Data-based vs. Model-consistent Estimates of Gaps and Trends in the Chilean Economy. *

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Abstract

Trend GDP and the output gap are key inputs for policy evaluation and forecasting in standard models of monetary policy. However, the measurement of these variables is plagued with difficulties. In this paper we propose two different approaches. First, a data-based approach, that starts with the primal and dual estimates of total factor productivity (TFP) growth, and then uses a variety of procedures to filter the inputs. Second, a model-consistent framework, that simultaneously estimates the macroeconomic dynamics and the underlying trends of the economy. We compare the difficulties in using each methodology, and we use them to construct measures of the output gap and potential growth for Chile.

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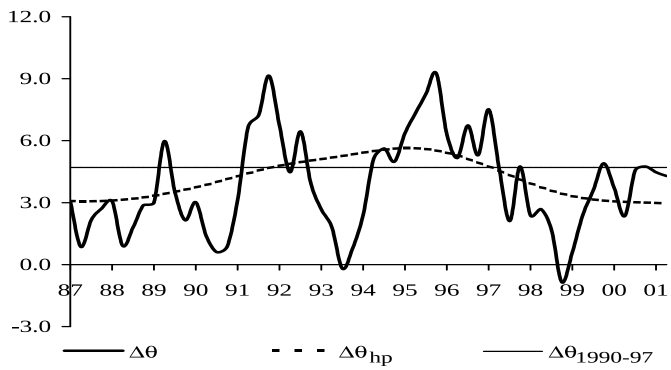
1 Introduction

Over the last few years the Chilean economy has experienced a marked deceleration in economic growth. Labor productivity growth averaged more that to 6% annually over 1994 to 1997, but since 2000 it reaches between 3% and 4%. Moreover, the time span since the outbreak of the Asian and Russian crises in 1997 and 1998 seems to indicate that more than purely cyclical factors are at play in determining the expansion of productivity and the recent rates of aggregate growth: monetary policy has shifted to a clearly more expansionary stance, and long term interest rates have declined sharply in real terms since 1999.

Figure 1: **Growth Rate of Aggregate Labor Productivity**



The debate on whether the current and forecasted rates of growth of the Chilean economy in the short terms reflect a shift in the underlying expansion of productivity or are only a symptom of weak aggregate demand reflects the difficulties in separating trends from cycle. The same can be said about the different opinions regarding the size of current slack in capacity utilization. This is unfortunate though, because these two variables are key inputs in the formulation of monetary and fiscal policy. On the one hand, the current slack in factor and goods markets determines the present underlying inflationary tendencies, through their impact on wages and markups. On the other hand, the expansion of capacity utilization over the next quarters or years affects the trends in these inflationary pressures.

Given that since 1999 monetary policy in Chile is guided by what has been called "forecast inflation targeting, in which the current stance of monetary policy is endogenous to the expected or forecasted path of prices, erring on one side or the other of the side of the output gap or trend growth can affect the achievement of the inflation target. Moreover, unlike supply shocks or relative price shocks such as oil prices and the exchange rate, which are immediately observed, the uncertainty about the true extent of underlying price and wage pressures is only lifted when it is too late to act with monetary policy. The transmission mechanism of monetary policy to inflation through the labor market is the one that is likely to have the longest lags.

On the fiscal policy side, difficulties are similar. The current framework in Chile aims at the achievement of a structural surplus of 1% of GDP over the cycle. Therefore, the yearly discussion of the budget requires an actual quantification of the size of the output gap, to fix the path of real expenditure. Again, using assumptions about the gap or trend growth over the short run can introduce a bias in fiscal policy, inducing too much or too little aggregate demand impulse, relative to what is deemed convenient.

Unfortunately, the construction of output gap measures is plagued with difficulties. A first approach, that we tackle on the second section of this paper and call data-based, relies on a two-step procedure. First, using traditional growth accounting exercises, a measure of total factor productivity (TFP) is obtained. Then, potential output is defined as the result of assuming that inputs (labor and capital) are at their normal or trend utilization rates. It is apparent then that this procedure is sensitive to assumptions in both steps. First, an accurate measurement of *actual* inputs used in the production process includes issues such as shifting quality and composition of both labor and capital, as well as time varying utilization rates that should not be accounted for as TFP fluctuations. Then, in the second step assumptions about the *trend* or *normal* use of inputs must be made, to go backwards and then estimate potential output.

In simple terms, and leaving aside the measurement issues related to quality trends, the data-based approach actually requires identifying a-priori the cyclical and trends component in the data. A typical case is the capital utilization rate, usually associated with the unemployment rate since Solow's classic exercise. Then, to construct potential output (an exercise that Solow did not pursue), the production function is evaluated at a "normal" utilization rate (i.e. the "natural" unemployment rate, defined in a particular way). Thus, it is paradoxical that the key identification assumption corresponds closely to the result of the calculation. By going through the two steps mentioned above we do not pretend to circumvent these difficulties, but hope that we will highlight the type of assumptions needed for this approach.

To complement the data-based estimates, the third section uses a simple empirical methodology to directly estimate the output gap from aggregate demand and aggregate supply macro models. Given that the output gap measures are typically used as inputs of a macro model such as the one we use, the simultaneous estimation through state-space techniques of the macroeconomic variables and the underlying unobserved level of the output gap provides an interesting alternative to the data-based approach.

This model-consistent estimate still requires some identification assumptions. The first one is the actual specification of the macroeconomic model, particularly the functional form and the excluded variables. This is unavoidable though, given that the output gap measures themselves are used in the context of specific models of the macroeconomy. The other main identification assumptions relate to the assumed volatility of trend output growth *vis-a-vis* actual growth.

The interest in potential growth is not novel in Chile. However, most of the research refers to the period previous to 1997, and in general does not acknowledge the importance of the specific identification issues that surround the estimation.

Combining two different analysis (growth accounting and regression analysis) Roldós (1997) examine economic growth determinants and the relation between economic growth and inflation pressures. He estimates a aggregate production function using a cointegrating vector, that relates total GDP and production factors - capital and labor - adjusted by quality indexes. These indexes measure the changes in composition of the production factors that make more productive aggregate factors. The factor shares, that are obtained from the estimation of the production function, allow to calculate the Solow's residual or the TFP (Total Factor Productivity). Through a Hodrick-Prescott filter the cyclical component of the TFP and employment is removed for estimating the potential output.

Roldós however did not find a positive correlation between the output gap and inflation, but rather a small negative one. He interprets these results as a product of the high average inflation over the nineties. He does not however control for movements in the exchange rate.

Rojas et al. (1997), make a similar exercise of a growth accounting model, considering not only capital and labor as production factors, but also the contribution of international trade to growth. They try to estimate the contribution of the increasing commercial integration of Chile in the last decades to effective and potential growth. The study calculates the potential output of the Chilean economy during 1960-1996.

Using a cointegration focus, this paper estimates a production function that considers capital and labor - corrected by grade of utilization and by quality indexes - and a variable of terms of trade that controls for the fluctuations of international prices faced by the economy. To calculate the potential output it is used the cointegration vector with the series of labor, capital, terms of trade and comercial integration filtered by HP.

As we see, it is typical to filter the series to obtain a measure of the gap. It is questionable however how much this differs from directly filtering the GDP data.

Other studies, that do not use filtering methods, still imply strong identification assumptions. Marfán and Artiagoitía (1989) use linear programming techniques to obtain a measure of the gap. However, they in fact impose a production function that is linear in capital.

García (1994) uses an indirect approach: he estimates a labor demand function to identify the parameters of the production function. Potential output is then defined as output at full employment. Hence, the gap is the mirror image of the unemployment rate. Jadresic and Sanhueza (1992) also identify the output gap in a similar way, by assuming an increase in the natural rate of unemployment by the late seventies and early eighties.

Coeymans (1999) does not estimate a measure of potential output, rather focusing on a sources of growth approach to measure trends in GDP. He estimates a production function, which growth determinants are centered in the aggregate supply factors: capital accumulation, hiring of new workers, TFP factor.[1] Assuming constant scale returns, this analysis shows an important cyclical component in productivity. The high correlation between productivity and external shocks (terms of trade, impact of international interest rate over financial services and external crisis index) reveals its importance as principal determinants of productivity cycles and output.

We will not dwell too far from previous efforts. However, we think it is important to acknowledge the importance of the assumptions behind the estimates of trends and gaps. That explains why we use two very different approaches.

The results of these two methodologies are different, as expected, and also are quantitatively different from simple filtering techniques such as Hodrick-Prescott. This reveals that, not unlike many of the other aspects surrounding monetary policy under Inflation Targeting, quite an amount of judgement must be used to evaluate what are the underlying inflationary pressures in the economy. The use of a unique mechanical procedure to estimate trends and gaps is therefore dangerous, in that it is very likely to introduce biases in the conduct of monetary policy.

The rest of the paper is structured as follows. The next section details the construction of data-based estimates of the gap. Section 3 uses the model-consistent approach. Section 4 concludes.

2 Data-based estimates of potential output and the output gap

In this section we construct estimates of total factor productivity (TFP) growth, and assess its contribution to the slowdown of aggregate growth over the last few years.

[1]It includes changes in the level of utilization of capital and labor, reallocation of resources from low to high productivity activities, and technical advance.

2.1 Dual and primal estimates of TFP: Notation

There are two possible strategies for the estimation of TFP, the primal and dual approaches. The differ in the data required, and in general are viewed as complementary. The primal approach relies on the calculation of "Solow's residual", through the use of aggregate GDP data, along with estimates for the capital stock and labor employment. The dual estimate instead focuses on the path of relative prices: wages and the cost of capital. The relationship between these two approaches can be easily seen by assuming a production function for value-added and using the income identity of national accounts, both in real terms.

$$Y = F(A, K, N) = C_k K + C_n N \tag{1}$$

The only assumption underlying Equation 1 is that output equals payments before direct taxation to the factors of production: labor ($C_n N$) and capital ($C_k K$). These include depreciation and eventually rents due to imperfect competition in labor or capital markets. Note that Y is cost-based value added, not including indirect taxes. No assumption is made about the shape of the production function, in particular the way technological change A affects the relative demands for capital and labor.

First order differentiation with respect to time, using the normalization $(\partial F/\partial K) = 1$, leads to:

$$\Delta Y = \Delta A + (\partial F/\partial K)\Delta K + (\partial F/\partial N)\Delta N = K\Delta C_k + C_k\Delta K + N\Delta C_n + C_n\Delta N \tag{2}$$

Dividing both sides of equation by Y, one obtains

$$\Delta y = \Delta a + \frac{\partial F}{\partial K}\frac{K}{Y}\Delta k + \frac{\partial F}{\partial N}\frac{N}{Y}\Delta n = \frac{C_k K}{Y}(\Delta c_k + \Delta k) + \frac{C_n N}{Y}(\Delta c_n + \Delta n) \tag{3}$$

Defining $\alpha = (C_k K/Y)$ as the share of capital in total costs, it is the case that

$$\Delta y = \Delta a + \alpha\Delta k + (1-\alpha)\Delta n = \alpha(\Delta c_k + \Delta k) + (1-\alpha)(\Delta c_n + \Delta n) \tag{4}$$

This formulation is correct under both perfect and imperfect competition, as long as markups enter as a wedge between marginal factor productivity and the reservation wage and cost of capital: $C_k = \overline{C_k}(1+\mu_k) = \partial F/\partial K$ and $C_n = \overline{C_n}(1+\mu_k) = \partial F/\partial N$. This means that both approaches should have the same measurement error.

Now, its possible to define the primal (Δa^{primal}) and dual (Δa^{dual}) estimates of TFP growth.

$$\underbrace{\Delta y - \alpha\Delta k - (1-\alpha)\Delta n}_{\Delta a^{primal}} \equiv \underbrace{\alpha\Delta c_k + (1-\alpha)\Delta c_n}_{\Delta a^{dual}}$$

The intuition for this identity is simple: TFP grows if real wages or the real return on capital are growing too, because in the steady state these relative prices (not adjusted for quality) should be constant.

Before moving to the primal and dual estimates of TFP growth, some measurement issues must be highlighted. These revolve around two main aspects: the changing quality of the inputs in production, and their varying utilization over the business cycle.

2.2 Dealing with quality trends

Capital

Over the last decade and a half, there has been a dramatic shift in the composition of gross fixed investment. In 1986, machinery and equipment (M&E) composed 43% of gross capital formation (in constant 1986 prices), while from 1995 onwards its share had stabilized around 60%. In nominal terms, the share of M&E first increased from close to 40% in the mid-eighties up to 50% in the early to mid-nineties, to then decline to slightly over 40% in recent years.[2]

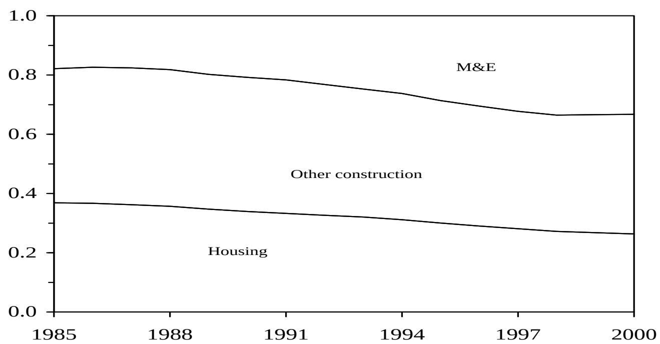
Also, within M&E there have been fairly large shifts over time. The imported component increased from 80% in the mid nineties, reached close to 90% in 1998, and then experienced a steep decline in 1999 and 2000, reaching 76%.[3]

[2]Official data on nominal investment reaches 1998, so we used estimates for investment deflators for 1999 and 2000. The methodology is described in the appendix.

[3]Again, the later data are estimates based on the path of capital imports quantum, which fell close to 35% in 1999. 2000 and 2001 have seen a modest recovery: third quarter data are only 11% higher than the 2000 average.

Accounting for these large shifts in the composition of investment is important. They have been substantial enough to matter in the composition of the capital stock over the last decade and a half. Recent estimates by Aguilar and Collinao (2000) show that the share of the capital stock accounted for by M&E increased from 18% in 1985 to 33% in 1997, having remained stable since. (Figure 4)

Figure 2: **Composition of the capital stock**



From Jorgenson's definition of the cost of capital, one sees that these shifts have potentially large effects on the dual estimate of TFP growth. Let $C_{k,i}$ represent the capital cost of brand i of capital, measured relative to the GDP deflator P, while R_i is the net return on capital, D_i the depreciation rate, P_i own deflator, and τ_i any tax-induced wedge:

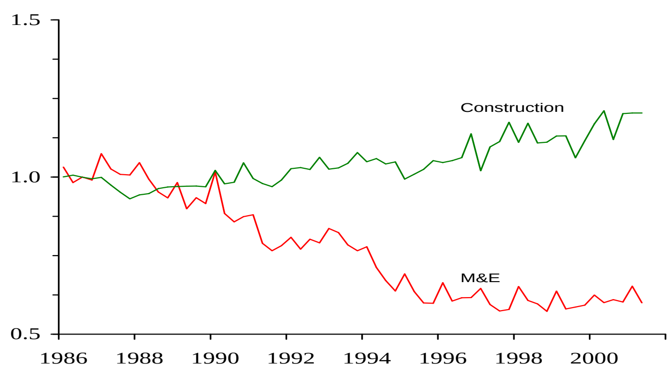
$$C_{k,i} = \tau_i \left(R_i + D_i\right) \frac{P_i}{P} \tag{5}$$

Abstracting from the importance of different tax treatments, τ_i, and if by arbitrage $R_i = \overline{R}$ for all i, still different rates of depreciation as well as different relative prices P_i/P for each brand of capital will have an important incidence in the cost of capital. M&E in particular, by having a high relative rate of depreciation, and an important imported component, must be treated differently that construction. Moreover, these facts affect not only the estimated path of the cost of capital for the dual estimate of TFP, but also the share of M&E in total costs, an input for primal growth accounting.

How important quantitatively are these factors in determining the path of the cost of capital? Figure 5 show quarterly M&E and construction deflators, normalized by the GDP deflator, for the period 1986 to 2001. They display a very different evolution. The increase of the construction deflator has been fairly stable, increasing by a little over 1% over the increase in the GDP deflator. Meanwhile, the appreciation of the real exchange rate had a large impact on the relative price of M&E, that fell around 40% between 1990 and 1996. Since then, it has remained stable: the depreciation of the nominal exchange rate has been compensated by not only a decrease in the dollar unit import value of capital goods, of close to 15% since its peak in early 1996, but also a reduction in the average tariff rate of 3 percentage points.

Thus, since the mid-eighties, the relative price of M&E *vis-a-vis* construction has declined 50%, although all this reduction occurred prior to 1997.

Figure 3: **Investment deflators (relative to the GDP deflator)**



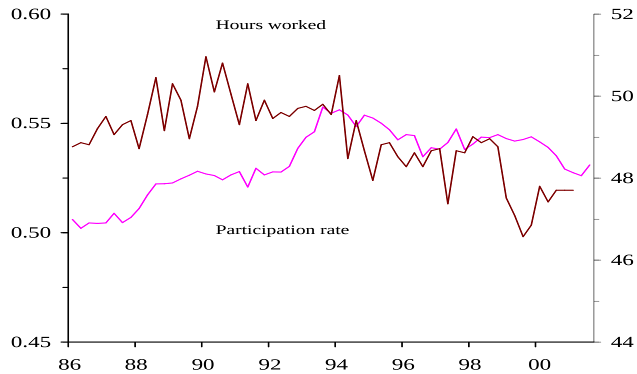
Labor

On the other hand, the quantity and quality of labor inputs changes over time, due to the increase in educational attainment, the sectoral reallocation of labor and secular trends in labor participation and hours worked. In simple terms, the actual labor input that enters the production function is a combination of the participation rate p, the employment rate $(1-u)$ (defining u as the unemployment rate), hours worked H, effort E and educational attainment S.

$$N = P \times (1-p) \times (1-u) \times H \times E \times S \tag{6}$$

All these factor have some importance in the case of Chile over the last decade and a half. Since the mid-eighties, the average years of schooling of the labor force have increased by over 10%. On the other hand, participation rates also shifted up, specially among women and in the early part of the nineties. Since 1999 though participation has declined by a couple of percentage points. Hours worked on the other hand show a downward trend since 1986. (Figure 6)

Figure 4: **Hours worked and average participation rate**



2.3 Dealing with utilization over the cycle

Over and above the changing quality of the inputs, their utilization over the cycle can introduce "false" movements in TFP. It must be assumed that some frictions exists that prevent the full utilization of the existing stock of capital or the labor force. We will not dwell on the reasons why this might be so, purely stating that this is a fact that must be taken into account to prevent a spurious relationship between TFP and the business cycle.

For the case of labor input, unemployment figures allow at least a partial disentangling of the labor utilization effect. However, three other factors, mentioned above, add to the complication: participation rates themselves are not exogenous, and do have a relationship with the business cycle due to the combination of "added-worker" and "discouraged-worker" effects. These two effects actually show interesting empirical dynamics over the cycle, depending on the persistence of the path of unemployment. C. García and G. Contreras (2001) show that an increase in unemployment initially increases participation ("added worker" effect). However, if this increase persists over time, participation start to drop below its initial level ("discouraged-worker" effect). Moreover, physical labor can be employed in varied intensity over the cycle, because of changes in hours worked and effort.

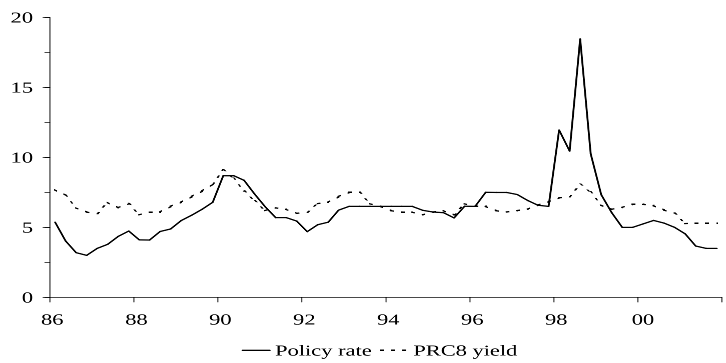
Above it was shown how these factors could affect labor input trends. They also have a quantitative impact over the cycle, particularly labor force participation.

The issues above are relevant for a correct interpretation of the primal TFP estimation. They also come into play for the case of the dual estimate. Real wages, once one controls for inflation fluctuations, move along with unemployment fluctuations in a significant way.[4]

[4]See Restrepo and García (2001), and Coeymans (1999)

Even more so, the real return on long-term bonds, which because of arbitrage is the variable used to construct the cost of capital, are sensitive to monetary policy shifts, that themselves react to perceived output deviations from trend and inflationary pressures. Therefore, the dual estimate of TFP growth will be polluted by the cyclical behaviour of the cost of capital and wages. (Figure 7)

Figure 5: **Interest rates**



2.4 Identifying assumptions

As we have seen, to construct the primal and dual measures of TFP one needs to identify separately the cyclical vs the trend components of each of the stock (for the primal) or price (for the dual) estimates. The key identification assumption we will make here is the estimation of the natural rate of unemployment, that as we'll see plays an important role in all the corrections for the estimation of TFP.

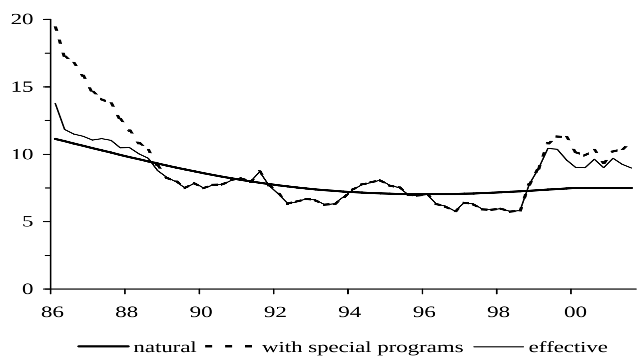
To obtain the natural rate of unemployment we filter the unemployment rate with the Hodrick-Prescott filter, setting $\lambda = 20000$ and restricting the sample up to the first quarter of 2000. The HP filter has a well known problem in dealing with end points, thus we exclude the last six quarters, fixing instead the natural rate of unemployment at 7,5% from then on (Figure 8).

We define the unemployment gap as $u - u_n$.

Capital

We assume that the effective use of capital over the cycle in a similar way to the gap between the effective unemployment rate and the natural rate. That way in a boom utilization

Figure 6: **Effective and natural rates of unemployment**



is over 100% and in a slump it falls below 100%. We further assume that both the utilization of M%E and construction move in tandem. These effective measures of the capital stock are defined as:

$$\widetilde{K_i} = K_i \times (1 + u_n - u) \tag{7}$$

Long term interest rates

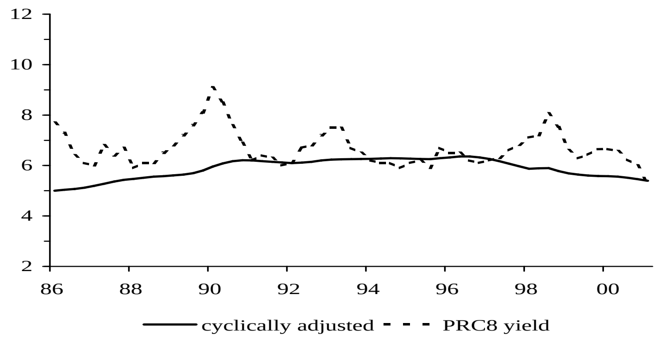
Long term interest rates in Chile have been highly volatile in the past, reflecting in part the impact of monetary policy decisions. However, for the dual calculation of TFP we are interested in more persistent factors that affect the demand for long term bonds, such as, precisely, growth prospects. Thus, we need to disentangle from the path of interest rates the movements associated with short term interest rates.

To do this we proceed in two steps. First, we use the Kalman filter to estimate a policy rule for short term interest rates, that includes the unemployment gap, the difference between inflation and the target and an autoregressive term.[5] We interpret the state variable that results as an indicator of the unobserved neutral stance for monetary policy. Second, we input the resulting neutral policy rate into an estimated equation for long term interest rates, that includes leads and lags of itself. In this manner we recover a path for long term interest rates that, hopefully, is unrelated to the cyclical situation of the economy.

Labor

[5]Details can be found in the Appendix.

Figure 7: **Long Term Interest Rate Cyclically Adjusted**



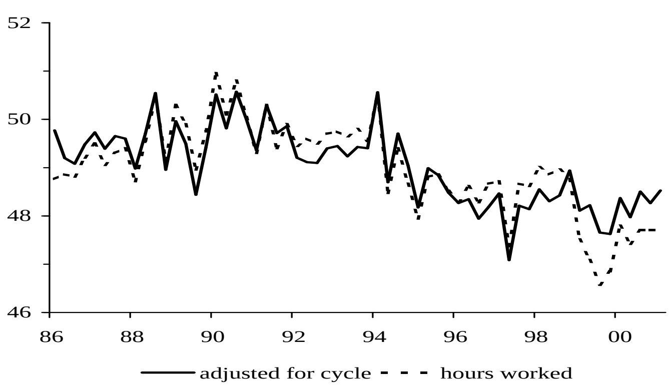
A similar correction needs to be performed for hours worked. We estimate a simple specification, regressing average hours worked with the unemployment gap, and a quadratic trend. The resulting equation, with Newey West standard errors below coefficients, is:

$$\ln H = \underset{(0.01)}{3.88} + \underset{(0.009)}{0.03} \ln trend - \underset{(0.001)}{0.008} (\ln trend)^2 - \underset{(0.155)}{0,778} (u - u_n)$$

$\overline{R^2} = 0,63,\ SE = 0,01,\ DW = 1,59$

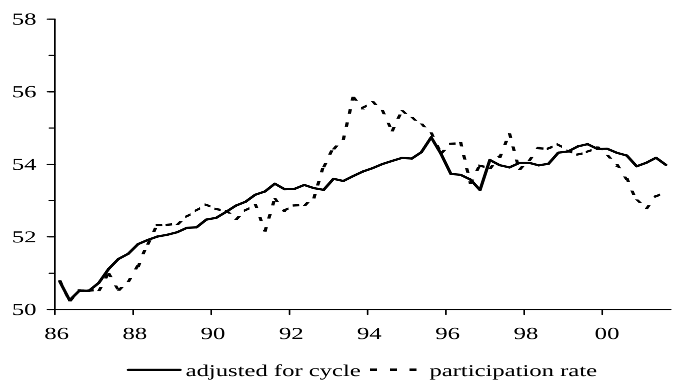
This shows that hours tend to be quite procyclical: a 1.3 percentage point increase in the rate of unemployment leads to a fall of 1 hour worked. (Figure 10)

Figure 8: **Cyclical correction of hours worked**



For the case of participation rates, Contreras and García estimate that the long run elasticity of the participation rate to unemployment is close to 1.[6] The short run dynamics however play an important role, but still the cyclical correction, using their estimates, shows a large procyclical component in the participation rate.

Figure 9: **Cyclical correction of the participation rate**



2.5 Estimation results

First, with regards to TFP estimates, the primal and dual approaches result in similar trayectories over the last fifteen years, that can be separated in three subperiods. We also constructed two measures of the primal TFP estimate, excluding inventory accumulation to reduce the cyclicality of output.

First, over the late eighties TFP growth was modest. This is a result of controlling for a varying utilization of capital, which incorrectly can be measured as TFP growth. A second period last over the nineties, from 1991 to 1995 for the case of the primal estimates, and 1989 to 1994 for the dual estimates. This was a period of high TFP growth. The third period is the last one, when TFP growth, although still positive, has slowed down.

The variety of adjustments discussed above do matter. For example, for the primal measure of TFP, an additional cumulative growth of 10% results if no adjustments are done. This is close to half the growth of adjusted TFP. However, the cyclical behavior of TFP is still an issue.

This matters for the estimation of the gap. Indeed, if one directly uses TFP for the

[6]Central Bank of Chile (2001).

Figure 10: **Primal and Dual TFP**

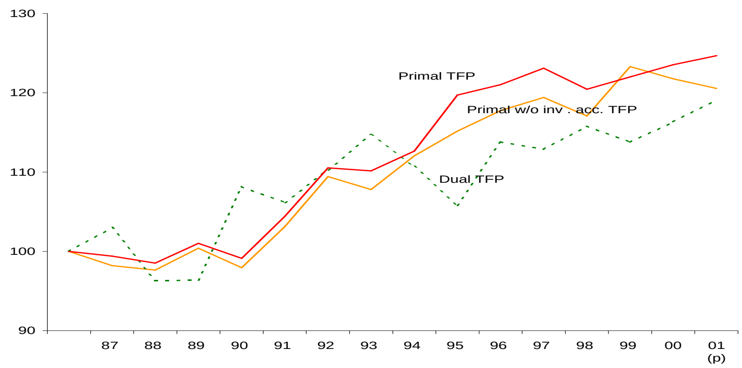
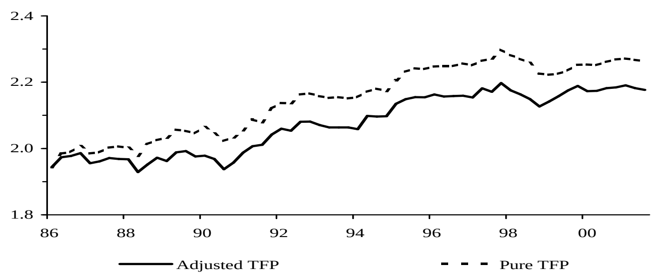


Figure 11: **Primal TFP**

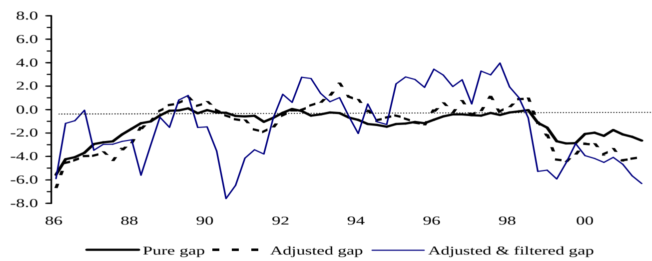


calculation of potential output, the gap over the mid eighties is close to zero, and the current slack is also small, specially the unadjusted series. As other authors have done, the filtering of TFP then appears as a reasonable option. Here we applied the HP filter, but with $\lambda = 10000$. The gap appears more procyclical.[7]

This exercise shows a last point that we want to highlight before moving to the model-consistent estimates. The filtering approach not only is required for the inputs, but also for the TFP measures that are finally obtained. This is a difficulty that should not be underestimated.

[7]The correlations between the three gaps presented and quarterly growth of seasonally adjusted GDP are -0.03, 0.01 and 0.40.

Figure 12: **Primal Output Gap**



3 Model-consistent estimates of the output gap

Given the difficulties surrounding the direct estimation of trend GDP starting from the data, in this section we propose the joint estimation of the output gap and the macroeconomic dynamics embedded in small macroeconometric models for the Chilean economy. This strategy still requires imposing some identification restrictions, which will be described below.

We use two models for the estimation, based on the aggregate demand and price blocks of a more complete model discussed in more detail elsewhere.[8] Each model will have a similar structure. First, an equation that describes the short run macroeconomic dynamics. Second, an equation for the unobserved state variable that captures underlying productivity. Third, a definition of trend GDP growth and/or the output gap.

3.1 Structure of the models

Aggregate demand model

Our first model (the AD, or aggregate demand model) consists of an aggregate output growth equation, relating the first difference of seasonally adjusted log output with an unobserved trend component γ, the stance of monetary policy given by the slope of the yield curve $r - rl$, long term interest rates rl, and external conditions, identified here with international interest rates rx and the log price of copper $\ln P_{cu}$. Two lags are included to capture the

[8] See García, Herrera and Valdés (2000).

dynamics. A disturbance term ϵ_y is also added to account for short-term fluctuations.

$$\Delta y = \gamma + \phi_r(r_{-1} - rl) + \phi_{rl} rl_{-2} + \phi_{rx} rx_{-2} + \phi_{cu} \ln P_{cu} + \phi_{y1}(\Delta y_{-1} - \gamma_{-1}) + \phi_{y2}(\Delta y_{-2} - \gamma_{-2}) + \epsilon_y \quad (8)$$

We think of the state variable γ as capturing underlying trends in output growth. Although the *level* of productivity should be smooth, we do not have any priors about the process that drives the growth rate of productivity. Thus we impose an autoregressive functional form.

$$\gamma = \rho\gamma_{-1} + \epsilon_\gamma \quad (9)$$

After estimating this small model, we cannot recover trend GDP, but we can infer its rate of change through time. For that it is necessary though to specify more carefully what we mean when we talk about trend growth. The definition we use is consistent with a neutral stance of monetary policy and stable relative prices such as the exchange rate, implying therefore:

$$r = rl = rx$$

Moreover, the price of copper should be at its equilibrium level, that we denote by $\ln \overline{P_{cu}}$.

This allows the following definition of trend output growth

$$\Delta \overline{y} = \gamma + (\phi_{rl} + \phi_{rx}) rx_{-2} + \phi_{cu} \ln \overline{P_{cu}} + \phi_{y1}(\Delta \overline{y}_{-1} - \gamma_{-1}) + \phi_{y2}(\Delta \overline{y}_{-2} - \gamma_{-2}) \quad (10)$$

This equation highlights some interesting issues. First, trend GDP growth is not static, but evolves through time, not only with the fluctuations of the unobserved underlying productivity component, but also with external conditions. This approach is not novel, at least in the inclusion of external conditions. Beechey et al. (2000) use it for the estimation of Australian trend growth, as do Rojas et al. (1997) , with a focus on terms of trade, for the Chilean economy. Although Coeymans' (1999) interpretation is closer to ours, in terms of adjudicating to the external conditions a role in trend growth, the model we propose assumes that the cost of international finance affects aggregate expenditure, while Coeymans interpret it as a determinant of TFP growth.

Aggregate supply models

Our second model focuses on the determination of prices. Inflation is determined by a Phillips curve (AS, aggregate supply), that relates the first difference of inflation with lags and leads of itself, and imported inflation (given by the sum of nominal exchange rate depreciation and international dollar inflation in US dollars). The first, restricted, version of the model imposes dynamic homogeneity on the inflationary process, to guarantee neutrality and a vertical Phillips curve in the long run. This implies adding-up restrictions on some of the right hand side regressors as well as the restriction of a zero constant. As this model is very simple, and there is evidence that inflation in Chile follows more complex dynamics[9], we also estimate an unrestricted version of the model. However, we cannot reject the null hypothesis of homogeneity and a zero constant.[10]

Thus, our AS model is as follows:

$$\Delta\pi = \xi_{l1} \sum_{i=2}^{4} \frac{\pi_{-i} - \pi_{-1}}{3} + \xi_f \sum_{i=1}^{2} \frac{\pi_{+i} - \pi_{-1}}{2} + \xi_e \sum_{i=1}^{2} \frac{\widehat{e}_{-i} + \pi^*_{-i} - \pi_{-1}}{2} + \xi_y \sum_{i=2}^{4} \frac{y_{-i} - \overline{y}_{-i}}{3} \quad (11)$$

The definition of trend GDP in this case is straightforward, as it is directly specified as the unobserved state variable:

$$\overline{y} = \rho\overline{y}_{-1} + \epsilon_{\overline{y}} \quad (12)$$

3.2 Estimation results

The models above are estimated using state-space techniques, imposing identification restrictions with respect to the volatility of the trend components of GDP growth. We assume throughout that trend output is smoother than actual output. Our choice for the dependent variable is total GDP minus mining, fishing and energy. These sectors are linked to natural resources, and their expansion over time responds more to exogenous factors.

Aggregate demand model

[9] See García and Restrepo (2001).

[10] The p-value of a χ^2 test of the joint hypothesis of a zero constant and adding-up constraint is 0.756.

Table 1: Estimation results - AD models

	OLS[a]		State-Space[b]					
			$\sigma(\epsilon_\gamma) = \sigma(\epsilon_y)/5$		$\sigma(\epsilon_\gamma) = \sigma(\epsilon_y)/20$		unrestricted $\sigma(\epsilon_\gamma)$	
ϕ_r	-0.404	*0.067*	-0.404	*0.087*	-0.405	*0.087*	-0.398	*0.083*
ϕ_{rl}	-1.501	*0.288*	-1.462	*0.308*	-1.479	*0.256*	-1.466	*0.295*
ϕ_{rx}	-0.564	*0.281*	-0.552	*0.220*	-0.552	*0.162*	-0.562	*0.208*
ϕ_{cu}	0.029	*0.011*	0.029	*0.010*	0.028	*0.005*	0.028	*0.007*
ϕ_{y1}	-0.288	*0.108*	-0.225	*0.173*	-0.267	*0.156*	-0.203	*0.171*
ϕ_{y2}	-0.282	*0.068*	-0.331	*0.180*	-0.290	*0.166*	-0.354	*0.246*
$\sigma(\epsilon_y)$	1.25%		1.05%		1.11%		0.99%	
$\sigma(\epsilon_\gamma)$							0.58%	
ρ			-0.493	*0.377*	-0.475	*0.717*	-0.555	*0.998*
$\overline{R}^2$	0.477							
Log-Likelihood	175.44		173.52		173.49		173.49	

[a] Newey-West standard errors in italics

[b] Standard errors in italics. OLS estimates used as initial conditions.

The results of the estimation are broadly consistent with single-equation estimates by least-squares. We performed a variety of estimations, using different assumptions for the variance of trend output growth.

As expected, the results show the sensitiveness of GDP growth to interest rates, both due to monetary policy actions and the shifts in the cost of external finance.[11] Also, the price of copper affects significantly GDP growth. The state-space estimates are similar to the OLS estimates, both in terms of their size and statistical significance. The state-space estimation also deliver some interesting results. First, the autocorrelation in the growth rate of the state variable (measured by ρ), although large, are not statistically different from zero. This implies that, at least in the context of the AD model, shocks to the underlying productivity growth show little persistence over time.

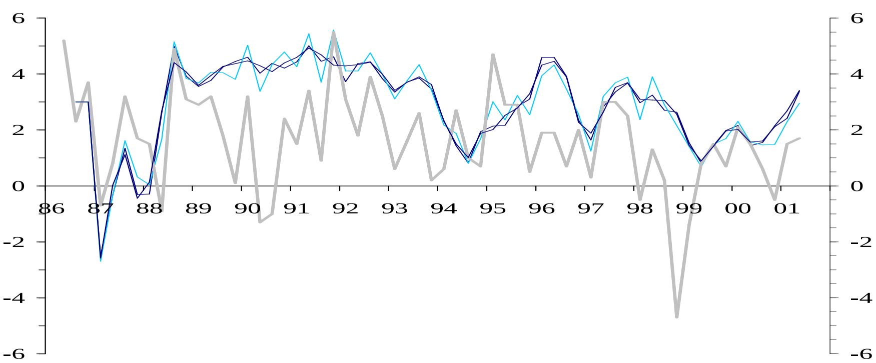
As a matter of fact, the state space estimation of the AD model differs very little from the OLS estimation, in that only a small fraction of the variation in the data can be attributed to

[11] rx was constructed using the 10 year T-bond as benchmark, deflated by US core inflation and adjusted for a measure of the sovereign spread and the incidence of capital controls over the nineties. More details can be found in the appendix.

the state variable. The path of trend GDP growth, as defined above, is similar if one considers the OLS estimation, one of the restricted versions of the AD model, and the unrestricted model that allows the variance of the state variable to be independent of the volatility of the error term. (Figure X shows, the path of actual output growth, the thick line, and the trend growth grates that result from the restricted and unrestricted state-space models, and the OLS estimation, in blue lines. The latter differ little from each other.)

From this exercise then one can conclude that, within the time span of the data, it is difficult to identify the relative importance of domestic factors, here associated with the state variable γ. Most of the variation of GDP growth can be accounted by monetary policy and external shocks.

Figure 13: **Actual and Trend Output Growth in AD models**



Aggregate supply model

From the results in the last section it is apparent that little information about trends is gained from direct observation of the path of output. With the AS models however, the inference about the size of the output gap and the growth rate of trend GDP depends on the informativeness of the path of inflation.

As before, some identification assumptions must be made, now related to the magnitude of the volatility of trend GDP $\sigma(\epsilon_{\overline{y}})$. We estimate the AS model with a variety of assumptions about this volatility. The table that follow show the resulting estimates.

The growth rates of trend output differ, but tend to be stable over time. However, some evidence of a slowdown in trend growth appears on some of the estimations. The measures of the gap too differ, in general show a positive gap over most of the nineties and a negative one since 1999. The magnitudes are important: close to 10% in recent quarters. This is

Table 2: Estimation results - AS models

	IV[a]		State-Space[b]					
			$\sigma(\epsilon_{\overline{y}}) = 1\%$		$\sigma(\epsilon_{\overline{y}}) = 0.3\%$		unrestricted $\sigma(\epsilon_{\overline{y}})$	
ξ_{l1}	0.462	*0.122*	0.416	*0.108*	0.460	*0.103*	0.303	*0.115*
ξ_{l2}	0.332	*0.113*	0.192	*0.085*	0.224	*0.059*	0.434	*0.126*
ξ_e	0.085	*0.037*	0.095	*0.049*	0.081	*0.045*	0.053	*0.052*
ξ_{vat}	0.700	*0.206*	0.668	*0.260*	0.693	*0.277*	0.655	*0.258*
ξ_y	0.048	*0.036*	0.054	*0.024*	0.043	*0.022*	0.025	*0.036*
$\sigma(\pi)$	0.73%		0.66%		0.68%		0.58%	
$\sigma(\overline{y})$							6.21%	
ρ			0.996	*0.005*	1.000	*0.004*	0.991	*0.011*
$\overline{R}^2$	0.477							
Log-Likelihood			207.71		200.23		211.04	

[a] Instrumental variables estimation. Instruments used for π_{+i} include lags of inflation, interest rates and the unemployment rate, among others. An HP trend is used as proxy for the gap.

[b] Standard errors in italics. IV estimates used as initial conditions.

Figure 14: **Trend Output Growth - AS model -** $\sigma_{\epsilon\overline{y}} \in [0.1\% - 1\%]$

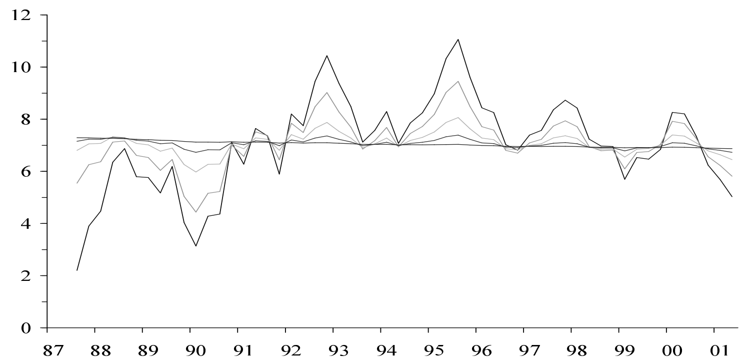
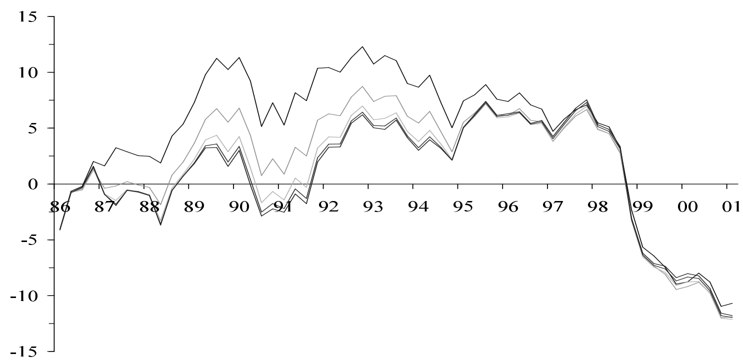


Figure 15: **Output Gap - AS model -** $\sigma_{\epsilon\overline{y}} \in [0.1\% - 1\%]$



probably related to the low passthrough from exchange rate depreciation up until now, that the state-space estimation interprets as a large, negative output gap.

4 Concluding remarks

We conclude first by discussing what we have learned from these exercises, in terms of the current size of the slack in factor markets. Then, we briefly touch upon an issue that is probably relevant for the discussion, but that escapes the scope of this work: sectoral shifts in production and employment.

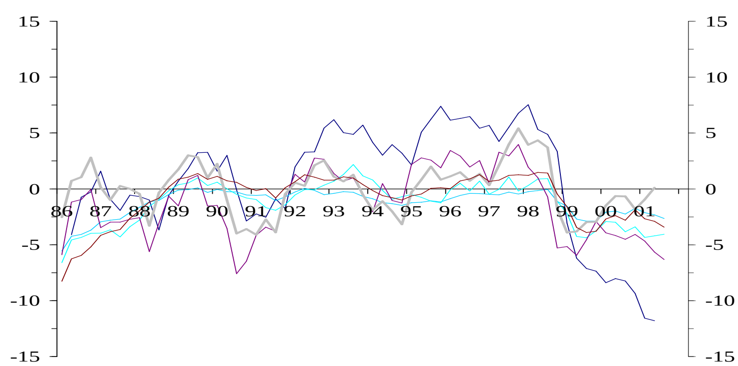
Table 3: Correlations between gaps

	HP $\lambda = 1600$	AS model	Primal adjusted	Primal n/adjusted	Primal filtered	Unemployment gap
HP	1.00	0.53	0.41	0.29	0.76	0.36
AS		1.00	0.77	0.59	0.75	0.66
Pr.adj.			1.00	0.90	0.57	0.92
Pr.n/adj.				1.00	0.40	0.99
Pr.filt.					1.00	0.49
Unemp						1.00

4.1 What have we learned?

The following figure displays several measures of the output gap that have been presented along this work. Some of them rely on growth accounting exercises, while another results of assuming a particular natural rate of unemployment. A third is the product of a state space estimation of a simple Phillips curve. As a benchmark, the thick gray line results from a simple HP filter.

Figure 16: **Output Gaps**



The first thing that is readily apparent is the wide dispersion in both the magnitude and the trends of the different estimates of the output gap. While some seem very procyclical, others are smoother. While some show a stable gap over the last quarters, others indicate an increasing slack. While some of them are not very correlated with each other, others are extremely so. This in particular is the case with the filtered primal TFP estimate and the unemployment gap, showing the importance of particular identification assumptions.

Secondly, all these measures indicate that current slack lies between 2% and 11% and, more importantly, that it has been mostly stable since 1999. The notable exception is the simple, $\lambda = 1600$, HP filter, that indicates actually a positive output gap for the second quarter of this year. The well-known sensitivity of this filter to end-points is the culprit for this somewhat counterintuitive result.

As a corollary, this is a warning to the mechanical application of statistical methods, loosely related to economic theory, for the measurement of trends and gaps. Some structure is needed to infer economically sensible conclusions about the measurement, through a particular method, of the output gap. A judgemental approach seems best: using a variety of methods provides a wider perspective on an issue that is key to the efficient conduct of monetary policy.

4.2 A final digression: sectoral shifts

An issue we have not dealt with at all is the role of sectoral shifts in the composition of output and employment. Have they been important in recent years in affecting productivity trends? We will briefly argue that, although they do seem to account for a share of the movements in aggregate productivity, this is only a fraction compared to increases in productivity within each sector.

To fix some notation, let $Y_t = \sum_i Y_{i,t}$ and $N_t = \sum_i N_{i,t}$ define aggregate output and employment, respectively. We define the sectoral employment and value added shares for the analysis that follows as $\beta_{i,t} = \frac{Y_{i,t}}{Y_t}$ and $\lambda_{i,t} = \frac{N_{i,t}}{N_t}$. On the other hand, we define sectoral labor productivity (LP) as $\theta_{i,t} = \frac{Y_{i,t}}{N_{i,t}}$. Equivalently, aggregate LP is represented by $\theta_t = \frac{Y_t}{N_t}$

Some identities are useful for what follows. First, there exists a relationship between sectoral value-added and employment shares, and aggregate and sectoral LP.

$$\theta_t \equiv \theta_{i,t} \frac{\lambda_{i,t}}{\beta_{i,t}}$$

It is possible to write aggregate LP as a weighted average of sectoral productivity, where the weights are employment shares:

$$\theta_t \equiv \sum_i \lambda_{i,t} \theta_{i,t}$$

From this, one obtains that the rate of productivity growth equals a weighted average of the sum between sectoral LP growth and the rate of change in the sectoral employment share. The weights are value-added shares.

$$\frac{\Delta\theta_t}{\theta_t} = \sum_i \beta_{i,t} \left[\frac{\Delta\theta_{i,t}}{\theta_{i,t}} + \frac{\Delta\lambda_{i,t}}{\lambda_{i,t}} \right]$$

However, it is also the case that aggregate LP equals the inverse of a weighted average of inverse sectoral LP's, but using value-added shares as weights:

$$\theta_t \equiv \left(\sum_i \frac{\beta_{i,t}}{\theta_{i,t}} \right)^{-1}$$

Equivalently, from the latter identity the rate of LP growth equals a weighted average of the difference between sectoral LP growth and the rate of change in value-added shares. The weights in this case are the sectoral employment shares.[12]

$$\frac{\Delta\theta_t}{\theta_t} = \sum_i \lambda_{i,t} \left[\frac{\Delta\theta_{i,t}}{\theta_{i,t}} - \frac{\Delta\beta_{i,t}}{\beta_{i,t}} \right]$$

The intuition for these results is simple. On the one hand, it must be the case that aggregate LP growth results partly from LP growth at the sectoral level. This however can be amplified if the sectors that display high LP growth are also increasing their share in total employment. The opposite occurs whenever the high-LP growth sectors are also increasing their share of value-added.

The following figures display both decompositions of aggregate productivity growth. The first one fixes

This shows that although shifts in the composition of output and employment do play in role in explaining short run fluctuations of aggregate productivity, the lion's share of its fluctuations are due to movements in sectoral labor productivity growth. Therefore an approach based on aggregates, such as the one discussed in this paper, seems like a good first take on the problem. The sectoral analysis is left for future work.

[12]Similar decompositions can be found in De Gregorio (1999).

Figure 17: **Decomposition of Labor Productivity Growth**

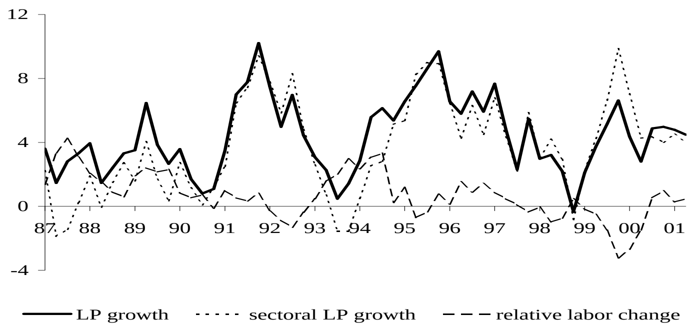
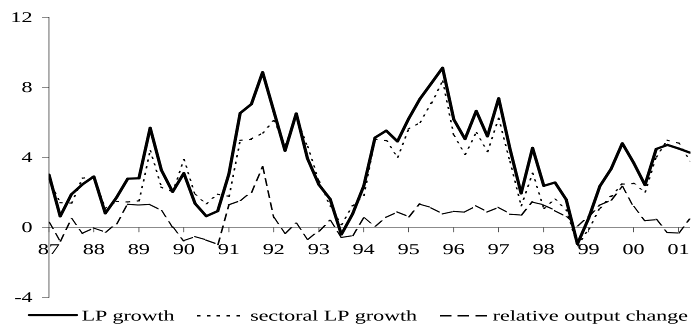


Figure 18: **Decomposition of Labor Productivity Growth**



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Appendix

[*To be written*]