetary policy and since 2000, with the implementation of an explicit fiscal rule. Another interesting issue we can highlight, besides the propensity to greater stability, is the degree of persistence of the output gap ($\theta_{1,t}$) and the response of this to changes in relative prices ($\theta_{4,t}$) have been reduced over time, while the contrary has occurred with the degree of response to expectations ($\theta_{2,t}$) and the monetary policy interest rate ($\theta_{3,t}$), which would be consistent with the logic of the inflation targeting framework. With respect to the parameters of the Phillips curve (figure 5), it is clear that these show a significant dependence on the state of the supply shocks. In particular, during the periods of high volatility of shocks, the parameters tend to also show high instability and when the state of these shocks is of low volatility the parameters are much more stable. It is important to mention that, as opposed to what is observed in the aggregate demand parameters, this dependence has been maintained throughout the entire period. These results suggest that the state of shocks is fundamental in explaining greater or lower degrees of uncertainty in the Phillips curve parameters. Another issue that can also be observed in figure 4 is that when the economy goes through a period of relative calm with respect to the supply shocks, the persistence of the inflation rate ($\phi_{1,t}$) and the importance of expectations in the determination of the inflation rate ($\phi_{2,t}$) are clearly greater towards the end of the period analyzed, while the trend is lower for the case of the response of inflation to the business cycle ($\phi_{3,t}$) and to the variations in the real exchange rate ($\phi_{4,t}$). Contrary to what was previously mentioned, when the supply shocks are highly volatile there is no definite trend for the Phillips curve parameters. Finally, the parameters associated with the real uncovered interest parity condition (figure 6) show a high degree of instability when the volatility of the external shocks is high, as well as when it is low. This would be related with the high degree of variability that these shocks have in both states (see table 5). With respect to the model parameters it is clear that these do not have a clear cut trend over time.

Figure 4: Time-Varying Parameters Estimated for the Aggregate Demand

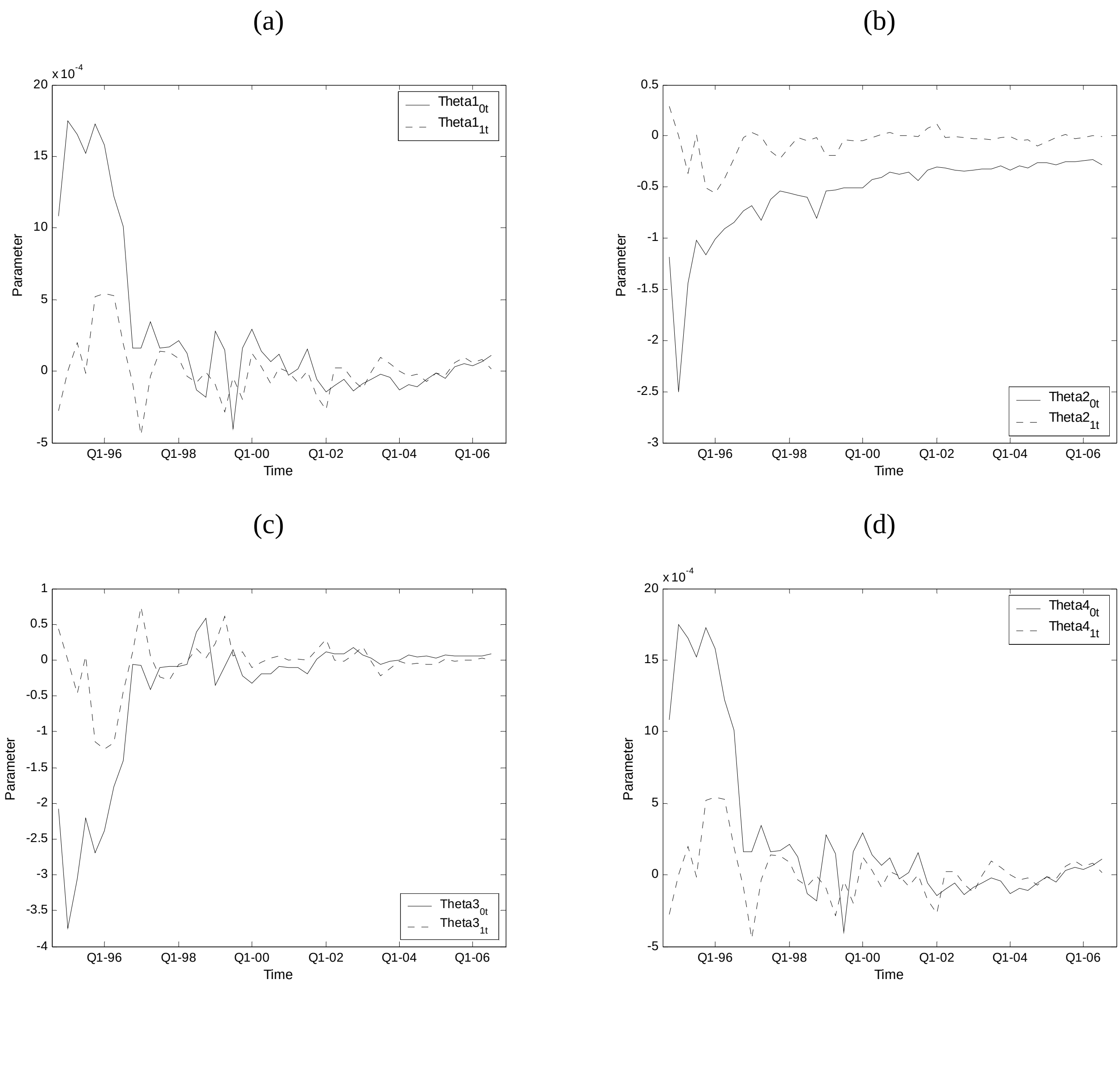


Figure 5: Time-Varying Parameters Estimated for the Phillips Curve

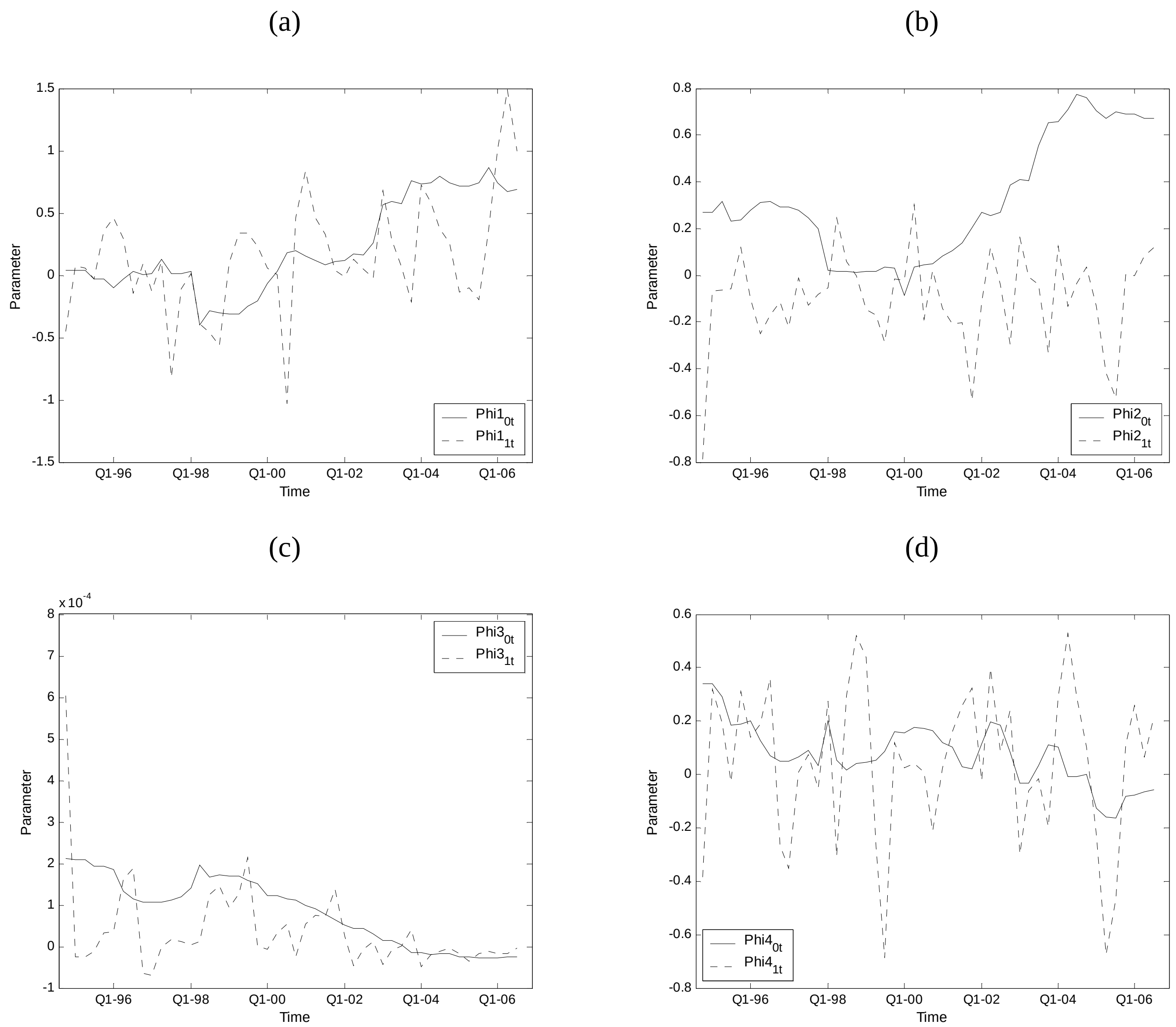
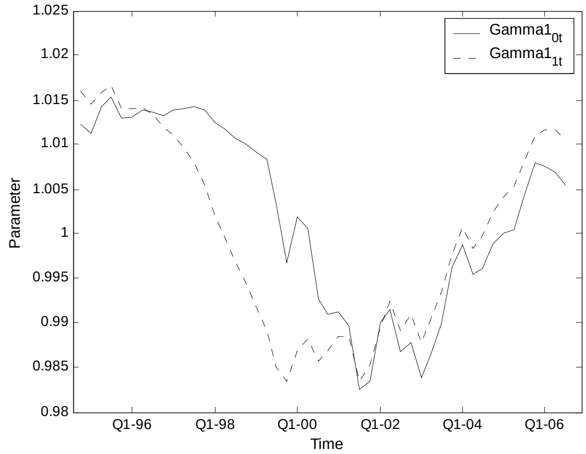
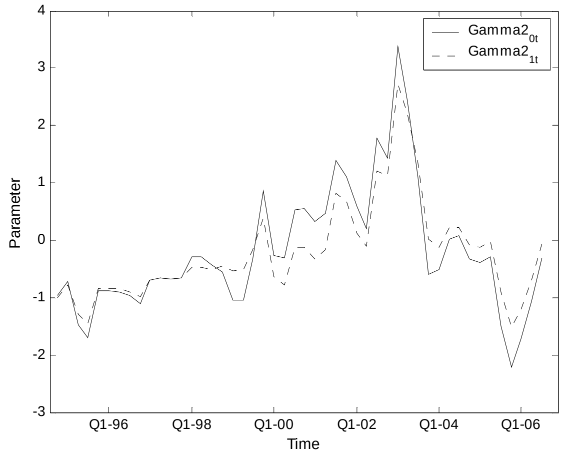


Figure 6: Time-Varying Parameters Estimated for the Real Uncovered Interest Parity

(a)



(b)



Based on the estimated parameters of the models presented in table 5, we calculated the decomposition of the conditional variance of the forecast error using the procedure proposed by Kim (1993). Figure 7 shows the decomposition for the set of equations associated with aggregate demand. As can be observed, total uncertainty in the behavior of the output gap has been relatively high throughout the entire period analyzed (note that the output gap is measured as the percentage deviation of output with respect to its trend). On average the forecast error variance was 0.021, which was 87.6% explained by uncertainty in the demand shocks (87.6%) and 12.4% by instabilities in the model parameters (12.4%) (see table 6). Note also that, consistent with the parameters behavior previously commented, total uncertainty showed significant spikes (to almost twice the average) in the mid 90s and during the 1998-1999 period. However, after 1999 total uncertainty declined on average a little over 60% with respect to the average observed between 1993 and 1995 and in something less than 25% with respect to that observed between 1995 and 1998. Something similar has occurred with the contributions of additive and multiplicative uncertainty to total uncertainty. Indeed, while parameter instability contributed approximately 15% to total uncertainty throughout the 90s, such contribution went down to a little over 10% in the period subsequent to 1999.

Figure 7: Decomposition of the Conditional Variance of the Forecast Error of the Output Gap

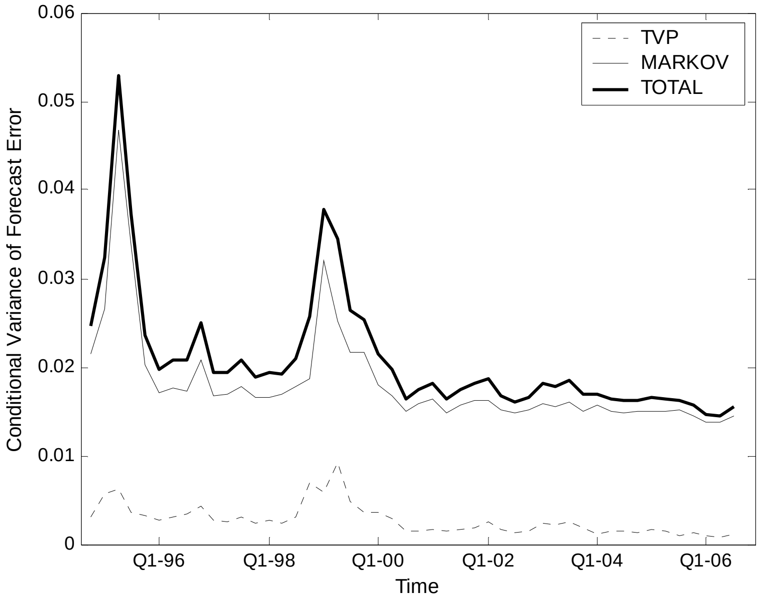


Table 6: Decomposition of the Conditional Variance of the Forecast Error of the Output Gap

	Conditional Variance of the Forecast Error			Percentage	
	TVP	MARKOV	TOTAL	TVP	MARKOV
1993-1995	0.00424	0.02566	0.02990	14.6%	85.4%
1996-1998	0.00353	0.01881	0.02234	15.5%	84.5%
1999-2006	0.00208	0.01616	0.01824	10.6%	89.4%
Total Sample	0.00279	0.01842	0.02121	12.4%	87.6%

The decomposition of the conditional variance of the forecast error for the inflation rate is shown in figure 8. The results in this case are similar to those found for the output gap as it relates to the magnitude and behavior (principally for the decade of the 90s). In effect, the total uncertainty associated to the inflation rate has been on average 0.015 for the entire period analyzed. This level of uncertainty is 69.9% explained by uncertainty in the supply shocks and 30.1% by parameter instability (see table 7). Note that the two recurrent periods of high uncertainty, as in the case of the output gap, are in the mid 90s and during the period 1998-1999, where uncertainty reached levels greater than twice the observed average for the entire period of analysis. An additional issue that can be noted from figure 8 is that in most sub periods, additive uncertainty explains a major part of total uncertainty. Nonetheless, for brief episodes in the mid-90s and during the Asian cries, the contribution pattern is reverted and it is uncertainty in the parameters that is most relevant (recall that the model parameters in a high volatility state in the supply shocks are much more unstable than in their counterparts of low volatility). Total inflation uncertainty, as in the case of the output gap, has been decreasing over time. And also the contribution of additive uncertainty grows with time.

Figure 8: Decomposition of the Conditional Variance of the Forecast Error of the Inflation Rate

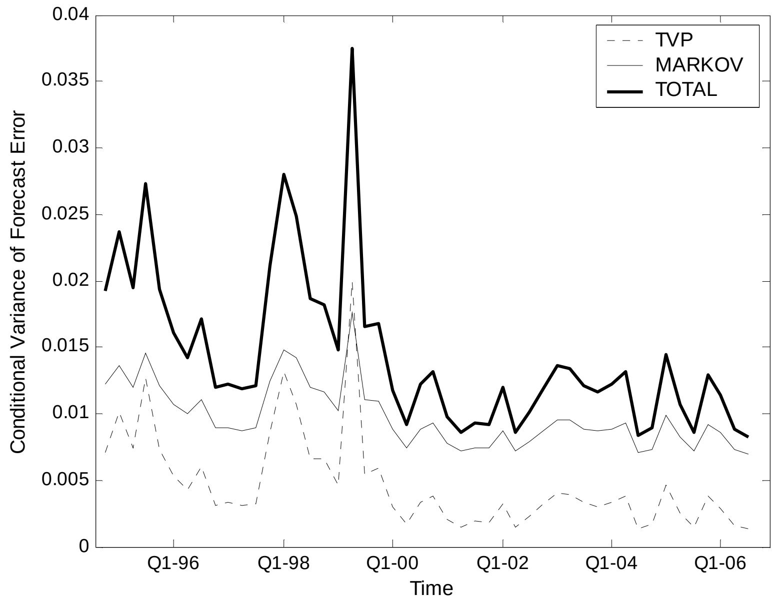


Table 7: Decomposition of the Conditional Variance of the Forecast Error of the Inflation Rate

	Conditional Variance of the Forecast Error			Percentage	
	TVP	MARKOV	TOTAL	TVP	MARKOV
1993-1995	0.01172	0.01428	0.02599	43.0%	57.0%
1996-1998	0.00612	0.01099	0.01711	33.7%	66.3%
1999-2006	0.00337	0.00869	0.01205	24.9%	75.1%
Total Sample	0.00545	0.01019	0.01563	30.1%	69.9%

Finally, figure 9 presents the decomposition of the conditional variance of the forecast error associated with the real exchange rate. In this figure it is observed that total uncertainty, measured by the variance, has been quite important throughout the period (approximately 11 on average) and basically explained (95%) by uncertainty in the shocks of the real uncovered interest parity condition or, according to the interpretation of the previous subsection, uncertainty in the risk premium that captures the effects of the unobservables of the exchange market sentiments. This result is consistent with what was mentioned before with respect to the nature of the exchange rate (an asset price). It is also important to highlight that total uncertainty has not shown a defined pattern over time (see table 8). As a matter of fact, since 1999 this uncertainty increased with respect to that observed between 1995 and 1998 but practically maintained the same levels in the mid 90s.

Summing up, overall uncertainty is dominated by additive uncertainty in all three set of equations (output gap, inflation and the real exchange rate). Moreover, results of the estimations of the behavioral equations (aggregate demand and aggregate supply) suggest that the variance of shocks is state-dependent and that such states could be considered as high volatility periods in the shocks and low volatility periods. For these two set of equations, total uncertainty has consistently declined during the current decade, bringing a rather long period of stability (so far) that coincides with the establishment of a *full-fledged inflation targeting* framework for the conduct of the Chilean monetary policy and an explicit rule for setting fiscal policy. On the other hand, it was observed that in particular periods, such as in the mid 90s and during the period

1998-1999, total uncertainty showed substantial increases in the output gap and the inflation rate, making clear the two states in the shocks variance and also indicating that during these periods the Chilean economy would have experienced a high volatility in such shocks. Finally, the asset price nature of the exchange rate is manifested in the behavior of both the parameters of the real uncovered interest parity condition and the uncertainty associated with the real exchange rate (whose source is basically exchange market shocks).

Figure 9: Decomposition of the Conditional Variance of the Forecast Error of the Real Exchange Rate

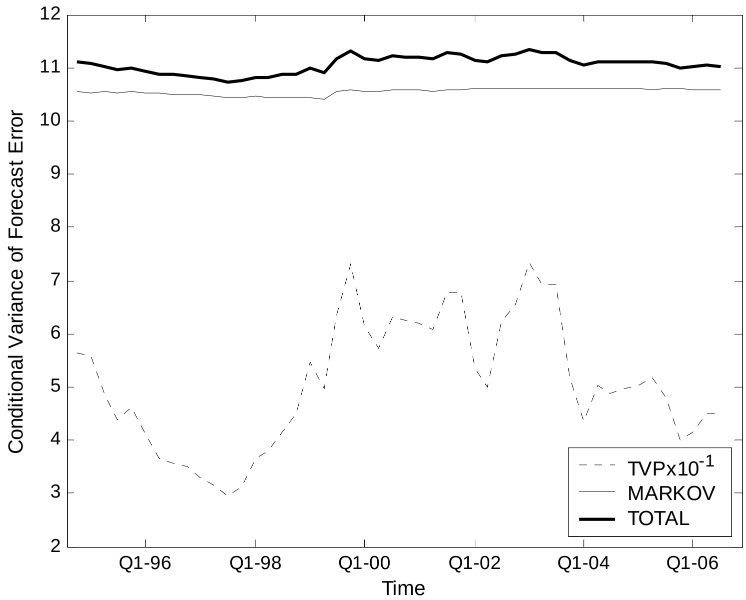


Table 8: Decomposition of the Conditional Variance of the Forecast Error of the Real Exchange Rate

	Conditional Variance of the Forecast Error			Percentage	
	TVP	MARKOV	TOTAL	TVP	MARKOV
1993-1995	0.54577	10.54444	11.09022	4.9%	95.1%
1996-1998	0.37324	10.46275	10.83625	3.4%	96.6%
1999-2006	0.56164	10.58610	11.14777	5.0%	95.0%
Total Sample	0.51542	10.55042	11.06592	4.6%	95.4%

We use the bootstrap method to verify whether the magnitudes of the variance of the forecast error found before for the distinct types of uncertainties are statistically different from zero, whether the differences between the variance of such error due to additive uncertainty and that due to multiplicative uncertainty are statistically significant and whether the assumption of Gaussian errors in the estimation introduces important biases.

Table 9: Bootstrap Decomposition of the Conditional Variance of the Forecast Error

	Gaussian ML			Bootstrap								
	TVP	MARKOV	TOTAL	TVP			MARKOV			TOTAL		
				Mean	[CI 95%	]	Mean	[CI 95%	]	Mean	[CI 95%	]
Output Gap												
1993-1995	0.00424	0.02566	0.02990	0.00585	0.00572	0.00598	0.05667	0.05542	0.05790	0.06251	0.06119	0.06384
1996-1998	0.00353	0.01881	0.02234	0.00548	0.00533	0.00564	0.02330	0.02264	0.02401	0.02878	0.02796	0.02961
1999-2006	0.00208	0.01616	0.01824	0.00193	0.00188	0.00197	0.01807	0.01749	0.01870	0.02000	0.01938	0.02066
Total Sample	0.00279	0.01842	0.02121	0.00342	0.00334	0.00351	0.02596	0.02524	0.02671	0.02938	0.02860	0.03020
Inflation Rate												
1993-1995	0.01172	0.01428	0.02599	0.01204	0.01199	0.01207	0.06555	0.02062	0.15871	0.07758	0.03267	0.18638
1996-1998	0.00612	0.01099	0.01711	0.00588	0.00586	0.00590	0.04010	0.01541	0.09914	0.04598	0.02129	0.09725
1999-2006	0.00337	0.00869	0.01205	0.00289	0.00288	0.00291	0.02381	0.01130	0.04950	0.02670	0.01420	0.05276
Total Sample	0.00545	0.01019	0.01563	0.00516	0.00514	0.00518	0.03479	0.01386	0.07986	0.03996	0.01903	0.08616
Real Exchange Rate												
1993-1995	0.54577	10.54444	11.09022	0.81944	0.77777	0.86192	9.21296	9.02628	9.40670	10.03222	9.83458	10.24011
1996-1998	0.37324	10.46275	10.83625	0.74982	0.70561	0.79519	9.20454	9.01600	9.39305	9.95438	9.75733	10.16217
1999-2006	0.56164	10.58610	11.14777	0.91215	0.86287	0.96336	9.19028	8.99863	9.38625	10.10250	9.89786	10.31116
Total Sample	0.51542	10.55042	11.06592	0.85864	0.81185	0.90699	9.19750	9.00743	9.39136	10.05616	9.85448	10.26448

Table 9 presents the results obtained from the bootstrap of the decomposition of the conditional variance of the forecast error for the three set of equations (mean estimation and 95% confidence intervals). Additionally, in the same table we present, for comparison purposes, the results found before under the assumption of the Gaussian errors in the estimation. The bootstrap re-sampling was done following the methodologies of Stoffer and Wall (1991) and Psaradakis (1998) for state-space models that use the Kalman filter and for the sampling of errors with Markov regime changes, respectively. There are three important issues we can highlight from the results in table 9. First, even though the bootstrap average estimations and the estimations based on the assumption of the Gaussian errors differ, the bias does not seem to be important in magnitude. This is true even though in some cases this bias is statistically significant (the Gaussian estimation falls outside the bootstrap interval boundaries). Second, the bootstrap estimations confirm what was mentioned before with respect to, on one hand, the observed decreasing trend of uncertainty over time for the output gap and the inflation rate, and on the other hand, and that uncertainty in the real exchange rate does not have a defined pattern. Finally, and more importantly, the bootstrap results suggest that uncertainty in all cases is statistically different from zero (no interval includes a level of zero uncertainty) and that the differences observed in the decomposition of the variance, that is, the contributions of the additive and multiplicative uncertainty to the total uncertainty, are statistically significant (no interval crosses).

To conclude this subsection we present a robustness analysis for the decomposition of the forecast error variance. In section 3 above we found evidence of important differences in the estimation of the output gap when we consider five output detrending methods. Given that aggregate demand and the Phillips curve utilize an output gap measure for its estimation, measurement errors in the estimation of this variable will be a part of the additive and multiplicative uncertainty without any possibility of discrimination. Tables 10 and 11 show the results of the decomposition of uncertainty in its two sources, additive and multiplicative, for these two equations and for each of the five filters used in section 3. The first row of both tables show the decomposition presented in the analysis of this subsection, where the gap was calculated using the HP filter, and hence, represents our benchmark. In the case of the output gap (table 10) it is observed that in general total uncertainty is quite similar for all filters and that differences, as is expected, arises in the contribution of each one of the types of uncertainty to total uncertainty. However, all detrending methods keep additive uncertainty as the most important source of uncertainty (its contribution varies from a minimum of 84.7% with the BK filter and a maximum of 90% with the Clark filter). With respect to the inflation rate (table 11) the difference between the filters can be observed in both the estimation of total uncertainty and the contributions of each type of uncertainty to total uncertainty. In the former case, the estimations are found in the range of 0.01374 and 0.02274 calculated using the BK filter and the quadratic trend, respectively, while the contributions of the additive uncertainty vary between 66.6% obtained using the BK filter and 73.5% using the Clark filter. It is important to highlight that in this case additive uncertainty is also the relevant source to explain total uncertainty of inflation, regardless of the method considered for the estimation of the output gap. These results strengthen the conclusions mentioned before with respect to the importance of this last type of uncertainty for the Chile's economy.

Table 10: Robustness Analysis for the Decomposition of the Conditional Variance of the Forecast Error of the Output Gap

	Conditional Variance of the Forecast Error			Percentage	
	TVP	MARKOV	TOTAL	TVP	MARKOV
Hodrick-Prescott	0.00279	0.01842	0.02121	13.2%	86.8%
Baxter-King	0.00314	0.01734	0.02048	15.3%	84.7%
Christiano-Fitzgerald	0.00304	0.01733	0.02037	14.9%	85.1%
Quadratic-Trend	0.00287	0.01901	0.02189	13.1%	86.9%
Clark	0.00200	0.01803	0.02003	10.0%	90.0%

Table 11: Robustness Analysis for the Decomposition of the Conditional Variance of the Forecast Error of the Inflation Rate

	Conditional Variance of the Forecast Error			Percentage	
	TVP	MARKOV	TOTAL	TVP	MARKOV
Hodrick-Prescott	0.00545	0.01019	0.01563	34.8%	65.2%
Baxter-King	0.00385	0.00988	0.01374	28.0%	72.0%
Christiano-Fitzgerald	0.00393	0.01006	0.01398	28.1%	71.9%
Quadratic-Trend	0.00761	0.01514	0.02274	33.4%	66.6%
Clark	0.00504	0.01397	0.01901	26.5%	73.5%

5. Final Remarks

Macroeconomic policy in Chile is currently of world class quality. The Central Bank of Chile has been operating within a full-fledged inflation targeting framework since 1999-2000 while fiscal policy has been bounded by an explicit budget rule that takes away pro-cyclical influences since 2001. As a result, inflation has stayed within the inflation target range most of the time and economic activity has grown steadily between 2 and 6% annually (with no recessions nor booms whatsoever). This rather stable period also shows in our findings here, in the sense that overall uncertainty concerning monetary policy has declined in the first seven years of the current decade. It has also implied a greater role for uncertainty attributed to shocks (and less to uncertainty linked to unstable parameters) in both the case of inflation and the output gap, as it could be expected. However, the prominence of additive uncertainty is a hallmark for the entire period, including both the tranquil first decade of the 21st century and the more volatile 90s. This means that the Central Bank of Chile should concentrate much of its future research effort in investigating the (stochastic) nature of shocks affecting the Chilean economy rather than perfecting further its models in search of more stable parameters.

The full-fledged inflation targeting scheme applied since 1999 came with a floating exchange rate and no explicit or implicit target for the exchange rate (as it was loosely the case during most of the 90s). But this important policy innovation did not change uncertainty surrounding the real exchange rate, which suggests (although not necessarily implies) that the floating regime has not brought more real exchange rate volatility.

Finally, results reported on uncertainty about the quality and completeness of output gap data reveal that, among other things, using the Hodrick-Prescott filter based on real time data could be misleading. So, the Central Bank of Chile should amplify its spectrum of filters for detrending real activity data and, more importantly, widen the menu of proxy variables to check for the economy’s temperature when making its monetary policy decisions.

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Appendix A: Estimation based on the Kalman filter and the EM algorithm (Kim and Nelson, 1999)

1. Kalman Filter

$\beta_{t|t-1}^{(i,j)}, P_{t|t-1}^{(i,j)}, \tau_{t|t-1}^{(i,j)}, f_{t|t-1}^{(i,j)}, H_{t|t-1}^{(i,j)}$

2. Hamilton's EM Algorithm

$$\Pr(S_t, S_{t-1} \mid \psi_{t-1}) = \Pr(S_t, S_{t-1}) \Pr(S_{t-1} \mid \psi_{t-1})$$

$$f(y_t \mid \psi_{t-1}) = \sum_{S_t} \sum_{S_{t-1}} f(y \mid S_t, S_{t-1}, \psi_{t-1}) \Pr(S_t, S_{t-1} \mid \psi_{t-1})$$

$$l(\theta) = l(\theta) + \ln(f(y_t \mid \psi_{t-1}))$$

$$\Pr(S_t, S_{t-1} \mid \psi_{t-1}) = \frac{f(y, S_t, S_{t-1}, \psi_{t-1})}{f(y_t \mid \psi_{t-1})} = \frac{f(y \mid S_t, S_{t-1}, \psi_{t-1}) \Pr(S_t, S_{t-1} \mid \psi_{t-1})}{f(y_t \mid \psi_{t-1})}$$

$$\Pr(S_t \mid \psi_t) = \sum_{S_{t-1}} \Pr(S_t, S_{t-1} \mid \psi_t)$$

3. Approximations

$$\beta_{t|t}^{j} = \frac{\sum_{i=1}^{2} \Pr(S_{t-1} = i, S_t = j \mid \psi_t) \beta_{t|t}^{(i,j)}}{\Pr(S_t = j \mid \psi_t)}$$

$$P_{t|t}^{j} = \frac{\sum_{i=1}^{2} \Pr(S_{t-1} = i, S_t = j \mid \psi_t) \left\{ P_{t|t}^{(i,j)} + (\beta_{t|t}^{j} - \beta_{t|t}^{(i,j)})(\beta_{t|t}^{j} - \beta_{t|t}^{(i,j)})' \right\}}{\Pr(S_t = j \mid \psi_t)}$$

4. Loglikelihood function

$$l(\theta) = \sum_{t=1}^{T} \ln(f(y_t \mid \psi_{t-1}))$$

Appendix B: Kalman filter with endogenous regressors (Kim, 2006)

$$\beta_{t|t-1} = E(\beta_t \mid v_t, \xi_t^*, \psi_{t-1}) = \beta_{t-1|t-1}$$

$$P_{t|t-1} = Var(\beta_t \mid v_t, \xi_t^*, \psi_{t-1}) = P_{t-1|t-1} + Q_\eta$$

$$\tau_{t|t-1} = x_t - E(x_t \mid v_t, \xi_t^*, \psi_{t-1}) = x_t - v_t'\beta_{t-1|t-1} - \xi_t^{*}'\rho\sigma_\varepsilon$$

$$H_{t|t-1} = Var(x_t \mid v_t, \xi_t^*, \psi_{t-1}) = v_t' P_{t|t-1} v_t + (1-\rho'\rho)\sigma_\varepsilon^2$$

$$\beta_{t|t} = E(\beta_t \mid v_t, \xi_t^*, \psi_{t-1}) = \beta_{t|t-1} + P_{t|t-1} v_t H_{t|t-1}^{-1} \tau_{t|t-1}$$

$$P_{t|t} = Var(\beta_t \mid v_t, \xi_t^*, \psi_{t-1}) = P_{t|t-1} - P_{t|t-1} v_t H_{t|t-1}^{-1} v_t' P_{t|t-1}$$

Appendix C: Loglikelihood function (Kim and Nelson, 1999)

$$\begin{aligned} f(x_t \mid \psi_{t-1}) &= \sum_{i=1}^{2}\sum_{j=1}^{2} f(x_t, S_t = i, S_{t-1} = j \mid \psi_{t-1}) \\ &= \sum_{i=1}^{2}\sum_{j=1}^{2} f(x_t \mid S_t = i, S_{t-1} = j \mid \psi_{t-1}) \Pr[S_t = i, S_{t-1} = j \mid \psi_{t-1}] \end{aligned}$$

where:

$$f(x_t \mid S_t = i, S_{t-1} = j, \psi_{t-1}) = (2\pi)^{-\frac{N}{2}} \mid f_{t|t-1}^{(i,j)} \mid^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\tau_{t|t-1}^{(i,j)}{}' f_{t|t-1}^{(i,j)-1} \tau_{t|t-1}^{(i,j)}\right\}$$