Monetary Policy and Key Unobservables in the G-3 and Selected Inflation-Targeting Countries [1]

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Abstract

Among the variables that play critical roles in the design of monetary policy, several are unobservable. These include such key variables as the neutral real rate of interest, the output gap, and the natural rate of unemployment. While individual central banks have undertaken efforts to estimate these unobservables, the approaches have generally been country specific and have not provided either systematic estimation or comparison across countries. We adopt a common estimation approach, applied to a parsimonious monetary-policy model, to provide consistent estimates of key unobservables for the U.S., the Eurozone, and Japan, and several inflation-targeting countries: Australia, Canada, Chile, New Zealand, Norway, Sweden, and the U.K. Doing so allows us to obtain comparable measures of unobservables across a range of countries. We exploit our estimates to investigate issues of commonalities and convergence across countries in these key but unobservable variables.

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1. Introduction

"In informal terms, we are uncertain
about where the economy has been,
where it is now and where it is going"
Donald Kohn
quoted in The Economist, Oct. 20, 2007

In recent years, the design of monetary policy has focused on gaps – the output gap, the interest rate gap, and the unemployment rate gap have all played a role in policy discussions. Standard models used for policy analysis are either specified in terms of such gaps or they imply important roles for these gap variables in the implementation of monetary policy. In the case of each, the gap is defined as the difference (often in percentage terms) between an observable variable, such as output or unemployment, and an unobserved variable, such as potential output or the natural rate of unemployment.

The presence of unobservables in the definitions of these gaps poses significant problems for central banks as they implement monetary policy. These problems are both conceptual in nature (what is the right definition of the output gap, potential output, or the neutral real interest rate?) and practical (which of many empirical strategies for estimating unobservables should be used?). These problems are compounded by the fact that real-time data used to estimate unobservables will be revised in the future, implying that the best estimates available at the time policy decisions must be taken may, in hindsight, diverge significantly from estimates based on subsequent vintages of data.

To estimate these key unobservables, economists have drawn on a variety of methodologies. Univariate approaches based on statistical methods designed to decompose a time series into trend and cycle have been widely used to estimate variables such as potential output or the natural rate of unemployment. In multivariate approaches, the joint behavior of several variables whose trend or cyclical elements may be related are employed. Multivariate strategies offer the possibility of bringing economic structure to bear on the estimation problem by incorporating restrictions implied by an economic model. For example, Okun's Law suggests a relationship between the output gap and the gap between unemployment and the natural rate of unemployment. Thus, the joint behavior of output and unemployment may provide information that is useful in estimating both these gaps. However, the results obtained by previous researchers studying different time periods or different economies are difficult to compare across countries since estimation methodologies often differ significantly. This hinders the ability to assess how business cycles might be linked across countries, how potential output or the neutral real interest rate in different countries might be related, and how closely related the various gaps might be across a sample of countries.

While the literature on international business cycles had employed common methods to estimate output gaps (Backus, Kehoe, and Kydland 1992), this work typically employed univariate statistical techniques (i.e., the H-P filter) to extract the cyclical component of output. A univariate approach ignores the information that is potentially available if one considers the joint behavior of several macro variables that are affected by the same set of unobservable variables. Variable definitions, sample periods, and the set of unobservables examined also vary across applications to individual countries. And while individual central banks have undertaken efforts to estimate these unobservable variables, their approaches have generally been country specific and have not provided either systematic estimation or comparison across countries.

Recently, Garnier and Wilhelmsen (2005) and Benati and Vitale (2007) have adopted a joint estimation approach to uncover important unobservables for several countries. Garnier and Wihelmsen focus on the U.S., the Euro area, and Germany, while Benati and Vitale study the U.S., the U.K., the Euro area, Sweden, and Australia. However, this approach has not been extended to include a larger number of inflation targeting economies nor has it included any emerging or developing economies. Yet many developing economies have adopted inflation targeting, and so unobservables such as the output gap and the neutral real interest rate play a particularly important role in their conduct of monetary policy.

Our objective is to provide a consistent approach to estimating potential output, the neutral rate of interest, and the natural rate of unemployment using data from several countries. This will then allow us to compare macroeconomic developments among these countries. The next section provides a brief discussion of the role of unobservables in the design of monetary policy. This discussion serves in part to motivate the variables – potential output, the neutral real interest rate and the natural rate of unemployment – on which our empirical analysis focuses. Section 3 then briefly sets out our empirical strategy. Section 4 discusses the model, the estimation approach, the data, and the basic results. Second 5 focuses on the cases of the U.S. and Chile and provides some robustness checks on our basic results. Section 6 then uses our estimated series on the key unobservables to provide evidence on the Great Moderation, the co-movements of the unobservables across the economies in our sample, and convergence of variables such as the neutral real interest rate. Section 7 concludes and discusses extensions.

2. The role and importance of unobservables in monetary policy

In this section, we discuss the role that key unobservables play in policy design. We then briefly review the way errors in estimating potential GDP and the natural rate of unemployment have contributed to critical policy mistakes.

2.1 Unobservable variables and policy design

The theoretical foundations both for monetary policy analysis and for the empirical models employed by central banks contain several important variables that are not directly observable. The output gap, where the output gap is the (log) difference between real GDP and an unobserved time-varying benchmark such as potential GDP, and the unemployment rate gap, the difference between the actual unemployment rate and the natural rate of unemployment, are typically the driving forces explaining inflation. Central banks may also need to monitor theses unobservables out of a direct concern for macroeconomic stability. Both potential GDP and the natural rate of unemployment must be inferred from observable macro variables. Policy makers must monitor difficult to measure expectations of inflation to ensure that private sector expectations are consistent with the central bank's inflation targets (i.e., the need to ensure expectations are anchored) and because movements in inflation expectations can contribute to fluctuations in actual inflation. And they need to adjust policy interest rates to reflect changes in the economy's neutral real rate of interest.

The critical role of these unobservable variables in designing monetary policy can be illustrated using a simple new Keynesian model. This benchmark model consists of a forward-looking Phillips Curve, an expectational IS relationship, and a specification of policy either in terms of an objective function (which the central bank is then assumed to maximize) or a decision rule (see Clarida, Gali, and Gertler 1999).

If the central bank's objective is to minimize volatility of inflation and the gap between output and potential output, optimal policy (under discretion) can be described in terms of what Svensson and Woodford (2005) have called a targeting rule. Such a rule involves ensuring that a weighted sum of the output gap and the inflation gap (inflation minus the inflation target) is always kept equal to zero. Intuitively, the output gap should be negative when inflation is above target as this will tend to produce a fall in inflation, acting to bring inflation back to its target level. And the output gap should be positive when inflation is below target. Just such a targeting relationship between the output gap and inflation is described by the Bank of Norway in its inflation report in discussing the desirable properties of future interest rate paths. The discussions of interest rate projections by the Reserve Bank of New Zealand in its monetary policy statements are consistent with a similar though implicit targeting rule. In following such a rule, the central bank knows its inflation target, and it has direct measures of both inflation and output (though there may be serious real-time measurement errors in the later, it is directly observable in principle), but it must estimate the level of potential output.

Potential output is not the only unobserved variable the central bank must estimate as it implements policy. To actually implement an optimal targeting rule, the central bank must still determine how to move its policy interest rate in order to maintain the required relationship between the output and inflation gaps. To determine the nominal interest rate that will implement the optimal policy requires knowledge of the relationship between interest rates and real spending, a relationship commonly summarized in new Keynesian models by an expectational IS curve. Using a standard specification of the IS relationship, one finds that the optimal interest rate will satisfy the following relationship (see Clarida, Gali, and Gertler 1999):

$$i_t = r_t^* + \left[1 + \frac{\sigma\kappa(1-\rho)}{\rho\lambda}\right] E_t\pi_{t+1} \tag{1}$$

where i is the nominal rate of interest, π is the inflation rate, and r^* is the neutral real interest rate, the rate consistent with a zero output gap.[4] The parameters σ, κ, λ, and ρ are, respectively, the inverse of the interest elasticity of aggregate demand, the output gap elasticity of inflation, the relative weight the policy maker places on output gap volatility relative to inflation volatility, and the degree of serial correlation in shocks to the inflation equation. Both the variables on the right side of equation (1) are unobservable or measurable only indirectly, for example via surveys or from asset prices or the term structure of interest rates.[5]

To solve for the equilibrium under the interest rate rule given by (1), the IS and Phillips curve relationships must also be specified. The ones underlying the derivation of (1) take the form

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - E_t\pi_{t+1} - r^*{}_t\right) \tag{2}$$

and

$$\pi_t = \beta E_t\pi_{t+1} + \kappa x_t + e_t. \tag{3}$$

It is clear from (1) that the neutral real interest rate will be of critical importance for getting the level of the policy rate right. Under an interest rate operating procedure for monetary policy, the *level* of the nominal rate when the inflation rate is equal to its target must be consistent with the

[4] There are numerous ways to write this relationship and to define the various unobservables. For example, it would be more in keeping with standard new Keynesian models to define r^* as the real interest rate consistent with output and the flexible-price equilibrium level of output being equal.

[5] If the inflation adjustment relationship incorporates lagged inflation, the targeting rule would also include further terms involving forecasts of future inflation rates and output gaps.

economy's equilibrium real rate of return. When inflation is equal to its (constant) target level, the Fisher relationship requires that the nominal interest rate equal the neutral rate plus the target inflation rate. Thus, while most of the recent literature has emphasized the importance of the Taylor Principle – the need to adjust the nominal rate more than one for one with changes in inflation – equally important is the need to fully adjust the nominal rate in response to changes in the neutral real interest rate. Woodford (2003) has labeled the equilibrium real interest associated with the absence of fluctuations due to nominal distortions as the Wicksellian real rate. An optimal monetary policy that maintains zero inflation to "undo" the real distortions created by nominal rigidities would ensure that the gap between the nominal interest rate and the Wicksellian rate remains equal to zero.

Unfortunately, this Wicksellian or neutral real rate is unobservable. It is, however, closely related to another key unobservable – the output gap. In the context of the simple model used to derive (1), the neutral real rate of interest is proportional to the growth rate of potential real output. Laubach and Williams (2003) use this relationship between these two unobservable variables to help them estimate the neutral real interest rate for the U.S.

Equations (2) and (3) serve also to highlight the key role of unobservable variables. The output gap appears in both, as does expected future inflation, while the neutral real interest rate appears in the IS relationship. For a central bank to actually use this simple framework for policy analysis requires that methods be developed for estimating potential output (to obtain an output gap measure), expected inflation, and the neutral real interest rate.

The difficulties in measuring the output gap go, in some sense, beyond the need to measure potential output, because the very definition of the output gap has evolved over the past twenty years. At the conceptual level, three distinct definitions have been employed. The first and most common definition of the output gap is in terms of the relationship between actual real GDP and potential GDP, where potential GDP is typically associated with the level of GDP that would be produced at full-employment of labor and capital at normal rates of utilization. This is the definition of the output gap that is most commonly used in models employed by central banks.

In recent years, the development of the new Keynesian Phillips curve has focused attention on a second definition of the output gap, a definition that the underlying theory identifies as the key variable driving inflation. This is the output gap measured as the gap between actual real GDP and the level of GDP that would be produced in the absence of nominal wage and price rigidities. This flexible-price output gap provides a measure of economic fluctuations that are due to nominal rigidities. It is these nominal rigidities that allow monetary policy to have real effects, but they also create real distortions. Standard new Keynesian models imply that monetary policy should aim at eliminating these distortions by minimizing fluctuations in the output gap.

However, stabilizing the flexible-price output gap is difficult, not least because the economy's equilibrium output that would arise if there were no nominal rigidities is clearly not observable, and it cannot be estimated using the (often) univariate statistical approaches employed to estimate potential output. Instead, any estimate must come from employing a dynamic stochastic general equilibrium (DSGE) model that can simulate the behavior of an economy that is not subject to nominal rigidities. Since the correct model of the economy is unknown, any estimate of the output gap will be subject to a great deal of uncertainty. Levin, Onatski, Williams, and Williams (2006) provide one example of a DSGE model that is estimated based on U.S. data and then used to construct a measure of the flexible-price output level and the associated flex-price output gap. To date, no central banks have employed such a definition of the output gap in their formal policy models.[6] Yet there is significant ongoing work at many central banks on developing DSGE models and their application to estimate flexible-price output levels, as well as other unobservables.

Finally, a third definition of the output gap is the gap between output and the welfare maximizing level of output. The gap defined in this manner is sometimes called the welfare gap. While it is this gap that, from a conceptual point of view, may be the most relevant for policy, it is also the hardest to measure. Standard new Keynesian models have the characteristic that the welfare gap and the flex-price output gap move together so that stabilizing one is equivalent to stabilizing the other, a property that Blanchard and Galí (2007) have labeled "the divine coincidence." In general, however, this relationship between the two gap measures holds only under very special conditions. If real wages are sticky or there are other labor market frictions or fluctuations in distortionary taxes, the flex-price output gap and the welfare gap will diverge.

Besides illustrating the general point that hard-to-measure variables are conceptually relevant for policy, equations (1) – (3) also highlight the variables that are among those that serve as the

[6] A possible exception are models that have developed from the Bank of Canada's Quarterly Projections Model (QPM), such as the Forecasting and Policy System model of the Reserve Bank of New Zealand. This model distinguishes between a long-run component, a short-run equilibrium component, and a cyclical component to output. The output gap is then defined relative to the short-run equilibrium level, and so might correspond to a flex-price output gap. However, the short-run equilibrium level of output is an estimate of a slow-moving trend, based on a multivariate filter. Variables (in addition to output) included in the trend estimation procedure include capacity utilization, unemployment, and inflation. QPM was replaced recently at the Bank of Canada by a new, open economy DSGE model, The Terms-of-Trade Economic Model (ToTEM); see Murchison and Rennison (2006).

primary focus of our study. These are the neutral real rate of interest, potential output, and expected inflation. For our purposes, we define the output gap as the log of real GDP minus the log of potential GDP, which is the common definition among central banks. While not appearing explicitly in (1), the natural rate of unemployment, which is linked to potential output, is also an unobservable variable that we incorporate into our analysis.

2.2 Unobservable variables and policy mistakes

Unobservable variables play a critical role in the design and implementation of optimal monetary policy, but these same variables have also been center stage for a numbers of accounts of past policy errors (see Sargent 2007 for an overview and discussion). For example, Orphanides (2002, 2002), Erceg and Levin (2003), Reis (2003), and Primiceri (2006) all argue that errors by either policy makers or the public in estimating key macro variables were central to an understanding of critical episodes in the inflation history of the U.S. over the past 40 years.

Orphanides has focused on the Federal Reserve's real-time overestimation of potential (trend) output following the productivity slowdown of the early 1970s. Simply put, overestimation of potential GDP implied an underestimation of the output gap. This in turn led to a policy stance that was, in retrospect, too expansionary and contributed to producing the Great Inflation of the 1970s. Orphanides and Van Norden (2002) have documented the difficulties of estimating the output gap when, for policy purposes, this must be done using real-time data.[7] McCallum (2001) has drawn the conclusion that policy makers should not respond strongly to movements in the estimated output gap.[8]

Primiceri (2006) has argued that the Fed's failure to estimate correctly potential output is only part of the story behind the Great Inflation.[9] If that were the only mistake, he argues that inflation would not have risen so much nor for so long. The second factor contributing to the persistence of high inflation was an underestimation by the Fed of the persistence of inflation. Initial increases in inflation were not expected to persist and so policy did not react strongly. Because potential output was overestimated, economic slowdowns that were thought to be

[7] The Reserve Bank of New Zealand (2004) provides a figure (figure 9, page 15) comparing their real-time quarterly output gap estimates and estimates prepared using final data (as of Nov. 2002) for the period 1997-2002. There are sizable differences between the two; for instance, the final series changes sign four times during the period shown, while the real time series changes sign three times *and never in the same quarter* as the final estimate series.

[8] Orphanides and Williams (2002) find that policy rules that respond to the change in the unemployment rate gap or the output gap perform well. One reason might be that differencing eliminates much of the error in measuring the level of the output gap.

associated with negative output gaps did not seem to lower inflation. Thus, policy makers concluded that inflation was unresponsive to economic activity so that a major recession would be needed to lower inflation. Thus, perceiving they faced a large sacrifice ratio if they tried to lower inflation, policy makers hesitated to try to bring inflation down. Primiceri develops a simple general equilibrium model in which the policy maker learns about the natural rate and the degree of inflation persistence and his model accounts for both the policy mistakes of the 1970s, as the Fed underestimated the natural rate of unemployment and overestimated the sacrifice ratio associated with lowering inflation, and then the disinflationary shift in policy under Volcker. Thus, both the difficulties in estimating unobservable variables and the fact that central banks do not know the true structure of the economy can contribute to policy errors.

It is important to note that the public also faces the need to estimate unobservable variables. Erceg and Levin (2003) focus on shifts in the Fed's implicit inflation target when these shifts are not publicly announced. In this case, the public becomes aware of the shift in target only gradually. Erceg and Levin characterize the Volcker disinflation as the result of a fall in the Fed's target inflation rate. Since this target change was not made explicit through any public announcement, agents overestimated inflation, leading to a significant contraction in real economic activity. While our focus is on estimating unobservable variables for use in designing monetary policy, the work of Erceg and Levin provides a reminder of the consequences that can occur when the central bank's inflation target is, from the perspective of the public, an unobservable.

3. Alternative approaches to estimating the neutral real rate, the output gap, and the natural rate of unemployment

There is a vast literature that has utilized a range of empirical techniques to estimate unobservable macro variables. Consequently, our survey will be brief and highly selective, focusing on those contributions of most direct relevance for our own empirical approach. For example, while a tremendous amount of work has employed univariate methods to estimate potential output or the natural rate of unemployment, we will not focus on these approaches. Instead, as an alternative to a univarate approach, we follow multivariate approaches that incorporate information from other macro variables, usually employing theory to guide the relationship between the variables or employing structural equations motivated by theory. We focus on multivariate approaches that are most directly relevant for the methods we use to obtain estimates of key unobservable variables. These approaches generally combine statistical

[9] Primiceri's model is actually expressed in terms of the natural rate of unemployment rather than potential output.

representations borrowed from the literature on identifying trend and cyclical components of a time series with relationships among variables implied by an economic model.

The general methodology we employ involves employing a multivariate Kalman filter to extract estimates of unobserved components from observed time series. The basic framework can be represented in quite general terms of a specification for the dynamic evolution of (i) a vector Z_t of unobserved factors and (ii) a vector of observed variables Y_t that are related to Z_t. The evolution of the unobserved variables is given in state-space form by

$$Z_{t+1} = AZ_t + u_{t+1}. \tag{4}$$

The measurement equations linking Y_t to Z_t take the form

$$Y_t = BY_{t-1} + CZ_t + DZ_{t/t} + GX_t + v_t, \tag{5}$$

where $Z_{t/t}$ is the time *t* estimate of the state vector Z_t and X_t is a vector of exogenous and observable variables. Both u_{t+1} and v_t are mean zero stochastic error terms. In section 4 we set out the specific formulations of equations (4) and (5) that we use in our empirical analysis.

Time *t* estimates of Z_t are updated using the Kalman filter. Since

$$Y_t - BY_{t-1} - (C+D)Z_{t/t-1} - GX_t$$

is the new information available from observing Y_t in period t, the equation for updating estimates of Z is given by

$$Z_{t/t} = Z_{t/t-1} + K\left[Y_t - BY_{t-1} - (C+D)Z_{t/t-1} - GX_t\right]. \tag{6}$$

The basic structure given by equations (4) - (6) has been used extensively to estimate a range of unobservable variables. Data on the observables Y_t and X_t are used to estimate the parameter matrices *A*, *B*, *C*, *D*, and *G*.

An early application of the Kalman filter approach to estimating potential GDP for the U.S. is provided by Kuttner (1994).[10] Kuttner lets Z_t consists of trend and cyclical components of output, with the trend following a random walk with drift and the cyclical component described by an AR(2) process. The vector Y_t consists of actual real output and inflation and reflects a Phillips curve relationship. Output is the sum of its trend and cyclical components and inflation is a function of lagged output growth and the cyclical component of output.

More recently, a related approach to estimating potential GDP and the output in the U.S. has been taken by Basistha and Nelson (2007). Like Kuttner, they adopt a latent variable approach and incorporate a Phillips Curve relationship. In addition, they also include the unemployment rate and allow trend and cyclical components of output to be correlated.

Laubach and Williams (2003) extend the Kuttner framework to incorporate the neutral real interest rate r^* as an additional unobserved variable. They assume r^* is a function of the growth rate of potential GDP and a stochastic component that follows an autoregressive process. They expand the set of measurement equations to include an IS relationship linking the output gap to the gap between the real interest rate and the neutral rate of interest.[11] While this specification allows for an integrated approach to estimating potential GDP and the neutral real interest rate, Laubach and Williams employ a separate univariate inflation forecasting equation to obtain the estimate of expected inflation they need to construct the real interest rate.

Fuentes, Gredig, and Larrain (2007) further extend the approach of Laubach and Williams by incorporating the unemployment rate and Okun's Law linking the output gap and the gap between the unemployment rate and the natural rate of unemployment. The latter is assumed to follow a random walk. They compare the resulting measures of the output gap for Chile with gap estimates obtained from structural VARs and production function approaches. Interestingly, the Kalman filter based estimates provided the best out-of-sample forecasts for inflation.

Each of these examples from the literature focused on a single country; the U.S. in the case of Kuttner (1994), Basistha and Nelson (2007), and Laubach and Williams (2003); and Chile in the case of Fuentes, Gredig, and Larraín (2007). Closest in formulation to our approach is a recent paper by Benati and Vitale (2007). They too focus on multiple unobservables – potential output, the natural rate of unemployment, the neutral real interest rate, and expected inflation, and they obtain estimates of each unobservable for five economies: the Euro area, the U.S., the U.K, Sweden, and Australia. Benati and Vitale allow for time-variation in the model parameters. We will restrict our attention to constant coefficient models.

Bjorksten and Karagedikli (2003) report estimates of the neutral real interest rate for seven countries (Australia, Canada, New Zealand, Sweden, Switzerland, the U.S., and the U.K.) using a methodology based on long- and short-term interest rates. However, to extract real interest

[10] Orphanides and Williams (2002) provide an overview of the literature that has attemped to estimate natural rates of unemployment and the neutral real interest rates for the U.S.
[11] They also allow the growth rate of potential GDP to follow a random walk.

rates, they assume expected inflation is equal to actual inflation. They find a marked decline since 1998 in neutral real rates for all seven countries.[12]

4. Empirical results

4.1 Our approach

Our approach, following the preceding literature, is based on a parsimonious new Keynesian specification. We use the core relationships in the new Keynesian model to guide our specification of the linkages between observable variables and the key unobservables as summarized in equation (5). The two relationships from the new Keynesian model that we draw upon are the IS equation and the Phillips curve. In addition, we make use of a Taylor rule to represent monetary policy and Okun's Law linking the unemployment gap and the output gap.

4.2 Model

We start with a simple backward-looking IS relationship, as in Rudebusch and Svensson (1999), where the output gap (x) is determined by its own lag, the lagged real interest rate gap (the difference between the ex-ante real interest rate, r, and the unobserved neutral real interest rate, r^*), and a serially uncorrelated error term (ε_1):

$$x_t = \alpha_1 x_{t-1} + \alpha_2 (r_{t-1} - r_{t-1}^*) + \varepsilon_{1,t} \tag{7}$$

The output gap is defined as the difference between actual output (y) and unobserved potential output or the natural level of output (y^*):

$$x_t = y_t - y_t^* \tag{8}$$

The second relationship is a standard Phillips curve specification for inflation. We specify this equation in terms of the inflation gap rather than the level of inflation, where the inflation gap $\overline{\pi}_t$ is the difference between actual inflation and either trend inflation (in the case of non-inflation targeting countries) or between actual inflation and the target rate of inflation (for inflation targeters). The inflation gap is determined by its own lag, the expected inflation gap, the lagged output gap, and a serially uncorrelated error term (ε_2) :

$$\overline{\pi}_t = \beta_1 \overline{\pi}_{t-1} + \beta_2 \overline{\pi}_t^e + \beta_3 x_{t-1} + \varepsilon_{2,t} \tag{9}$$

The inflation gap is an observable variable, given by :

$$\overline{\pi}_t = \pi_t - \pi_t^T \tag{10}$$

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[12] See also Basdevant, Bjorksten, and Karagedikli (2004).

where π_t is actual inflation and π_t^T is the trend or target rate. Similarly, the inflation expectations gap is defined as the difference between observed (estimated) inflation expectations and trend or target inflation:

$$\overline{\pi}_t^e = \pi_t^e - \pi_t^T \tag{11}$$

We specify a standard Taylor rule that relates the observed ex-ante real interest rate (r) to the ex-ante real natural rate (r^*), the real interest rate lag, the inflation expectations gap, the lagged output gap, and a serially uncorrelated error term(ε_3):

$$r_t = r_t^* + \delta_1(r_{t-1} - r_{t-1}^*) + \delta_2\overline{\pi}_t^e + \delta_3 x_{t-1} + \varepsilon_{3,t} \tag{12}$$

Equations (7) - (12) comprise our basic model. As an extension of this model, we add Okun's Law that relates the observed unemployment rate (u) to the unobserved natural rate of unemployment (u^*), the lagged gap between the observed unemployment rate and the natural rate of unemployment, the output gap, and a serially uncorrelated error term (ε_4):

$$u_t = u_t^* + \gamma_1(u_{t-1} - u_{t-1}^*) + \gamma_2 x_{t-1} + \varepsilon_{4,t} \tag{13}$$

Now we turn to the transition equations of the model corresponding to equation (4) in the schematic formulation of section 3. As in Laubach and Williams ((2002), potential output is taken to follow an I(2) process and unobserved potential output growth (g) follows a random walk:

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_{5,t} \tag{14}$$

$$g_t = g_{t-1} + \varepsilon_{6,t} \tag{15}$$

where ε_5 and ε_6 are serially uncorrelated error terms.

We specify random-walk processes for both the neutral real rate of interest and the natural rate of unemployment:

$$r_t^* = r_{t-1}^* + \varepsilon_{7,t} \tag{16}$$

$$u_t^* = u_{t-1}^* + \varepsilon_{8,t} \tag{17}$$

where ε_7 and ε_8 are serially uncorrelated error terms.

4.3 Estimation method

We follow closely Laubach and Williams' (2002) procedure in estimating our model, adapting it to our specification. As they note, maximum-likelihood estimates of the standard deviations of the innovations to the transition equations of the unobservables, equations (14)-(17), are likely to be biased toward zero due to the pile-up problem discussed by Stock (1994). Hence we also use the Stock-Watson (1998) median unbiased estimator to obtain estimates of the signal-to-noise ratios reflected by the ratios of the corresponding residual variances $\lambda_g = \sigma_6 / \sigma_5$, $\lambda_r = (1-$

$\delta_1)\ \sigma_7 / \sigma_3$, and $\lambda_u = (1\text{-}\gamma_1)\sigma_8 / \sigma_4$. We impose these ratios when estimating the remaining model parameters by maximum likelihood.

We also follow Laubach and Williams (2002) closely in the subsequent sequential-step estimation procedure. In the first step (following Kuttner 1994) we apply the Kalman filter to estimate jointly the IS relationship (after substituting equation (8) into (7)) and the Phillips curve (after substituting equations (10) and (11) into (9)). In this stage we omit the real interest rate gap from the IS equation and assume that potential output growth (*g*) is constant. From the latter preliminary estimation we obtain a preliminary potential output level series from which we compute an estimate of the (preliminary) constant potential output growth. Then we estimate equation (14) to test for structural breaks in the level of g. Using Stock and Watson's (1998) Table 3, we determine a positive value for λ_g when the null of no-structural break is rejected.

In the second step we apply the Kalman filter to estimate jointly the IS relationship, the Phillips curve, the Taylor rule (equation (12)), and the transition equations for potential output level (equation (14)) and potential output growth (equation (15)). At this stage we impose a preliminary constant neutral interest rate (r^*) in the IS relation and the Taylor rule. We also impose the λ_g estimate obtained in the first step. From the latter preliminary estimation we obtain an estimate of the (preliminary) constant neutral rate interest rate. Then we estimate equation (12) to test for structural breaks in the level of r^*. Using Stock and Watson's (1998) Table 3, we determine a positive value for λ_r when the null of no-structural break is rejected.

In step 3 we estimate jointly the IS relationship, the Phillips curve, the Taylor rule, and Okun's Law (equation (13)), in addition to transition equations (14), (15), and (16). We impose a preliminary constant natural unemployment rate in Okun's Law. We also impose the λ_g and λ_r estimates obtained in the first and second first steps. From the latter preliminary estimation we obtain an estimate of the (preliminary) constant neutral unemployment rate. Then we estimate equation (13) to test for structural breaks in the level of u^*. Using Stock and Watson's (1998) Table 3, we determine a positive value for λ_u when the null of no-structural break is rejected.

Final step 4 comprises Kalman-filter estimation of the full model, imposing the estimates for λ_g, λ_r, and λ_u obtained sequentially in the preceding steps. This yields the final estimates for our model coefficients and time series of unobservables. As in Laubach and Williams, we compute confidence intervals and standard errors for the parameters and unobservables applying Hamilton's (1986) Monte Carlo method.

4.4 Data

Our sample covers 10 countries: the G-3 group comprised by the U.S., the Eurozone, and Japan, all of them with central banks that do not target explicitly or exclusively inflation; a group of 6 industrial countries with inflation-targeting central banks, comprised by New Zealand, Canada, United Kingdom, Australia, Sweden, and Norway; and Chile, an emerging economy with an inflation-targeting central bank.[13]

Time coverage of each country sample is determined by the availability of quarterly data. While the standard sample covers the 1970-2006 period. One exception on the long side is the U.S. (1960-2007) and on the short side are New Zealand (1974-2006), Norway (1979-2006), and in particular Chile (1986-2006). Data sources and definitions are reported in the Data Appendix.

4.5 Estimation results

Here we report estimation results for our state-space model in its base version (without Okun's Law) for all countries. This implies omitting step 3 of the estimation method describe above. Thus, the model consists of equations (7)–(12) and (14)–(16). In section 5 below we report empirical results based on the extended model that includes equations (13) and (17) for the U.S. and Chile and the corresponding full 4-step estimation procedure.

Tables 1-3 report country estimates for the two key ratios of the standard deviations of the residuals (λ_g and λ_r), all structural model parameters, and standard deviations of equation residuals. We report results for the full sample available for each country (ending in 2006:4) and a shorter data sample (1986:2 – 2006:4) for 9 countries and only for the shorter sample in the case of Chile. Figures 1-3 depict the estimated time series of observables and unobservables for each country, consistent with the full-sample estimations.

Our estimation strategy is the following. When obtaining estimation results from the last (third) step, we report them directly. When not obtaining estimation results at either the second or third stages, we conduct a grid search of estimation results for an interval of values of standard deviation ratios (λ_g and λ_r), as reported on the footnotes of the tables. Therefore we report a varying number of results for each country. For example, for the U.S. (Table 1) we report only one set of results for each sample period, as we obtained estimates for all model parameters. In contrast, for Japan (Table 1) due to estimation problems we report a second set of results for each sample period, based on pre-determined median values for λ_g and λ_r, corresponding to an interval of values over which a grid search was conducted.

[13] An attempt was made to include Israel (with 1986-2006 data) but we were not able to attain

While the estimation results differ in significant ways across the 10 countries, we point out the following general findings (abstracting from country-specific exceptions).

(i) The potential growth rate and the neutral real interest rate are typically not constant – not even for the shorter 1986-2006 sample – as reflected by non-zero λ_g and λ_r and as depicted in figures 1-3. This has implications for construction of output gap measures as well as for the specification of Taylor rules.

(ii) Point values and significance levels of structural parameter estimates vary from country to country, and sometimes from sample to sample for a given country. For example, parameter estimates conform to our priors in the full-sample estimations for the U.S., Canada, and Chile. At the other extreme is Japan, where parameter estimates were hard to obtain and, when estimated over a grid search, often did not conform to expected signs or significance levels. Thus, significant differences emerge among the 10 countries.

(iii) The IS equation reflects generally very large output-gap inertia (reflected in the large and significant parameter estimate of its own lag). However the sensitivity of the output gap to the lagged real interest rate gap ranges from negative and significant to positive and significant.

(iv) The Phillips curve generally reflects small but significant inflation-gap reversion, suggesting partial reversal of quarterly inflation shocks. (The exception is Chile, which reflects positive inflation-gap persistence). Expected inflation shocks affect inflation gaps positively, significantly, and by a large magnitude in many countries. The lagged output gap raises inflation significantly, positively, and by a sizable magnitude in most countries.

(v) The Taylor rule reflects significant, large inertia in central-bank rate real-interest rate innovations in all countries, less Japan. Most central banks raise nominal interest rates in response to a lagged inflation shocks but not enough to satisfy the Taylor principle (i.e., because we have specified the Taylor rule in terms of real interest rates, the Taylor Principle requires that $\delta_2 \geq 0$) . The exception is Chile, where the coefficient estimate was found to be not significantly different from zero.[14] Finally, we obtained a wide range for the interest rate gap response to a lagged output gap shock: monetary policy ranges from counter-cyclical (U.S.) to a-cyclical (Sweden) and to pro-cyclical (Japan).

convergence of our estimation model.

[14] It is likely that Chile's exceptional experience reflects the peculiarity that the policy interest rate was set directly in real (i.e., inflation-indexed) terms during most of the sample period (1986-2000).

(vi) Judging by conformity of parameter point estimates and significance levels to priors, the best country results were obtained for the U.S. (1960-2007) and Chile (1986-2006).

Our estimates for unobservables reveal the following results:

(i) Estimated time series for potential output growth reveal smooth behavior, but *g* changes over time in most countries (except the Eurozone and Australia), consistent with positive country estimates for λ_g.

(ii) With relatively stable potential output growth, the variance of country output gaps is largely determined by the variance in actual output growth rates.

(iii) Similar to potential output growth, the neutral real rate of interest also follows a smooth pattern in all countries, coherent with positive country estimates for λ_r.

(iv) Generally we have obtained precise estimates for our three unobservables, as reflected by the narrow confidence intervals depicted in the figures.

(v) We obtain similar estimates for potential output growth and the neutral real rate of interest rates across the long and short samples for most countries. The exceptions are Australia and Norway, for which we obtain neutral interest rates well above actual levels in the shorter samples.

(vi) We also obtain similar estimates for output gaps across the long and short samples in many countries. However, strong departures from the latter are found in New Zealand, U.K., Australia, and Sweden, where the dynamic pattern, sign, and/or magnitudes of output gap estimates differ significantly in the 1986-2006 sample from those observed in the larger samples. This is likely to reflect small-sample bias. Hence we will conduct our tests of great moderation, co-movements, and convergence across countries on our large-sample estimates of unobservables.

Before using the results that we have obtained in this section to examine further the behavior of the key unobservables, we extend the basic model to incorporate Okun's Law for two of the countries in our sample: the U.S. and Chile.

5. Extensions for the U.S. and Chile

In this section we extend our basic model to include the unemployment gap (Okun's Law) and apply it to the U.S. and Chile, for which we obtained the best results for the basic model. We also test for robustness of the basic model results for the U.S. by replacing four-step-ahead inflation forecasts by eight-step-ahead forecasts.[15]

5.1 Results for the U.S.

For the extended model with Okun's Law for the U.S., we proceed in the following way. When estimating freely all parameter values and unobservables, λ_u was estimated in the fourth step at a value of zero, implying a constant 5.6% natural rate of unemployment for the U.S. during 1960-2007. Following the approach adopted for countries in section 4, we pursue next a grid search over alternative pre-set values of λ_u. The model parameter estimates consistent with $\lambda_u = 0$ and $\lambda_u = 0.4$ (the median value of our grid search) are reported in columns 1 and 2 of Table 4. Figure 4 depicts the grid-search results for the unobservables. The findings can be summarized as follows:

(i) Parameter estimates are generally similar for the extended model (in both columns 1 and 2 of Table 4) to those reported for the basic model (Column 1, Table1).

(ii) In the IS curve, the output gap becomes more sensitive to the lagged interest rate gap.

(iii) The coefficient of lagged inflation in the Phillips curve now turns positive, with a corresponding reduction in size of the two other Phillips curve coefficients.

(iv) For the newly introduced Okun's Law, parameter estimates exhibit expected signs and are highly significant. The parameter estimate for the lagged unemployment gap reflects large unemployment inertia. The coefficient estimate of the lagged output gap is very large (-0.95) when the natural unemployment rate is estimated as constant and declines to -0.35 when the natural unemployment rate is variable, consistent with a value of λ_u set at 0.4.

(v) Figure 4 depicts estimation ranges for unobservables for λ_u varying between 0.08 and 0.72. The estimates for both potential output growth and the natural interest rate are robust to changes in λ_u, reflected in their narrow ranges depicted in Figure 4. Moreover, the estimated values and dynamics of both potential growth and the natural interest rate for the extended model are very close to those depicted for the basic model (upper panel, Figure 1a).

(vi) However, the range of estimates for the output gap for different values of λ_u is larger. In addition, the median value for the new output gap estimate is less close to the estimate for the basic model. This should not come as a surprise, as the extended

[15] We did not obtain model convergence when using eight-step-ahead inflation forecasts for Chile. We also conducted sensitivity analyses for the Phillips curves in both countries, by replacing 1-period inflation lags by four-quarter lags, obtaining virtually unchanged results.

model imposes a close relation between the output gap and the unemployment gap. Okun's Law implies that the latter gaps are almost a mirror image of each other.

(vii) The largest range of estimates depicted in Figure 4 is the one for the newly estimated natural rate of unemployment. For the median value of λ_u, the natural rate varies over time between 5.1% and 7.2%. Over the full range of λ_u values, the natural rate varies over time between 4.8% and 8.1%. This is consistent with recent findings of King and Morley (2007), who estimate the natural rate as the steady-state of a VAR and attribute most of the volatility in observed unemployment to movements in the natural rate.

Now we turn back to the parsimonious model, replacing the 4-step-ahead inflation forecast for the U.S. by an eight-step-ahead forecast. This change affects the measurement of inflation expectations in the three structural model equations. We obtain the following results for parameter estimates (column 3, Table 4):

(i) The IS curve parameter estimates are not much modified (cf. column 1, Table 1). The parameter estimate for the inflation expectations gap in the Phillips curve declines almost by half in size but remains very significant. The parameter estimate for the inflation-forecast gap in the Taylor rule stays significant but is now more negative (from -0.13 to -0.22), implying a corresponding decline in the nominal interest setting reaction to an inflation expectations shock, from +0.87 to +0.78. Both results – for the Phillips curve and the Taylor rule – may suggest that 4-quarter-ahead inflation expectations describe both the inflation process and interest rate setting behavior during 1960-2007 in the U.S. better than 8-quarter-ahead inflation expectations.

(ii) Finally, with regard to the unobservables, the output gap, the neutral interest rate, and potential output growth exhibit similar patterns and values than those based on four-step-ahead inflation forecasts.

5.2 Results for Chile

For the extended model with Okun's Law for Chile, we proceed in a similar way as we did for the U.S. However, the difference is that when estimating freely all parameter values and unobservables, the estimates for λ_g, λ_r, and λ_u are estimated at zero at the fourth stage estimation. Therefore we conduct separate grid searches over alternative pre-set values of the three signal-to-noise coefficients. The model parameter estimates consistent with $\lambda_g = \lambda_r = \lambda_u = 0$, and with $\lambda_g = 0.082$, $\lambda_r = 0.080$, and $\lambda_u = 0.4$ (the median values of our grid searches) are reported in columns 1 and 2, respectively, of Table 5. Figure 5 depicts in each row the corresponding grid-search results for the unobservables. The findings are the following:

(i) Parameter estimates are generally very similar for the extended model (comparing columns 1 and 2 of Table 5) to those reported for the basic model (corresponding columns 1 and 2, Table 3).

(ii) The one important exception is the IS curve, where the output gap becomes more sensitive (and significant) to the lagged interest rate gap in the extended model with when the natural rates are allowed to vary over time (i.e., the λ_i's are set at positive values).

(iii) The coefficient of lagged inflation in the Phillips curve now turns positive, with a corresponding reduction in size of the two other Phillips curve coefficients.

(iv) For the newly introduced Okun's Law, parameter estimates exhibit expected signs and are highly significant. The parameter estimates for the lagged unemployment gap reflects moderate unemployment inertia. The coefficient estimate of the lagged output gap is large (around -0.6).

(v) The estimation ranges depicted in the three rows of Figure 5 are relatively narrow for all unobservable variables. Obviously the widest range in each row is for the unobservable over which the grid search is conducted.

(vi) The general dynamic pattern of three unobservables (potential output growth, output gap, and neutral interest rate) estimated for the extended model are similar to those obtained for the basic model. However there are differences in the estimated levels.

(vii) Similar to the results for the extended model applied to the U.S., the differences in output gap estimates are not surprising as the extended model imposes a close relation between the output gap and the unemployment gap. Again, Okun's Law implies that the latter gaps are almost a mirror image of each other.

(viii) However, in contrast to the U.S., the range for the new estimates of the natural rate of unemployment is not as large in Chile. For the median value of λ_u, the natural rate varies over time between 7.7% and 8.1%. Over the full range of λ_u values, the natural rate varies over time between 7.5% and 8.5%. This is consistent with recent findings by Restrepo (2006) based on different models of estimation for the NAIRU in Chile.

6. Great moderation, co-movements, and convergence in industrial countries

The Great Moderation – the attainment of low inflation and low volatility in key macroeconomic variables since the 1990s, in stark contrast to the high inflation and real instability of the mid 1970s and early 1980s – has been documented in academic research and policy evaluations.[16] At the same time, there is a presumption that rising world trade and financial integration should lead to stronger business cycle co-movement across countries, as well as stronger convergence in real variables, like growth and real interest rates, particularly among industrial countries. In this section we exploit our country time-series estimates of unobservables, in addition to the series of selected observables, to test for the great moderation, co-movements, and convergence in our sample of 9 industrial countries, using quarterly data for 1970-2006.[17]

6.1 Common trends in key unobservables

We start by describing the trends in potential output growth (Figure 6) and the neutral real interest rate (Figure 7) across the nine countries.

The most striking feature of the potential output growth estimates is the large reduction in cross-country variation observed between 1970 and 2006. Leaving out Japan, country point estimates of potential growth ranged from nil (New Zealand) to 4% (Canada) in the early 1970s. In contrast, the range of potential growth estimates for 2006 narrowed down to an interval defined by the Eurozone's constant potential growth rate (2.4%) and Australia's constant rate (3.2%). The most striking increase in potential growth is New Zealand's growth miracle, with potential growth rising from nil to 3.2%, in sharp contrast to Japan's meltdown in potential output growth rate from 4.5% to 1.8% during the last four decades. Sweden and the U.K. exhibit a slight trend increase in potential growth, with the opposite pattern observed in Canada, Norway and the U.S. Similar to the case of growth, the cross-country dispersion in neutral real interest rates has declined strongly during the last four decades (Figure 7). In the early 1970s, neutral rates ranged from -1.9% (U.K.) to 3.1% (Eurozone). By 2006, the range had narrowed to an interval from 1.5% (Japan) to 3.1% (Eurozone), with New Zealand being an exception. Six countries exhibit a U-shaped dynamic pattern of their neutral real rates of interest. This reflects the strong monetary adjustment in response to the "great inflation" of the late 1970s, with real policy rates peaking during the 1980s and early 1990s at levels of up to 6.5% (Australia in 1990). The stabilization success of the 1980s and 1990s that greatly contributed to the great moderation, led to the subsequent reduction in neutral rates observed in the 1990s and 2000s. The exception to the latter trend is New Zealand, where the neutral real interest rate has kept rising, attaining 4.8% in 2006.

[16] For example, the IMF's most recent W*orld Economic Outlook*, devotes a well-documented chapter to the great moderation.

[17] We use our shorter time series for New Zealand and Norway, and we drop Chile, due to the lack of quarterly data before 1986.

6.2 Great moderation

To investigate the great moderation, we report volatility trends for seven key variables: inflation, output growth, potential output growth, the output gap, the real interest rate, the natural real interest rate, and the interest rate gap. Three of these are observables (inflation, output growth, and the real interest rate) and four are unobservables (potential output growth, the output gap, the natural real interest rate, and the interest rate gap). We compute rolling standard deviations for the latter variables using a window of 74 quarters and report the associated confidence intervals obtained by bootstrap techniques.[18] [19]

This approach is informative about the great moderation, reflected in increased stability of key macro variables. We focus on both the level of the rolling standard deviation and the varying width of the confidence interval. The results are depicted separately for each variable in Figures 8.1-8.g. The nine smaller figures on each page show rolling point estimates of the standard deviation and their estimated time-varying confidence intervals for each country, while the larger bottom figure depicts the nine point estimates for each country and the corresponding country mean to better represent the common volatility trend across our sample countries. The findings are the following.

(i) The volatility of inflation has declined in all countries, except Norway; the mean volatility of inflation fell from 4.0% in 1970-1987 to 2.2% in 1988-2006 (Figure 8.a).[20] Moreover, this trend is also significant as reflected by the confidence intervals, which follow closely the point estimates of the standard deviations, narrowing around point estimates toward the end of the sample period. The exception is again Norway, where point estimates decline while confidence intervals rise after 1988. The largest reductions in inflation volatility are observed in Australia, Canada, and New Zealand, roughly from 6.0% to circa 2.2%. The Euro zone exhibits the lowest inflation volatility during most of the sample span.

[18] We use a window size of 74 quarters (or 18.5 years), which is half our 37-year sample coverage from 1970 to 2006. We choose this rather large window to show more clearly long-term volatility trends, avoiding excessive noise in standard deviations that shows up when using conventional 40-quarter (10-year) rolling windows.

[19] We apply a bootstrap technique for estimating time-varying confidence intervals because of its superior asymptotic properties in small samples, in comparison to standard confidence intervals. Hall's confidence intervals are calculated using the stationary bootstrap method of Politis and Romano (1994). This technique guarantees stationary artificial series by allowing a random block size (indeed, it follows a geometric distribution) when re-sampling the data. We set the mean of the block size at 3 and perform 2000 replications.

[20] It is well known that the correlation between the first and second moment of inflation is very large. Hence the declining trends in inflation volatility described here are matched by declining trends in inflation levels.

(ii) The reduction of the volatility of output growth in all nine countries is remarkable, reflected both by declining point estimates and narrowing confidence intervals. The country average level of output growth volatility fell roughly by half, from 5.0% in 1970-1987 to 2.7% in 1988-2006. The largest growth stabilization was recorded in New Zealand, from 14% in the 1970s and 1980s to 5% in the 1990s and 2000s. Australia, Sweden, and the U.K. also exhibit large reductions in growth volatility. Again the Eurozone exhibits the highest level of stability throughout the last 37 years.

(iii) Now we turn to our first unobservable, potential output growth.[21] As all estimated unobservables, potential growth is estimated either as a constant (in the Eurozone and Australia) or, if variable (in the other countries), it exhibits a smooth pattern over time, without high-frequency volatility. Therefore its volatility – like that of the neutral rate of interest, reported below – is lower by an order of magnitude to the volatilities exhibited by our observable variables. The country average volatility (for the seven countries where potential output varies over time) declines only marginally over time. Opposite trends are observed in different countries; New Zealand records a strong trend decline in potential growth volatility, while a growing trend is observed in Japan up to 2000, partially reverted thereafter.

(iv) There is a slight reduction in the country average volatility of the output gap (our second unobservable), from 1.6% in 1970-1987 to 1.4% in 1988-2006. There are moderate to large reductions in the volatility of the output gap in six countries, no clear trends in two countries, and a slight trend rise in one country (Australia). The U.K. exhibits the most stable output gap throughout the full 1970-2006 period.

(v) A general pattern of declining volatility is also found for the actual real interest rate: the country average volatility falls from 3.8% to 2.3%. The largest reductions in interest rate volatility are recorded in New Zealand and the U.K. Norway does not exhibit a trend reduction because its interest rate volatility is already low from the sample start. The exception is Sweden, influenced by its sharp rise in interest rate volatility in the third quarter of 1992, as a result of its short high-interest rate hike.

(vi) Like in the case of potential output growth, the results for the volatility of our estimated neutral real rate of interest are mixed. The average country volatility of the neural rate declines by half, from 1.2% in 1970-1987 to 0.6% in 1988-2006. The largest decline in the volatility of the neutral rate is recorded by the U.K., while the

[21] We should recall that the descriptive statistics discussed below for our estimates of unobservable are obviously conditional on our estimates, and therefore should be taken with caution, in comparison to those reported for observables like inflation, actual growth, and actual interest rates.

volatility rises in Norway. Japan records the lowest neutral rate volatility, close to zero, throughout the full sample period.

(vii) The results for the interest rate gap largely mimic those of the actual interest rate as the natural interest rate exhibits very low variability compared to the actual rate.

The evidence presented here is strongly supportive of a great moderation in key macro variables in industrial countries. The strong trend reduction in volatilities of three observed variables (inflation, output growth, and the real rate of interest) and the moderate decline in volatilities of the unobservable neutral interest rate and the two unobservable gap measures (the output gap and the interest rate gap), as well as the narrowing of the corresponding confidence intervals, are proof of the gains attained in macroeconomic stability during the past 15 years. The narrowing of country differences in volatilities that came about with the reduction in country volatilities during the last four decades also suggests stronger co-movements across countries, which is our next topic.

6.3 Testing for co-movements

Now we focus on co-movements of key variables across countries. We look at the same variables as above, less inflation. Cross-country correlations are reported for each variable for the full sample period (1970s-2006) in Table 6. We focus on pair-wise regional patterns. The findings follow:

(i) Output growth correlations among the G3 are low but significant. The correlations between the G3 and relevant third countries (Canada and European economies) are generally larger.

(ii) Our estimates for potential output growth in the Eurozone and Australia are constant, so we focus on correlations of third countries with the U.S. Japan, Canada, and Norway display large correlations with the U.S. The large and negative correlations of the New Zealand, Sweden, and the U.K. with the U.S. reflect their opposite potential output growth trends.

(iii) Output gap correlations between the Eurozone and every included country are either large and negative or zero, reflecting highly non-synchronous business-cycle conditions of the Eurozone with other industrial countries. This stands in contrast to the U.S., whose output gap is highly and positively correlated with most countries.

(iv) Among the G3, actual real interest rates are positively correlated. The same is true for most pair-wise correlations, except Japan's. This reflects the common, long cycle of low-high-low real interest rates observed in most countries during the last four decades.

(v) Even stronger correlations are observed in the case of neutral real interest rates, again except Japan, reflecting the common world trend in monetary policy observed in most industrial countries.

(vi) Cross-country interest rate gap correlations are similar to actual interest rate correlations, but often smaller and less significant.

In order to describe cross-country co-movements, we follow the approach adopted above in documenting volatility trends. Here we focus on rolling correlations of key variables between the U.S. and the eight industrial economies. We report point estimates of correlation coefficients and their confidence intervals for 74-quarter windows during 1970-2006, using the above mentioned stationary bootstrap technique. Our results are the following.

(i) There is no common trend in output growth correlations with the U.S. While output growth correlations with the U.S. rise in Canada, the U.K., Australia, and Sweden, they decline in Japan, New Zealand, and Norway.

(ii) Potential output growth correlations turn from positive (and mostly significant) to negative (and significant) in New Zealand, Canada, U.K., and Sweden.

(iii) Except the Eurozone and Japan, output gap correlations of all other countries with the U.S. rise over time, confirming increasing cyclical synchronization between small and medium-sized industrial economies and the U.S. economy.

(iv) Actual real interest rate correlations with the U.S. display a U-shaped pattern over the last four decades, reaching their lowest values during the 1980s-early 1990s and rising to high levels again in the late 1990s - 2000s. This suggests rising monetary integration (or declining independence) during the last decade.

(v) Regarding neutral real interest rate correlations with the U.S., the U-shaped pattern is confirmed in most economies, while in Japan and Norway correlations turn from negative and significant and positive and significant. New Zealand displays the opposite pattern, positive and significant to negative and significant.

(vi) The country pattern of interest rate gap correlations with the U.S. replicates that of actual interest rate correlations, reflecting the smoothness of neutral rates.

Country averages of the rolling correlation coefficients of country variables with those of the U.S. display slightly rising trends for the output gap, the actual interest rate, the neutral interest rate, and the interest rate gap (Figure 9, lower panels). The opposite is observed regarding average trends in actual and potential output growth with the U.S., which decline over time.

6.4 Convergence across countries

In this section we test for convergence in key variables across countries. It is important to note that rising correlations over time do not imply convergence in levels. Therefore we carry out this final set of exercises on convergence, complementing the previous evidence on increasing co-movements.

We test for convergence across countries using the following simple autoregressive models for the difference in country j's variable *v* with respect to that of the U.S. or the Eurozone:

$$v_{j,t} - v_{us,t} = \alpha_0 + \sum_{i=1}^{p} \alpha_i (v_{j,t-i} - v_{us,t-i}) + u_{j,us,t} \tag{18}$$

$$v_{j,t} - v_{eurozone,t} = \alpha_0 + \sum_{i=1}^{p} \alpha_i (v_{j,t-i} - v_{eurozone,t-i}) + u_{j,eurozone,t}$$

In the AR(p) model, we have convergence across countries if the AR polynomial is stationary.[22] To test for stationarity we use a grid bootstrap method to estimate confidence intervals for the parameters of interest (Hansen 1999).[23]

The variable *v* could represent observable variables (output growth and the interest rate), our estimates for unobservables (potential output growth and the neutral rate of interest), and our estimated unobservable gaps (the output gap and the interest rate gap). We will not test for convergence in levels of cross-country gap measures, as they tend to zero by construction.

The convergence tests for actual output growth and interest rates reveal the following results.

(i) For actual growth convergence with the U.S., we find that all countries are characterized by an AR(1) models, except Sweden with an AR(2) process. We find (weak) evidence of convergence with the U.S. for all countries, although α_j is only significant in New Zealand, Sweden, Norway, and Chile. For the remaining countries we are not able of rejecting a white-noise process.[24] For all countries we obtain small half-lives of shocks (HLS), on average of only 0.6 quarters.

(ii) When we examine actual growth convergence with the Eurozone, the relationships are characterized by higher-order AR processes in Japan, U.K., Sweden, and Norway. We find evidence of convergence with the Eurozone for all countries. The smallest HLS is 0.19 quarters (Australia) and the highest is 2.33 quarters (U.K.); the average HLS is 1.08 quarters.

[22] For example, convergence of an AR(1) model requires that $|\alpha_1| < 1$; convergence of an AR(2) model requires that $\alpha_1 + \alpha_2 < 1$, $\alpha_2 - \alpha_1 < 1$, and $\alpha_2 > -1$. Hamilton (1994) provides a more detailed discussion of stationarity conditions.

[23] The technique works as follows. Pick a grid over the parameters of interest and calculate the confidence interval by bootstrap at each parameter value, then smoothen the estimated function for the confidence interval using a kernel regression, and finally obtain the confidence interval estimated by the kernel for a given value of the parameter. Lag lengths (p lags) are determined by using the AIC, HQC, and BIC criteria.

(iii) Turning to convergence of actual interest rates with U.S. interest rates, we estimate for almost all countries an AR(1) process, except Chile with an AR(2) process. We find that all countries converge to the U.S. (and all estimated parameters are significant). As above, we also estimate HLS coefficients, which are much larger than those obtained for growth convergence. HLS coefficients range from 1.8 quarters (Sweden) to 7.5 quarters (Chile), with an average HLS of 3.65 quarters.

(iv) For interest rate convergence with the Eurozone, we estimate for all countries an AR(1) process, less Canada with an AR(2). All countries' interest rates converge to the Eurozone's. Our HLS estimates range from 0.83 quarters (Sweden) to 5.5 quarters (Chile), with an average HLS of 2.55 quarters.

Our estimates for the two key unobservables reveal the following results.

(i) We did not find country convergence of our estimated country unobservables (the potential output growth rate and the neutral real rate of interest) with either the U.S. or the Eurozone. This simply reflects the fact that country differentials in unobservables – with either the U.S. or the Eurozone – are not stationary in the 1970-2006 sample.

7. Conclusions and Possible Extensions

It is well recognized that the conduct of monetary policy is crucially dependent on several key unobservables – the output gap, the neutral real rate of interest, and expected inflation being the most critical. Individual central banks have developed methodologies for estimating these variables, and several researchers have attempted to estimate them by focusing on a single country (usually the U.S.) or on a small number of developed economies. We have extended this literature by providing new estimates of key unobservables for ten economies, including several inflation targeters and, among this group, one emerging market economy (Chile).

We adopted a very parsimonious model that we employed for all ten countries. This undoubtedly was the reason that our estimation results for the ten economies were mixed. However, for both the longer sample and the shorter sample periods, the evidence pointed to time variation in trend output growth, the neutral real rate of interest, and (for the U.S. and Chile) the natural rate of unemployment. This time variation has important implications for the conduct of monetary policy. For example, if trend growth of potential output were constant, then policy rules that focus on the growth rate of output relative to the growth rate of potential (speed limit policies of the type analyzed in Walsh 2003) might serve to eliminate (or at least

[24] All autocorrelations and partial correlations are not significantly different from zero.

significantly reduce) measurement problems in estimating the level of potential output. But if the growth rate of potential output is also subject to stochastic variation, as we find it to be, the problem of estimating the level of potential cannot be eliminated by simply focusing on growth rates.

Similarly, time variation in the neutral real interest rate implies that simple Taylor rules for the policy interest rate that very commonly assume the equilibrium real interest rate is constant, may lead to policy errors.

Consistent with notions of a great moderation, measures of inflation volatility showed a marked and common decline over the past decade. Output growth also declined in volatility. However, little of this decline in output growth volatility seems due to a decline in the volatility of the growth rate of potential output. The volatility of the latter has fallen slightly over the past twenty years, but this decline is small relative to the overall reduction in output growth volatility. Given these results, it is perhaps surprising that the volatility of the output gap displays only a modest decline over the sample. This reflects, in part, a rise in the average output gap volatility among our sample countries over the past decade. This is an interesting finding since it offers evidence consistent with standard theoretical models that greater inflation stability should come at the cost of some increase in output gap volatility. The failure of output gap volatility to fully reflect the decline in output growth volatility suggests that there may have been an increase in the volatility of the *level* of potential output over this period.

We find evidence that the volatility of the neutral real interest rate has declined when we look at the average across the sample economies. However, this masks significant differences among the individual economies.

Interestingly, we find neutral real interest rates to be more highly correlated across countries than either actual real rates or Wicksellian interest rate gaps. The notable exception to this finding is Japan. While neutral real rates were highly correlated across countries, this did not reflect a common pattern of convergence to the level of the U.S. or Eurozone neutral real rates. In fact, the neutral real rate differentials were non-stationary, indicating no long-run tendency to converge.

There are several extensions of the analysis that would be interesting to pursue. We would like to extend the approach to allow for richer and potentially different dynamics across the set of countries. Undoubtedly, one reason for some of our mixed results for individual countries arises from our use of a common specification of dynamics across all countries, particularly since our parsimonious model incorporated a fairly simple dynamic structure.

It would also be useful to extend the sample to include more emerging market and developing economies. Many of these economies have adopted inflation targeting frameworks in which the output gap and the neutral real interest rate are central to the design of policy. These economies also are small open economies, making them candidates for exploring issues of convergence and co-movements among these countries and the large industrialized economies.

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Data Appendix

Variable	Description	Source	Countries
Inflation measure	Consumer price index	IFS	Japan, New Zealand, Canada, U.K., Australia, Sweden, and Norway
	Core consumer price index	INE and BCCh	Chile
	Price index for personal consumption expenditures	LW	U.S.
	Consumption deflactor	ECB	Eurozone
Inflation targets	A composite measure which join the HP-filtered inflation rate and the observed inflation targets for inflation targeters. For non-inflation targerters (Japan, U.S. and the Eurozone) we use the HP-filtered series for the inflation measure	Authors' construction	All countries
Inflation expectation	Calculations based on four step-ahed forecasts stemming from an AR(4) for the actual inflation rate	Authors' construction	All countries
GDP	Seasonally adjusted real GDP	OECD	Japan, New Zealand, Canada, U.K., Australia, Sweden, and Norway
		ECB	Eurozone
		BCCh	Chile
		LW	U.S.
UR	Seasonally adjusted unemployment rate	OECD	Japan, New Zealand, Canada, U.K., Australia, Sweden, and Norway and U.S.
		ECB	Eurozone
		INE	Chile
Interest rate	Short-term nominal interest rate. The real interest rate is calculated as the difference of the nominal interest rate and our estimation of the inflation expectations.	OECD	Japan, New Zealand, Canada, U.K., Australia, Sweden, and Norway
		ECB	Eurozone
	Real monetary policy rate. Previous to 1994 indexed CBC's 90-day bond rate. Since 2001, official nominal MPR less expected inflation from inflation reports.	BCCh	Chile
	Monetary policy rate	LW	U.S.

BCCh: Central Bank of Chile
ECB: European Central Bank
IFS: International Financial Statistics
LW: Lauch and Williams (2003)

Table 1: Parameter estimates for the G3

Parameters	U.S.		Eurozone	
	1960:01 2007:02	**1986:01 2007:02**	**1970:02 2006:04**	**1986:02 2006:04**
	(1)	**(2)**	**(3)**	**(4)**
λ_g	0.0475	0.0612	0.0000	0.0000
λ_r	0.0215	0.1399	0.0214	-
α_1	0.9492 (0.0351)	1.2285 (0.1193)	0.9365 (0.0582)	0.9740 (0.0183)
α_2	-0.0710 (0.0226)	-0.1355 (0.0844)	0.0264 (0.0325)	- (-)
β_1	-0.0838 (0.0565)	-0.0502 (0.0849)	0.0144 (0.0650)	-0.2482 (0.0794)
β_2	0.8039 (0.0486)	1.2426 (0.1173)	0.6498 (0.0459)	1.1070 (0.0899)
β_3	0.4172 (0.1189)	-0.3384 (0.1346)	-0.0279 (0.0272)	0.0481 (0.0593)
δ_1	0.8632 (0.0233)	0.0251 (0.1427)	0.3652 (0.0490)	- (-)
δ_2	-0.1329 (0.0289)	-0.9141 (0.1119)	-0.5706 (0.0506)	- (-)
δ_3	0.1272 (0.0752)	2.2387 (0.5900)	1.0071 (0.1251)	- (-)
σ_y	0.4831 (0.0951)	0.1947 (0.0462)	0.3581 (0.0498)	0.4267 (0.3034)
σ_π	0.6790 (0.0319)	0.7292 (0.0406)	0.7362 (0.0468)	0.4680 (0.0401)
σ_r	1.1502 (0.0317)	0.0000 (5081.2000)	0.6101 (0.0384)	- (-)
σ_{y^*}	0.6543 (0.1044)	0.4367 (3687.0000)	0.4776 (0.1334)	0.1833 (0.1583)

Note: Standard errors in parentheses.
(1) The estimations are from the third step.
(2) The estimations are from the third step.
(3) The estimations are from the third step.
(4) The estimations are from the first step. We did not obtain estimations after the first step due to the matrix singular problem.

Table 1 (cont.): Parameter estimates for the G3

Parameters	Japan 1970:02 2006:04 (1)	(2)	Japan 1986:02 2006:04 (3)	(4)
λ_g	0.0000	0.0400	0.0000	0.0400
λ_r	-	0.0400	-	0.0400
α_1	0.8227 (0.0707)	1.0603 (0.0285)	0.9753 (0.0077)	1.0784 (0.0446)
α_2	- (-)	0.0562 (0.0282)	- (-)	0.1030 (0.0494)
β_1	-0.2137 (0.0478)	0.0557 (7711.2291)	-0.4258 (0.0920)	-0.0802 (1557.7195)
β_2	0.6607 (0.0309)	0.1374 (0.0672)	1.3892 (0.1317)	-0.0728 (0.1139)
β_3	2.2984 (0.4361)	0.5016 (0.0583)	0.0563 (0.0308)	0.4485 (0.1613)
δ_1	- (-)	0.0236 (0.0238)	- (-)	0.0616 (0.0761
δ_2	- (-)	-0.7107 (0.0336)	- (-)	-0.8420 (0.0616)
δ_3	- (-)	-2.2838 (0.9590)	- (-)	-1.2997 (0.3804)
σ_y	0.4647 (0.1000)	0.2167 (0.0924)	0.7196 (0.4655)	0.2091 (0.0762)
σ_π	1.3389 (0.1248)	2.2620 (0.1502)	1.0289 (0.0859)	1.3858 (0.1207)
σ_r	- (-)	0.3874 (0.0688)	- (-)	0.1678 (0.0396)
σ_{y^*}	0.8164 (0.1592)	0.8946 (6510.0068)	0.3170 (0.6622)	0.8532 (1304.8673)

Note: Standard errors in parentheses.
(1) The estimations are from the first step, since λ_r is not estimated in the second step when we impose λ_g =0 due to the matrix singular problem.
(2) The estimations are from the third step, where λ_r and λ_g are obtained across a grid search in the interval [0.005; 0.075].
(3) The estimations are from the first step, since λ_r is not estimated in the second step when we impose λ_g =0 due to the matrix singular problem.
(4) The estimations are from the third step, where λ_r and λ_g are obtained across a grid search in the interval [0.005; 0.075].

Table 2: Parameter estimates for New Zealand and Canada

Parameters	New Zealand				Canada			
	1974:02 2006:04		1986:02 2006:04		1970:02 2006:04	1986:02 2006:04		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
λ_g	0.0544	0.0544	0.0757	0.0757	0.0484	0.0000	0.0484	0.0484
λ_r	0.0000	0.0544	0.0871	0.0757	0.0698	-	0.0698	0.8198
α_1	0.914 (0.0643)	0.9462 (0.0505)	0.7153 (0.1345)	0.6256 (0.0927)	0.9598 (0.0582)	0.9916 (0.0187)	0.8788 (0.0946)	0.8773 (0.0813)
α_2	-0.0091 (0.0281)	0.0203 (0.0396)	0.2821 (0.0729)	0.2577 (0.0643)	-0.0790 (0.0291)	- (-)	0.0305 (0.0342)	0.0369 (0.0464)
β_1	-0.1923 (0.0703)	-0.1983 (0.0685)	-0.2158 (0.221)	-0.1067 (44246.3385)	0.0844 (14868.7)	-0.3020 (0.0759)	-0.2260 (10298.66)	-0.2260 (10274.93)
β_2	1.4305 (0.0897)	1.4288 (0.0834)	1.2403 (0.214)	-0.1816 (0.2006)	0.0223 (0.0747)	1.2527 (0.1191)	-0.1199 (0.0886)	-0.2318 (0.0878)
β_3	0.5697 (0.2459)	0.5743 (0.2219)	0.9306 (0.2942)	1.1411 (0.1346)	0.6890 (0.1242	0.0739 (0.2246)	0.7680 (0.1301)	0.8807 (0.1433)
δ_1	0.7038 (0.0491)	0.5875 (0.0472)	0.1262 (0.0621)	0.1475 (0.0651)	0.7370 (0.0420)	- (-)	0.2825 (0.0697)	0.1968 (0.0684)
δ_2	-0.3204 (0.0857)	-0.3742 (0.0779)	-0.6219 (0.1614)	-0.5968 (0.1567)	-0.2602 (0.0635)	- (-)	-0.9390 (0.0883)	-0.9290 (0.0796)
δ_3	-0.2211 (0.142)	-0.1838 (0.1383)	-0.1313 (0.1412)	-0.2096 (0.1815)	0.3684 (0.1469)	- (-)	2.2223 (0.4015)	1.5811 (0.3244)
σ_y	1.1969 (0.3918)	1.183 (0.3701)	1.0281 (0.1749)	1.0015 (0.1928)	0.4408 (0.0978)	0.5978 (0.9679)	0.2605 (0.0624)	0.2982 (0.0724)
σ_π	1.5029 (0.1417)	1.4946 (0.1309)	1.5014 (0.2658)	1.5179 (0.2073)	1.3423 (0.0707)	1.1695 (0.0833)	1.3798 (0.1163)	1.2553 (0.1032)
σ_r	2.1501 (0.0847)	2.0427 (0.0697)	1.5995 (0.1426)	1.6071 (0.1417)	1.0691 (0.0576)	- (-)	0.4273 (0.0557)	0.3548 (0.0415)
σ_{y^*}	1.9964 (0.7595)	2.0137 (0.6157)	0.9803 (0.2825)	0.9577 (37739.0760)	0.5649 (11505.45)	0.0000 (185845.55)	0.5019 (7749.17)	0.4724 (7613.58)

Note: Standard errors in parentheses.
(1) The estimations are from the second step. We did not obtain estimations in the third step due to the matrix singular problem.
(2) The estimations are from the third step, where λ_r is obtained across a grid search in the interval [0.0444; 0.1244].
(3) The estimations are from the third step.
(4) The estimations are from the third step, where λ_r and λ_g are obtained across a grid search in the interval [0.0275; 0.9775].
(5) The estimations are from the third step.
(6) The estimations are from the first step, since λ_r is not estimated in the second step when we impose λ_g =0 due to the matrix singular problem.
(7) The estimations are from the third step, where λ_g and λ_r are obtained in the estimation with the sample 1970-2006.
(8) The estimations are from the third step, where λ_r is obtained across a grid search in the interval [0.0098; 2.0198].

Table 2 (cont.): Parameter estimates for U.K. and Australia

	U.K.				Australia			
Parameters	1970:02 2006:04		1986:02 2006:04		1970:02 2006:04	1986:02 2006:04		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
λ_g	0.0275	0.0275	0.0000	0.0275	0.0000	0.0069	0.0069	0.0569
λ_r	-	0.0900	0.0000	0.0600	0.0522	0.0000	0.0522	0.0522
α_1	0.8796 (0.0575)	0.6669 (0.1249)	0.9776 (0.0345)	0.9854 (0.0156)	0.9363 (0.0415)	0.9669 (0.05)	0.9906 (0.0432)	0.9291 (0.1031)
α_2	- (-)	0.0407 (0.0195)	-0.036 (0.0388)	-0.0490 (0.0427)	0.0022 (0.0321)	-0.0237 (0.0303)	-0.0036 (0.0453)	0.0062 (0.0247)
β_1	-0.1142 (0.0601)	-0.169 (0.0759)	-0.2266 (0.1021)	-0.2245 (0.1017)	-0.2231 (0.0553)	-0.4366 (0.1275)	-0.4316 (0.1214)	-0.2872 (15262.2512)
β_2	0.9532 (0.0391)	0.8837 (0.0545)	1.3391 (0.1148)	1.3271 (0.0984)	1.0026 (0.0979)	1.3629 (0.117)	1.3581 (0.1111)	-0.4366 (0.1165)
β_3	1.0792 (0.3806)	2.4103 (0.7842)	0.2045 (0.1227)	0.2063 (0.0891)	0.3114 (0.114)	0.3246 (0.1883)	0.3191 (0.1497)	1.2311 (0.1387)
δ_1	- (-)	0.4519 (0.0331)	0.8953 (0.0555)	0.7431 (0.0694)	0.7168 (0.0481)	0.8507 (0.0526)	0.7758 (0.0706)	0.7554 (0.0773)
δ_2	- (-)	-0.7096 (0.046)	-0.1097 (0.0935)	-0.0995 (0.0805)	-0.3327 (0.0496)	-0.2668 (0.1081)	-0.2945 (0.0932)	-0.3612 (0.0946)
δ_3	- (-)	0.6368 (0.2713)	0.0282 (0.0403)	0.0523 (0.0497)	0.0438 (0.0752)	0.1157 (0.0976)	0.1345 (0.0931)	0.6577 (0.3732)
σ_y	0.6381 (0.1161)	0.4554 (0.1017)	0.1404 (0.159)	0.4713 (0.1190)	1.0046 (0.1178)	0.616 (0.1922)	0.6615 (0.1262)	0.3051 (0.0995)
σ_π	1.77 (0.1337)	1.5288 (0.1767)	0.8628 (0.0611)	0.8644 (0.0599)	2.0193 (0.1177)	1.4473 (0.1283)	1.4495 (0.126)	1.4616 (0.1383)
σ_r	- (-)	1.6097 (0.0818)	0.788 (0.0568)	0.7557 (0.0563)	1.6796 (0.0757)	0.9827 (0.083)	0.9362 (0.0731)	0.8986 (0.0756)
σ_{y^*}	0.6383 (0.1174)	0.7789 (0.284)	0 (4092)	0.0000 (3802.0530)	0.0000 (12158.6)	0.2168 (0.3725)	0.0008 (157.37)	0.5638 (11804.8755)

Note: Standard errors in parentheses.

(1) The estimations are from the first step, since λ_r is not estimated in the second step when we impose λ_g =0.0275 due to the matrix singular problem.

(2) The estimations are from the third step, where λ_r is obtained across a grid search in the interval [0.0444; 0.1244].

(3) The estimations are from the second step. We did not obtain estimations in the third step due to the matrix singular problem.

(4) The estimations are from the third step, where λ_r is obtained in the estimation with the sample 1970-2006 and λ_r is obtained across a grid search in the interval [0.055; 0.065].

(5) The estimations are from the third step.

(6) The estimations are from the second step. We did not obtain estimations in the third step due to the matrix singular problem.

(7) The estimations are from the third step, where λ_r is obtained in the estimation with the sample 1970-2006.

(8) The estimations are from the third step, where λ_r and λ_g are obtained across a grid search in the interval [0.0275; 0.9775].

Table 2 (cont.): Parameter estimates for Sweden and Norway

Parameters	Sweden			Norway			
	1970:02 2006:04	1986:02 2006:04		1979:02 2006:04		1986:02 2006:04	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
λ_g	0.0262	0.0000	0.0262	0.0677	0.0677	0.1186	0.1186
λ_r	0.0315	-	0.0315	0.000	0.040	0.000	0.040
α_1	0.9177 (0.0478)	0.9913 (0.0274)	0.9403 (0.0522)	0.9236 (0.0405)	0.9375 (0.0613)	0.0072 (0.2289)	-0.7573 (0.1780)
α_2	-0.0452 (0.0190)	- (-)	-0.0558 (0.0110)	-0.0958 (0.0658)	-0.0050 (0.0208)	-0.1925 (0.1273)	0.5243 (0.2039)
β_1	-0.1680 (16775.9)	-0.3390 (0.0888)	-0.0646 (11442.1623)	-0.3339 (0.0444)	-0.1700 (16845.7489)	-0.3609 (0.0572)	-0.3064 (25668.8387)
β_2	-0.3429 (0.0594)	1.3353 (0.1098)	-0.2998 (0.1031)	1.4904 (0.0928)	-0.2921 (0.0500)	1.2578 (0.0891)	-0.3553 (0.0531)
β_3	1.3183 (0.1133)	0.2620 (0.3898)	1.3436 (0.1289)	0.3326 (0.1267)	1.5101 (0.0997)	0.2158 (0.2943)	1.1926 (0.0766)
δ_1	0.5615 (0.0292)	- (-)	0.3929 (0.0581)	0.7958 (0.0485)	0.6415 (0.0615)	0.8777 (0.0708)	0.9868 (0.0115)
δ_2	-0.4751 (0.1683)	- (-)	-0.7365 (0.4081)	-0.3852 (0.0842)	-0.5778 (0.0919)	-0.4053 (0.1509)	-0.4583 (0.1064)
δ_3	-0.4555 (0.3784)	- (-)	-0.5290 (1.0947)	-0.1139 (0.0560)	-1.1227 (0.2599)	-0.2346 (0.2826)	-0.4790 (0.1677)
σ_y	0.3447 (0.1196)	0.6823 (0.4974)	0.1191 (0.1642)	0.9041 (0.2227)	0.2402 (0.0890)	0.7054 (0.1376)	0.3312 (0.1086)
σ_π	1.9639 (0.1272)	1.6336 (0.1287)	1.7076 (0.1579)	1.3759 (0.0770)	1.4408 (0.0894)	1.4839 (0.1064)	1.2810 (0.0977)
σ_r	2.6759 (0.0620)	- (-)	3.1712 (0.1470)	1.1974 (0.0727)	1.0270 (0.0693)	1.2259 (0.0822)	0.3762 (0.2991)
σ_{y^*}	0.9841 (14365.4)	0.0000 (106856.8391)	0.5951 (8956.6283)	0.7633 (0.4370)	1.1710 (16930.1425)	0.5428 (3.2854)	0.8557 (21516.3255)

Note: Standard errors in parentheses.
(1) The estimations are from the third step.
(2) The estimations are from the first step, since λ_r is not estimated in the second step when we impose λ_g =0.00 due to the matrix singular problem.
(3) The estimations are from the third step, where λ_r is obtained in the estimation with the sample 1970-2006.
(4) The estimations are from the second step. We did not obtain estimations in the third step due to the matrix singular problem.
(5) The estimations are from the third step, where λ_r is obtained across a grid search in the interval [0.0050; 0.0750].
(6) The estimations are from the second step. We did not obtain estimations in the third step due to the matrix singular problem. (7) The estimations are from the third step, where λ_r is obtained across a grid search in the interval [0.0050; 0.0750].

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Table 3: Parameter estimates for Chile

	Chile	
Parameters	**1986:02 2006:04**	
	(1)	**(2)**
λ_g	0.0000	0.0820
λ_r	0.0000	0.0800
α_1	1.0771 (0.0540)	0.9412 (0.1074)
α_2	-0.2461 (0.1245)	-0.1076 (0.0961)
β_1	0.4639 (0.0697)	0.4325 (0.0946)
β_2	0.5078 (0.1612)	0.5940 (0.1959)
β_3	0.0142 (0.0251)	0.2756 (0.2216)
δ_1	0.6996 (0.1242)	0.6552 (0.0861)
δ_2	-0.0151 (0.2658)	0.1188 (0.2049)
δ_3	0.0733 (0.0809)	0.3680 (0.2525)
σ_y	1.2847 (0.9877)	1.0436 (0.2924)
σ_π	1.8274 (0.1110)	1.7188 (0.1230)
σ_r	1.3993 (0.0750)	1.2777 (0.0833)
σ_{y^*}	0.0001 (8810.1)	0.7456 (0.3177)

Note: Standard errors in parentheses.
(1) The estimations are from the second step. We did not obtain estimations in the third step due to the matrix singular problem. (2) The estimations are from the third step, where λ_g and λ_r are obtained across a grid search in the intervals [0.062; 0.102] and [0.06; 0.10], respectively.

Table 4: Parameter estimates for alternative models for the U.S., 1960-2007

Parameters	**U.S. 1960:1 - 2007:2 (1)**	**(2)**	**(3)**
	Extended Model (with Okun's Law)		**8-step ahead inflation forecasts**
λ_g	0.0475	0.0475	0.0586
λ_r	0.0215	0.0215	0.0304
λ_u	0.0000	0.4000	---
α_1	0.9539 (0.0302)	0.9558 (0.0331)	0.9503 (0.0441)
α_2	-0.0252 (0.0100)	-0.0681 (0.0213)	-0.0546 (0.0216)
β_1	0.1097 (0.0599)	0.0602 (0.0593)	0.0680 (0.1031)
β_2	0.6525 (0.0525)	0.7032 (0.0474)	0.4514 (0.0482)
β_3	0.3926 (0.1876)	0.2820 (0.0968)	0.4337 (0.1427)
γ_1	0.4956 (0.0999)	0.5635 (0.0879)	-
γ_2	-0.9466 (0.3234)	-0.3523 (0.1010)	-
δ_1	0.8756 (0.0316)	0.8697 (0.0256)	0.7880 (0.0262)
δ_2	-0.1478 (0.0286)	-0.1353 (0.0298)	-0.2193 (0.0201)
δ_3	0.1731 (0.1587)	0.1250 (0.0825)	0.1910 (0.1075)
σ_y	0.2411 (0.0780)	0.4731 (0.1053)	0.5176 (0.1060)
σ_π	0.8223 (0.0385)	0.7750 (0.0340)	0.8250 (0.0411)
σ_u	0.0442 (0.0643)	0.1253 (0.0144)	-
σ_r	1.1552 (0.0283)	1.1498 (0.0316)	1.2768 (0.0352)
σ_{y^*}	0.7969 (0.3020)	0.6656 (0.1485)	0.6293 (0.1288)

Note: Standard errors in parentheses.
(1) The estimations are from the fourth step of the extended model with Okun's Law.
(2) The estimations are from the fourth step, where λ_u is obtained across a grid-search in the interval [0.08; 0.72].
(3) The estimations are from the third step of the modified standard model with eight-step-ahead inflation forecast.

Table 5: Parameter estimates for an alternative model for Chile, 1986-2007

Parameters	Chile	
	1986.2 - 2006.4	
	(1)	(2)
	Extended Model (with Okun's Law)	
λ_g	0.0000	0.0820
λ_r	0.0000	0.0800
λ_u	0.0000	0.4000
α_1	1.0033 (0.0515)	1.0329 (0.0433)
α_2	-0.0644 (0.0425)	-0.1583 (0.0685)
β_1	0.4501 (0.0803)	0.4533 (0.0842)
β_2	0.5191 (0.1703)	0.5182 (0.1687)
β_3	0.1474 (0.1614)	0.1173 (0.1420)
γ_1	0.2501 (0.1791)	0.2045 (0.2190)
γ_2	-0.6591 (0.3348)	-0.5356 (0.2237)
δ_1	0.7821 (0.0600)	0.6996 (0.0724)
δ_2	0.0205 (0.2750)	-0.0073 (0.2139)
δ_3	0.3329 (0.2328)	0.2654 (0.1804)
σ_y	0.5644 (0.2246)	0.5899 (0.1810)
σ_π	1.8052 (0.1135)	1.8071 (0.1175)
σ_u	0.1935 (0.0971)	0.2151 (0.0671)
σ_r	1.3852 (0.0743)	1.3135 (0.0704)
σ_{y^*}	1.2730 (0.6356)	1.1429 (0.5518)

Note: Standard errors in parentheses.
(1) The estimations are from the fourth step.
(2) The estimations are from the fourth step, where λ_g, λ_r, and λ_u are obtained across a grid-search in the intervals [0.062; 0.102], [0.06; 0.10], and [0.08; 0.72], respectively.

Table 6: Cross country correlations of key variables, 1970:2-2006:4[1]

actual output growth	USA	Euro	Japan	New Zealand	Canada	United Kingdom	Australia	Sweden	Norway
USA	1.00	**0.24**	**0.19**	**0.24**	**0.50**	**0.29**	**0.30**	0.18	**0.15**
Euro	-	1.00	**0.31**	**0.25**	**0.32**	**0.37**	0.10	**0.32**	**0.30**
Japan	-	-	1.00	-0.07	0.13	**0.28**	-0.02	-0.01	-0.08
New Zealand	-	-	-	1.00	**0.20**	0.08	0.12	**0.24**	**0.26**
Canada	-	-	-	-	1.00	**0.27**	**0.31**	0.11	0.08
United Kingdom	-	-	-	-	-	1.00	0.05	**0.28**	-0.01
Australia	-	-	-	-	-	-	1.00	0.08	0.01
Sweden	-	-	-	-	-	-	-	1.00	**0.22**
Norway	-	-	-	-	-	-	-	-	1.00

potential output growth	USA	Euro	Japan	New Zealand	Canada	United Kingdom	Australia	Sweden	Norway
USA	1.00	0.00	**0.82**	**-0.61**	**0.55**	**-0.64**	0.00	**-0.73**	**0.66**
Euro*	-	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
Japan	-	-	1.00	**-0.83**	**0.58**	**-0.90**	0.00	**-0.75**	0.27
New Zealand	-	-	-	1.00	-0.56	**0.85**	0.00	**0.70**	-0.30
Canada	-	-	-	-	1.00	**-0.34**	0.00	-0.05	0.16
United Kingdom	-	-	-	-	-	1.00	0.00	**0.80**	-0.31
Australia*	-	-	-	-	-	-	1.00	0.00	0.00
Sweden	-	-	-	-	-	-	-	1.00	-0.34
Norway	-	-	-	-	-	-	-	-	1.00

output gap	USA	Euro	Japan	New Zealand	Canada	United Kingdom	Australia	Sweden	Norway
USA	1.00	**-0.28**	**0.27**	0.29	**0.66**	**0.50**	**0.53**	**0.47**	**0.32**
Euro	-	1.00	**-0.77**	**-0.75**	-0.20	0.04	**-0.56**	**-0.60**	**-0.57**
Japan	-	-	1.00	**0.48**	0.11	0.02	**0.48**	**0.42**	0.21
New Zealand	-	-	-	1.00	0.26	-0.12	**0.42**	**0.44**	**0.80**
Canada	-	-	-	-	1.00	**0.38**	**0.66**	**0.52**	**0.34**
United Kingdom	-	-	-	-	-	1.00	**0.40**	**0.34**	0.04
Australia	-	-	-	-	-	-	1.00	**0.65**	**0.27**
Sweden	-	-	-	-	-	-	-	1.00	**0.38**
Norway	-	-	-	-	-	-	-	-	1.00

Note: Bold numbers indicate significant correlation coefficients based on Hall's confidence intervals calculated using the stationary bootstrap technique while underlined numbers indicate significant correlation coefficients based on t-distribution.
1. The sample period is 1974:2-2006:4 and 1979:2-2006:4 for New Zealand and Norway, respectively.
** The potential output growth estimate is constant for the Eurozone and Australia.*

Table 6 (cont.): Cross country correlations of key variables, 1970:2-2006:4[1]

actual interest rate	USA	Euro	Japan	New Zealand	Canada	United Kingdom	Australia	Sweden	Norway
USA	1.00	**0.49**	0.26	0.24	**0.71**	**0.39**	**0.52**	0.22	0.13
Euro	-	1.00	**0.52**	**0.48**	**0.63**	**0.55**	**0.69**	**0.65**	**0.60**
Japan	-	-	1.00	0.06	0.22	-0.09	0.18	0.25	**0.62**
New Zealand	-	-	-	1.00	**0.27**	**0.61**	**0.57**	0.29	0.16
Canada	-	-	-	-	1.00	**0.53**	**0.60**	0.32	**0.39**
United Kingdom	-	-	-	-	-	1.00	**0.66**	**0.37**	**0.40**
Australia	-	-	-	-	-	-	1.00	**0.38**	**0.30**
Sweden	-	-	-	-	-	-	-	1.00	**0.33**
Norway	-	-	-	-	-	-	-	-	1.00

natural interest rate	USA	Euro	Japan	New Zealand	Canada	United Kingdom	Australia	Sweden	Norway
USA	1.00	**0.64**	**0.37**	0.17	**0.90**	**0.76**	**0.78**	**0.48**	**0.91**
Euro	-	1.00	0.14	**0.60**	**0.83**	**0.74**	**0.73**	**0.68**	**0.60**
Japan	-	-	1.00	**-0.63**	0.21	-0.20	-0.15	**-0.38**	**0.99**
New Zealand	-	-	-	1.00	**0.45**	**0.78**	**0.74**	**0.82**	**-0.63**
Canada	-	-	-	-	1.00	**0.90**	**0.91**	**0.77**	**0.77**
United Kingdom	-	-	-	-	-	1.00	**0.99**	**0.89**	**0.57**
Australia	-	-	-	-	-	-	1.00	**0.89**	**0.54**
Sweden	-	-	-	-	-	-	-	1.00	-0.05
Norway	-	-	-	-	-	-	-	-	1.00

interest rate gap	USA	Euro	Japan	New Zealand	Canada	United Kingdom	Australia	Sweden	Norway
USA	1.00	**0.41**	0.21	0.18	**0.58**	0.16	**0.39**	0.17	-0.24
Euro	-	1.00	**0.52**	**0.31**	0.13	0.04	**0.26**	**0.54**	**0.42**
Japan	-	-	1.00	0.07	-0.04	-0.39	-0.05	**0.23**	**0.39**
New Zealand	-	-	-	1.00	-0.10	**0.24**	**0.23**	0.11	0.14
Canada	-	-	-	-	1.00	**0.22**	**0.27**	0.01	-0.11
United Kingdom	-	-	-	-	-	1.00	**0.34**	0.01	0.09
Australia	-	-	-	-	-	-	1.00	0.06	-0.09
Sweden	-	-	-	-	-	-	-	1.00	**0.27**
Norway	-	-	-	-	-	-	-	-	1.00

Note: Bold numbers indicate significant correlation coefficients based on Hall's confidence intervals calculated using the stationary bootstrap technique while underlined numbers indicate significant correlation coefficients based on t-distribution.
1. The sample period is 1974:2-2006:4 and 1979:2-2006:4 for New Zealand and Norway, respectively.

7.a Convergence of Actual Output Growth, 1970:2-2006:4[1]

Convergence with the U.S.						
Country	I(0)	Order	Drift	AR coefficients		HLS
	(1)	(2)	(3)	(4)		(5)
Eurozone	Yes	1	0	0.1260	-	0.3346
Japan	Yes	1	0	0.1105	-	0.3146
New Zealand	Yes	1	0	**-0.1836**	-	0.6122
Canada	Yes	1	0	-0.0273	-	0.5128
U.K.	Yes	1	**-0.6998**	-0.1092	-	0.3129
Australia	Yes	1	0	-0.0684	-	0.5365
Sweden	Yes	2	0	-0.0162	**0.1742**	1.6091
Norway	Yes	1	0	**-0.3124**	-	0.7272
Chile	Yes	1	**2.6974**	**0.2233**	-	0.4623
				Average HLS		0.6024

Convergence with the Eurozone								
Country	I(0)	Order	Drift	AR coefficients				HLS
	(1)	(2)	(3)	(4)				(5)
Japan	Yes	4	0	0.0797	0.1253	**0.2402**	**-0.1871**	1.7196
New Zealand	Yes	1	0	**-0.2010**	-	-	-	0.6253
Canada	Yes	1	**0.6894**	**0.2240**	-	-	-	0.4632
U.K.	Yes	3	0	-0.0377	**0.1720**	**0.1801**	-	2.3339
Australia	Yes	1	0	0.0244	-	-	-	0.1866
Sweden	Yes	3	0	**-0.1829**	**0.2448**	**0.1837**	-	1.5852
Norway	Yes	3	0	**-0.2135**	**0.3031**	0.1768	-	1.1330
Chile	Yes	1	**3.3966**	**0.2991**	-	-	-	0.5742
						Average HLS		1.0776

Notes: Significant estimates in boldface.
1. Except Chile, for which the sample is 1986-2006.

(1) We use the grid bootstrap (Hansen, 1999) for autoregressive models to compute confidence intervals for all AR coefficients.
(2) We use AIC, BIC and HQC criteria to determine lag lengths.
(3) The value of the constant in the AR model.
(4) Estimated AR coefficients.
(5) Half-life of a unit shock (HLS) coefficient, which is defined as HLS=abs(log(1/2)/log(α)) for AR(1) model (with α ≥ 0). The HLS for AR(p) models can be calculated directly from the impulse response functions.

We did not find convergence for the unobservables (natural rate of interest and potential output growth) in both cases (with U.S. and Eurozone), since the series are not I(0) (stationary). In these cases we have that HLS coefficients are explosive (∞ or a large number).

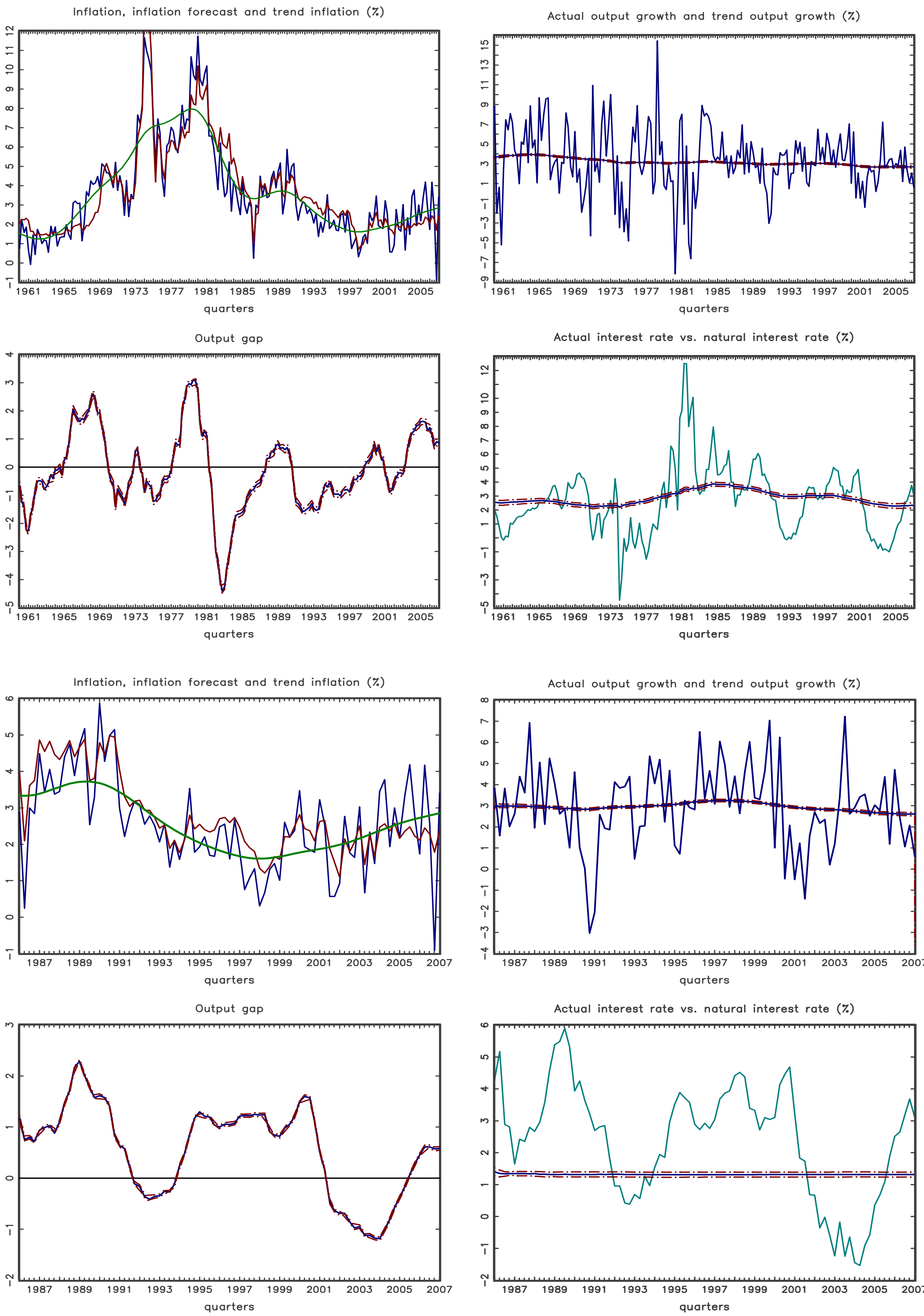
7.b Convergence of the Actual Rate of Interest, 1970:2-2006:4[1]

Convergence with the U.S.						
	I(0)	Order	Drift	AR coefficients		HLS
Country	(1)	(2)	(3)	(4)		(5)
Eurozone	Yes	1	0	**0.8650**	-	4.7794
Japan	Yes	1	0	**0.8274**	-	3.6584
New Zealand	Yes	1	0	**0.7494**	-	2.4027
Canada	Yes	1	0	**0.7571**	-	2.4910
U.K.	Yes	1	0	**0.7625**	-	2.5562
Australia	Yes	1	0	**0.7107**	-	2.0296
Sweden	Yes	1	0	**0.6806**	-	1.8014
Norway	Yes	1	0	**0.8826**	-	5.5503
Chile	Yes	2	0	**0.7066**	**0.2182**	7.5142
				Average HLS		3.6425

Convergence with the Eurozone						
Country	I(0)	Order	Drift	AR coefficients		HLS
	(1)	(2)	(3)	(4)		(5)
Japan	Yes	1	0	**0.7554**	-	2.7410
New Zealand	Yes	1	0	**0.7060**	-	1.9910
Canada	Yes	2	0	**1.0074**	**-0.2645**	2.9601
U.K.	Yes	1	0	**0.6695**	-	1.7275
Australia	Yes	1	0	**0.5953**	-	1.3363
Sweden	Yes	1	0	**0.4365**	-	0.8361
Norway	Yes	1	0	**0.8115**	-	3.3185
Chile	Yes	1	0	**0.8813**	-	5.4856
				Average HLS		2.5495

Notes: Significant parameters in boldface.
1. Except Chile, for which the sample is 1986-2006.
(1) We use the grid bootstrap (Hansen, 1999) for autoregressive models to compute confidence intervals for all AR coefficients.
(2) We use AIC, BIC and HQC criteria to determine lag lengths.
(3) The value of the constant in the AR model.
(4) Estimated AR coefficients.
(5) Half-life of a unit shock (HLS) coefficient, which is defined as HLS=abs(log(1/2)/log(α)) for AR(1) model (with $\alpha \geq 0$). The HLS for AR(p) models can be calculated directly from the impulse response functions.

Figure 1.a: Inflation, output, and interest rate in the U.S., 1960:1-2007:2 and 1986:1-2007:2



Note: actual inflation, inflation forecast and inflation trend in blue, red and green lines, respectively, in the up-left figure of the panels.

Figure 1.b: Inflation, output, and interest rate in the Eurozone, 1970:2-2006:4

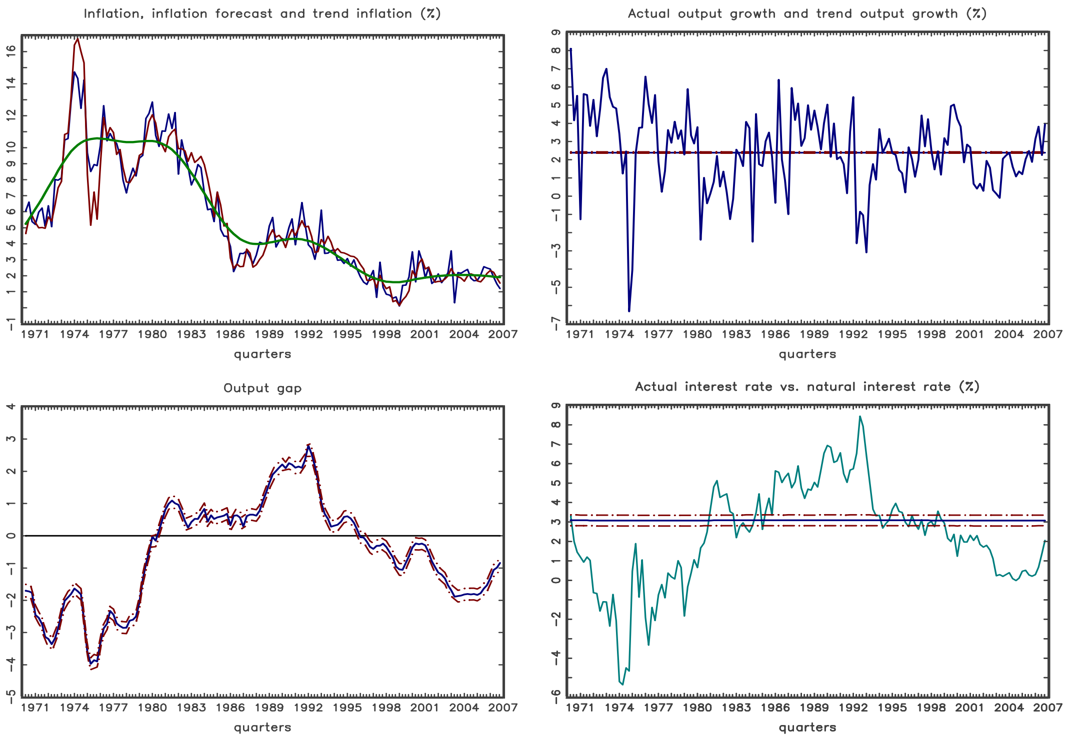
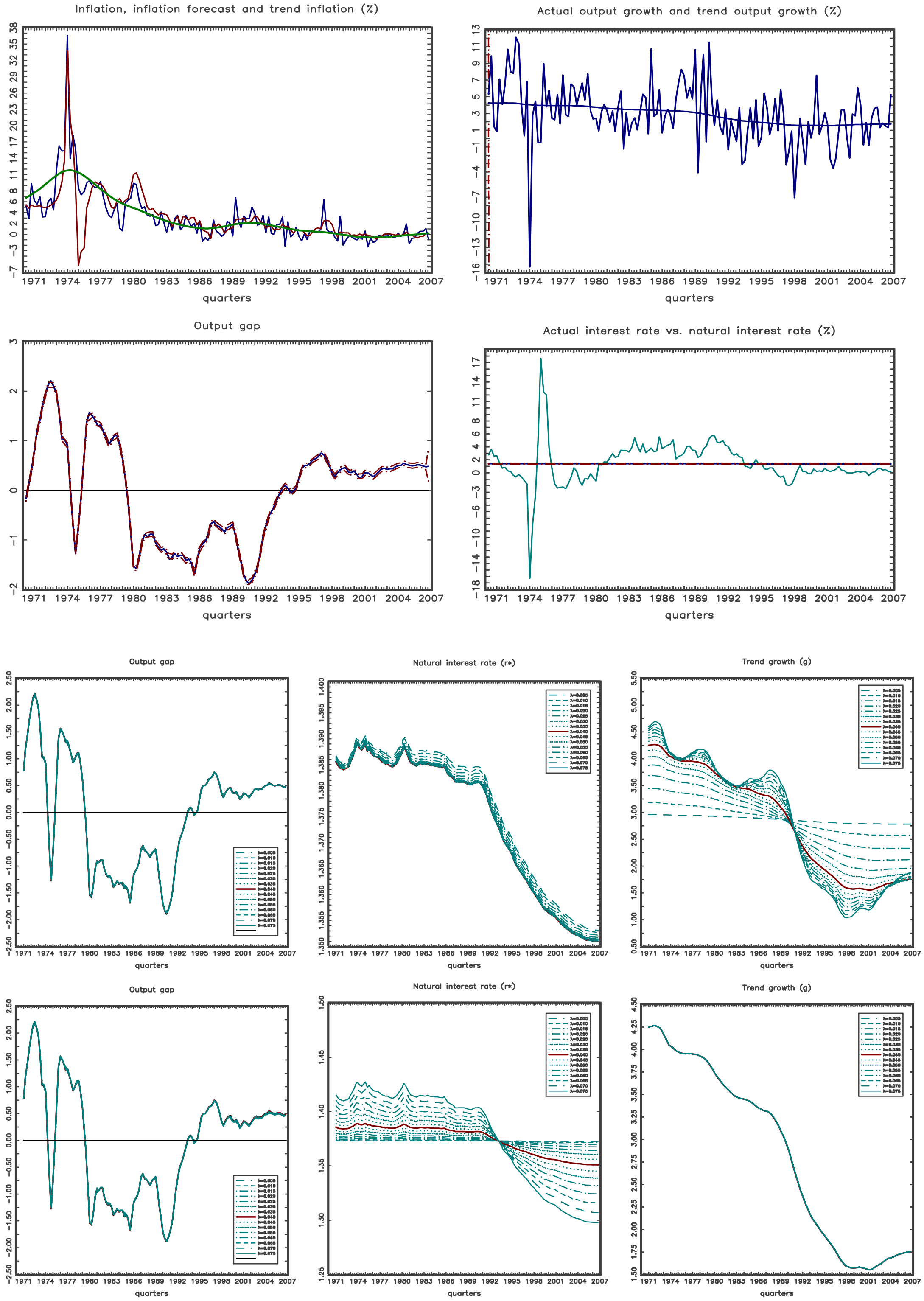


Figure 1.c: Inflation, output, and interest rate in Japan, 1970:2-2006:4



Note: the second panel shows the unobservables for different grid values for λ_g (first row) and λ_r (second row).

Figure 2.a: Inflation, output, and interest rate in New Zealand, 1974:2-2006:4/1986:2-2006:4

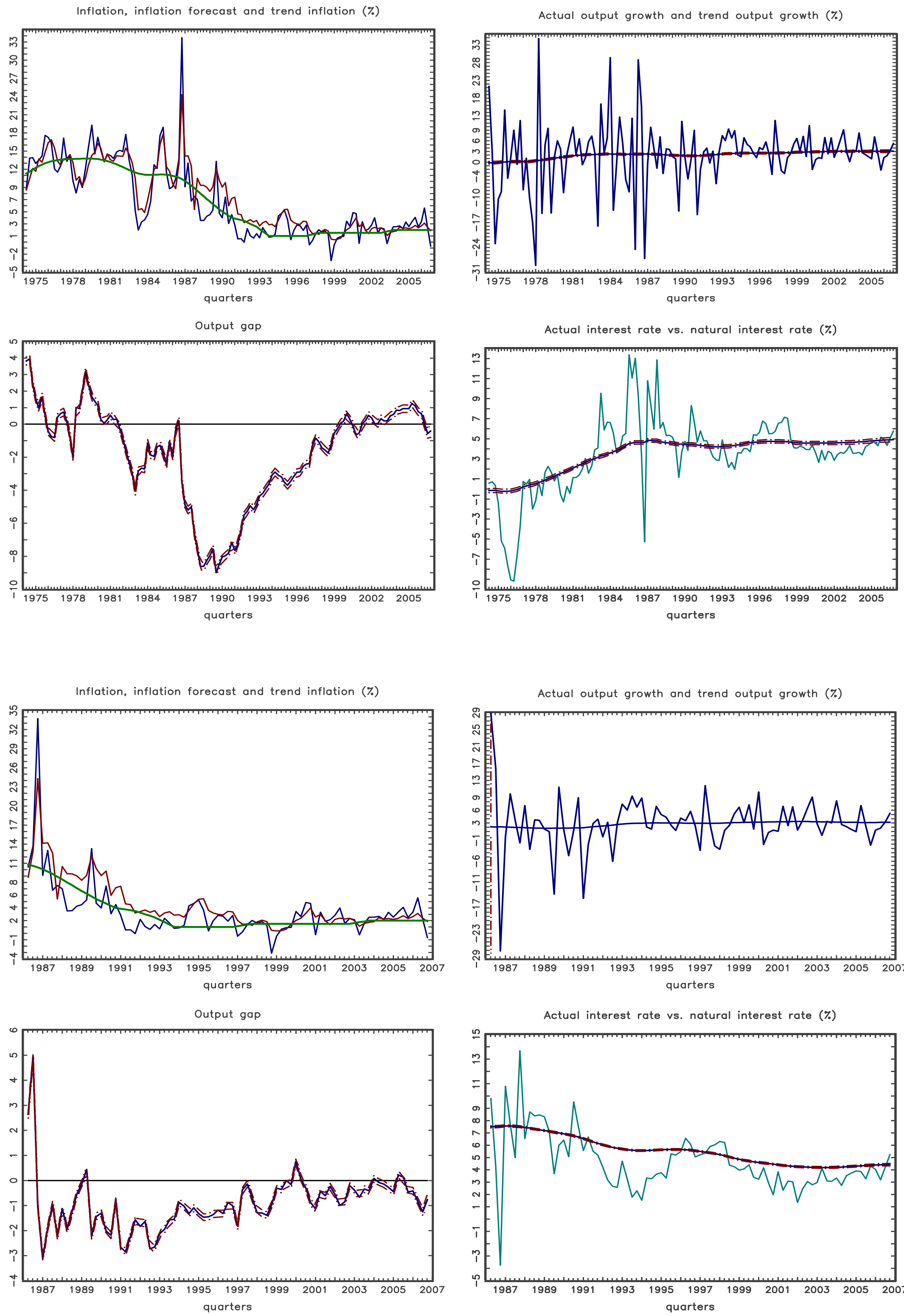


Figure 2.b: Inflation, output, and interest rate in Canada, 1970:2-2006:4/1986:2-2006:4

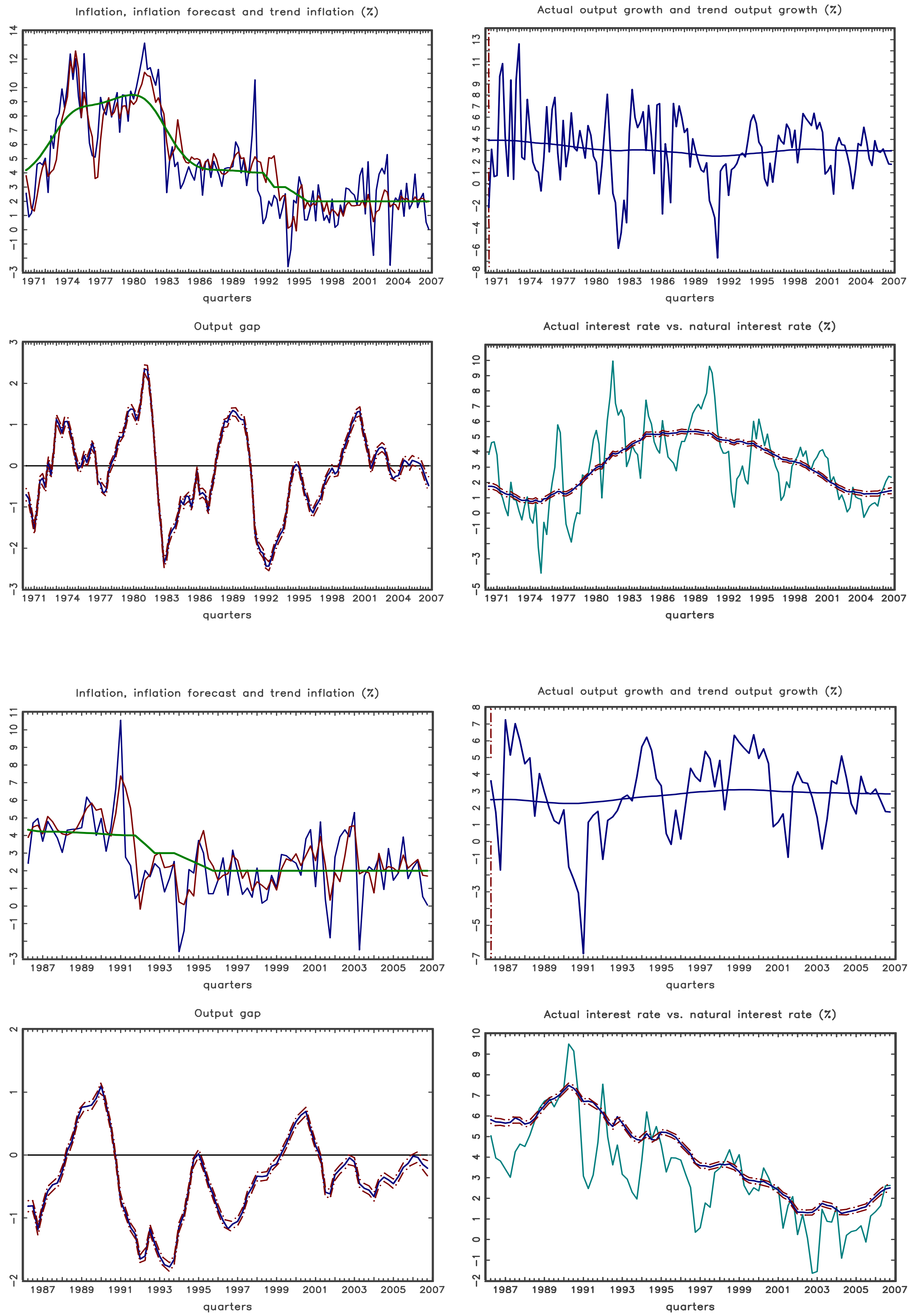


Figure 2.c: Inflation, output, and interest rate in the U.K., 1970:2-2006:4/1986:2-2006:4

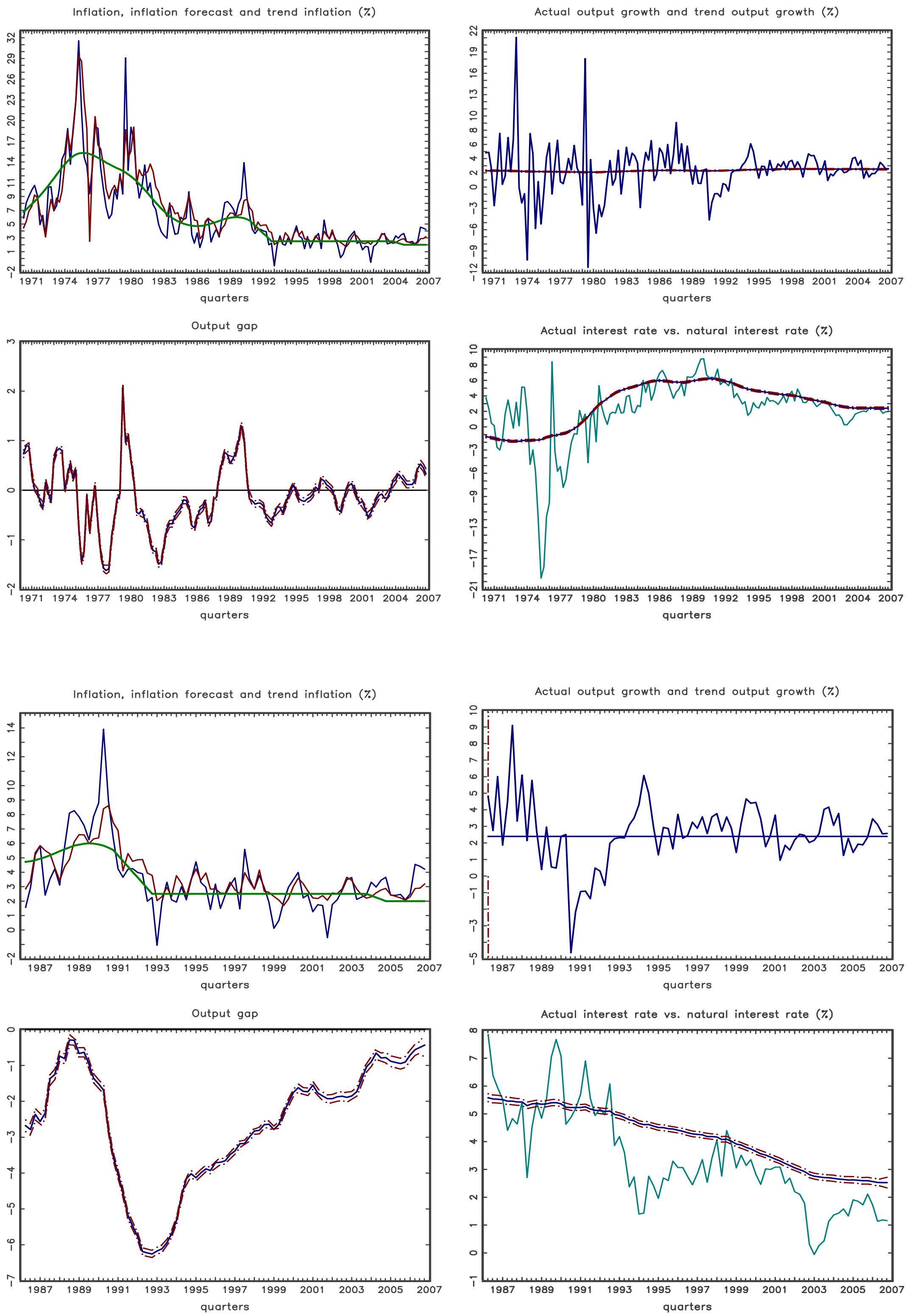


Figure 2.d: Inflation, output, and interest rate in Australia, 1970:2-2006:4/1986:2-2006:4

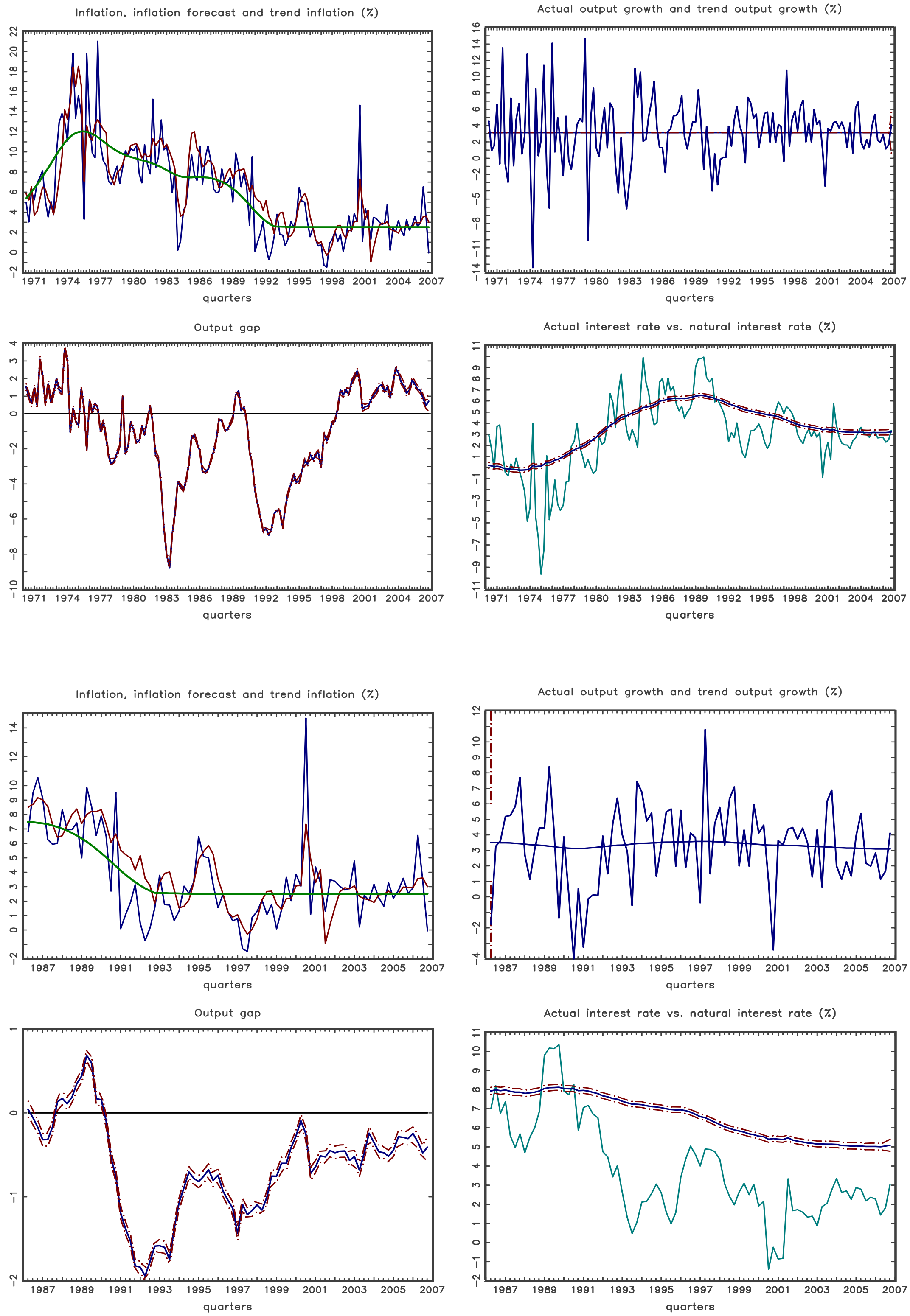


Figure 2.e: Inflation, output, and interest rate in Sweden, 1970:2-2006:4/1986:2-2006:4

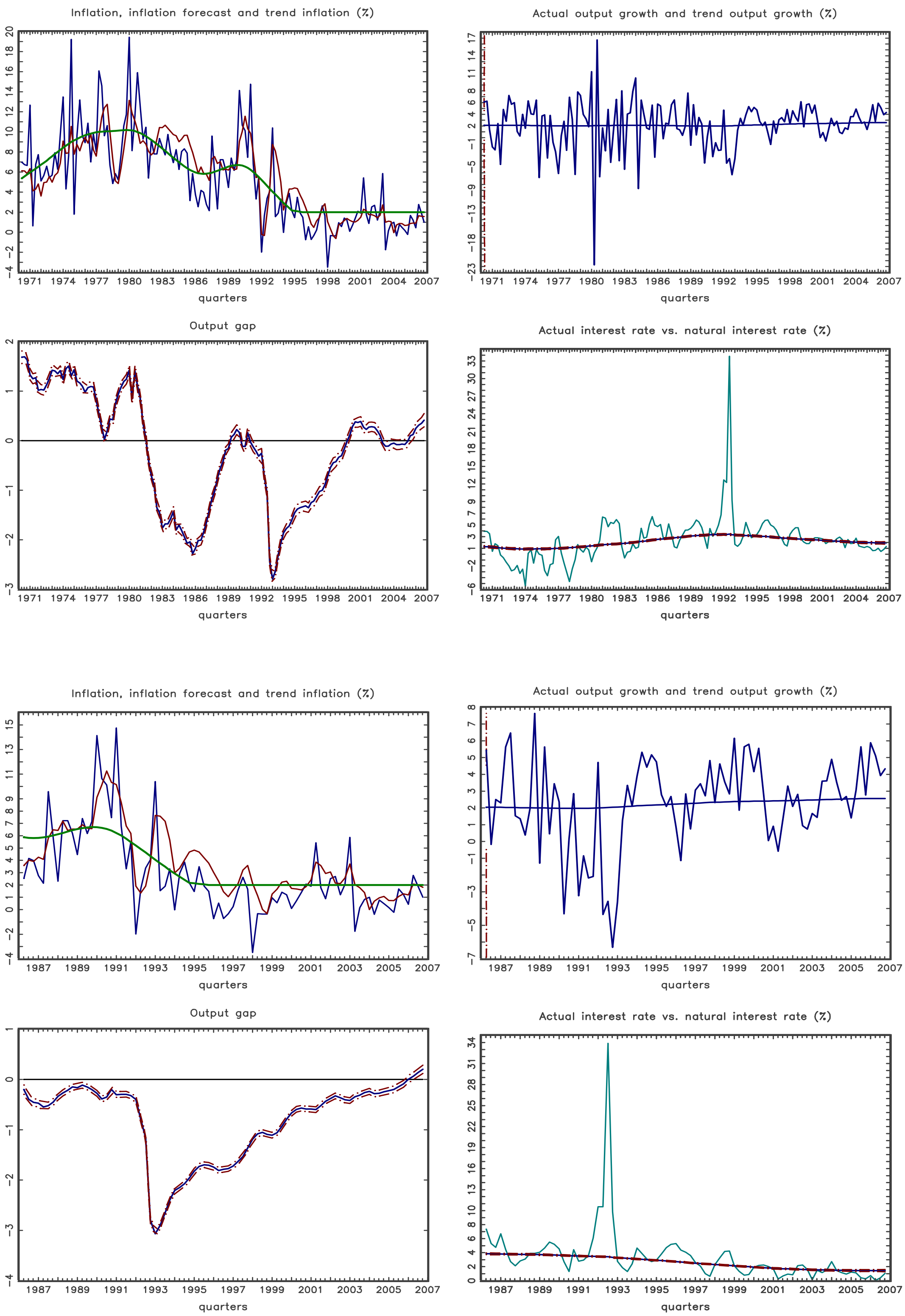


Figure 2.f: Inflation, output, and interest rate in Norway, 1979:2-2006:4/1986:2-2006:4

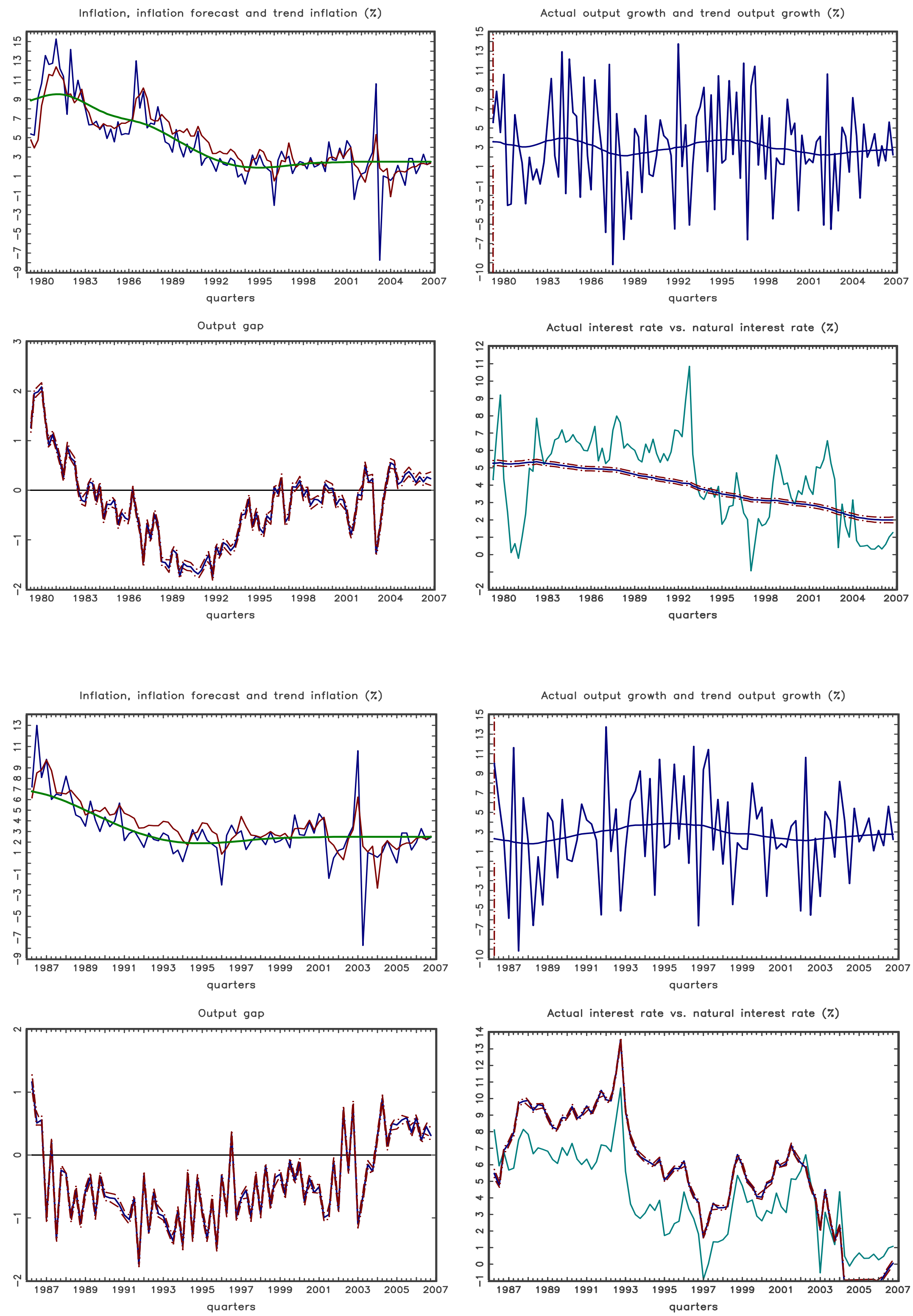


Figure 3: Inflation, output, and interest rate in Chile, 1986:2-2006:4

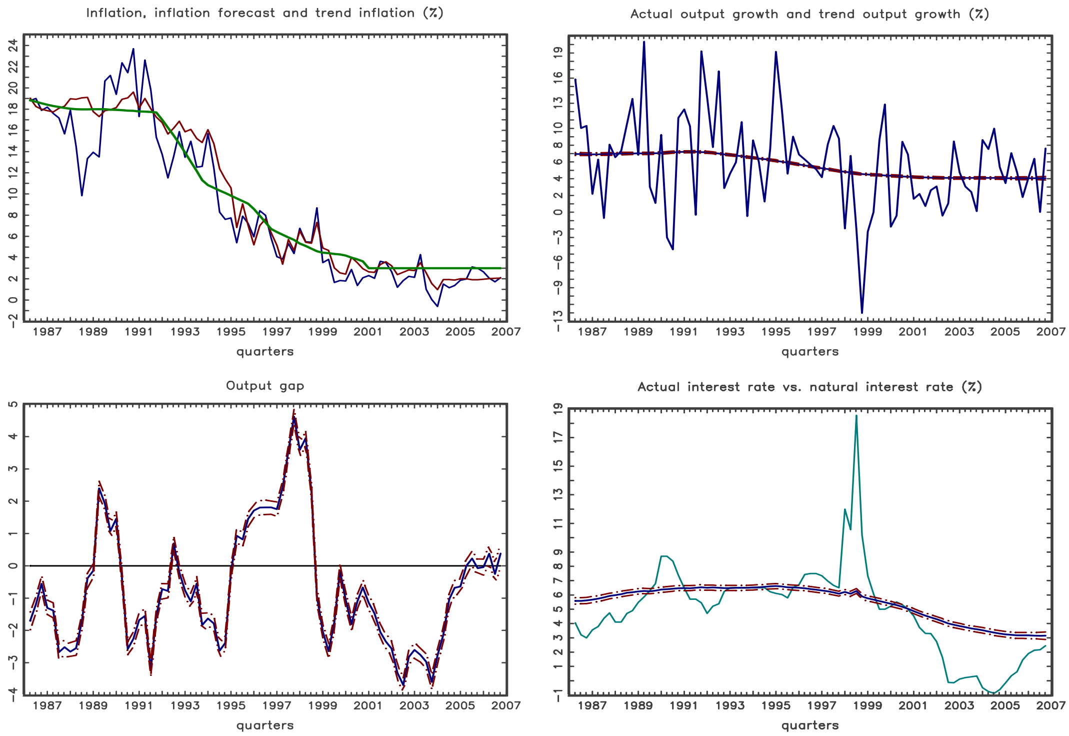
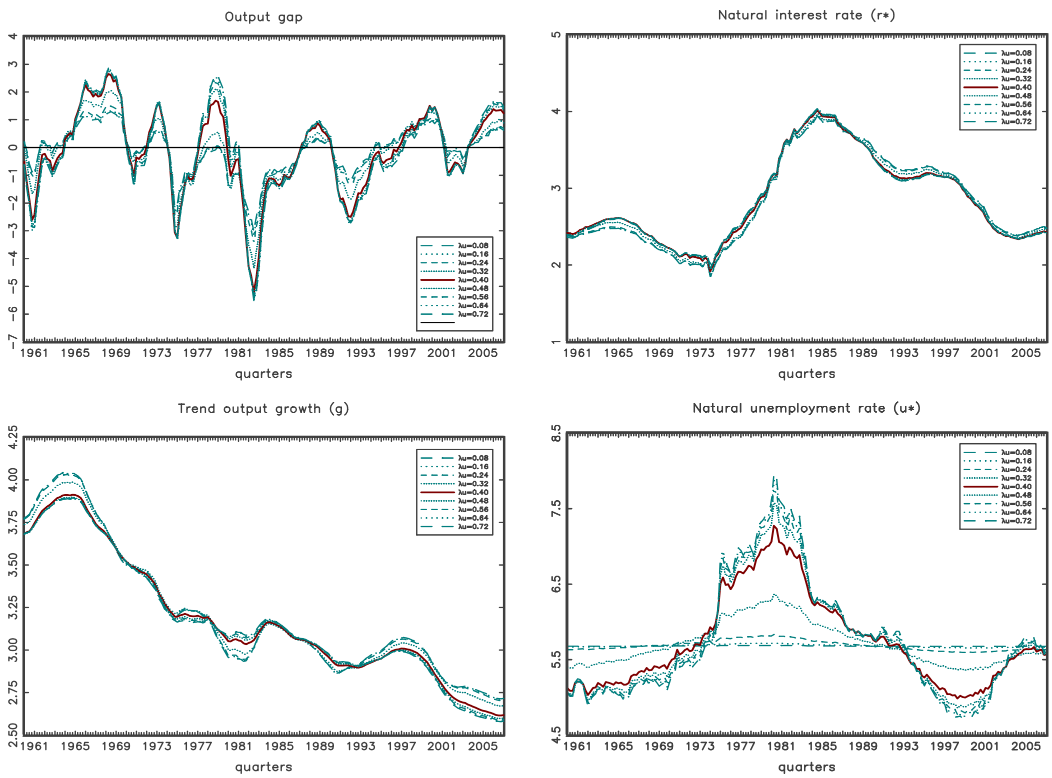
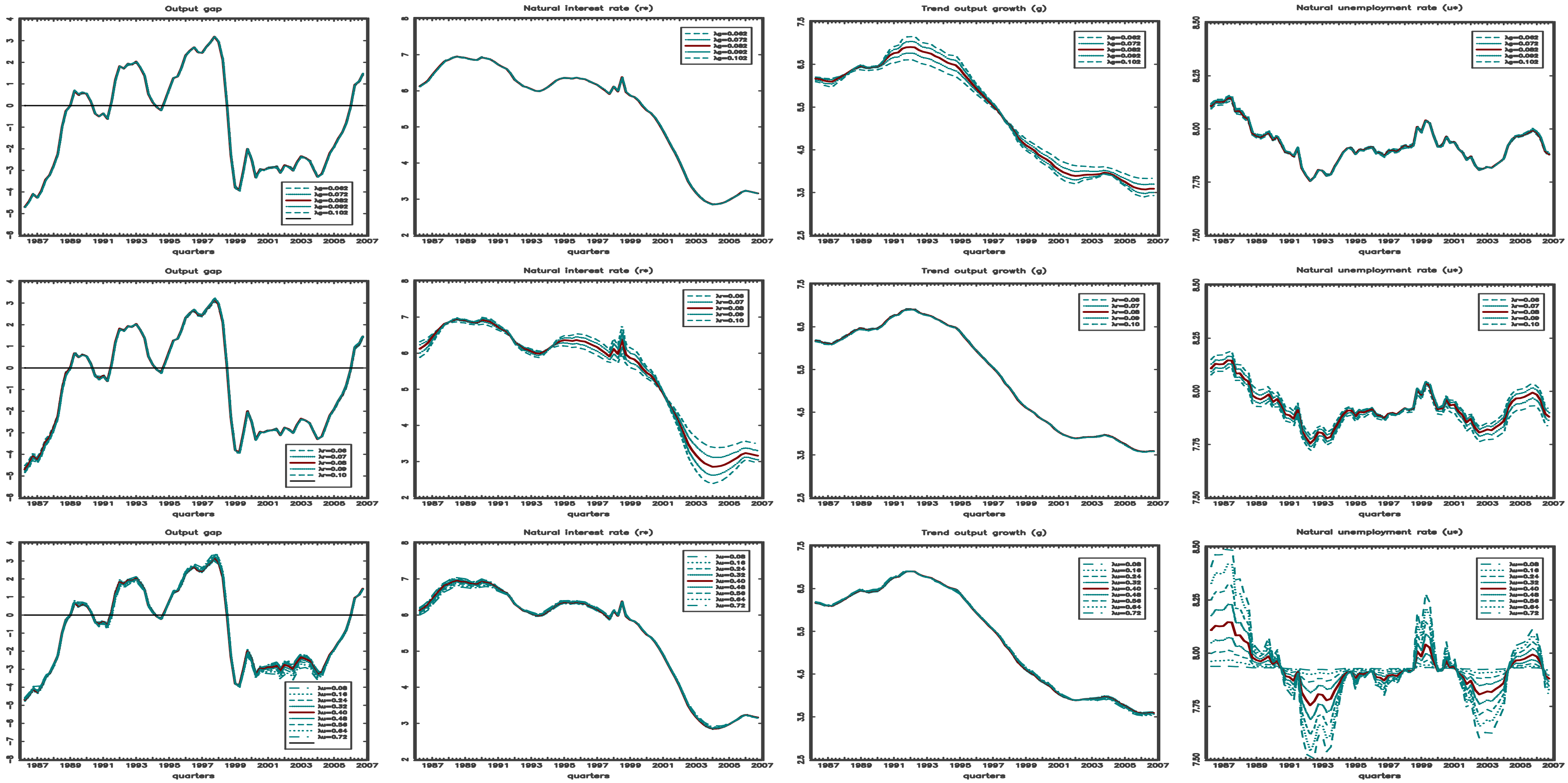


Figure 4: Potential growth rate, output gap, neutral interest rate, and natural unemployment rate in the U.S., 1960:1 - 2007:2
(Grid-search results for extended model)



Note: the panel shows the unobservables for different grid values of λ_u .

Figure 5: Potential growth rate, output gap, neutral interest rate, and natural unemployment rate in Chile, 1986:2 - 2006:4 (Grid-search results for extended model)



Note: the panel shows the unobservables for different grid values of λ_g (first row), λ_r (second row) and λ_u (third row).

Figure 6: Potential output growth in nine countries, 1970:2-2006:4

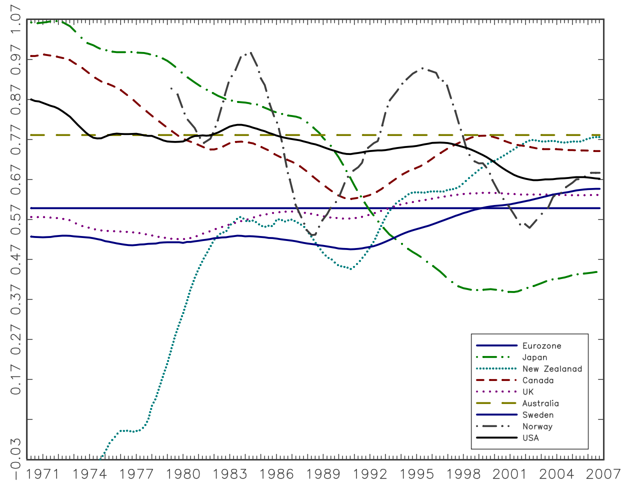
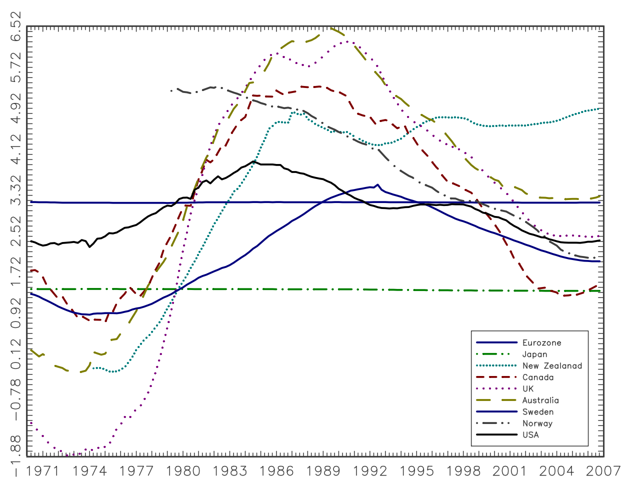
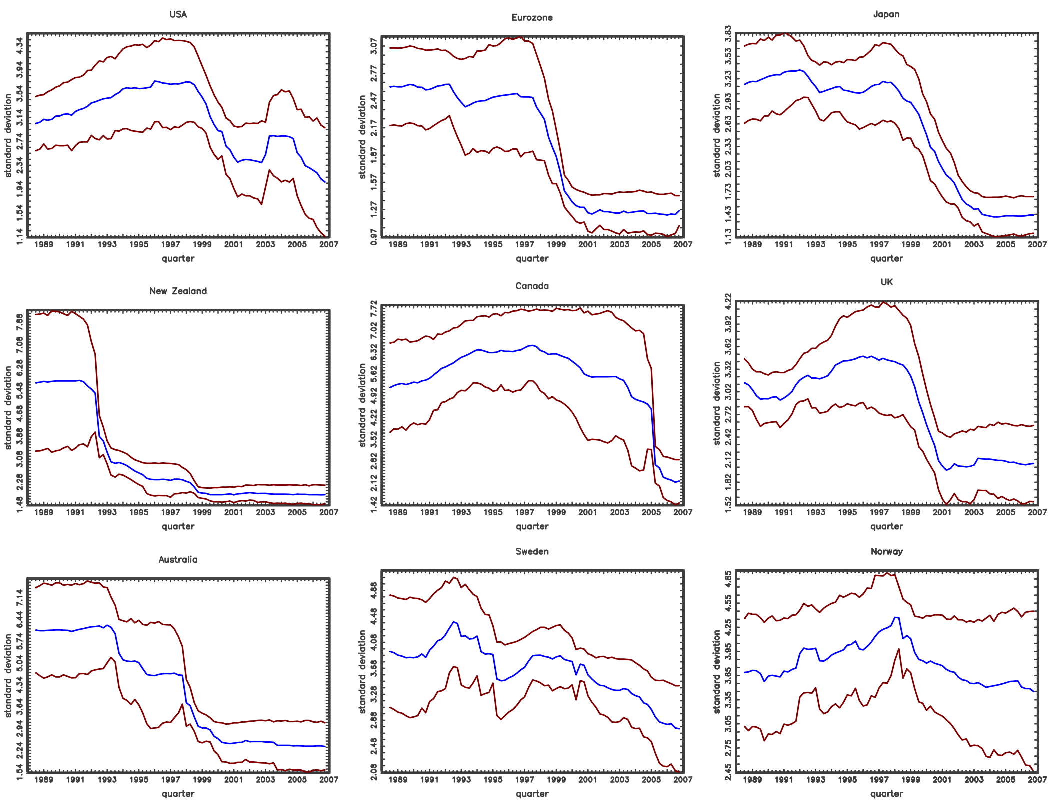


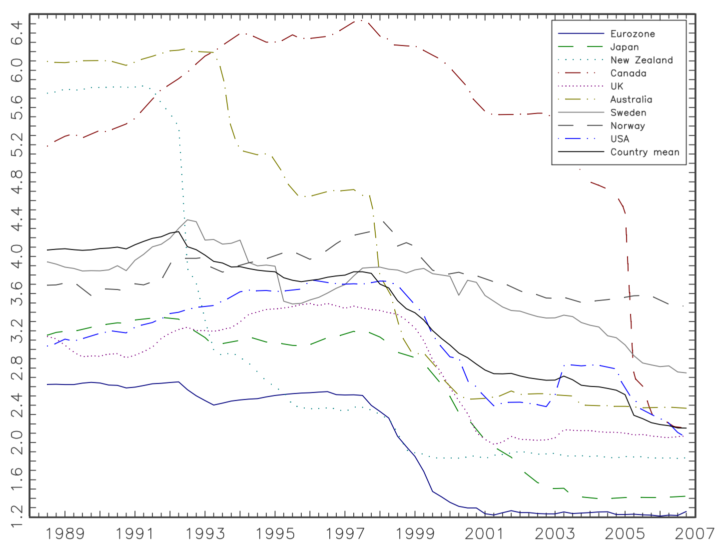
Figure 7: Neutral real interest rate in nine countries, 1970:2-2006:4



Note: the sample period for New Zealand and Norway begins in 1974:2 and 1979:2, respectively.

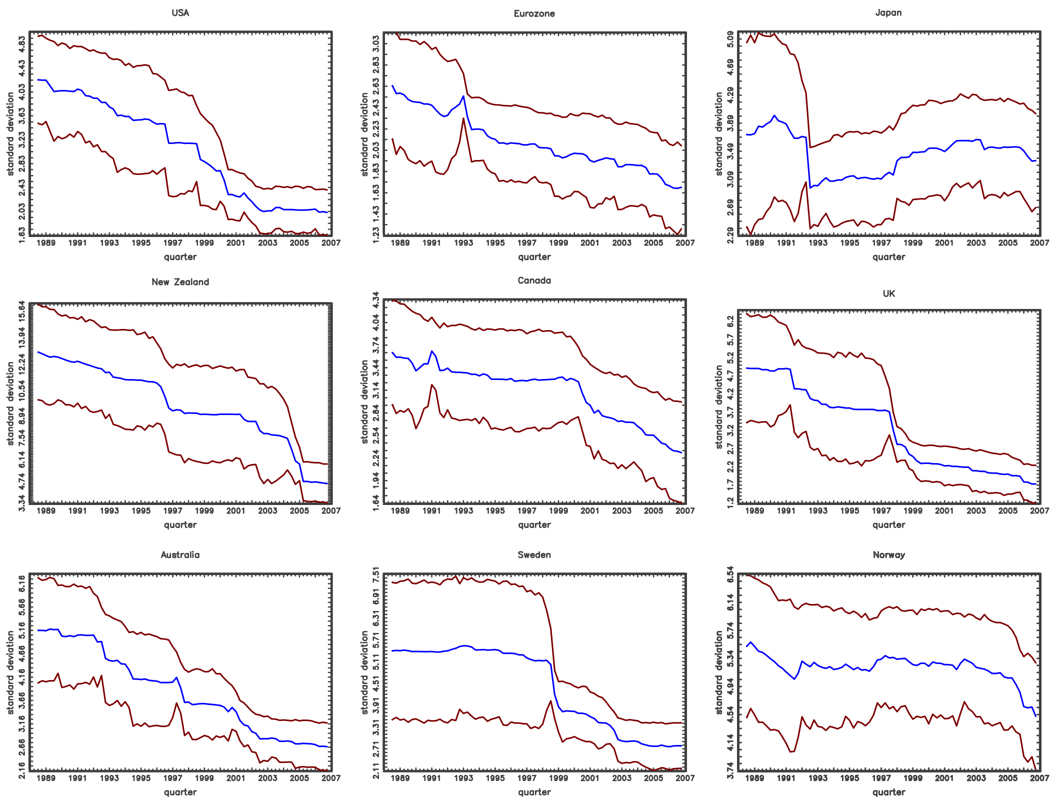
Figure 8.a: Inflation volatility trends in nine countries, 1970:2-2006:4





Notes:
The sample period for New Zealand and Norway begins in 1974:2 and 1979:2, respectively.
The window size for the rolling estimations is 74 quarters. For instance, the first point estimate corresponds to 1988:3 which is based in the period 1970:2-1988:3.

Figure 8.b: Actual output growth volatility trends in nine countries, 1970:2-2006:4


USA
Eurozone
Japan
New Zealand
Canada
UK
Australia
Sweden
Norway
standard deviation
quarter

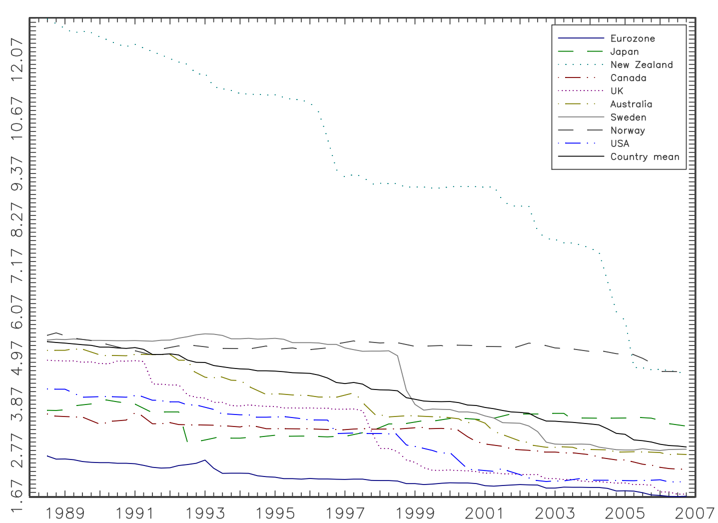
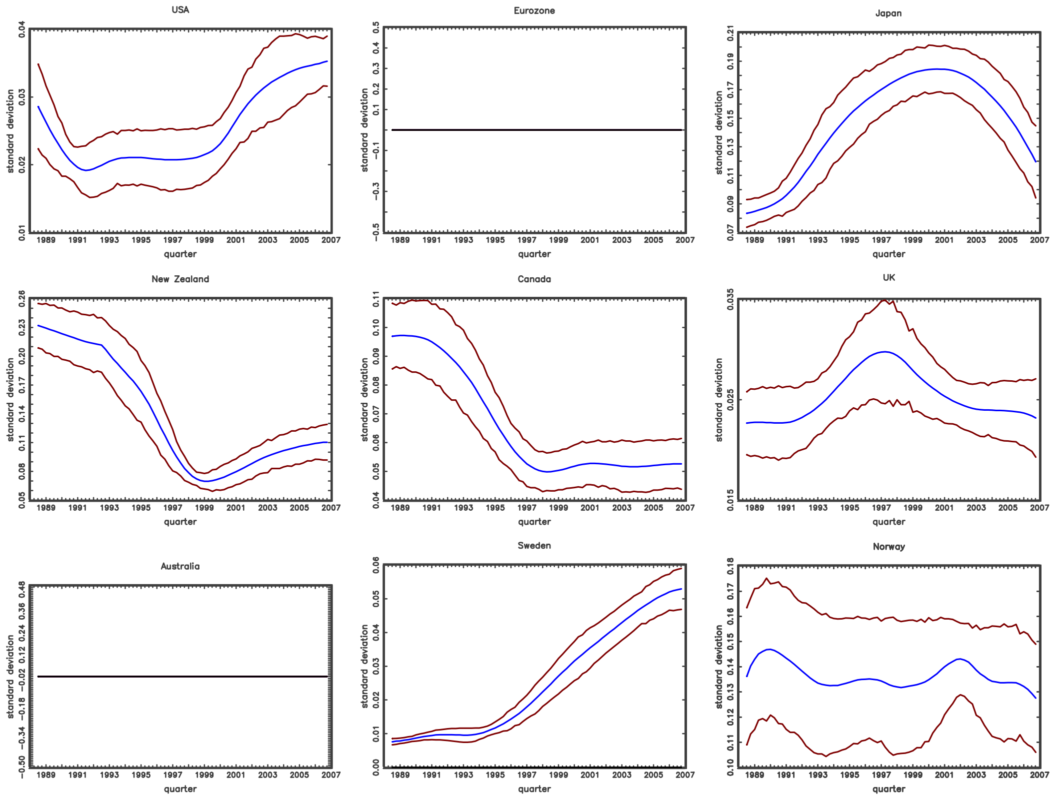

Eurozone
Japan
New Zealand
Canada
UK
Australia
Sweden
Norway
USA
Country mean

Figure 8.c: Potential output growth volatility trends in nine countries, 1970:2-2006:4



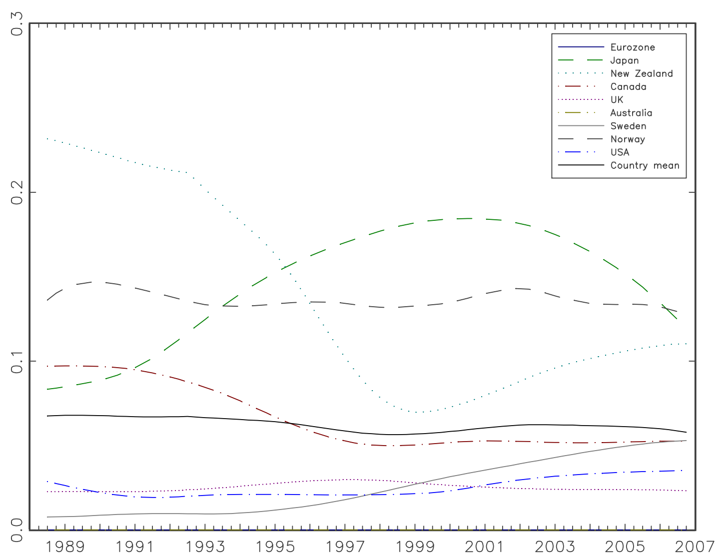
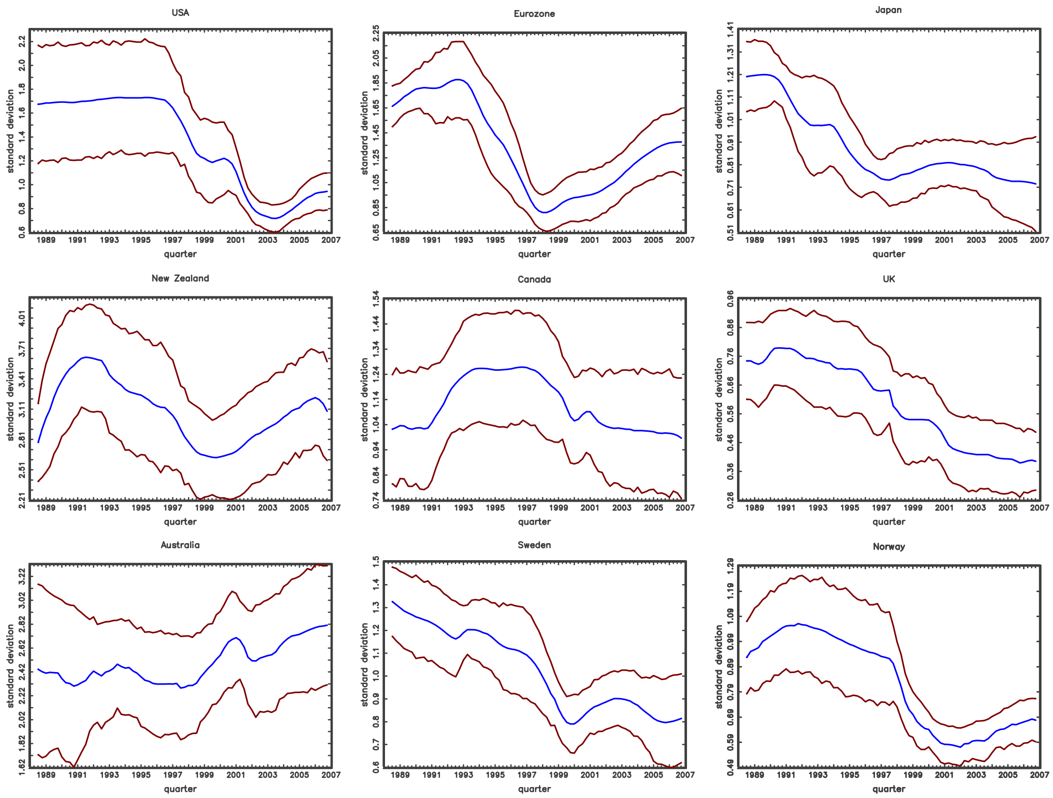


Figure 8.d: Output gap volatility trends in nine countries, 1970:2-2006:4



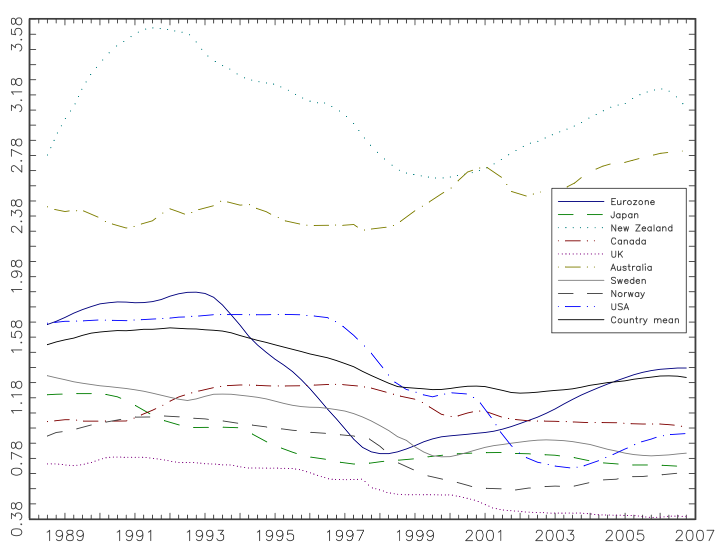
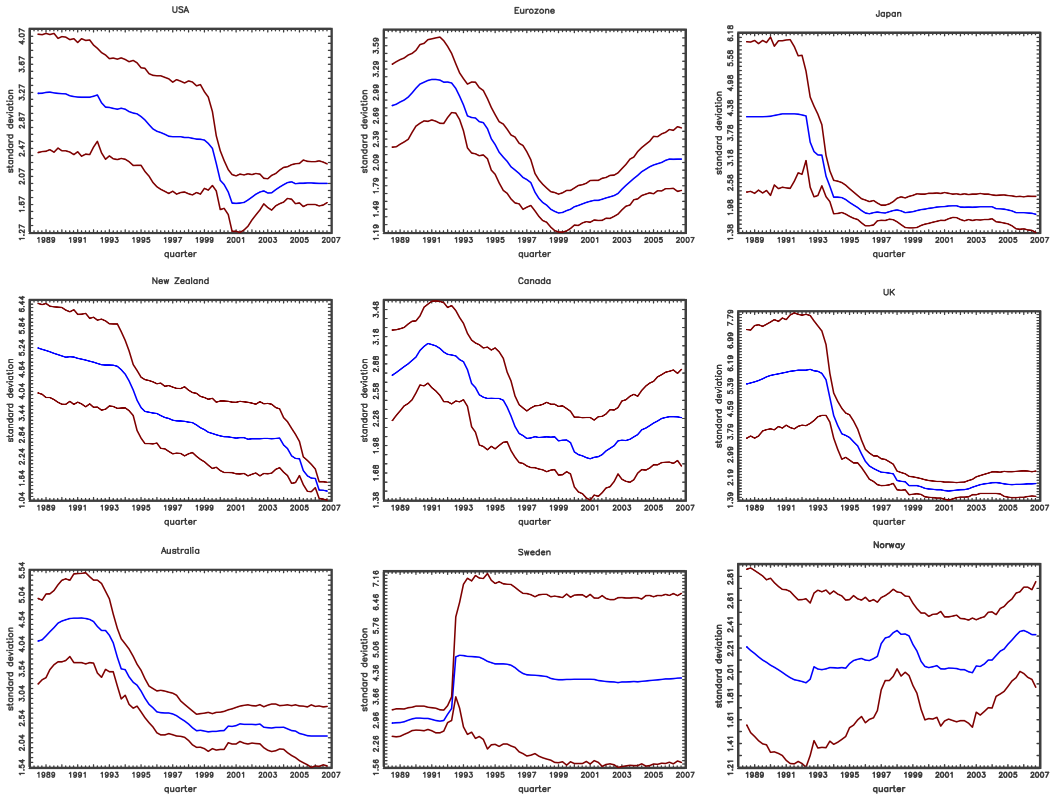


Figure 8.e: Actual interest rate volatility trends in nine countries, 1970:2-2006:4



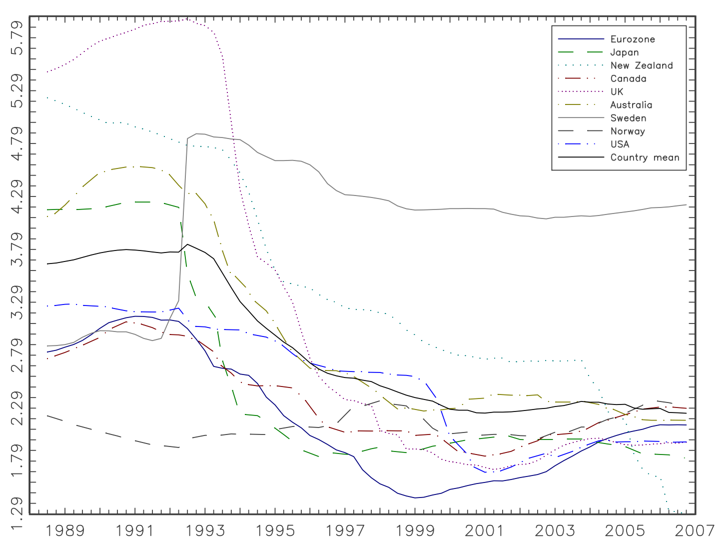
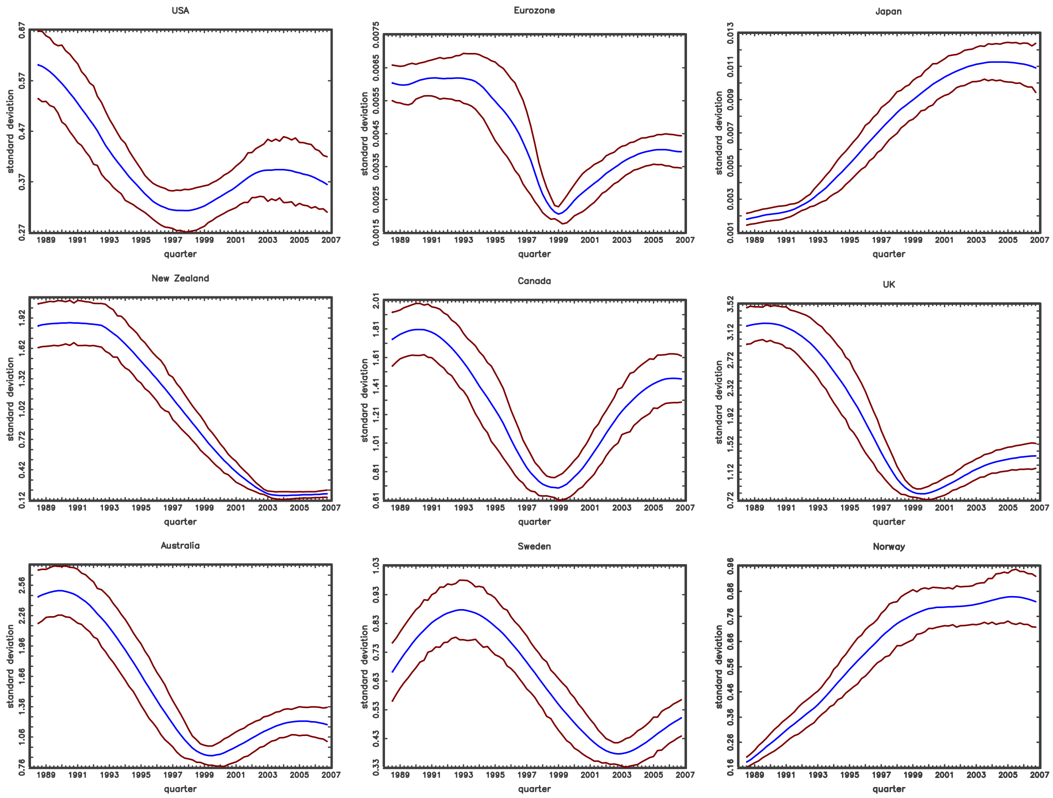


Figure 8.f: Natural interest rate volatility trends in nine countries, 1970:2-2006:4



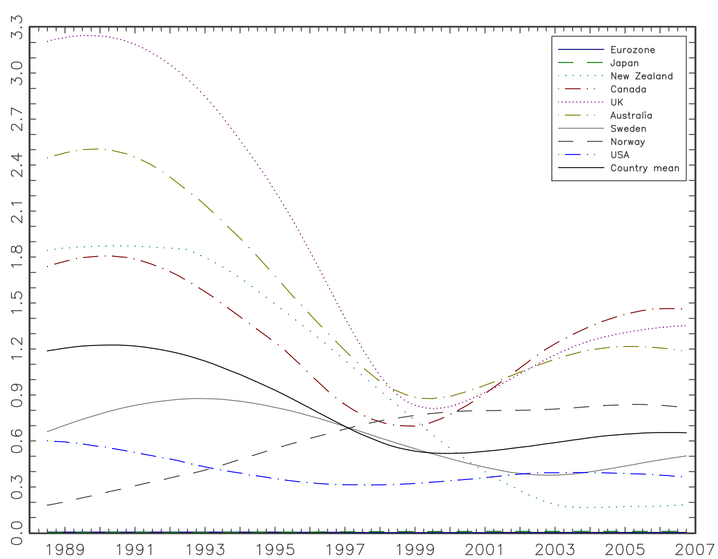


Figure 8.g: Interest rate gap volatility trends in nine countries, 1970:2-2006:4

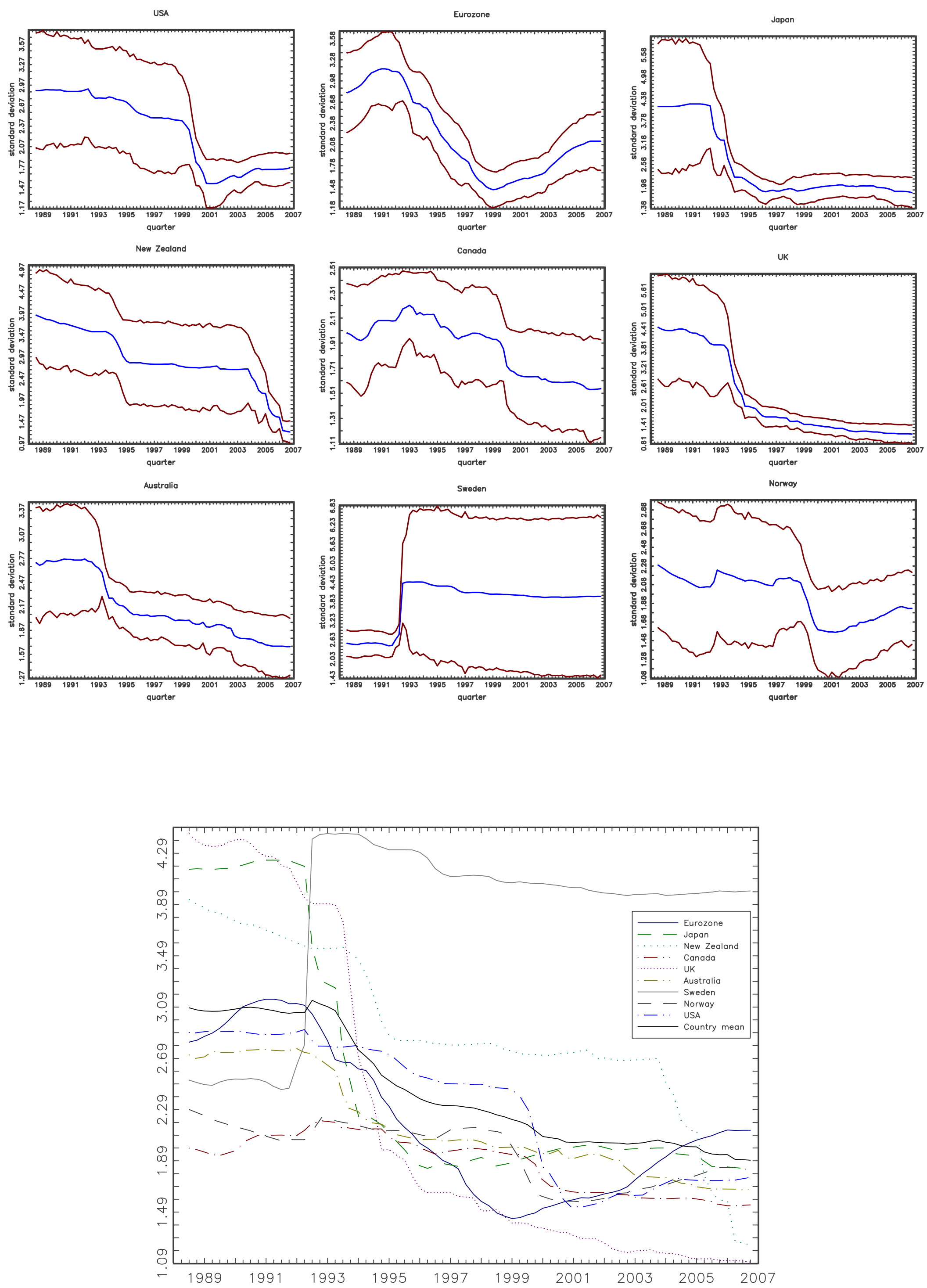
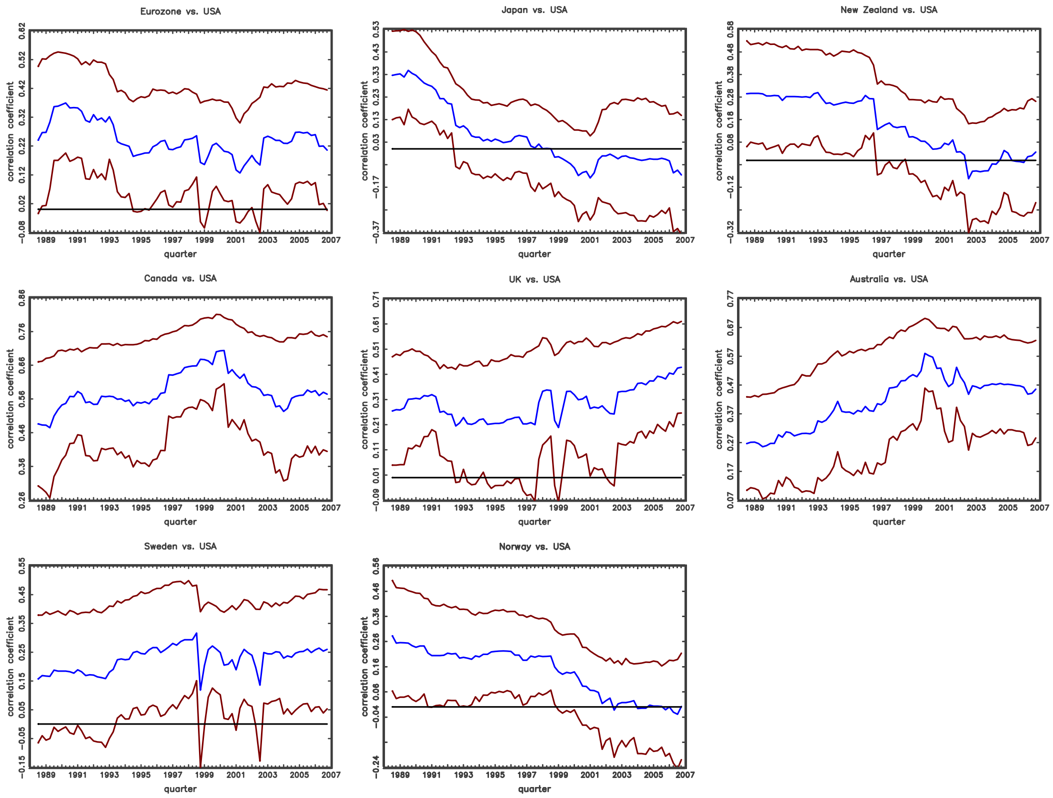


Figure 9.a: Actual output growth correlation with U.S., 1970:2-2006:4



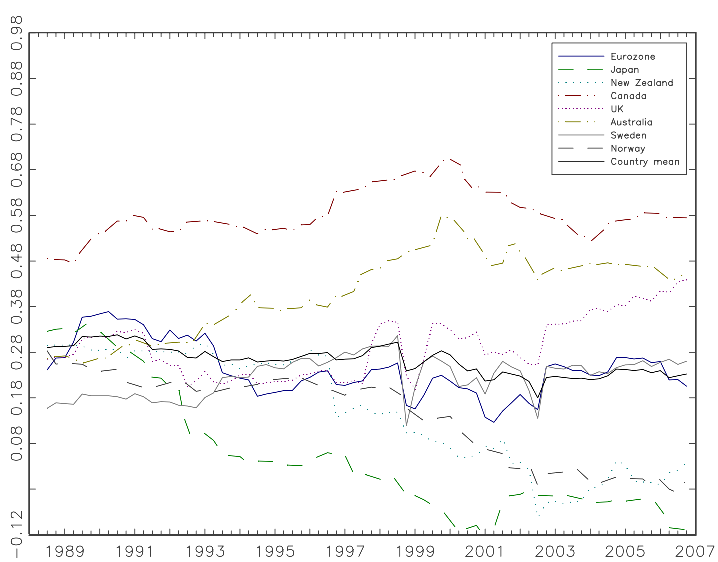
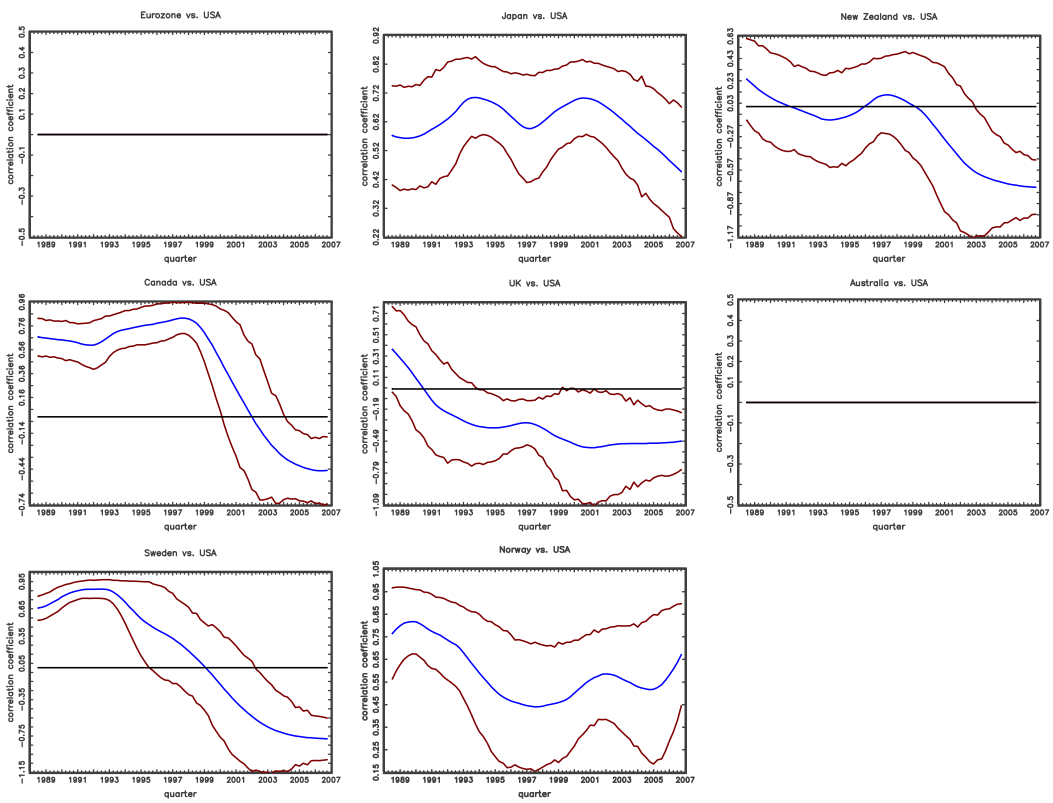


Figure 9.b: Potential output growth correlation with U.S., 1970:2-2006:4



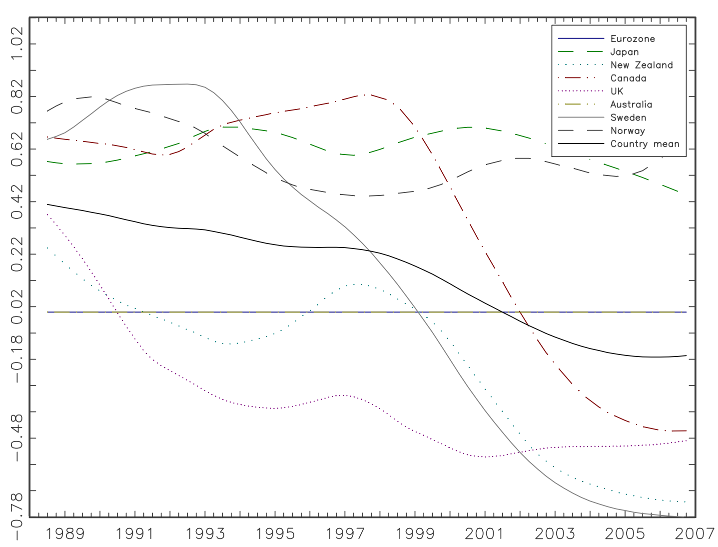
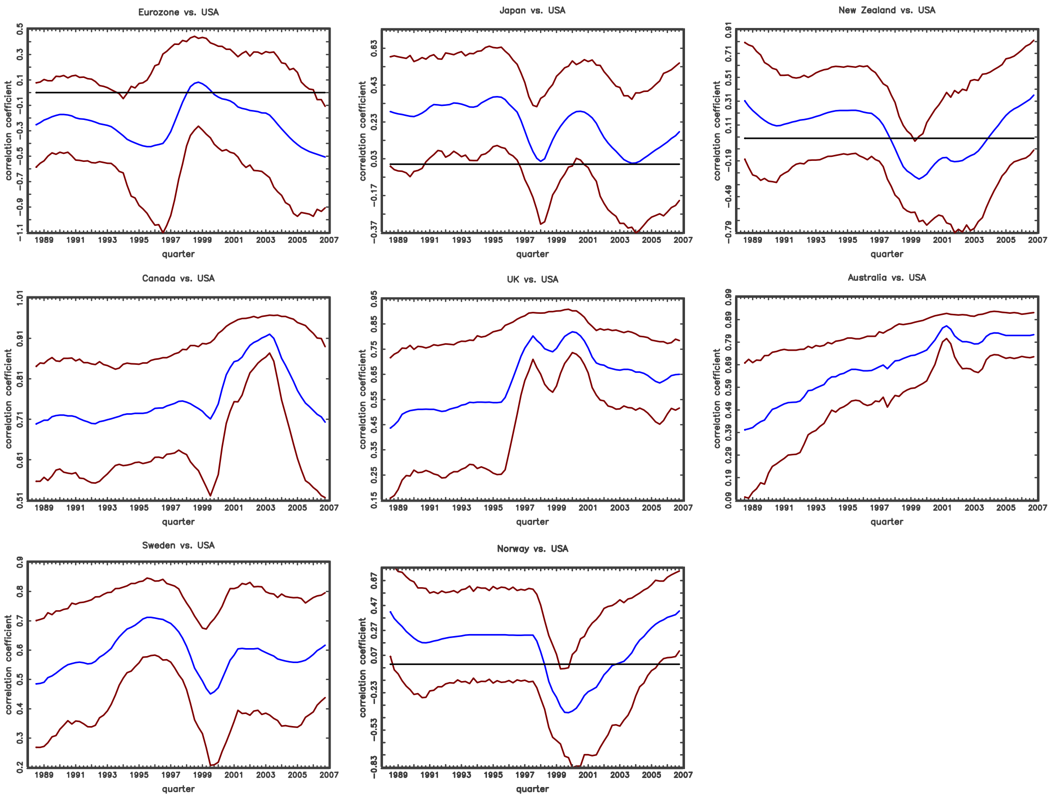


Figure 9.c: Output gap correlation with U.S., 1970:2-2006:4



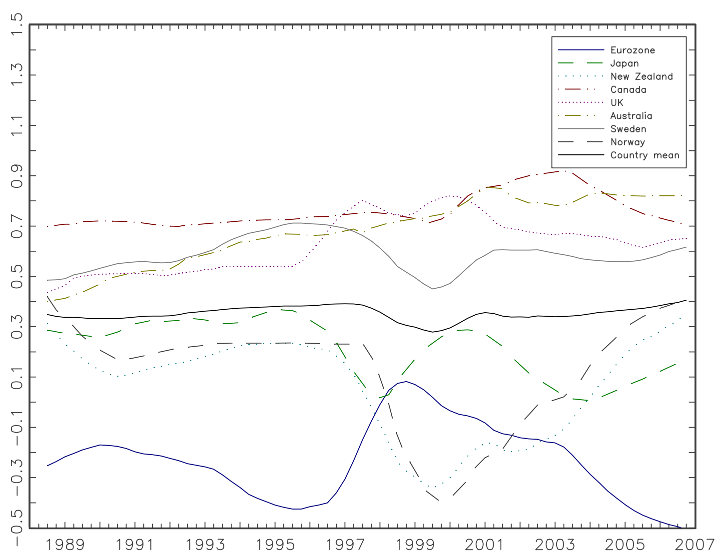
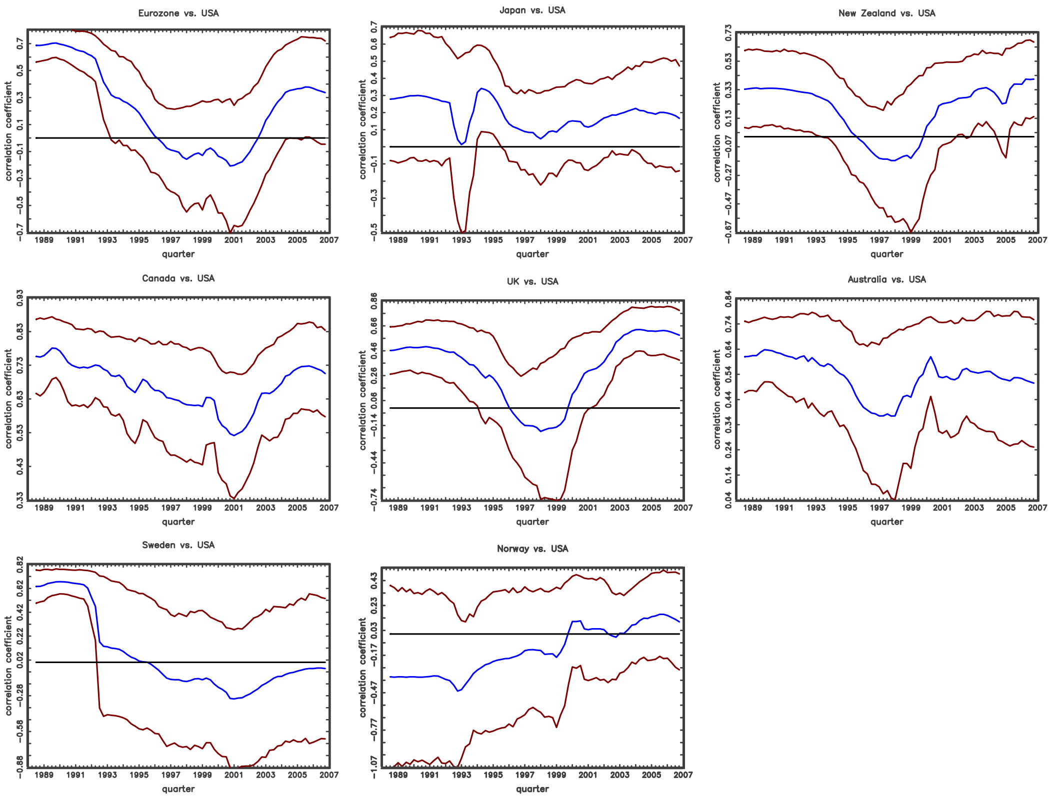


Figure 9.d: Actual interest rate correlation with U.S., 1970:2-2006:4



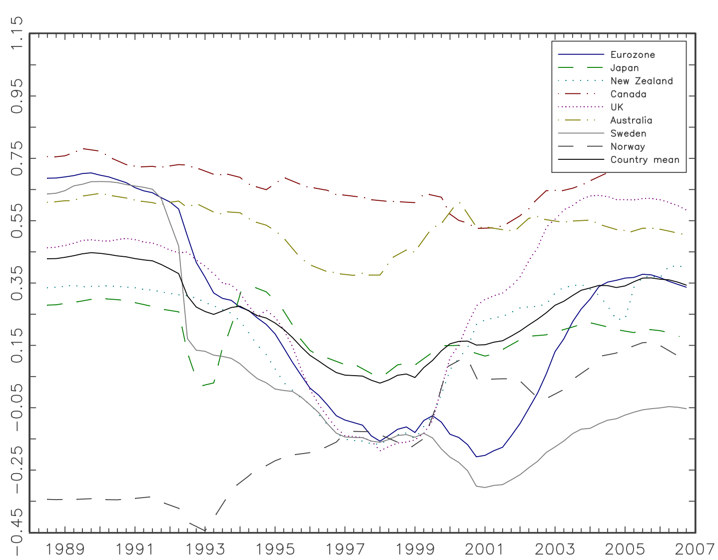
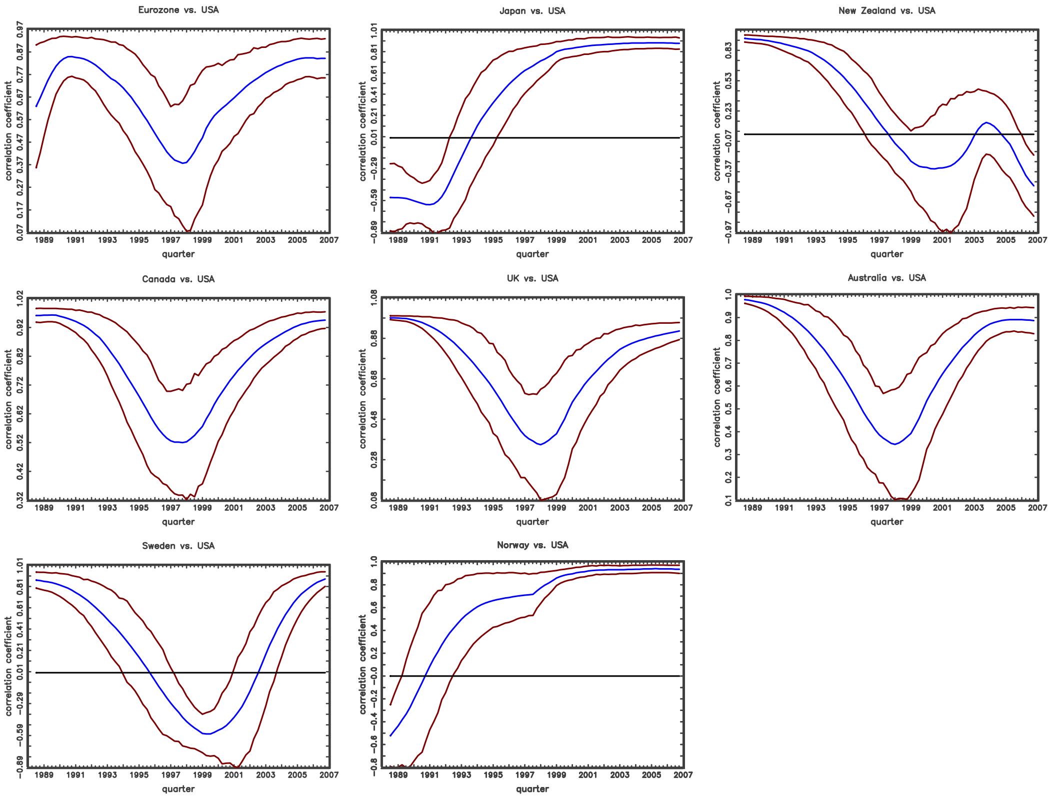


Figure 9.e: Natural interest rate correlation with U.S., 1970:2-2006:4



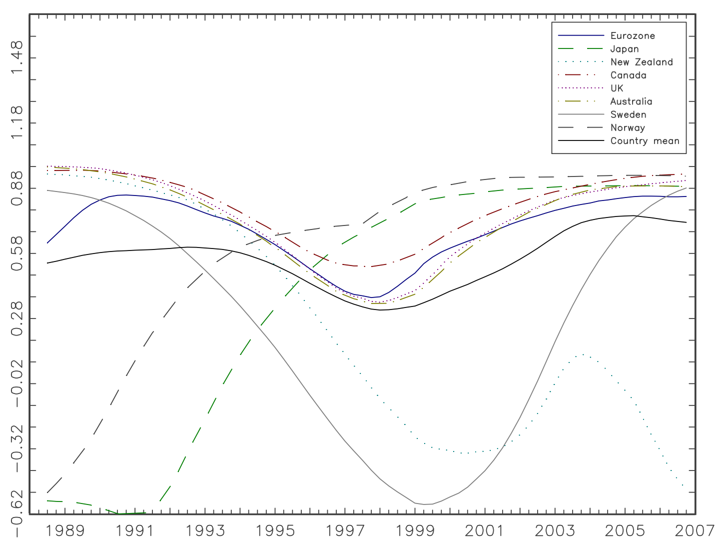


Figure 9.f: Interest rate gap correlation with U.S., 1970:2-2006:4