Abstract

Providing unemployment insurance is particularly problematic in countries with high informality because workers can claim unemployment benefits and work in the informal sector at the same time. This paper shows that the implementation of an unemployment insurance savings account (UISA) scheme can be very successful in this context. In particular, about 50 percent of a new born welfare gains between the benchmark economy and the full information economy can be delivered with a simple UISA scheme. In addition, that UISA scheme would decrease dramatically the size of the informal sector. The last result, however, depends heavily on the government ability to control households savings.

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1 Introduction

In many developing countries, a large share of labor market relationships cannot be monitored by governments. The informal sector in these countries produces between 25 to 76 percent of gross domestic product (GDP) (Schneider and Enste, 2000). This feature of labor markets presents a challenge for the provision of unemployment insurance (UI) in the region. For instance, if individuals in Mexico could claim unemployment benefits whenever they are not formally working, more than half of the labor force would be qualified for UI. Since receiving UI would be compatible with working in the informal sector, this sector would be even more attractive and the government capacity to oversee labor markets would become even more impaired.

The goal of this paper is to study the design of optimal UI in economies with high informality. Besides informality, the model considered here contains several key features. First, individuals income in formal and informal labor markets increase during their life cycle. This feature is key to quantify the role played by savings. Second, individuals must exert effort to find and keep formal jobs, and that effort can not be monitored by the government. Third, some individuals are more prone to work in the informal sector because their cost of searching for a effort is higher.

The model is used to study the optimal UI scheme among the class of UISA parameterized by a replacement rate, which determines UI payments, an initial contribution to the saving account, a minimum level of savings at which the payment is suspended, and a maximum level of savings at which the contributions are suspended. The quantitative results are derived from an economy calibrated to Mexico, with labor income taxes and severance payments that resembles that country.

The optimal UISA provides large welfare gains compared to the benchmark economy, of the order of 12 percent in terms of consumption equivalent units. Those gains are about 50 percent of the gains that would be obtained if the government would be given complete control of all the individuals actions (full information). The optimal UISA scheme also produces a drastic reduction of the size of the informal sector.
This generates an increase in revenues collected by the government, and allows the government to reduce labor income taxes from 27 to 23 percent.

The reduction in the size of the informal sector under the optimal UISA is mainly caused by the fact that young individuals start their lives with an stock of savings in their account that they can use, only partially, every time they are unemployed. The partial insurance component makes them search hard. They exert effort because that is the only way of increasing future consumption.\footnote{This is the same mechanism in Hopenhayn and Nicolini (1997).} The fact that they receive some payments during unemployment allows them to search for a formal job, instead of taking an informal job.\footnote{This resembles the mechanism in Alvarez-Parra and Sanchez (2009).}

The analysis of the mechanism suggest that the results may depend on the government ability to control the individuals’ savings. To analyze that possibility, the optimal UISA is recomputed under the assumption that individuals can keep aside part of their salary or unemployment payments in the form of hidden savings. The optimal scheme in this case has a much higher replacement ratio, a much lower initial savings. Under this system, there are also large welfare gains compared to the benchmark economy, but the reduction of the informal sector is much less significant.

\subsection*{1.1 Related literature}

Hopenhayn and Nicolini (1997) answer the following question: What is the optimal design of unemployment insurance if search effort cannot be monitored? They study the repeated moral hazard: “The optimal long-term scheme involves a replacement ratio that decreases throughout the unemployment spell and a wage tax after reemployment that (...) increases with the length of the unemployment spell.”

One of the important assumptions in the work of Hopenhayn and Nicolini is that employment is an observable state. In particular, this assumption implies that workers cannot claim UI benefits and, at the same time, work in the informal sector. However,
recent research has focused on studying the optimal UI if there is an informal sector that allows unemployed individuals to secretly work in the informal sector and, simultaneously, ask for unemployment benefits. In particular, Alvarez-Parra and Sanchez (2009) argue that the existence of a hidden labor market modifies the optimal UI in a nontrivial way. The optimal contract has two phases: (i) unemployment benefits decrease very slowly to encourage workers to search for a job instead of working in the informal sector, and (ii) after several months of unemployment, benefits decline abruptly to zero. Since the optimal design involves payments and taxes that depend on the history of workers’ labor market decisions and earnings, an unemployment insurance saving account (UISA) seems a reasonable scheme to implement such a contract.

The analysis in this paper will focus on incentives and will not consider general equilibrium effects. In particular, the labor demand side will be omitted to simplify the analysis. In a recent study, D’Erasmo and Moscoso-Boedo (2012) propose a firm dynamics model with formal and informal sectors. Their model predicts that countries with high costs of formality are characterized by low allocative efficiency and large output shares produced by low-productivity firms in the informal sector. The design of UI has not been studied in that framework.

2 The Environment

The economy is populated by a large number of ex-ante identical individuals with names in the unit interval. Each period, a new identical generation is born while the population growth is constant at $\rho \geq 0$. Time is discrete and each of these households has the following lifetime profile.

The first $N$ periods are working periods in which agents can participate in labor markets and work. When an individual reaches age $N + 1$, he retires from the labor market. Once retired, individuals survive to the next period with probability $\rho$.

Workers will be ex-post heterogenous. At age $n = 1$, before the worker enters to the labor market, he privately observes a preference shock, $\theta$, that determines the disutility
of search effort, where $\theta \in \{\overline{\theta}, \underline{\theta}\}$ with probabilities $g(\theta)$ and $g(\overline{\theta})$. This margin of ex-post heterogeneity is key for the analysis herein, and it is discussed in more detail below.

In each period and given $\theta$, the worker’s utility depends on consumption $c$ and the corresponding effort level $e$, according to the utility function $u(c) - \theta e$. Lifetime expected utility for an individual with preference shock $\theta$ is represented by

$$E \left( \sum_{t=1}^{\infty} \beta^t [u(c_t) - \theta e_t] \right)$$

where $\beta \in (0,1)$ is the discount factor and $E$ is the expectation operator.

**Labor Market Decisions**

We define the labor market decisions faced by an individual during his working periods. An individual of working age $n = 1, ..., N$ can either work in the formal sector, work in the informal sector, or be unemployed. The employment decisions in those three different states are the following.

First, consider a worker who enters his working period $n$ with an offer in the formal sector. The worker’s wage offered, denoted $\omega_n$, is his productivity in the formal sector. As the offer is accepted, a worker who exerts effort $e$ keeps this job in the formal sector next period with probability $q_f(e)$.

Second, consider a worker who enters his working period $n$ with an offer in the informal sector. The worker’s wage will equal his productivity in the informal sector, $\omega_n < \omega_n$. If accepted this offer, he decides how much effort $e$ to exert to receive an offer in the formal sector next period, with probability $q_i(e)$. Importantly, informal job offers are not only low productivity jobs but also unobservable to third parties.

Finally, consider a worker who enters his working period $n$ as unemployed; i.e., he does not receive either a formal or an informal job offer. The worker decides how much effort $e$ to exert to receive an offer in the formal sector next period, with probability $q_u(e)$.

We assume that a worker can receive offers in both sectors at the same time. The
technology to find a job in the formal sector satisfy \( q_f(e) > q_u(e) > q_i(e) \) for all \( e \). So effort is most productive to find a formal job if the worker is already in this sector. On the hand, working in the informal sector makes effort less productive than being unemployed.

The probabilities for a worker of age \( n \) to find a job in the informal sector are all \textit{exogenous} but conditional upon the current employment status. The conditional probability of having an offer in the informal sector next period if the worker has worked informally during the current period is \( p_{i,n} \); that is, \( 1 - p_{i,n} \) is the informal separation rate. The conditional probability of having an offer in the informal sector next period if the worker has been unemployed in the current period is \( p_{u,n} \). Finally, \( p_{f,n} \) is the conditional probability of having an offer in the informal sector next period if the worker has worked in the formal sector during the current period. We assume that \( p_{i,n} > p_{u,n} > p_{f,n} \) for all \( n \); i.e. informality is persistent as informal offers tomorrow are most likely if working informally today.

An active worker’s employment status is denoted by \( \{f, i, u\} \), which denote formal, informal, and unemployed states, respectively.

\section*{Financial Markets}

Both agents and the government have access to financial markets. Agents must also undertake consumption-savings decisions. That is, an agent must allocate his resources (which will include financial income, as detailed below) between consumption and savings. The individual can save at the gross interest rate \( R \) and, once retired (i.e., age \( n \geq N + 1 \)), this is the only decision he must make.

\section{The full-insurance economy}

We first study the key features of an economy in which there is no friction, i.e. both effort and informal jobs opportunities are observable, and so full-insurance is attainable. To
characterize the optimal allocation we consider the problem of a planner who allocates consumption and effort while it has access to credit markets at the gross interest rate \(R = \beta^{-1}\). The idea is to solve the problem backwards. So we first consider the problem faced by retired workers and then the problem faced by the representative worker faced at date 1 before his type \(\theta\) has realized.

**The Retired Worker’s Problem**

Consider a worker who retires at age \(n \geq N + 1\). Suppose that this individual reaches this period with \(m\) asset holdings and denote \(H(m)\) as his expected lifetime utility. As retired agents survive with probability \(\rho\), \(H\) must solve

\[
H(m) = \max_{m' \geq 0} \left[ u(mR - m') + \beta \rho H(m') \right]
\]

where \(m'\) denote next period asset holdings. Notice that retired agents do not exert any effort; i.e. \(e_t = 0\) for all \(t \geq N + 1\).

In our setting with CRRA, as the budget constraint is homogeneous of degree one in this case then \(H\) has closed form solution; i.e. \(H(w) = A(m)^{1-\sigma}\).

**The Ex-Ante Representative Worker’s Problem**

As the worker’s problem is time-consistent, we take \(H\) as given. Let \(s_n \in f, i, b, u\) be the worker’s job opportunity where \((f, i, b, u)\) denote to have opportunities that formal, informal, both and unemployment (none), respectively. Let \(s^n\) denote a partial history up to age \(n\) and \(\pi(s^n)\) is the corresponding (endogenous) probability.

\[
\max_{((\tau_n, e_n)_{n=1}^N, m)} \int_{\theta} \sum_{s_n} \beta^{n-1} \pi(s^n) [u(w_{\theta,n}(s^n) + \tau_n(\theta, s^n)) - \theta e_n(\theta, s^n)] g(\theta) d\theta
\]

\[
+ \int_{\theta} \sum_{s^N} \beta^N \pi(s^N) H(m(\theta, s^N)),
\]
subject to

\[
\int_{\theta} \sum_{n=1}^{N} \sum_{s^n} (1 + x)^{-(n-1)} \pi(s^n) \tau_n(\theta, s^n) g(\theta) \, d\theta \\
+ \int_{\theta} \sum_{s^N} (1 + x)^{-N} \pi(s^N) m(\theta, s^N) g(\theta) \, d\theta = m_0 - S_{III}.
\]

where $S_{III}$ is defined below.

That is, in this stationary environment, the fictitious planner can manipulate all the cross-sectional resources to maximize the ex-ante utility of the representative worker. In what follows we assume that $R = \beta^{-1} = (1 + x)$. More about this assumption below.

**Lemma 1** In this setting with full insurance, first order conditions with respect to consumption and period $N + 1$ asset holdings imply that $c_n(\theta, s^n) = c^*$ and $m(\theta, s^N) = m^*$ for all $n = 1, ..., N$, all $s^n$ and all $\theta$.

If we let $\lambda$ be the Lagrange multiplier corresponding to the constraint (1), then it follows that $\lambda = u'(c^*) = H'(m^*)$. This fact makes the planner’s problem (1) simplify as follows.

**Lemma 2** Problem (1) reduces to solve

\[
\max_{(\tau_n, w_n, e_n)_{n=1}^{N}} \int_{\theta} \sum_{n=1}^{N} \sum_{s^n} R^{-(n-1)} \pi(s^n) [\lambda w_n(s^n) - \theta e_n(s^n)] g(\theta) \, g(\theta) \\
+ \lambda R^{-N} m^* - \lambda \frac{1 - R^{-N+1}}{1 - R^{-1}} c^* + \frac{1 - R^{-N+1}}{1 - R^{-1}} u(c^*) + R^{-N} H(m^*).
\]

That is, it reduces to maximize the expected discounted flow of income net of the cost of effort, in utility units.

This problem becomes non-standard as the planner must decide whether the worker should make use of the job opportunity. Notice that when the opportunity is a formal job, either alone or jointly with an informal opportunity, the optimal choice is to make
the worker work since both the worker’s productivity (his “wage”) is high and also the technology to find a high productivity, formal job opportunity next period is best.3

The difficulty here is that it is not obvious what the optimal choice is when only a low productivity, informal job opportunity arrives. This is key as it will determine the optimal size of the informal sector. To tackle this we solve the problem backwards. Consider the last period $N$ when an informal opportunity is the only option. As the worker need not do any effort as he will be retired next period, the optimal choice is to make the individual work in the informal sector as there is some income. A similar logic follows for a high productivity, formal job.

Consider now period $N-1$ and suppose first that a formal job opportunity is drawn, either alone or jointly with an informal offer. As mentioned, it is optimal to allocate the worker to this formal job. The value of that decision is defined

$$
\phi_{N-1}(f) = [\lambda w_{N-1,f} - \theta \tilde{e}_{N-1}(\theta, f)] \\
+ \beta q_f(\tilde{e}_{N-1}(\theta, f)) \lambda w_f + \beta(1 - q_i(\tilde{e}_{N-1}(\theta, f))) p_i \lambda w_i
$$

where $\tilde{e}_{N-1}(\theta, f)$ is uniquely determined by

$$
\theta = \beta q_f(\tilde{e}_{N-1}(\theta, f)) \lambda (w_f - p_i w_i).
$$

If the worker is allocated to work in the informal sector, the value of that decision is

$$
\phi_{N-1}(i^a) = [\lambda w_i - \theta \tilde{e}_{N-1}(\theta, i^a)] \\
+ \beta q_i(\tilde{e}_{N-1}(\theta, i^a)) \lambda w_f + \beta(1 - q_i(\tilde{e}_{N-1}(\theta, i^a))) p_i \lambda w_i
$$

3More precisely, suppose that a worker faces both job opportunities, formal and informal. Optimality makes necessary to compare allocate the worker to the formal job compared to be allocated in the informal sector. This needs to compute optimal effort levels. Take the optimal effort level under the alternative of working in the informal sector. Now, evaluate the payoff of working in the formal sector using that effort level. Since in the formal job both the productivity is higher and the technology to find higher productivity job is better, the payoff of choosing for the formal job is higher; i.e. the planner would never allocate a worker to an informal job if the he has the chance to be allocated in the formal sector.
where $\tilde{e}_{N-1}(\theta, \bar{r})$ is uniquely determined by

$$\theta = \beta q'_i(\tilde{e}_{N-1}(\theta, \bar{r})) \lambda (w_f - p_i w_i).$$

Alternatively, if the worker is not making use of that job opportunity, he remains unemployed and its value is

$$\phi_{N-1}(\bar{r}) = \max \left\{ \phi_{N-1}(\bar{r}), \phi_{N-1}(\bar{r}) \right\}$$

where $\tilde{e}_{N-1}(\theta, r)$ is uniquely determined by

$$\theta = \beta q'_u(\tilde{e}_{N-1}(\theta, r)) \lambda (w_f - p_u w_u).$$

The optimal choice then satisfies

$$\phi_{N-1}(i) = \max[\phi_{N-1}(\bar{r}), \phi_{N-1}(\bar{r})]$$

and so $\tilde{e}_{N-1}(\theta, i)$ is corresponding level of effort.

Finally, notice that in the case that there is no job opportunity, the worker remains unemployed and the optimal values and an effort level satisfy

$$\phi_{N-1}(u) = \phi_{N-1}(\bar{r})$$

$$\tilde{e}_{N-1}(\theta, u) = \tilde{e}_{N-1}(\theta, \bar{r})$$

Consider now any working period $n \leq N - 1$ and notice that as we are solving this backwards, all the decisions regarding allocation of effort and jobs have been already made for any working period $t > n$. Let $\phi_{n+1}(s')$ be the next period net wealth for $s' \in \{f, i, u\}$.

If the worker receives a formal job opportunity (either alone or jointly with an informal job opportunity), the planner optimally allocates the worker to this high productivity job and therefore

$$\phi_{n}(f) = [\lambda w_f - \theta \tilde{e}_{n}(\theta, f)] + \beta q_f(\tilde{e}_{n}(\theta, f)) \phi_{n+1}(f)$$

$$+ [1 - q_f(\tilde{e}_{n}(\theta, f))] [p_f \phi_{n+1}(i) + (1 - p_f) \phi_{n+1}(u)]$$
where $\tilde{e}_n(\theta, f)$ is uniquely determined by

$$\theta = \beta q'_f(\tilde{e}_n(\theta, f)) \lambda (\phi_{n+1}(f) - (p_f \phi_{n+1}(i) + (1 - p_f)\phi_{n+1}(u))).$$

Suppose now that worker receives an informal job opportunity. The planner, again, must first decide whether make the work take the job or not. Therefore, if the worker is allocated to this low productivity, informal job the value is

$$\phi_n(\tilde{r}) = [\lambda w_i - \theta \tilde{e}_n(\theta, \tilde{r})] + \beta q_i(\tilde{e}_n(\theta, \tilde{r}))\phi_{n+1}(f) + [1 - q_i(\tilde{e}_n(\theta, \tilde{r}))][p_i\phi_{n+1}(i) + (1 - p_i)\phi_{n+1}(u)]$$

where $\tilde{e}_n(\theta, \tilde{r})$ is uniquely determined by

$$\theta = \beta q'_i(\tilde{e}_n(\theta, \tilde{r})) \lambda (\phi_{n+1}(f) - (p_i \phi_{n+1}(i) + (1 - p_i)\phi_{n+1}(u))).$$

Alternatively, if the worker is not allocated to the informal job, the worker stays unemployed and so both values coincide as before and satisfy

$$\phi_n(\tilde{r}) = \phi_n(u) = -\theta \tilde{e}_n(\theta, u) + \beta q_u(\tilde{e}_n(\theta, u))\phi_{n+1}(f) + [1 - q_u(\tilde{e}_n(\theta, u))][p_u\phi_{n+1}(i) + (1 - p_u)\phi_{n+1}(u)]$$

where $\tilde{e}_n(\theta, \tilde{r}) = \tilde{e}_n(\theta, u)$ and are uniquely determined by

$$\theta = \beta q'_u(\tilde{e}_n(\theta, u)) \lambda (\phi_{n+1}(f) - (p_u \phi_{n+1}(i) + (1 - p_u)\phi_{n+1}(u))).$$

As before, the optimal choice then satisfies

$$\phi_n(i) = \max\{\phi_n(\tilde{r}), \phi_n(\tilde{r})\}$$

and so $\tilde{e}_n(\theta, i)$ is corresponding optimal level of effort.

The analysis above allows to solve for the full-insurance allocation. Importantly, it makes transparent how to characterize the optimal size of the informal sector which, probably confronting some of the conventional wisdom, is not necessarily 0. This follows because optimality in this setting dictates that some workers might indeed be
allocated to work in the informal sector as the future costs of this low productivity job are not "sufficiently high" to overcome the instantaneous/current payoff.

In what follows, we study the impact of alternative systems of unemployment insurance. The key issue here is that governmental agencies must deal with two informational: both effort and job opportunities in the informal sector are unobservable.

4 The UI Economy

Here we describe the UI system that capture the some of the main features of the status quo in the economy under analysis. We assume that an individual who has been working in the formal sector at age \( n \), and losses his or her job at age \( n + 1 \), receives a severance payment as unemployment protection for one period, denoted by \( b_n = b \omega_{n-1} \). The replacement ratio in the benchmark economy will be referred to as \( b \). Otherwise, the worker receives nothing during his working periods.

Importantly, in what follows we assume that rejected formal job offers are observable by the government. We assume that the worker has no right to file up for severance payment in the case that he rejects a formal offer. Both assumptions coupled together permits to simplify the analysis significantly as there is no need to keep track of the history of employment in the formal sector. That is, if we define \( m \) as the amount of asset at the beginning of the period, this not only includes assets privately accumulated by the worker in the past but it also includes the severance payment in the case that he has lost a formal job and got the severance payment.

When the worker retires, the government provides a retirement payment of \( d \).

Now we describe the decision problem faced by a representative worker. We solve the worker’s problem backwards and so we begin by studying the problem of a retired agent.
4.1 Retired Workers

Consider an individual with age \( n \geq N + 1 \). A retired agent with \( m \) asset holdings has expected lifetime utility \( H(m) \). As retired agents receive \( d \) as retirement payments and survive with probability \( \rho \), \( H \) must solve

\[
H(m) = \max_{m' \geq 0} \left[ u(d + mR - m') + \beta \rho H(m') \right]
\]

where \( m' \) denote next period asset holdings. Notice that retired agents do not exert any effort; i.e. \( e_t = 0 \) for all \( t \geq N + 1 \).

4.2 Active Workers

At \( n = 1 \) and before entering the labor force, a worker receives an unobservable preference shock \( \theta \) that represents how much the worker dislikes the effort exerted to find a job in the formal sector in the next period. In what follows, agents are indexed with their corresponding \( \theta \).

Suppose that a worker with working age \( n < N \) receives an offer in the informal sector. The worker must decide whether to accept the offer (\( a \), accept) or not (\( r \), reject).

In this last case as this offers are unobservable, the government cannot distinguish if the worker indeed received an informal offer or if he received none. This makes possible to file for unemployment insurance while still working in the informal sector.

If this individual has \( m \) assets, his maximized lifetime utility, \( V_{i,n}^i(\theta, m) \), solves

\[
V_{i,n}^i(\theta, m) = \max \left\{ V_{i,a,n}^i(\theta, m), V_{i,r,n}^i(\theta, m) \right\}
\]

Here, \( V_{i,a,n}^i(\theta, m) \) denotes the value of accepting the informal job offer and it satisfies

\[
V_{i,a,n}^i(\theta, m) = \max_{e, m'} u(mR - m' + \omega_i) - \theta e + \\
\beta \left[ (1 - q_i(e)) \left( p_{i,n} V_{i+1,n}^i(\theta, m') + (1 - p_{i,n}) V_{i+1,n}^u(\theta, m') \right) + \\
q_i(e) \left( p_{i,n} V_{i+1,n}^b(\theta, m') + (1 - p_{i,n}) V_{i+1,n}^f(\theta, m') \right) \right]
\]

where the corresponding policy functions for effort levels and savings are given by \( e_{i,n}^i(\theta, m) \) and \( m_{i,n}^i(\theta, m) \), respectively.
The value of rejecting the informal job offer, $V_{n}^{i,r}$, satisfies

$$V_{n}^{i,r}(\theta, m) = \max_{e,m'} \{u(mR - m') - v(\theta e) + \\
\beta [ (1 - q_{u}(e))(p_{u,n}V_{n+1}^{i}(\theta, m') + (1 - p_{u,n})V_{n+1}^{u}(\theta, m')) + \\
q_{u}(e)(p_{u,n}V_{n+1}^{b}(\theta, m') + (1 - p_{u,n})V_{n+1}^{f}(\theta, m'))] \}$$

where the corresponding policy functions for effort levels and savings are given by $e_{n}^{i,f}(\theta, m)$ and $m_{n}^{i,r}(\theta, m)$, respectively.

Suppose, on the other hand, that the worker with working age $n < N$ receives an offer in the formal sector. If the worker enters the period with $m$ assets, his or her maximized lifetime utility, $V_{n}^{f}(\theta, m)$, must solve

$$V_{n}^{f}(\theta, m) = \max \{ V_{n}^{f,a}(\theta, m), V_{n}^{f,r}(\theta, m) \}$$

where $V_{n}^{f,a}(\theta, m)$ and $V_{n}^{f,r}(\theta, m)$ denote the value of accepting or rejecting the offer, respectively.

Here $V_{n}^{f,a}$ must solve

$$V_{n}^{f,a}(\theta, m) = \max_{e,m'} \{ u(mR - m' + \omega_{n}(1 - \tau)) - v(\theta e) + \\
\beta [ (1 - q_{f}(e))(p_{f,n}V_{n+1}^{f}(\theta, m' + b_{n}) + (1 - p_{f,n})V_{n+1}^{u}(\theta, m' + b_{n})) + \\
q_{f}(e)(p_{f,n}V_{n+1}^{b}(\theta, m') + (1 - p_{f,n})V_{n+1}^{f}(\theta, m'))] \}.$$  

Notice that the assumptions on observability are key here. If the worker accepts a formal offer, he can get unemployment benefits if he receives no formal offer next period, either no offer or unobservable informal. Notice that if he gets a formal offer next period, instead, there is no way he can get unemployment benefits. In particular, rejections are assumed to be observable and so the worker has no rights to receive those benefits.

The corresponding policy functions for effort levels and savings are given by $e = e_{n}^{f,a}(\theta, m)$ and $m' = m_{n}^{f,a}(\theta, m)$, respectively.

Observe that

$$V_{n}^{f,r}(\theta, m) = V_{n}^{i,r}(\theta, m),$$

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and evidently the corresponding policy functions coincide.

Suppose, now, that the worker with working age \( n < N \) receives offers in both the formal and informal sectors. If the worker enters the period with \( m \) assets, his or her maximized lifetime utility, \( V^b_n(\theta, m) \), must solve

\[
V^b_n(\theta, m) = \max \left\{ V^{f,a}_n(\theta, m), V^{i,a}_n(\theta, m), V^n_n(\theta, m) \right\}.
\]

Consider an unemployed worker (i.e., an individual with no offer), with working age \( n \) and savings \( m \) (which could include severance payments). Notice that

\[
V^n_n(\theta, m) = V^{f,r}_n(\theta, m) = V^{i,r}_n(\theta, m).
\]

It is important to highlight that, in both types of economies, the value of rejecting all the job offers available and receive none must necessarily coincide in equilibrium. The key difference is that in the first case the worker decides to be unemployed while in the second he is forced to be unemployed.

Finally, consider the decision’s problem faced by an agent at working age \( n = N \) (i.e., the period just before retirement). If employed in the formal sector, the worker solves

\[
V^f_N(m) = \max \left\{ V^{f,a}_N(m), V^{f,r}_N(m) \right\}
\]

where

\[
V^{f,a}_N(m) = \max_{m'} \left\{ u(\omega_N(1 - \tau) + mR - m') + \beta H(m') \right\},
\]

\[
V^{f,r}_N(m) = \max_{m'} \left\{ u(mR - m') + \beta H(m') \right\}.
\]

If employed in the informal sector, the worker solves

\[
V^i_N(m) = \max \left\{ V^{i,a}_N(m), V^{i,r}_N(m) \right\},
\]

where

\[
V^{i,a}_N(m) = \max_{m'} \left\{ u(\omega_N + mR - m') + \beta H(m') \right\},
\]

\[
V^{i,r}_N(m) = V^{f,r}_N(m) = \max_{m'} \left\{ u(mR - m') + \beta H(m') \right\}.
\]
Notice that during the last period of working age, the worker does not exert any effort to find a job in the formal sector as he will be retired in the next period. That is, the worker only decides how much to consume and save, and so \( \theta \) is immaterial.

5 The UISA Economy

This section evaluates policy reforms aimed at protecting unemployed workers. In particular, the analysis considers the implementation of alternative savings accounts systems in this framework.

The UISA system considered herein can be characterized by six parameters.

- A lower bound for active savings in the worker’s account, \( s \); that is, an agent that is not working in the formal sector can withdraw resources from his or her savings account only if the current savings balance is above \( s \).
- An upper bound for savings in the worker’s account, \( S \); that is, an agent working in the formal sector must contribute to his or her savings account if the current savings balance is below \( S \).
- A contribution made to the worker’s saving account during employment in the formal sector if the total savings balance is smaller than \( S, \psi \), as a proportion of his wage.
- A replacement ratio (on the age specific wage), \( b \).
- An initial transfer to the saving account made by the government, \( s_0 \).
- A general tax paid in the formal market, \( T \).

Funds accumulated by the government on behalf of the workers are invested at the gross interest rate \( R \). Here, \( \hat{\cdot} \) denotes functions and variables for the UISA economy.

We will consider two alternative settings in terms of the access to credit markets by the workers.

A large literature studied optimal long-term insurance contracts under moral hazard
assuming that agents cannot borrow or save.\textsuperscript{4} Rogerson (1985a) shows that preventing the agent from entering the asset market is critical - even in the presence of liquidity constraints - since in the optimal contract, the agent is actually willing to save. It can be shown that when the planner can perfectly control the agents’ asset holdings, and can contractually restrict their acquisition of additional assets and liabilities, the efficient allocation is the same as the one where the agents have no access to the credit market. In many situations, however, the planner cannot have perfect control over the agents’ wealth and consumption.\textsuperscript{5}

Below we analyze both cases to better understand to which extend hidden savings affect the design of the optimal UISA and its impact. First, we describe a setting in which agents can secretly save (but not borrow) at a given interest rate. Then, we discuss the particular case of perfect control of the agents’ asset holdings and consumption which reduces to restrict individual savings to zero.

5.1 UISA I: Private Access to Credit Markets - Hidden Savings

Here we analyze a UISA economy in which worker have access to unobservable savings. This is true for instance when there are hidden storage or investment opportunities as some forms of asset accumulation that are typically not observable by the government. We discuss how the access to credit markets can affect the impact of this system.

5.1.1 Active Workers

As before, the government will provide funds to the agents when they just enter the job market. Workers are still forced to deposit a fraction of their wages into a savings account. Agents can later withdraw from these accounts while not working in the formal market, as long as they have available funds. But now workers will also have


\textsuperscript{5}Indeed, perfect monitoring is common in most of the literature on optimal allocation with private information. Two notable exceptions are Cole and Kocherlakota (2001) and Abraham and Pavoni (2008).
access to private savings account in which their balances \( m \) cannot be observed by any kind of governmental agency. Importantly, this means that the design of the UISA cannot be contingent on asset holdings, \( m \). To simplify, we assume that private returns equal the returns in the UISA; i.e. \( R = \widetilde{R} \).

Consider a worker with working age \( n < N \). Suppose first that he receives an offer in the informal sector. His maximized lifetime utility, \( \widetilde{V}_i^j \), must solve

\[
\widetilde{V}_i^j(\theta, s, m) = \max \left\{ \tilde{V}_i^{ja}(\theta, s, m), \tilde{V}_i^{jr}(\theta, s, m) \right\}.
\]

The value of accepting the informal job offer, \( \tilde{V}_i^{ja} \), satisfies

\[
\tilde{V}_i^{ja}(\theta, s, m) = \max_{e, m'} u\left( m \tilde{R} + \omega_n + b \omega_n I(s > \bar{s}) - m' \right) - \theta e + \beta \left\{ (1 - q_i(e)) \left[ p_{i,n} \tilde{V}_{n+1}^j(\theta, s', m') + (1 - p_{i,n}) \tilde{V}_{n+1}^u(\theta, s', m') \right] + q_i(e) \left[ p_{i,n} \tilde{V}_{n+1}^h(\theta, s', m') + (1 - p_{i,n}) \tilde{V}_{n+1}^f(\theta, s', m') \right] \right\},
\]

where

\[
s' = \tilde{R} \max \{ s - b \omega_n I(s > \bar{s}), 0 \},
\]

and the corresponding policy function for effort levels and private savings are \( \tilde{e}_n^{ja}(\theta, s, m) \) and \( \tilde{m}_n^{ja}(\theta, s, m) \), respectively.

Alternatively, the value of rejecting the informal job offer, \( \tilde{V}_i^{jr} \), satisfies

\[
\tilde{V}_i^{jr}(\theta, s, m) = \max_{e, m'} u\left( \tilde{R} m + \omega_n I(s > \bar{s}) - m' \right) - \theta e + \beta \left\{ (1 - q_u(e)) \left[ p_{u,n} \tilde{V}_{n+1}^i(\theta, s', m') + (1 - p_{u,n}) \tilde{V}_{n+1}^u(\theta, s', m') \right] + q_u(e) \left[ p_{u,n} \tilde{V}_{n+1}^h(\theta, s', m') + (1 - p_{u,n}) \tilde{V}_{n+1}^f(\theta, s', m') \right] \right\},
\]

where

\[
s' = \tilde{R} \max \{ s - b \omega_n I(s > \bar{s}), 0 \}
\]

and the corresponding policy function for effort levels and private savings are \( \tilde{e}_n^{jr}(\theta, s, m) \) and \( \tilde{m}_n^{jr}(\theta, s, m) \).
Suppose that the worker receives an offer in the formal sector. His maximized expected lifetime utility, $\tilde{V}_f$, must solve

$$\tilde{V}_f(\theta, s, m) = \max \{ \tilde{V}_f^{fa}(\theta, s, m), \tilde{V}_f^{fr}(\theta, s, m) \}.$$  

The value of accepting the formal job offer, $\tilde{V}_f^{fa}$, must solve

$$\tilde{V}_f^{fa}(\theta, s, m) = \max \left\{ u\left( \bar{R} m + \omega_n \left( 1 - \psi I(s < \breve{s}) - \bar{s} \right) - \theta e \right) + \beta \left( 1 - q_f(e) \right) \left[ p_{ff, n} \tilde{V}_{n+1}^{i}(\theta, s', m') + (1 - p_{ff, n}) \tilde{V}_{n+1}^{u}(\theta, s', m') \right] + q_f(e) \left[ p_{ff, n} \tilde{V}_{n+1}^{i}(\theta, s', m') + (1 - p_{ff, n}) \tilde{V}_{n+1}^{f}(\theta, s', m') \right] \right\},$$

where

$$s' = \bar{R} \left( s + \psi \omega_n I(s < \breve{s}) \right)$$

and the corresponding policy functions are $\tilde{e}_f^{fa}(\theta, s, m)$ and $\tilde{m}_f^{fa}(\theta, s, m)$.

Suppose that the worker receives both offers, formal and informal. His maximized expected lifetime utility, $\tilde{V}_b(\theta, s, m)$, must solve

$$\tilde{V}_b(\theta, s, m) = \max \{ \tilde{V}_f(\theta, s, m), \tilde{V}_i(\theta, s, m), \tilde{V}_u(\theta, s, m) \}.$$  

Finally, suppose that the worker receives no offer and he is consequently unemployed. His maximized expected lifetime utility is

$$\tilde{V}_u(\theta, s, m) = \tilde{V}_f^{fr}(\theta, s, m) = \tilde{V}_i^{fr}(\theta, s, m).$$  

Consider now the problem faced by an individual at working age $N$ (i.e., the period just before retirement). If the worker receives an job offer in the formal sector, he solves

$$\tilde{V}_f^{fN}(s, m) = \max \{ \tilde{V}_f^{fa}(s, m), \tilde{V}_f^{fr}(s, m) \}.$$  

If the worker accepts the job offer, $\tilde{V}_f^{fa}(s)$ solves

$$\tilde{V}_f^{fa}(s, m) = \max \left\{ u\left( \bar{R} m + \omega_N \left( 1 - \phi I(s < \breve{s}) - \bar{s} \right) - \theta e \right) + \beta H(s') \right\},$$

6Here, again, the value of $\theta$ is inmaterial
where

\[ s' = \tilde{R} \left( s + \phi_\omega N I(s < \tilde{s}) \right). \]

Alternatively, if the worker rejects the formal job offer, \( \tilde{V}^{fr}_N \) solves

\[ \tilde{V}^{fr}_N (s, m) = \max_{m'} u(\tilde{R}m + b \omega_\omega I(s > \tilde{s}) - m') + \beta H(s'), \]

where

\[ s' = \tilde{R} \max \{s - b\omega N I(s > \tilde{s}), 0\}. \]

If the worker receives an offer in the informal sector, he or she solves

\[ \tilde{V}^{i}_N(s) = \max \{ \tilde{V}^{i\omega}_N(s), \tilde{V}^{ir}_N(s) \}. \]

If the worker accepts the job offer, \( \tilde{V}^{i\omega}_N(s) \) solves

\[ \tilde{V}^{i\omega}_N(s,m) = \max_{m'} u(\tilde{R}m + \bar{\omega}_\omega I(s < \tilde{s}) - m') + \beta H(s'), \]

where

\[ s' = \tilde{R} \max \{s - b\omega N I(s > \tilde{s}), 0\}. \]

If the worker rejects the offer, \( \tilde{V}^{ir}_N(s,m) \) solves

\[ \tilde{V}^{ir}_N(s,m) = \tilde{V}^{fr}_N(s,m) = \tilde{V}^{ir}_N(s,m), \]

where

\[ s' = \tilde{R} \max \{s - b\omega N I(s > \tilde{s}), 0\}. \]

and \( \tilde{V}^{ir}_N(s,m) \) is the value of receiving no job offer and so being unemployed.

The value of rejecting all the job offers available and receive none must necessarily coincide in equilibrium. The key difference is that in the first case the worker decides to be unemployed while in the second he is forced to be unemployed.
5.2 UISA I: No Access to Private Credit Markets

Here we briefly discuss the environment in which agents do not have access to credit markets or, equivalently, that the government can decide their levels of financial wealth. In the setting describe in the previous section, this reduces to restrict $m' = 0$. So agents cannot manipulate asset holdings and that means that government have extra room to provide incentives.

6 Calibration

Since the goal is to evaluate the quantitative impact of policy reforms, we must select the key parameter of the model. This section explains the key choices, describe the parameters, and compare the model with the data. A detailed explanation of all the measurement in the model is presented in the Appendix.

To make the environment immune to the differences in returns stemming from the differences in the real return, $R$, with respect to the physical return, $(1 + x)$, we assume that $R = \beta^{-1} = 1 + x$.

The utility function is of the standard constant relative risk aversion (CRRA) form,

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

with relative risk aversion parameter $\sigma > 0$. The functions describing the probability of getting formal job offers are

$$q_i(e) = 1 - \exp(-\xi_i e),$$

$$q_u(e) = 1 - \exp(-\xi_U e),$$

$$q_f(e) = 1 - \exp(-\chi e).$$

The model is calibrated to Mexico. The value of the parameters are set using two strategies. First, there is a group of parameters that can be obtained directly from data or taken from previous literature. Whenever possible, we follow that strategy.
For instance, the lifecycle profile of formal wages is obtained from the estimations in Polachek (2007), while the profile of informal wages is calibrated to capture the life cycle profile of informality.

For the rest of the parameters, we search for values that imply that the model replicates specific targets as closely as possible. The artificial economy is generated by simulating various lifecycle profiles of individuals. Each individual begins with initial assets $m_0$. In the initial period, we assume there is an equal number of individuals in each offer state (formal offer, informal offer and no offer). Table 1 presents the resulting parameters and the basis for the calibration. The parameters that cannot be determined ex-ante will be determined jointly in the calibration procedure. For those parameters, the reference in Table 1 is to the moment that is likely to be more affected by that parameter. Thus, the parameters that are important to determine the probability of transitions across labor market outcomes are associated with the size of the employment and unemployment sectors and the value of transition probabilities. For instance, the parameter $\chi$ in the function $q_f$ determines the probability of keeping a job offer from the current employer; its associated statistic is the probability for a
worker of making a transition from a formal job to unemployment.

Table 1: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma = 2 )</td>
<td>Coefficient of relative risk aversion</td>
<td>Standard</td>
</tr>
<tr>
<td>( \beta = 0.96^{1/4} )</td>
<td>Discount factor</td>
<td>Standard</td>
</tr>
<tr>
<td>( d = 0.7 )</td>
<td>Retirement payment</td>
<td>Standard</td>
</tr>
<tr>
<td>( \rho = 0.9875 )</td>
<td>Retirees survival probability</td>
<td>Expected life after retirement</td>
</tr>
<tr>
<td>( \tau = 0.27 )</td>
<td>Labor income tax rate</td>
<td>Mexican tax rate</td>
</tr>
<tr>
<td>( \delta = 1 )</td>
<td>Severance payment is ( b_n = \delta w_{n-1} )</td>
<td>Mexican severance payment</td>
</tr>
<tr>
<td>( \xi_U = 0.0030 )</td>
<td>Parameter in fn ( q_u )</td>
<td>Unemployment rate</td>
</tr>
<tr>
<td>( \xi_I = 0.0015 )</td>
<td>Parameter in fn ( q_i )</td>
<td>Size of informal sector</td>
</tr>
<tr>
<td>( \chi = 0.065 )</td>
<td>Parameter in fn ( q_f )</td>
<td>F-U transition rate</td>
</tr>
<tr>
<td>( p_i = 0.97 )</td>
<td>Prob. informal offer, given informal at t-1</td>
<td>I-U transition rate</td>
</tr>
<tr>
<td>( p_f = 0.50 )</td>
<td>Prob. informal offer, given formal at t-1</td>
<td>F-I transition rate</td>
</tr>
<tr>
<td>( p_u = 0.60 )</td>
<td>Prob. informal offer, given unempl. at t-1</td>
<td>U-I transition rate</td>
</tr>
<tr>
<td>( w : ) (see Figure 1)</td>
<td>Formal sector wage</td>
<td>Mexican profile, Polachek (2007)</td>
</tr>
<tr>
<td>( R = \frac{1}{\beta} )</td>
<td>UI Gross interest rate</td>
<td>UI Asset</td>
</tr>
<tr>
<td>( \tilde{R} = R )</td>
<td>UI( \tilde{S})A Gross interest rate</td>
<td>UI( \tilde{S})A Asset</td>
</tr>
<tr>
<td>( \tilde{\tilde{R}} = 1 )</td>
<td>( UI\tilde{S}A_{\tilde{I}} ) Gross interest rate</td>
<td>Hidden Asset</td>
</tr>
<tr>
<td>( \theta = 0.010 )</td>
<td>Minimum value of ( \theta )</td>
<td>Lifecycle profile of U,E,I</td>
</tr>
<tr>
<td>( \overline{\theta} = 0.025 )</td>
<td>Maximum value of ( \theta )</td>
<td>Lifecycle profile of U,E,I</td>
</tr>
</tbody>
</table>

Table 2 shows that the predictions of the benchmark UI economy match closely with unemployment, informality, and employment rates, both at the aggregate level as well as by age group. For instance, the aggregate informality rate is 52.7 percent in the data and 48.8 percent in the model. This table also compares the percentage of individuals, based on age group, in each employment sector calculated from the data and the model. The model can effectively reproduce the shape of the lifecycle profile of formal and informal employment.
Table 2: Lifecycle Patterns: UI Economy and Data

<table>
<thead>
<tr>
<th></th>
<th>Unemployment Rate</th>
<th>Informality Rate</th>
<th>Formality Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UI Data</td>
<td>UI Data</td>
<td>UI Data</td>
</tr>
<tr>
<td>All Ages</td>
<td>3.7%</td>
<td>2.7%</td>
<td>48.8%</td>
</tr>
<tr>
<td></td>
<td>52.7%</td>
<td>47.5%</td>
<td>44.5%</td>
</tr>
<tr>
<td>20-24</td>
<td>5.2%</td>
<td>4.8%</td>
<td>41.6%</td>
</tr>
<tr>
<td></td>
<td>46.3%</td>
<td>53.2%</td>
<td>48.9%</td>
</tr>
<tr>
<td>25-39</td>
<td>3.0%</td>
<td>2.2%</td>
<td>44.6%</td>
</tr>
<tr>
<td></td>
<td>48.4%</td>
<td>52.4%</td>
<td>49.4%</td>
</tr>
<tr>
<td>40 +</td>
<td>4.0%</td>
<td>1.0%</td>
<td>58.9%</td>
</tr>
<tr>
<td></td>
<td>63.5%</td>
<td>37.1%</td>
<td>35.3%</td>
</tr>
</tbody>
</table>

In this economy, formality decreases with age for two reasons; first, being on the formal track pays off more when one is young since the favorable probability distribution can be used for more periods. As agents get older, they accumulate enough assets to make it possible to substitute effort. Figure 2 shows how agents accumulate differently according to their type. High $\theta$ workers accumulate aggressively form the very beginning to be able to quit search relatively sooner, while low $\theta$ workers accumulate less at the beginning to start saving aggressively later and indeed they end up with more assets (as they are better endowed to find better high productivity formal jobs).

Figure 2: Asset Accumulation by Type(UI)
7 Policy Evaluation

7.1 Full Insurance

Table 3 compares labor market outcomes in the Full insurance (FI) economy with the UI benchmark economy (UI).

<table>
<thead>
<tr>
<th></th>
<th>UI</th>
<th>FI</th>
<th>UI</th>
<th>FI</th>
<th>UI</th>
<th>FI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment rate</td>
<td>3.7%</td>
<td>5.4%</td>
<td>48.8%</td>
<td>32.7%</td>
<td>47.5%</td>
<td>61.9%</td>
</tr>
<tr>
<td>20-24</td>
<td>5.2%</td>
<td>9.9%</td>
<td>41.6%</td>
<td>56.2%</td>
<td>53.2%</td>
<td>33.9%</td>
</tr>
<tr>
<td>25-39</td>
<td>3.0%</td>
<td>4.8%</td>
<td>44.6%</td>
<td>38.6%</td>
<td>52.4%</td>
<td>56.7%</td>
</tr>
<tr>
<td>40+</td>
<td>4.0%</td>
<td>3.9%</td>
<td>58.9%</td>
<td>11.5%</td>
<td>37.1%</td>
<td>84.6%</td>
</tr>
</tbody>
</table>

First of all, notice the size of the informal sector in the FI economy is non-negligible, i.e. 32.7%. The intuition behind this quantitative outcome can be grasped as follows. The technology to find and keep high productivity formal jobs is indeed better as the worker is either employed in the formal sector or even unemployed. However, for a fraction of the population, the cost in terms of utility of exerting the required effort is “too” high (i.e. high $\theta$). Optimality dictates that almost a third of the workers should work informally as well as some redistribution from the ex-post high productivity workers goes to the ex-post low productivity workers.

Now we compare the outcomes in both economies. While agent type is optimally correlated with sector (i.e. low $\theta$ agents work formally, and high $\theta$ agents work informally), consumption is not. On the other hand, in the UI $\theta$’s correlation with sector is also high, but so is consumption; i.e. insurance is limited.

In contrast to the UI economy, in the FI economy effort and formality optimally rise with age (i.e., it is more efficient to exert effort when the formal-informal wage premium is higher and this occurs at older ages; see Figure 1).
Table 4 shows that, compared with the UI setting, optimality makes informal workers exert almost no effort with a consequent very low probability of finding a formal job and transit from informality to formality. On the other hand, optimality dictates that both formal workers and unemployed must exert higher levels of effort to find a formal job compared to the incentives provided in the UI setting.

<table>
<thead>
<tr>
<th>Formally Employed</th>
<th>Unemployed</th>
<th>Informally Employed</th>
</tr>
</thead>
<tbody>
<tr>
<td>UI</td>
<td>FI</td>
<td>UI</td>
</tr>
<tr>
<td>All Ages</td>
<td>95.8%</td>
<td>97.4%</td>
</tr>
<tr>
<td>20-24</td>
<td>96.4%</td>
<td>95.2%</td>
</tr>
<tr>
<td>25-39</td>
<td>96.5%</td>
<td>98.1%</td>
</tr>
<tr>
<td>40+</td>
<td>95.0%</td>
<td>97.2%</td>
</tr>
</tbody>
</table>

Table 5 displays the participation of different types of agents in both labor markets and in the pool of unemployed. UI overallocates low $\theta$ workers in the informal sector, while underallocates this type in the formal sector and in the unemployed pool. Regarding high $\theta$ workers, the same pattern is observed but the lower participation in the formal sector is exacerbated.

<table>
<thead>
<tr>
<th>Unemployment Rate</th>
<th>Informality Rate</th>
<th>Formality Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>UI</td>
<td>FI</td>
<td>UI</td>
</tr>
<tr>
<td>Low $\theta$</td>
<td>2.9%</td>
<td>4.6%</td>
</tr>
<tr>
<td>High $\theta$</td>
<td>4.9%</td>
<td>6.5%</td>
</tr>
</tbody>
</table>

Table 6 displays that UI can only provide partial insurance regarding consumption smoothing across $\theta$’s. It also shows that UI provide relative poor incentive to exert effort to find formal jobs and so the probabilities are significantly lower than optimal.
Table 6: Consumption and Formal Offer Probability

<table>
<thead>
<tr>
<th></th>
<th>Average Consumption</th>
<th>Average Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UI</td>
<td>FI</td>
</tr>
<tr>
<td>Low $\theta$</td>
<td>0.52</td>
<td>0.50</td>
</tr>
<tr>
<td>High $\theta$</td>
<td>0.39</td>
<td>0.50</td>
</tr>
</tbody>
</table>

7.2 Liquidity Provision

In this class of life-cycle models in which agents are restricted to borrow, liquidity provision can potentially help alleviate some allocative distortions. In this section we compare the UI Economy (UI) to a version in which liquidity is provided at date 1, denoted $\text{UI}_L$. The liquidity policy consists of determining parameters, $(\bar{m}_0, \tau)$. The first refers to a liquidity transfer that leaves the agent with initial assets $\bar{m}_0$ and so the net transfer is $l_0 = \bar{m}_0 - m_0$. The second, $\tau$, is determined such that the budget surplus remains unchanged compared to the benchmark economy $S^{\text{UI}} = S^{\text{UI}_L}$. The optimal policy is $\bar{m}_0 = 0.22$, $\tau = 0.28$. The amount of liquidity is limited since it increases informality and requires higher payroll taxes (which raises informality even more) to be financed.

Table 7 shows that the level of informality is not a consequence of lack of liquidity. Indeed, the provision of liquidity increases informality not only at the aggregate levels but also for all the ages. The same happens with the levels of unemployment and so the level of formal employment decreases at all ages.

Table 7: Lifecycle Patterns: UI and $\text{UI}_L$

<table>
<thead>
<tr>
<th></th>
<th>Unemployment Rate</th>
<th>Informality Rate</th>
<th>Formality Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UI</td>
<td>UI$_L$</td>
<td>UI</td>
</tr>
<tr>
<td>All Ages</td>
<td>3.7%</td>
<td>3.8%</td>
<td>48.8%</td>
</tr>
<tr>
<td>20-24</td>
<td>5.2%</td>
<td>5.4%</td>
<td>41.6%</td>
</tr>
<tr>
<td>25-39</td>
<td>3.0%</td>
<td>3.1%</td>
<td>44.6%</td>
</tr>
<tr>
<td>40+</td>
<td>4.0%</td>
<td>4.1%</td>
<td>58.9%</td>
</tr>
</tbody>
</table>
The unemployment rate is slightly higher because agents substitute effort with liquidity; especially the youngest agents, for whom the liquidity effect is strongest. Table 8 makes this evident as effort and so probabilities decrease at all ages. In general, the provision of liquidity makes workers exert less effort to look for a formal job.

### Table 8: Probability of a Formal Offer

<table>
<thead>
<tr>
<th></th>
<th>Formally Employed</th>
<th>Unemployed</th>
<th>Informally Employed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$UI$</td>
<td>$UI_L$</td>
<td>$UI$</td>
</tr>
<tr>
<td>All Ages</td>
<td>95.8%</td>
<td>95.7%</td>
<td>22.7%</td>
</tr>
<tr>
<td>20-24</td>
<td>96.4%</td>
<td>96.3%</td>
<td>56.2%</td>
</tr>
<tr>
<td>25-39</td>
<td>96.5%</td>
<td>96.4%</td>
<td>27.4%</td>
</tr>
<tr>
<td>40+</td>
<td>95.0%</td>
<td>94.9%</td>
<td>13.7%</td>
</tr>
</tbody>
</table>

As a direct consequence, Table 9 shows that the provision of liquidity makes the participation in the informal sector and unemployment increase as well as the participation in the formal sector decrease.

### Table 9: UI and UI\_L Sector Shares by $\theta$

<table>
<thead>
<tr>
<th></th>
<th>Unemployment Rate</th>
<th>Informality Rate</th>
<th>Formality Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$UI$</td>
<td>$UI_L$</td>
<td>$UI$</td>
</tr>
<tr>
<td>Low $\theta$</td>
<td>2.9%</td>
<td>3.0%</td>
<td>20.2%</td>
</tr>
<tr>
<td>High $\theta$</td>
<td>4.9%</td>
<td>4.9%</td>
<td>91.6%</td>
</tr>
</tbody>
</table>

Table 10 shows its impact in terms of consumption smoothing across $\theta$'s and the average probability of finding a formal job is basically negligible.

### Table 10: Consumption and Formal Offer Probability

<table>
<thead>
<tr>
<th></th>
<th>Average Consumption</th>
<th>Average Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$UI$</td>
<td>$UI_L$</td>
</tr>
<tr>
<td>Low $\theta$</td>
<td>0.52</td>
<td>0.51</td>
</tr>
<tr>
<td>High $\theta$</td>
<td>0.39</td>
<td>0.39</td>
</tr>
</tbody>
</table>
Finally, Figure 4 makes it evident that the provision has no significant impact on savings for any type of worker.

7.3 UISA

In what follows, we quantify the impact of implementing the UISA system. First in this section we analyze the case in which the government can control savings and so consumption. The optimal policy is: $\bar{s} = 0.01, s = 2.0, \psi = 0.06, b = 0.43, s_0 = 5.0, \bar{\tau} = 0.23$. Higher liquidity provision is feasible because it can be financed by the UISA’s larger formal sector.

Table 11 compares the impact of implementing the UISA system with the benchmark case of the UI economy. The system provides incentives such that informality reduces drastically to make formality increase significantly while unemployment also increases slightly. Notice that workers consume out of their UISA accounts when young, which is why effort and formality are so low for the young. The resources in the UISA do not grow because agents cannot accumulate savings over $\bar{s}$. 

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Table 11: Lifecycle Patterns, UI and UISA

<table>
<thead>
<tr>
<th>Age</th>
<th>UI Unemployment Rate</th>
<th>UISA Unemployment Rate</th>
<th>UI Informality Rate</th>
<th>UISA Informality Rate</th>
<th>UI Formality Rate</th>
<th>UISA Formality Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Ages</td>
<td>3.7%</td>
<td>4.7%</td>
<td>48.8%</td>
<td>22.9%</td>
<td>47.5%</td>
<td>72.4%</td>
</tr>
<tr>
<td>20-24</td>
<td>5.2%</td>
<td>8.1%</td>
<td>41.6%</td>
<td>87.5%</td>
<td>53.2%</td>
<td>4.5%</td>
</tr>
<tr>
<td>25-39</td>
<td>3.0%</td>
<td>4.8%</td>
<td>44.6%</td>
<td>15.1%</td>
<td>52.4%</td>
<td>80.1%</td>
</tr>
<tr>
<td>40+</td>
<td>4.0%</td>
<td>2.7%</td>
<td>58.9%</td>
<td>1.1%</td>
<td>37.1%</td>
<td>96.2%</td>
</tr>
</tbody>
</table>

In addition to liquidity provision, the UISA policy reduces moral hazard, thus effort and formality are in general higher, and so is consumption; see Table 12 and 13.

Table 12: Probability of a Formal Offer

<table>
<thead>
<tr>
<th>Age</th>
<th>UI Formally Employed</th>
<th>UISA Formally Employed</th>
<th>UI Unemployed</th>
<th>UISA Unemployed</th>
<th>UI Informally Employed</th>
<th>UISA Informally Employed</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Ages</td>
<td>95.8%</td>
<td>97.4%</td>
<td>22.7%</td>
<td>52.0%</td>
<td>1.5%</td>
<td>1.6%</td>
</tr>
<tr>
<td>20-24</td>
<td>96.4%</td>
<td>90.0%</td>
<td>56.2%</td>
<td>2.5%</td>
<td>4.9%</td>
<td>0.0%</td>
</tr>
<tr>
<td>25-39</td>
<td>96.5%</td>
<td>97.9%</td>
<td>27.4%</td>
<td>61.4%</td>
<td>2.1%</td>
<td>4.7%</td>
</tr>
<tr>
<td>40+</td>
<td>95.0%</td>
<td>97.2%</td>
<td>13.7%</td>
<td>64.2%</td>
<td>0.9%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Table 13: Consumption and Formal Offer Probability

<table>
<thead>
<tr>
<th>Age</th>
<th>Average Consumption UI</th>
<th>Average Consumption UISA</th>
<th>Average Probability UI</th>
<th>Average Probability UISA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low ( \theta )</td>
<td>0.52</td>
<td>0.60</td>
<td>68.7%</td>
<td>84.5%</td>
</tr>
<tr>
<td>High ( \theta )</td>
<td>0.39</td>
<td>0.60</td>
<td>2.9%</td>
<td>82.3%</td>
</tr>
</tbody>
</table>

Table 14 displays the impact on labor allocation. It is evident that the main impact of the implementation of the UISA system comes from the high \( \theta \) workers. They work less informally to end up finding a formal job as long as they can consume out of their saving accounts.
Figure 4: Asset Accumulation by Type (UI and UISA)

Table 14: UI and UISA Sector Shares by $\theta$

<table>
<thead>
<tr>
<th>Unemployment Rate</th>
<th>Informality Rate</th>
<th>Formality Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UI</strong></td>
<td><strong>UISA</strong></td>
<td><strong>UI</strong></td>
</tr>
<tr>
<td>Low $\theta$</td>
<td>2.9%</td>
<td>3.9%</td>
</tr>
<tr>
<td>High $\theta$</td>
<td>4.9%</td>
<td>5.9%</td>
</tr>
</tbody>
</table>

Figure 5 shows the differences on asset accumulation by type. Under the UISA young workers deplete their saving accounts when young, thus substituting away from effort and raising informality for this age group. Once their savings are sufficiently depleted, the design of the UISA provides incentives for effort and formality through the amelioration of moral hazard.

7.4 UISA - Hidden Savings

Here we analyze the case in which the government cannot control savings; i.e. UISA with hidden savings. The optimal policy is $\underline{s} = 0.01, \bar{s} = 1.0, \psi = 0.01, b = 0.9, s_0 = 1.28, \tau = 0.24$. 

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The UISA system still takes workers away from informality to formality but the impact is much lower, 5% approximately at the aggregate level. However, the impact differs according to their ages. Notice that indeed UISA make young workers participate more in the informal sector then in UI system as savings can be hidden. Workers move to the formal sector as they get older. See Table 15.

Table 15: Lifecycle Patterns: UI and UISA_H

<table>
<thead>
<tr>
<th></th>
<th>Unemployment rate</th>
<th>Informality rate</th>
<th>Formality rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UI</td>
<td>UISA_H</td>
<td>UI</td>
</tr>
<tr>
<td>All Ages</td>
<td>3.7%</td>
<td>3.4%</td>
<td>48.8%</td>
</tr>
<tr>
<td>20-24</td>
<td>5.2%</td>
<td>6.1%</td>
<td>41.6%</td>
</tr>
<tr>
<td>25-39</td>
<td>3.0%</td>
<td>2.5%</td>
<td>44.6%</td>
</tr>
<tr>
<td>40+</td>
<td>4.0%</td>
<td>3.3%</td>
<td>58.9%</td>
</tr>
</tbody>
</table>

Table 16 shows that indeed young workers exert less effort to find a formal job due to liquidity provision, and this translates in lower probabilities and lower participation in the formal market. As workers get older and the policy’s liquidity effect dissipates, the policy’s moral hazard amelioration effect raises effort levels, the corresponding probabilities and participation in the formal market.

Table 16: Probability of a Formal Offer

<table>
<thead>
<tr>
<th></th>
<th>Formally Employed</th>
<th>Unemployed</th>
<th>Informally Employed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UI</td>
<td>UISA_H</td>
<td>UI</td>
</tr>
<tr>
<td>All Ages</td>
<td>95.8%</td>
<td>97.2%</td>
<td>22.7%</td>
</tr>
<tr>
<td>20-24</td>
<td>96.4%</td>
<td>96.6%</td>
<td>56.2%</td>
</tr>
<tr>
<td>25-39</td>
<td>96.5%</td>
<td>97.9%</td>
<td>27.4%</td>
</tr>
<tr>
<td>40+</td>
<td>95.0%</td>
<td>96.6%</td>
<td>13.7%</td>
</tr>
</tbody>
</table>

Compared to the UI system, the informality rate decreases and the formality rate increases for both types, low and high \( \theta \). The unemployment rate decreases for low \( \theta \)’s but increases for high \( \theta \)’s. See Table 17.
<table>
<thead>
<tr>
<th>Unemployment rate</th>
<th>Informality rate</th>
<th>Formality rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>UI</td>
<td>UISA$_H$</td>
<td>UI</td>
</tr>
<tr>
<td>Low $\theta$</td>
<td>2.9%</td>
<td>2.0%</td>
</tr>
<tr>
<td>High $\theta$</td>
<td>4.9%</td>
<td>5.4%</td>
</tr>
</tbody>
</table>

Notably, Table 18 shows that UISA with hidden savings makes the dispersion of consumption across different $\theta$'s increase compared to the UI system while consumption levels are a bit higher.

<table>
<thead>
<tr>
<th>Average Consumption</th>
<th>Average Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>UI</td>
<td>UISA$_H$</td>
</tr>
<tr>
<td>Low $\theta$</td>
<td>0.52</td>
</tr>
<tr>
<td>High $\theta$</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Figure 6 below shows the evolution of private savings. The similarities between the savings in the UI and UISA with hidden savings economies are striking. The opportunity to secretly accumulate assets makes it possible to undo what the UISA system intends the workers to do with their asset holdings. In the UISA system, when consumption and saving can be monitored, the government can design the system such that incentives are provided to exert effort and so find formal jobs. Once savings cannot be observed (and so is consumption), workers substitute away from effort to avoid paying the utility cost; that is, although they obtain less utility from consuming less, they increase their utility by paying less for exerting effort.

### 7.5 Welfare gains

Table 19 below summarizes the main parameters characterizing each system.
Figure 5: Asset Accumulation by Type (UI and UISA$_H$)

![Graph showing asset accumulation by type (UI and UISA$_H$)](image)

Table 19: Unemployment Insurance Policies

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>UI</th>
<th>UI$_L$</th>
<th>UISA$_H$</th>
<th>UISA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>Payroll Tax</td>
<td>0.27</td>
<td>0.28</td>
<td>0.24</td>
<td>0.23</td>
</tr>
<tr>
<td>$b$</td>
<td>Replacement Rate</td>
<td>1.0</td>
<td>1.0</td>
<td>0.9</td>
<td>0.43</td>
</tr>
<tr>
<td>$s$</td>
<td>UISA Lower Bound</td>
<td>-</td>
<td>-</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$\bar{s}$</td>
<td>UISA Upper Bound</td>
<td>-</td>
<td>-</td>
<td>1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$\psi$</td>
<td>UISA Contribution</td>
<td>-</td>
<td>-</td>
<td>0.01</td>
<td>0.06</td>
</tr>
<tr>
<td>$l_0$</td>
<td>Liquidity Transfer</td>
<td>-</td>
<td>0.08</td>
<td>1.14</td>
<td>4.86</td>
</tr>
</tbody>
</table>
Table 20: Welfare Comparison

<table>
<thead>
<tr>
<th></th>
<th>UI</th>
<th>UI$_L$</th>
<th>UISA$_{H}$</th>
<th>UISA</th>
<th>FI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formality</td>
<td>47.5%</td>
<td>45.8%</td>
<td>52.2%</td>
<td>72.4%</td>
<td>61.9%</td>
</tr>
<tr>
<td>Unemployment</td>
<td>3.7%</td>
<td>3.8%</td>
<td>3.4%</td>
<td>4.7%</td>
<td>5.4%</td>
</tr>
<tr>
<td>Informality</td>
<td>48.8%</td>
<td>50.4%</td>
<td>44.4%</td>
<td>22.9%</td>
<td>32.7%</td>
</tr>
<tr>
<td>Welfare Gains, CE Units</td>
<td>0.0%</td>
<td>1.1%</td>
<td>9.2%</td>
<td>11.9%</td>
<td>24.4%</td>
</tr>
<tr>
<td>Share of Total Gains</td>
<td>0.0%</td>
<td>4.5%</td>
<td>37.7%</td>
<td>48.8%</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

Beginning with the UI$_L$, welfare gains come exclusively from liquidity provision. Since the liquidity policy does not ameliorate moral hazard, informality is persistent and revenues cannot be increased significantly, which is why the feasible transfer is modest and welfare gains are small. Sector shares for the UISA$_{H}$ system are similar to the UI system. The welfare gains relative to the UI$_L$ come from the larger liquidity transfer, which is made possible by the UISA’s reduction of moral hazard and higher formality. The UISA system differs from the UISA$_{H}$ because savings are a poor substitute for effort: agents have no control in smoothing consumption so the incentives to substitute away from effort are lower. This is why formality is so high, and why such high liquidity provision is feasible, thus raising welfare. Finally, note that effort is excessive in the UISA system, relative to the FI. The UISA system eliminates the correlation between $\theta$ and consumption, but at the cost of also eliminating the correlation between $\theta$ and effort. In the FI, the planner can do the former without having to incur the cost of the latter: resources that are unequally and thus efficiently produced (in terms of $\theta$) can be equally distributed for consumption among different types.
8 Conclusions

To be completed.
References


9 Appendix: Measurement

At any date \( t \) as population growth is constant, we normalize the population with working age \( n \leq N \) as \((1 + x)^{-\langle n-1 \rangle}\). As agents retire they survive with probability \( \rho \) for \( n = N + k \) for all \( k \geq 1 \), the size of the population of that age is \( \rho^k (1 + x)^{-(N+k-1)} \).

Let \( F_n^j(\theta, m, s, es) \) denote the fraction of individuals in the economy \( j \in \{FI, UI, UISA\} \) with preference shock \( \theta \), asset holdings \( m \), savings \( s \), age \( n \), and employment status \( es \in \{f, i, u\} \) (i.e., formal employee, informal employee, unemployed). Denote \( c_n^j(\theta, m, es) \) as the corresponding consumption’s policy functions.

In order to carry out measurement, we assume that the law of large number holds and so \( F_n^j(\theta, m, s, es) \) also denote the date-1 probability that a worker reaches age \( n \) with with preference shock \( \theta \), asset holdings \( m \), savings \( s \) and employment status \( es \in \{f, i, u\} \) in the economy \( j \in \{FI, UI, UISA\} \).

Recall that all workers are assumed to receive a job offer in the formal sector as they enter the job market.

9.1 Fiscal Accounting

UI Economy

Now we compare the effect of a fiscal reform taking into account the impact on the fiscal budget. Total taxes collected by the government and its corresponding expenditures in the economy \( UI \) are

\[
T^{UI} = \sum_{n=1}^{N} \sum_{\theta} \int_m (1 + x)^{-(n-1)} F_n^{UI}(\theta, m, f) \ w_n \tau \ g(\theta) dm,
\]

\[
G^{UI} = \sum_{n=1}^{N-1} \sum_{\theta} \int_m b (1 + x)^{-n} F_n^{UI}(\theta, m, f)(1 - q_f(e_n^f(\theta, m, s)))) \ g(\theta) dm
+ \sum_{k \geq 1} \rho^k (1 + x)^{-(N+k-1)} d,
\]

since \( F_n^{UI}(\theta, m, s, f)(1 - q_f(e_n^f(\theta, m, s))) \) is the fraction of workers that at working age \( n + 1 \) who get fired from the formal sector and so collect unemployment benefits \( b \). That is,
they were employed in the formal sector the previous period \( n \) and were exerting effort \( e_n^f(\theta, m, s) \). Each of them gets fired with probability \( 1 - q_f(e_n^f(\theta, m, s)) \).

So we define the fiscal surplus in the UI economy as

\[
S_{UI} = T_{UI} - G_{UI}
\]

Notice that under our assumptions the implicit amount of net wealth left to the agents at date 1 is \( m^0 - S_{UI} \).

**UISA Economy**

Total taxes collected to finance transfers to workers entering the labor market are

\[
T^{UISA} = \sum_{n=1}^{N} \sum_{\theta} \int_m (1 + x)^{-(n-1)} E_{n}^{UISA}(\theta, m, s, f) w_n \tilde{\tau} g(\theta) \, dm,
\]

while the expenditures needed to finance those transfers are

\[
G^{UISA} = (s_0 - m_0) + \sum_{k \geq 1} \rho^k (1 + x)^{-(N+k-1)} \, d,
\]

since \( s_0 \) is uncontingent with respect to \( \theta \).

Notice that this means that the government can force the workers to deposit their initial wealth in the unemployment insurance saving account.

To make both systems comparable, we restrict to UISA to

\[
T^{UISA} - G^{UISA} = S_{UI}.
\]

### 9.2 Some Key Concepts

**On the Design of the Optimal UISA**

The optimality criterion is the following. The set of alternative policy reforms that we consider belongs to unemployment protection schemes that can be parametrized by

\[
\Gamma = (s, \bar{s}, \psi, b, s_0, \tilde{\tau}).
\]

Let \( \tilde{V}_{1}^{es}(\theta, m_0 \mid \Gamma) \) be the utility of the representative worker with preference shock \( \theta \), initial asset holdings \( m_0 \), and employment offer status \( es \).
Definition 1  We define the optimal UISA scheme, $\Gamma^*$, as the one that solves

$$\max_\Gamma \sum_\theta \int_{es} \tilde{V}^*_1(\theta, m_0 | \Gamma) d es g(\theta),$$

subject to

$$T^{UISA}(\Gamma) - G^{UISA}(\Gamma) = S^{UI}.$$

That is, in this stationary environment, the fictitious planner can manipulate all the cross-sectional resources to maximize the ex-ante utility of the representative worker. In addition, to make the comparison with the UI system feasible, it must generate $S^{UI}$.

Labor Market

It is useful to first compute the levels of formal employment, informal employment, and unemployment for each economy $j \in \{FI, UI, UISA\}$ The level of employment in the formal sector is given by

$$F^j = \sum_{n=1}^{N} \sum_\theta \int_{m} (1 + x)^{-\theta - 1} F^j_n(\theta, m, s, f) g(\theta) dm.$$

The corresponding level of employment in the informal sector, our measure of informality in the economy, is given by

$$I^j = \sum_{n=1}^{N} \sum_\theta \int_{m} (1 + x)^{-\theta - 1} I^j_n(\theta, m, s, i) g(\theta) dm.$$

Finally, the unemployment level for each economy is defined as

$$U^j = \sum_{n=1}^{N} \sum_\theta \int_{m} (1 + x)^{-\theta - 1} U^j_n(\theta, m, s, u) g(\theta) dm.$$

These represent total numbers of workers for each employment status. We can translate the numbers into shares by simply writing

$$S^j_f = \frac{F^j}{F^j + I^j + U^j},$$

$$S^j_i = \frac{I^j}{F^j + I^j + U^j},$$

$$U^j = \frac{U^j}{F^j + I^j + U^j}.$$
where \( S_f^j, S_i^j, \) and \( U^j \) stand for the fraction of workers employed in the formal sector, the fraction of workers employed in the informal sector, and the unemployment rate, respectively, for the economy \( j \).

**Welfare Comparisons**

Let \( \nu(j, j') \) be the percentage change in consumption needed to make an ex-ante representative worker indifferent between the allocations in the economies \( j \) and \( j' \), which deliver ex-ante expected utility \( V^j \) and \( V^{j'} \). As the utility function of the representative worker is assumed to be homogeneous of degree \( (1 - \sigma) \) with respect to consumption for \( \sigma > 0 \) (i.e. CRRA preferences), \( \nu(j, j') \) can be directly computed as follows

\[
\nu(j, j') = \left[ \frac{V^{j'} + \int_\theta \int_m \int_s \left( V^j_e(\theta, m, s) - \beta^N H^j(\theta, m, s)d\theta dmds \right)}{\int_\theta \int_m \int_s V^j_e(\theta, m, s)d\theta dmds} \right]^{\frac{1}{1-\sigma}}
\]

where

\[
V^j_e(\theta, m, s) = \sum_{n=1}^N \beta^{u-1} \sum_{es \in \{f, i, u\}} F^j_n(\theta, m, s, es) \theta e^j_n(\theta, m, s, es)d\theta dmds
\]

\[
V^j_c(\theta, m, s) = \sum_{n=1}^N \beta^{u-1} \sum_{es \in \{f, i, u\}} F^j_n(\theta, m, s, es) u(c^j_n(\theta, m, s, es))d\theta dmds
\]