Incorporating Financial Sector Risk into Monetary Policy Models: Application to Chile

October 31, 2008 Draft

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ABSTRACT

This paper builds a model of financial sector vulnerability and integrates it into a macroeconomic framework, typically used for monetary policy analysis. The main question to be answered with the integrated model is whether or not the central bank should include explicitly the financial stability indicator (FSI) in the interest rate reaction function. Contingent claims analysis (CCA) used to create an aggregate financial stability indicator, the Distance to Distress (DTD). It is found in general, that including Distance to Distress in the reaction function reduces inflation volatility while increasing the variability of output. The preliminary conclusions are that it is better to include DTD in the interest rate reaction function if exchange rate pass-through is higher; if financial vulnerability (DTD) has a larger impact on the exchange rate, as well as on GDP (or the reverse, there is more impact of GDP on bank's equity and thus DTD –endogeneity).

Prepared for the Central Bank of Chile's 12th Annual Conference on "Financial Stability, Monetary Policy and Central Banking" November 6 and 7, 2008

JEL Codes: E32, E61, E62, E63, F41. **Keywords**: Financial vulnerability, monetary policy, central banking.

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I. INTRODUCTION

This article builds a model of financial sector vulnerability and integrates it into a macroeconomic framework, typically used for monetary policy analysis. The integration of the analysis of financial sector vulnerability into macroeconomic models is an area of important and growing interest for policymakers, in both developed and emerging markets. However, estimating the effect of shocks to vulnerability on the risk of banks in a coherent manner requires both a model of banking sector risk and a tractable methodology for simulating shocks and estimating their effect on various risk measures.

Financial stability models and monetary policy models, by their nature, are very different frameworks. There is keen interest in putting together these two types of analysis. Monetary policy models are widely used by central banks to understand the transmission mechanisms of interest rates to the macroeconomy and inflation.

Market-based financial stability incicators (FSIs)² summarize both the credit channel and credit risk transmission from distressed borrowers in the economy. FSIs provide information on the banking sector's financial condition which is related to the quantity of credit extended and the possible or expected effects of this channel on the real economy and GDP (credit expansion and the "financial accelerator"). FSIs also capture the reduced financial soundess of banks when borrowers default in periods of economic distress which leads to lower banking sector assets, higher banking asset volatility. This is a reflection of the economic condition of borrowers and of the real economy.

Since the economy and interest rates affect financial sector credit risk, and the financial sector affects the economy, an important issue is whether market-based financial stability indicators should be included in monetary policy models and, if so, how.³ Including an aggregate credit risk indicator in the GDP gap equation and testing whether the coefficient is significant or not is an important first step to get a better understanding of how the financial sector credit risk affects GDP. The most important question is whether or not the central bank should include explicitly the financial stability indicator in the interest rate reaction function. The alternative would be to react only indirectly to financial risk by reacting to inflation and GDP gaps, since they already include the effect financial factors have in the economy. An alternative could also be designed where the central bank only reacts directly to financial risk whenever financial stability indicator breaches a predetermined threshold.

² The term FSI used here is an indicator derived from forward-looking market information, including indicators from the CCA model. This should not be confused with the accounting ratio financial stability indicators.

³ Bernanke, Gertler and Gilchrist (1999) introduce financial frictions into a business cycle model.

This paper uses contingent claims analysis (CCA) tools developed in finance to estimate the riskiness of banks and to construct the financial stability indicators. The basis of CCA is that the liabilities of a financial institution or firm derive their value from assets which are stochastic. The expected variation (volatility) of assets over a future horizon, relative to the promised payments on liabilities provides a measure of financial distress risk. CCA methodology is frequently used to estimate the probability that an entity (in our case, banks, but also corporations or even governments) will default on its obligations. Due to its explicit focus on risk and probability of distress or default, and its link to market prices of equity, CCA has many advantages. Equity data by nature incorporate the forward-looking expectations of the market in a way that static indicators of bank risk, such as nonperforming loan ratios and provisioning cannot. The high frequency of observations, at least for equity and interest rate data, allow for much faster updating of risk measures than data available only at monthly or quarterly frequencies. The CCA financial risk indicators are calculated for individual banks and then aggregated into a system-wide financial stability indicator.

The CCA system-wide FSI is modeled jointly with a practical five equation dynamic, stochastic macroeconomic model used to set monetary policy. The macro model closely resembles the one proposed by Berg, Karam and Laxton (2006) as a useful toolkit applicable to the analysis of monetary policy in many small open economies. As they claim, "in the new Keynesian synthesis, there has been a convergence between the useful empirically motivated IS/LM models developed in several policymaking institutions and dynamic stochastic general equilibrium approaches that take expectations seriously and are built on solid microeconomic foundations."⁴

The specific model used here consists of an equation for the output gap (IS), an equation for inflation (Phillips curve or aggregate supply), an equation for the exchange rate (interest parity condition), a yield curve relating short and long-run interest rates, and the Central Bank reaction function (Taylor rule). Indeed, the primary tool for macroeconomic management is the interest rates set by the central bank as a reaction to the deviations of inflation from the target and the output gap (Taylor, 1993). It is worth noting that several equations include in the right hand side the expected levels of the dependent variables. In addition to the macro equations, a CCA module is included, which interacts with the macro equations affecting each other in several ways. Moreover, the model contains a steady state to which the variables converge, thanks to the reaction of monetary authorities.

Finally, in order to assess the inclusion of risk indicators in the monetary authorities reaction function, we construct efficiency frontiers mapping inflation and output volatilities, after the artificial economy is hit with stochastic shocks drawn from a normal distribution.

⁴ Berg, Karam and Laxton (2006), page 3.

Section II presents the background of CCA, discusses the data used in the analysis. Section III lays out the macroeconomic framework, as well as the equations required to simulate Distance to Distress (*DTD*), which will be included in the macro setting. Section IV presents the results of the simulations and, finally, Section V concludes and presents possible extensions in this line of research.

II. RISK MEASURES FROM CONTINGENT CLAIMS ANALYSIS

A. Background

The contingent claims approach (CCA) provides a methodology to combine balance sheet information with widely used finance and risk management tools to construct markedto-market balance sheets that better reflect underlying risk. The risk adjusted balance sheets use option pricing tools to value the liabilities which are modeled as claims on stochastic assets. It can be used to derive a set of risk indicators that can serve as barometers of risk for firms, financial sector vulnerability, and sovereign risk.

A contingent claim is any financial asset whose future payoff depends on the value of another asset. The prototypical contingent claim is an option – the right to buy or sell the underlying asset at a specified exercise price by a certain expiration date. A call is an option to buy; a put is an option to sell and the value of each is contingent on the price of the underlying asset to be bought or sold. Contingent claims analysis is a generalization of the option pricing theory pioneered by Black-Scholes (1973) and Merton (1973). Since 1973, option pricing methodology has been applied to a wide variety of contingent claims. In this paper we focus on its application to the analysis of credit risk and guarantees against the risk of default, and their links to macroeconomic and financial developments.

The contingent claims approach is based on three principles: (i) the values of liabilities are derived from assets; (ii) liabilities have different priority (i.e. senior and junior claims); and, (iii) assets follow a stochastic process. The liabilities consist of senior claims (such as senior debt), subordinated claims (such as subordinated debt) and the junior claims (equity or the most junior claim). For a bank, as the value of its total assets decline, the debt that it owes to other institutions becomes riskier, and its value declines, while and credit spreads on its risky debt rise.

Balance sheet risk is the key to understanding credit risk and the probability of crisis. Default happens when assets cannot service debt payments, that is, when assets fall below a distress barrier comprising the total value of the firm's liabilities. Uncertain changes in future asset value, relative to promised payments on debt, is the driver of default risk. Figure 1 illustrates the key relationships. The uncertainty in asset value is represented by a probability distribution at time horizon T. At the end of the period the value of assets may be above the promised payments indicating that debt service can be made, or below the

promised payments leading to default. The area below the distribution in Figure 1 is the "actual" probability of default. The asset-return probability distribution used to value contingent claims is not the "actual" one but the "risk-adjusted" or "risk-neutral" probability distribution, which substitutes the risk-free interest rate for the actual expected return in the distribution. This risk-neutral distribution is the dashed line in Figure 1 with expected rate of return r, the risk-free rate. Thus, the "risk-adjusted" probability of default calculated using the "risk-neutral" distribution is larger than the actual probability of default for all assets which have an actual expected return (μ) greater than the risk-free rate r (that is, a positive risk premium).⁵



Figure 1: Distribution of Asset Value and Probability of Default

The calculation of the actual probability of default is outside the CCA/Merton Model but such a probability can be calculated by combining the CCA/Merton model with an equilibrium model of underlying asset expected returns to produce estimates that are consistent for expected returns on all derivatives, conditional on the expected return on the asset. One does not have to know expected returns to use the CCA/Merton models for the purpose of value or risk calculations, but for calibration into actual probabilities such data are necessary. The value of assets at time t is A(t). The asset return process is $dA/A = \mu_A dt + \sigma_A \varepsilon \sqrt{t}$, where μ_A is the drift rate or asset return, σ_A is equal to the standard

⁵ See Merton (1992, pp.334-343; 448-450).

deviation of the asset return, and ε is normally distributed, with zero mean and unit variance. The probability distribution at time T is shown in (a) below.

Default occurs when assets fall to or below the promised payments, B_t . The probability of default is the probability that $A_t \le B_t$ which is:

 $\operatorname{Prob}(A_{t} \leq B_{t}) = \operatorname{Prob}\left(A_{0} \exp\left[\left(\mu_{A} - \sigma_{A}^{2}/2\right)t + \sigma_{A}\varepsilon\sqrt{t}\right] \leq B_{t}\right) = \operatorname{Prob}\left(\varepsilon \leq -d_{2,\mu}\right)$ Since $\varepsilon \square N(0,1)$, the "actual" probability of default is $N(-d_{2,\mu})$,

where $d_{2,\mu} = \frac{\ln(A_0/B_t) + (\mu_A - \sigma_A^2/2)t}{\sigma_A\sqrt{t}}$. is distance to distress with a drift of mu and $N(\Box)$ is

the cumulative standard normal distribution.

The probability distribution of assets (dashed line in Figure 1) has drift of the risk-free interest rate, r. Risk adjusted probability of default is $N(-d_2)$, where

 $d_2 = \frac{\ln(A_0/B_t) + (r - \sigma_A^2/2)t}{\sigma_A \sqrt{t}}$. This is distance to distress with drift of r, the risk-free rate.

See Annex 1 for more information on the Merton Model, how to link actual and riskadjusted probabilities of default, and extension of the CCA model.

Financial fragility is intimately related to probability of default. Shocks to prices or liquidity frequently end up being converted into credit risk in a crisis, as banks' debtors income flows weaken and they run into difficulties servicing their loans to banks. Default is hard to handle in traditional macro models in part due to assumptions which usually exclude such possibility.⁶ In addition, flow-of-funds and accounting balance sheets cannot provide measures of risk exposures which are forward-looking estimates of losses. CCA, on the other hand, is a framework that explicitly includes and estimates the probability of default.

Since there is a nonzero chance of default, the value of debt is risky and therefore less than the value of risk free debt:

Risky debt + guarantee against default \equiv Risk-free debt

The value of "risky" debt can therefore be modeled as the default-free value of the debt less the expected loss:

Risky debt \equiv Risk-free debt - Guarantee Against Default

⁶ Transverality conditions exclude the possibility of default.

Given that this guarantee derives its value from the underlying (stochastic) asset, the value of the risky debt can be modeled as a contingent claim. This identity holds both conceptually and in terms of value. If the debt is collateralized by a specific asset, then the guarantee against default can be modeled as a put option on the asset with an exercise price equal to the face value of the debt. The debt holder is offering an implicit guarantee as it is obligated to absorb the losses if there is default. However, often a third party is the guarantor, as is the case when government guarantees the deposit liabilities of banks or the pension-benefit promises of firms.⁷

Using the Black-Scholes-Merton differential equation for pricing contingent claims, the value of risky debt is a function of the default free value of debt (i.e. distress barrier) at time 0, asset level at time 0, volatility of the asset, the time horizon until the expiration date of the claim, and the risk-free interest rate. Since 1973, the Merton Model methodology has been applied to a wide variety of corporations and financial institutions, as well as sovereigns.⁸

Banks do not frequently default⁹, and regulators are likely to be interested less in the probability of such an event than they are in the possibility that bank assets will fall below a level at which the authorities might be expected to intervene. One useful threshold is a minimum capital threshold. This barrier would be the default barrier plus say 8% of assets. The CCA model can be used of this analysis. This three layer model would give "distance-to-minimum capital" as well as "distance to distress." Annex 1 provides more details on this extension of the CCA model.

B. Calculating Risk Indicators for Individual Banks or Financial Institutions

Domestic equity markets provide pricing and volatility information for the calculation of implied assets and implied asset volatility in corporate, bank and non-bank financial institutions. The simplest method solves two equations for two unknowns, asset value and asset volatility (shown in Annex 1). Levonian (1991) used explicit option prices on bank equity to measure equity volatility and calibrate Merton Models for banks. Moody's-KMV has successfully applied its version of the CCA model to measure the implied assets values and volatilities and to calculate expected default frequencies (EDFs) for over 35,000 firms and financial institutions in 55 countries around the world KMV (1999 and 2001).

⁷ The CCA framework is an extension of Merton's models of risky debt (1974) and deposit insurance (1977).

⁸ See Gray and Malone (2008).

⁹ This has not been the case for many banks in the last Sub-Prime Crisis.

For unlisted corporates and banks, the relationship between the accounting information and the risk indicators, of companies with traded equity, can be used as a guide to map accounting information of companies without traded equity to default probabilities and risk indicators for institutions that do not have traded equity. (An example is Moody's RiskCalc for corporate sectors in many countries and for banks in the U.S.)

The CCA model for banks and financial institutions uses a time series of the daily market capitalization, the volatility of the market capitalization, and the distress barrier (derived from book values of deposits and debt) to estimate a time series of the implied market value of bank assets and asset volatility. Several useful risk indicators can be calculated for each bank or institution: (i) distance to distress; (ii) the risk adjusted and actual probabilities of default; (iii) the expected losses (put option) to depositors and debt holders; (iv) potential size of financial guarantees of the public sector; and, (v) sensitivity of risk indicators to changes in underlying bank assets, asset volatility or other factors. The steps used to calculate the implied assets and asset volatility of the individual bank or financial institution, and the risk indicators, is shown in Figure 2.



Figure 2: Calibrating Bank CCA Balance Sheets and Risk Indicators

C. A Distance-to-Distress Indicator for Chile

A useful indicator of banking or financial sector risk over time is a graph of average distance-to-distress (*DTD*) where $dtd = d_2$.¹⁰ Figure 3 shows the estimated time pattern of DTD for the Chilean banking system from 1998 to 2007. This procedure used historical volatility of market capitalization calculated with GARCH(1,1).



Figure 3: Distance to Distress for the Banking System

It is clear in the figure that the period of higher risk for the banking system coincides with the fallout from the LTCM/Russian crisis, between late 1998 and early 1999. Since then, the Chilean banking system has gradually reduced its risk, though this trend appears to have leveled off in late 2005¹¹. Other periods where markets assessed suddenly higher risk for the Chilean banks are easily discerned, for example, the decline in world stock markets

¹⁰ Derivation of the CCA risk indicators shown in Figure 3 are taken from Gray, Echeverría and Luna (2006) who used daily market capitalization data for the banks obtained by the Central Bank of Chile from the Bolsa de Santiago. Bank debt was obtained from the Superintendence of Banks and Financial Institutions' (SBIF) database. Financial practitioners use various methods for estimating the volatility of daily asset returns. Two frequently used methods model daily volatility either as a GARCH(1,1) or as a moving average process. The results shown here were obtained using the GARCH(1,1) methodology for all banks in the sample, but the results of the moving-average model are similar. A detailed technical analysis is developed in Echeverría, Gómez and Luna (2008).

¹¹ As we see below, this leveling-off has occurred at a very low level of risk.

following the collapse of the internet bubble in 2000 and the period preceding the Brazilian presidential elections in the third quarter of 2002.

In Figure 3, it is also clear the relation between the distance to distress of the banking system with both GDP annual growth and the output gap. The regressions with output and output gap as the dependent variable with DTD as one of the independent variables are shown in Annex 2. DTD has a significant impact on both output and output gap. Other systemic risk indicators that could be used are described in detail in Gray, Merton and Bodie 2007 and 2008.¹² (also see Goodhart et al. 2006a and 2006b, Gray and Walsh 2008, Gray and Malone 2008, Haldane et al. 2007, and Segoviano 2006a, 2006b).

III. LINKING MACROFINANCE INDICATORS TO A SIMPLE DYNAMIC, STOCHASTIC MACROECONOMIC POLICY MODEL

In this section, we will lay out an integrated, "macrofinance policy model" in which macrofinancial outputs are incorporated directly into macroeconomic policy models. Our focus here will be on a modular exposition of the parts of the model and the equations that comprise these parts, as well as giving intuition for how they are linked together and can be used for the analysis of a wide range of policies.

Distance-to-distress for the banking system is included in the GDP gap equation, the parity condition, and in the policy rate reaction function. The model parameters are then estimated using historical data, including the distance-to-distress indicator but some of them are also calibrated.¹³ The approach can be used to examine the tradeoffs between GDP, inflation, with and without the inclusion of distance-to-distress for the banking system in the monetary authorities reaction function.¹⁴

The five equation dynamic, stochastic macroeconomic model used to set monetary policy was already briefly described. This model, close to the one by Berg, Karam, and Laxton (2006), was built in the Central Bank of Chile at the start of the implementation of

¹² Examples of forward-looking indicators of systemic risk from the CCA model are distance-to-distress (*DTD*), expected loss (i.e. implicit put option), or the default probability weighted by the assets of individual financial institutions. The skewness from equity put options is another indicator that can indicate financial system distress and also periods of excessive exuberance. See Gray, Merton, and Bodie 2008a and 2008b.

¹³ A related issue is whether an indicator of market risk appetite such as the VIX should be included in monetary policy models along with the risk indicator. This could help estimate the impact of the credit risk indicator on the GDP gap, adjusted for changes in risk appetite.

¹⁴ There are several other interesting routes to take in linking risk analytics more closely with macroeconomic models. These include incorporating default risk and a risk premium into the Mundell-Fleming model to separate out the effects of changes in interest rates due to changes in the market for liquidity, and changes in interest rates due to changes in the risk premium on debt (See Gray and Malone, forthcoming IMF WP).

fully fledged inflation targeting in 2000. An application of it to the design of monetary policy using efficiency frontiers is found in Herrera, García and Valdés (2002). It is one specimen of a class of macroeconomic policy models that can be used for policy analysis in small open economies.

Module 1: Output, Inflation, exchange rate, and a Taylor rule

The first module of our macrofinance policy model consists of equation for the most important macro variables. Thus, there is an equation for the output gap, an equation for inflation, an equation for inflation expectations, and a Taylor rule for setting the domestic policy rate. The domestic policy rate is a short term interest rate set by the central bank.

Equation for output gap

$$ygap_{t} = \beta_{1}ygap_{t-1} + \beta_{2}ygap_{t-2} + \beta_{3}ygap_{t-3} + \beta_{4}(r_{t-1} - \bar{r}_{t-1}^{eq}) + \beta_{5}(rl_{t-2} - \bar{r}l_{t-2}^{eq}) + \beta_{6}(q_{t-4} - q_{t-4}^{eq}) + \beta_{7}\min(ldtd_{t}, 0) + \varepsilon_{t}^{y}$$

$$(1)$$

Where *ygap* corresponds to the output gap, *r* is the short-run real interest rate, *rl* the long-run real interest rate, *q* is the real exchange rate and *ldtd* is the natural logarithm of distance to default (ln(*DTD*)), which is also modeled here. As was explained in detail above, distance to default (*DTD*) is a financial risk indicator that could reflect, in general, the financial conditions that the economy faces. Finally, ε_t^y is a shock to GDP.

Phillips curve

$$\Delta \pi_{t} = \alpha_{1} \left[(\pi_{t-2} + \pi_{t-3} + \pi_{t-4})/3 - \pi_{t-1} \right] + \alpha_{2} \left[(\pi_{t+1}^{e} + \pi_{t+2}^{e})/2 - \pi_{t-1} \right] + \alpha_{3} \left[(s_{t-1} - s_{t-4} + pf_{t-1} - pf_{t-4})/3 - \pi_{t-1} \right] + \alpha_{4} \left[(ygap_{t-1} + ygap_{t-2})/2 \right] + \varepsilon_{t}^{\pi}$$

$$(2)$$

Where π_t stands for inflation, s_t is the nominal exchange rate and pf_{t-1} is the foreign price level., inflation expectations, lags of the rate of nominal exchange rate depreciation, the local currency debt risk premium

Exchange rate equation (interest parity condition)

$$q_{t} = \delta_{1}q_{t-1} + \delta_{2}q_{t+1} + (1 - \delta_{1} - \delta_{2})q^{eq} + (r - rf) + \delta_{4}\min(ldtd_{t}) + \varepsilon_{t}^{q}$$
(3)

The real exchange rate depends on the domestic policy rate, the foreign policy rate, the sovereign spread for domestic debt, the sovereign spread for foreign debt

According to uncovered interest rate parity, the expected change in the spot exchange rate should be related to the differential between the domestic and foreign interest rates, plus some risk premium. In practice, it has been found that a rise in domestic interest rates is usually associated with a subsequent appreciation, rather than a depreciation of the exchange rate as standard economic theory predicts.

Long term interest rate (yield curve)

$$(rl_{t} - rl^{eq}) = \delta_{1}(rl_{t+1}^{e} - rl^{eq}) + \delta_{2}(rl_{t-1} - rl^{eq}) + (1 - \delta_{1} - \delta_{2})(r_{t} - rl^{eq}) + \varepsilon_{t}^{rl}$$
(4)

This equation describes the relationship between long run (rl_t) and short run (r_t) interest rates

Reaction Function (Taylor rule)

$$r_{t} = \rho(r_{t-1}) + (1-\rho) \left\{ r l^{eq} + \theta \left[\gamma(\pi_{t+1} + \pi_{t+2} + \pi_{t+3} + \pi_{t+4} - 4\overline{\pi}) / 4 + (1-\gamma)(ygap_{t-1}) \right] \right\} + \delta_{3} \min(ldtd_{t}, 0) + \varepsilon_{t}^{r}$$
(5)

The monetary policy interest rate depends on its own lag, expected inflation gap ,output gap, and distance to default. While including a measure of financial stability in the Taylor rule for setting interest rates may be able to improve efficiency (welfare), in particular if financial stability affects output, there may be better ways to target financial stability than the interest rate.

Module 2: Distance to Distress Model for the Banking System

This module completes the whole system to be simulated

The value of assets AA is derived from the Black & Scholes (B&S) model,

$$AA = (EE + BB * \exp(-r * t) * cdfd2) / cdfd1$$
(6)

where *EE* is the value of the Equity (or the same, the value of the call option). *BB* is the value of the debt in the B&S model, but here it is also the default barrier, where 'r' is the risk free interest rate and 't' is time -fixed in the model to one year. Finally cdfd2 = N(d2), where N() is the normal cumulative distribution function and d2 is derived from the B&S model explained in section II (the same is true for cdf1 = N(d1)).

The transformation of N(d2) and N(d1) into cdfd2 and cdfd1 is required because the software used to solve the model does not have an explicit function for the cumulative normal distribution function. The following two equations are used to build such approximations:

The cumulative distribution function of d2 $cdfd2 = 1 - (1/2.506628) * EXP(-1/2*(d2^2)) * (0.4361836*(1/(1+0.33267*d2)))$ $+ (-0.1201676) * (1/(1+0.33267*d2))^2$ (7) $+ 0.937298*(1/(1+0.33267*d2))^3)$

And the cumulative distribution function of d1 $cdfd1 = 1 - (1/2.506628) * EXP(-1/2*(d1^2)) * (0.4361836*(1/(1+0.33267*d1)))$ $+ (-0.1201676) * (1/(1+0.33267*d1))^2 + 0.937298*(1/(1+0.33267*d1))^3)$ (8)

Where d1 and d2 definitions are in section II and correspond to:

$$d1 = d2 + Sa^*\sqrt{t}$$
 and (9)

$$d2 = (log(AA/BB) + (r - (Sa^2)/2)*t)/(Sa*\sqrt{t}) + dtd_shk$$
(10)

Note that d2 is equal, precisely, to Distance to Default (DTD = d2)

It is apparent from equation 10 that assets volatility, *Sa* and assets value, *AA* are crucial for finding *DTD*. Thus, the system of non linear equations requires an equation for *Sa* to have a solution:

$$Sa = (Se^*EE) / (AA^*cdfd1)$$
⁽¹¹⁾

Where, Se stands for volatility of the equity.¹⁵

As the reader may recall, distance to default affects GDP in equation 1 of the macro model. In the following equation, GDP growth affects banks' capital, *EE* and through it distance to default, as well as GDP making the system completely endogenous:

$$EE = \rho EE(-1) + 0.01*((y(-1) - y(-4))/3 - (y_eq(-1) - y_eq(-4))/3)$$
(12)

Finally, another measure of risk, described in Gray, Merton and Bodie (2008) and Gray and Malone (2008) that could be used here is the *spread put*, which is a function of the value of the *Put option*, the default barrier, the risk free rate and time:

¹⁵ A through explanation is found in Gray and Malone (2008).

spread _ put = $-1/t * \log(1 - PUT / BB * exp(-r*t)) - 0.00925382$

IV. STOCHASTIC SIMULATIONS AND POLICY ANALYSIS

In order to understand how the model works, we obtained impulse responses (Figure 4). Next, we assess different alternatives of monetary policy as well as calibrations of the model by building efficiency frontiers with the volatilities of GDP and inflation (García, Herrera and Valdés, 2002; Laxton and Pesenti, 2003).

Responses of GDP, inflation, the exchange rate, and the monetary policy interest rate r, as well as the *CCA* derived risk indicator *ldtd*, are obtained after a shock of 100 basis points is given to every variable (Figure 4).

- After a shock to inflation (cost-push shock), the monetary policy rate (MPR) goes up, reducing the output gap (*ygap*) and appreciating de currency (RER falls). The RER fall also contributes to depress the output. On the other hand, *DTD* barely moves.
- In the case of a positive shock to the output gap, GDP and inflation increase, and so do interest rates, while the exchange rate falls in agreement with economic intuition. The system takes around six years to return to equilibrium after the shock.
- The shock to the real exchange rate *q* has a very moderate impact on inflation and GDP. The movements observed in the other variables are barely noticeable.
- A Monetary Policy Interest Rate hike generates a large appreciation of the currency followed by a depreciation before the real exchange rate converges back to equilibrium. Given that GDP also falls after the shock, these two variables put downward pressure to inflation, which exhibits an extremely persistent negative deviation from its steady state level.
- Once the long-run real interest rate (*rl*) is hit by the respective shock output and both output and inflation decrease. Given that monetary authority reacts to economic developments through a Taylor rule, a reduction of the short-run interest rate is called for. As a result, the real exchange rate exhibits a large positive swing.
- Finally, a negative shock to distance to default (*DTD*) is implemented causing a recession. Due to the fact that *DTD* is included in the policy reaction function, the original shock is followed by a reduction in the MPR. Moreover, arbitrage through the uncovered interest parity results in a large real depreciation.
- In general, the model works as expected according to standard economic intuition signs and magnitudes are sensible. While there is strong interaction among macro

variables, *DTD* movements are small. However, *DTD* has a large impact on MPR, MPR and OPG.



Figure 4: Impulse Responses

Source: authors' calculations

The efficiency frontiers are built combining the volatility of inflation and GDP that results after the economy is hit repeatedly by shocks drawn from a normal distribution. Indeed, standard deviations of the variables were computed between periods 20 and 25 after the shock hits the artificial economy. The purpose of the exercise is to compare frontiers that were obtained using different policy rules or even different calibrations of the model. Whenever a frontier is closer to the origin, one says that the policy choice is better for the central bank and the society as a whole.

Each of the figures below includes two frontiers. One obtained with the baseline model that includes a traditional Taylor Rule in which there is only inflation and *GDP* gaps (theta=0.5, rho=0.6 and gamma =0.6), i.e., authorities do not react to the risk indicator (blue line). The other corresponds to an alternative reaction function for monetary policy that also includes *ldtd* in it (with a coefficient equal to 0.5) in a non-linear way (red line). Namely, besides reacting to inflation and *GDP* gaps the monetary authority also reacts to distance to default, but only when it is below a certain critical value, indicating that the banking system is close to default.

Reaction size to *DTD* in the policy rule

Compared to the frontier obtained with the standard Taylor rule (blue line in the left panel of Figure 5), a central bank's reaction function that also includes *dtd* implies a lower level of inflation volatility while volatility of *GDP* increases (red line in the same chart). The reduction of inflation volatility ranges from 5 to 10 basis points for the same level of output volatility (Figure 5). In general, a stronger reaction to *DTD* moves the frontier upward and leftward. In other words, volatility of inflation decreases while volatility of production grows the opposite happens when the coefficient of DTD is smaller. The red line moves down (red line of the right panel in Figure 5).



Figure 5: Efficiency frontier

Source: authors' calculations

• GDP impact on bank equity (endogeneity)

A comparison of this graph with the previous one shows that if there is no feedback from GDP to bank equity and thus to *dtd* (endogeneity), the rule without distance to default leads to a frontier that is closer to the origin (Figure 6, right panel). Thus, the policy that excludes *dtd* is preferred to the alternative. Regarding inflation, some gain can still be obtained by including *dtd* in the monetary rule, given that part of the red line goes further towards the left, i.e. inflation volatility is lower. However, if volatility of both inflation and GDP are taken into account, one can say that the closest points to the origin are in the blue, therefore, the central bank faces smaller combinations of volatilities.



Figure 6: Efficiency frontier and endogeneity of bank's equity

Source: authors' calculations

No effect of DTD on real exchange rate

In this experiment the impact of *DTD* on the risk premium and the exchange rate was reduced (Figure 7, right panel). Again, the blue line, which represents the frontier obtained with no *dtd* in the reaction function, includes points that are closest to the origin, closer than points in the red line (rule with *dtd*). Thus this policy of not reacting to *dtd* should be preferred by the central bank.



Figure 7: Efficiency frontier and interest parity condition

Source: authors' calculations

Previous: Lower pass-through

Again a policy in which the Central Bank reacts to *DTD* is more efficient in economies where the pass-though of inflation to exchange rate is low (see Figure 8). This is an important issue in economies that have small levels of inflation or small price inertia, in this case it would be preferred that the Central Bank reacts to *DTD*.





Source: authors' calculations

In summary, if pass-through is higher, if financial vulnerability or *DTD* has a larger impact on the exchange rate, and on GDP (or GDP also has a larger impact on bank's equity

and through it on *DTD* –more endogeneity), it is better to include *DTD* in the reaction function.

V. CONCLUSIONS

This main objective of this paper is the integration of the analysis of financial sector vulnerability into macroeconomic models, which is an area of important and growing interest for policymakers in both developed and emerging markets. This paper uses contingent claims analysis (CCA) tools, developed in finance, to construct financial stability indicators and incorporate them in a standard monetary policy model. The economy and interest rates affect financial sector credit risk, while the financial sector affects the economy. Market-based financial stability incicators (FSIs) summarize both the credit channel and credit risk transmission from distressed borrowers in the economy.

The new framework is simple, but powerful for monetary policy analysis. Indeed, the model has the main variables analyzed by policymakers, but is small enough to understand easily how it works. Even though, it is an artificial economy used to be stochastically simulated, the empirical evidence supports the model. In addition, impulse responses behave in accordance with economic intuition.

The main question to be answered with the integrated model is whether or not the central bank should include explicitly the financial stability indicator in the interest rate reaction function. The alternative is to react only indirectly to financial risk by reacting to inflation and GDP gaps, since they already include the effect financial factors have in the economy. In order to reach the objective, efficiency frontiers are built with the volatility of inflation and output obtained from the stochastic simulations. It is found in general, that a including *DTD* in the reaction function reduces inflation volatility while increasing the variability of output.

A set of exercises were also performed in which some of the parameters of the model were calibrated to reflect and assess actual differences among economies regarding exchange rate pass-through, the relation between financial risk and exchange rate (through the parity condition and the endogeneity of the financial indicator: its effect on GDP and the impact GDP on banks equity and distance to default.

The conclusions are that it is better to include DTD in the interest rate reaction function if exchange rate pass-through is higher; if financial vulnerability (DTD) has a larger impact on the exchange rate, as well as on GDP (or the reverse, there is more impact of GDP on bank's equity and thus DTD –endogeneity).

Finally, this paper is the first to address the subject of forward-looking financial stability indicators in a monetary policy model and there are a number of refinements and extensions that could be introduced in the future.

A non exhaustive list of refinements and extensions includes:

- Combinations of financial scenarios (strong, normal, fragility) could be incorporated.
- The effects of interest rates on the FSI/DTD indicator and feedbacks of the indicator on interest rates could be included.
- Different FSIs could be tested. Note that forward-looking market based indicators can indicate financial vulnerability and when these indicators are abnormally high could be indicators of exuberance and might be incorporated into monetary policy analysis during boom/exuberance periods.¹⁶
- Extensions could include FSIs which are market-based indicators of capital adequacy. (Distance to minimum capital is described at the end of Annex 1 and ways that capital adequacy might be included in the monetary policy model equation is described in Annex 3.)
- Changes in the dynamics of the macro model should be tested (GEM).
- Empirical data in other countries could be used and the model or its extensions can be applied to other economies.

¹⁶ See Gray, Merton, Bodie (2008a and 2008b).

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Annex 1– Estimating Implied Assets and Volatility, Default Probabilities and Extensions of the Merton Model

This Annex provides details on estimating implied assets and asset volatility and extensions of the Merton Model.

Calculating Implied Assets and Implied Asset Volatility

The value of assets is unobservable, but it can be implied using CCA. In the Merton Model for firms, banks and non-bank financials with traded equity use equity, J, and equity volatility, σ_{J} , and the distress barrier in the following two equations to solve for the two unknowns A, asset value, and σ_{A} , asset volatility. (See Crouhy, Mark and Galai (2000)).

$$J = A_0 N(d_1) - \overline{B} N(d_2)$$
$$J\sigma_J = A\sigma_A N(d_1)$$

Extensions of the Merton Model

Numerous extensions of the original Merton Model have been developed that relax certain assumptions in the original model. Restrictions of the model include the assumptions that: (i) default can occur only at the maturity date of the debt; (ii) there is a fixed default barrier; (iii) there is a constant risk-free rate; and, (iv) asset volatility is constant. Cossin and Pirotte (2001) provide a good summary of extensions of the Merton Model. Black and Cox (1976) extended the Merton Model to relax the assumptions (i) and (ii) above by introducing a "first passage time" model where default can occur prior to the maturity of the debt if the asset falls below a specified barrier function for the first time.

Although the strict theoretical condition in the Merton Model for default is that the value of assets is less than the required payments due on the debt, in the real world, default typically occurs at much higher asset values, either because of a material breach of a debt covenant or because assets cannot be sold to meet the payments ("inadequate liquidity") or because the sovereign decides to default and induce a debt renegotiation rather than sell assets. To capture these real-world conditions for default in the model, we specify a market value of total assets at which default occurs. We call this level of assets that trigger default the "distress barrier." This barrier can be viewed as the present value of the promised payments discounted at the risk-free rate. The approach used in the KMV model sets the barrier level equal to the sum of the book value of short-term debt, promised interest payments for the next 12 months, and half of long-term debt (see Crouhy, et. al. (2000) and KMV (1999, 2001)).

In the 1990s the KMV model was based the VK model (Vasicek and Kealhofer) which has multiple layers of liabilities and several confidential features. MKMV's EDF (expected default frequency) credit measure is calculated using an iterative procedure to solve for the asset volatility. This distance-to-distress was then mapped to actual default probabilities using a database of detailed real world default probabilities for many firms. The MKMV distance-to-distress and the CEDF (cumulative expected default probabilities) are calculated as follows:

$$DD_{KMV} = f\left(\frac{\ln\left(A_0 / B_t\right) + \left(\mu_A - \sigma_A^2 / 2\right)t}{\sigma_A \sqrt{t}}\right)$$
$$CEDF_t = f\left(DD_{KMV}(t)\right)$$

Note that this definition of DD_{KMV} includes the real drift of the asset, μ_A , whereas the distance-to-distress from the Merton approach has *r* for the asset drift. Since MKMV estimates the actual default probabilities, the risk neutral default probabilities are calculated from the correlation of the implied asset with the market, the market Sharpe Ratio, and time horizon.

The Merton Model has been extended to include stochastic interest rates as well. Shimko, Tejima, and Van Deventer (1993) include a Vasicek interest rate term structure model which relaxes assumption (iii) above allowing the risk free interest rate to change and including the correlation of asset return with the interest rate. There are two stochastic factors, the asset and the interest rate and this model is frequently called the STV model. This closed form model is a very useful extension by including the impact of changing interest rate term structures. Longstaff and Schwartz (1995) take the Black and Cox (1976) model and add in stochastic interest rates, similar to the way STV includes interest rates.

The CreditGrades model (2002) includes a diffusion of a firm's assets and a first passage time default with a stochastic default barrier. The model was modified to incorporate equity derivatives (Stamicar and Finger 2005). Recent research has studied the relationship between the volatility skew implied by equity options and CDS spreads (Hull et. al. 2004). They establish a relationship between implied volatility of two equity options, leverage and asset volatility. This approach is, in fact, another way of implementing Merton's Model to get spreads and risk-neutral default probabilities directly from the implied volatility of equity options. A similar approach using several equity options is discussed in Zou (2003).

Financial support for liquidity and potential credit risk from the authorities is likely to be provided before "default" barrier is reached. A minimum capital barrier, or simply a capital barrier, can be defined in addition to the default barrier. The default barrier plus 4% of market value of assets will be used as the "minimum 4% capital" barrier. The default

barrier plus 8% of market value of assets will be used as the "minimum 8% capital" barrier. Figure A1 below shows the area between the minimum capital barrier and the default barrier. The area represents the probability of falling below minimum capital but not as far as default. The value of this area is calculated as the implicit put option below the minimum capital barrier minus the implicit default put option. We will call the value of the area as the "capital barrier put option" or "capital barrier expected loss."

This is particularly relevant to the central bank as it is a measure loss directly related to liquidity support/financial support which would be needed to get the bank asset level above the minimum capital level.

Figure A1 Volatile Assets Relative to Debt Distress Barrier and "Minimum Capital Barrier"



Annex 2 Regression Results of Output and Output Gap on Distance to Distress of the Banking System

The first regression is on:

 $\Delta y_t = c + \alpha_1 r_{t-1} + \alpha_2 \Delta dt d_{t-1} + \alpha_3 \Delta e_{t-1} + \alpha_4 \Delta y_{t-1} + \varepsilon_t$

Dependent Variable: DLOG(YS,0,3) Sample (adjusted): 1998M05 2007M02 Included observations: 106 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	0.011	0.002	4.830	0.000
R(-1)	-0.001	0.000	-3.723	0.000
DLOG(TCR(-1),0,3)	0.046	0.019	2.438	0.017
DLOG(DTDS(-1),0,3)	0.012	0.003	3.551	0.001
DLOG(YS(-1),0,3)	0.463	0.074	6.283	0.000
R-squared	0.574	Mean dependent var		0.009
Adjusted R-squared	0.557	S.D. dependent var		0.013
S.E. of regression	0.008	Akaike int	fo criterion	-6.677
Sum squared resid	0.007	Schwarz	criterion	-6.552
Log likelihood	358.890	F-statistic	;	34.036
Durbin-Watson stat	1.912	Prob(F-st	atistic)	0.000

 $gap_{t} = c + \alpha_{1} \Delta dtd_{t-1} + \alpha_{2} \Delta e_{t-1} + \alpha_{4} gap_{t-1} + \varepsilon_{t}$

Dependent Variable: YGAP Sample (adjusted): 1998M02 2007M02 Included observations: 109 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	-1.736	0.470	-3.691	0.000
DLOG(TCR(-3),0,3)	4.134	1.639	2.522	0.013
LOG(DTDS(-1))	0.934	0.256	3.653	0.000
YGAP(-1)	0.513	0.082	6.275	0.000
YGAP(-3)	0.225	0.072	3.113	0.002
R-squared	0.661	Mean dep	Mean dependent var	
Adjusted R-squared	0.648	S.D. depe	S.D. dependent var	
S.E. of regression	0.712	Akaike in	Akaike info criterion	
Sum squared resid	52.766	Schwarz	Schwarz criterion	
Log likelihood	-115.126	F-statistic	F-statistic	
Durbin-Watson stat	1.842	Prob(F-st	Prob(F-statistic)	

These regressions show that changes in DTD are significant in explaining both GDP quarterly growth (equation #1) and the output gap (equation #2) with the expected (positive) sign.

Annex 3– Extensions to Include Capital Adequacy

The central bank may expand its set of policy instruments to better accommodate its multiple objectives. Additional tools that can be used to target financial stability include the reserve requirements for banks and other measures of capital adequacy, such as Value-at-Risk based measures advocated in Basel II. The distance to minimum capital concept described at the end of Annex 1 could be a forward-looking indicator related to capital adequacy. As described by Gray and Malone (2008) a rule could be specified for targeting such a measure of capital adequacy, C, as follows:

 $C_{t} = \phi_{1}C_{t-1} + (1 - \phi_{1})[\eta_{2}ygap_{t} + \eta_{3}fsi_{t}] + \varepsilon_{10,t}$

The closer the parameter ϕ_1 is to one, the more continuity is built into the capital adequacy requirement. As in the case of interest rates, some continuity is important, because significant changes in capital adequacy requirements, or interest rates, in a short amount of time can also potentially contribute to instability as banks move en masse to comply with new requirements. The second term in the above rule, which is multiplied by the coefficient $1 - \phi_1$, allows the central bank to use capital adequacy requirements, or other variables that affect the risk profile of the banking sector, to respond to deviations of inflation, output, and financial stability from their targets.

Lower capital adequancy requirements, by stimulating lending, may be able to contribute to higher investment that stimulates output when output is below target. Likewise, more stringent capital adequacy requirements can help increase the financial stability indicator when it is below target, by lowering the probability of banking sector instability or widespread defaults.