Inflation Dynamics in a Small Open Economy Model under Inflation Targeting

Some Evidence from Chile

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This Draft: November, 2007

Abstract

The paper estimates a small open economy DSGE model, written along the lines of Galí and Monacelli (QJE 2005) and Lubik and Schorfheide (JME 2007), on Chilean data for the inflation targeting period (1999-2007). We study the specification of the policy rule followed by the Central Bank, the dynamic response of inflation to domestic and external shocks, and how these dynamics change under different policy parameters. We use the DSGE-VAR methodology (Del Negro and Schorfheide 2007) to assess the robustness of the conclusions to the presence of model misspecification.

JEL CLASSIFICATION: C32

KEY WORDS: Bayesian Analysis, Small Open Economy DSGE Models, Model Misspecification

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1 Introduction

Following the influential work of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003), many central banks are building and estimating dynamic stochastic general equilibrium (DSGE) models with nominal rigidities and using them for policy analysis. This new generation of sticky price (and wage) models typically emphasizes that relative price distortions caused by firms' partial inability to respond to changes in the aggregate price level lead to an inefficient use of inputs and in turn to welfare losses. In such an environment monetary policy can partially offset these relative price distortions by stabilizing aggregate inflation. In an open economy environment the policy problem is more complicated because domestic price movements are tied to exchange rate and terms-of-trade movements.

DSGE models can be used at different stages of the policy making process. If the structure of the theoretical model is enriched up to a point that the model is able to track historical time series, DSGE model can be used as a tool to generate multivariate macroeconomic forecasts. Monetary policy is typically represented by an interest rate feedback rule and the innovations in the policy rule can be interpreted as modest, unanticipated changes in monetary policy. These impulse response can then be used to determine, say, what interest rate change is necessary to keep inflation rates near a target level over the next year or two. Finally, one can use DSGE models to qualitatively or quantitatively analyze more fundamental changes in monetary policy, i.e., inflation versus output targeting, fixed versus floating exchange rates.

An important concern in the use of DSGE models is that some of the cross-equation restrictions generated by the economic theory are misspecified. This misspecification potentially distorts forecasts as well as policy predictions. In a series of papers (Del Negro and Schorfheide (2004), Del Negro, Schorfheide, Smets, and Wouters (2007), and Del Negro and Schorfheide (2007)), we developed an econometric framework that allows us to gradually relax the cross-coefficient restrictions and construct an empirical model that can be regarded as structural vector autoregression and retains many of the features of the underlying DSGE model, at least to the extent that they are not grossly inconsistent with historical time series. We refer to the empirical model as DSGE-VAR.

Based on a small open economy model developed by Gali and Monacelli (2005) and modified for estimation purposes by Lubik and Schorfheide (2007), we present estimation results for such a DSGE-VAR in this paper for the Chilean economy, using data on output growth, inflation, interest rates, exchange rates, and terms of trade. Throughout the

1990's monetary policy transitioned toward an inflation targeting regime. This transition was completed only in 1999, which leaves a fairly short sample for the estimation of an empirical model for monetary policy analysis. An important advantage of the DSGE-VAR framework is that it allows us to estimate a vector autoregressive system with a short time series. Roughly speaking, this estimation augments actual observations by hypothetical observations, generated from a DSGE model, to determine the coefficients of the vector autoregression. Over time, as more actual observations become available, our procedure will decrease or increase the fraction of actual observations in the combined sample, depending on whether or not the data contain evidence of model misspecification.

The empirical analysis is divided in four parts. We begin by estimating both the DSGE model as well as the DSGE-VAR. The DSGE-VAR produces estimates of the coefficients of the underlying theoretical model along with the VAR coefficients. Our discussion first focuses on the monetary policy rule estimates. Starting from a prior that implies a strong reaction of the Central Bank to inflation movements, we find that since 1999 the central bank did not react in a significant way to exchange rate or terms of trade movements, which is consistent with the official policy statements. In the second part, we study the fit of our small scale DSGE model. Not surprisingly based on our earlier work, the fit of the empirical vector autoregressive model can be improved by relaxing the theoretical cross-coefficient restrictions. More interestingly, due to the short sample size the fraction of DSGE model generated observations in the mixed sample that is used for the estimation of the VAR is much higher, than, say in estimations that we have conducted for the U.S. As a consequence, the dynamics of the DSGE-VAR closely resemble those of the underlying DSGE model, which is documented in the third part of the empirical analysis. Here, we are focusing specifically in how the various structural shocks affect inflation movements.

In the final part of the empirical analysis we study the effect of changes in the monetary policy rule. Conceptually, this type of analysis is very challenging. If one beliefs that the DSGE model is not misspecified, then one can determine the behavioral responses of firms and households, by resolving the model under alternative policy rules. Empirical evidence of misspecification of cross-equation restrictions, on the other hand, raises questions about the reliability of the DSGE model's policy implications. In Del Negro and Schorfheide (2007) we have developed tools that allow us, under particular invariance assumptions, to check for the robustness of the DSGE model conclusions to presence of misspecificaton. We apply some of these tools to ask what would happen the variability of inflation if the central bank would respond more or less to inflation as well as terms of trade movements.

There is a substantial amount of empirical literature on the Chilean economy (Chumacero 2005, Céspedes and Soto 2006; these two papers also provide a survey of the existing literature) which studies many of the issues analyzed in the paper: the specification of the policy rule, the dynamics of inflation, the responses of domestic variables to external shocks. To our knowledge for most of this literature the estimation period comprises the 1990s, a period of convergence toward full fledged inflation targeting (see Banco Central de Chile 2007). Because of concerns about structural change between the early phase of inflation targeting and the current one, we do not use the early period in the estimation. This choice makes our results not directly comparable with those of the previous literature. A particularly close paper to ours is the one of Caputo et al. (2007), who estimate a somewhat more sophisticated small open economy DSGE model using Bayesian methods on Chilean data. Again, their use of 1990s data makes the results not directly comparable. In future work it would be interesting though to apply some of the techniques used in our paper to a larger scale small open economy DSGE model such as theirs.

The remainder of the paper is organized as follows. Section 2 contains a description of the small open economy model. The DSGE-VAR framework developed in Del Negro and Schorfheide (2007) is reviewed in Section 3. The data set used for the empirical analysis is discussed in Section 4. Empirical results are summarized in Section 5 and Section 6 concludes.

2 A Small Open Economy Model

We now describe a simple small-open-economy DSGE model for the Chilean economy. The model has been previously estimated with data from Australia, Canada, New Zealand, and the United Kingdom in Lubik and Schorfheide (2007). It is a simplified version of the model developed by Galí and Monacelli (2005) to which we refer for details. We restrict our exposition to the key equilibrium conditions, represented in log-linearized form. All variables below are measured in percentage deviations from a stochastic balanced growth path, induced by a technology process, A_t, that follows an AR(1) process in growth rates:

$$\Delta \ln A_t = \gamma + \widetilde{z}_t, \quad \widetilde{z}_t = \rho_z \widetilde{z}_{t-1} + \sigma_z \epsilon_{z,t}. \tag{1}$$

Here Δ denotes the temporal difference operator.

We begin with a characterization of monetary policy. We assume that monetary policy is described by an interest rate rule, where the central bank adjusts its instrument in response to movements in CPI inflation [check] and output growth. Moreover, we allow for the possibility of including nominal exchange rate depreciation or changes in the terms of trade in the policy rule:

$$\widetilde{R}_t = \rho_R \widetilde{R}_{t-1} + (1-\rho_R)\left[\psi_1 \widetilde{\pi}_t + \psi_2(\Delta\widetilde{y}_t + \widetilde{z}_t) + \psi_3 \Delta\widetilde{x}_t\right] + \sigma_R \epsilon_t^R. \tag{2}$$

Since $\widetilde{y}_t$ measures percentage deviations from the stochastic trend induced by the productivity process A_t, output growth is given by $\Delta\widetilde{y}_t + \widetilde{z}_t$. We use $\Delta\widetilde{x}_t$ to represent either exchange rate or terms of trade changes. In order to match the persistence in nominal interest rates, we include a smoothing term in the rule with $0 < \rho_R < 1$. ϵ_t^R is an exogenous policy shock which can be interpreted as the non-systematic component of monetary policy.

The household behavior in the home country is described by a consumption Euler equation in which we use equilibrium conditions to replace domestic consumption by a function of domestic output $\widetilde{y}_t$, foreign output $\widetilde{y}_t^*$ and terms of trade $\widetilde{q}_t$:

$$\begin{aligned} \widetilde{y}_t \;=\; & I\!\!E_t \widetilde{y}_{t+1} - [\tau + \alpha(2-\alpha)(1-\tau)]\left(\widetilde{R}_t - E_t\widetilde{\pi}_{t+1}\right) + \rho_z \widetilde{z}_t \\ & -\alpha\left[\tau + \alpha(2-\alpha)(1-\tau)\right] I\!\!E_t[\Delta\widetilde{q}_{t+1}] + \alpha(2-\alpha)\frac{1-\tau}{\tau} I\!\!E_t[\Delta\widetilde{y}^*_{t+1}], \end{aligned} \tag{3}$$

where $0 < \alpha < 1$ is the fraction of imported goods consumed by domestic households and τ is their intertemporal substitution elasticity. Terms of trade are defined as the relative price of exports in terms of imports. Notice that the equation reduces to its closed economy variant when $\alpha = 0$.

Optimal price setting of domestic firms leads to the open economy Phillips curve:

$$\widetilde{\pi}_t = \beta E_t \widetilde{\pi}_{t+1} + \alpha\beta E_t \Delta\widetilde{q}_{t+1} - \alpha\Delta\widetilde{q}_t + \frac{\kappa}{\tau + \alpha(2-\alpha)(1-\tau)}\left(\widetilde{y}_t - \widetilde{\overline{y}}_t\right), \tag{4}$$

where $\widetilde{\overline{y}}_t = -\alpha(2-\alpha)\frac{1-\tau}{\tau}\widetilde{y}_t^*$ is potential output in the absence of nominal rigidities. Again, the closed economy variant obtains when $\alpha = 0$. The slope coefficient $\kappa > 0$ is a function of underlying structural parameters, such as labor supply and demand elasticities and parameters capturing the degree of price stickiness. Since we do not use any additional information from the underlying model we treat κ as structural.

In order to study exchange rate policies we introduce the nominal exchange rate e_t via the definition of the CPI. Assuming that relative PPP holds, we have:

$$\widetilde{\pi}_t = \Delta\widetilde{e}_t + (1-\alpha)\Delta\widetilde{q}_t + \widetilde{\pi}_t^*, \tag{5}$$

where $\widetilde{\pi}_t^*$ is a world inflation shock which we treat as an unobservable. An alternative interpretation, as in Lubik and Schorfheide (2005), is that $\widetilde{\pi}_t^*$ captures misspecification, or deviations from PPP. Since the other variables in the exchange rate equation are observed, this relaxes the potentially tight cross-equation restrictions embedded in the model.

Instead of solving endogenously for the terms of trade, we add a law of motion for their growth rate to the system:

$$\Delta\widetilde{q}_t = \rho_q \Delta\widetilde{q}_{t-1} + \sigma_q \epsilon_{q,t}. \tag{6}$$

As discussed in Lubik and Schorfheide (2007), this specification is not fully consistent with the underlying structural model. Since firms do have a certain modicum of market power, the prices of internationally traded products are not exogenous to the economy even if its size relative to the rest of the world goes to zero. However, it turns out that the specification with exogenous terms of trade generates a better empirical fit.

Equations (2) to (5) form a rational expectations system that determines the law of motion for domestic output, inflation, and interest rates, as well as the nominal exchange rate. We treat technology growth $\widetilde{z}_t$, Equation (1), and the terms of trade $\widetilde{q}_t$, Equation (6) as exogenous. Moreover, we assume that rest-of-the-world output and inflation, $\widetilde{y}_t^*$ and $\widetilde{\pi}_t^*$, follow exogenous autoregressive processes:

$$\widetilde{\pi}_t^* = \rho_{\pi^*}\widetilde{\pi}_{t-1}^* + \sigma_{\pi^*}\epsilon_{\pi^*,t}, \quad \widetilde{y}_t^* = \rho_{y^*}\widetilde{y}_{t-1}^* + \sigma_{y^*}\epsilon_{y^*,t}. \tag{7}$$

The rational expectations model comprised of Equations (2) to (7) can be solved with standard techniques, e.g., Sims (2002). We collect the DSGE model parameter in the vector θ defined as

$$\theta = [\psi_1, \psi_2, \psi_3, \rho_R, \alpha, \beta, \tau, \rho_z, \rho_q, \rho_{\pi^*}, \rho_{y^*}, \sigma_R, \sigma_z, \sigma_{\pi^*}, \sigma_{y^*}].$$

Moreover, we assume that the innovations $\epsilon_{R,t}$, $\epsilon_{z,t}$, $\epsilon_{q,t}$, $\epsilon_{\pi^*,t}$, and $\epsilon_{y^*,t}$ are independent standard normal random variables. We stack the innovations in the vector ϵ_t.

3 The DSGE-VAR Approach

To capture potential misspecification of stylized small-open economy model described in the previous section we will embed it into a vector autoregressive specification that allows us to relax cross-coefficient restrictions. We refer to the resulting empirical model as DSGE-VAR. We have developed this DSGE-VAR framework in a series of papers including Del Negro and Schorfheide (2004), Del Negro, Schorfheide, Smets, and Wouters (2007), and Del Negro and Schorfheide (2007). The remainder of this section will review the setup in Del Negro and Schorfheide (2007), which is used in the subsequent empirical analysis.

Let us write Equation (2), which describes the policymaker's behavior, in more general form as:

$$y_{1,t} = x_t'\beta_1(\theta) + y_{2,t}'\beta_2(\theta) + \epsilon_{1,t}\sigma_R, \tag{8}$$

where $y_t = [y_{1,t}, y_{2,t}']'$ and the $k \times 1$ vector $x_t = [y_{t-1}', \ldots, y_{t-p}', 1]'$ is composed of the first p lags of y_t and an intercept. Here $y_{1,t}$ corresponds to the nominal interest rate $\widetilde{R}_t$ and the subvector $y_{2,t}$ is composed of output growth, inflation, exchange rate depreciation, and terms of trade changes:

$$y_{2,t} = [(\Delta\widetilde{y}_t + \widetilde{z}_t), \widetilde{\pi}_t, \Delta\widetilde{e}_t, \Delta\widetilde{q}_t].$$

The vector-valued functions $\beta_1(\theta)$ and $\beta_2(\theta)$ interact with x_t and $y_{2,t}$ to reproduce the policy rule.

The solution of the linearized DSGE model presented in Section 2 generates a moving average representation of $y_{2,t}$ in terms of the ϵ_t's. We proceed by approximating this moving average representation with a p-th order autoregression, which we write as

$$y_{2,t}' = x_t'\Psi^*(\theta) + u_{2,t}'. \tag{9}$$

Ignoring the approximation error for a moment, the one-step ahead forecast errors $u_{2,t}$ are functions of structural innovations ϵ_t. Assuming that under the DSGE model the law of motion for $y_{2,t}$ is covariance stationary for every θ, we define the moment matrices

$$\Gamma_{XX}(\theta) = I\!E_\theta^D[x_t x_t'] \quad \text{and} \quad \Gamma_{XY_2}(\theta) = I\!E_\theta^D[x_t y_{2,t}'].$$

In our notation $I\!E_\theta^D[\cdot]$ denotes an expectation taken under the probability distribution for y_t and x_t generated by the DSGE model conditional on the parameter vector θ. We define the VAR approximation of $y_{2,t}$ through

$$\Psi^*(\theta) = \Gamma_{XX}^{-1}(\theta)\Gamma_{XY_2}(\theta). \tag{10}$$

The equation for the policy instrument (8) can be rewritten by replacing $y_{2,t}$ with expression (9):

$$y_{1,t} = x_t'\beta_1(\theta) + x_t'\Psi^*(\theta)\beta_2(\theta) + u_{1,t}, \tag{11}$$

Let $u_t' = [u_{1,t}, u_{2,t}']$ and define

$$\Sigma^*(\theta) = \Gamma_{YY}(\theta) - \Gamma_{YX}(\theta)\Gamma_{XX}^{-1}(\theta)\Gamma_{XY}(\theta). \tag{12}$$

If we assume that the u_t's are normally distributed, denoted by $u_t \sim \mathcal{N}(0, \Sigma_*(\theta))$, then Equations (9) to (12) define a restricted VAR(p) for the vector y_t. While the moving-average representation of y_t under the linearized DSGE model does in general not have an exact VAR representation, the restriction functions $\Psi^*(\theta)$ and $\Sigma^*(\theta)$ are defined such that the covariance matrix of y_t is preserved. Let $I\!E^{VAR}_{\Psi,\Sigma}[\cdot]$ denote expectations under the restricted VAR. It can be verified that

$$I\!E^{VAR}_{\Psi^*(\theta),\Sigma^*(\theta)}[y_t y_t'] = I\!E^{D}_{\theta}[y_t y_t']. \tag{13}$$

This point is important since we will assess the affect of policy rule changes on the volatility of inflation and output in the subsequent empirical analysis.

To account for potential misspecification we now relax the DSGE model restrictions and allow for VAR coefficient matrices Ψ and Σ that deviate from the restriction functions $\Psi^*(\theta)$ and $\Sigma^*(\theta)$. Thus,

$$\begin{aligned} y_{1,t} &= x_t'\beta_1(\theta) + x_t'\Psi\beta_2(\theta) + u_{1,t}, \\ y_{2,t}' &= x_t'\Psi + u_{2,t}', \end{aligned} \tag{14}$$

and $u_t \sim \mathcal{N}(0,\Sigma)$. Our analysis is cast in a Bayesian framework in which initial beliefs about the DSGE model parameter θ and the VAR parameters Ψ and Σ are summarized in a prior distribution. Our prior distribution for Ψ and Σ is chosen such that conditional on a DSGE model parameter θ

$$\begin{aligned} \Sigma|\theta &\sim \mathcal{IW}\Big(T^*\Sigma^*(\theta), T^*-k\Big) \\ \Psi|\Sigma,\theta &\sim \mathcal{N}\left(\Psi^*(\theta), \frac{1}{T^*}\Big[(B_2(\theta)\Sigma^{-1}B_2(\theta)')\otimes\Gamma_{XX}(\theta)\Big]^{-1}\right), \end{aligned} \tag{15}$$

where $\mathcal{IW}$ denotes the inverted Wishart distribution, $\mathcal{N}$ is a multivariate normal distribution, $B_1(\theta) = [\beta_1(\theta), 0_{k\times(n-1)}]$, and $B_2(\theta) = [\beta_2(\theta), I_{(n-1)\times(n-1)}]$.

A few remarks are in order. First, the distribution of prior mass around the restriction functions $\Psi^*(\theta)$ and $\Sigma^*(\theta)$ is controlled by the hyperparameter T^*, which we will re-parameterize in terms of multiples of the actual sample size T, that is $T^* = \lambda T$. Large values of λ imply that large discrepancies are unlikely to occur and the prior concentrates near the restriction functions. We consider values of λ on a finite grid Λ and use a data-driven procedure to determine an appropriate value for this hyperparameter. A natural criterion to select λ in a Bayesian framework is the marginal data density

$$p_\lambda(Y) = \int p(Y|\Psi,\Sigma,\theta)p_\lambda(\Psi,\Sigma,\theta)d(\Psi,\Sigma,\theta). \tag{16}$$

Here $p_\lambda(\Psi,\Sigma,\theta)$ is a joint prior distribution for the VAR coefficient matrices and the DSGE model parameters. This prior is obtained by combining the prior in (15) with a prior density for θ, denoted by $p(\theta)$:

$$p_\lambda(\Psi, \Sigma, \theta) = p(\theta) p_\lambda(\Sigma|\theta) p_\lambda(\Phi|\Sigma, \theta). \tag{17}$$

We define

$$\hat{\lambda} = \text{argmax}_{\lambda \in \Lambda}\; p_\lambda(Y). \tag{18}$$

As discussed in Del Negro, Schorfheide, Smets, and Wouters (2007), $\hat{\lambda}$ and the marginal likelihood ratio $p_{\lambda=\hat{\lambda}}(Y)/p_{\lambda=\infty}(Y)$ provide an overall measure of fit for the DSGE model. If there is a large discrepancy between the autocovariances implied by the DSGE model and the sample autocovariances, $\hat{\lambda}$ will be small and the marginal likelihood ratio will be large.

Second, holding the innovation matrix $\Sigma^*(\theta)$ constant, $\Gamma_{XX}(\theta)$ tends to be large – hence the prior variance of Ψ^Δ small – whenever θ implies that the endogenous variables are highly persistent. We view this as an attractive feature of the prior. Since due to the presence of transversality conditions DSGE model solutions are restricted to be stationary, our prior steers us away from VAR parameterizations that imply non-stationarity and explosiveness. Third, our prior is also computationally convenient. We use Markov-Chain-Monte Carlo methods described in Del Negro and Schorfheide (2007) to generate draws from the joint posterior distribution of Ψ, Σ, and θ as well as to evaluate the marginal data density $p_\lambda(Y)$. We refer to empirical model comprised of the likelihood function associated with the restricted VAR in Equation (14) and the prior distributions $p_\lambda(\Psi, \Sigma|Y)$, given in (15), and $p(\theta)$ as DSGE-VAR(λ).

Finally, a word on identification of structural shocks. Up to this point, we expressed the VAR in terms of one-step ahead forecast errors u_t. However, both for understanding the dynamics of the DSGE-VAR and for the purpose of policy analysis, it is more useful to express the VAR as a function of the structural shocks ϵ_t. It turns out that in our setup the monetary policy shock is identified through exclusion restrictions:

$$\begin{aligned} y_{1,t} &= x_t'\beta_1(\theta_) + [x_t'\Psi + u_{2,t}']\beta_2(\theta) + \epsilon_{1,t}\sigma_R \\ y_{2,t}' &= x_t'\Psi + u_{2,t}'. \end{aligned}$$

According to the underlying DSGE model, $u_{2,t}$ is a function of the monetary policy shock $\epsilon_{1,t}$ and other structural shocks $\epsilon_{2,t}$. We assume that the shocks $\epsilon_{2,t}$ have unit variance and are uncorrelated with each other and the monetary policy shock. We express $u_{2,t}$ as

$$u_{2,t}' = \epsilon_{1,t} A_1 + \epsilon_{2,t}' A_2. \tag{19}$$

Straightforward matrix algebra leads to the following formulas for the effect of the structural shocks on $u'_{2,t}$:

$$A_1 = \left[\Sigma_{11} - \beta_2'\Sigma_{22}\beta_2 - 2(\Sigma_{12} - \beta_2'\Sigma_{22})\beta_2\right]^{-1}(\Sigma_{12} - \beta_2'\Sigma_{22}) \tag{20}$$

$$A_2'A_2 = \Sigma_{22} - A_1'\left[\Sigma_{11} - \beta_2'\Sigma_{22}\beta_2 - 2(\Sigma_{12} - \beta_2'\Sigma_{22})\beta_2\right]A_1. \tag{21}$$

While the above decomposition of the forecast error covariance matrix identifies A_1, it does not uniquely determine the matrix A_2. To do so, we follow the approach taken in Del Negro and Schorfheide (2004). Let $A'_{2,tr}A_{2,tr} = A_2'A_2$ be the Cholesky decomposition of $A_2'A_2$. The relationship between $A_{2,tr}$ and A_2 is given by $A_2' = A'_{2,tr}\Omega$, where Ω is an orthonormal matrix that is not identifiable based on the estimates of $\beta(\theta)$, Ψ, and Σ. However, we are able to calculate an initial effect of $\epsilon_{2,t}$ on $y_{2,t}$ based on the DSGE model, denoted by $A_2^D(\theta)$. This matrix can be uniquely decomposed into a lower triangular matrix and an orthonormal matrix:

$$A_2^{D'}(\theta) = A_{2,tr}^{D'}(\theta)\Omega^*(\theta). \tag{22}$$

To identify A_2 above, we combine $A'_{2,tr}$ with $\Omega^*(\theta)$. Loosely speaking, the rotation matrix is constructed such that in the absence of misspecification the DSGE model's and the DSGE-VAR's impulse responses to $\epsilon_{2,t}$ coincide. To the extent that misspecification is mainly in the dynamics as opposed to the covariance matrix of innovations, the identification procedure can be interpreted as matching, at least qualitatively, the short-run responses of the VAR with those from the DSGE model.

Since the matrix Ω does not affect the likelihood function, we can express the joint distribution of data and parameters as follows

$$p_\lambda(Y, \Psi, \Sigma, \Omega, \theta) = p(Y|\Psi, \Sigma)p_\lambda(\Psi, \Sigma|\theta)p(\Omega|\theta)p(\theta),$$

where $p(\Omega|\theta)$ is a point-mass centered at $\Omega^*(\theta)$. We use MCMC techniques described in Del Negro and Schorfheide (2007) to generate draws from the joint posterior distribution of Ψ, Σ, Ω, and θ.

4 Data

For our empirical analysis we compiled a data set comprised of observations on output growth, inflation, interest rates, exchange rates, and the terms of trade. Unless otherwise noted, the raw data are taken from the on-line database maintained by the Banco de Chile and seasonally adjusted. Output growth is defined as the log difference of real GDP, scaled by 400 to convert it into annualized percentages. To construct the inflation series, we pass the consumer price index extracted from the Central Bank database through the X12 filter (using the default settings in EVIEWS) to obtain a seasonally adjusted series. We then compute log differences, scaled by 400. The MPR serves as our measure of nominal interest rates.[1] Annualized depreciation rates are computed from log differences of the Chilean Pesos / US Dollar exchange rate series. Finally, annualized quarter-to-quarter percentage changes in the terms of trade are computed from the export and import price indices maintained by the Central Bank.

While we compile a data set that contains quarterly observations from 1986 to 2007, we restrict the estimation sample to the the period from 1999:I to 2006:IV and hence to the most recent monetary policy regime. Between 1991 and 1999 the Central Bank applied a partial inflation targeting approach that involved two nominal anchors: an exchange rate band as well as an inflation target. In 1999 the central bank implemented a floating exchange rate and the institutional arrangements for full inflation targeting. Official bank publications state that the operating objective of monetary policy is to keep annual inflation projections around 3.0% annually over a horizon of about two years. Indeed, the average inflation rate in our estimation sample is 2.8%. We plot the path of the inflation rate and the nominal interest rate in Figure 1 for the period 1986 to 2007. Throughout the 1990s, Chile experienced a decade-long disinflation process, and with the adoption of the 3% target inflation rate in 1999, inflation and nominal interest rates stabilized at a low level.

The average growth rate of real output, 4.4% during our sample period, provides an estimate of γ in (1). The average inflation rate can be viewed as an estimate of the target inflation rate π_* and the average nominal interest rate can be linked to the discount factor β, because our model implies $R_* = \gamma/\beta + \pi_*$. It turns out that the sum of average inflation and output growth is 7.2% and exceeds the average nominal interest rate, which is about 5.6%. Hence the sample averages are inconsistent with the model's steady state implications. Rather than estimating the steady state parameters jointly with the remaining DSGE model parameters and imposing the steady state restrictions, we decided to demean our observations and fit the DSGE model and the DSGE-VAR to demeaned data.

To provide further details on the features of our data set, we plot the Peso-USD exchange rate in Figure 2 together with percentage changes in the terms of trade. Both series exhibit very little autocorrelation and are very volatile. According to our DSGE model, the exchange

[1]Before 2001 the MPR is constructed following the same approach as in Chumacero (2005).

rate fluctuations are a function of inflation differentials and terms of trade movements:

$$\Delta\widetilde{e}_t = \widetilde{\pi}_t - \widetilde{\pi}_t^* - (1-\alpha)\Delta\widetilde{q}_t$$

The ROW inflation rate π_t^* is treated as a latent variable. In Figure 3 we plot the exchange rate depreciation as well as the "observable" exchange rate determinants $\widetilde{\pi}_t - (1-\alpha)\Delta\widetilde{q}_t$ for $\alpha = 0.2$. The difference between the two series can be interpreted as ROW inflation.

5 Empirical Results

5.1 Estimating the Policy Rule

This section investigates the feedback rule followed by the Central Bank in the recent period. As discussed before, Chile witnessed significant movements in the nominal exchange rate since it entered the freely floating regime in 1999. Moreover, it was subject to large swings in the terms of trade. Did the Central Bank respond to these movements in order to pursue the inflation target? Table 1 addresses these questions. The Table estimates the coefficients of the policy rule (2) under three different specifications. Under the first specification, which we refer to as *Baseline*, policy only responds to inflation and real output growth, in addition to the lagged interest rate. Under the second and third specification, called *Response to FX* and *Response to ToT* respectively, policy responds also to the exchange rate depreciation. Finally, under the *Response to ToT* specification the terms of trade also enter the feedback rule, in addition to real output growth, inflation, and the nominal exchange rate. We further consider a fourth specification where policy responds to year-over-year inflation (*Response to Y-o-Y Inflation*) as opposed to current quarter inflation. We consider this latter specification because the inflation target is stated in terms of year-over-year inflation, as opposed to quarter-to-quarter.

The first column of the top panel of Table 1 shows the prior mean and standard deviation for the policy parameters. On the ground that in 1999 Chile entered the full-fledged inflation target regime, and that in the previous decade it had acquired a reputation as inflation fighter by bringing down inflation, we posit a fairly high prior on ψ_1, the response to inflation. The prior is centered at 2.5 with a standard deviation of .5. The prior mean is higher than what is usually assumed for the U.S. The priors on ψ_2 and ψ_3, the response to real output growth and exchange rate depreciation respectively, and ρ_r, the persistence parameter, are similar to those used in Lubik and Schorfheide (2005). The priors on ψ_2 and ρ_r are also similar to that used in the estimation of DSGE models for the U.S. The prior on ψ_4, the response to a terms of trade depreciation, is centered at zero with a fairly large standard deviation, .5, since we did not want to impose any strong a priori view on the sign or the magnitude of the response.

The remaining columns of the top panel of Table 1 show the estimates of ψ_1, ψ_2, ψ_3, and ρ_r according to the four specifications. These estimates are obtained from the estimation of the (four different specification of the) DSGE model. The estimates of ψ_1 range from 1.8 to 2.3. The estimates of ψ_2 range from .15 to .2 and indicate a small, and barely significant, response of interest rates to output growth. The main focus of the section lies in the responses to nominal depreciation and to the terms of trade. From the third and fourth column of Table 1 we can see that the magnitude of the response to the exchange rate depreciation is small, less than .1. The response to terms of trade movement is also small at .08. From these estimates we conclude that the response of the Central Bank to nominal exchange rate and terms of trade movement has been small, if not zero. Further confirmation of this finding comes from the comparison of the marginal likelihoods, the measure of model's fit in a Bayesian context, across these different specification. The marginal likelihoods show that the best fit is achieved by the *Baseline* specification, where policy does not respond to the nominal exchange rate or terms of trade. If we are willing to put equal a priori probability on the different specifications we can compute the posterior odds of the alternative specifications relative to the Baseline. The Table shows that these posterior odds are fairly small, although the *Response to FX* model is not as clearly rejected by the data as the others. Finally, the results show that the specification where the authorities respond to year-over-year inflation is soundly rejected: Its posterior odd relative to the Baseline is minuscule. This result should not be interpreted as contradicting the statement that the Central Bank target is the year-over-year inflation, but simply providing information on the rule the Central Bank follows to achieve this target.

As is well known, there are pros and cons associated with full information estimation if one is interested in the parameters of a particular equation in the system, in this case the policy rule. On the one hand, if the cross-equation restrictions imposed by the model are correct the full information estimates are more efficient than those from other instrumental variable estimators. On the other hand, to the extent that these cross-equation restrictions are invalid, the full information estimates may not be credible, and limited information methods may be preferable. In this context, DSGE-VAR strikes a compromise between full and limited information estimation, as it allows for deviations from the cross-equation restrictions. In the case at hand it may such a compromise may be necessary, since the amount of data available is limited and therefore estimators that completely ignore the restrictions ($\lambda = 0$) may not be efficient. At the same time, the model used here imposes quite strong restrictions (exogeneity of the terms of trade, for one) and therefore one may not want to dogmatically impose the restrictions ($\lambda = \infty$). For these reasons, the lower panel of Table 1 show the estimates of ψ_1, ψ_2, ψ_3, and ρ_r according to the three specifications (*Baseline*, *Response to FX*, *Response to ToT*) of interest using DSGE-VAR with $\lambda = 2$. We will justify the choice of $\lambda = 2$ in the next section. For now, notice that for each specification the marginal likelihood of the DSGE-VAR (lower panel) is substantially higher than that of the corresponding DSGE model (upper panel), validating some of the concerns about the cross-equation restrictions.

Under DSGE-VAR the estimates of ψ_1 andψ_2, the response to inflation and output growth respectively, are very similar across all specifications, with ψ_1 about 2.75 and ψ_2 about .125. Most importantly, the results regarding the response to exchange rate depreciation and the terms of trade from the DSGE models are confirmed. The response to both is quite small, about .08. In addition, even under the DSGE-VAR the alternative specifications, *Response to FX* and *Response to ToT*, are rejected relative to the *Baseline*, with posterior odds similar to those in the upper panel. In summary, we seem to have some robust evidence that the Central Bank did not respond to movements in the nominal exchange rate or the terms of trade in the recent period.

5.2 The Fit of the Small Open Economy DSGE Model

This section discusses the fit of the small open economy DSGE model, and its implications in terms of the estimation for the DSGE model parameters. We ask the question: How much does fit improve as we relax the cross-equation restrictions imposed by the DSGE model? The answer to this question sheds some light on the extent to which the data is at odds with the restrictions imposed by the model. Importantly from a policy perspective, this analysis is also informative as to whether forecasting should be conducted with models that are relatively rich in terms of structure, or with models that are loosely parameterized such as VARs with relatively flat priors.

Table 2 shows the log marginal likelihood for the DSGE model as well as for the DSGE-VAR, where λ varies in a grid from .75 to 5. As discussed in section 3, high values of λ

correspond to tightly imposed cross-equation restrictions, while low values indicate a relatively flat prior on the VAR parameters. The table also shows the posterior odds relatively to the best-fitting model, computed under the assumption that we have equal a priori weight on the different specifications. Table 2 shows that the best fit is achieved for values of λ around 2 – hence, in the notation of section 3, $\hat{\lambda} = 2$. To put this number in perspective we use the dummy observations interpretation of λ discussed in Del Negro and Schorfheide (2004): λ equal to 2 means that in forecasting with the VAR one should use twice as many dummy observations generated from the DSGE model as the length of the actual time series available. In other words, the weight of the DSGE model prior in the best fitting model is not negligible. The Table also shows that the posterior odds of the DSGE model relative to DSGE-VAR($\hat{\lambda}$) are very small, indicating that from a statistical point of view there is evidence that the cross-equation restrictions are violated in the data. The next section investigates whether this statistical evidence is economically meaningful, that is, whether it translates into large differences in terms of the dynamic response of the endogenous variables to different shocks. However, as λ increases from 2 to 5 the marginal likelihoods declines very gradually, suggesting that DSGE-VARs with prior tighter than $\hat{\lambda}$ do not fare poorly at all.

Interestingly, the Table shows that the posterior odds of DSGE-VARs with less tight DSGE priors (for instance, $\lambda = .75$) are even smaller than those of the DSGE model. This evidence suggest that some structure is needed to perform analysis, and forecasts, with this data set: unrestricted VARs perform very poorly. Of course, there may be prior information other than that coming from this specific small open economy DSGE model which can help in describing the data, e.g, the Minnesota priors or priors from different DSGE models. In this sense, the evidence here should not be interpreted as saying that this small open economy DSGE model is a particularly good one. Rather, it suggests that some prior information from this model is better than none, especially given how short the sample is.

Table 3 provides the estimates of the DSGE model non-policy parameters (the policy parameters were already described in the previous section). The first column shows the prior mean and standard deviations. The parameter α measures the fraction of foreign produced goods in the domestic consumption basket. In 2006 imports as goods as a fraction of total domestic demand in Chile was about 30%. Restricted to consumer goods, this fraction was 10%. We decided to center our prior at the 30% value allowing for substantial variation. The parameter r^* can be interpreted as the growth adjusted real interest rate. While our observations on average GDP growth, inflation, and nominal interest rates between 1999 and 2007 suggest that this value is negative, we view this as a temporary phenomenon and center our prior for r^* at 2.5%. In the closed-economy version of our model ($\alpha = 0$) the parameter κ would correspond to the slope of the Phillips curve, which captures the degree of price stickiness. According to our prior, κ falls with high probability in the interval 0 to 1, which encompasses large nominal rigidities as well as the case of near flexible prices. τ captures the inverse of the relative risk aversion. We center our prior at 2, which implies that the consumers are slightly more risk averse than consumers with a log utility function. Finally, the priors for the parameters of the exogenous processes were chosen based with pre-sample evidence in mind.

The second column shows the posterior mean and standard deviations obtained from the estimation of the DSGE model. In light of the DSGE model misspecification discussed above it is important to ask whether accounting for deviations from the cross-equation restrictions affects the inference about the DSGE parameters. Therefore, the third column shows the estimates obtained using DSGE-VAR($\hat{\lambda}$). The data provide little information on r^*, which enters the log-linear equations through the discount factor β, and the slope of the Phillips curve κ. The estimated import share is about 10%. While this estimate is influenced based on the output, inflation, interest rate, and exchange rate dynamics, it remains broadly consistent with data on import quantities. Finally, the posterior mean of τ decreases compared to its mean and its standard deviation shrinks from 0.2 to 0.1. The estimated standard deviation of the monetary policy shock is around 60 to 70 basis points. Overall, the parameter estimates obtained from the state-space representation of the DSGE model and the DSGE-VAR are very similar.

Since the DSGE model itself exhibits very little endogenous propagation, the dynamics of the data are mostly captured by the estimated autocorrelation parameters of the exogenous shock processes. The terms of trade are purely exogenous in the DSGE model and, hence, the posterior means of ρ_q and σ_q measure the autocorrelation and innovation standard deviation in our terms of trade series. The foreign inflation process π_t^* is captured by the difference of the two series plotted in Figure 3 and the estimates of ρ_{π^*} and σ_{π^*} capture its persistence of volatility. The remaining sources of cyclical fluctuations are a foreign demand shock $\widetilde{y}_t^*$ and a technology growth shock $\widetilde{z}_t$. The estimated autocorrelations of these shocks are 0.93 and 0.72 (DSGE) and 0.89 and 0.64 (DSGE-VAR). In general we observe that the shock-standard-deviation and autocorrelation estimates obtained with the DSGE-VAR are slightly smaller. The reason is that the DSGE-VAR can capture model misspecification by deviating from cross-equation restrictions, whereas the directly estimated DSGE model has to absorb this misspecification in the exogenous shock processes.

5.3 The Determinants of Inflation

This section discusses the impulse responses of the endogenous variables to internal and external shocks. Given that the Central Bank is in an inflation targeting regime, the discussion will focus on the determinants of inflation dynamics. Specifically, from section 5.4 we learned that the Central Bank seemingly does not respond to exchange rate or terms of trade movements. Did this policy manage to insulate the economy, and inflation in particular, from external shocks?

Figure 4 shows the impulse response functions to the five shocks described in section 2: policy shocks (*Money*), *Technology*, terms of trade (*ToT*), foreign output (y^*) and foreign inflation (π^*). There are two lines for each plot, one black and one gray. Both are impulse responses computed using the DSGE model. The difference between the two consists in the underlying estimates of the DSGE model parameter. The gray line uses the DSGE model estimates, and the black line uses the DSGE-VAR's estimates. In general, the main difference between the black and the gray impulse responses is that the latter are more pronounced, reflecting the larger estimated standard deviation of shocks documented in Table 3.

In terms of the determinants of inflation, the interesting feature of Figure 4 is that the shocks that move the terms of trade and the nominal exchange rate depreciation, namely *ToT* and π^* shocks, barely affect inflation. According to the DSGE model identification, the shocks that move inflation around are largely domestic, namely *Technology* and to a lesser degree *Money* shocks. Notably, these shocks have very little effect on the exchange rate depreciation (and of course on the terms of trade, given that the DSGE model considered here treats these as exogenous). These findings indicate that the monetary authorities have been successful in terms of isolating inflation from foreign disturbances.

It is somewhat surprising that *Money* shocks have a significant effect on inflation, given that these shocks are avoidable. One possibility is that the Central Bank, in the attempt to respond to future rather than current inflation, makes errors in forecasting inflation. From the model's perspective these errors appear as policy shocks. Another possible explanation is that the policy reaction function is misspecified: Policy responds to some other variable not included in the reaction function. While this is certainly a possibility, we know that the missing variable cannot be the exchange rate, since the impulse responses to *Money* shocks in the *Response to FX* model look exactly as those in Figure 4.

Figure 4 shows that the impulse responses are generally not very persistent, reflecting the fact that the DSGE model is not very rich in terms of frictions. Moreover, the DSGE impulse responses are computed under stark identification assumptions, e.g., exogeneity of the terms of trade. These limitations, as well as the evidence of misspecification discussed in the previous section, suggest that we may want to compare the DSGE model impulse responses to those from the DSGE-VAR and check whether relaxing the cross-equation restrictions leads to very different dynamics. In comparing the DSGE model impulse responses with those from the DSGE-VAR, one should bear in mind that in principle some differences may arise from the fact that the DSGE model does not have an exact finite VAR representation (see Ravenna 2007, among others). Figure A-1 in the Appendix shows that in the case considered here this is not an issue. Figure A-1 compares the DSGE impulse responses with those obtained from the finite VAR representation of the DSGE model, e.g., DSGE-VAR($\lambda = \infty$). The two are virtually identical. This implies that if the data were generated by the DSGE model at hand, the DSGE-VAR would recover the "true" impulse response functions.

Figure 5 compares the impulse responses computed from DSGE-VAR($\lambda = \infty$) (black), which are identical to the black lines shown in Figure 4, to those from DSGE-VAR($\hat{\lambda}$) (gray).[2] The Figure shows that by and large the differences between the DSGE-VAR($\lambda = \infty$) and the DSGE-VAR($\hat{\lambda}$) impulse responses lies in the dynamic of the nominal exchange rate, which is somewhat more volatile and persistent than according the DSGE model. Interestingly, the terms of trade impulse responses are not very different either. Note that the assumption of exogeneity of the terms of trade is not strictly imposed on the DSGE-VAR. Hence, if the data were substantially at odds with this assumption, we would see differences between the gray and black impulse responses in the last column. While we see some differences, these are small relative to the magnitude of movements in the terms of trade.

In summary, Figure 5 suggests that the misspecification found in section 5.2 is not very important from an economic point of view. This result must be interpreted with caution, however. The identification in the DSGE-VAR is by construction linked to that in the DSGE model. While this may be a virtue, as it ties the DSGE-VAR impulse responses to

[2] We do not show the posterior bands for simplicity of exposition. These are available from the authors upon request.

those of the underlying DSGE model, it can also be a drawback: There may be other DSGE models, and other identification schemes, that are equally capable of describing the data. By construction, DSGE-VAR is not going to be able to uncover such models. Finally, the data may simply not be informative enough, because of the shortness of the sample, to point out the deficiencies of this model.

5.4 A Look at Alternative Policy Rules

This section discusses alternative policy rules. Should the Central Bank respond more or less aggressively to inflation? Could a policy rule different than that currently pursued further dampen inflation variability? How does the presence of misspecification change the answers to these questions?

Figure 6 describes how the impulse responses change as the parameter ψ_1 in the policy reaction function varies from 1.25 (light gray) to 2.75 (dark gray, historical estimate), to 3.5 (black). Although each plot has three lines, visually it appears as if it had only two. This is because raising the reaction to inflation from its estimated value of 2.75 (according to DSGE-VAR) to 3.5 has virtually no impact on the dynamics. Hence responding more aggressively to inflation would not have any effect on the Chilean economy, at least according to this estimated model. Conversely, a much weaker response to inflation ($\psi_1 = 1.25$) would have serious effects, especially on inflation. The response to *Technology* shocks would be much more pronounced. Moreover, the response to π_* shocks, which historically has been negligible, would become sizable. This result suggests that a strong response to inflation has been a key ingredient in isolating the Chilean economy from external shocks.

Figure 7 shows how the variance of inflation changes as psi_1 varies in a grid ranging from 1 to 3.5. Specifically, we show that the increase (or decrease, when negative) in the variance relative to the historical policy rule $\psi_1 = 2.75$. In Figure 7 the variance differentials are computed using the DSGE model. The solid (dashed-and-dotted) gray lines represent the posterior mean (90% posterior bands) differentials under the DSGE model estimates of parameters (second column of Table 3). The solid (dashed-and-dotted) black lines represent the posterior mean (90% posterior bands) differentials under the DSGE-VAR estimates of parameters (third column of Table 3). Consistently with Figure 6, under both sets of estimates the variance of inflation increases substantially as ψ_1 decreases below 1.5, while not much happens as ψ_1 increases from 2.75 to 3.5. The magnitude of the increase in the variance differential differs substantially under the two sets of estimates. Under the DSGE the shocks are estimated to be more persistent and more variable than under DSGE-VAR, hence the effect of changes in policy on the variability of inflation is larger. One can view the higher persistent and variability of the exogenous shocks under the DSGE model estimates as a consequence of the model's misspecification, as discussed in section 5.2, and therefore not trust the outcomes of the policy analysis exercise under these estimates. In any case, these results highlight the sensitivity of the policy exercises to the estimates of the processes followed by the exogenous shocks, a point made in Del Negro and Schorfheide (2007).

Figure 8 shows the expected changes in the variability of inflation under three different approaches to performing the policy experiment. Under all three approaches the experiment is the one just described, that is, varying psi_1 in a grid ranging from 1 to 3.5. The first approach (black line) also is the same one described in the previous paragraph: It amounts to performing the experiment using the DSGE model under the DSGE-VAR estimates of the non-policy parameters. The second approach (dark gray line) is called *DSGE-VAR/Policy-Invariant Misspecification* and is described in detail in Del Negro and Schorfheide (2007). This approach to policy assumes that while the cross-equation restrictions change with policy, the deviations from the cross-equation restrictions outlined in Figure 5 are policy invariant. More specifically in terms of the DSGE-VAR notation, the matrices that embody the cross-equation restriction ($\Psi^*(\theta)$ and $\Sigma^*(\theta)$) change with ψ_1, but the deviations (Ψ^Δ and Σ^Δ) do not.[3] This approach may be appealing if one thinks that these deviations capture low or high frequency movements in the data that are not going to be affected by policy. The variance differential under this alternative approach is about the same as under the DSGE model (and so are the bands, which we do not show to avoid cluttering the figure). This is not surprising given that the deviations from the cross-equation restrictions are small, particularly for inflation.

The second approach (light gray line) is called *DSGE-VAR/Backward-Looking Analysis* and is again described in detail in Del Negro and Schorfheide (2007). Under this approach the DSGE-VAR is treated as an identified VAR: The change in ψ_1 only affects the policy rule (e.g., Sims 1999), but does not affect the remaining equation of the system. Under this approach the cross-equation restriction are completely ignored. The light gray line shows that the outcome from this approach is quantitatively different from that of the other two approaches. The rationale for ignoring the cross-equation restrictions when the deviations are small, especially in economic terms, is questionable, however.

[3] As discussed in Del Negro and Schorfheide (2007), we work with the moving average rather than the VAR representations. So literally we treat the deviations from the DSGE-VAR(∞) impulse responses in Figure 5 as policy invariant.

6 Conclusion

We estimate the small open economy DSGE model used in Lubik and Schorfheide (2005) on Chilean data for the inflation targeting period, 1999-2007, using data on the policy rate, inflation, real output growth, nominal exchange depreciation, and log differences in the terms of trade. We also estimate on the same a Bayesian VAR where the prior comes from the small open economy DSGE model, following the DSGE-VAR methodology proposed in Del Negro and Schorfheide (2004). The purpose of the DSGE-VAR is to check whether the answers provided by the DSGE model are robust to the presence of misspecification, where misspecification is defined as deviations from the cross-equation restrictions imposed by the model.

We first focus on the estimated policy rule for the Central Bank. Since the Chilean economy has been exposed to large movements in the exchange rate and especially the terms of trade, we ask whether the Central Bank responded to these movements the in order to pursue the inflation target. We find that the answer is no. According to our estimates, the Central Bank mainly responded to inflation, and to a much less degree to output growth. We also find that the DSGE-VAR provides similar answers to these questions.

We study the degree of misspecification in the DSGE model by comparing its fit, as measured by the marginal likelihood, to that of the DSGE-VAR for various degree of relaxation of the cross-equation restrictions. We find that some degree of misspecification exists, as the fit improves from a statistical point of view when the cross-equation restrictions are relaxed, but is relatively small in the sense that in the best-fitting DSGE-VAR the weight of the DSGE prior is high. This finding may be in part due to the short data sample, as the DSGE model itself is quite simple. Whatever the reason for this result, its implication is that a good model for the Chilean economy should have strong a priori restrictions, possibly coming from a DSGE model. We suspect that a loosely parameterized model is unlikely to give good forecasts or sharp policy advice.

Next, we use the DSGE model and the DSGE-VAR to investigate the determinants of inflation. In particular, we ask whether the policy pursued by the Central Bank managed to insulate the economy, and inflation in particular, from external shocks. We find that both approaches give the same answer: yes. We find that the sources of inflation variability mainly lie in domestic shocks. We also find that the misspecification of the DSGE model, if statistically significant, is not large from an economic point of view: the dynamic responses of the variables to shocks are very similar.

Finally, we use the DSGE model to conduct policy exercises. We study the effect of changing the response to inflation in the feedback rule on the variance of inflation. We find that increasing the response from the historical value would produce little change, but that a substantial decrease would lead to a spike in volatility. Quantitatively, the answer depends heavily on which estimates of the non-policy parameters are used. In line with the results described above, we find that accounting for misspecification makes little quantitative difference in terms of the outcome of the policy exercise , at least to the extent that the cross-equation restrictions are not completely ignored.

An important caveat to the policy analysis exercise is that the DSGE model used here has many restrictive assumptions, and hence may not capture some the important policy trade-offs. In spite of this, we believe that a few lessons can be learned from this exercise, which are likely to carry over to more sophisticated models: First, the outcome of policy experiment is very sensitive to the estimates for the parameters describing the law of motion of the exogenous shocks. Second, the presence of misspecification – that is, the fact that the DSGE model is rejected relative to a more loosely parameterized model – does not necessarily imply that the answers to policy exercises obtained from the DSGE model are not robust. The DSGE-VAR methodology provides ways of checking the robustness of the policy advice under different assumptions about misspecification, and we hope this can be useful in applied work at Central Banks.

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Table 1: Which Policy Rule?

Parameter	Prior	(1) Baseline	(2) Response to FX	(3) Response to ToT	(4) Response to Y-o-Y Inflation
DSGE					
ψ_1	2.50 (0.50)	2.31 (0.52)	2.04 (0.58)	1.89 (0.57)	1.82 (0.39)
ψ_2	0.25 (0.13)	0.18 (0.10)	0.20 (0.11)	0.19 (0.09)	0.14 (0.07)
ψ_3	0.25 (0.12)		0.09 (0.03)	0.07 (0.03)	0.08 (0.03)
ψ_4	0.00 (0.50)			-0.08 (0.05)	
ρ_r	0.50 (0.20)	0.45 (0.11)	0.42 (0.11)	0.40 (0.11)	0.44 (0.09)
Marginal Likelihood		-585.52	-588.32	-589.24	-593.52
Posterior Odds relative to Baseline DSGE		1	.061	.024	3.35e-04
DSGE-VAR($\lambda = 2$)					
ψ_1		2.76 (0.46)	2.75 (0.47)	2.68 (0.49)	
ψ_2		0.13 (0.06)	0.13 (0.06)	0.13 (0.07)	
ψ_3			0.09 (0.04)	0.08 (0.04)	
ψ_4				-0.08 (0.07)	
ρ_r		0.50 (0.10)	0.49 (0.10)	0.49 (0.10)	
Marginal Likelihood		-572.89	-575.71	-577.30	
Posterior Odds relative to Baseline DSGE		1	.060	.012	

Notes: We report means and standard deviations (in parentheses).

Table 2: The Fit of the Small Open Economy DSGE Model

Specification	λ	Log Marginal Likelihood	Posterior Odds relative to DSGE-VAR($\hat{\lambda}$)
DSGE		-585.52	3.27e-06
DSGE-VAR:			
	5	-575.40	0.081
	3	-573.45	0.571
	2.5	-573.02	0.878
$\hat{\lambda}$	2	-572.89	1.00
	1.5	-574.21	0.267
	1	-582.24	8.69e-05
	.75	-600.89	6.91e-13

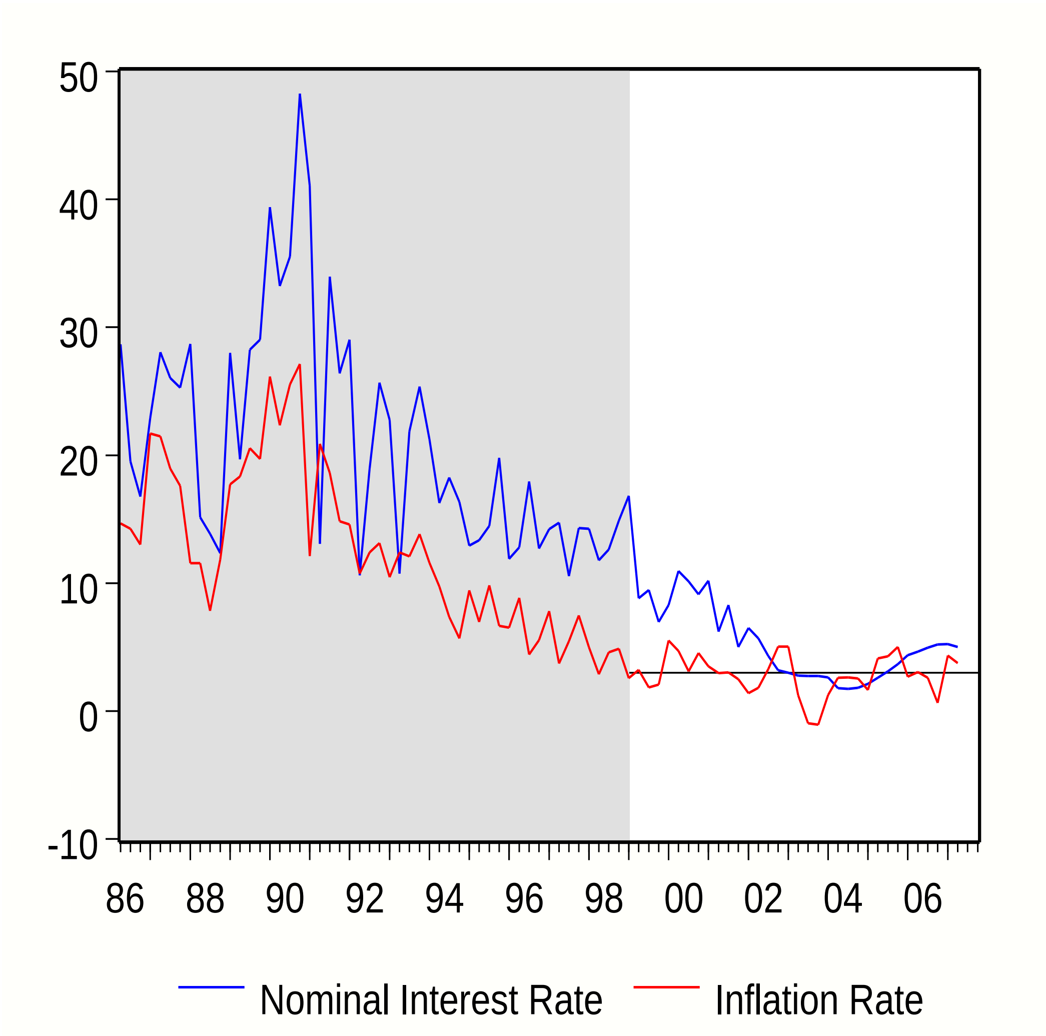
Notes: The difference of log marginal data densities can be interpreted as log posterior odds under the assumption of that the two specifications have equal prior probabilities. We report odds relative to the DSGE-VAR ($\lambda = 2$).

Table 3: DSGE Model Parameters

Paramter	Prior	DSGE	DSGE-VAR($\lambda = 2$)
α	0.30 (0.10)	0.08 (0.02)	0.10 (0.03)
r^*	2.50 (1.00)	2.59 (1.08)	2.46 (0.99)
κ	0.50 (0.25)	0.60 (0.22)	0.78 (0.24)
τ	0.50 (0.20)	0.37 (0.09)	0.39 (0.10)
ρ_z	0.20 (0.10)	0.72 (0.06)	0.64 (0.06)
ρ_q	0.50 (0.10)	0.38 (0.08)	0.42 (0.08)
ρ_{y^*}	0.85 (0.05)	0.93 (0.03)	0.89 (0.04)
ρ_{π^*}	0.70 (0.15)	0.34 (0.11)	0.38 (0.13)
σ_z	1.88 (0.99)	0.89 (0.16)	0.82 (0.12)
σ_q	4.39 (2.29)	4.62 (0.54)	3.21 (0.48)
σ_{y^*}	1.88 (0.99)	7.72 (2.77)	3.84 (1.70)
σ_{π^*}	1.88 (0.99)	5.24 (0.65)	3.35 (0.58)
σ_r	0.63 (0.33)	0.68 (0.12)	0.58 (0.12)

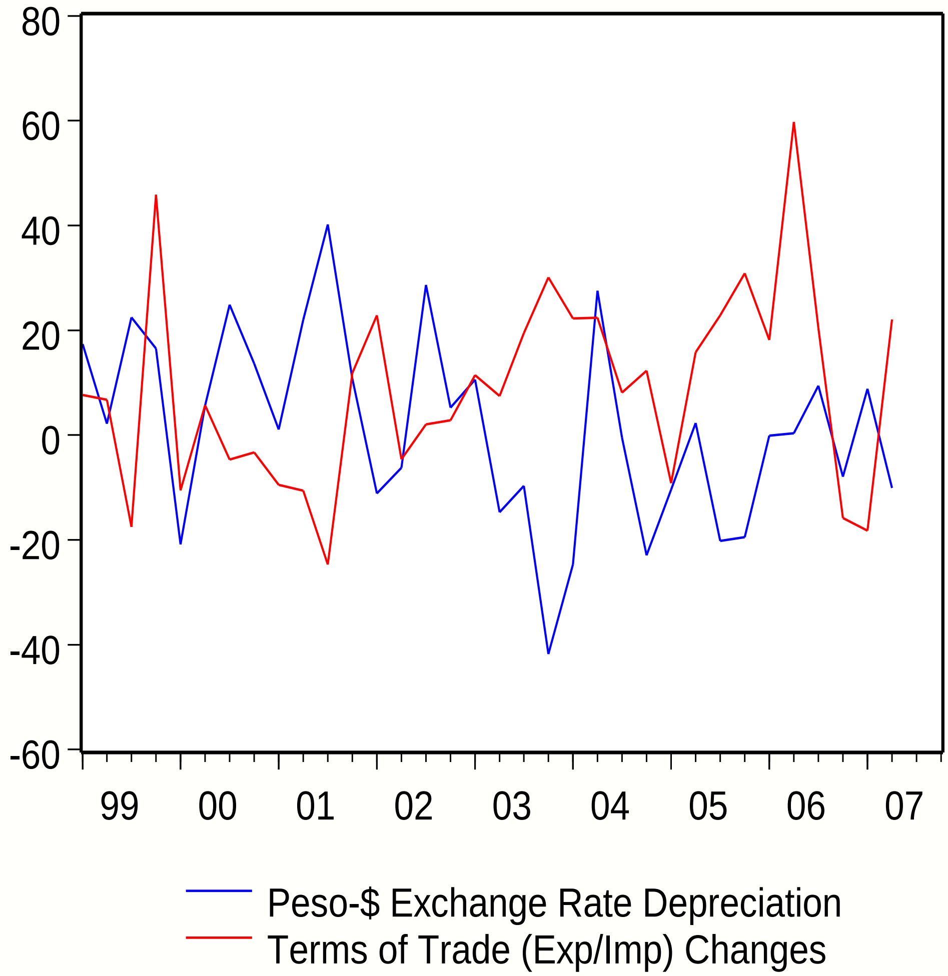
Notes: We report means and standard deviations (in parentheses).

Figure 1: Interest Rates and Inflation in Chile



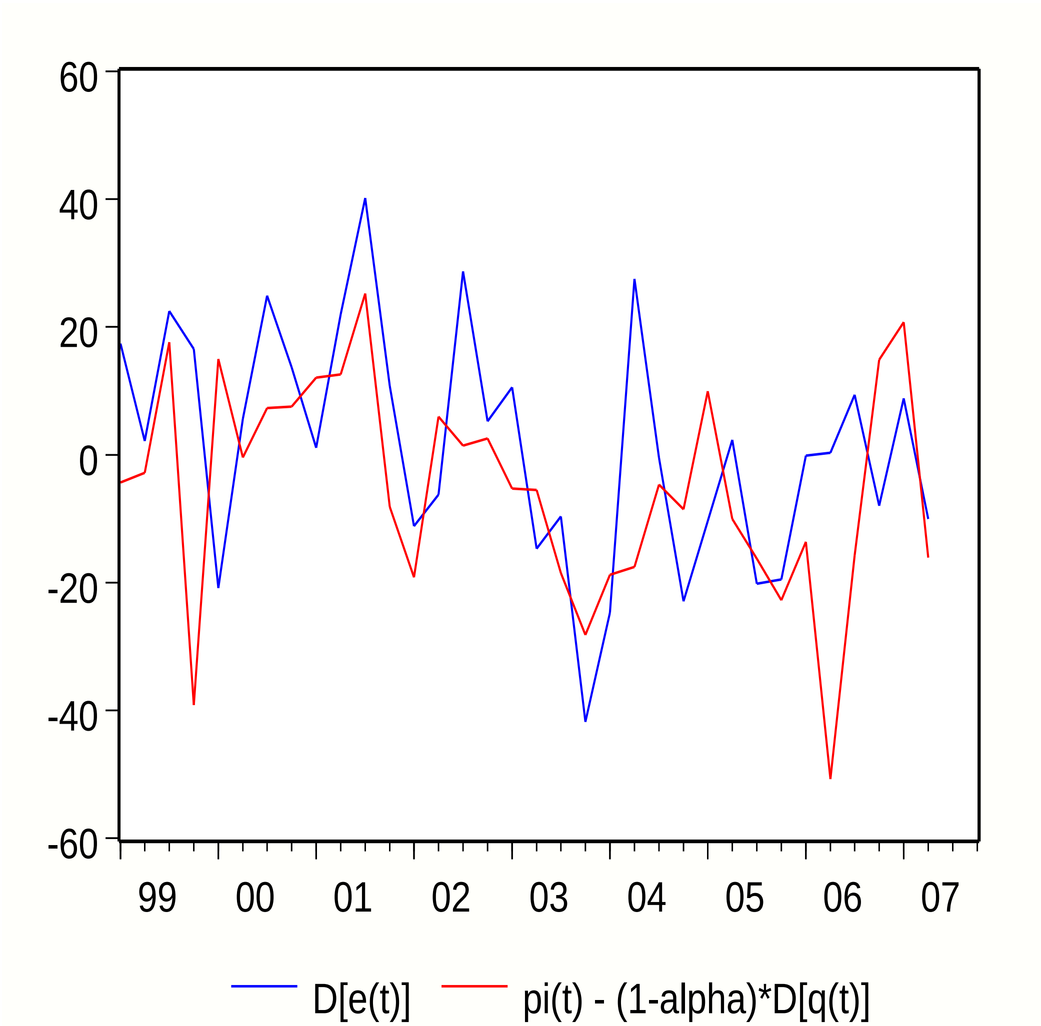
Notes:

Figure 2: Exchange Rate and Terms of Trade Dynamics



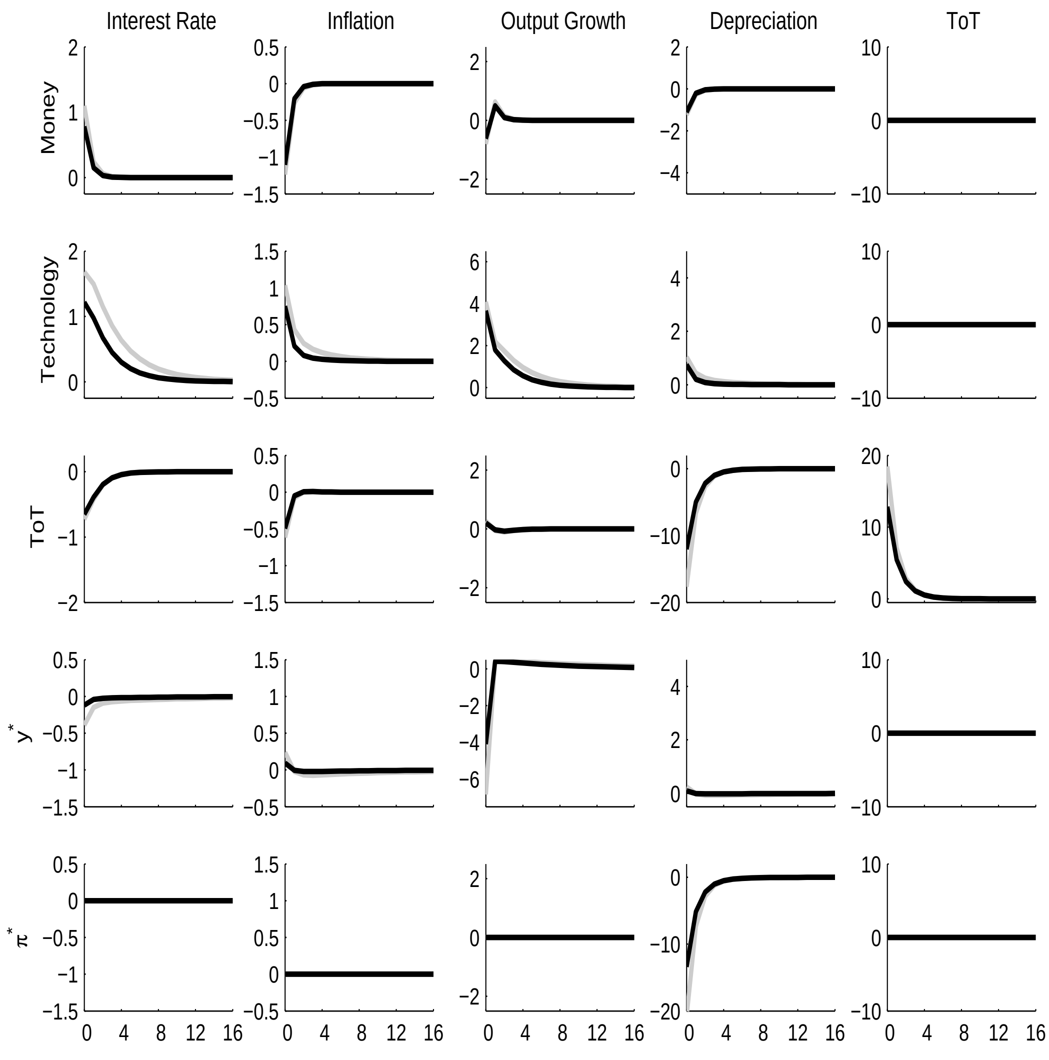
Notes:

Figure 3: Exchange Rate Movements and PPP



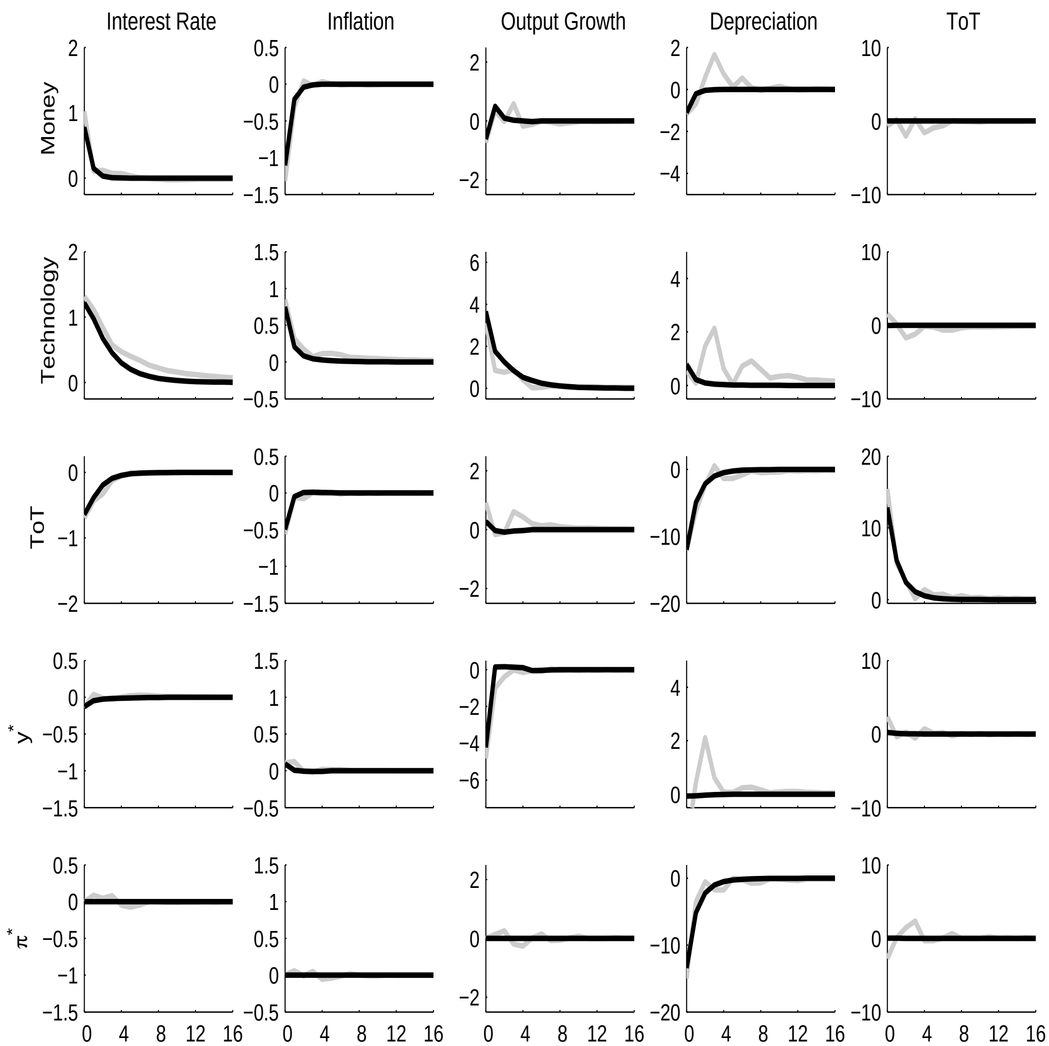
Notes:

Figure 4: DSGE Model Impulse Responses: DSGE vs DSGE-VAR($\lambda = 2$) Parameter Estimates



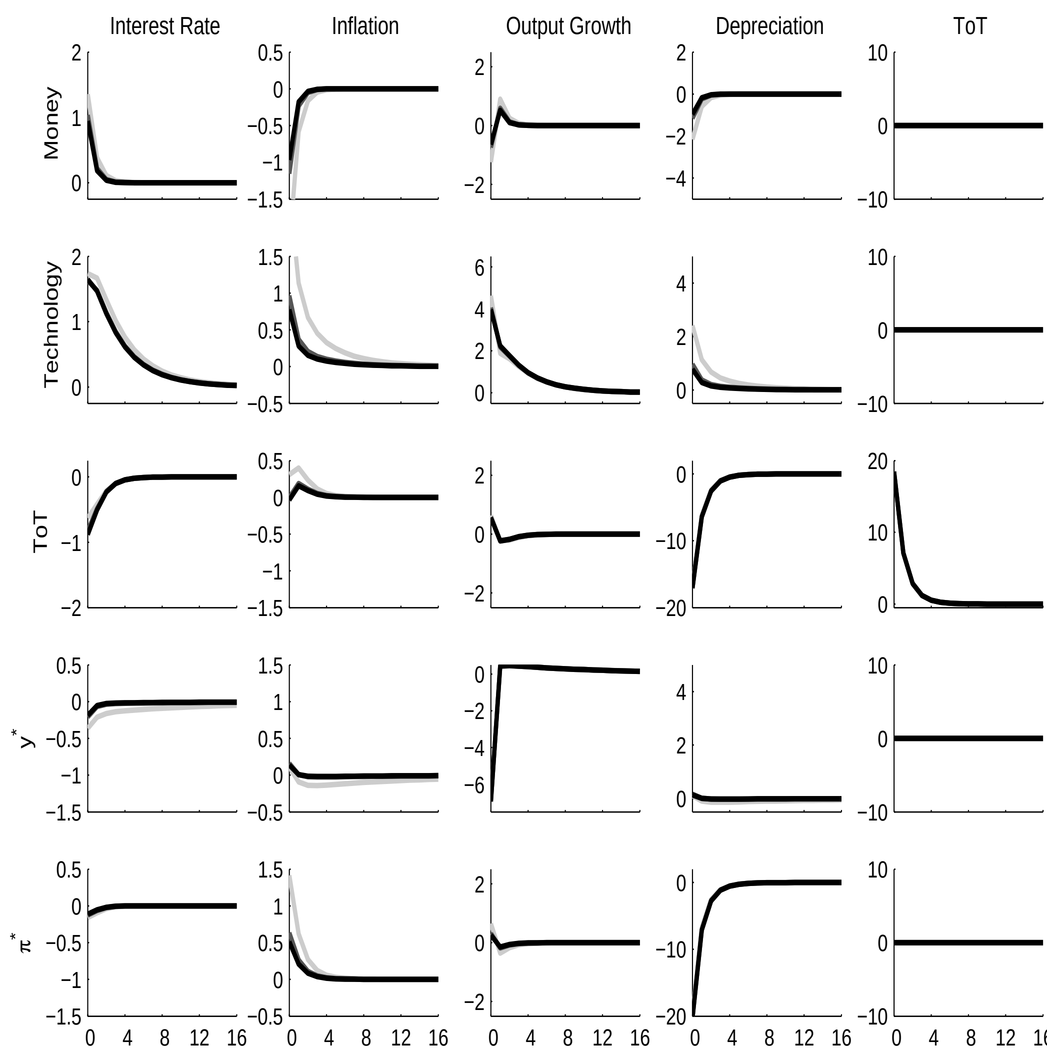
Notes: The figure depicts impulse responses from the DSGE-VAR($\lambda = 2$) (black) and the DSGE (gray) based on the respective posterior estimates summarized in Tables 1 and 3.

Figure 5: Impulse Responses: DSGE-VAR($\lambda = \infty$) versus DSGE-VAR($\lambda = 2$)



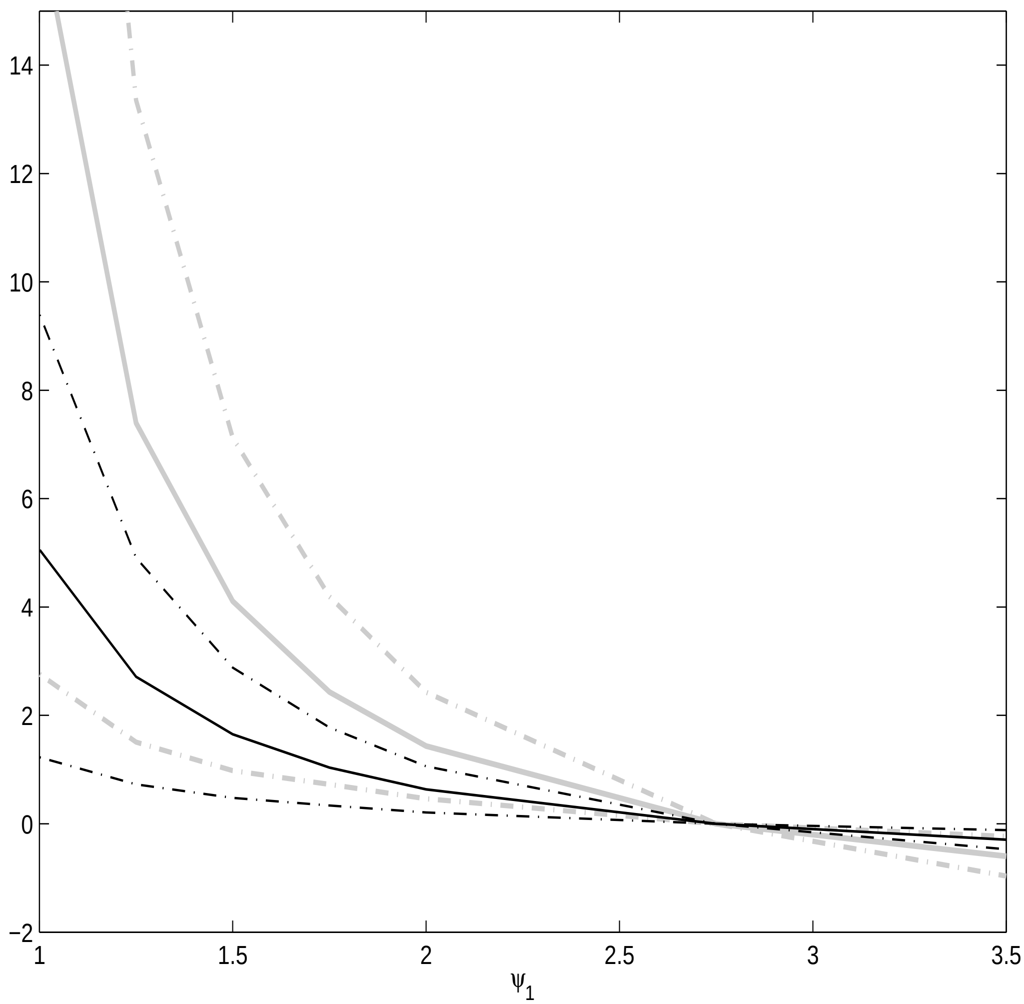
Notes: The figure depicts impulse responses from the DSGE-VAR($\lambda = 2$) (black) and the DSGE-VAR(∞) (gray) based on the DSGE-VAR($\lambda = 2$) posterior estimates summarized in Tables 1 and 3.

Figure 6: DSGE Model Impulse Responses as Function of ψ_1



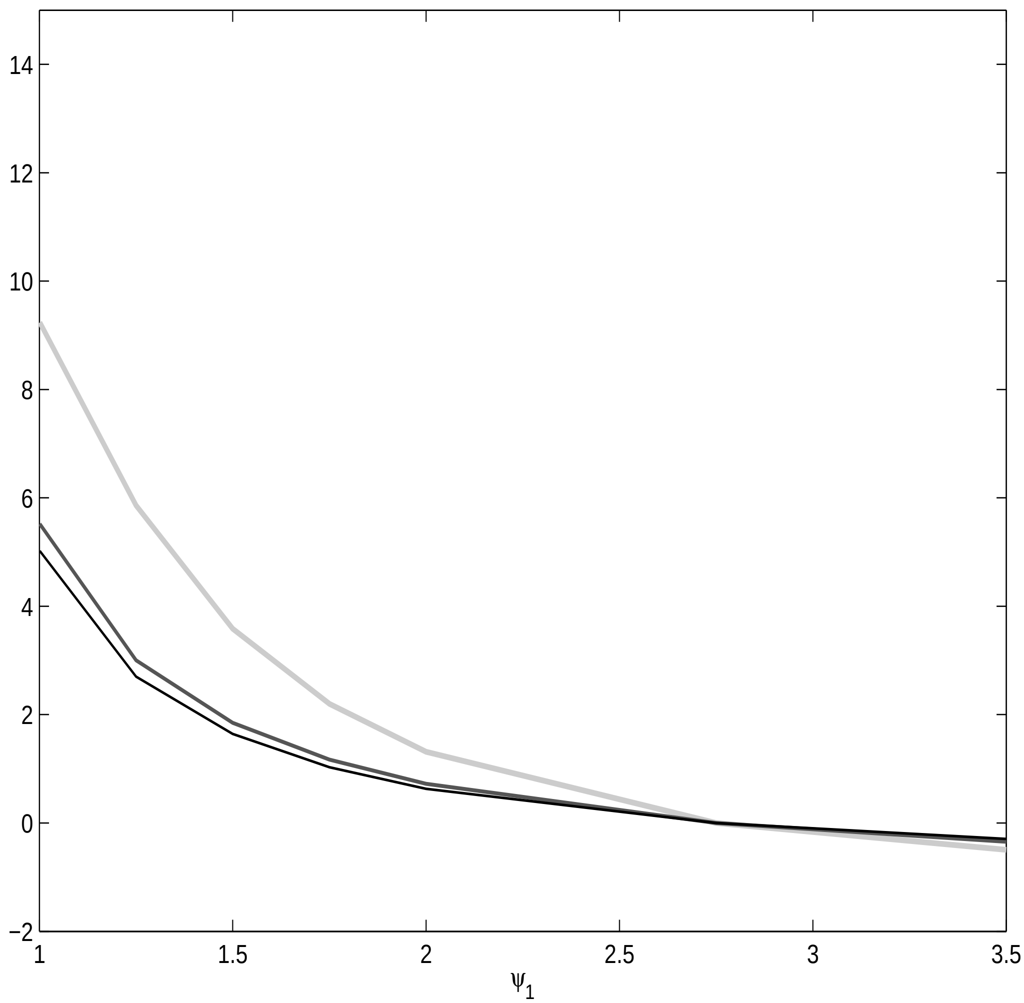
Notes: The figure plots the posterior mean of the DSGE model impulse responses computed for three different values of the response to inflation in the policy rule, ψ_1: 3.5 (black), 2.75 (dark gray), and 1.25 (light gray). The remaining policy parameters ψ_2, ψ_3, ψ_4, and ρ_R are kept at the baseline values of 0.125, 0, 0, and 0.5, respectively. For all impulse responses we use the DSGE model posterior estimates of the non-policy parameters $\theta_{(np)}$, summarized in Table 3.

Figure 7: Comparative Performance of Policy Rules: Benchmark DSGE versus DSGE-VAR($\lambda = 2$) Parameter Estimates



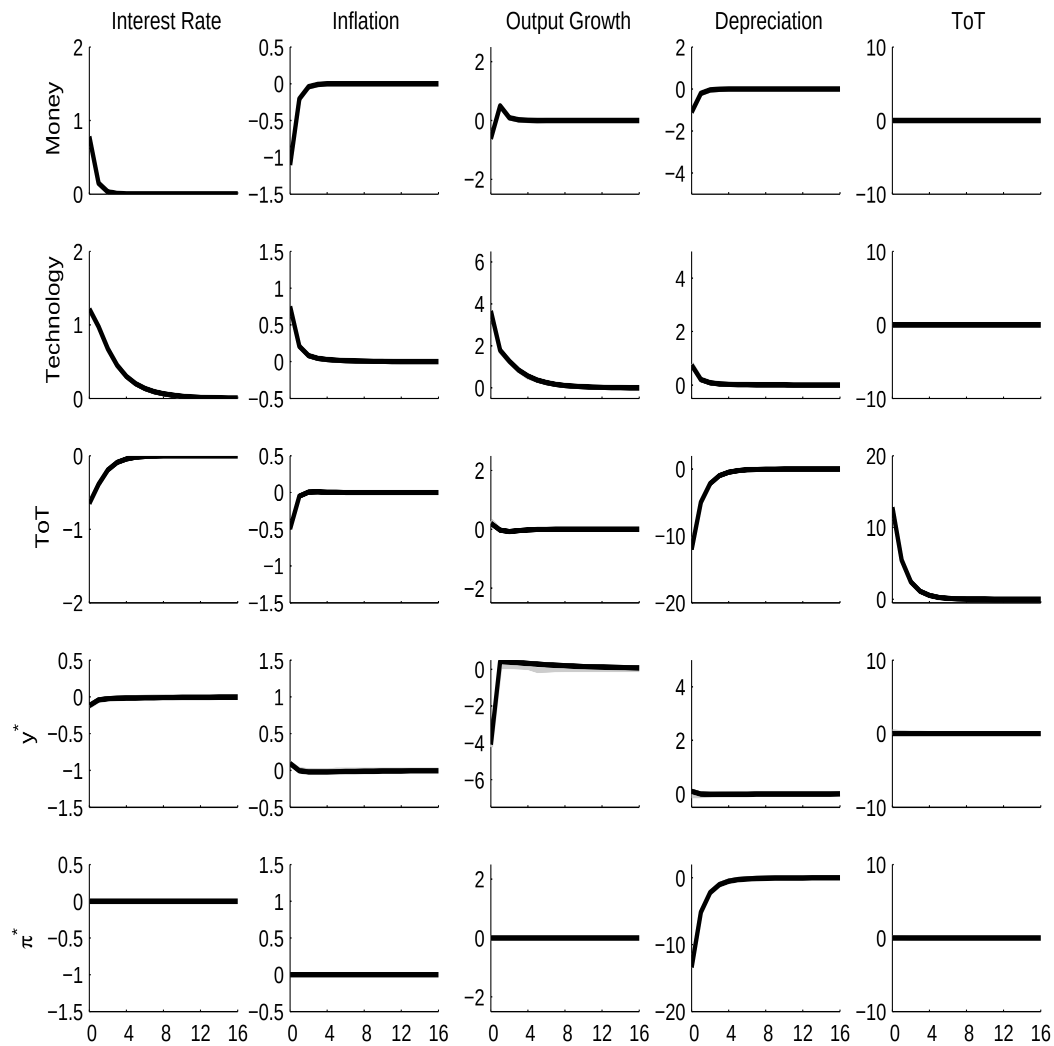
Notes: Posterior expected variance differentials as a function of ψ_1 relative to baseline policy rule $\psi_1 = 2.75$. The remaining policy parameters ψ_2, ψ_3, ψ_4, and ρ_R are kept at the baseline values of 0.125, 0, 0, and 0.5, respectively. Negative differentials signify a variance reduction relative to baseline rule. Differentials are computed using DSGE-VAR posterior (gray) and DSGE model (black) posterior estimates of the non-policy parameters $\theta_{(np)}$, summarized in Table 3.

Figure 8: Comparative Performance of Policy Rules: DSGE versus DSGE-VAR/Policy-Invariant Misspecification and DSGE-VAR/Backward-Looking Analysis



Notes: Posterior expected variance differentials as a function of ψ_1 relative to baseline policy rule $\psi_1 = 2.75$. The remaining policy parameters ψ_2, ψ_3, ψ_4, and ρ_R are kept at the baseline values of 0.125, 0, 0, and 0.5, respectively. Negative differentials signify a variance reduction relative to baseline rule. Differentials are computed using the DSGE-VAR/Backward-Looking Analysis (light gray), the DSGE-VAR/Policy-Invariant Misspecification scenario (dark gray) and the DSGE model (black), where the latter uses the DSGE-VAR($\lambda = 2$) posterior estimates of the non-policy parameters $\theta_{(np)}$, summarized in Table 3.

Figure A-1: Impulse Responses: DSGE-VAR($\lambda = \infty$) versus DSGE



Notes: The figure depicts impulse responses from the DSGE-VAR(∞) (black) and the DSGE (gray) based on the DSGE-VAR($\lambda = 2$) posterior estimates summarized in Tables 1 and 3.