Defining Inflation Targets and the Output-Inflation Tradeoff

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October 2006

Abstract

This paper shows the equivalence between different approaches to the inflationary objective. Defining a range and the percentage of time expected to be within such range is the same as defining a target for inflation projection in a given horizon. Both forms are similar to defining a target in terms of the expected value and desired variance for inflation. Likewise, it shows that the tolerance to deviations from the inflation target, directly associated to the policy horizon, depends on the costs of inflation deviations from the target, and also on output deviations from full employment. Therefore, setting the target in terms of an inflationary objective does not overlook the importance of unemployment nor of the output gap in monetary policy decisions. Finally the paper presents some empirical evidence and provides simple guidelines to define the target and to choose the optimal policy horizon.

1 Introduction

The main objective of the great majority of central banks in the world is controlling inflation. In some cases financial stability is added; in others, employment or economic development objectives are also included. In the case of financial stability, it involves two dimensions. The first one is stability of the domestic financial system, which in simple terms may be described as avoiding financial crises. But it also

*I am grateful to Luis Felipe Céspedes, Eduardo Engel, Jordi Galí and Rodrigo Valdés for their valuable discussions and comments, and Christopher Neilson and Alexis Montesinos for superb research assistance. The content of this paper is my sole responsibility.

refers to the normal functioning of payments with the rest of the world, which in simple terms means avoiding balance of payments crises.

In the specific case of the inflationary objective, there is a growing movement towards inflation targeting schemes. In these regimes, the central bank has a publicly-announced numeric objective on inflation, which may be either a specific number or a range. While the objective is quite clear, generally the percentage of time it is sought to meet the range is usually not made explicit, since there are inflationary or deflationary shocks conducive to inflation deviations from the target.[1] More explicit, actually, is the horizon in which the target is sought to be met and deviations corrected. Much imprecision is due precisely to uncertainty about the economy's behavior and the shocks it is exposed to.

It is important to emphasize that the range refers to an inflationary objective pursued in the current period. Expressed in terms of projection, the objective is not a range but rather a specific value. Projection is not exact either, which also admits a certain variability, but of a much lesser magnitude and, therefore, no range for the projection is specified.

As mentioned above, the target is fixed for a time horizon. The logic of this is that it is recognized that inflation cannot be controlled in the short term, since monetary policies act with lags. Furthermore, and as discussed in this paper, a gradual adjustment of inflation is allowed when inflation deviates from the target in order to avoid costs in terms of output required to return inflation to its objective. In other words, even if monetary policy does not operate with lags, it is desirable to have a monetary policy leading to gradual adjustments.[2] Moreover, in general, when the target is specified in the projection horizon, explicit reference is made to a precise point, which always corresponds to the center of the target range.

This paper intends to clarify some issues around the definition of inflationary objectives and the conduct of monetary policy under inflation targeting schemes. This paper presents a minimal model to discuss these issues and to present some empirical evidence. This paper shows that:

- The inflationary objective can be seen as described by a desired distribution for inflation. This may be thought of as defining an average value for inflation and a variability (variance). But in practice the target is defined by a mean value or a range.
- Setting the target as a mean and a variance is equivalent to defining the target in terms of a range and the percentage of time one expects it to be in the range.

[1] These schemes are known (Svenssson, 1997) as "flexible inflation targets" because deviations gradually adjust to the target.

[2] This is valid for supply shocks, which are the ones analyzed herein. A more general model should admit demand shocks, which require a different policy response. For the sake of simplicity, this paper omits demand shocks, as they do not change conclusions at all.

This is comparable to setting the target around an inflationary projection for the future, where the time frame ahead is known as "policy horizon" and depends on the inflation target variance.[3] The greater the fraction of time inflation is sought to be within the range, the shorter the necessary policy horizon

- A flexible inflation-targeting scheme, where the target is defined with a time horizon, reflects an objective function of the central bank, which values price and output stability. In particular, a direct relationship also exists between the policy horizon and tolerance to inflation deviations from the target on one hand, and the importance attributed by authorities to output deviations from full employment, on the other. In addition, the policy horizon depends on the degree of backward lookingness of price setting and on the slope of the Phillips curve.[4]

This paper continues in section 2 with a comparison between the inflation objective defined in terms of the target range and projected inflation in the policy horizon. Section 3 presents empirical evidence showing what are the implications of the actual stochastic process of inflation regarding the definition of the parameters of the inflation target. In section 4 this objective is rationalized as the result of a minimization of losses, which depends on unemployment and inflation deviations. Finally, section 5 presents some concluding remarks.

2 Target range and inflation forecast

In this section the central bank is assumed to take an inflation process as given and then accommodate to it, showing some equivalences that are useful to understand the formulation of the target. In the next section this discussion is complemented with empirical evidence.

Consider a central bank whose target is to attain inflation within a range between $\overline{\pi}$ and $\underline{\pi}$, with a center equal to $\pi^* = (\overline{\pi} + \underline{\pi})/2$. In the case of Chile the range goes from 2 to 4%. In countries such as Canada, Israel and New Zealand, this range

[3] Here it is assumed that the central bank controls the process of inflation; therefore, actual inflation adjusts to target inflation. Consequently, the variance of target inflation is equal to the variance of actual inflation, and both expressions are used indistinctly in addition to simply inflation variance.

[4] The optimal policy horizon has been discussed by Smets (2000), who estimates a model for the Euro area, which allows for the calibration of the optimal horizon. His results are similar to those found here regarding the lengthening of the policy horizon with a stronger inflation-aversion, a steeper Phillips curve, an a more indexed economy, discussed in sectiontres. In this paper, however, I use a very simple structure to derive the results and to provide empirical guidelines. In Appendix-A I solve for a more general case, although the results are qualitatively the same.

1 to 3%, while in South Africa it goes from 3 to 6%. Some countries define just the center of this range, including the United Kingdom with 2% and Norway and Iceland with 2.5%, without specifying a range. Table 1 presents the 21 countries reported in IMF (2005) and Batini and Laxton (2005), that have formally adopted inflation targets. The table presents the date (in year and quarter) of adoption of the target, the current range for the target, the specific indicator of inflation used as a target, the horizon for the target, and finally who is selects the target.

Most countries use CPI, and only a few some measure of core inflation. Except for Norway and the UK, where the government decides the target, in all of the other countries the cental bank decides the inflation target or it does it coordinated with the government. In general, the law gives the mandate of price stability, while the central bank and the government interprets this mandate in terms of a specific target.

One might think that what matters is that the inflation target is matched by a probability distribution of inflation. In simple terms, the objective should be an expected value and a variance for inflation. It will be assumed that inflation has a known symmetric distribution and, more concretely, in the empirical evidence I will consider a normal distribution, fully defined by its expected value and its variance. In the following section we will prove that this is precisely the case. But in practice, central banks do define a range. It is easier, from the point of view of communicating to the public, to set the target in terms of a range than stating its variance. On the other hand, defining the distribution requires much more information and certitude, which do not exist in practice. As should be clear from this section, in order for equivalence to exist not only the range must be defined but also the fraction of time inflation is expected to be within the range, or, in other words, its probability of being within the range, which we will define by x. In practice, this x value is not defined, although when a policy horizon is defined, information on the tolerated variability for inflation is being implicitly provided.

Once a target range is known, the first question is what this range means. Central banks are reluctant to be more specific about such range. However, it is useful to think that what the central bank wants is to be x% of the times within this range. For example, one could think that the central bank wants to be 75% or more than half the time within this range. It suffices to specify the range and the percentage of the time one intends to be within it for it to be equivalent to establishing the center of the range and the variance.

Figure 1 shows this equivalence. Given the values $\overline{\pi}$ and $\underline{\pi}$ the value for x, the distribution must be such that the area below the curve between $\overline{\pi}$ and $\underline{\pi}$ equals x. That defines the variance of the probability distribution function. Alternatively, if the central bank fixes π^* and the variance, for any x there will be only one pair of $\overline{\pi}$ and $\underline{\pi}$ that will define the range. Hence, for a central bank to define a target range and how strictly it intends to meet it is similar to setting an expected value and a variance for inflation. Therefore, we have established that a central bank whose objective is to maintain inflation within a range between $\underline{\pi}$ and $\overline{\pi}$ in $x\%$ of the time, is the same as a central bank that sets as its target an inflation rate which in average is π^* with a variance of σ_π^2.[5]

Now, as shown by Svensson (1997), the inflation target may be operationalized by setting the objective in terms of an inflation projection over a given horizon. In general, this period ranges between four and eight quarters, that is, one to two years. A first reason for this is that monetary policy affects inflation with lags. A second reason for defining a long policy horizon is that adjusting inflation rapidly to its target entails undesired costs in terms of reduced activity and high unemployment, even if inflation is perfectly controllable. In other words, unemployment is considered in inflation targeting models. In fact, in the following section it is assumed, for simplicity purposes, that the central bank determines inflation without lags and also that adjustment is gradual.

Below we show another relevant equivalence: between the target inflation variance and the policy horizon. Let's suppose that inflation follows the first order autoregressive process given by:

$$\pi_t - \pi^* = \rho(\pi_{t-1} - \pi^*) + \epsilon_t, \tag{1}$$

where ϵ_t is an i.i.d. random shock with zero mean and variance σ_ϵ^2, and ρ the autocorrelation coefficient, which is between zero and one. The expected value of inflation is π^* and its non-conditional variance are:[6]

$$\sigma_\pi^2 = \frac{\sigma_\epsilon^2}{1-\rho^2}. \tag{2}$$

In making its monetary policy decisions, the central bank aims at projecting inflation into the future. The central bank observes ϵ_t, but from $t+1$ thereafter the best it can do is to assume that this shock is zero. The inflation projection one period ahead will be $\rho\pi_t + (1-\rho)\pi^*$. Now then, T periods ahead, the projection is:

$$\mathrm{E}_t\pi_{t+T} = \rho^T\pi_t + (1-\rho^T)\pi^*. \tag{3}$$

As the horizon is increased (greater T), ρ^T approaches zero and the projection approaches π^*. We will consider then that the central bank announces it wishes inflation to be around π^* in a period T. More precisely, it wants the forecast of inflation to converge to π^*. In this respecto, the objective of the central bank

[5] Strictly speaking, this requires knowing the whole distribution, but those technical details of the discussion are omitted, and for simplicity purposes it will suffice to continue assuming a normal distribution.

[6] It suffices to take a variance on both sides of (1), where the unconditional variance of inflation and past inflation is the same, and equal to σ_π^2.

is the convergence of $\mathrm{E}_t\pi t + T$, where the relevant information set contains π_{t-1}. Therefore, the forecast, $\rho^T\pi_t+(1-\rho^T)\pi^*$, fully known in t, but with an unconditional mean and variance, and on this variable is based the operational objective of the central bank.

Given that only as T goes to infinity the projection converges to π^*, it is assumed that a tolerance margin is allowed, expressed as a variance of the projection, equal to s. In consequence, the variance of the projected of inflation that is obtained from (3) is:

$$\rho^{2T} = \frac{s}{\sigma_\pi^2}, \tag{4}$$

that is:

$$T = \frac{\log s - \log \sigma_\pi^2}{2 \log \rho}. \tag{5}$$

In the latter expression it should be noted that, given that $s < \sigma_\pi^2$ and $\rho < 1$, both the numerator and the denominator are negative; accordingly, T is well defined, since it is positive. It can be observed that the greater the variance of target inflation, σ_π^2, or in other words, the greater the range for a given x the longer the policy horizon in which projected inflation is expected to converge around π^*. Likewise, the greater ρ, the greater the policy horizon, because the increased persistence of inflation slows down its return to the center of the target range.

Summarizing, it has been established that defining an inflationary objective in terms of its mean and variance is equivalent to defining a range where inflation is expected to remain during a given percentage of the time. This, in turn, is directly related to the projection horizon in which inflation is expected to converge towards its expected value. Therefore, if one knows the inflation distribution and inflation follows an AR(1) process such as the one described in (1), we have established that all three definitions of the target shown below are equivalent:

1. The inflation target is given by an expected value of π^* and a variance of σ_π^2.
2. The inflation target is given by the range $\underline{\pi}$ to $\overline{\pi}$, in which it is expected to be $x\%$ of the time.
3. Projected inflation is expected to be around π^* with a variance of s in a horizon of T periods ahead.

Completely defining the parameters of one of the three forms indicated above, one may determine the parameters of the other definitions. Therefore, if one knew exactly the economy's behavior it would be impossible to separate the inflation-targeting decision from the policy horizon. However, in reality this is not known with accuracy, which explains the lack of numeric precision in all parameters of the objective function. Moreover, it may be argued that specifying them may lead to inconsistency, precisely due to the uncertainty existing regarding the actual structure of the economy. For example, the target may be defined as in 2 or 3 above, but the value T may be inconsistent with the target specified in 2, and this is simply the result of not knowing the economy's structure well.

Alternatively to defining exactly all of the inflation target's parameters, central banks have moved to increasing transparency and providing public explanations of their deviations from the target through their inflation reports, also called monetary policy reports. The governor of the Bank of England writes a formal letter to the Minister of Finance to give account of why deviations occur. All these forms replace a more mechanical and explicit behavior in respect of the inflation target, in a world with much more uncertainty than assumed in the models, with a public and transparent rendering of accounts. There are risks and contingencies that cannot be predicted with central banks' projection models, nor can all policy responses to more complex scenarios than simple inflation deviation—particularly those associated to financial stability—be anticipated. The foregoing suggests the need to balance a good definition of the rule, whereby the central bank may be evaluated with proper flexibility in a reality with much uncertainty.[7]

3 Empirical evidence on parameters of the inflation target

It is possible to derive the implicit values of the target for countries that have formally adopted inflation target (IT) regimes using the equivalences described in section 2. This has been achieved with a minimal structure, based on the univariate process of inflation, without additional specifications.

I will proceed estimating an AR(1) process for inflation for all inflation targeters since the beginning of the regime as defined in equation (1), using the declared target. Given the estimated persistence parameter (ρ) and variance of the shock (σ_ϵ^2) it is possible to compute the implicit fraction of time, x, that the central bank will be within the range, as presented in figure 1. In addition, using equation (5) it is possible to compute the implicit horizon (T). The only unknown variable is s, the deviation of the forecast from the center of the band. In practice discussions on inflation forecast are done centered on how many basis points the inflation forecast is away from the mid point of the target. For this reason, I assume three alternative values for s starting from the distribution of forecast and assuming that with 90 percent probability the central bank wants the forecast to be $\pm.1$, $\pm.2$ and $\pm.3$ from the mid point.

There has been extensive discussion on the persistence of inflation and problems of measurement, and, for this reason, the results I present here are illustrations of

[7] In this regard, it suffices to recall the difficulties for central banks to forecast GDP evolution.

the implicit parameters of the inflation target. In particular, traditional measures of persistence use more than one lag, and the persistence parameter is the sum of coefficients of the lagged variable. In this paper, however, and to be more consistent with the simple framework I have presented, I will estimate AR(1) processes. An important issue regarding measures of persistence are the potential breaks in the intercept. As known from Perron (1990), ignoring structural breaks in the intercept could induce an upward bias in the persistence parameter. This could be particularly important in the case of inflation since the inflation objective of central banks could have been changing, without changes in persistence.[8] Most of the evidence, however, has shown that there is a structural break around the adoption of the inflation target. The estimations of this paper starts with the adoption of the target, and hence this should not be a serious problem, except for the reduction in the length of the time series.

Another important issue is the measurement of inflation. In general, the measured used is month-to-month or quarter-to-quarter inflation. Here I use monthly measures of 12-month inflation. It is well known that this procedure may induce higher persistence. Indeed, it is possible that we may accept the hypothesis of a unit root of inflation, when actually the process for inflation may be I(0). The reason for the measure I use is to be consistent with actual definition of the inflation in an inflation target regime. Central banks' objective is in general 12-months inflation on a monthly or quarterly basis. Hence, the measure I use is the one consistent with the actual definition of the targets.[9]

The estimations and computations of the implicit parameters of the inflation target are presented in table 2. The first two columns indicate the beginning of the application of the inflation target and the current range for the target. Several countries define the target for periods of approximately one year. In contrast, an stable inflation target is defined as one in which the objective is defined without an ending date. This former case is indicated with an * in the table. The AR(1) processes have been estimated from the beginning of the regime until the last available observation, taken from the IFS, using quarterly data. The results of the estimation of the autocorrelation coefficient (ρ) and the variance of the shock (σ_ϵ^2) are presented in the next two columns. The column under x indicates the percentage of the time

[8]For recent discussions on this issue in the context of industrial countries, in particular in the Euro area, see the Inflation Persistence Network of the European Central Bank at http://www.ecb.int/home/html/researcher_ipn.en.html. For aggregate evidence estimating aggregate parameters of inflation persistence and structural breaks see Gadzinski and Orlandi (2004), Levin and Piger (2004), and O'Reilly and Whelan (2004). A summary is presented in Altissimo, et al. (2006).

[9]Indeed, it is possible that quarter-to-quarter inflation displays very low persistence, although 12-months measure do actually are highly persistent. If the central bank objective were month-to-month inflation the horizon should be very short, although the variance of the objective large. See, for example García and Valdés (2005) for the Chilean case, or Benati (2006) for a discussion of persistence across monetary regimes for some industrial countries.

that inflation is expected to be within the range given the estimated AR(1) process. Iceland, Norway and the United Kingdom specify a point for the target, so x cannot be defined. For the rest of the countries, there is wide dispersion in the implicit value of x.

For Australia, Colombia, Israel, Philippines, and South Africa, the value of x is between 10 and 35 percent, which is less than "most of the time". This is the result of the fact that the variance of ϵ is relatively large. However, three out of these five countries (Australia, Colombia and Philippines), are, together with Korea, the only countries in this sample that have a range of one percentage point width.

The remaining 13 countries, with the exception of the Czech Republic that is borderline, inflation is expected to be within the range more than 50 percent of the time, and Switzerland, the country with the less volatile inflation a relatively low persistence, inflation is within the target 0 to 2 percent almost all the time. The median estimation of x is about 48%.

As discussed before, the implicit horizon varies with the tolerance of the inflation forecast being away from the mid point of the range. The greater the tolerance the shorter the horizon. There is wide variety in the implicit horizon and depends crucially on the persistence of inflation. The more persistence the longer the horizon. The media horizon for $\pm.3$ is about two years and for $\pm.1$ it lengthen to somewhat more than three years. Countries with high persistence. Countries with the largest persistence would have an implicit horizon for $\pm.3$ tolerance between seven to eight years, which is much longer than the stated horizon, which implies that given the persistence and variance of the inflation process tolerance must be greater. This is the case of Brazil, Philippines and Sweden. The results are very sensitive to the persistence parameter, and the parameters reported here are somewhat higher than those found in other studies for industrialized countries.

In table 2 I present the estimations of persistence from Levine and Piger (2004). These estimations were done for the period 1984–2003, and the first column assumes no break, and the other two assume a break in mean inflation. The need to add a break for a longer period is that the persistence parameter would be biased upward if the inflation target has changed. My estimations, in contrast, are done for the period with stable inflation target. Indeed, assuming no break gives persistence parameters relatively high, which decline substantially, much below the estimations of the paper, when the estimates are done using classical statistics or bayesian methods. Overall, Levine and Piger (2004) conclude that persistence has not changed significantly over a long period of time, and they are in average about 0.7 for industrial countries. Similar results are found in Gadzinski and Orlandi (2004), but much higher estimates are computed by O'Reilly and Whelan (2004) for the euro area using the harmonized CPI index. Overall, there could be an upward bias in the estimations of this paper, although this could be the result of the estimation method and the length of this period rather than ignoring a break because I use a stable period in the objectives of monetary policy. From the point of view of this paper, this could result in somewhat shorter policy horizons.

Other estimations of the optimal are discussed in Smets (2000). Previous studies have found shorter horizon while Smets (200) find values between three and four years for inflation targeting. The results of table 2 suggests horizons between two and three years according to the estimates of inflation persistence. To reach this values I have just used the actual definition of the inflation target range and the autocorrelation coefficient of an AR(1) process fitted to inflation. In the next section I provide an structural interpretation using a simplified macroeconomic model.

4 The output-inflation tradeoff

While the foregoing section considered that the central bank takes inflation process as given, this section takes a step forward and adds structure to the economy in order to understand where inflation comes from and how it relates to the output gap. Such gap may be also associated to unemployment. What this section does is deriving the expression (1) from the fundamental parameters of the economy, which, in this case, are given by preferences between unemployment and inflation, and the Phillips curve. Here I will show that the inflationary process is endogenous. The value of ρ is determined by the authority, which gradually adjusts inflation to reduce its costs in terms of output. The existence of demand shocks is omitted.

In order to proceed I will use the model presented in De Gregorio (1995), which enables to derive the optimal evolution of inflation from a function of social loss from inflation and output gaps plus a Phillips curve that includes indexation. To such effect, I will assume that there is an optimal inflation rate π^*, but the central bank implements gradual adjustments of inflation to reduce welfare losses.

The social loss function is given by:[10]

$$\mathrm{L} = a(y - \bar{y})^2 + (\pi - \pi^*)^2, \tag{6}$$

where y is GDP and $\bar{y}$ its full employment level, or it rather corresponds to an output level consistent with a non-accelerating-inflation rate of unemployment. It should be noted that here there are no time inconsistency problems, which recommend, for example, that the loss function be different to the socially optimal loss (Rogoff, 1985).[11]

[10]This function of preferences may be formally derived from the utility of the representative consumer, see Woodford (2003), chap. 6. However, it should be warned that in case there is indexation, the utility function will be somewhat different, but the main qualitative results in this paper should not change.

[11]Strictly speaking, the present value of losses rather than the value in each period should be

Inflation is determined by the following Phillips curve:

$$\pi_t = \alpha\pi_{t-1} + (1-\alpha)\mathrm{E}_{t-1}\pi_t + \delta(y-\bar{y}) + \nu. \tag{7}$$

The term ν corresponds to an inflationary shock i.i.d with zero mean and variance σ_ν^2. The Phillips curve contains persistency given by the term $\alpha\pi_{t-1}$, which may be interpreted as the result of indexation of prices and salaries. A simple case is that of some regulated utility prices, which are indexed to past inflation. It could also represent the outcome of decisions on prices and salaries overlapped according to the Taylor extension (1980) proposed by Fuhrer and Moore (1995).[12] The parameter α, which takes values between zero and one, represents the degree of indexation. The Phillips curve's slope is δ and, to simplify the notation, its inverse is defined as θ, i.e. $\theta \equiv 1/\delta$.

Solving for the output gap in the Phillips curve, and replacing it in the objective function, we have that the first-order condition for the central bank's optimization is given by (subindex t is eliminated, and subindex -1 is used for a one-period lag):

$$\pi - \pi^* = \frac{1}{1+a\theta^2}[\alpha a\theta^2(\pi_{-1}-\pi^*) + (1-\alpha)a\theta^2(\mathrm{E}_{t-1}\pi - \pi^*) + a\theta^2\nu]. \tag{8}$$

Taking expectations from the above expression to solve for rational expectations of inflation and replacing this expression in the same first-order condition, we obtain the following optimal inflation:

$$\pi - \pi^* = \frac{1}{1+\phi}(\pi_{-1}-\pi^*) + \frac{\nu}{1+\phi\alpha}, \tag{9}$$

where:

$$\phi = \frac{1}{a\theta^2\alpha}. \tag{10}$$

Optimal inflation has the same form as the one assumed in (1), where the autocorrelation coefficient and the error depend on the fundamental parameters of the model and on the inflationary shock. That is:

$$\rho = \frac{1}{1+\phi} \quad \text{and} \quad \epsilon = \frac{\nu}{1+\phi\alpha\theta^2}. \tag{11}$$

minimized. The solution of said problem is significantly more complex and the details are presented in Appendix-A. The results are qualitatively the same, in particular the signs of the partial derivatives of the persistence of inflation with respect to the main parameters.

[12] For more details, see Walsh (2003), chapter 5.3. The existence of indexation is what complicates the solution of the problem in case an intertemporal loss function is assumed. In the event of $a = 0$ the static and the intertemporal solutions are the same.

It should be noted that expected inflation is equal to the central value of the target, π^*, and the variance is:

$$\begin{aligned}\sigma_\pi^2 &= \frac{1+\phi^2+2\phi}{(1+\phi\alpha\theta^2)^2(\phi^2+2\phi)}\sigma_\nu^2 \\ &= \frac{1}{\left(1+\frac{1}{a\theta^2}\right)^2(\phi^2+2\phi)}\sigma_\nu^2 + \frac{1}{\left(1+\frac{1}{a\theta^2}\right)^2}\sigma_\nu^2. \end{aligned} \tag{12}$$

In the foregoing section it was shown that increasing the variance of target inflation is similar to extending the policy horizon or increasing the target range, all this without changing the expected value of the inflation target. The effects on the credibility of target definition are not considered either, which implicitly assumes that the public knows well the central bank's loss function and the economy's structure given by the Phillips curve.

Now using the analysis of this section, the following results may be verified:

- What happens when $a = 0$, i.e, when the central bank has no concern at all for unemployment? In this case the value of ρ would be zero and expected inflation would adjust to π^* in each period. Therefore, the policy horizon collapses to zero, namely, it attempts to meet the inflation projection in each period. In this case inflation would be equal to π^*, because the monetary policy would fully offset the effect of an inflationary shock. As a increases, the policy horizon extends or, similarly, the inflation target variance becomes greater.
- The greater the volatility of inflationary shocks, the greater the variance of target inflation, which, in turn, generates a longer policy horizon.
- Something similar occurs when indexation increases, measured by α, as this also produces a slower adjustment and greater variability of the inflation target, i.e., the target range increases.
- When the Phillips curve's slope decreases, that is, δ falls, or θ rises, the Phillips curve becomes flatter. In this case the output gap has a lesser impact on inflation. Therefore, the central bank will accept a greater inflation variance, or a longer policy horizon, as the central bank does not want to vary the output gap too much in order to approach optimal inflation.

Why does all this happen? Because although the central bank defines its objective in respect of an inflation target, its decisions consider costs in terms of the unemployment caused by the attainment of the target. In other words, having an inflation target does not mean that unemployment costs are disregarded.

It should be noted that in this exercise monetary policy operates without lags, and that the inflation target is not intended to be met in the short term. Namely, the authority could always try to attain the target, in which case $\pi = \pi^* + \epsilon$ and the expected inflation would be π^*. However, the authority will not implement this policy. The reason for this is that, given the existence of indexation and prices that mechanically follow the past, we will have that inflation, in its expected value, will deviate from $\alpha\pi_{t-1} + (1-\alpha)\mathrm{E}_{t-1}\pi_t = \alpha\pi_{t-1} + (1-\alpha)\pi^*$, originating output fluctuations which, to the extent that $a > 0$ result in welfare losses. Although expectations are rational, the Phillips curve has persistence that leads to deviations from full employment.

The advantage of specifying the macroeconomic stability objective in terms of an inflation target avoids the inconveniences of defining two objectives that might be inconsistent. For example, defining a limit for output variation along with an explicit inflation target could turn both objectives incompatible with the economy's structure. There are additional complications when the target is specified in terms of activity level, since full employment output is hardly known, and it could lead to the traditional inflationary acceleration to the extent that a very low unemployment is pursued, or similarly, to inflationary deceleration in the event the full employment output is underestimated.

However, the essential reason for choosing an inflation-targeting regime is that otherwise inflation would be undetermined. Monetary policy deals with prices and inflation. Defining an inflation target enables to anchor inflation.

Summing up, inflation persistence declines, and the policy horizon shortens, due to a decline in a, that is an increase in inflation aversion, a decline in α, the degree of indexation, or an increase in the slope of the Phillips curve ($\delta = 1/\theta$), that is a decline in θ. The decline in inflation persistence should also be accompanied by a decline in the variance of inflation, as long as the variance of the inflation shock (σ_ν^2) remains constant.

5 Concluding remarks

In general, as shown in more detail herein, it may be said that targeting inflation on the basis of a range in which one expects inflation to be most of the time is similar to fixing an objective for projected inflation in a given policy horizon, or indicating an expected value and a variance for target inflation. In any case, the definition is not quite accurate, since the whole economy's structure is not known with certainty so as to define the parameters of the inflation target rule. Likewise, some flexibility must be allowed to address situations that are impossible to anticipate.

On the other hand, defining the monetary authority's objective in terms of an inflation target does not mean that the business cycle, particularly unemployment, is irrelevant to it. This is reflected in the fact that the target is not intended to be met always and under any circumstances, and is well reflected also in the fact that the target is established in the context of a policy horizon which generally is of one to two years.

This paper has used a minimum analytical model to clarify the points, but has omitted some relevant aspects of the monetary policy practice, although they should not change the conclusions of this paper. The economy is subject to much more shocks than merely inflationary ones. Credibility of the central bank has not been considered, but the decisions made by the central bank and the formulation of its objective reveal information about its ability to contain inflation, as well as its commitment with the target. Incorporating these aspects adds much more complexity, but generally, they will look in the direction of rigorously meeting the target, since gained credibility makes adjustments less costly. In terms of the model in section 3, credibility can be thought of as reducing inertia and the degree of indexation, thus enabling a faster return of inflation to the target range when deviations occur.

The need for gradualism has been justified by inflation persistence. Nevertheless, excessive activism, in the sense of having a very short policy horizon, can be viewed as leading to volatility of interest rates and asset prices that could provoke undesired instability in the financial system. In a more general dynamic stochastic model, one could conceive that the optimal monetary policy could have a variable policy horizon depending on the nature and magnitude of shocks. In practice, however, this may be solved with escape clauses that allow for deviations from the target in exceptional situations. This may happen, for example, when financial stability is threatened. In such case, extending the policy horizon to be able to better accommodate the potential financial risks may prove optimal.

The analysis has been presented in the context of a closed economy. Extension to an open economy, and interactions with the exchange rate should not change the main conclusions of this discussion, but they certainly add new sources of fluctuation caused by shocks in the international scenario. In any event, incorporating open-economy elements could provide an additional reason to have a medium term horizon. In case we have a very short horizon or a very narrow range, the principal mechanism of monetary policy passthrough to inflation would be the exchange rate rather than the effects via aggregate demand. This, in turn, could generate deviations of the exchange rate that might affect the external equilibrium of the economy, a very relevant aspect for emerging economies subject to strong fluctuations in external financing.

No distinction has been made here between core inflation, which is calculated for a subset of goods in the CPI, and actual CPI inflation. In general, inflation targets refer to actual-rather than core-inflation, although the latter tends to be more stable. While theory tends to prefer core inflation, there are reasons, not fully developed from an analytical standpoint, that explain why central banks prefer actual CPI (headline) inflation. In the first place, there is a problem of credibility and understanding by the public of the various alternatives for core inflation, as opposed to actual inflation. Secondly, isolating volatile products from the inflationary target, such as fuels and related products would reinforce second-round effects of a shock on such prices. In other words, if the central bank wishes to minimize second-round effects, it should be willing to respond with monetary policy to cost shocks, even if the response is not immediate. Accordingly, fixing the target in terms of headline inflation provides an anchor to all prices.

A policy of flexible inflation targets does consider full employment among its objectives. However, it is still preferable to organize monetary policy around a flexible inflation target. This permits to organize the monetary policy decision-making process, as well as communication of the commitment and meeting of the inflationary objective to the public. All this, in turn, strengthens monetary policy credibility, a crucial element to minimize the costs of attaining macroeconomic stability.

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Table 1: **Inflation Targets**

Country	Beginning (1)	Current Range (%) (2)	Target (3)	Current Horizon (quarters) (4)	Selects Target (5)
Australia	1993Q1	2-3	CPI	open	CB–G
Brazil	1999Q2	2.5-6.5	CPI	4	CB–G
Canada	1991Q1	1-3	CPI	6-8	CB–G
Chile	1999Q3	2-4	CPI	4 - 8	CB
Colombia	1999Q3	4.5-5.5	CPI	4	CB–G
Czech Republic	1998Q1	2-4	CPI	6-8	CB
Hungary	2001Q3	2.5-4.5	CPI	4	CB
Iceland	2001Q1	2.5	CPI	all times	CB–G
Israel	1997Q2	1-3	CPI	year end	G
Korea	1998Q2	2.5-3.5	Core CPI	4	CB–G
Mexico	2002Q1	2-4	CPI	4	CB
New Zealand	1990Q1	1 - 3	CPI	6-8	CB–G
Norway	2001Q1	2.5	Core CPI	8	G
Peru	2002Q1	1.5-3.5	CPI	4	CB
Philippines	2002Q1	5-6	CPI	4	CB–G
Poland	1999Q1	1.5-3.5	CPI	5-7	CB
South Africa	2000Q1	3-6	CPIX	4	CB
Sweden	1993Q1	1-3	CPI	4-8	CB
Switzerland	2000Q1	0-2	CPI	open	CB
Thailand	2000Q2	0-3.5	Core CPI	4	CB
United Kingdom	1992Q4	2	CPI	all times	G

Source: Columns (1) and (2) from Batini and Laxton (2005), (3) and (5) from Tuladhar (2005) and column (4) from individual country sources.

Notes: CB: central bank; G: government; CPI: consumer price index.

Table 2: **Implicit Parameters of the Inflation Target**

	Beginning	Current Range (%)	ρ	$\sigma_\epsilon^2 \times 1000$	x (%)	Current Horizon	T $\pm.1$ (months)	T $\pm.2$	T $\pm.3$
Australia	1993Q1	2-3	0.84	0.09	22.4	–	60	48	41
Australia*	1994Q3	2-3	0.84	0.09	22.4	–	60	48	41
Brazil	1999Q2	2.5-6.5	0.90	0.41	33.3	12	128	108	97
Canada	1991Q1	1-3	0.77	0.04	65.9	24	35	27	22
Canada*	1995Q1	1-3	0.66	0.04	77.7	24	20	15	12
Chile	1999Q3	2-4	0.68	0.12	61.3	12-24	42	33	28
Chile*	2001Q1	2-4	0.82	0.05	59.4	12-24	47	37	31
Colombia	1999Q3	4.5-5.5	0.73	0.19	19.5	12	36	29	25
Czech Rep.	1998Q1	2-4	0.76	0.11	46.1	24	38	30	26
Hungary	2001Q3	2.5-4.5	0.79	0.06	56.3	12	41	32	27
Iceland	2001Q1	2.5	0.87	0.17	–	–	82	68	59
Iceland*	2003Q1	2.5	0.54	0.06	–	–	14	11	9
Israel	1997Q2	1-3	0.73	0.37	27.7	6	39	32	28
Korea	1998Q2	2.5-3.5	0.86	0.05	52.1	12	64	51	43
Korea*	1999Q1	2.5-3.5	0.79	0.04	63.6	12	40	31	26
Mexico	2002Q1	2-4	0.72	0.07	59.4	12	29	23	19
New Zealand	1990Q1	1 - 3	0.81	0.02	78.7	24	40	30	24
New Zealand*	1993Q1	1 - 3	0.89	0.04	53.3	24	88	70	59
Norway*	2001Q1	2.5	0.72	0.06	–	24	28	22	18
Peru*	2002Q1	1.5-3.5	0.52	0.11	58.3	12	15	12	10
Philippines	2002Q1	5-6	0.89	0.29	10.5	12	113	95	84
Poland	1999Q1	1.5-3.5	0.85	0.15	33.3	21	71	58	51
South Africa	2000Q1	3-6	0.85	0.31	34.8	12	78	65	57
South Africa*	2001Q1	3-6	0.85	0.31	34.8	12	78	65	57
Sweden	1993Q1	1-3	0.93	0.03	50.6	24	134	106	90
Sweden*	1995Q1	1-3	0.93	0.03	50.6	24	134	106	90
Switzerland*	2000Q1	0-2	0.60	0.01	97.9	–	12	8	6
United Kingdom*	1992Q4	2	0.81	0.02	–	–	40	30	24
Median			*0.80*	*0.10*	*48*	*12*	*40*	*31*	*26*

Source: Table 1, Mishkin and Schmidt-Hebbel (2005) and author's calculations.

Notes: x corresponds to the percentage of the time inflation is within the target given its distribution, and T the length of the horizon given the tolerance to deviations of the inflation forecast, where $\pm.\alpha$ indicates the range in which inflation forecast should be 90% of the times. * indicates stable inflation target. Mexico and Poland are adopted recently stable inflation targets (Mishkin and Schmidtt-Hebbel, 2005), but do not have enough data for estimation. The median is computed with the first line for each country. Thailand is excluded since the estimated ρ is greater than one.

Table 3: **Persistence of Inflation**

	This Paper	Levin and Piger (2004)		
		No Break	Classical	Bayesian
Australia	0.84	0.82	0.34	0.53
Canada	0.68	0.72	-0.04	0.34
New Zealand	0.81	0.99	0.50	0.57
Sweden	0.93	0.94	0.43	0.35
Switzerland	0.60	0.95	0.93	0.74
United Kingdom	0.81	0.72	0.55	0.53

Source: Table 2 and Levine and Piger (2004).

Notes: The values reported from Levine and Piger (2004) corresponds to the 50th percentile.

Appendix-A Central Bank with Infinite Horizon

This appendix extends the results presented in section 4 by incorporating an infinite horizon in the monetary authorities loss function.

$$\Omega_t = \sum_{i=0}^{\infty} \beta^i \mathrm{E}_{t+i}\Big\{a(y_{t+i}-\overline{y})^2 + (\pi_{t+i}-\pi^*)^2\Big\} \tag{A.1}$$

$$y_t - \overline{y} = \theta\big[\pi_t - (1-\alpha)\mathrm{E}_{t-1}\pi_t - \alpha\pi_{t-1} - \nu_t\big] \tag{A.2}$$

The monetary authority minimizes (A.1) subject to (A.2):

$$\min_{\pi_t} \quad \Omega_t \quad \text{s.a. Phillips} \tag{A.3}$$

Expanding the objective function for term in π_t we have that:

$$\begin{aligned}\Omega_t = \mathrm{E}_t\Bigg\{ & a(\theta\big[\pi_t - (1-\alpha)\mathrm{E}_{t-1}\pi_t - \alpha\pi_{t-1} - \nu_t\big])^2 + (\pi_t - \pi^*)^2 \ldots \\ & +\beta\mathrm{E}_{t+1}\Big\{a(\theta\big[\pi_{t+1} - (1-\alpha)\mathrm{E}_t\pi_{t+1} - \alpha\pi_t - \nu_{t+1}\big])^2 + (\pi_{t+1}-\pi^*)^2\Big\} + \ldots \Bigg\},\end{aligned}$$

and the first order condition is:

$$\frac{\partial L}{\partial \pi_t} = a\theta^2\Big[\pi_t - (1-\alpha)\mathrm{E}_{t-1}\pi_t - \alpha\pi_{t-1} - \nu_t - \alpha\beta\big(\mathrm{E}_t\pi_{t+1} - \alpha\pi_t\big)\Big] + (\pi_t - \pi^*) = 0, \tag{A.4}$$

which simplifies to:

$$\pi_t-\pi^* = \frac{a\theta^2}{a\theta^2(1+\alpha^2\beta)+1}\Big[(1-\alpha)\mathrm{E}_{t-1}(\pi_t-\pi^*)+\alpha(\pi_{t-1}-\pi^*)+\alpha^2\beta\mathrm{E}_t(\pi_{t+1}-\pi^*)+\nu_t\Big] \tag{A.5}$$

Note that if $\beta = 0$ as in the finite horizon case, equation (A.5) simplifies to equation (8).

Taking expectations as of $t-1$ we have:

$$\begin{aligned}
\mathrm{E}_{t-1}(\pi_t-\pi^*) &= \frac{a\theta^2}{a\theta^2(1+\alpha^2\beta)+1}\Big[(1-\alpha)\mathrm{E}_{t-1}(\pi_t-\pi^*)+\alpha(\pi_{t-1}-\pi^*)+\alpha^2\beta\mathrm{E}_{t-1}(\pi_{t+1}-\pi^*)\Big] \\
\mathrm{E}_{t-1}(\pi_t-\pi^*) &= \frac{a\theta^2\alpha}{a\theta^2\alpha(1+\alpha\beta)+1}\Big[(\pi_{t-1}-\pi^*)+\alpha\beta\mathrm{E}_{t-1}(\pi_{t+1}-\pi^*)\Big] \qquad \text{(A.6)}
\end{aligned}$$

We can use the method of undetermined coefficients to solve for the following solution, which presumes to be a first order autorregressive process of order 1:

$$\mathrm{E}_{t-1}(\pi_t-\pi^*) = \varrho(\pi_{t-1}-\pi^*) \tag{A.7}$$

Where the autocorrelation coefficient depends on $\{\alpha, \beta, \theta, a\}$.

Substituting this AR(1) process into the first order condition (A.6) yields the following equation for the solution for ϱ:

$$a\alpha^2\beta\theta^2\varrho^2 - [a\theta^2\alpha(1+\alpha\beta)+1]\varrho + a\alpha\theta^2 = 0. \tag{A.8}$$

The solutions for ϱ of this equation, denoted by r_1 and r_2, are:

$$r_{1,2} = \frac{-\phi_2 \pm \sqrt{\phi_2^2 - 4\phi_1\phi_3}}{2\phi_3} \tag{A.9}$$

With $\phi_1 = \frac{a\theta^2\alpha}{a\theta^2\alpha(1+\alpha\beta)+1}$, $\phi_2 = -1$ and $\phi_3 = \phi_1\alpha\beta$

For the roots of equation (A.9) to be real and distinct, $\phi_2^2 - 4\phi_1\phi_3 > 0$.

Result 1:

$$\phi_2^2 - 4\phi_1\phi_3 > 0$$

Proof:

$$
\begin{aligned}
0 &< \phi_2^2 - 4\phi_1\phi_3 \\
&= 1 - 4\phi_1^2\alpha\beta \\
&= 1 - 4\left[\frac{a\theta^2\alpha}{a\theta^2\alpha(1+\alpha\beta)+1}\right]^2 \alpha\beta \\
&= \frac{\Big[a\theta^2\alpha(1+\alpha\beta)+1\Big]^2 - 4\big(a\theta^2\alpha\big)^2\alpha\beta}{\Big[a\theta^2\alpha(1+\alpha\beta)+1\Big]^2}
\end{aligned}
\tag{A.10}
$$

The denominator is positive since it is squared. Taking the numerator:

$$
\begin{aligned}
= \; & \Big[a\theta^2\alpha(1+\alpha\beta)\Big]^2 + 2a\theta^2\alpha(1+\alpha\beta) + 1 - 4(a\theta^2\alpha)\alpha\beta \\
& \Big[a\theta^2\alpha\Big]^2\Big[(\alpha\beta)^2 + 2\alpha\beta + 1\Big] + 2a\theta^2\alpha(1+\alpha\beta) + 1 - 4(a\theta^2\alpha)\alpha\beta \\
& \Big[a\theta^2\alpha\Big]^2(\alpha\beta)^2 + \cancel{2\Big[a\theta^2\alpha\Big]^2\alpha\beta} + \Big[a\theta^2\alpha\Big]^2 + 2a\theta^2\alpha(1+\alpha\beta) + 1 - \cancel{4}(a\theta^2\alpha)\alpha\beta \\
& \big(a\theta^2\alpha\big)^2\big((\alpha\beta)^2 - 2\alpha\beta + 1\big) + 2a\theta^2\alpha(1+\alpha\beta) + 1 \\
& \underbrace{\big(a\theta^2\alpha\big)^2\big(1-\alpha\beta\big)^2}_{+} + \underbrace{2a\theta^2\alpha(1+\alpha\beta)+1}_{+} > 0
\end{aligned}
$$

therefore, the roots of equation (A.9) are real and distinct. $\|$

With the previous results we can find the values of $r_{1,2}$.

Defining Φ as:

$$
\Phi = \phi_2^2 - 4\phi_1\phi_3 = \frac{\big(a\theta^2\alpha\big)^2\big(1-\alpha\beta\big)^2 + 2a\theta^2\alpha(1+\alpha\beta) + 1}{\Big(a\theta^2\alpha(1+\alpha\beta)+1\Big)^2}
\tag{A.11}
$$

Replacing in equation (A.9):

$$
\begin{aligned}
r_1 &= \frac{-\phi_2 + \sqrt{\phi_2^2 - 4\phi_1\phi_3}}{2\phi_3} \\
&= \frac{1+\sqrt{\Phi}}{2a\theta^2\alpha^2\beta}\Big(a\theta^2\alpha(1+\alpha\beta)+1\Big) \\
&= \frac{a\theta^2\alpha(1+\alpha\beta) + 1 + \sqrt{\big(a\theta^2\alpha\big)^2\big(1-\alpha\beta\big)^2 + 2a\theta^2\alpha(1+\alpha\beta)+1}}{2a\theta^2\alpha^2\beta} \\
&= \frac{a\theta^2\alpha + a\theta^2\alpha^2\beta + 1 + \sqrt{+}}{2a\theta^2\alpha^2\beta}
\end{aligned}
$$

The following inequality holds given that α and β are smaller than 1:

$$\frac{a\theta^2\alpha + a\theta^2\alpha^2\beta + 1 + \sqrt{+}}{2a\theta^2\alpha^2\beta} > \frac{2a\theta^2\alpha^2\beta + 1 + \sqrt{+}}{2a\theta^2\alpha^2\beta} > 1$$

This shows that $r_1 > 1$, and hence we can rule out this explosive solution as not possible in a rational expectations equilibrium. We must verify that $r_2 < 1$ and whether we must impose restrictions on the parameter space.

$$\begin{aligned} r_2 &= \frac{-\phi_2 - \sqrt{\phi_2^2 - 4\phi_1\phi_3}}{2\phi_3} \\ &= \frac{1-\sqrt{\Phi}}{2a\theta^2\alpha^2\beta}\Big(a\theta^2\alpha(1+\alpha\beta)+1\Big) \end{aligned}$$

$$r_2 = \frac{a\theta^2\alpha(1+\alpha\beta) + 1 - \sqrt{\left(a\theta^2\alpha\right)^2\left(1-\alpha\beta\right)^2 + 2a\theta^2\alpha(1+\alpha\beta)+1}}{2a\theta^2\alpha^2\beta} < 1 \qquad \text{(A.12)}$$

$$\begin{aligned} a\theta^2\alpha(1+\alpha\beta) + 1 - \sqrt{\left(a\theta^2\alpha\right)^2\left(1-\alpha\beta\right)^2 + 2a\theta^2\alpha(1+\alpha\beta)+1} &< 2a\theta^2\alpha^2\beta \\ \sqrt{\left(a\theta^2\alpha\right)^2\left(1-\alpha\beta\right)^2 + 2a\theta^2\alpha(1+\alpha\beta)+1} &> a\theta^2\alpha(1+\alpha\beta)+1-2a\theta^2\alpha^2\beta \\ \sqrt{\left(a\theta^2\alpha\right)^2\left(1-\alpha\beta\right)^2 + 2a\theta^2\alpha(1+\alpha\beta)+1} &> a\theta^2\alpha(1-\alpha\beta)+1 \\ \Big(a\theta^2\alpha\big(1-\alpha\beta\big)\Big)^2 + 2a\theta^2\alpha(1+\alpha\beta)+1 &> \Big(a\theta^2\alpha(1-\alpha\beta)+1\Big)^2 \\ 2a\theta^2\alpha(1+\alpha\beta)+1 &> 2a\theta^2\alpha(1-\alpha\beta)+1 \\ 1+\alpha\beta &> 1-\alpha\beta \end{aligned}$$

Which is true for all $\alpha,\ \beta \in [0,1] \Rightarrow r_2 < 1$.

Therefore the solution is:

$$\mathrm{E}_{t-1}(\pi_{t-1}-\pi^*) = r_2(\pi_{t-1}-\pi^*) \qquad \text{(A.13)}$$

where:

$$\varrho = r_2, \qquad \text{(A.14)}$$

given by (A.12).

Finally, to characterize the process of inflation we need to find the shock to the AR(1) process. Using (A.12) in equation (A.5) we find equilibrium inflation after a bit of algebra:

$$\pi_t - \pi^* = \frac{a\theta^2}{a\theta^2(1+\alpha^2\beta)+1}\left[(1-\alpha)\mathrm{E}_{t-1}(\pi_t - \pi^*) + \alpha(\pi_{t-1}-\pi^*) + \alpha^2\beta\mathrm{E}_t(\pi_{t+1}-\pi^*) + \nu_t\right]$$

$$\pi_t - \pi^* = \frac{a\theta^2}{a\theta^2(1+\alpha^2\beta)+1}\left[(1-\alpha)(\pi_{t-1} - \pi^*)r_2 + \alpha(\pi_{t-1}-\pi^*) + \alpha^2\beta(\pi_t-\pi^*)r_2 + \nu_t\right],$$

which yields:

$$\pi_t - \pi^* = \underbrace{\frac{a\theta^2\{(1-\alpha)r_2+\alpha\}}{a\theta^2\alpha(1+\alpha\beta(1-r_2))+1}}_{\varrho}(\pi_{t-1}-\pi^*) + \underbrace{\frac{a\theta^2}{a\theta^2\alpha(1+\alpha\beta(1-r_2))+1}}_{\varepsilon_t}\nu_t. \tag{A.15}$$

Using the fact the $\varrho = r_2$, we have that

$$\pi_t - \pi^* = \varrho(\pi_{t-1} - \pi^*) + \varepsilon, \tag{A.16}$$

where

$$\varrho = \frac{a\theta^2\alpha(1+\alpha\beta)+1-\sqrt{(a\theta^2\alpha)^2(1-\alpha\beta)^2+2a\theta^2\alpha(1+\alpha\beta)+1}}{2a\theta^2\alpha^2\beta}, \tag{A.17}$$

and

$$\varepsilon_t = \frac{a\theta^2}{a\theta^2\alpha(1+\alpha\beta(1-r_2))+1}\nu_t. \tag{A.18}$$

Equation (A.16) is very similar to (9), and it can be easily shown that the later is a particular case for $\beta = 0$. Given the AR(1) nature of the process, the sacrifice ratio is again independent of the persistence parameter, although this is not the proper welfare measure in this model since it would require discounting, and the evaluation of the inflationary losses.

Result 2: $\partial\varrho/\partial a$, $\partial\varrho/\partial\theta$, and $\partial\varrho/\partial\alpha$, are all positive, as it was the case of the text for $\beta = 0$.

Proof: ϱ can be written as:

$$\varrho = \frac{1+\alpha\beta}{2\alpha\beta} + \frac{1}{2a\theta^2\alpha^2\beta} - \frac{1}{2\alpha\beta}\sqrt{(1-\alpha\beta)^2 + \frac{2(1+\alpha\beta)}{a\theta^2\alpha} + \frac{1}{(a\theta^2\alpha)^2}}.$$

Differentiating with respect to a we have that:

$$\frac{\partial\rho}{\partial a} = -\frac{1}{2a^2\theta^2\alpha^2\beta} - \frac{1}{4\alpha\beta}(\psi)^{-\frac{1}{2}}\left(-\frac{2(1+\alpha\beta)}{a^2\theta^2\alpha} - \frac{2}{a^3(\theta^2\alpha)^2}\right),$$

where

$$\psi = (1-\alpha\beta)^2 + \frac{2(1+\alpha\beta)}{a\theta^2\alpha} + \frac{1}{(a\theta^2\alpha)^2} = \frac{1 + 2a\theta^2\alpha(1+\alpha\beta) + (a\theta^2\alpha)^2(1-\alpha\beta)^2}{(a\theta^2\alpha)^2},$$

collecting terms,

$$\begin{aligned}\frac{\partial\rho}{\partial a} &= \frac{1}{2a^2\theta^2\alpha^2\beta}\left[\psi^{-\frac{1}{2}}\left(1+\alpha\beta+\frac{1}{a\theta^2\alpha}\right)-1\right] \\ &= \frac{1}{2a^2\theta^2\alpha^2\beta}\left[\psi^{-\frac{1}{2}}\frac{(1+\alpha\beta)a\theta^2\alpha+1}{a\theta^2\alpha}-1\right],\end{aligned}$$

because ψ has a $1-\alpha\beta$, it is easy to see that the first term in the square bracket is greater than one, and hence we have proven that $\partial\varrho/\partial a$ is greater than zero for all positive values of a and θ, and for all α and β between zero and one.

For θ we have that:

$$\frac{\partial\rho}{\partial\theta} = -\frac{2}{2a\theta^3\alpha^2\beta} - \frac{1}{4\alpha\beta}(\psi)^{-\frac{1}{2}}\left(-\frac{4(1+\alpha\beta)}{a\theta^3\alpha} - \frac{4}{\theta^5(a\alpha)^2}\right),$$

and collecting terms we arrive at:

$$\frac{\partial\rho}{\partial\theta} = \frac{1}{a\theta^3\alpha^2\beta}\left[\psi^{-\frac{1}{2}}\left(1+\alpha\beta+\frac{1}{a\theta^2\alpha}\right)-1\right],$$

which, for the same reasons as in the case of a, is greater than zero for all positive values of a and θ, and for all α and β between zero and one.

Finally for α, we have that:

$$\frac{\partial\rho}{\partial\alpha} = -\frac{1}{2\beta\alpha^2} - \frac{1}{a\theta^2\beta\alpha^3} + \frac{1}{2\alpha\beta}(\psi^{-\frac{1}{2}})\left((1-\alpha\beta)\beta + \frac{1}{a(\theta\alpha)^2} + \frac{1}{(a\theta^2)^2\alpha^3}\right) + \frac{1}{2\beta\alpha^2}(\psi)^{-\frac{1}{2}}$$

Collecting terms,

$$\frac{\partial\rho}{\partial\alpha} = \frac{1}{2\beta\alpha^2}(\psi^{\frac{1}{2}}-1) + \frac{1}{\alpha^3\beta\ a\theta^2}\left(\frac{\psi^{-\frac{1}{2}}}{2}\left(1+(1-\alpha\beta)\alpha^2\beta+\frac{1}{a\theta^2\alpha}\right)-1\right).$$

This expression cannot be signed, but calibrating for reasonable parameters of β, α, and θ, simulations show that the derivative is also positive. $\|$

Using L'Hopital it is easy to show that ϱ goes to ρ defines in (10) when β goes to zero.

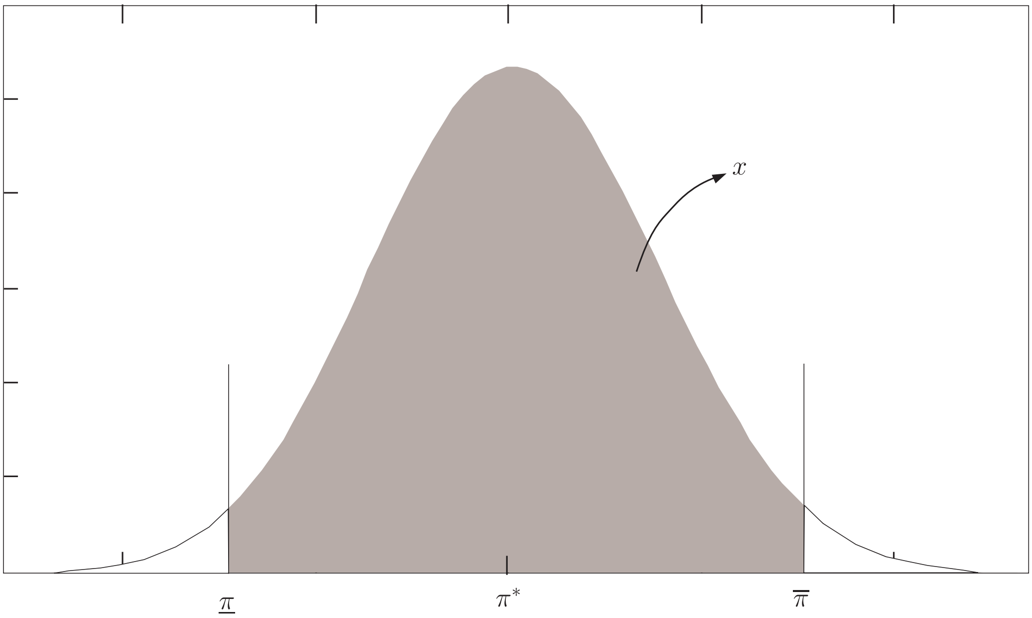

Figure 1: Distribution of Inflation

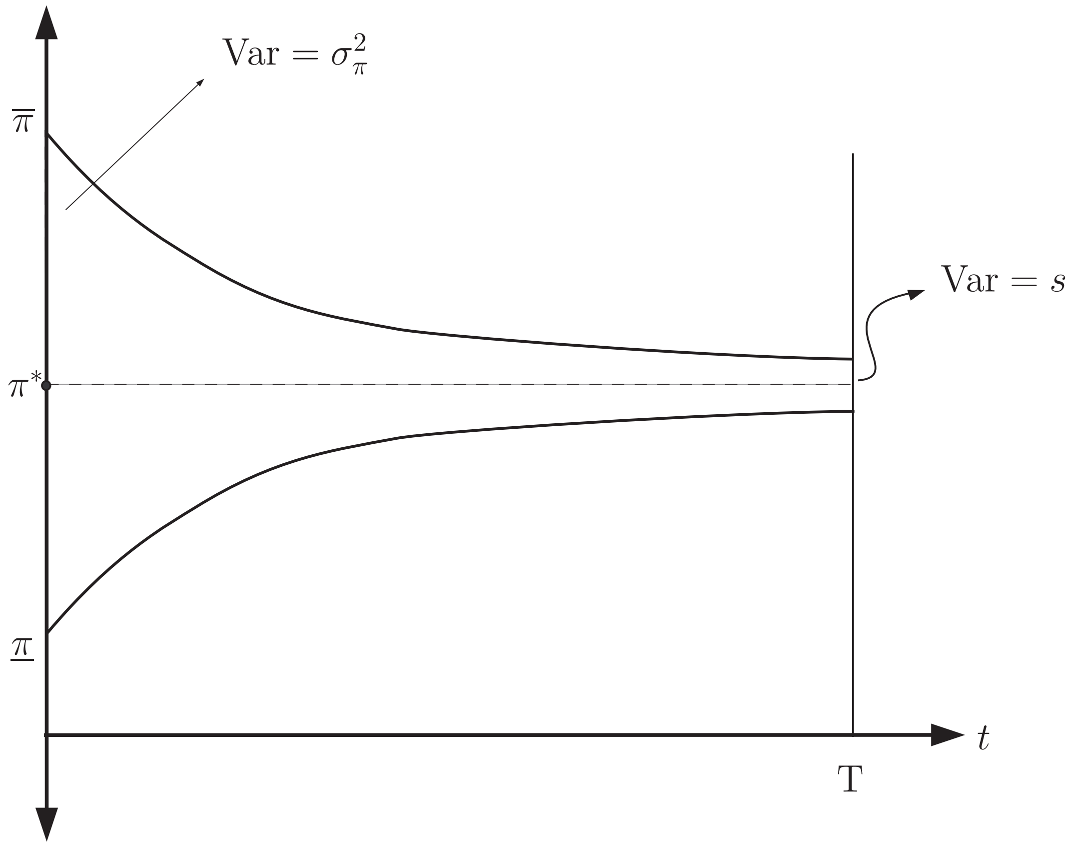

Figure 2: Inflation Target and Policy Horizon