Learning, Endogenous Indexation and Disinflation in the New-Keynesian Model

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Abstract

This paper introduces adaptive learning and endogenous indexation in the New-Keynesian Phillips curve and studies disinflation performance under inflation targeting policies. The analysis is motivated by the disinflation performance of Latin American inflation-targeting countries, in particular, the Chilean gradual disinflation experience with announced short-term inflation targets. At the start of the disinflation episode price-setters expect highly persistent inflation and indexation is complete. As the central bank acts to bring inflation under control, price-setting firms revise their beliefs regarding the degree of persistence. Thus, adaptive learning lowers the cost of disinflation and a gradual approach to disinflation can take advantage of this beneficial effect. Firms that choose the rate for indexation also re-assess the likelihood that announced inflation targets determine steady-state inflation and adjust indexation of contracts accordingly. A strategy of announcing and immediately achieving relatively modest short-term targets for inflation is found to influence the likelihood that firms switch from backward-looking indexation to the central bank's announced targets more effectively than a strategy with a long-run target that is achieved only gradually. Learning introduces additional dynamics into the model. A sophisticated central bank with complete information regarding the learning process of price-setters would be able to improve policy performance further by exploiting these dynamics. As such information is typically not available in practice an alternative approach with central bank learning is proposed.

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1 Introduction

Disinflation episodes and strategies have been studied extensively under the assumption of rational expectations. However, this assumption attributes substantial knowledge and ”computing” capacity to market participants. More recently, many contributions to the monetary policy literature have allowed departures from rational expectations. In this vein, the present paper introduces adaptive learning in the New-Keynesian Phillips curve and assesses inflation stabilization under inflation targeting strategies. The analysis is motivated by the disinflation performance of Latin American inflation-targeting central banks, in particular, the Chilean central bank’s experience with gradual disinflation based on announced short-term inflation targets.

A novel element of the analysis is endogenous indexation. At the start of the disinflation episode indexation is complete and price-setters expect highly persistent inflation. As price-setting firms learn over time they re-assess the likelihood of announced inflation targets and adjust indexation of contracts accordingly. The findings in the paper confirm that learning and endogenous indexation help lower the costs of disinflation. A gradual disinflation approach can take advantage of these beneficial effects. An interesting new result is the finding that announcing and achieving short-term targets for inflation may reduce disinflation costs relative to the announcement of a long-run inflation target that will only be achieved after many years of gradual disinflation. A sophisticated central bank with complete information regarding the learning process of price-setters would be able to improve policy performance. As such information is typically not available in practice, an alternative approach with central bank learning is proposed.

The paper proceeds as follows. Section 2 contrasts the costs of disinflation under the simplest forward-looking New-Keynesian Phillips curve and a traditional accelerationist Phillips curve. The latter relationship will re-emerge later in the paper at the start of disinflation given a structural New-Keynesian Phillips curve with complete backward-looking indexation and adaptive expectations. Section 3 emphasizes some lessons from the Chilean disinflation experience. The New-Keynesian Phillips curve with adaptive learning and endogenous indexation is introduced in section 4. Section 5 compares immediate and gradual disinflation strategies. Section 6 assesses the performance of temporary inflation targets with learning and endogenous indexation. Section 7 presents alternative approaches to optimal policy design with learning and section 8 concludes.

2 Costly disinflation

As is well-known the macroeconomic policy goals of stabilizing output and inflation do not come into conflict in the simplest, micro-founded New-Keynesian Phillips curve (cf. Walsh (2003), Woodford (2003)). This controversial property–sometimes termed the ”divine coincidence”–is based on the following specification

$$(\pi_t - \pi^S) = \beta E_t[(\pi_{t+1} - \pi^S)] + \lambda x_t \tag{1}$$

where π_t refers to the inflation rate and x_t to the output gap. This relationship is derived assuming that price-setting firms only adjust prices optimally once they receive a signal to do so as proposed by Calvo (1983). There is a constant probability $1-\theta$ of such a signal. The slope parameter λ is a function of θ and the discount factor β.

In the periods when firms do not adjust prices optimally, price changes are tied to the steady-state inflation rate denoted by π_t^S. Steady-state inflation and average inflation are equal to the central bank's inflation target π^*. If the central bank adjusts the inflation target downwards inflation expectations as well as actual inflation follow immediately. Thus, inflation is achieved without any loss of output.

This property stands in contrast to the conventional wisdom that setting monetary policy in order to achieve the inflation target at any and all times is suboptimal due to resulting output variations. This concern is embodied in the traditional accelerationist Phillips curve:

$$\pi_t = \pi_{t-1} + \lambda x_t \tag{2}$$

Clearly, this inflation-output relationship does not exhibit the divine coincidence property. Disinflation is costly and requires tough policies by the central bank. The sacrifice ratio, that is the cumulative output loss per percentage point of disinflation, is constant and equal to the inverse of λ. If the above equation were to constitute a structural relationship, then the sacrifice ratio could not be affected by policy and would be independent of π^* whether or not π^* is announced.

This paper revisits questions regarding the cost of disinflation previously analyzed in models with rational expectations in an environment with adaptive learning following recent contributions by Orphanides and Williams (2005, 2006) and Gaspar, Smets and Vestin (2005,2006).[1] Similar to recent analyzes of the New-Keynesian models it incorporates indexation to past inflation by price-setters that are only rarely able to fully re-optimize price-setting in a forward-looking manner. An novel element is endogenous indexation triggered by learning. Firms actively consider whether lagged inflation or the policy-makers announced inflation target constitutes a more likely long-run mean of inflation based on the most recent data. Once the probability of the inflation target is high enough, these firms use the next optimal price-adjustment opportunity (i.e. the signal in the Calvo framework) to also switch the index for indexation in future periods without such adjustment opportunities.

Before proceeding to investigate the impact of learning, endogenous indexation and inflation targets on the cost of disinflation some instructive elements of the Chilean dis-

[1] See also the influential monograph by Evans and Honkapohja (2001).

inflation experience are reviewed.

3 The Chilean Experience

The adoption of inflation targeting and its success in stabilizing inflation in several Latin American countries provides a set of fascinating case studies concerning the design of monetary stabilization policies. The case of Chile is particularly interesting. There exists an impressive literature studying the Chilean experience using the most sophisticated modeling and empirical techniques (cf. Aguirre and Schmidt-Hebbel (2005), Caputo, Liendo and Medina (2007), Caputo, Medina and Soto (2006), Cespedes, Ochoa and Soto (2005), Herrera (2002), Lefort and Schmidt-Hebbel (2002) and Schmidt-Hebbel and Werner (2002) and others).

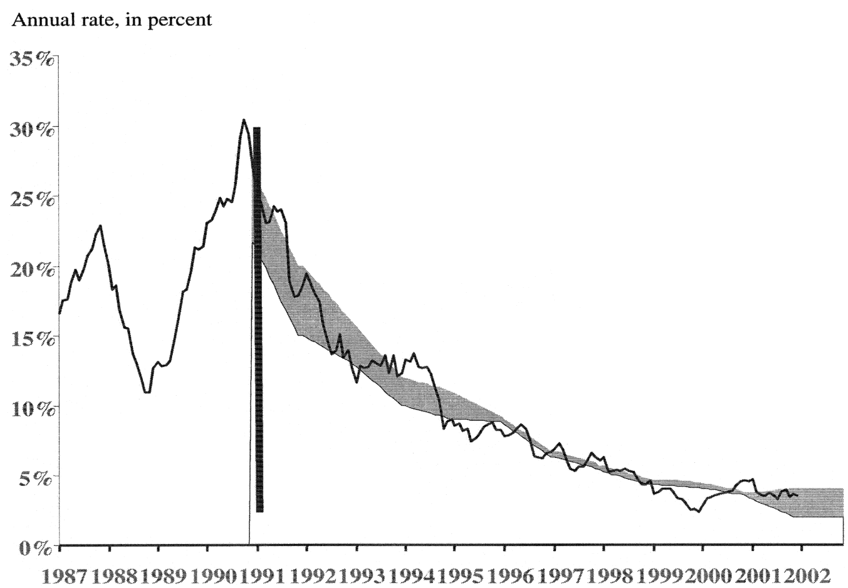
The Chilean disinflation stands out as a very gradual disinflation. Nevertheless, the central bank was very successful in terms of achieving pre-announced temporary inflation targets. Associated output losses appear to have been limited. Aguirre and Schmidt-Hebbel (2005) write that the Chilean central bank initially identified two major difficulties in searching for a disinflation strategy: low policy credibility and widespread backward-looking price indexation in goods, labor, and financial markets. These authors argue that the central bank was able to overcome the consequences of backward-looking price indexation and related inflation inertia, to improve policy credibility, and to influence private-sector inflation expectations, by adopting a forward-looking inflation target as the explicit nominal anchor for conducting monetary policy.

The central bank's first official target was publicly announced in September 1990 and set for a range of 15 to 20 % for the rate of annual CPI inflation between December 1990 and December 1991. From 1991 to 1999 inflation targets were set on an annual basis for the following calendar year as target ranges or as point targets.**Figure 1** reports the inflation targets along with actual inflation.

Aguirre and Schmidt-Hebbel argue that Chile's annual inflation targets during 1991-2000, even though announced for the short-term, were observationally equivalent to hard policy targets in full-fledged IT regimes and provide some evidence. Empirical investigations of New-Keynesian Phillips curves for Chile such as Cspedes, Ochoa and Soto (2005) report evidence of structural change during the late 1990s. This change is exhibited in a higher weight of expected future inflation - and a correspondingly lower weight of lagged inflation - when producers set their prices. These authors provide evidence that the extent of indexation declined over time. For a sample from 1990 to 2000 they estimate a degree of backward-looking indexation around 0.85, essentially indistinguishable from the limiting case of complete indexation. With the sample extended to 2005, however, the degree of indexation declines to around 0.66.[2].

[2]Further interesting findings on more elaborate New-Keynesian models with additional rigidities in

Figure 1: Inflation Targets and Actual Inflation in Chile, 1990-2002*



Source: Banco Central de Chile.

Note: *This figure is taken from Schmidt-Hebbel and Werner (2002) and uses data up to 2002. It will be replaced with an updated figure using data from 1990 to 2007 supplied to the author by the Central Bank of Chile.

In the remainder of this paper, we explore possible links between the particular inflation targeting strategy and the degree of inflation persistence perceived by price setters as well as endogenous reduction in backward-looking indexation.

4 Adaptive learning and endogenous indexation

Indexation in the New-Keynesian Phillips curve

Christiano, Eichenbaum and Evans (2001, 2005) have shown that the basic New-Keynesian Phillips curve can be extended to incorporate indexation to past inflation. Price-setting firms that do not receive a signal to adjust prices optimally implement a pricing rule based on past inflation. Assuming that a share κ of price-setting firms indexes to past inflation while the remaining $(1-\kappa)$ firms index to the steady-state inflation rate the basic New-Keynesian Phillips curve takes the following form:

$$\pi_t - (\kappa\pi_{t-1} + (1-\kappa)\pi^S) = \beta E_t[(\pi_{t+1} - (\kappa\pi_t + (1-\kappa)\pi^S))] + \lambda x_t \qquad (3)$$

Solving for current inflation, it follows that inflation at time t depends on a weighted

labor markets and the importing sector are available from Caputo, Liendo and Medina (2007) and Caputo, Medina and Soto (2006) average of past inflation and expected future inflation with the weight depending on the share of firms that implement backward-looking indexation:

$$\pi_t = \frac{\kappa}{1+\beta\kappa}\pi_{t-1} + \frac{\beta}{1+\beta\kappa}E_t[\pi_{t+1}] + \frac{\lambda}{1+\beta\kappa}x_t + \frac{(1-\kappa)(1-\beta)}{1+\beta\kappa}\pi^S \tag{4}$$

This specification has been estimated for many countries including Chile. It is useful to note that in the limiting case of complete indexation, $\kappa = 1$, the inflation equation simplifies to

$$\pi_t = \frac{1}{1+\beta}\pi_{t-1} + \frac{\beta}{1+\beta}E_t[\pi_{t+1}] + \frac{\lambda}{1+\beta}x_t \tag{5}$$

and is independent of steady-state inflation π^S.

Introducing adaptive learning

Expectations play a key role in determining inflation dynamics. Since the 1980s research on inflation dynamics and monetary policy has relied extensively on the assumption of rational expectations and explored its implications for policy design. However, a drawback of the rational expectations approach is that it imputes a probably unrealistic extent of knowledge to market participants. More recently, researchers have started to consider departures from rational expectations assuming that economic agents behave like econometricians in forming expectations. This approach, often-called adaptive expectations or least-squares learning, has been widely applied following the influential monograph of Evans and Honkapohja (2001).

Orphanides and Williams (2005, 2006) and Gaspar, Smets and Vestin (2005, 2006), for example, study monetary policy performance when price-setting market participants form expectations about future inflation in a least-squares regression fashion. Departing from rational expectations requires researchers to choose from a variety of least squares learning specifications. Branch and Evans (2006) provide a useful, short exposition of alternative approaches and assess how well they fit survey expectations.

Following this line of research price-setting firms are assumed to estimate the following reduced-form regression for inflation:

$$\pi_t = \gamma_t \pi_{t-1} + \varepsilon_t \tag{6}$$

The parameter γ_t carries a time subscript to allow for high and low inflation episodes with time-varying degrees of inflation persistence and indexation. Thus, market participants consider such variations in their regression model of inflation.[3] Recursive estimation

[3] For comparison, we also consider recursive least squares under the assumption that γ is constant.

then implies the following updating equations:

$$\begin{aligned} c_t &= c_{t-1} + \Sigma_{t-1}\pi_{t-1}F^{-1}(\pi_t - c_{t-1}\pi_{t-1}) \\ \Sigma_t &= \Sigma_{t-1} - \Sigma_{t-1}X_tF^{-1}X_t'\Sigma_{t-1} + \sigma^{\gamma} \\ & \quad \text{where } F = X_t\Sigma_{t-1}X_t' + \sigma^{\varepsilon} \end{aligned} \tag{7}$$

c_t denotes the price setters' estimate of the inflation persistence parameter in period t and Σ_t its variance. For a derivation based on the Kalman filter see Harvey (1992). These updating equations are also consistent with Bayes rule assuming normal shock distributions and beliefs (see Zellner (1971)).

The price setters' expectation of future inflation assuming least squares learning $E_t^{LS}[\pi_{t+1}]$ corresponds to

$$E_t^{LS}[\pi_{t+1}] = c_{t-1}\pi_t. \tag{8}$$

As in Gaspar, Smets and Vestin (2006) it is assumed that the estimate c_{t-1} does not yet incorporate the most recent inflation observation π_t.[4] Using equation 8 to substitute out the expectation of future inflation in the Phillips curve one can solve for the following reduced-form inflation equation:

$$\pi_t = \frac{\kappa}{1+\beta(\kappa - c_{t-1})}\pi_{t-1} + \frac{\lambda}{1+\beta(\kappa - c_{t-1})}x_t + \frac{(1-\kappa)(1-\beta)}{1+\beta(\kappa - c_{t-1})}\pi^S \tag{9}$$

As pointed out by others adaptive learning, specifically the time-varying estimate c_{t-1}, exerts an influence on the observed degree of inflation persistence. In addition, the degree of inflation persistence depends on the specific policy strategy. We come back to the question of the consistency of the price-setting firms beliefs and observed inflation persistence in the next section.

Learning and endogenous indexation

So far, the degree of backward-looking indexation κ has been treated as constant. A novel element of this paper is to introduce a time-varying degree of indexation κ_t and to link the determination of κ_t with the learning process of price setting firms. To this end it is necessary to define how firms that get to decide on the proper index to be applied in the future assess the likely steady-state inflation rate.

Firms are assumed to consider two alternative values for the steady-state–namely the central bank's announced inflation target π^* and the preceding period's rate of inflation π_{t-1}. $s_t = Prob(\pi^S = \pi^*)$ denotes the probability that the announced inflation target is the better indicator of future steady-state inflation. Every time a new inflation observation

[4] Alternative approaches would be either to use only lagged information, i.e. $E_t^{LS}[\pi_{t+1}] = c_{t-1}^2\pi_{t-1}$ or to use $E_t^{LS}[\pi_{t+1}] = c_t\pi_t$. The latter specification would require solving a more complicated fixed point problem.

becomes available this probability is updated as follows:

$$s_{t+1} = \frac{s_t e^{(-0.5\sigma^{-1}(\pi_t-\pi^*)^2)}}{s_t e^{(-0.5\sigma^{-1}(\pi_t-\pi^*))^2} + (1-s_t)e^{(-0.5\sigma^{-1}(\pi_t-\pi_{t-1}))^2}} \tag{10}$$

This updating equation is consistent with Bayes rule given normal shocks and beliefs.[5]

The degree of indexation κ_t is allowed to vary between complete indexation, i.e. $\kappa_t = 1$, and a minimal value of $\underline{\kappa}$ measuring the exogenous degree of indexation, i.e. $\kappa_t \in [\underline{\kappa}, 1]$. At any point in time there is a probability of θ that a firm receives a signal to adjust the current price optimally. Whenever such a signal arrives, the firm is also allowed to choose the rate for indexation that will apply to future periods without price adjustment signals. It has two choices, the past inflation rate, π_{t-1}, and the announced inflation target, π^*.

The firm only chooses to switch the rate for indexation when there is overwhelming evidence in favor of such a switch. Specifically, if the current choice of indexation rate is π_{t-1}, it will only switch to π^* if the probability of π^* is greater than a trigger value $\bar{S}$, i.e. if $s_t > \bar{S}$. Similarly, if the current choice of indexation rate is π^* the firm will only switch back to π_{t-1} if the probability of π_{t-1} is greater than the same trigger value, i.e. $1 - s_t > \bar{S}$.

We note that all firms face the same information regarding inflation. Thus, s_t is symmetric across firms. Since the probability of a price- and index-adjustment signal is θ, a share of θ firms switches the rate of indexation at any point in time if there is overwhelming evidence for such a shift. Thus, κ_t is governed by the following process:

$$\kappa_t = \begin{cases} \kappa_{t-1} + \theta & \text{if } s_t > \bar{S} \text{ and } \kappa_t \leq 1 - \theta \\ \kappa_{t-1} - \theta & \text{if } (1 - s_t) > \bar{S} \text{ and } \kappa_t \geq \theta + \underline{\kappa} \\ \kappa_{t-1} & \text{else} \end{cases} \tag{11}$$

Since the share of firms using backward-looking indexation varies over time, the reduced-form inflation equation (9) needs to be re-written:

$$\pi_t = \frac{\kappa_{t-1}}{1+\beta(\kappa_{t-1} - c_{t-1})}\pi_{t-1} + \frac{\lambda}{1+\beta(\kappa_{t-1} - c_{t-1})}x_t + \frac{(1-\kappa_{t-1})(1-\beta)}{1+\beta(\kappa_{t-1} - c_{t-1})}\pi^S \tag{12}$$

As a short-hand we will denote the time-varying, reduced-form parameters by $\delta_{(1,2,3),t}$ and write the inflation equation as follows:

$$\pi_t = \delta_{1,t}\pi_{t-1} + \delta_{2,t}x_t + \delta_{3,t} \tag{13}$$

[5] See Wieland (2000).

It remains to describe the determination of the output gap x_t and policy objectives in order to study disinflation under inflation targeting policies.

5 Inflation targeting: Immediate versus gradual disinflation

An inflation-targeting central bank is typically assumed to design policy according to the following per-period loss function l(.):

$$l(\pi_t, x_t) = (\pi_t - \pi^*)^2 + \alpha x_t^2 \tag{14}$$

The parameter α determines the central bank's preference for minimizing output losses relative to inflation deviations from the target π^*. To keep the analysis simple the central bank is assumed to control the output gap x_t directly and to observe the model parameters and the beliefs of the price setters c_{t-1}. Thus, the central bank observes $\delta_{(1,2,3),t}$ in equation (13). However, the central bank does not attempt to exploit the dynamic learning process of price-setters in conducting policy.[6] Thus, the dynamic optimization problem of the central bank corresponds to:

$$\begin{aligned} &\underset{x_t}{\text{Min}} \, E_t \left[\sum_{t=1}^{\infty} \beta^{t-1} (\pi_t - \pi^*)^2 + \alpha x_t^2 \right] \\ &\text{s.t. } \pi_t = \delta_{1,t}\pi_{t-1} + \delta_{2,t} x_t + \delta_{3,t} \end{aligned} \tag{15}$$

The limiting cases of strict inflation targeting, $\alpha = 0$, and pure output stabilization, $\alpha \to \infty$, are easily considered.

The latter policy implies always setting $x_t = 0$. Inflation persistence would be governed exclusively by the time-varying parameter $\delta_{1,t}$ and inflation–depending on the beliefs of price setters regarding inflation persistence–could even spiral out of control. In contrast, strict inflation targeting would ensure that the inflation target is met at all times independent of the price setting firms' beliefs. It would imply the following policy rule for the output gap

$$x_t = -\delta_{4,t}(\delta_{1,t}\pi_{t-1} + \delta_{3,t} - \pi^*) \tag{16}$$

with $\delta_{4,t} = \delta_{1,t}^{-1}$. Note, with an inflation target of zero, $\delta_{3,t} = 0$ at all times.

The dynamically optimal policy given central bank preferences that assign a positive (but not infinite) weight α to the output gap falls in between these two extremes, i.e. $0 < \delta_{4,t} < \delta_{1,t}^{-1}$. Orphanides and Wieland (2000) provide an analytical formula for the case of $\delta_{1,t} = 1$. Dynamically optimal policies for alternative values of $\delta_{1,t}$ are easily

[6] We return to this proposal in the last section under the heading of a ”sophisticated” central bank. This terminology is taken from Gaspar, Smets and Vestin (2006).

computed numerically with the algorithm provided along with that paper.[7]

Having specified the complete macroeconomic model we can proceed to evaluate alternative disinflation strategies. This requires defining an initial scenario with high inflation. Initial inflation is set at 20 percentage points, $\pi = 0.2$, not too dissimilar from the average inflation rate of Chile prior to the start of inflation targeting. Firms that cannot always adjust prices optimally in such a period of high and highly variable inflation will opt for indexation. Thus, the initial scenario is characterized by complete indexation $\kappa_0 = 1$. Similarly, beliefs of price-setters in such a period are best characterized by a unit root, i.e. $c_0 = 1$. These initial conditions represent an equilibrium if policy aims exclusively at stabilizing output, i.e. $x_0 = 0$. Inflation then follows a random walk.

Given the above-mentioned initial conditions the reduced-form inflation equation (13) simplifies further to

$$\pi_t = \pi_{t-1} + \lambda x_t \tag{17}$$

corresponding exactly to equation (2), the accelerationist Phillips curve discussed in section 2. The cost of disinflation is very high and the sacrifice ratio given these beliefs is constant. The parameter values used in the subsequent simulations are summarized in Table 1.

Table 1: Parameter values and initial beliefs

Parameter	Value	Economic interpretation
β	0.99	Discount factor.
λ	0.5	Slope of Phillips curve.
κ_t	$\kappa_0 = 1$	Degree of indexation to $t-1$ inflation.
χ_t	$\chi_0 = 1$	Price setters initial belief regarding inflation persistence.
Σ_t	$\chi_0 = 10$	Price setters initial variance.
s_t	$\sigma_0 = 0.1$	Price/index setters initial belief regarding $Prob(\pi^S = \pi^*)$.
π_*	$0.2/0$	Initial inflation is at 0.2, long-run inflation target is 0.
$\underline{\kappa}$	0.05	Degree of minimal exogenous indexation.
S	0.8	Trigger probability for switching the rate for indexation.
θ	0.1	Probability of index-adjustment signal.
σ	0.01	Variance of noise (added later).
σ_γ	1	Belief regarding variability of γ.

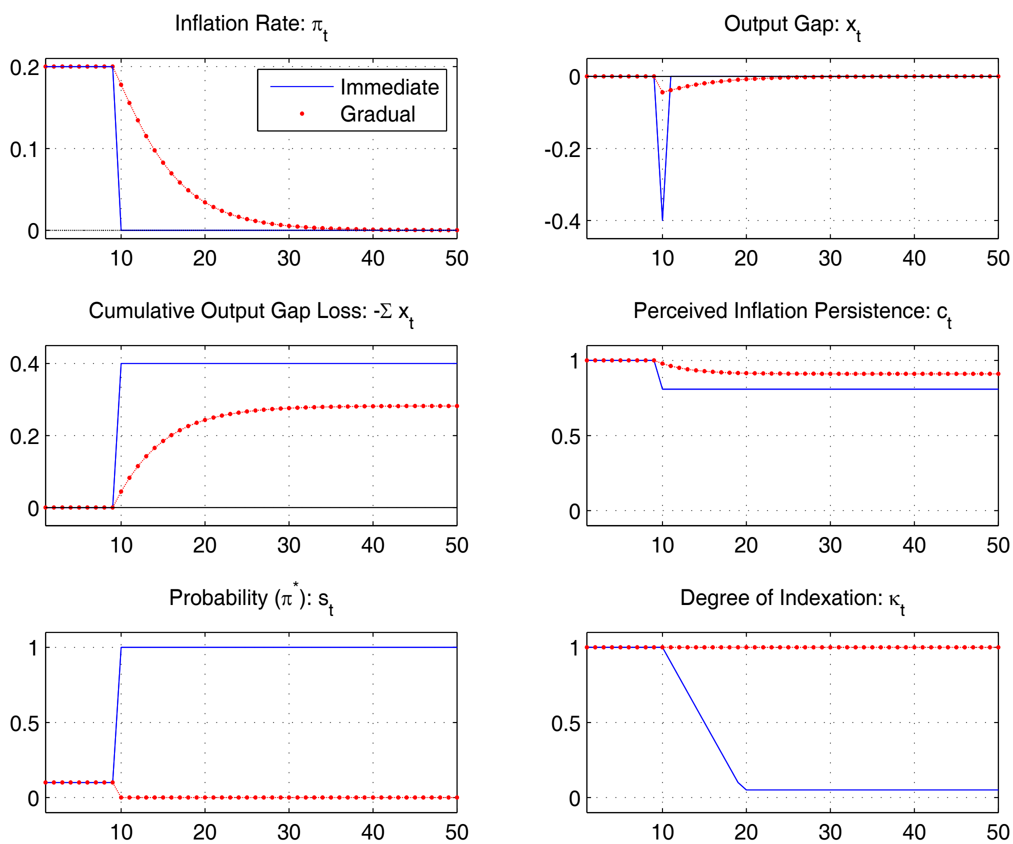
These initial conditions set the stage for the entry of an independent inflation-targeting central bank. [8] As a first step, we contrast the *immediate* disinflation approach that would be implemented under strict inflation targeting with a more *gradual* approach that is realized with a positive weight on output in the central bank's preferences. Given the parameter values defined in Table 1 the optimal policy coefficient for immediate disinflation that corresponds to the inverse of the slope of the reduced-form inflation equation

[7] The matlab code is available from www.volkerwieland.com.

[8] For a fascinating account of the implications of learning for inflation and stabilization when money growth and inflation are determined by the government's budget constraint rather than an independent central bank the reader is referred to Sargent, Williams and Zha (2007).

is equal to $\delta_{4,0} = \delta_{1,0}^{-1} = 2$. While this policy achieves the long-run inflation target of zero percent immediately, it also results in an output loss of 40%. This outcome is shown by the solid blue line in **Figure 2** . The cumulative output loss due to the disinflation of 20 percentage points is within the first period, i.e. period 10. Clearly, this approach is more of academic interest and could not be implemented in practice.

Figure 2: Immediate versus Gradual Disinflation



Interestingly, aggressive disinflation drives down price setter's estimate of inflation persistence, c_t, to about 0.8. Furthermore, it convinces firms that get to choose the rate for indexation that the future steady-state rate of inflation will coincide with the policymaker's inflation target. As a result, the share of firms that use backward-looking indexation to the preceding inflation rate declines to the minimum exogenous degree of indexation, $\underline{\kappa}$ within about 10 periods. Unfortunately, the strict inflation targeting approach cannot take advantage of the subsequent reduction in the cost of disinflation due to the beneficial evolution of the perceived degree of inflation persistence, c_t as well as the degree of indexation, κ_t.

The gradual approach to disinflation (shown by the red dotted line in **Figure 2**) is computed with a weight α on output in the central bank's preferences such that the policy response coefficient δ_4 corresponds initially to 0.22, that is about one tenth of the policy response needed to meet the target immediately. Consequently, the initial output decline is much smaller but will be sustained for a much longer time. The inflation rate declines gradually and reaches the long-run target of zero by about period 35. The cumulative sum of output gap losses is much smaller under the gradual approach and converges to about 28% after about 25 periods. The reason for the decline in the sacrifice ratio is to be found in adaptive learning. As price-setters observe the downward drift of inflation they revise their estimate of inflation persistence downwards. This reduction in c_t adds disinflationary impetus and reduces the costs of disinflation.

However, due to the slow decline in inflation firms adjusting the rate of price indexation see now reason to switch from backward-looking indexation to the announced inflation target. The announced target is just too far way and progress towards it too slow to change the probability weights on lagged inflation versus the announced target. As a result endogenous indexation does not come into play in terms of reducing the costs of disinflation under such a gradual strategy.

6 Inflation targeting: Temporary inflation targets

Two strategic aspects of the Chilean disinflation experience have been emphasized in section 3. One aspect was the very gradual disinflation while the second aspect concerned the announcement of short-lived, temporary inflation targets. These temporary targets appear to have been pursued very actively. Thus, we investigate whether such announced temporary targets,π_t^*, could have a beneficial effect on learning and the degree of indexation thereby further lower the costs of disinflation.

Cespedes, Ochoa and Soto (2005) took into account temporary targets in estimating New-Keynesian Phillips curves for Chile under the assumption of rational expectations. They show that the forward-looking Phillips curve then needs to account for the current and future inflation target as follows:

$$\pi_t = \frac{\kappa}{1+\beta\kappa}\pi_{t-1} + \frac{\beta}{1+\beta\kappa}E_t[\pi_{t+1}] + \frac{\lambda}{1+\beta\kappa}x_t + \frac{(1-\kappa)}{1+\beta\kappa}(\pi_t^* - \beta\pi_{t+1}^*) \tag{18}$$

The reduced-form inflation equation in our model with adaptive learning and endogenous indexation is then modified accordingly:

$$\begin{aligned} \pi_t \quad = \quad & \frac{\kappa_{t-1}}{1+\beta(\kappa_{t-1}-c_{t-1})}\pi_{t-1} + \frac{\lambda}{1+\beta(\kappa_{t-1}-c_{t-1})}x_t \\ & + \frac{(1-\kappa_{t-1})}{1+\beta(\kappa_{t-1}-c_{t-1})}(\pi_t^* - \beta\pi_{t+1}^*) \end{aligned} \tag{19}$$

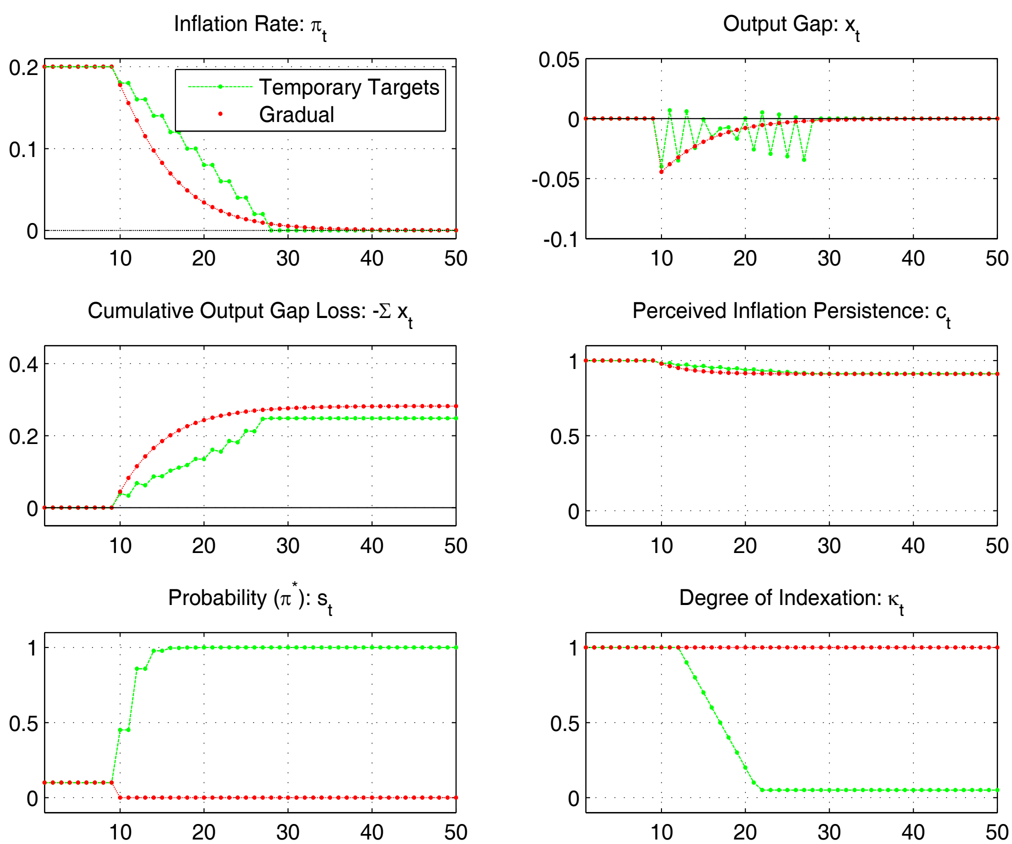
As an example, we consider a gradual, albeit mechanical reduction in the inflation π_t^*

by 2 percentage points every 2 years. The pursuit of this temporary inflation target, however, is as vigorous as possible. After deciding on the inflation target for the next period, the central bank acts in order to meet this temporary target. Consequently, the optimal policy response conditional on the temporary target corresponds to the policy response under strict inflation targeting, i.e. the policy rule corresponds to:

$$x_t = -\delta_{4,t}(\delta_{1,t}\pi_{t-1} + \delta_{3,t} - \pi_t^*) \tag{20}$$

with $\delta_{4,t} = \delta_{1,t}^{-1}$, and $\delta_{(1,2,3)}$ consistent with equation (19).

Figure 3: Temporary Inflation Targets versus Gradual Disinflation



The disinflation performance under such announced, temporary targets is shown by the green, dashed-dotted line in **Figure 3**. It is contrasted to the previously discussed gradual disinflation with an announced long-run target shown again by the red-dotted line. By announcing and meeting the temporary inflation targets the central bank succeeds in raising the likelihood of the announced target relative to past inflation as the better variable for indexing. Thus, firms that receive a signal allowing for an adjustment of the rate applicable for indexation in future periods fairly quickly switch to the announced inflation targets of the central bank. As a result the degree of backward-looking indexation declines fairly rapidly by period 20.

Since the temporary inflation targets are only lowered gradually the central bank can take advantage of lower disinflation costs in the later stages of the disinflation process. As a result, the cumulative loss of output is smaller than under the gradual disinflation with an announced long-run target. The perceived degree of persistence, c_t however, declines similarly under the two strategies. Thus, its impact in terms of reducing the costs of disinflation remains similar under both strategies.

So far, this analysis has been conducted in the absence of unforeseeable shocks. Next, white noise shocks, ε_t, with variance $\sigma^2 = 0.01$ are introduced.

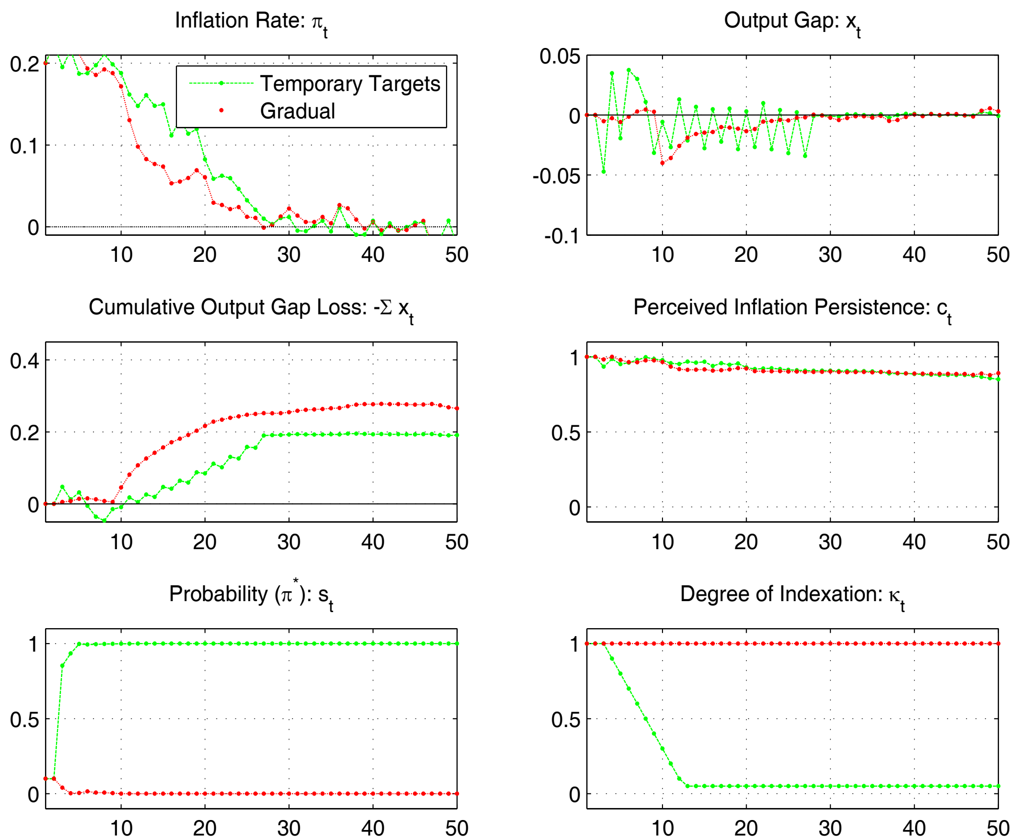
$$\pi_t = \frac{\kappa}{1+\beta\kappa}\pi_{t-1} + \frac{\beta}{1+\beta\kappa}E_t[\pi_{t+1}] + \frac{\lambda}{1+\beta\kappa}x_t + \frac{(1-\kappa)}{1+\beta\kappa}(\pi_t^* - \beta\pi_{t+1}^*) + \varepsilon_t \qquad (21)$$

The timing of expectations formation, policy actions and shocks is such that the shocks are realized after time t expectations have been formed and policy has been set. Thus, the shocks introduce noise in inflation that cannot be affected by current policy actions. However, such variations in the rate of inflation lead to variations in the output gap if policy responds to past inflation. More interestingly, such variation accelerates the speed of learning and further reduces the costs of disinflation as shown in **Figure 4**.

Of course, this is just a single draw of shocks. The strategy with temporary inflation targets need not always outperform the gradual disinflation strategy in terms of output losses.[9]

[9] A sensitivity study will be included in the next version of the paper.

Figure 4: Shocks Accelerate Learning



7 A "sophisticated" central bank versus one that learns

A "sophisticated" central bank

The results obtained so far suggest that policy performance could be further improved by allowing the central bank to exploit the dynamic learning process of price-setters in designing policy. Such an approach has been studied under the heading of "sophisticated" central banking by Gaspar, Smets and Vestin (2006). Such a sophisticated central bank would solve the following dynamic optimization problem:

$$\underset{x_t}{\text{Min}} E_t \left[\sum_{t=1}^{\infty} \beta^{t-1} (\pi_t - \pi^*)^2 + \alpha x_t^2 \right] \tag{22}$$

$$\text{s.t. } \pi_t = \delta_{1,t}\pi_{t-1} + \delta_{2,t}x_t + \delta_{3,t}$$

and s.t. equations $(7), (8), (10)$ and (11).

The optimal policy takes into account the nonlinear updating equations that determine the recursive estimation of the degree of inflation persistence, i.e. equations (7) and (8). This effect has been studied by the above authors. In our model, an additional policy channel arise due to the nonlinear dynamics of endogenous indexation. This channel is represented by equations (10) and (11) in the above optimization problem.

This optimization problem could by set up in two different ways. The first approach would be to specify the output gap as policy control variable given an announced, long-run target π^*. The second approach, inspired by the positive performance of announced temporary targets in the preceding section, would involve the optimal choice of temporary targets π_t^*. A choice of temporary target would automatically imply a given output gap according to the strict inflation targeting policy derived previously in the paper.

The above optimization problem can be expressed as nonlinear dynamic programming problem with four state variables: $(\pi_{t-1}, c_{t-1}, \Sigma_{t-1}, s_{t-1})$. Numerical approximation of such a problem is complicated but within reach of current methodology. However, optimal policy design in this manner relies on rather courageous information assumptions regarding the central bank's knowledge of private sector expectations formation. Not only is it assumed to observe the private sector's beliefs, it also knows the exact learning dynamics. The optimal policy of such an extremely knowledgeable central bank forms a useful benchmark for model-based comparison but it does not represent a strategy that could be implemented in practice.

A central bank that learns

An alternative approach that could be implemented with the information available to central banks in practice takes again recourse to learning. The central bank may learn about inflation dynamics via recursive estimation or least squares learning. Contrary to the price-setting firms in the model that were assumed to learn about the reduced-form relationship between current and lagged inflation, the central bank can spend more resources on learning. Certainly, central bank econometricians estimate Phillips curves on a regular basis including the effect of policy on the inflation process via the output gap x_t in the Phillips curve.

In the model studied in this paper, central bank learning can be applied to the inflation equation derived under recursive least squares learning by price-setting firms, that is,

$$\pi_t = \delta_{1,t}\pi_{t-1} + \delta_{2,t}x_t + \delta_{3,t} \tag{23}$$

Following Wieland (2006)[10] central bank beliefs regarding these three time-varying parameters may be summarized by a vector $d_t = (d_{1,t}, d_{2,t}, d_{3,t})$ and associated covariance

[10] Other related work on central bank learning of interest in this context includes Cogley, Colacito and Sargent (2005), Ellison (2006), Svensson and Williams (2006) and Wieland (2000a,b).

matrix $\Sigma_{d,t}$.

$$Var\left[\begin{pmatrix}\delta_{1,t}\\ \delta_{2,t}\\ \delta_{3,t}\end{pmatrix}\right] = \Sigma_d = \begin{pmatrix} v_t^1 & v_t^{12} & v_t^{13}\\ v_t^{12} & v_t^2 & v_t^{23}\\ v_t^{13} & v_t^{23} & v_t^3\end{pmatrix}. \tag{24}$$

The vector of state variables that characterize central bank beliefs contains nine variables, the three means, three variances and three covariances. The associated updating equations for recursive least squares with time-varying parameters:[11]

$$\begin{aligned}\begin{pmatrix} d_{1,t}\\ d_{2,t}\\ d_{3,t}\end{pmatrix} &= \begin{pmatrix} d_{1,t-1}\\ d_{2,t-1}\\ d_{3,t-1}\end{pmatrix} + \Sigma_{t-1}X_tF^{-1}(\pi_t - d_{1,t-1}\pi_{t-1} - d_{2,t-1}x_t - d_{3,t-1})\\ \Sigma_{d,t} &= \Sigma_{d,t-1} - \Sigma_{d,t-1}X_tF^{-1}X_t'\Sigma_{d,t-1} + \sigma_d \text{ where } F = X_t\Sigma_{d,t-1}X_t' + \sigma^{\eta}\end{aligned} \tag{25}$$

F refers to the conditional variance of inflation.

In contrast to the sophisticated central bank discussed above, the information requirements for such a learning central bank are much less stringent. Only inflation and output observations are needed. Potential output could be subsumed in the time-varying intercept. Thus, an area of fruitful future analysis would be to re-assess the disinflation policies in the preceding section under the assumption that the central bank learns about the time-varying parameters governing the inflation process according to updating process defined above.[12]

8 Conclusion

This paper investigated disinflation with different inflation targeting strategies in an economy with adaptive learning by price-setters and endogenous indexation. As the central bank acts to bring inflation under control, price-setting firms revise their beliefs regarding the degree of persistence. Thus, adaptive learning lowers the cost of disinflation and a gradual approach to disinflation can take advantage of this beneficial effect. Firms that choose the rate for indexation also re-assess the likelihood that announced inflation targets determine steady-state inflation and adjust indexation of contracts accordingly. A strategy of announcing and immediately achieving relatively modest short-term targets for inflation is found to influence the likelihood that firms switch from backward-looking indexation to the central bank's announced targets more effectively than a strategy with

[11] A derivation of the updating equations using Bayes rule or Kalman filter see Zellner (1971) and Harvey (1992) respectively.

[12] Computing a dynamically policy which takes into account central bank learning dynamics is probably not feasible however, with more than two unobserved time-varying parameters.

a long-run target that is achieved only gradually.

In future work it would be of interest to compute the fully optimal policy taking advantage of the learning dynamics in the model. Such a policy would form a useful benchmark for comparison with simpler, practically implementable policies. Such an implementable policy with central bank learning is proposed here.

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