Monetary Policy Credibility and the Macroeconomy

Martin Melecky
World Bank

Diego Rodríguez Palenzuela
European Central Bank

Ulf Söderström*
Bocconi University, IGIER, and CEPR

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Abstract

We quantify the effects of monetary policy transparency and credibility on macroeconomic volatility in an estimated model of the Euro area economy. In our model, private agents are unable to distinguish between temporary shocks to the central bank's monetary policy rule and persistent shifts in the inflation target, and therefore use optimal filtering techniques to construct estimates of the future monetary policy stance. We find that the macroeconomic benefits of credibly announcing the current level of the time-varying inflation target are reasonably small as long as private agents correctly understand the stochastic processes governing the inflation target and the temporary policy shock. If, on the other hand, private agents overestimate the volatility of the inflation target, the overall gains of announcing the target can be substantial. We also show that the central bank can help private agents in their learning process by responding more aggressively to deviations of inflation from the target.

Keywords: Credibility; Transparency; Imperfect information; Private sector learning; Inflation targeting.

JEL Classification: E32, E52, E58.

*Corresponding author: Ulf Söderström, IGIER, Bocconi University, Milan, Italy; ulf.soderstrom@unibocconi.it. This paper was prepared for the 11th Annual Conference of the Central Bank of Chile on "Monetary Policy under Uncertainty and Learning," in Santiago, November 15–16, 2007. We are grateful for comments from Jim Bullard, Andy Levin, Luca Sala, Paul Söderlind, Antonella Trigari, John Williams, and Tony Yates; and from participants at seminars at Bocconi University, the Bank of Finland, the Swiss National Bank Research Conference on "Expectations and Monetary Policy" in Zurich, September 2007, and the Third Banca d'Italia-CEPR Conference on Money, Banking and Finance on "Monetary Policy Design and Communication" in Rome, September 2007. We also thank Eric Swanson for sharing his Matlab code. All views, conclusions, and opinions expressed in this paper reflect solely those of the authors and do not represent those of the World Bank or the European Central Bank.

1 Introduction

During the last twenty years many central banks have adopted inflation targeting as a strategy for monetary policy, with an explicit numerical target for some measure of the inflation rate. One important benefit of this strategy is that of increasing monetary policy credibility and anchoring private sector inflation expectations at the numerical target (see, for instance, Leiderman and Svensson, 1995, or Bernanke, Laubach, Mishkin, and Posen, 1999). As economic theory suggests that private decisions are partly determined by agents' expectations concerning the future, inflation targeting, by anchoring inflation expectations, should be expected to simplify private agents' decisions, thereby reducing macroeconomic volatility and increasing overall welfare.

Several authors have produced empirical evidence that inflation targeting coupled with central bank independence has had the effect of anchoring inflation expectations. For instance, Levin, Natalucci, and Piger (2004) find that private sector inflation forecasts in the United States (where monetary policy is not guided by an inflation target) are highly correlated with a moving average of lagged inflation, while this correlation is essentially zero in a number of countries with formal inflation targets. Gürkaynak, Levin, and Swanson (2006) and Gürkaynak, Levin, Marder, and Swanson (2007) show that long-term inflation expectations tend to be less responsive to macroeconomic announcements in countries with independent inflation-targeting central banks, such as Canada, Sweden, or the United Kingdom after 1997, than in countries where the central bank is either not independent or does not have an explicit inflation target, for instance the U.S. or the U.K. before formal independence in 1997.

However, there is no strong evidence that this effect on inflation expectations has reduced macroeconomic volatility in general. While many economies, for instance the U.K. and Sweden, have performed well after the introduction of inflation targets, other economies without formal inflation targets, in particular the U.S., have shown similar, or even more impressive, performance.[1]

This paper aims at better understanding the links between, on the one hand, monetary policy credibility and communication and, on the other, private sector expectations and macroeconomic volatility. We study an empirical dynamic stochastic general equilibrium (DSGE) model of the euro area, estimated by Smets and Wouters (2003). In our specification of the model, private agents observe changes in the monetary policy stance (the central bank's interest rate instrument), but are unable to distinguish between temporary deviations from the central bank's monetary policy rule and permanent shifts in the inflation target. Agents therefore use the Kalman filter to construct optimal estimates of the current inflation objective and the temporary monetary policy shock and to make forecasts of the future path of monetary policy, and they update these estimates and forecasts as more information arrives. This learning behavior affects private agents' decisions and therefore all endogenous variables in the economy, with consequences for macroeconomic volatility in general.

[1] Cecchetti and Ehrmann (1999) and Levin, Natalucci, and Piger (2004) instead suggest that the introduction of a formal inflation target may lead to *higher* volatility in output, as the central bank shifts its preference toward stabilizing inflation and the economy moves along a fixed inflation/output volatility frontier. However, they do not find strong empirical support for this hypothesis.

Within this model, we first quantify the macroeconomic benefits of credibly announcing the (time-varying) level of the central bank's inflation objective. Such an announcement enables private agents to directly observe movements in the central bank's inflation objective and temporary deviations from the monetary policy rule. We then study the design of optimized rules for monetary policy within our framework, assuming a standard objective function for the central bank. In particular, we analyze whether rules optimized for the full information specification of the model need to be altered if agents do not observe the central bank's inflation objective.

Our results suggest that the macroeconomic benefits of credibly announcing the current level of the time-varying inflation target may be reasonably small as long as private agents correctly understand the stochastic processes governing the unobservable inflation target and the temporary policy shock. While economic volatility decreases substantially after shocks to monetary policy, these shocks account for a very small fraction of overall volatility in the economy. Therefore, the overall gains from announcing the inflation target are fairly small. However, if private agents overestimate the volatility of the inflation target, the overall gains of announcing the target can be large.

We also find that optimized monetary policy rules tend to respond more aggressively to inflation when private agents have imperfect information. By responding more aggressively to inflation, the central bank helps private agents in their learning process, thus reducing the deviation of inflation from the target with small consequences for volatility in remaining macroeconomic variables.

Our model setup is closely related to those of Erceg and Levin (2003) and Andolfatto, Hendry, and Moran (2005). Erceg and Levin (2003) study inflation persistence and the cost of disinflation in a model where private agents cannot distinguish between temporary and permanent monetary policy shocks which follow stationary autoregressive processes, as in our setup. Their model is able to generate substantial persistence in inflation and large costs of disinflation as a consequence of the learning behavior of private agents, properties that are present also in our model. Andolfatto, Hendry, and Moran (2005) study the properties of inflation expectations in a model where the temporary shock follows an autoregressive process but the permanent shock follows a Bernoulli process. They show that common econometric tests tend to reject the rationality of inflation expectations when private agents learn about the properties of monetary policy shocks over time. We present similar evidence that private sector forecast errors are large and persistent when agents learn about the underlying shocks. Relative to these contributions, our purpose is somewhat broader, as we try to understand the overall costs of imperfect information about monetary policy in terms of macroeconomic volatility, and we also study the appropriate design of monetary policy.

Moran (2005) uses a similar model to study the welfare effects of reducing the inflation target when agents learn about the inflation target shift using Bayesian updating. The welfare benefits are significant when comparing steady states, but if also taking the transitional period of learning into account, the benefits are much smaller.

A number of recent contributions study the consequences for monetary policy of private sector learning about the general structure of the economy in the stylized "New Keynesian"

model framework developed by Clarida, Galí, and Gertler (1999), Woodford (2003), and others. For instance, Nunes (2005) uses a model where a proportion of private agents learn about the economic structure, and finds that his model explains well the transitional dynamics of the economy after a disinflationary shock. Gaspar, Smets, and Vestin (2005, 2006a, 2006b) show that optimal monetary policy responds more persistently to shocks when private agents learn about the structure of the economy than with rational expectations, in order to reduce the persistence and volatility of inflation. Similarly, Molnár and Santoro (2006) show that optimal monetary policy responds more aggressively and more gradually to shocks under private sector learning than when private agents have rational expectations. We will present similar results in our framework.

Also in a New Keynesian framework, Orphanides and Williams (2007) study monetary policy in a small estimated model where the central bank learns about the natural rates of unemployment and interest and private agents learn about the structure of the economy. They show that the explicit communication of the central bank's inflation objective substantially improves macroeconomic performance under a suboptimal policy, while the gains are fairly modest under the optimal policy. Rudebusch and Williams (2006) instead study how the publication of the central bank's interest rate projections can better align private sector expectations when private agents either do not observe the coefficients in the monetary policy rule or the central bank's target level for inflation. Finally, Aoki and Kimura (2007) show that the learning processes of the central bank and the private sector implies that higher-order beliefs become relevant, leading to an increase in macroeconomic persistence and volatility. They also show that private sector learning can reduce macroeconomic volatility over time, and announcing the inflation objective can help the central bank to estimate the natural rate of interest.

In contrast to these papers, as well as those cited earlier, we study an estimated medium-sized DSGE model often used for quantitative analysis. In particular, we show that while announcing the inflation target reduces the volatility due to shocks to monetary policy, this volatility is small relative to that from the remaining shocks in the model.

Finally, similar models have also been used by Beechey (2004) and Gürkaynak, Sack, and Swanson (2005) to study the relationship between monetary policy and the yield curve. Beechey uses a stylized model with optimizing agents to study the effects on the yield curve of central bank private information concerning macroeconomic shocks and the central bank's preferences, following Ellingsen and Söderström (2001, 2005). In her model, the central bank sets monetary policy optimally given a quadratic loss function, and private agents use a Kalman filter to construct estimates of the unobservable shocks. Gürkaynak, Sack, and Swanson (2005) use a small macroeconometric model (without complete microfoundations) to study the effects of macroeconomic announcements on the yield curve. They rationalize the large response of long-term forward rates found in case studies by a model where the central bank's inflation target moves with actual inflation, but the target is unobservable to the private sector, and private agents use a signal extraction methodology to estimate the current inflation target from observed movements in the short-term interest rate.[2] We

[2] A similar model is also used by Gürkaynak, Levin, and Swanson (2006).

deviate from these authors by studying an estimated medium-scale DSGE model. While our model is also suited to study the behavior of the yield curve, we focus here on macroeconomic volatility in general.

Our paper is organized as follows. We present the structure of the model economy, following Smets and Wouters (2003), and discuss the restrictions on the private sector's information set and the Kalman filter used to construct estimates of the two monetary policy shocks in Section 2. We then present the results concerning volatility in private expectations and the macroeconomy in Section 3, and we study the design of optimized rules for monetary policy in Section 4. Finally, we summarize our findings and conclude in Section 5.

2 Model

We use the dynamic stochastic general equilibrium model developed and estimated on quarterly euro area data by Smets and Wouters (2003).[3] We here present briefly the log-linearized version of the model; we refer to Smets and Wouters (2003) for a more extensive discussion.

2.1 The structural model

Households choose consumption, labor supply, and holdings of a one-period bond to maximize lifetime utility, which depends on consumption relative to an external habit level and leisure. Utility maximization subject to a standard budget constraint gives the log-linearized consumption Euler equation

$$C_t = \frac{h}{1+h}C_{t-1} + \frac{1}{1+h}\mathrm{E}_t C_{t+1} - \frac{1-h}{\sigma_c(1+h)}\left[R_t - \mathrm{E}_t\pi_{t+1} + \mathrm{E}_t\varepsilon^b_{t+1} - \varepsilon^b_t\right], \tag{1}$$

where C_t is aggregate consumption, R_t is the nominal one-period interest rate (measured at a quarterly rate), π_t is the one-period rate of inflation, $h \in [0,1)$ determines the importance of habits, $\sigma_c > 0$ is related to the intertemporal elasticity of substitution, and ε^b_t is a general preference shock.

Households act as price-setters in the labor market, but wages are set in a staggered fashion: a fraction $1-\xi_w$ of wages are reset in a given period, and the remaining fraction is partially indexed to past inflation. This gives the log-linearized real wage equation

$$\begin{aligned} W_t &= \frac{\beta}{1+\beta}\mathrm{E}_t W_{t+1} + \frac{1}{1+\beta}W_{t-1} + \frac{\beta}{1+\beta}\mathrm{E}_t\pi_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\pi_t + \frac{\gamma_w}{1+\beta}\pi_{t-1} \\ &- \frac{(1-\beta\xi_w)(1-\xi_w)\lambda_w}{[\lambda_w + (1+\lambda_w)\sigma_l](1+\beta)\xi_w}\left[W_t - \sigma_l L_t - \frac{\sigma_c}{1-h}\left(C_t - hC_{t-1}\right) - \varepsilon^l_t - \eta^w_t\right], \end{aligned} \tag{2}$$

where W_t is the real wage, L_t is aggregate labor demand, $\beta \in [0,1]$ is a discount factor, γ_w is the degree of wage indexation, σ_l measures the elasticity of labor supply, λ_w is the steady-state wage markup, ε^l_t is a labor supply shock, and η^w_t is a wage markup shock.

Households also own the capital stock, which is rented to firms producing intermediate

[3] This model is based on Christiano, Eichenbaum, and Evans (2005). Other versions of the model include Smets and Wouters (2005, 2007), Levin, Onatski, Williams, and Williams (2005), and Del Negro, Schorfheide, Smets, and Wouters (2005).

goods at the rental rate r_t^k. They can increase the supply of capital by either investing in new capital or by changing the utilization rate of installed capital, and both actions are costly in terms of foregone consumption. The optimal choice of the capital stock, investment and the utilization rate give the log-linearized conditions

$$I_t = \frac{1}{1+\beta} I_{t-1} + \frac{\beta}{1+\beta} \mathrm{E}_t I_{t+1} + \frac{1}{\varphi_i (1+\beta)} Q_t + \frac{1}{1+\beta} \left[\beta \mathrm{E}_t \varepsilon_{t+1}^i - \varepsilon_t^i \right], \tag{3}$$

$$Q_t = -\left[R_t - \mathrm{E}_t \pi_{t+1} \right] + \beta (1-\tau) \mathrm{E}_t Q_{t+1} + \left[1 - \beta (1-\tau) \right] \mathrm{E}_t r_{t+1}^k + \eta_t^q, \tag{4}$$

$$K_t = (1-\tau) K_{t-1} + \tau I_{t-1}, \tag{5}$$

where I_t is investment, Q_t is Tobin's Q, K_t is the total capital stock, φ_i is the second derivative of the investment adjustment cost function, τ is the depreciation rate of capital, ε_t^i is a shock to the investment cost function, and η_t^q is a shock that captures variations in the external finance premium.

There is a single final good which is produced under perfect competition using a continuum of intermediate goods. These intermediate goods, in turn, are produced under monopolistic competition using capital and labor inputs with a Cobb-Douglas technology. Prices on intermediate goods are staggered as in Calvo (1983), so a fraction $1 - \xi_p$ of prices are reset in a given period. The remaining prices are partially indexed to past inflation.[4] The optimal price-setting behavior then implies that aggregate inflation is determined by the New Keynesian Phillips curve

$$\begin{aligned} \pi_t = & \frac{\beta}{1+\beta\gamma_p} \mathrm{E}_t \pi_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p} \pi_{t-1} \\ & + \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p (1+\beta\gamma_p)} \left[\alpha r_t^k + (1-\alpha) W_t - \varepsilon_t^a + \eta_t^p \right], \end{aligned} \tag{6}$$

where γ_p is the degree of indexation to past inflation, α is the Cobb-Douglas parameter on capital, ε_t^a is a technology shock, and η_t^p is a price markup shock. Profit optimization also gives the labor demand function

$$L_t = -W_t + (1+\psi) r_t^k + K_{t-1}, \tag{7}$$

where ψ is the inverse of the elasticity of the capital utilization cost function.

Finally, market clearing implies that

$$Y_t = \alpha \varphi_y \psi r_t^k + \varphi_y \varepsilon_t^a + \varphi_y \alpha K_{t-1} + \varphi_y (1-\alpha) L_t, \tag{8}$$

where Y_t is the aggregate level of output, and φ_y is equal to 1 plus the share of the fixed cost in production, and the resource constraint gives

$$Y_t = c_y C_t + \tau k_y I_t + (1 - c_y - \tau k_y) \varepsilon_t^g + \frac{[1-\beta(1-\tau)] k_y \psi}{\beta} r_t^k, \tag{9}$$

[4] More recent models instead assume that the prices that are not reoptimized are indexed in part to past inflation and in part to the (non-zero) inflation target or steady-state inflation (see, for instance, Smets and Wouters, 2007). This assumption would imply that changes in the perceived inflation target have a direct effect on price-setting and therefore on welfare (see below).

where c_y and k_y are the steady-state ratios of consumption and capital to output, and ε_t^g is government spending.[5]

There are eight structural shocks in the model. Three of these—the price and wage markup shocks η_t^p and η_t^w, and the equity premium shock η_t^q—are assumed to be white noise with variances $\sigma_p^2, \sigma_w^2, \sigma_q^2$. The remaining five shocks—to preferences, the investment adjustment cost, technology, labor supply, and government spending—are assumed to follow the stationary autoregressive processes

$$\varepsilon_t^j = \rho_j \varepsilon_{t-1}^j + \eta_t^j, \qquad j = b, i, a, l, g, \tag{10}$$

where $\rho_j \in [0, 1)$, and the innovations η_t^j are white noise with variance σ_j^2

2.2 Monetary policy

For the specification of monetary policy, we depart slightly from Smets and Wouters (2003) by assuming that monetary policy is set according to the interest rate rule[6]

$$R_t = (1 - g_r)\left\{\pi_t^* + g_\pi\left[\pi_{t-1} - \pi_t^*\right] + g_y\left[Y_{t-1} - Y_{t-1}^n\right]\right\} + g_r R_{t-1} + \varepsilon_t^r. \tag{11}$$

Thus, the nominal one-period interest rate R_t is a linear combination of the deviation of the previous period's rate of inflation π_{t-1} from the central bank's current inflation objective π_t^*, the previous period's output gap (the log deviation of real output Y_t from its natural, or flexible price/wage, level Y_t^n), and the previous period's interest rate.[7] There are two exogenous elements in the policy rule: the inflation objective π_t^* and the monetary policy shock ε_t^r. In general, these are assumed to follow stationary AR(1) processes:

$$\pi_t^* = \rho_* \pi_{t-1}^* + \eta_t^*, \tag{12}$$

$$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r, \tag{13}$$

where $\rho_*, \rho_r \in [0, 1)$ and η_t^* and η_t^r are white noise processes with variances σ_*^2 and σ_r^2. However, we will assume that the inflation target is very persistent (close to a random walk) while the monetary policy shock is (almost) white noise.[8]

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[5]The last term on the right-hand-side of equation (9) is due to the capital utilization costs. This term is not in the original Smets and Wouters (2003) model, but was added by Onatski and Williams (2004).

[6]Smets and Wouters (2003) instead specify their monetary policy rule as

$$\begin{aligned} R_t = {} & (1 - g_r)\left\{\pi_t^* + g_\pi\left[\pi_{t-1} - \pi_t^*\right] + g_y\left[Y_t - Y_t^n\right]\right\} \\ & + g_{\Delta\pi}\left[\pi_t - \pi_{t-1}\right] + g_{\Delta y}\left[\left(Y_t - Y_t^n\right) - \left(Y_{t-1} - Y_{t-1}^n\right)\right] + g_r R_{t-1} + \varepsilon_t^r, \end{aligned}$$

and obtain the estimates $g_\pi = 1.684$, $g_y = 0.099$, $g_{\Delta\pi} = 0.140$, $g_{\Delta y} = 0.159$, and $g_r = 0.961$. Also, they estimate the autoregressive coefficient of the inflation target to $\rho_* = 0.924$. Using this rule instead of our rule (11) gives very similar qualitative results. We have also experimented with rules containing the current rate of inflation and output gap, and rules with persistent monetary policy shocks rather than gradual behavior, as advocated by Rudebusch (2002). Again, the results with these rules are similar to those presented here.

[7]The presence of the past inflation rate and output gap in the policy rule implies that monetary policy only responds to predetermined variables. Thus, using the terminology of Svensson and Woodford (2004), the policy rule is an "operational" or "explicit" instrument rule, as opposed to an implicit instrument rule that includes non-predetermined variables. Such rules are also recommended by McCallum (1997).

[8]Time variation in the inflation target could be due to true time-variation in the preferred rate of inflation for an individual central banker, time variation in the composition of the monetary policy committee (and thus

2.3 Parameterization

For the structural parameters, we use the calibrated or estimated values from Smets and Wouters (2003), summarized in Table 1. These estimates were obtained using quarterly data from the Euro Area from 1980:2 to 1999:4. For the monetary policy parameters, we will in Section 3 use a fairly standard calibration of the policy rule (11), with $g_\pi = 2.0$, $g_y = 0.2$ and $g_r = 0.9$, also reported in Table 1, while in Section 4 we will choose the policy rule parameters to minimize a standard objective function for the central bank. The inflation objective π_t^* is assumed to be a near-random walk, with $\rho_* = 0.99$, while the temporary monetary policy shock ε_t^r is essentially white noise, with $\rho_r = 0.01$. Thus, changes in the inflation objective are highly persistent (the half-life of a shock is close to 70 quarters), while other deviations from the policy rule are entirely temporary. The standard deviations of the two monetary policy shocks are set to the Smets and Wouters (2003) estimates: $\sigma_* = 0.017$ and $\sigma_r = 0.081$ percentage points, respectively. Thus, innovations to the temporary shock are almost five times as volatile as those to the inflation target.[9] However, as the model is estimated on a sample with changing monetary regimes and high inflation in Europe, the estimated volatility of the inflation target is likely an upper bound on the true volatility.

2.4 Private sector information

Our key assumption is that private agents are unable to distinguish between the two exogenous shocks to the monetary policy rule, the inflation objective π_t^* and the temporary monetary policy shock ε_t^r. However, they are perfectly informed about all other aspects of the economy. In particular, as they can observe the interest rate R_t, private agents can use the policy rule (11) to back out the combination

$$\widehat{\varepsilon}_t = (1 - g_r)(1 - g_\pi)\pi_t^* + \varepsilon_t^r, \tag{14}$$

and then use the Kalman filter to calculate optimal estimates of the inflation target π_t^* and the policy shock ε_t^r.[10] The Kalman filter is thus characterized by the state equation

$$\begin{aligned} \begin{bmatrix} \pi_{t+1}^* \\ \varepsilon_{t+1}^r \end{bmatrix} &= \begin{bmatrix} \rho_* & 0 \\ 0 & \rho_r \end{bmatrix} \begin{bmatrix} \pi_t^* \\ \varepsilon_t^r \end{bmatrix} + \begin{bmatrix} \eta_{t+1}^* \\ \eta_{t+1}^r \end{bmatrix} \\ &\equiv F \begin{bmatrix} \pi_t^* \\ \varepsilon_t^r \end{bmatrix} + \begin{bmatrix} \eta_{t+1}^* \\ \eta_{t+1}^r \end{bmatrix}, \end{aligned} \tag{15}$$

in the average preferred inflation rate of the committee), or time variation in the committee's concerns for the zero lower bound of interest rates. We assume that the inflation target is close to a random walk, so changes in the inflation target are not expected to be reversed immediately, but are seen as close to permanent.

[9] Andolfatto, Hendry, and Moran (2005) instead model the inflation target as a Bernoulli process, so occasional shifts in the inflation target are followed by long periods of a constant target. Our specification implies that the inflation target changes in every period, but with a very low variance. One advantage of this specification is that the Kalman filter produces optimal forecasts of the future temporary shock and inflation target.

[10] As mentioned earlier, this specification is similar to those of Erceg and Levin (2003) and Andolfatto, Hendry, and Moran (2005).

and the observation equation

$$\begin{aligned}\widehat{\varepsilon}_t &= \left[\begin{array}{cc}(1-g_r)(1-g_\pi) & 1\end{array}\right]\left[\begin{array}{c}\pi_t^* \\ \varepsilon_t^r\end{array}\right] \\ &\equiv H'\left[\begin{array}{c}\pi_t^* \\ \varepsilon_t^r\end{array}\right]. \end{aligned} \tag{16}$$

Optimal forecasts of the future inflation target and policy shock are then calculated as

$$\left[\begin{array}{c}\widehat{\mathrm{E}}_t\pi_{t+1}^* \\ \widehat{\mathrm{E}}_t\varepsilon_{t+1}^r\end{array}\right] = (F-\kappa H')\left[\begin{array}{c}\widehat{\mathrm{E}}_{t-1}\pi_t^* \\ \widehat{\mathrm{E}}_{t-1}\varepsilon_t^r\end{array}\right] + \kappa H'\left[\begin{array}{c}\pi_t^* \\ \varepsilon_t^r\end{array}\right], \tag{17}$$

where κ is the Kalman gain,[11] and the optimal estimates of the current target and policy shock are given by

$$\left[\begin{array}{c}\widehat{\mathrm{E}}_t\pi_t^* \\ \widehat{\mathrm{E}}_t\varepsilon_t^r\end{array}\right] = F^{-1}\left[\begin{array}{c}\widehat{\mathrm{E}}_t\pi_{t+1}^* \\ \widehat{\mathrm{E}}_t\varepsilon_{t+1}^r\end{array}\right]. \tag{18}$$

Although private agents' estimates of π_t^* and ε_t^r do not enter the model explicitly, these estimates will affect private expectations of future monetary policy, and therefore indirectly affect all other endogenous variables. As agents learn over time, private expectations are in general biased predictors of future outcomes. This bias may lead private agents to make inefficient decisions, and therefore the economy may experience inefficient volatility relative to the case of perfect information. If the central bank instead were to announce the current level of the inflation target, π_t^*, private agents would be able to perfectly infer the realization of the shock ε_t^r, and the perfect-information equilibrium is attainable. We will next study the effects on macroeconomic volatility of announcing the inflation target, that is, moving from the equilibrium with imperfect information to that with perfect information.

3 Macroeconomic dynamics and volatility

We now study the dynamics of our model economy, first in terms of impulse responses and optimal forecasts after the two monetary policy shocks, and then in terms of the volatility of

[11] To determine the Kalman gain κ, let Σ be the variance-covariance matrix of $\left[\begin{array}{cc}\eta_{t+1}^* & \eta_{t+1}^r\end{array}\right]'$ and let $P_{t+1|t}$ denote the mean-squared error of the forecast of $\xi_{t+1} \equiv \left[\begin{array}{cc}\pi_{t+1}^* & \varepsilon_{t+1}^r\end{array}\right]'$, that is,

$$P_{t+1|t} = \mathrm{E}\left[\left(\xi_{t+1} - \widehat{\mathrm{E}}_t\xi_{t+1}\right)\left(\xi_{t+1} - \widehat{\mathrm{E}}_t\xi_{t+1}\right)'\right].$$

Starting from the unconditional mean-squared error, given by

$$\mathrm{vec}(P_{1|0}) = (I - F\otimes F)^{-1}\,\mathrm{vec}(\Sigma),$$

the Kalman gain matrix and the mean-squared error are found by iterating on

$$\begin{aligned}\kappa_t &= FP_{t|t-1}H\left(H'P_{t|t-1}H\right)^{-1}, \\ P_{t+1|t} &= \left(F-\kappa_t H'\right)P_{t|t-1}\left(F-\kappa_t H'\right)' + \Sigma.\end{aligned}$$

See Hamilton (1994, Ch. 13) for details. Thus, the Kalman gain depends on all elements of F, H, and Σ, that is on $g_\pi, g_r, \rho_*, \rho_r, \sigma_*$, and σ_r.

simulated time series.

3.1 The effects of monetary policy shocks

Figures 1–2 show impulse responses and optimal forecasts after one-standard-deviation-sized innovations to the inflation objective and the temporary monetary policy shock, respectively. The solid lines represent the impulse responses (and forecasts) in the benchmark case of full information (when all shocks are observable), the dash-dotted lines represent optimal forecasts with imperfect information, and the dashed lines show the effects of shocks on the economy when there is imperfect information and agents learn over time.[12]

Consider first the case of *full information*, represented by the solid lines in Figures 1–2. Figure 1 shows impulse responses and forecasts after a negative shock to the inflation target π_t^*. With full information, private agents immediately notice that the inflation target has decreased, so the perceived target jumps down to its new level and agents adjust their expectations accordingly. As a consequence there is a fall in inflation in the initial period, and the central bank is able to increase the real interest rate with only a slight increase in the nominal interest rate, which is soon reversed. This leads to a decrease in consumption, investment, output, employment, and the real wage, and therefore a fall in inflation. When inflation and the time-varying inflation target are close, they move back together to the initial level, and the nominal interest rate follows them back. The real interest rate is therefore close to its neutral level, and all real variables return toward steady state. There is thus a hump-shaped response of all variables, with the maximum effect on output (around 4.5 basis points) after four to six quarters.

After a positive innovation to the temporary monetary policy shock ε_t^r in Figure 2, the interest rate increases by the full amount of the shock (32 basis points), and the real interest rate increases even more as inflation falls. This leads to a reduction in all real variables, which motivates the fall in inflation. Again, all responses are hump-shaped, and the maximum effects on output (-20 basis points) and inflation (-5.5 basis points) occur after three quarters. Inflation and the interest rate return to steady state after 12 to 14 quarters and the output gap after around 20 quarters. (Note that the monetary policy shocks have no effect on the natural level of output, so changes in the level of output are exactly mirrored in changes in the output gap.)

Introducing *imperfect information*, private agents use the Kalman filter to make optimal estimates of the current and future inflation target and policy shock, and adjust their expectations accordingly. Figure 1 shows that after a negative inflation target shock a persistent increase in the interest rate is necessary to reduce inflation expectations. Private agents observe the small increase in the nominal interest rate, and they attribute this partly to a negative inflation target shock and partly to a positive temporary policy shock. As they know that the inflation target is much less volatile than the temporary shock, their optimal estimate of the inflation target initially falls very little (by 0.09 basis points) while the estimate

[12] In all figures and tables, the inflation and interest rates are measured on an annualized basis. Appendix A outlines how we simulate the model and construct impulse responses and optimal forecasts with imperfect information.

of the temporary shock increases more (by 0.67 basis points).

In future periods, the dash-dotted lines show that agents forecast the inflation target to return slowly to its initial level and the temporary shock to jump back in the next period, leading to a gradual return of the interest rate. These small movements in the interest rate imply that agents expect very small effects on all variables. As agents attribute almost all of the interest rate movement to a small temporary policy shock, they forecast small effects on the economy.

In practice, as time goes by the central bank increases the interest rate further, and when agents update their information set they find it increasingly likely that the inflation target has in fact decreased. Therefore inflation falls further and all real variables continue to fall as the real interest rate increases. As agents learn, the perceived and actual inflation target slowly converge and the perceived temporary monetary policy shock approaches zero. This slow learning process implies that all variables respond more gradually and persistently to the inflation target shock than with full information, and the maximum effects on output and inflation now occur after 12 to 14 quarters. As in Erceg and Levin (2003) and Nunes (2005), the presence of imperfect information substantially increases the real cost of disinflation.

After a temporary policy shock in Figure 2 private agents again observe an increase in the nominal interest rate and attribute almost all of this (32 basis points) to a positive temporary shock and very little (four basis points) to a negative inflation target shock. In the initial period, the main difference compared with the full information case is a larger fall in inflation, as private agents believe that the inflation objective is lower. Thus, the same increase in the interest rate leads to a larger increase in the real interest rate with imperfect information, and therefore a larger effect on real variables.

Private agents then forecast the inflation target to return gradually to its initial level, whereas the temporary shock is expected to disappear in the following period. Thus, agents expect inflation to remain low for a long time.

As agents learn over time, the monetary policy tightening leads to a deeper recession than under full information, and the central bank needs to lower the interest rate below the initial level to stimulate the economy. The real variables then return toward steady state, often with some overshooting, while inflation and the interest rate return very slowly to the initial level together with the perceived inflation target.

As in Andolfatto, Hendry, and Moran (2005) we note that private agents' forecasts under imperfect information (represented by the dash-dotted lines) deviate substantially from the true responses (the dashed lines) for all variables. This is because agents' forecasts are based on information available at the time of the shock, while the actual outcomes are also affected by the private sector's learning behavior over time.

To summarize, imperfect information about the two policy shocks implies that agents optimally attribute almost all unexpected movements in the nominal interest rate to the more volatile temporary shock, and very little to the persistent inflation target shock, which is less volatile. After shocks to the inflation target, private expectations therefore deviate substantially from the actual path of the economy, while the effects of temporary shocks have are very similar to the full information case. In order to persuade private agents that

the inflation target is lower the central bank needs to tighten policy more, resulting in a deeper recession. The learning process implies that all variables respond more gradually to an inflation target shock with imperfect than with full information.

3.2 Imperfect information and macroeconomic volatility

It is clear from the impulse responses and forecasts in Figures 1–2 that imperfect information about the two monetary policy shocks has large effects on the dynamic behavior of the economy and private sector forecasts, in particular after shocks to the inflation target. This impression is confirmed by Panel (a) of Table 2, which shows the variance in some key macroeconomic variables in the model that is due to the two monetary policy shocks.[13]

Conditional on the two monetary policy shocks, most variables are considerably more volatile under imperfect information than with full information, with the exception of inflation and the interest rate. The variance of the real variables due to monetary policy shocks is 20 to 25 percent larger with imperfect information than with full information, while inflation and the nominal interest rate are considerably less volatile with imperfect information. Going back to Figures 1 and 2 reveals that this effect on volatility is mainly due to the effect of shocks to the inflation target, where the response of all real variables is considerably more gradual with imperfect information, leading to larger volatility. As inflation target shocks have a smaller impact on inflation and the interest rate with imperfect information than with full information, these are also less volatile. Thus, imperfect information about the monetary policy shocks has an important impact on macroeconomic volatility, conditional on the two monetary policy shocks.

However, as the remaining eight shocks are observable to the private sector and therefore are not affected by the information restrictions, the total effect of imperfect information on macroeconomic volatility depends on the overall contribution of the monetary policy shocks to volatility. The impulse responses to these eight shocks are shown in Figures 3 to 6. It is clear that some of these structural shocks, in particular to technology and labor supply, have very large effects on the real variables compared with the monetary policy shocks.

Panel (b) of Table 2 reports the effects of imperfect information on aggregate volatility. This panel reveals that imperfect information has small effects on the volatility of macroeconomic variables once we take into account all structural shocks: the variance of most real variables increases by less than two percent. The largest effects are in terms of inflation and interest rate volatility, which is lower with imperfect information, and on the volatility of inflation around the target, which is substantially higher. This is because actual inflation adjusts slowly to changes in the inflation target when private agents cannot direcly observe the target (see Figure 1). Nevertheless, the overall effects of imperfect information on macroeconomic volatility—and thus the potential benefits of credibly announcing the central bank's target for inflation—seem modest.

[13]The reported variances are averages across 1,000 simulated samples of 5,000 observations (after discarding the initial 500 observations). Inflation and the interest rate are in annualized terms, so $\bar{\pi}_t = 4\pi_t$ and $\bar{R}_t = 4R_t$.

3.3 The role of private sector information about monetary policy shock processes

The above results suggest that there are small effects of imperfect information on macroeconomic volatility, and therefore that the gains of announcing the inflation target are small. However, as discussed earlier, the response of private expectations to the unobservable shocks depends crucially on the perceived volatility of the shocks. In the benchmark calibration, the temporary shock is considerably more volatile than the inflation target shock. Private agents therefore attribute a small fraction of the unexpected movement in the interest rate to the inflation target and a large fraction to the temporary shock, with a small effect on overall volatility as a result.

If the central bank is unwilling to announce its inflation target, it may not be reasonable to assume that private agents know the true variance of the target. In this section, we therefore analyze an alternative scenario where private agents overestimate the variance of the inflation target. In particular, we set the perceived standard deviation of the inflation target five times larger than the actual standard deviation, so the perceived standard deviation is $\hat{\sigma}_* = 0.085$, which is of similar magnitude as the standard deviation of the temporary policy shock. In this situation, private agents will attribute a greater part of the unexpected movements in the interest rate to inflation target shocks than when they know the true variance of the inflation target.

To illustrate how private agents' perceptions affect the speed with which they update their forecasts as new information arrives, Figures 7–8 show how the sensitivity of the optimal forecasts for the inflation target and the temporary policy shock to the observed interest rate depends on the perceived coefficients in the monetary policy rule and the persistence and volatility of the two monetary policy shocks.[14] Figure 7 reveals that private agents' inflation target forecast is more sensitive to unexpected changes in the observed interest rate when either the central bank is more responsive to inflation deviations from target or when the inflation target process is seen to be more persistent or volatile.[15] A larger central bank response to the lagged interest rate or more persistence or volatility in the temporary policy shock instead reduce the effect of new information on the inflation target forecast. Figure 8 shows the opposite pattern for the sensitivity of the temporary shock forecast. In our benchmark calibration (marked by vertical lines in the figures), private agents' forecast are particularly sensitive to the perceived volatility of the inflation target.

Figures 9–10 show impulse responses and optimal forecasts after innovations to the two monetary policy shocks when private agents overestimate the variance of the inflation target. (The responses under full information are of course the same as in Figures 1–2.) After an inflation target shock in Figure 9, the larger movements in the perceived inflation target implies that inflation falls faster than when private agents know the variance of the inflation

[14]The figures thus plot the two updating coefficients in the Kalman gain κ in equation (17) as a function of $g_\pi, g_r, \rho_*, \rho_r, \sigma_*$, and σ_r. Rudebusch and Williams (2006) also discuss how the private sector's information set affects the optimal updating scheme in a model where private agents are unable to observe the inflation target and the central bank helps private agents by publishing its forecast for the interest rate.

[15]Note that the inflation target forecast responds negatively to the observed interest rate, as an interest rate increase signals a decrease in the target.

target. The increase in the nominal interest rate then translates into a larger increase in the real interest rate than when private agents know the true variance of the inflation target, with a deeper and less gradual recession as a result. The central bank reduces the nominal interest rate toward the new target level more quickly, and as the perceived inflation target approaches the true target, all real variables and inflation return to their steady-state levels earlier than before. Thus, the negative humps in the impulse responses are deeper but less persistent than before. Private agents' forecasts also respond more and with larger persistence.

After a temporary policy shock in Figure 10, there are now larger differences compared with the full information case, as the initial interest rate increase is translated into a much larger fall in the perceived inflation target, leading to lower inflation, a higher real interest rate and a deeper initial recession. The central bank then quickly reduces the interest rate, and all variables return toward steady state with some over-shooting. Again, private sector forecasts respond more quickly and all variables are expected to return more slowly to steady state than when private agents know the true variance of the inflation target.

In general, when private agents overestimate the volatility of the inflation target, both shocks have larger but less persistent effects on all variables. As private agents' estimate of the inflation target is more sensitive to shocks, actual inflation also responds more to these shocks, translating into larger movements in the real interest rate and the other real variables.

Table 3 shows that all variables are now considerably more volatile than with full information, in particular inflation, the output gap, and the interest rate, but also the other real variables, whose variances increase by more than seven percent relative to the full information case. Thus, allowing for imperfect information not only regarding the shocks to the monetary policy rule but also to the variance of these shocks, our model generates fairly large effects of imperfect information on macroeconomic volatility. As a consequence, in this case the gains in terms of macroeconomic stability from announcing the central bank's inflation target are reasonably large.

4 Optimized monetary policy rules and imperfect credibility

We now study the properties of optimized rules for monetary policy within our framework. We assume that the central bank aims to stabilize inflation around the inflation target, the output gap, and the interest rate by minimizing the loss function

$$L_t = \text{Var}\left(\bar{\pi}_t - \bar{\pi}_t^*\right) + \lambda_y \text{Var}\left(Y_t - Y_t^n\right) + \lambda_r \text{Var}\left(\bar{R}_t\right), \tag{19}$$

where $\bar{\pi}_t, \bar{\pi}_t^*$, and $\bar{R}_t$ measure inflation, the inflation target and the nominal interest rate at an annualized basis, so, for example, $\bar{\pi}_t \equiv 4\pi_t$. While this objective function does not represent the welfare of a representative household in our economy, it is consistent with the mandates of most central banks.[16] We assume that the central bank preference parameters

[16] A proper welfare analysis would instead use an approximation of the representative household's utility as the central bank loss function (see, for instance, Woodford, 2003). In this case, the assumptions concerning firms' price-setting would be important for the effects of imperfect information on the macroeconomy. If, as in our model, prices are indexed only to past inflation, the inflation target does not direcly affect private sector behavior, and therefore the utility-based loss function would not depend on the volatility of the inflation

are $\lambda_y = 0.5$ and $\lambda_r = 0.1$, so the central bank attaches a larger weight to inflation stability than to output gap stability, and a small weight to stability in the interest rate.[17]

We first choose the coefficients in the central bank's policy rule (11) to minimize the central bank loss function when private agents have perfect information about the inflation target and the temporary monetary policy shock. We then evaluate this optimized rule in the case of imperfect information concerning the inflation target. Finally, we discuss what deviations from the optimized benchmark rule tend to improve on the outcome of monetary policy when private agents do not have full information about the inflation target.

The coefficients that minimize the value of the loss function (19) in the case of full information are given by $g_\pi = 7.915$, $g_y = 1.748$, $g_r = 0.917$, and Panel (a) of Table 4 reports the variances of inflation, the output gap, and the interest rate for the three alternative models, along with the value of the loss function (19). For comparison, Panel (b) reports the corresponding results for the calibrated rule analyzed in Section 3.

Compared with typical parameterizations of monetary policy rules, the optimized rule responds more aggressively to both inflation and the output gap, while the degree of interest rate smoothing is very similar.[18] As a consequence, in the model with full information the optimized rule stabilizes inflation and the output gap considerably more than the calibrated rule, at the cost of larger interest rate variability.

We then implement the rule optimized for the full information model in the models with imperfect information. As in Section 3, Panel (a) of Table 4 shows that the presence of imperfect information (when agents know the true variance of the inflation target) leads to modest increases in the volatility of output around the natural level and of inflation around target. Thus, the value of the loss function is slightly higher than with full information, although the difference in the two loss function values is approximately equivalent to a permanent output gap of only 0.043 percent.[19] Assuming that private agents also overestimate the variance of the inflation target leads to a further increase in volatility and loss, but again the effects are modest: the difference relative to the full information case is now equivalent

target. If instead prices were indexed to the (perceived) inflation target, changes in the target would have direct welfare effects.

[17]The interest rate stabilization objective can be seen as a proxy for stability on financial markets. For instance, Tinsley (1999) argues that interest rate volatility may increase term premia and therefore lead to higher long-term interest rates. From a theoretical perspective, Woodford (2003) shows that the welfare-maximizing policy aims at reducing interest rate volatility when there are money transaction frictions or when the central bank wants to avoid the zero lower bound of nominal interest rates.

[18]It is not uncommon for optimized policy rules to be more aggressive than estimated rules. This result is often attributed to the fact that the optimized rules do not take into account different sources of uncertainty that may make policy more cautions. See, for instance, Rudebusch (2001).

[19]To see this, consider the quadratic version of the loss function (19) given by

$$L_t = (1-\widehat{\beta})\mathrm{E}_t \sum_{j=0}^{\infty} \widehat{\beta}^j \left[\left(\overline{\pi}_{t+j} - \overline{\pi}^*_{t+j}\right)^2 + \lambda_y \left(Y_{t+j} - Y^n_{t+j}\right)^2 + \lambda_r \overline{R}^2_{t+j} \right],$$

which approaches the specification in equation (19) as the central bank discount factor $\widehat{\beta}$ approaches one. A permanent output gap of x percent then implies a value of the loss function of $(1-\widehat{\beta})\sum_{j=0}^{\infty}\widehat{\beta}^j\lambda_y x^2 = \lambda_y x^2$. Denoting by L_0 the loss under full information and by L_1 the loss under imperfect information, the permanent output gap that would be equivalent to moving from full information to imperfect information is given by $x = \sqrt{L_1/\lambda_y} - \sqrt{L_0/\lambda_y}$.

to a permanent output gap of 0.063 percent. However, comparing with the calibrated rule in Panel (b) reveals that the central bank is able to substantially reduce the effects of imperfect information by optimizing the policy rule.[20]

To analyze the effects of imperfect information on the optimized policy rule, we study the performance of six alternative rules, where we let one policy rule coefficient at a time deviate by 50% from the optimized rule while keeping the remaining coefficients at their optimized levels.[21] The results are reported in Table 5.

By construction, any deviations from the optimized rule will increase loss in the full information model, but Panel (a) of Table 5 shows that the effects of deviating from the optimized coefficients of inflation or the output gap are very small. On the other hand, it is more costly to deviate from the optimized coefficient of the lagged interest rate: reducing the interest rate coefficient by 50% increases loss substantially, and increasing the coefficient to 0.99 almost even more so.

Panel (b) shows the results for the model where private agents have imperfect information, but know the true variance of the inflation target. Now, deviations from the optimized rule do not necessarily increase loss, as the rule is optimized for the full information model. Nevertheless, also with imperfect information all deviations from the optimized rule increase loss, and the results are similar to the case of full information.

Finally, Panel (c) shows the results when agents have imperfect information about the monetary policy shocks and overestimate the variance of the inflation target. In this case, the central bank is better off responding more aggressively to inflation than under full information (although the gains are small), whereas responding more aggressively to the output gap has barely no effects on central bank loss.[22] As before, a large coefficient on the lagged interest rate is detrimental to central bank loss, even more so than in the other two cases. The more aggressive response to inflation implies that inflation follows the inflation target more closely, at the cost of a small increase in interest rate volatility. Under imperfect information when private agents overestimate the volatility of the inflation target, the inflation gap is more volatile than under full information. By responding more aggressively to the inflation deviation from target, the central bank helps private agents to learn the inflation target more quickly (see Figure 7), which tends to reduce overall volatility.[23] It is also clear, however, that the aggressive policy rule is not a perfect substitute for announcing the inflation target: moving from imperfect information to full information would reduce the value of the loss function considerably more than responding more aggressively to inflation.

[20] A similar result is obtained by Orphanides and Williams (2007).

[21] The coefficient of the lagged interest rate is not allowed to be larger than 0.99.

[22] For smaller deviations from the optimized coefficients, it is also beneficial to respond more aggressively to the output gap when agents overestimate the variance of the inflation target, and to inflation when agents know the true variance of the target.

[23] Similar results are obtained by Molnár and Santoro (2006) and Orphanides and Williams (2007) in models where private agents learn about the processes for inflation, output (or unemployment), and the interest rate.

5 Concluding remarks

The aim of this paper was to measure the effects of monetary policy transparency and credibility on macroeconomic volatility and welfare. To this aim we use an estimated DSGE model of the euro area economy where private agents are unable to distinguish between persistent movements in the central bank's inflation target and temporary deviations from the monetary policy rule.

Our model implies that the macroeconomic benefits of credibly announcing the current level of the time-varying inflation target are reasonably small as long as private agents correctly understand the stochastic processes governing the inflation target and the temporary policy shock. While economic volatility decreases substantially after shocks to monetary policy, these shocks account for a small fraction of overall volatility in the economy. The overall gains from announcing the time-varying inflation target are therefore fairly small. However, if private agents overestimate the volatility of the inflation target, the overall gains of announcing the target can be substantial.

We have also demonstrated that the central bank can help private agents in their learning process by responding more aggressively to inflation. Assuming a standard objective function for monetary policy, our results suggest that the optimal response to inflation is more aggressive when private agents have imperfect information and overestimate the volatility of the inflation target than when private agents have full information.

As our model is derived from the optimizing behavior of private agents, our framework can also be used to study the welfare effects of imperfect monetary policy credibility and transparency, for instance, using a linear-quadratic approximation of welfare in our model, following Benigno and Woodford (2003) and Altissimo, Cúrdia, and Rodríguez Palenzuela (2005). We plan to pursue this avenue in future work.

A Simulating the model with learning

The solution of the model is given by

$$z_t = Az_{t-1} + B\eta_t, \tag{A1}$$

z_t is a vector that includes the variables in the sticky price/wage model (13 equations), the Kalman filter variables $\mathrm{E}_t\pi^*_{t+1}, \mathrm{E}_t\varepsilon^r_{t+1}, \mathrm{E}_t\pi^*_t$, and $\mathrm{E}_t\varepsilon^r_t$ (4 equations), the flexible price/wage model (9 equations), and the 10 shock processes, including π^*_t and ε^r_t, while η_t is a vector that includes the 10 innovations.

Under imperfect information, the shocks to the inflation target (η^*_t) and the monetary policy rule (η^r_t) are not directly observable to private agents. Instead, in each period t private agents observe the interest rate R_t, use the Kalman filter to update their estimate of π^*_t and ε^r_t, and then adjust their expectations of future monetary policy, inflation, and output accordingly. As time goes by, the observed interest rate differs from agents' expectations, so agents continue to update their information and adjust their expectations. To capture this process we feed in the change in agents' estimate of π^*_t and ε^r_t as new "shocks" in each period by calculating

$$\begin{aligned}
\begin{bmatrix} \hat{\mathrm{E}}_t\eta^*_t \\ \hat{\mathrm{E}}_t\eta^r_t \end{bmatrix} &= \begin{bmatrix} \hat{\mathrm{E}}_t\pi^*_t \\ \hat{\mathrm{E}}_t\varepsilon^r_t \end{bmatrix} - \begin{bmatrix} \hat{\mathrm{E}}_{t-1}\pi^*_t \\ \hat{\mathrm{E}}_{t-1}\varepsilon^r_t \end{bmatrix} \\
&= F^{-1}\begin{bmatrix} \hat{\mathrm{E}}_t\pi^*_{t+1} \\ \hat{\mathrm{E}}_t\varepsilon^r_{t+1} \end{bmatrix} - \begin{bmatrix} \hat{\mathrm{E}}_{t-1}\pi^*_t \\ \hat{\mathrm{E}}_{t-1}\varepsilon^r_t \end{bmatrix} \\
&= \left[F^{-1}\left(F - \kappa H'\right) - I\right]\begin{bmatrix} \hat{\mathrm{E}}_{t-1}\pi^*_t \\ \hat{\mathrm{E}}_{t-1}\varepsilon^r_t \end{bmatrix} + F^{-1}\kappa H'\begin{bmatrix} \pi^*_t \\ \varepsilon^r_t \end{bmatrix},
\end{aligned} \tag{A2}$$

and we add the shocks $\mathrm{E}_t\eta^*_t, \mathrm{E}_t\eta^r_t$ in the innovation vector η_t, and the forecasts $\mathrm{E}_t\pi^*_t, \mathrm{E}_t\varepsilon^r_t$ among the shock processes in the vector z_t. (These $\mathrm{E}_t\pi^*_t, \mathrm{E}_t\varepsilon^r_t$ coincide with those from the Kalman filter.) This gives a total of 26 endogenous variables and 12 autoregressive shocks in the vector z_t and 12 innovations in the vector η_t.

Finally, we need to modify the model solution (A1) to take into account the effect of learning on the endogenous variables: while the central bank responds to the true π^*_t, ε^r_t, private agents respond to $\mathrm{E}_t\pi^*_t, \mathrm{E}_t\varepsilon^r_t$. We do this by reshuffling the matrices A and B so that the columns corresponding to $\pi^*_t, \varepsilon^r_t, \eta^*_t$, and η^r_t in the private sector equations (all equations except the interest rate rule) are moved to the positions of $\mathrm{E}_t\pi^*_t, \mathrm{E}_t\varepsilon^r_t, \mathrm{E}_t\eta^*_t$, and $\mathrm{E}_t\eta^r_t$. Simulating the model with the learning shocks described above then gives the evolution of the economy.

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Table 1: Parameter values

Parameter	Value	Description
Calibrated parameters		
β	0.99	Discount factor
τ	0.025	Depreciation rate of capital
α	0.30	Capital share in production
k_y	8.8	Capital/output ratio
c_y	0.60	Consumption/output ratio
λ_w	0.5	Average wage markup
Estimated structural parameters		
φ_i	6.771	Investment adjustment cost parameter
σ_c	1.353	Coefficient of relative risk aversion
h	0.573	Consumption habit parameter
σ_l	2.400	Elasticity of labor supply
φ_y	1.408	Fixed cost in production
ψ	0.169	Inverse elasticity of capital utilization
ξ_w	0.737	Calvo wage parameter
ξ_p	0.908	Calvo price parameter
γ_w	0.763	Rate of wage indexation
γ_p	0.469	Rate of price indexation
Estimated autoregressive parameters		
ρ_b	0.855	Preference shock
ρ_i	0.927	Investment cost shock
ρ_a	0.823	Productivity shock
ρ_l	0.889	Labor supply shock
ρ_g	0.949	Government spending shock
Estimated standard deviations		
σ_b	0.336	Preference shock
σ_i	0.085	Investment cost shock
σ_q	0.604	Equity premium shock
σ_a	0.598	Productivity shock
σ_p	0.160	Price markup shock
σ_w	0.289	Wage markup shock
σ_l	3.520	Labor supply shock
σ_g	0.325	Government spending shock
σ_*	0.017	Inflation objective
σ_r	0.081	Temporary monetary policy shock
Calibrated monetary policy parameters		
g_π	2.0	Coefficient on inflation
g_y	0.2	Coefficient on output gap
g_r	0.9	Coefficient on lagged interest rate
ρ_*	0.99	Persistence in inflation objective
ρ_r	0.01	Persistence in temporary monetary policy shock

The estimated parameter values are from Smets and Wouters's (2003) estimates using euro area data from 1980:2 to 1999:4.

Table 2: Variances of simulated data under full and imperfect information

	C_t	Y_t	I_t	L_t	W_t	$\bar{\pi}_t$	$Y_t - Y_t^n$	$\bar{R}_t$	$\bar{\pi}_t - \bar{\pi}_t^*$
(*a*) *Monetary policy shocks only*									
Full information	0.20	0.23	0.89	0.18	0.086	0.15	0.23	0.40	0.031
Imperfect information	0.25	0.29	1.12	0.22	0.10	0.084	0.29	0.32	0.16
(*b*) *All shocks*									
Full information	3.67	4.36	22.41	3.05	0.65	0.20	1.29	0.73	0.083
Imperfect information	3.72	4.42	22.63	3.10	0.67	0.14	1.34	0.64	0.21

This table reports simulated variances (averages over 1,000 simulated series of 5,000 observations) in the models with full information and with imperfect information. Inflation and the interest rate are in annualized terms: $\bar{\pi}_t = 4\pi_t$ and $\bar{R}_t = 4R_t$.

Table 3: Variances of simulated data when private agents overestimate the volatility of the inflation target

	C_t	Y_t	I_t	L_t	W_t	$\bar{\pi}_t$	$Y_t - Y_t^n$	$\bar{R}_t$	$\bar{\pi}_t - \bar{\pi}_t^*$
(*a*) *Monetary policy shocks only*									
Full information	0.20	0.23	0.89	0.18	0.086	0.15	0.23	0.40	0.031
Imperfect information	0.46	0.59	2.43	0.51	0.16	0.47	0.59	0.58	0.41
(*b*) *All shocks*									
Full information	3.67	4.36	22.41	3.05	0.65	0.20	1.29	0.73	0.083
Imperfect information	3.93	4.70	23.90	3.38	0.73	0.52	1.64	0.90	0.45

This table reports simulated variances (averages over 1,000 simulated series of 5,000 observations) in the models with full information and with imperfect information when private agents overestimate the volatility of the inflation target: $\hat{\sigma}_* = 5\sigma_*$. Inflation and the interest rate are in annualized terms: $\bar{\pi}_t = 4\pi_t$ and $\bar{R}_t = 4R_t$.

Table 4: Performance of optimized and calibrated monetary policy rules

	Simulated variances				Loss
	$\bar{\pi}_t$	$Y_t - Y_t^n$	R_t	$\bar{\pi}_t - \bar{\pi}_t^*$	
(*a*) *Optimized rule*					
Full information	0.17	0.38	1.82	0.046	0.42
Imperfect information, $\hat{\sigma}_* = \sigma_*$	0.15	0.40	1.82	0.077	0.46
Imperfect information, $\hat{\sigma}_* = 5\sigma_*$	0.20	0.41	1.83	0.089	0.48
(*b*) *Calibrated rule*					
Full information	0.20	1.29	0.73	0.083	0.80
Imperfect information, $\hat{\sigma}_* = \sigma_*$	0.14	1.34	0.64	0.21	0.94
Imperfect information, $\hat{\sigma}_* = 5\sigma_*$	0.52	1.64	0.90	0.45	1.36

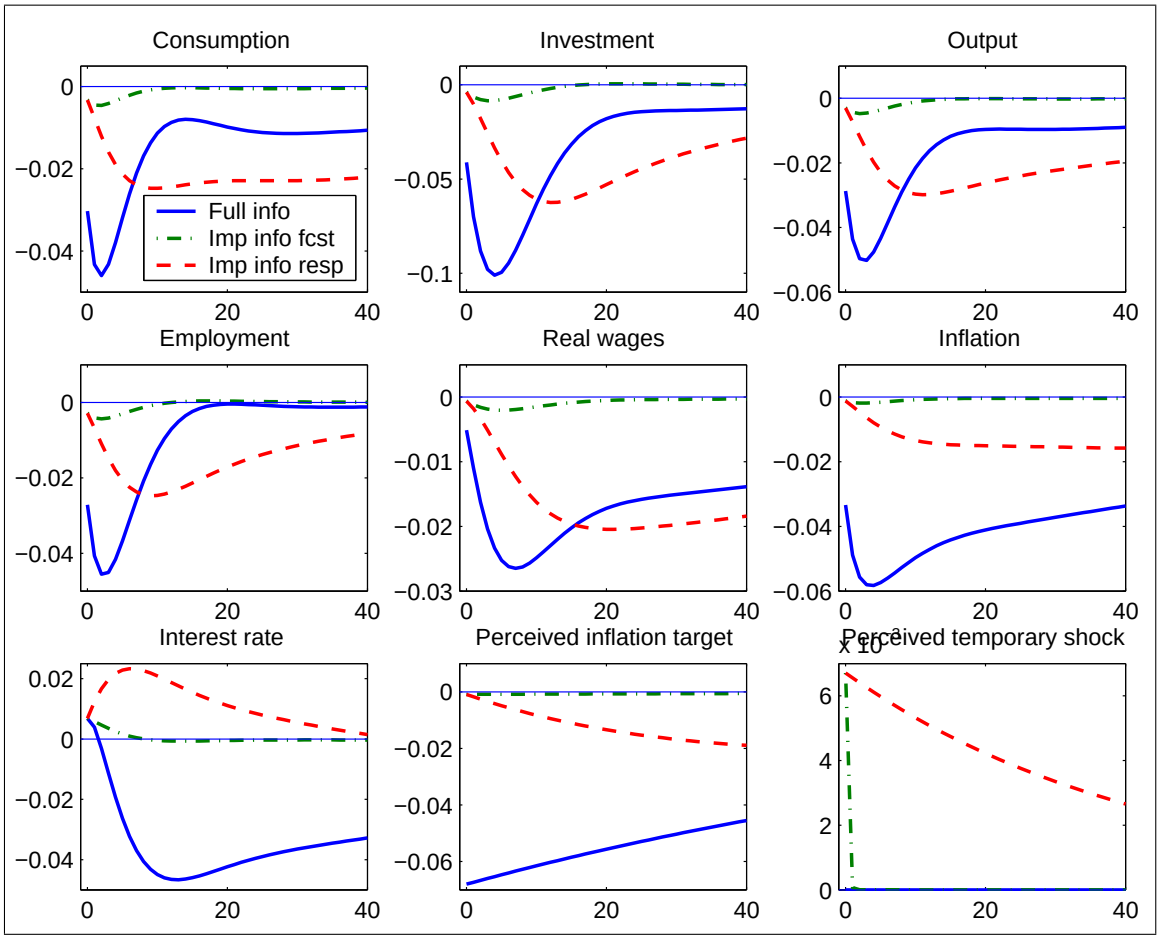
This table reports simulated variances (averages over 1,000 simulated series of 5,000 observations) in the models with full information and with imperfect information. The optimized rule is the parameterization of the policy rule (11) that minimizes the loss function (19) with $\lambda_y = 0.5$ and $\lambda_r = 0.1$ under full information, and is given by $g_\pi = 7.915$, $g_y = 1.748$, $g_r = 0.917$. The calibrated rule is given by $g_\pi = 2.0$, $g_y = 0.2$, $g_r = 0.9$.

Table 5: Performance of alternative monetary policy rules

	Simulated variances				Loss
	$\bar{\pi}_t$	$Y_t - Y_t^n$	$\bar{R}_t$	$\bar{\pi}_t - \bar{\pi}_t^*$	
(*a*) *Full information*					
Optimized rule	0.16	0.38	1.81	0.046	0.42
Large g_π	0.18	0.39	1.91	0.036	0.42
Small g_π	0.12	0.38	1.70	0.075	0.43
Large g_y	0.14	0.29	2.29	0.057	0.43
Small g_y	0.19	0.58	1.28	0.035	0.45
Large g_r	0.27	1.88	0.67	0.16	1.17
Small g_r	0.17	0.14	5.74	0.054	0.70
(*b*) *Imperfect information,* $\hat{\sigma}_* = \sigma_*$					
Optimized rule	0.14	0.40	1.82	0.076	0.46
Large g_π	0.17	0.41	1.95	0.055	0.46
Small g_π	0.092	0.39	1.68	0.13	0.49
Large g_y	0.13	0.30	2.30	0.086	0.47
Small g_y	0.17	0.62	1.28	0.066	0.50
Large g_r	0.23	2.04	0.58	0.31	1.39
Small g_r	0.17	0.14	5.80	0.058	0.71
(*c*) *Imperfect information,* $\hat{\sigma}_* = 5\sigma_*$					
Optimized rule	0.20	0.41	1.82	0.088	0.47
Large g_π	0.20	0.41	1.90	0.062	0.46
Small g_π	0.17	0.41	1.77	0.15	0.56
Large g_y	0.17	0.31	2.31	0.092	0.48
Small g_y	0.24	0.63	1.27	0.092	0.53
Large g_r	0.87	3.02	0.81	0.83	2.41
Small g_r	0.17	0.14	5.78	0.056	0.70

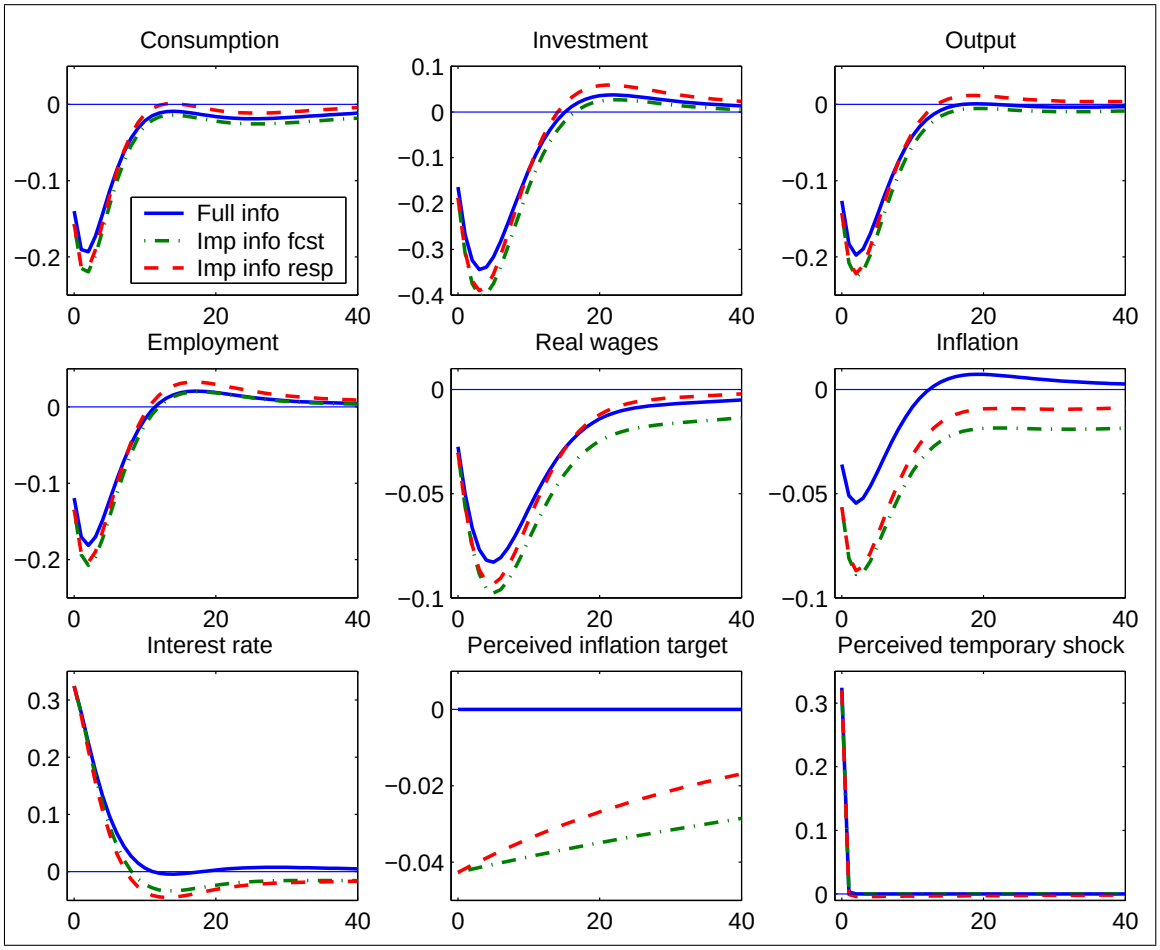
This table reports simulated variances (averages over 1,000 simulated series of 2,500 observations) in the models with full information and with imperfect information for different parameterizations of the monetary policy rule (11). The optimized rule is the parameterization that minimizes the loss function (19) with $\lambda_y = 0.5$, $\lambda_r = 0.1$ under full information, and is given by $g_\pi = 7.915$, $g_y = 1.748$, $g_r = 0.917$. "Large" and "small" coefficients are 50% larger or smaller than the optimized coefficients, with the exception of "large g_r," which is equal to 0.99.

Figure 1: Impulse responses and private sector forecasts after an inflation target shock



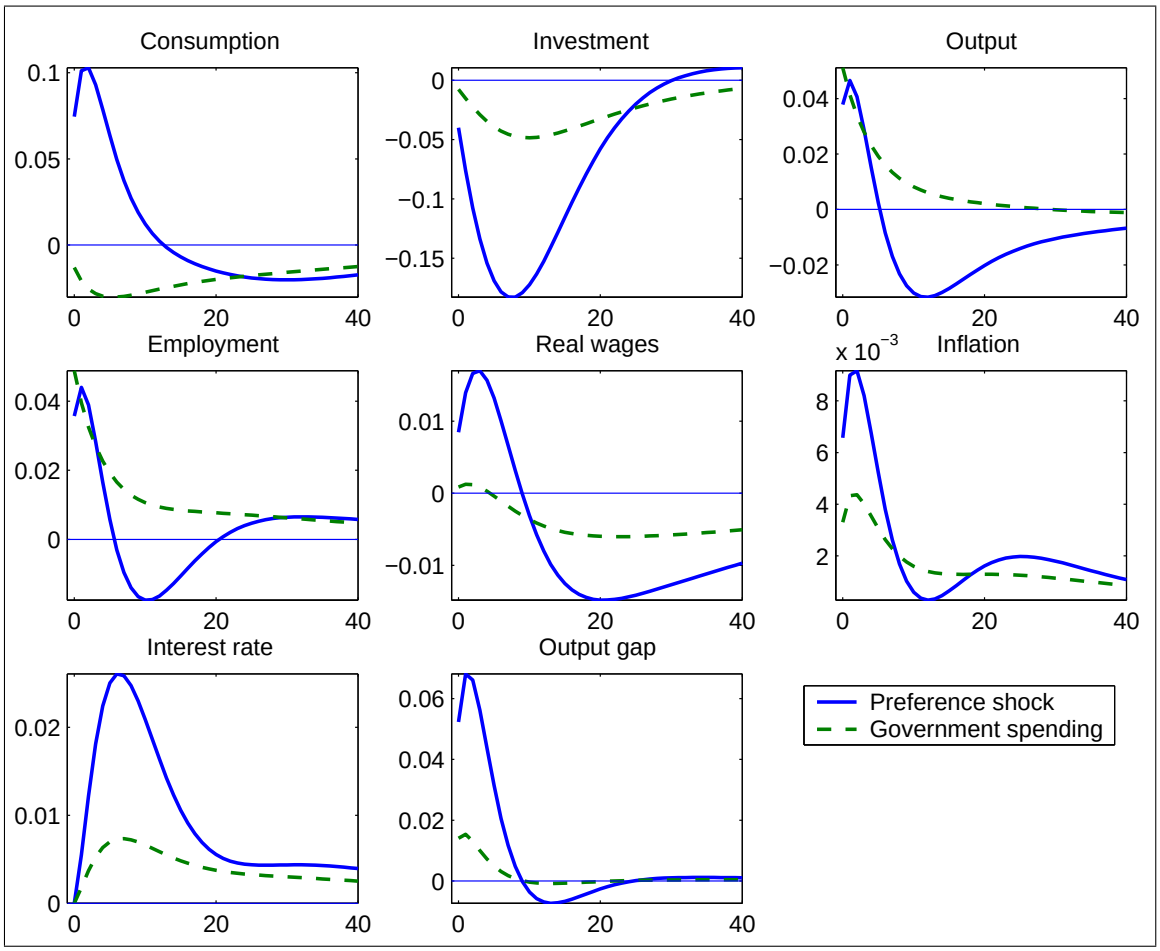
This figure shows impulse responses and optimal private sector forecasts after a negative innovation (of one standard deviation) to the inflation target π_t^*.

Figure 2: Impulse responses and private sector forecasts after a temporary monetary policy shock



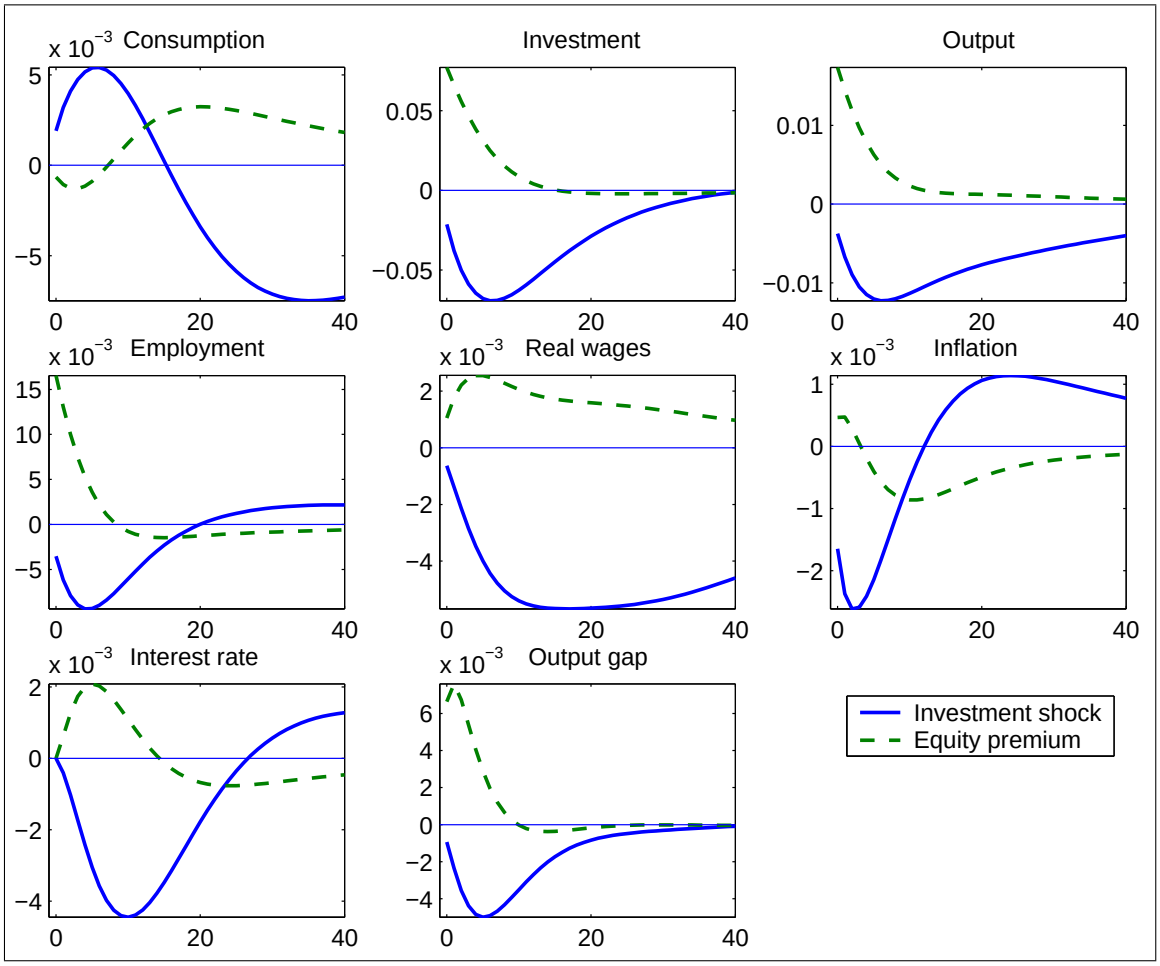
This figure shows impulse responses and optimal private sector forecasts after an innovation (of one standard deviation) to the temporary monetary policy shock ε_t^r.

Figure 3: Impulse responses to preference shock and government spending



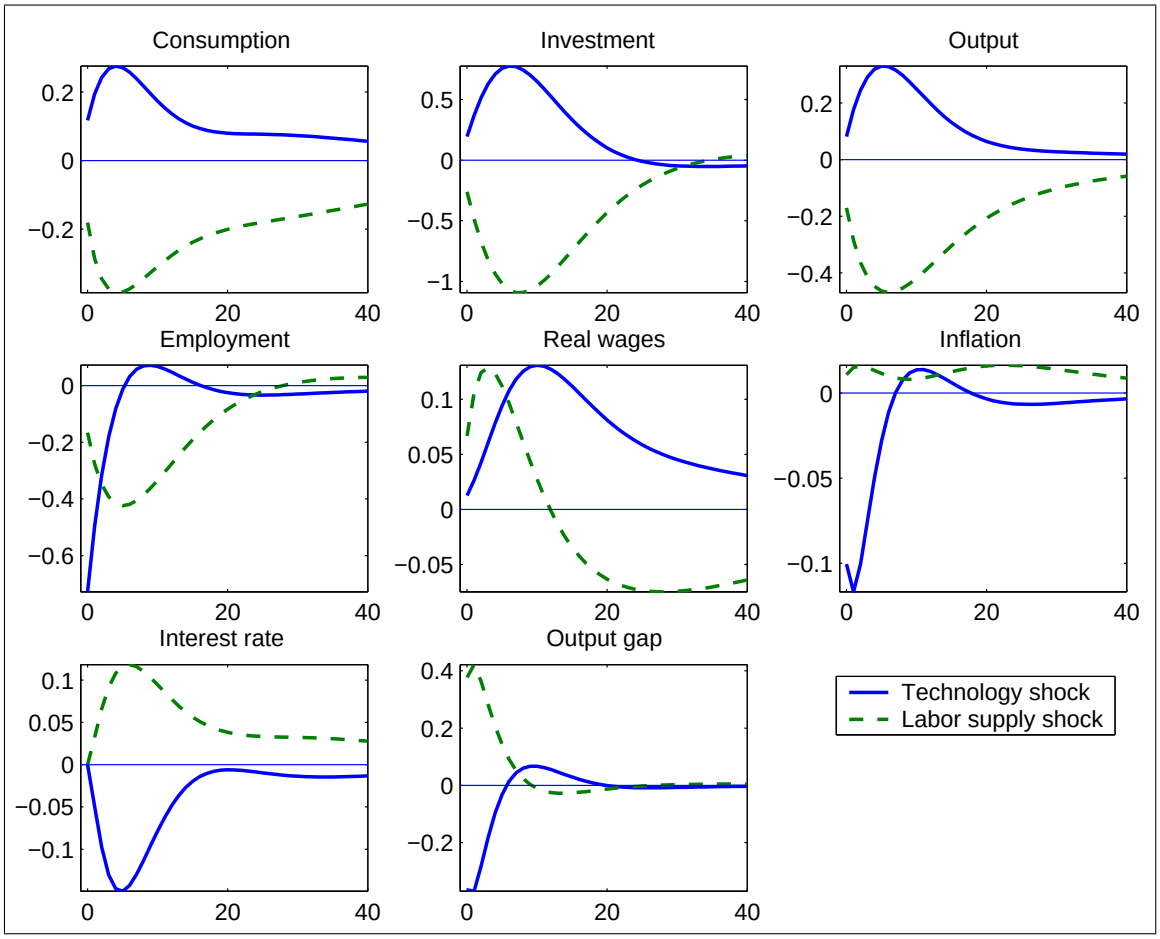
This figure shows the responses to innovations (of one standard deviation) to the preference shock ε_t^b and government spending ε_t^g.

Figure 4: Impulse responses to investment shock and equity premium



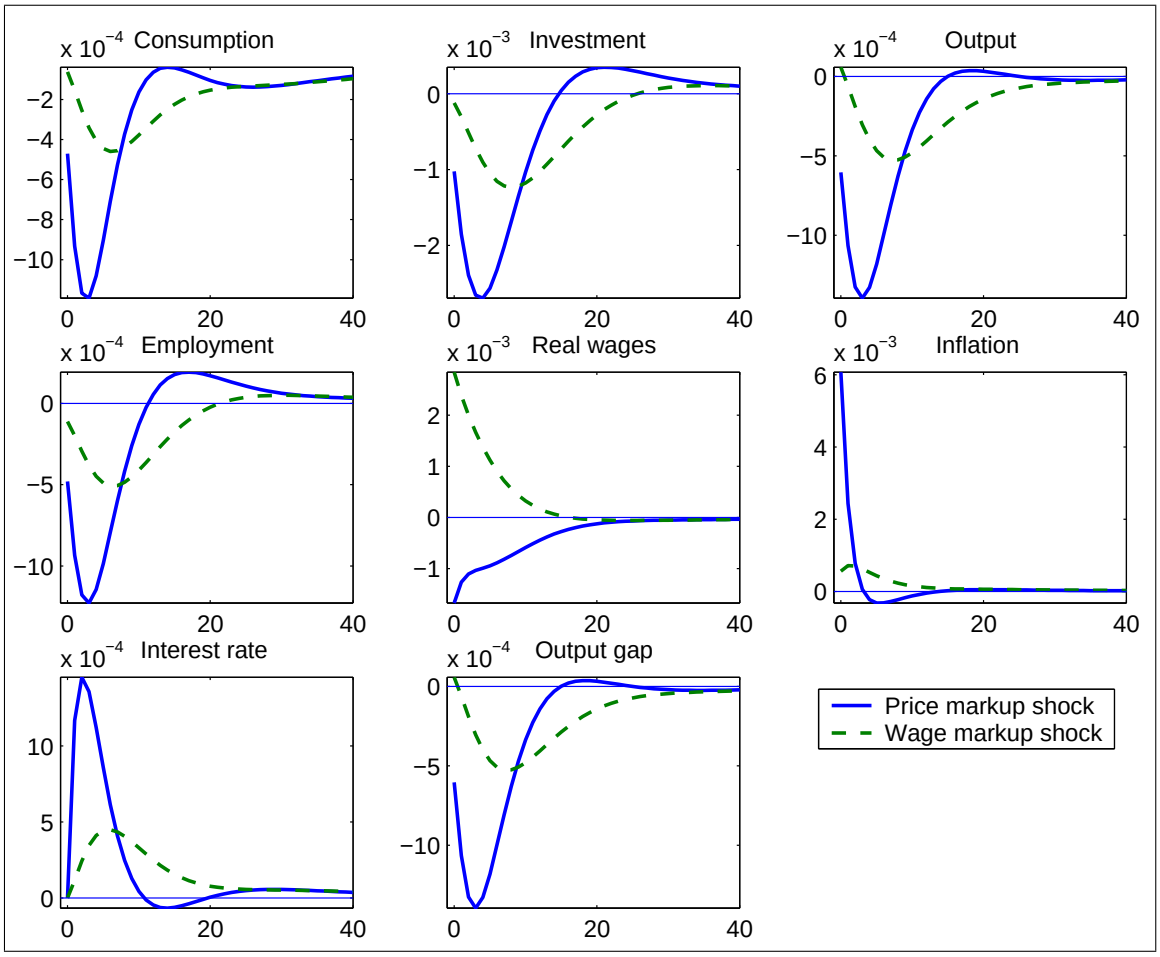
This figure shows the responses to innovations (of one standard deviation) to the investment adjustment cost shock ε_t^i and the equity premium η_t^q.

Figure 5: Impulse responses to technology and labor supply shocks



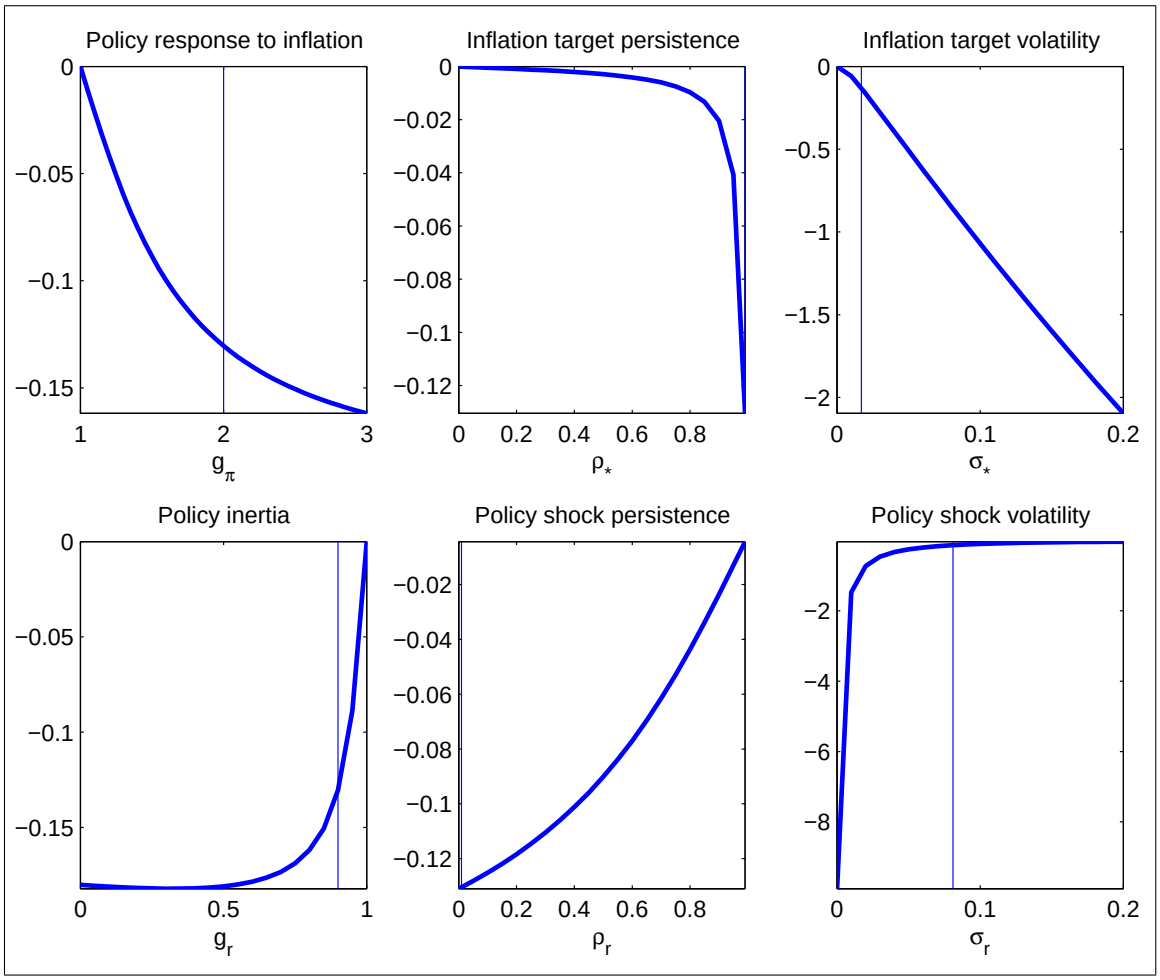
This figure shows the responses to innovations (of one standard deviation) to the technology shock ε_t^a and the labor supply shock ε_t^l.

Figure 6: Impulse responses to price and wage markup shocks



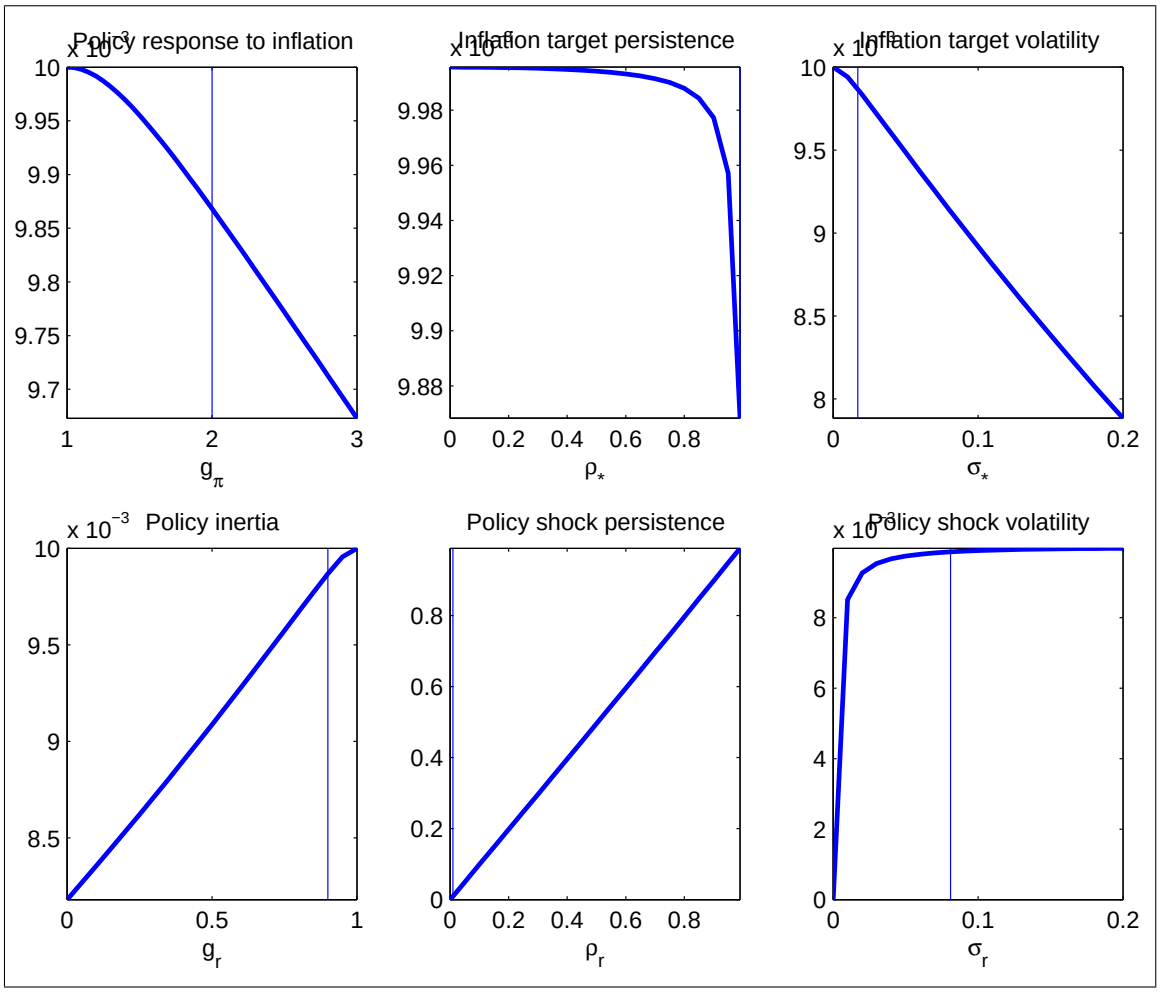
This figure shows the response to innovations (of one standard deviation) to the price markup η_t^p and the wage markup η_t^w.

Figure 7: Sensitivity of inflation target forecast to new information



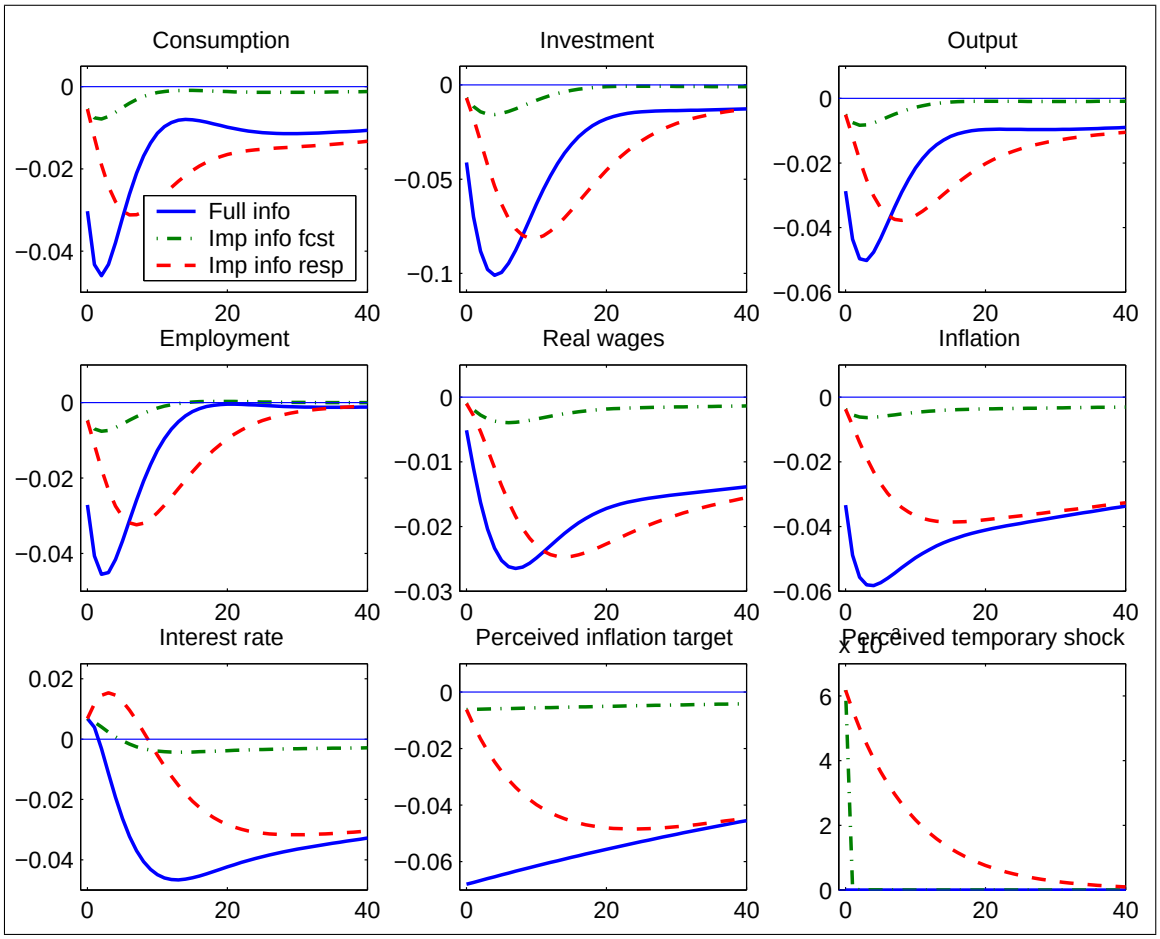
This figure shows the optimal updating coefficient (the Kalman gain) for the inflation target forecast as key parameters vary from the benchmark calibration. Vertical lines denote benchmark values.

Figure 8: Sensitivity of temporary policy shock forecast to new information



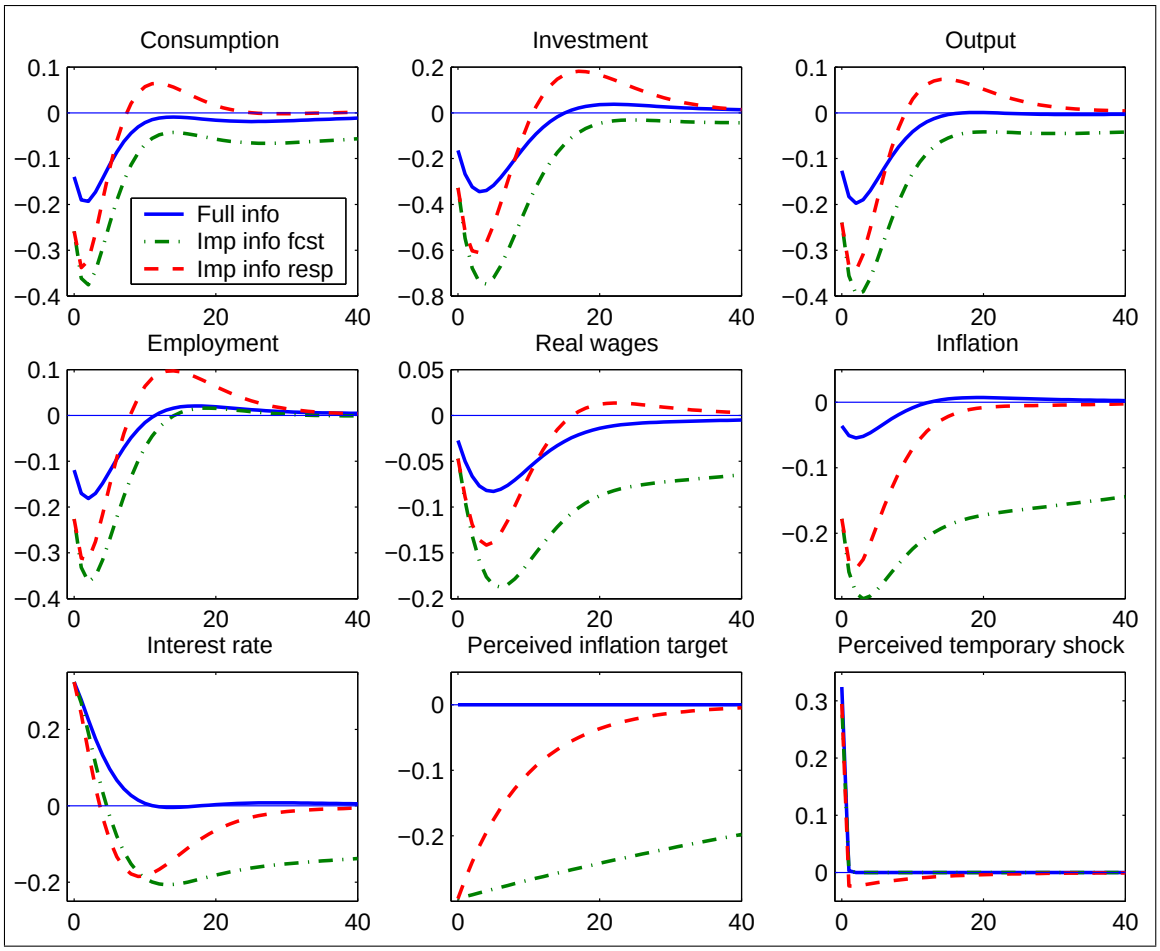
This figure shows the optimal updating coefficient (the Kalman gain) for the temporary policy shock forecast as key parameters vary from the benchmark calibration. Vertical lines denote benchmark values.

Figure 9: Impulse responses and private sector forecasts after an inflation target shock when private agents overestimate the volatility of the inflation target



This figure shows impulse responses and optimal private sector forecasts after a negative innovation (of one standard deviation) to the inflation target π_t^* when private agents overestimate the volatility of the inflation target: $\hat{\sigma}_* = 5\,\sigma_*$.

Figure 10: Impulse responses and private sector forecasts after a temporary monetary policy shock when private agents overestimate the volatility of the inflation target



This figure shows impulse responses and optimal private sector forecasts after an innovation (of one standard deviation) to the temporary monetary policy shock ε_t^r when private agents overestimate the volatility of the inflation target: $\hat{\sigma}_* = 5\,\sigma_*$.