Accounting for Labor Gaps

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December 2014

Abstract

We develop a balanced growth model with labor supply and search and matching frictions in the labor market to study the impact of economic policy variables on the two margins which constitute the (total) labor input: the extensive one (the rate of employment) and the intensive one (the hours worked per worker). As it is standard in the literature, the taxes have an impact mainly on the hours worked while the labor market institutions (the replacement rate and the bargaining power of workers) influence the rate of employment; however there is also an interaction between the two margins. The model focuses on two countries (US and France) and it is solved in perfect foresight: the aim is to use it to perform counterfactual experiments about the evolution of the policy variables and to compare welfare levels implied by policy changes.

JEL Classification : E20, E60, J22, J60

Keywords: Taxes, labor market institutions, hours, labor market search

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1 Introduction

The evolution of hours worked during the post WWII period has been characterized by sharp differences across developed economies: if we look at the evolution of the US and three selected countries among the European ones (France, Germany (1) and UK (2)), we observe a sharp decline of hours in the two continental European countries, at least till the mid eighties, while in the UK the decline has been less important and in the US they have remained relatively flat (see Figure 1).

Figure 1: Total hours 1960-2010

The heterogeneity in the evolution of aggregate worked hours has already been highlighted in the literature(3). The main finding of these contributions is the following: "I determine the importance of tax rates in accounting for these differences in labor supply for the major advanced industrial countries and find that tax rates alone account for most of these differences in labor supply", Prescott (2004) [23]. According to this view, the welfare impact of a tax reform in France could therefore be considerable. Prescott (2004) [23] for example obtains that if France adopted American tax rates on labor income (i.e., reducing the effective tax rate on labor by 20 percentage points) "the welfare of the French people would increase by 19 percent in terms of lifetime consumption equivalents.". Such large welfare gains would undoubtedly call for cutting French (and more broadly, European) taxes

(1) Data for Germany refer to the actual country as it exists from 1990; the series for the years which precede the reunification are reconstructed using data from the West and the East Germany.

(2) These countries are also among those retained by Prescott (2004) [23] in his seminal paper on the labor wedge and taxes.

down to US levels. At the general equilibrium, the corollary of such policy will be naturally to reduce the Government expenditures and transfers, as it is done in Prescott (2004) [23].

The simple neoclassical growth model with endogenous labor supply can be considered as a parsimonious approach to evaluate quantitatively the impact of a tax reform on the level of aggregate hours worked at the general equilibrium: in this case the labor wedge is reduced to the tax wedge.

However, this approach does not allow to distinguish between hours worked per employee and the number of employees, whereas these two margins are also characterised by different evolutions: considering the employment rate, while the American "jobs miracle" seems to be a peculiar feature of only one side of the Atlantic Ocean till the end of the 2000s, Germany and the UK show comparable evolutions between each other, whereas France shows a particularly bad performance. Considering hours per worker instead, the steady decline in Germany is even more pronounced than that one observed in France, while the UK stands remarkably closer to the evolution of the US from the mid eighties (see Figure 2).

Figure 2: The decomposition of the total hours 1960-2010

The objective of our contribution is to develop a dynamic general equilibrium model with

\[ \text{Employment rate 1960-2010} \]

\[ \text{Hours per worker 1960-2010} \]

\[ \text{Figure 2: The decomposition of the total hours 1960-2010} \]

\[ \text{Employment rate 1960-2010} \]

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\[ \text{The objective of our contribution is to develop a dynamic general equilibrium model with} \]

\[ \text{From a methodological point of view, Prescott’s evaluation of the welfare gains coming from the tax cuts takes into account the contraction in Government expenditure and the transition necessary to reach a new steady state with a smaller welfare State.} \]

\[ \text{In particular, from the mid nineties, the employment rate continuously increases in Germany. This features is magnified since the beginning of the implementation of the Hartz reforms. These new laws on the labor market make easier the part-times, and also reduce the labor costs. Thus, the hours per worker do not increase, whereas the employment rate is, at the end of sample larger than the one observed in the US, for the first time since the end of the WWII.} \]
search and matching frictions in order to decompose the labor wedges in two components: the "hours worked per employee wedge" and the "employment wedge". From a theoretical point of view, the originality of our paper is to propose a theory for the allocation of time allowing to identify the relative contribution of taxes and labor market institutions’ changes by decomposing the aggregate hours of work into the intensive and the extensive margins. We then propose to analyze the decentralized allocation of a general equilibrium model with matching frictions, wage bargaining and efficient bargain on the number of hours worked per employee.\footnote{The model is closed to the first contributions of Langot (1995) [11], Merz (1995) [17] and Andolfatto (1996) [1].}

With this type of theory of hours and employment allocation, we can distinguish between the contribution of taxes and the contribution of changes in labor market institutions (such as unemployment benefits and the bargaining power of the workers) on the labor wedge.\footnote{Ohanian et al. (2008) [19] show that the labor wedge computed with a model that merge hours worked per employee and the employment rate, is not independent from the labor market institutions, such as the unemployment benefits or the bargaining power. This in accordance with an other feature of the data observed since the end of the seventies: the differences across countries in aggregate hours are due to quantitatively important differences along the extensive margin. This calls for a theory that distinguish the two margins on the labor market.}

Whereas taxes are the first and the only candidate for Prescott (2004) [23] to explain the European aggregate hour gaps with the US, another important strand of literature focuses on the low employment rate in Europe. These studies encompass the ones that focus on unemployment\footnote{see e.g. Mortensen, Pissarides (1999) [18], Blanchard, Wolfers (2000) [3] or Ljungqvist, Sargent (2007) [14].} and those that analyze the non-participation.\footnote{see e.g. Gruber, Wise (2005) [8].} The main result of these studies is that incomes perceived during inactivity generate a large distortion on the employment rate: unemployment benefits, pensions or subsidized education induce an implicit tax on the labor supply. Hence, beyond the unemployment gap, the European countries are characterized by an employment gap. This leads us to focus on the employment rate, rather than on the restrictive measure of the unemployment rate\footnote{In this respect, we agree with the view presented in Rogerson (2006) [25]: the unemployment rate alone can not be interesting for the explanation of the gap between Europe and US. Given the institutional arrangements, only the employment rate can account for this gap.}, and on the impact of the distortions implied by the shifts of the labor market institutions. Hence, our contribution consists in distinguishing the elasticities of the hours worked per worker and of the employment rates to long run changes in taxation and labor market institutions: as it is suggested by Ljungqvist, Sargent (2007) [14], the standard neoclassical growth model used by Prescott (2004) [23] cannot account for the observed impact of both taxes and labor market institutions.

From a methodological point of view, we depart from Prescott (2004) [23] and Ohanian
et al. (2008) [19]: they only compute wedges in the static first-order condition governing labor supply in a calibrated version of the growth model.\(^{(11)}\)

In our paper, we instead follow McDaniel (2011) [16]: rather than simply focusing on the static first-order conditions, we solve for the time series of choice variables given country-specific tax rates, labor market institutions and productivity series, under perfect foresight.\(^{(12)}\) With this method, we encompass the measures based on the static wedges and the others, as the one proposed by Pissarides (2007) [21], where the steady state equilibrium variables depend on rate of growth of the exogenous variables. With this general equilibrium approach, we take into account the dynamics of the Solow residual and of the taxes on capital: these two components can not be ignored in a analysis of the long run evolutions of the input factors.

Finally, we also depart from Prescott (2004) [23] with respect to the modelling choice of the Government expenditures. Indeed, Prescott’s evaluation is done under the simplifying (caricatural) assumption that all government expenditures can be substituted by private consumption: hence, in Prescott’s view, the Government size is "excessive" by definition, because its optimal size is zero. This view is contestable: one can distinguish between “individual” goods public expenditures (education, health, etc.) and those intrinsically “collective” (army, justice, collective equipments). Unlike the first category, the optimal size of the collective public spending can not be zero because they are not "perfect" substitute to private consumption, as they cannot be made by the household herself.\(^{(13)}\) As documented in Figure 3, Euro zone countries, and particularly France, feature collective public spending (in proportion to GDP) comparable to other OECD countries.\(^{(14)}\) By contrast, individual Government spending is much larger. Hence, in our evaluation of a tax cut reform, we will only reduce the individual Government spending which induce a misallocation of consumption. In the model, we then distinguish these two type of Government spending by imposing that the optimal 'size' of the collective good is strictly positive.

What do we learn from our methodology?

Firstly, from a theoretical point of view, we show that:

\(^{(i)}\) there is a non-trivial interaction between the two labor margins leading to a substitution between hours per worker and employment,

\(^{(11)}\) Prescott (2004) computes this static wedge in two points (the early seventies and the mid-nineties), whereas Ohanian et al. (2008) compute this wedge at an annual frequency, on a more larger set of OECD countries.

\(^{(12)}\) From this last important point, we depart from Langot, Quintero-Rojas (2008) [13] who propose a search and matching model, but only account for the static wedges.

\(^{(13)}\) This view finds some empirical support in Ragan (2013) [24] and Rogerson (2007) [26]. They show that it is necessary to introduce these “collective” public spending in the utility function of the agent to account for labor market outcomes heterogeneity among OECD countries.

\(^{(14)}\) Data come from Langot et al. (2014) [12].
In each panel of Figure 2, for each year, the central mark is the median value over a sample of 32 OECD countries. The edges of the box are the 25th and 75th percentiles, the whiskers extend to the most extreme data points not considered outliers, and outliers are plotted individually.

(ii) taxes and labor market institutions do not have the same impact on these substitutions.

More precisely, under the assumption of a bargaining process which is efficient with respect to the hours worked, an arbitrary decrease in this intensive margin reduces the reservation wage of the worker leading to a rise in the employment rate at the equilibrium: the analysis of the intensive margin leads to an equilibrium relation between hours worked and employment which is decreasing in the plane \((h, N)\). Concerning the extensive margin, an arbitrary decrease of the employment rate is perceived by the agents as a wealth loss which can be compensated by an increase of the work effort by the members of the household having the chance to be in employment: the analysis of the extensive margin also leads to an equilibrium decreasing relation between hours worked and employment in the plane \((h, N)\). A decrease in taxes on labor increases the opportunity cost of leisure: the supply of hours worked increases whereas the reservation wage decreases. Whereas the first effect induce a positive wealth effect leading to a decline of the number of individuals at a work, the second decline of the reservation wage induces incentives to hire. We show that for realistic calibration of the model parameters, these two effects offset: only the number of hours worked is affected by changes in the labor taxes. The story is not the same for the labor market institutions. Indeed a change in the labor market institution has a direct impact only on the extensive margin: lowering wage pressures (by reducing the bargaining power of the workers or their unemployment benefits) rises the hirings. For all the households, this is perceived
as a positive wealth effect, leading them to reduce their hours worked by employee. Hence, the magnitude of such change in institutions can be large for the two labor margins.\(^{(15)}\)

Secondly, from a quantitative point of view, several points must be stressed:

\((i)\) the choice of these countries allows us to control for some experiments: the US workers are the 'non-treated' group because the tax wedge and the labor market institutions are stable over all the period, whereas the French workers form the group experimenting different treatments. In France, the taxes increase over the period and the labor market institutions shift in favor of the worker at the beginning of the first socialist government (the beginning of the 80s).

\((ii)\) Given our calibration strategy, which only restricts the averages of the simulated series of hours per worker and employment to match their empirical counterparts, we show that the model allows to predict the slope of the continuous decline in hours per worker, and the big changes in the employment rate after 1980.

\((iii)\) Finally, we can compute the welfare gains associated to a change in the policy, conditionally to year of entry in the labor market. For a French worker, given that the distortions induced by taxes and labor market institutions increase during all the period, the gains to adopt the US tax-benefits system raise from almost 8% of consumption in 1960, to 18% in 2010.

The paper is organized as follows. Section 2 documents the data used. In Section 3, the search and matching model is exposed. Section 4 brings the model to the data, after calibrating the key parameters of the model. Section 5 concludes.

\section{The basic facts}

In this section, we describe the data we use. Considered the differences among the evolutions of the two margins composing total labor input, we decide to focus on two countries at the extreme: the US and France\(^{(16)}\); we keep the UK as an 'intermediate' case lying in between the two. We can therefore show why the choice of these countries is interesting in the perspective of a test of the theory: since the time series of the exogenous variables (taxes, labor market institutions and technological progress) have not the same dynamics, we expect

\(^{(15)}\)Our analysis then extends the one by Fang, Rogerson (2009) [6] who study from a qualitative point of view the implications a model of labor supply and the search and matching frictions (without capital accumulation) on the interactions between the two margins of labor input at the steady state.

\(^{(16)}\)We prefer not to focus on Germany because of data availability, since it would be necessary to distinguish between West and East Germany till the reunification.
that it will be the case for the endogenous variables which are the hours per worker and the employment rate.

**2.1 Hours per worker and employment rates**

We present in this part the basic facts we want to account for with our model\(^{(17)}\). Figures 1 and 2 give a complete description of the dynamics of hours worked and employment in the selected countries. Table 1 summarize these data.\(^{(18)}\)

<table>
<thead>
<tr>
<th>Ratios in 2008 relative to 1960</th>
<th>US</th>
<th>FR</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{Nh}{Pop \times 365+14}$</td>
<td>1.00</td>
<td>0.66</td>
<td>0.85</td>
</tr>
<tr>
<td>$N$</td>
<td>1.10</td>
<td>0.95</td>
<td>1.03</td>
</tr>
<tr>
<td>$\frac{Nh}{h}$</td>
<td>0.91</td>
<td>0.70</td>
<td>0.82</td>
</tr>
</tbody>
</table>

For UK we report the ratios in 2008 relative to 1971

Total hours worked show a very different evolution along the last fifty years in the selected countries: while they have remained flat in the US, we observe an important decline in our representative European continental country (France), with the UK staying in between the experience of the US and its continental neighbor.

This evolution is in fact originated, as we have seen, by the path of the two margins which compose the total labor input: the intensive and the extensive one. In the US the employment rate showed an increasing trend between the mid eighties and the end of the nineties, while in France the employment rate decreased between the eighties and the mid of the nineties; the UK recently closed the "employment gap" with respect to the US which started to be observed in the mid seventies, as we can see in Figure 2. If we look at the intensive margin, we observe a sharp decline in France, while in the US hours per worker declined only slightly; the UK seems to share the evolution of its continental neighbor, but only till the mid eighties, when the decline slowed down.

**2.2 Taxes, labor market institutions and technological progress**

In this model we have three sets of exogenous variables: the set of tax rates (on labor income, on consumption, on capital revenues and on investment), a set of variables summing

\(^{(17)}\)We use the dataset constructed by Ohanian et al. (2012) [20]. We are really thankful to A. Raffo for having shared with us the last version of their dataset. A version of it can be found at the address http://www.sciencedirect.com/science/article/pii/S0304393211001139.

\(^{(18)}\)We choose 2008 as terminal point of the sample in order to purge our basic static from the last recession.
up the labor market institutions (the replacement rate and an indicator of the level of unionization, representing the bargaining power of the worker), and finally the Solow residual of the production function, representing the technological progress in labor productivity.

2.2.1 Taxes on labor

For what it concerns the data sources, the tax rates are taken from McDaniel (2007) [15](19).

Figure 4: Tax wedge $\frac{1+\tau_c}{1-\tau_w}$

We report in Figure 4 the evolution of this first exogenous variable. We observe that the tax wedge is stable over all the period in US. In 1960, it is necessary to produce 1.4$ in order to consume 1$. In 2010, the situation is unchanged. At the opposite, in France, these tax wedges increase rapidly between 1960 and 1985, and continue to grow after but at a lower rate. Whereas in 1960, it was necessary to produce 1.6$ in order to consume 1$, in 2010, it would be necessary to produce for a amount of 2.2$ in order to obtain 1$ of consumption. In the UK, the evolution of the tax wedge is comparable to what happened in France till the mid 1970s, while after that date it stopped to grow and its level today stays in between those of US and France.

2.2.2 Taxes on capital

Considering that our model is a general equilibrium one, it is important not omit the taxes on capital. These taxes modify the relative demand between capital and employment. There

are two type of taxes: on the revenues of the existing capital, and on the investment goods. The tax rates on capital revenues and on investment are taken from McDaniel (2007) [15] (20). We report in Figures 5 the evolution of this set of exogenous variables. As it is shown

Figure 5: Taxes on capital and investment 1960-2010

in Figure 2, the policy choices are not the same in the two side of the Atlantic. In the US and UK, the taxes are based largely on the capital incomes, whereas in France, the taxes are heavier on the investment. Remark that the gaps on the capital income taxes decline over all the sample, whereas the gaps in the investment taxes are persistent.

2.2.3 Labor market institutions

The replacement rate is taken from the OECD. Since it is available only for uneven years, we linearly interpolated the missing values. The bargaining power of the worker is considered as an average of two indicator, the union density and the union coverage. The data are taken from the Database on Institutional Characteristics of Trade Unions, Wage Settings, State Intervention and Social Pact (ICTWSS). (21)

Two statistical indicators are available to give an indirect measure of the bargaining power of the employee during the wage bargaining process: the union coverage and the union density. These two indicators are are closely linked to the bargaining power: a large union coverage or a high union density give to the worker the possibility to maker counter-offer during the bargaining process. We choose to evaluate the worker bargaining power by

(20) latest update: 2012; http://www.caramcdaniel.com/researchpapers
(21) the database is compiled by the Amsterdam Institute for Advanced Labour Studies (AIAS)
the average of the union coverage and the union density. Indeed, even if we observe a decline in the union density, the institutional agreements can conserve an "historical" coverage (the memory effect).

Figure 6: Labor market institutions 1960-2010

![Replacement rate](image1.png)  
![Bargaining power of the workers](image2.png)

We report in Figures 6 the evolution of these exogenous variables. A rapid look to these data suggests that the weight of the labor market institutions is very different in our selected countries: the unemployment benefits are more than two times lower in 2008 in the two English-speaking countries than in France; the path of the unemployment benefit level in UK is impressive: while at the beginning of the sample the replacement rate was more similar to the French one, starting from the eighties we observe a very strong policy intervention towards a decrease of it. In France we observe a large rise of the generosity at the beginning of the first socialist government (the beginning of the 80s). It is also important to notice that this rise of the generosity of the unemployment benefits is also accompanied by an increase of the eligibility: older workers (55 years old and more) can perceive benefits until their retirement age, without any constraint to search for a job. Given that this rise of the replacement rate is accompanied by less stringent eligibility rules, it can be viewed as an increase of the non-wage incomes for all the 16-65 years old. This French policy experience can thus be analyzed in a model which do not distinguish between unemployment and non-participation (retirement, pre-retirement...)

Considering the bargaining power of workers, we observe a continuous decline in the US, while the UK shows a very sharp change of direction in the eighties with respect to the evolution observed in the seventies. In France, we observe an continuous increase in the bargaining power of the workers from 1960 to 1990, and a stabilization at a high level after.
2.2.4 Technological progress

We recover the Solow residuals, measuring the labor augmenting technological process, from the production function as

\[ A = \left( \frac{Y}{K^{1-\alpha}} \right)^{1/\alpha} \frac{1}{Nh} \]

The Figure 7 reports the logarithm of the TFP time series. This figure suggests that there exists some "break" in these time series.

Figure 7: Solow residual in log

For what it regards the 'European' economies, there is a vast literature which asserts the existence of a period of technological catching-up after the material destructions of the WWII period. In order to capture this feature, we proceed in a very simplistic manner: we identify a linear trend for the TFP using only data starting from the mid\(^{(22)}\) or end\(^{(23)}\) of the eighties and then we use it to deflate the whole sample data. In this way our deflated TFP can keep the path of a technological catch up. This identification of the technological progress in Europe then consists to compute the expectations of the agents by setting the rate of growth of the technological progress at its long run value, the catch-up period being a transitory period during which the level of the technological progress is under its long run value. The convergence toward this long run trend is effective in the middle of the eighties.

For the US, we observe a break in 1990: the TFP seems to be higher after this year. Hence, as for the catch-up story in France, we assume that the TFP rate of growth is not

\(^{(22)}\)for France
\(^{(23)}\)for UK
affected by this episode. This rate of growth is estimated over the period 1960-1990. We then identify 1990-2010 as a transitory period where the level of the TFP is beyond its long run value.

3 Dynamic perfect foresight model

The model we use is a neoclassical growth model with search and matching frictions in the labor market. It is composed by a representative household, a representative firm and Government which runs a balanced budget every period.

3.1 Labor market

In the labor market the evolution of the stock of employment is given by the new matches $M_t$ which add to the non-destroyed jobs $(1 - s)N_t$:

$$N_{t+1} = (1 - s)N_t + M_t$$

where the matching function is $M_t = \Upsilon_t^\psi (1 - N_t)^{1-\psi}$. We highlight here that the separation rate $s$ is fixed and it differs from country to country. The labor market tightness is given by
\[ \theta_t = \frac{N_t}{1 - N_t} \] while \( f_t \) and \( q_t \) indicate respectively the job finding and the job filling probability:

\[
\begin{align*}
    f_t &= \Upsilon \theta_t^\psi \\
    q_t &= \Upsilon \theta_t^{\psi-1}
\end{align*}
\]

### 3.2 Households

The economy is populated by a large number of households, each of them consists of a continuum of identical infinitely lived agents. Each agent can be either employed or non-employed (thus free for occupying a job). Agents pool their incomes inside the household, so that they are fully insured against non-employment idiosyncratic risk. Agents consume and save by accumulating physical capital that they rent to firms. Agents pay taxes on their wage income, capital income, investment decisions and consumption. When they are non-employed they receive benefits from the Government. Investment is subject to capital adjustment costs.\(^{(24)}\)

Government expenditure \((G_t)\) can be distinguished in roughly two different types, the collective services expenditure and the individual services one. We consider then that the part of consumption that Government uses for individual services \((G^{ind}_t)\) is a substitute for private consumption (according to a degree given by the parameter \(\gamma\): if \(\gamma = 1\) then it is a perfect substitute), while the part which is affected to collective services \((G^{col}_t)\) enters also in the utility function of household, but in a separate way. This hypothesis seems appropriate since we want to evaluate the welfare consequences of a reduction in tax rates if the 'European' type of economy should shift to an 'American' taxing system: if Government consumption expenditure is financed by taxes, it is not reasonable to try to evaluate welfare under the hypothesis that Government expenditures are pure waste. The term \(\pi\) indicates the presence of a 'subsistence' term in consumption: this term is important to match the 'catching up' of the European countries with respect to the development levels of the US in the aftermath of WWII.\(^{(25)}\)

The benefits perceived during the periods of non-employment are expressed as a fraction (given by the replacement rate \(\rho\)) of the wage bill.

The program of a household is given by:

\[
W^h(N_t, K_t) = \max_{c_t, K_{t+1}} \left\{ \log(c_t + \gamma G^{ind}_t - \pi) + \chi \log(G^{col}_t) + N_t(-\sigma_l h^{1+\eta}) + (1 - N_t)\Gamma^u + \beta W^h(N_{t+1}, K_{t+1}) \right\}
\]

\(^{(24)}\)While the presence of these costs help in smoothing the reaction of the variable physical capital, the results of the model stay even if they are not present.

\(^{(25)}\)See for example Rogerson (2006) [25]
s.t.

\[ K_{t+1} = K_t(1 - \delta) + I_t \]

\[ I_t(1 + \tau_{i,t}) + c_t(1 + \tau_{c,t}) + \frac{\Phi}{2} (K_{t+1} - (1 + g)K_t)^2 = (1 - \tau_{w,t}) \left[ w_t h_t N_t + (1 - N_t)\bar{b}_t \right] \]

\[ + \pi_t + (1 - \tau_{k,t}) r_t K_t \]

\[ N_{t+1} = (1 - s)N_t + f_t(1 - N_t) \]

where, at the symmetric equilibrium, we have \( \bar{b}_t = \rho_t w_t h_t \). With the two first constraints, we obtain

\[ K_{t+1}(1 + \tau_{i,t}) + c_t(1 + \tau_{c,t}) + \frac{\Phi}{2} (K_{t+1} - (1 + g)K_t)^2 \]

\[ = (1 - \tau_{w,t}) \left[ w_t h_t N_t + (1 - N_t)\bar{b}_t \right] + \pi_t + [(1 - \tau_{k,t}) r_t + (1 - \delta)(1 + \tau_{i,t})]K_t \]

The first order conditions for the household are therefore given by

\[ \frac{1}{(C_t - \bar{c})} = \lambda_t(1 + \tau_{c,t}) \]

\[ \lambda_t[(1 + \tau_{i,t}) + \Phi (K_{t+1} - (1 + g)K_t)] \]

\[ = \lambda_{t+1}[(1 - \tau_{k,t+1}) r_{t+1} + (1 - \delta)(1 + \tau_{i,t+1}) + (1 + g)\Phi (K_{t+2} - (1 + g)K_{t+1})] \]

where \( C_t = c_t + G^\text{ind}_t \). The equation (3.1) shows how the 'subsistence' term changes the individual choices: its reduces the wealth effect when the economy is above its long run steady state. Indeed, with the exogenous growth, this component disappears because \( C_t \to \infty \) when \( t \to \infty \), whereas \( \bar{c} \) is constant.

### 3.3 Firms

There is a representative firm which produces using a Cobb-Douglas technology combining capital \( K_t \) and labor input \( N_t h_t \): \( Y_t = K_t^{1-\alpha}(A_t N_t h_t)^\alpha \). The technological progress \( A_t \) is labor augmenting, according to a balanced growth path. In order to hire workers the firm posts vacancies \( V_t \), the unit cost of keeping a vacancy open is given by \( \omega \), so that the total costs paid by the firm are given by the wage bill, the rental cost of capital and the vacancy posting costs.

The firm’s program is given by

\[ V^f(N_t) = \max_{V_t, K_t} \left\{ K_t^{1-\alpha}(A_t N_t h_t)^\alpha - w_t h_t N_t - r_t K_t - \omega_t V_t + \beta \frac{\lambda_{t+1}}{\lambda_t} V^f(N_{t+1}) \right\} \]
s.t. \( N_{t+1} = N_t(1 - s) + q_t V_t \). The first order condition for the firm are given by the following equations

\[
\begin{align*}
  r_t &= (1 - \alpha)K_t^{-\alpha}(A_t N_t h_t)^\alpha \\
  \frac{\omega_t}{q_t} &= \beta \frac{\lambda_{t+1}}{\lambda_t} \left( \alpha \frac{Y_{t+1}}{N_{t+1}} - w_{t+1} h_{t+1} + (1 - s) \frac{\omega_{t+1}}{q_{t+1}} \right)
\end{align*}
\] (3.3, 3.4)

### 3.4 Wage bargaining

Wage and hours are set by the firm and the worker simultaneously, according to a Nash bargaining scheme. Differently from most models, we allow for a time-varying bargaining power of the firm \( \epsilon_t \)

\[
\max_{w_t, h_t} \left( \frac{\partial W^h}{\partial N_t} \right)^{1-\epsilon_t} \left( \frac{\partial V^f}{\partial N_t} \lambda_t \right) \epsilon_t 
\]

The result of the bargaining process is given by the wage and the hour equations.

\[
\begin{align*}
  w_t h_t &= \left( \frac{1 - \epsilon_t}{1 - \rho_t \epsilon_t} \right) \frac{Y_t}{N_t} + \omega_t \left\{ \frac{1 - s}{q_t} \left( 1 - \frac{\phi_{t+1} (1 - \tau_{w,t+1})}{\phi_t (1 - \tau_{w,t})} \right) + \frac{\phi_{t+1} (1 - \tau_{w,t+1}) \theta_t}{\phi_t (1 - \tau_{w,t})} \right\} \\
  + \left( \frac{\epsilon_t}{1 - \rho_t \epsilon_t} \right) \frac{1 + \tau_{c,t}}{1 - \tau_{w,t}} (C_t - \bar{v}) \left( \Gamma^u + \sigma_t h_t^{1+\eta} \right) \\
  \sigma_t h_t^{\eta} &= \alpha \frac{Y_t}{N_t h_t} \frac{1}{(1 + \tau_{c,t}) C_t - \bar{v}}
\end{align*}
\] (3.5, 3.6)

where \( \phi_t = \frac{1 - \epsilon_t}{\epsilon_t} \). Then, the marginal labor cost per employee \( (w_t h_t) \) expresses the opportunity cost of working as the sum of the bargained surplus \( (BS) \) and the reservation wage \( (RW) \). The \( BS \) is composed by two components: the marginal productivity of the employee and the cost of the search activity\(^{(26)}\). During the bargaining process, the firm-worker pair shares the returns on the search process. For the worker, this is equal to the discounted time duration to find a job offer; for the firm, returns are instead equivalent to the discounted time duration to find a worker. These relative time spans cannot be proxied by the ratio of the average duration for these two search processes \( (\theta_t = \frac{C_t}{q_t}) \) as it would be the case when bargaining power and taxes are constant. Indeed, if workers expect that tomorrow their bargaining powers are close to zero \( (\phi_{t+1} \approx 0) \), the evaluation of the current match surplus is only driven by the search costs saved by the firm if the job is not destroyed: \( (1 - s) \frac{\omega_t}{q_t} \). At the opposite, when the bargaining power of the worker increases \( (\phi_{t+1} > \phi_t) \), the match value

\(^{(26)}\)Note that in the simple case where the bargaining power and the taxes are constant over time, we simply have \( BS = \alpha \frac{Y_t}{N_t} + \omega \theta_t \).
must be depreciated by the firm (it expects a decrease of its bargaining power), whereas the relative time spans must be over-evaluated by the worker because its bargaining power increases. Thus, the value of the search cost is a function of the bargaining power which itself changes over time, and is affected by the time-varying distortions induced by the taxes: this explain why BS is a function of dynamics of \( \epsilon \) and \( \tau \). The RW is given by the marginal rate of substitution of consumption for employment \( (C_t - \bar{c}) \left( \Gamma_u + \sigma_t \frac{h_t^{1+\eta}}{1+\eta} \right) \). Given that the non-employment benefit are proportional to the average wage, it raises the two components of this wage at the symmetric equilibrium.

Since we are assuming an efficient bargaining process, the equilibrium number of hours (the intensive labor supply) is determined jointly with wages. The equation (3.6) shows that, at the symmetric equilibrium, the solution is such that the marginal rate of substitution of consumption for an hour worked is equal to the marginal product of an hour worked, net of the tax wedge. This expression does not introduce any labor market institutions because we assume an efficient bargaining process over the hours worked, so that the hours contracts are only affected directly by the different taxes.

### 3.5 Market clearing

To close the model, the market clearing conditions on the goods market must be satisfied

\[
Y_t = \left( c_t + \frac{G^{ind}_t}{C_t} \right) + G^{col}_t + I_t + \omega_t V_t + \frac{\Phi}{2} \left( K_{t+1} - (1 + g)K_t \right)^2
\]

whereas the Government budget constraint is balanced at each date through lump-sum transfers given to the agents:

\[
TR_t = \tau_c c_t + \tau_w w_t N_t + \rho_t w_t h_t (1 - N_t) + \tau_i I_t + \tau_k r_t K_t - \rho_t w_t h_t (1 - N_t) - G^{col}_t - G^{ind}_t
\]

### 3.6 Stationarized FOCs

The model we described is a neoclassical growth model which allows for balanced growth path; in the economy we have two sources of growth: population is growing at rate \( g_n \), as well as technological progress, which is growing at the constant rate \( g_A \). Each of the three countries is characterized by a different rate of growth, but here we just stress that in order to have a stationary model, we deflate all growing variables by the common rate of growth \( g = g_A + g_n \). Here there are the equations that compose the model, where the convention is to indicate with \( \hat{X} \) a variable \( X \), for \( X \in \{A,C,Y,K,I,w,\omega\} \) which is deflated by the rate
of growth, ie. $\tilde{X}_t = X_t/(1 + g)^t$.\(^{(27)}\)

4 Quantitative results

The model is solved in perfect foresight: the path of all exogenous variables is known to the agents from the beginning.

First of all, we present our calibration strategy. Secondly, we plot the simulated series and the actual ones for the three countries, once the model is simulated with the identified parameters and the exogenous variables are specific to each country.

We then choose to focus mainly on the two countries at the extreme of the spectrum: the US and France. We inspect the functioning of the model by considering a steady state version to study the impact of a permanent change in policy variable on both the intensive margin (hours per worker) and the extensive one (in this case represented by labor market tightness).

Finally we come back to the fully dynamic model in order to study the whole impact of changes of policy variables: what would have been the evolution of the extensive and intensive margins in France with the US policies for taxation and labor market institutions? What would have been the impact of respectively US taxes and labor market institutions?

4.1 Identification of parameters

In order to solve the model, we need to identify some parameters. The set of all parameters is given by $\Theta = \{\beta, \delta, \sigma_l, \eta, \alpha, g_A, r, \Upsilon, \psi, \omega, \Gamma^u, s, g_n, \Phi\}$. We choose to calibrate the following subset of parameters

$$\Theta_1 = \{\beta, \delta, \alpha, \eta, \psi, s, g_A, g_n, \omega, \Phi\}$$

In particular, we set $\beta = 0.98$, $\delta = 0.05$ and $\alpha = 0.3$ according to standard values in the literature when the period of reference is one year. In particular for the evidence about the depreciation rate, we follow Gomme, Rupert (2007) \([7]\). In the long run, we have the following restrictions:

$$1 = \frac{\beta}{1 + g} \left[1 - \delta + \frac{1 - \tau_k}{1 + \tau_i} r \right]$$

$$r = (1 - \alpha) \frac{Y}{K}$$

$$I = (\delta + g) K$$

\(^{(27)}\)See the Appendix A for a complete description of the equation of the stationarized model.
For an observed value for $E[I/Y] \approx 0.17$ in the US, we obtain a gross interest rate $r \approx 12.35\%$, and thus $r - \delta \approx 7.35\%$, using the two last equations (the demand of capital). Remark that our value of $\alpha$ is such that the first equation (the supply of capital) is also satisfied, for the average values of the tax rates on capital and investment. Nevertheless, this standard calibration leads, as usual, to an over-estimation of the interest rate.$^{(28)}$ We then have chosen to reduce the interest rate of an amount which correspond to risk premium ($\kappa = 20\%$) which is paid by the firm when the uncertainty on its investment projects is taken into account by the financing contract. This leads us to $r(1 - \kappa) - \delta \approx 4.88\%$, which is more close to the long run value of the asset returns.

Our parameter $\eta$ is set to a value of 0.6 which implies a labor supply elasticity of 1.66. The parameter $\psi$, which represents the elasticity of the matching function with respect to vacancies, is set to a value of 0.5, which is an average of the range of possible values identified by Petrongolo, Pissarides (2001) $^{[22]}$ and it is widely adopted in the literature. For what it regards the country-specific separation rate $s$, we use the estimation results in Elsby et al. (2008) $^{[5]}$.

We then use information from our data to give a value to $g_A$ and $g_n$. Using our computation of the Solow residual for each country (see section 2.2.4), we fit a linear trend which is our $g_A$. Similarly, we fit a linear trend to the population 15-64 and we call it $g_n$. The total rate of growth of all non-stationary variables will therefore be $g = g_A + g_n$. In order to account for the technological catching-up after the material destructions of the WWII period in European countries, we identify a linear trend for the Solow residuals using only data starting from the mid/end eighties and then we use it to deflate the whole sample data. In this way our deflated Solow residuals for France can keep the path of a technological catch up. In order to catch on the other way the technological acceleration of the 1990s in the US, we identify in this case a linear trend for the Solow residuals using data till the beginning of the 1990s and use it to de-trend the whole sample data.

Before considering the value of $\omega$, let us remind that what matters in the FOC for the firm is a value which is given by the ratio $\omega/Y$ $^{(29)}$; we can use the information about the value of labor market tightness to calibrate the value of $\omega$ in order to match the average of $\theta$ over the period, while to find the the value of $\Upsilon$, as we will see below, we use additional information. We acknowledge that the implied value of total vacancy posting costs in terms of total wage bill which result from the simulation of the model for the three economies are very different: we get a value of $\omega V/q = 0.33whN$ for the US, $\omega V/q = 0.09whN$ for France

$^{(28)}$See Gomme and Rupert (2007) for a more complete discussion on this point.

$^{(29)}$the FOC for the firm is in fact $\frac{\omega \theta^{1-\psi}}{Y} = \beta \frac{\Lambda t+1}{N_t} \left( \alpha \frac{Y_{t+1}}{N_{t+1}} - w_{t+1} h_{t+1} + (1 - s) \frac{\omega_{t+1} \theta_{t+1}^{1-\psi}}{Y_{t+1}} \right)$
and $\frac{\omega}{q} = 0.2whN$ for UK \textsuperscript{(30)}. The first value is larger than the calibration of Mortensen, Pissarides (1999) \textsuperscript{(18)}, while the other two are lower: given that these author calibrate a representative country of the OECD, one can consider that our calibration procedure leads to parameters in the range of the admissible values.

Table 2: Calibrated parameters

<table>
<thead>
<tr>
<th>Common</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$\eta$</th>
<th>$\alpha$</th>
<th>$\psi$</th>
<th>$\Phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.98</td>
<td>0.05</td>
<td>0.66</td>
<td>0.3</td>
<td>0.5</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$g_A$</th>
<th>$s$</th>
<th>$g_n$</th>
<th>$\frac{e^{cont}}{Y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.017</td>
<td>0.12</td>
<td>0.0136</td>
<td>0.098</td>
</tr>
<tr>
<td>FR</td>
<td>0.019</td>
<td>0.15</td>
<td>0.0069</td>
<td>0.085</td>
</tr>
<tr>
<td>UK</td>
<td>0.024</td>
<td>0.17</td>
<td>0.0033</td>
<td>0.090</td>
</tr>
</tbody>
</table>

Table 3: Calibrated parameter (2)

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>Target $E(\theta)$</th>
<th>Implied $E(\theta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>23475</td>
<td>0.61</td>
</tr>
<tr>
<td>FR</td>
<td>1956</td>
<td>0.63</td>
</tr>
<tr>
<td>UK</td>
<td>3800</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Data for vacancies for US are taken from the Help Wanted Composite Index proposed by Barnichon \textsuperscript{(2)}, for France from Dares, for UK from the OECD

We then remain with four parameters to identify:

$$\Theta_2 = \{\sigma_l, \Gamma^u, \tau, \Upsilon\}$$

Considering the evidence about the effect of technological catch up that can be highlighted for the European countries, the value of $\tau$ should be different from zero only for France and the UK: as in Rogerson (2006) \textsuperscript{(25)}, Ohanian et al. (2008) \textsuperscript{(19)} and McDaniel (2011) \textsuperscript{(16)}, we introduce a consumption subsistence term in order to capture the fact that the level of hours worked was higher at the beginning of the sample in the countries which experienced a lower level of productivity with respect to the US \textsuperscript{(31)}.

\textsuperscript{(30)}If we look at the cost of posting one vacancy, normalised by the wage, we get $\frac{\omega}{w} = 0.54$ for the US, $\frac{\omega}{w} = 0.1$ for France and $\frac{\omega}{w} = 0.18$ for UK.

\textsuperscript{(31)}The presence of the term $\tau$ is important from a quantitative point of view to match the fact that in countries which were relatively poorer than the US hours worked were much higher at the beginning of the sample period and decreased very strongly in the catching-up period; see the Appendix D for a more detailed discussion.
We therefore choose to identify three parameters concerning the US and four parameters for France and UK. We fix the parameters by using three restrictions: we target four moments, as in McDaniel (2011) [16], which are the average value over the period of hours per worker \( (E(h)) \) and of the rate of employment \( (E(N)) \) in the US, France and the UK. We add an additional restriction: we want the preferences parameters \( \sigma_l, \Gamma^u \) to be the same in the three countries, so that any difference in the behavior of the economic variables predicted by the model will be therefore guided by difference in policy variables or "technological" conditions (both the Solow’s residuals and the matching technology efficiency). In the case of France and UK, for which we need to estimate four parameters, we make use of one additional information, i.e. the change in the level of hours worked in the first half of the sample, since it is known that the effect of the consumption subsistence term is important at the beginning of the sample and then its relevance diminishes as long as the productivity catch up process goes on in the European economies.

We report in Table 4 the values of the identified parameters, which are the solution of this just-identified system (six unknown and six moments).

<table>
<thead>
<tr>
<th>parameter</th>
<th>U.S.</th>
<th>FR</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_l )</td>
<td>5.5</td>
<td>5.5</td>
<td>5.5</td>
</tr>
<tr>
<td>( \Gamma^u )</td>
<td>-0.09</td>
<td>-0.09</td>
<td>-0.09</td>
</tr>
<tr>
<td>( \Upsilon )</td>
<td>0.55</td>
<td>0.36</td>
<td>0.46</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.35</td>
<td>0.1</td>
<td></td>
</tr>
</tbody>
</table>

### 4.2 The fit of the model

Given that the parameters are set in order to match some average values \( \{h; N\} \) and the slope over the first years of \( h \) in France and UK, the ability of the model to fit the observed data must be done using additional moments. We have some "natural" experiments in the historical data. Moreover, we report the simulated value of the ratio \( I/Y \) implied by the model and its counterpart in the data. Moreover, we report the simulated value of the ratio \( I/Y \) implied by the model and its counterpart in the data.

First, for the US, the taxes remain stable, whereas the labor market institutions (LMI) shift slowly in favor of the firms. When the simulation is performed using these country-specific policy variables, the model then predict a increase in the employment rate and a small decline in the hours per worker. The composition of these adjustments of the two labor market margins lead to a predicted total hours which slightly increase during the period. These results for the US economy are reported in figure 9 and is compared to the observed data. Given that there is no break point in the time series of the taxes and the LMI in
the US, the model reproduces the small and continuous changes observed simultaneously in the hours per worker and employment rate. The Mean Square Errors (MSE) for the hours worked and for employment are respectively equal to $6.3781 \times 10^{-5}$ and $7.6451 \times 10^{-4}$. Finally, we slightly under-estimate the dynamics of the ratio $I/Y$, but its size and its long run pattern seem to be well approximated by the theory.

Figure 9: The US economy

In France, the tax wedge does experiment at least three regimes. At the beginning of the sample, till 1985, it increases rapidly. Between 1985 and 2000, its increase is less great, whereas after 2000, we observe a significant decline (see Figure 4). In response to these tax rates, the model predicts that each French worker works fewer hours, with a small recovery
after 2000. This prediction is not rejected by the data (see Figure 10).

Figure 10: The French economy

On the French labor market, while the bargaining power of the workers remains stable over all the sample, it is not the case for the replacement rate: it largely increases in 1981 and 1985, then it remains stable and it begins to decline after 2002 (see Figure 6). In response to these large changes in the LMI, the employment rate predicted by the model largely declines at the beginning of the eighties, and increases at the end of the sample. These predictions are consistent with the data (see Figure 10), even if the elasticity of the model slightly overestimates the changes in the employment rate in France. The MSE for the hours worked and for employment are respectively equal to $2.9847 \times 10^{-4}$ and $8.7555 \times 10^{-4}$. Finally, remark

23
that the ratio $I/Y$ is well reproduced: it declines at the begin of the simulation, as in the sample, and then fluctuate around its means after 1985. This corresponds to the continuous increase of tax rate on capital before 1985, whereas after, its fluctuates around its mean.

The overall fit for the UK is worse than for the other two economies: the MSE for the hours worked and for employment are respectively equal to $2.7089 \times 10^{-4}$ and $0.0043$. We can remark that the model does a fairly good job in reproducing the dynamics for hours but only starting from the mid eighties, once the tax wedge on consumption and labor income stabilizes. For what it regards the employment rate, the model captures a tendency to decrease till the end of the seventies, and a recovery afterwards, mirroring the change in the evolution of labor market institutions; however it overestimates the elasticity of employment rate to these changes and it doesn’t capture the evolutions of the nineties.

4.2.1 The flow value of non-employment

As it has been pointed out by Shimer (2005) [27] and Hall (2005) [10] the performance of the search and matching model in reproducing the variability of employment is tightly linked to the mechanisms underlying the wage process, in particular the evolution of the so called "flow value of non-employment".\(^{(32)}\) We check then the evolution over time of the implied flow value of non-employment produced by our model. We report for the reader’s convenience the term which enters in the wage equation which represents the flow value of non-employment (or reservation wage):

$$RW_t = \left( \Gamma - \sigma_i \frac{h_t^{1+\eta}}{1+\eta} \right) \left( C_t - \bar{c} \right) \left( \frac{1 + \tau_c}{1 - \tau_w} \right) + \frac{\rho_t w_t h_t}{\text{replacement rate}}$$

We plot the results implied by our model\(^{(33)}\) in Figure 12.

Figure 12 puts in evidence two things. First of all, the implied flow value of non-employment remains inside some range which is widely accepted in the literature which is $[0.4; 0.943]$, the highest value being provided by Hagedorn, Manovskii (2008) [9] whereas the lowest being the one proposed by Shimer (2005) [27]. Nevertheless, it seems that the general equilibrium value of this flow value of non-employment is closer to the one proposed by Hagedorn, Manovskii (2008) [9]: this is not surprising because our model does not restrict this value to the replacement rate, as it is suggested in Shimer (2005) [27]. Finally remark that the levels of flow value of non-employment are quite similar. But it is important to have

\(^{(32)}\)A recent paper by Chodorow-Reich, Karabarbounis (2014) [4] focuses on the empirical counterparts of the elements we found in the flow value of non-employment when wages are set through a Nash bargaining mechanism.

\(^{(33)}\)We plot the measure of the reservation wage normalized with respect to the wage bill $w_t h_t$.
Figure 11: The UK economy

- Hours per worker
- Employment rate
- Total hours
- I/Y
Figure 12: Flow value of non-employment
in mind that in the US and in UK, the bargaining power of the workers is very low, leading the real wage to be close to this flow value of non-employment\(^{(34)}\) whereas in France, the bargaining power of the workers is large, implying that the real wage is larger than this flow value of non-employment.

Secondly we remark that the evolution of the flow value of non-employment for our countries is driven by different forces: in France the weight of the non-employment benefit part is much more important while the the value of "home staying" decreased (market hours decreased). In the US, the flow value of non-employment increases because the number of hours worked per employee rises over the sample: this endogenously increases the reservation wage of the US workers. In the UK the overall flow value of non-employment started to decrease after the mid eighties, mainly driven by the decrease of the non-employment benefit. At the opposite, in France, the flow value of non-employment decreases over all the sample: before the large increase of the replacement ratio at the beginning of the eighties, this decline of the reservation wage explains the small increase of the employment rate. After the eighties, this component of the flow value of the non-employment is dominated by the dynamics of the replacement rate: the employment rate declines after the beginning of the eighties. It is only during the 2000s that the large decline of the reservation wage, due to the dynamics of the hours worked, explains the small increase of the employment rate. Thus, the endogeneity of the reservation wage, and more precisely its strong link with the number of hours worked, underline the interest of our general equilibrium approach.

4.3 Inspecting the mechanisms of the model

In this subsection, we propose to inspect the mechanisms of the model in two steps: firstly, we focus on a steady state analysis, secondly we analyze the dynamics when only particular exogenous variables vary over the sample of the simulations.

Because of clarity concerns (and considered the worst performance of the model in fitting the data for the UK), we decide to concentrate on the two countries which lie at the extreme of the spectrum for what it regards the evolution of policy variables: the US and France.

4.3.1 Steady state analysis

Before considering the counterfactual experiments in the fully dynamic model, we look at final steady state and study the comparative static effects of a change in a policy variable (tax rates, replacement rate or bargaining power). For this purpose we can start from the steady state equations which represent our model and reduce the size of the system to only two

\(^{(34)}\)The observation of low bargaining power also supports the views of Hagedorn and Manovskii (2008) [9] who calibrate this parameter at a value equals to 0.061.
equations in two variables (hours and labor market tightness) to plot the two relations that determine the equilibrium. In doing this, we follow Fang, Rogerson (2009) [6] who also give a diagrammatic representation of the equilibrium of a search and matching model with both the extensive and the intensive margin of labor input. Differently from their exercise, our model includes capital as well as the replacement rate and is calibrated to match empirical data. At the steady state, we can reduce the system of steady state equations to three equations: (i) the hours per worker equation, (ii) the combination of the wage equation with the job creation condition, and (iii) the aggregate market clearing. They are respectively given by:

\[
Ch^{1+\eta} = \frac{\alpha (1 - \tau_w)}{\sigma_l (1 + \tau_c)} \left( \frac{r}{1 - \alpha} \right)^{\frac{(1-\eta)}{\alpha}} Ah
\]  

\[
\frac{\omega \theta^{1-\psi}}{\Upsilon} \left[ \frac{1}{\beta} - (1 - s) \right] + \left( \frac{1 - \epsilon}{1 - \rho \epsilon} \right) \omega \theta = \frac{\epsilon}{1 - \rho \epsilon} \left\{ (1 - \rho) \alpha Ah \left( \frac{r}{1 - \alpha} \right)^{\frac{(1-\eta)}{\alpha}} - C \left( \frac{1 + \tau_c}{1 - \tau_w} \right) \left( \Gamma^u + \sigma_l h^{1+\eta} \right) \right\}
\]

\[
C + \omega \theta \left( 1 - \frac{1}{1 + \frac{n+s}{\theta \omega \psi}} \right) = Ah \left( \frac{r}{1 - \alpha} \right)^{\frac{(1-\eta)}{\alpha}} \left( \frac{r}{1 - \alpha} - (g + \delta) \right) \]  

We substitute the expression for consumption which comes from the aggregate market clearing condition in the two other relations, two obtain a system in two equations and two variables that can be represented in the space \((\theta, h)\):

\[
\sigma_l h^{1+\eta}C(\theta, h) = \alpha Ah \left( \frac{1 - \tau_w}{1 + \tau_c} \right) \left( \frac{r}{1 - \alpha} \right)^{\frac{(1-\eta)}{\alpha}}
\]

\[
\frac{\omega \theta^{1-\psi}}{\Upsilon} \left[ \frac{1}{\beta} - (1 - s) \right] + \left( \frac{1 - \epsilon}{1 - \rho \epsilon} \right) \omega \theta = \left\{ (1 - \rho) \alpha Ah \left( \frac{r}{1 - \alpha} \right)^{\frac{(1-\eta)}{\alpha}} - C \left( \frac{1 + \tau_c}{1 - \tau_w} \right) \left( \Gamma^u + \sigma_l h^{1+\eta} \right) \right\}
\]

The first equation can be interpreted as the locus where the "intensive margin" is at the equilibrium, whereas the second is the locus where the "extensive margin" is at the equilibrium. These two relations can be interpreted as showing a trade off between the two margins of labor input for households as well as firms. Hence, our analysis can be viewed as an extension to general equilibrium framework to the one proposed by Fang, Rogerson (2009) [6]. The general equilibrium approach implies that the consumption is a function of the equilibrium values of \(h\) and \(\theta\), denoted by \(C(\theta, h)\) and implicitly defined by the equation (4.3).

**Proposition 1** For an equilibrium employment rate larger than 1/3, the resource constraint (4.3) always implies that \(\epsilon_{C|h} > 0\) and \(\epsilon_{C|\theta} > 0\).
Proof. See appendix C.1. Given that an employment rate equal to 1/3 is largely below what has been observed for all countries along all the time span, so that we can confidently say that in our model an increase in tightness implies an increase in consumption.

**Proposition 2** If the resource constraint (4.3) leads to $\epsilon_{C|h} > 0$ and $\epsilon_{C|\theta} > 0$, then the equilibrium intensive margin (equation 4.1) defines a negative relationship between hours worked $h$ and the labor market tightness $\theta$.

**Proof.** Differentiating the equations (4.1) and (4.3) leads to $\left(\eta + \epsilon_{C|h}\right) \frac{dh}{h} = -\epsilon_{C|\theta} \frac{d\theta}{\theta}$, where $\epsilon_{C|h} = C''_h \frac{h}{C(\theta,h)}$ and $\epsilon_{C|\theta} = C''_\theta \frac{\theta}{C(\theta,h)}$, where $C(\theta,h)$ is the consumption compatible with the resource constraint. The optimal choice of the intensive margin shows that the labor market tightness acts as wealth for the agent decisions: a high $\theta$ implies a high employment rate and then lower incentives to work for each individual in the household.

**Proposition 3** For $\frac{\eta}{1+\eta} > \rho$, there exists a value for $\Gamma^u < 0$ such that the equilibrium extensive margin (equation 4.2) defines a negative relationship between hours worked $h$ and the labor market tightness $\theta$.

**Proof.** See appendix C.2. The optimal choice of the extensive margin shows that a high $h$ implies an higher gap between the disutility at work and at home, leading to lower incentives to work for an additional worker in the household. This can be view as an increase of the wage reservation dues to the scarcity of leisure when $h$ increase.

**Comparative statics: counterfactual experiments.** Let’s now perform a comparative statics analysis: what would be on the final steady state the impact of a reduction in tax
rates? We can for example apply the U.S. tax rates on consumption and labor income to France and check the functioning of the model. We expect both the labor supply and the labor market equilibrium curves to shift upward. The overall effect on hours per worker is unambiguously positive, while the effect on the extensive margin is almost null. We can see graphically this mechanism in the left panel of the Figure 14. Indeed, for our calibration, the tax reduction gives incentives to work longer and at the same time, it also reduce the labor costs, inducing a rise in employment. But, this last effect is damped by the increase in the reservation wage of worker who now want to work longer. Hence, the reservation wage dynamics evicts the positive effect of the tax reduction on the employment rate.

Figure 14: SS comparative statics - France

![Graph](image)

What if we simulate the French economy with the benefits system of the U.S.? We see in the graphical representation in the right panel of the Figure 14 that a change in the replacement rate doesn’t affect the labor supply curve, but only the labor market equilibrium curve: it shifts toward the right so that the effect on the employment rate is strongly positive. Remark that when the workers are numerous to work, the generated wealth effect leads them to reduce their individual effort at work. Thus, the reservation wage is reduced and then the impact of the reduction of the replacement rate amplified. Considering that the 'intensive margin' curve is very flat, the effect on hours per worker (which overall decrease) is quantitatively less important.
4.3.2 The driving forces of the model

Before proceeding to the counterfactual experiments we may want to analyze the impact of the different driving forces of the model. In order to disentangle the effect of the evolution of the exogenous variables, we proceed as in McDaniel (2011) by "switching off" the effects caused by the different variables: we compare the outcome of the "full" benchmark model with that of

(i) a model in which only taxes vary;

(ii) a model in which taxes and technology vary, but labor market institutions do not.

In particular we fix the level of the constant exogenous variables to that one they attain in 2010, so that all the versions of the model share the same final steady state.

To sum up, while in the benchmark model the driving forces behind the evolution of the endogenous variables are the labor market institutions, the tax rates and technology, in model (i) the driving force is just the evolution of taxes, while in model (ii) the evolutions are driven by taxes and technology, while we abstract from labor market institutions.

If we look at hours for the US in Figure 15, we realize that the evolution of hours is mainly driven by taxes, while we completely miss the evolution of the employment rate by not considering the labor market institutions: even for the US case, the dynamics of the labor can not be negligible to account for the employment rate. This comes from the large decline of the bargaining power of US worker during the sample.

The effect of technology can be interpreted in the following way: when we "switch off" technology, we are actually feeding the model with the final level of productivity, which is higher than what observed for most of the sample, at least till the beginning of the 2000s (see Figure 8), so that the US citizens of the first counterfactual experiments are in general richer, so that till the beginning of the 2000s they work "less" than in the benchmark experiment.

If we look at the evolution of hours for France in Figure 16, we see clearly the explicative power of the tax wedge: hours remain flat till 1975 and then they start to decrease, reflecting the evolution of the tax wedge that we can see in Figure 4; on the other hand, the evolution of employment is not at all explained in this case. When we add the technological process of Figure 8, we introduce, as for the US, a "wealth effect" coming from the fact that agents are richer at the end of the period, but also a different temporal path, since in our framework agents know that their productivity increases till roughly the beginning of the 1980s: we have therefore two forces, the "wealth effect" and the "inter-temporal substitution effect" for households; from the point of view of the firms, it is less convenient to hire workers when they are less productive, so that from this point of view we expect that the growing productivity would help increase employment.
Figure 15: Counterfactual experiments for US: disentangling the contribution of exogenous variables

Hours if only taxes vary

Employment if only taxes vary

Hours if taxes and technology vary

Employment if taxes and technology vary
Figure 16: Counterfactual experiments for FR: disentangling the contribution of exogenous variables

Hours if only taxes vary

Employment if only taxes vary

Hours if taxes and technology vary

Employment if taxes and technology vary
4.4 The dynamic model: counterfactual experiments

Once we understood the main forces at work in the model, we can perform a counterfactual experiment with the fully fledged dynamic version. We consider France as our center of interest and we ask which would have been the evolution of the two margins of labor input with the path of policy variables which characterized the post WWII history of the US.

Let us start by comparing the evolution of the simulated variables for France when the economy is fed with the complete set of policy variables which characterize the US. This means that in our first experiment the two economies differ only with respect to the path of technological progress and few parameters: the separation rate, the rate of growth of population and the matching efficiency. In Figures 17 we see the results of the simulations of the French economy with both US taxes and labor market institutions. If we look at the evolution of hours per worker, we can notice that starting from the mid 1970s, once completed the technological catch-up process, the evolution of hours would have mimicked that of the US: this is natural once we set the preference parameters to be the same in the two countries. If we look at the evolution of the employment rate, it seems that with the labor market institutional arrangements which characterize the US, France would have observed a spectacularly high employment rate: at the end of the simulation, the employment will be 87.5% in this "fictive" France, whereas it is equal to 62.5% in the "real" economy.(35)

This simulation then gives the relative importance of the labor gaps: when both hours and employment gaps are simultaneously decrease, it appears that the employment rate is the more sensitive, showing that the weight of the employment gap is the largest.

If we consider now a situation in which we would have the evolution of the tax rates of the US in France, while keeping the labor market arrangements of this country, we see that the amount of hours worked would have been even higher than one predicted for the US: the agents, suffering less "distortions" on the intensive margin, would have tried to compensate the "low" labor input coming from the extensive margin. The important point here is that a simple reduction of the tax rates can not have a significant impact on employment because the decline of the labor cost they induce is evicted by the rise in reservation wage of the workers who work now longer.

If we look at Figure 19 we see that in this case we would have observed an employment level even higher than in the US. Agents in France, being strongly taxed, choose to work less than a American worker. The general equilibrium effects magnify these two direct effects. First, the large "chance" to be employed is perceived as a wealth for the agent, that reduces

(35) Speaking about the model functioning, the gap between the France with the US policy and the US comes from the fact that the two countries differ with respect to structural parameters which are the separation rate, the population growth rate and the matching efficiency, so that the overall "employment rate elasticity" differs between the two countries.
Figure 17: Counterfactual: US taxes and LMI in France

![Graph showing hours per worker (h) and employment rate (N) for the period 1965 to 2005, with data and model curves differentiated by line style.]

Hours per worker (h)  
Employment rate (N)

Figure 18: Counterfactual: France with US taxes

![Graph showing hours per worker (h) and employment rate (N) for the period 1965 to 2005, with data and model curves differentiated by line style.]

Hours per worker (h)  
Employment rate (N)
her incentive to work longer. Second, when worker reduces her hours worked, its reservation
decrease, leading to magnify the rise in the employment rate.

Figure 19: Counterfactual: France with US LMI

<table>
<thead>
<tr>
<th>Hours per worker (h)</th>
<th>Employment rate (N)</th>
</tr>
</thead>
</table>

4.5 Welfare comparisons

Once we performed the counterfactual experiments and we saw what would have been
the evolution of the total hours for France with the policy variables of the US, we can ask
ourselves what would have been the welfare implications of such an alternative historical
path.

To evaluate these welfare gains we compute the welfare of France in three cases: we
consider the situation in which taxes and labor market institutions would have been those
of the US, the case in which the taxes were the actual French ones but the LMI those of
the US and viceversa the case in which we keep the LMI of France but we use US taxes. In
any case the residual variable of adjustment in the Government budget constraint is given
by lump-sum transfers.

The 'reference' value of welfare is computed as that one that would have been chosen by
a benevolent social planner: we know that the first best economy coincides with a situation
in which the tax rates and the replacement rate would have been all set to zero, while the
bargaining power of the firms would have been set as to equalize the elasticity of vacancies
in the matching function. To sum up, a "first best" economy would be characterized by
the following conditions: $\tau_c = \tau_w = \tau_k = \tau_{inv} = \rho = 0$ and $\epsilon = \psi$, while the Government
consumption expenditure in collective goods would remain but it would be financed in a
non-distortive way through lump-sum taxes. The planner observe the same dynamics of the technological shocks than the private agents.

We therefore compute the rate at which we should "tax" the social planner in order to have an equivalent welfare level than the one of the market economy at each date; we define a factor \( \lambda \) entering the social welfare function as following

\[
\log((1 - \lambda_t)C_t) + \zeta \log(G^\text{col}_t) + N_t \left(-\sigma_l \frac{h_t^{1+\eta}}{1+\eta}\right) + (1 - N_t)\Gamma^u + \beta W_{t+1} = W^\text{actual}_t \text{ or } W^\text{cf}_t
\]

and we compute it so that at each date the 'taxed' planner welfare level is equivalent to the one of the 'actual' market economy (where the decentralized economy is either the benchmark case or one of the counterfactual experiments).

The value of \( \zeta \) is derived by considering the choice that a social planner would have made, i.e. considering the FOCs with respect to privately consumed goods and collective goods: \( \zeta = \frac{G^\text{col}_t}{C_t} \). In order to find the value of \( \zeta \) we consider the time series for \( G^\text{col}_t \) and we compute the mean value over the period, i.e. \( \zeta = \frac{\overline{G}^\text{col}}{\overline{C}} \).

Table 5: Value of \( \zeta \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>US</th>
<th>FR</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta )</td>
<td>0.139</td>
<td>0.1219</td>
</tr>
</tbody>
</table>

The results are shown in Figures 20.

This exercise can add some information with respect to the analogous one made by Prescott (2004) [23]. If we look at Figure 20, we observe the smallest welfare gap with respect to a 'first best' economy if all US policy variables would have characterized the French economy: in this case, the welfare gap would have increased till a level of 6%. Let us now consider a year, as for example 2005: at this time the welfare gain in changing the taxation scheme from the French to the American one, conditional on the fact that we keep the labor market institutions which characterize France, is measured by the difference between the line with circles and that one with stars: the vertical distance gives 7.5 percentage points. The idea is that if we do not distinguish between the intensive and the extensive margin of total hours worked, we would measure this gap looking at the distance between the circle line and the dashed-one, which would imply an overestimate of welfare gaps of around 2 percentage points. Anyway, it is an evaluation of the welfare gains of shifting to a 'US style tax system' which is clearly below the 19% suggested by Prescott (2004) [23]. Our results suggest that there is a complementarity between these two policies: if only one of two labor gap is reduced, then the other margin react in the opposite direction, damping the total.
effect of the reform. At the opposite, if tax rates and labor market institutions are reformed, then it is possible to exhibit positive effect on both extensive and intensive margin.

Finally, we can also consider the percentage points of output growth which has been 'lost' in France with respect to what could have happened with a set of policy variables as in the US: in Figure 21 we plot the history of output in the actual French economy and in the three counterfactual experiments normalised at their respective level in 1980.

5 Conclusions

In this paper we developed a dynamic perfect foresight model of neoclassical growth with labor market frictions which can account for the long run evolutions of both the extensive and the intensive margin of labor supply. We calibrated it to reproduce the evolutions of these two margins for two representative countries such as the United States and France. These two countries showed in fact very different evolutions with respect to the aggregate labor supply. We highlighted that there are non trivial interactions between the two margins, and we confirmed that quantitatively the evolution of the tax wedge can explain the path of hours worked per worker while labor market institutions cannot be ignored if we want to explain the evolution of employment. 

\footnote{We limit ourselves to consider only the bargaining power of workers and the replacement rate}
The interest of the model relies on the fact that it allows us to perform counterfactual experiments and to evaluate the welfare losses or gains of implementing some reforms: in this sense the country of interest is France, and we ask ourselves which would be the welfare gains of switching towards an "American-style" system for taxes and institutions. One of the potential policy implication of our exercise is that it seems pointless to advocate for a "liberalization" of labor market institutions if we do not think of diminishing the tax wedge at the same time.
References


Appendix

A Stationarized FOCs

The set of equations used to solve the dynamic paths of the model is then:

\[(1 + n)N_{t+1} = (1 - s)N_t + \min\left[\min(V_t, 1 - N_t), TV_t^{\psi}(1 - N_t)^{1-\psi}\right]\]

\[(1 + g)\hat{K}_{t+1} = (1 - \delta)\hat{K}_t + \hat{I}_t\]

\[\hat{Y}_t = \hat{K}_t^{1-\alpha}(\hat{A}_tN_th_t)^\alpha\]

\[\hat{H}_t = \hat{C}_t + \hat{I}_t + \hat{G}_t^{\text{col}} + \hat{\omega}V_t\]

\[\frac{(\hat{C}_{t+1} - \bar{c})}{(\hat{C}_t - \bar{c})} = \frac{\beta}{(1 + g)(1 + \tau_{c,t})(1 + \tau_{i,t+1})} \left[r_{t+1}(1 - \tau_{k,t+1}) + (1 - \delta)(1 + \tau_{i,t+1})\right]\]

\[r_t = (1 - \alpha)\hat{K}_t^{1-\alpha}(\hat{A}_tN_th_t)^\alpha\]

\[\frac{\hat{\omega}_t}{f_t} = \frac{\beta}{(1 + g)(\hat{C}_{t+1} - \bar{c})(1 + \tau_{c,t+1})} \left[\alpha \frac{\hat{Y}_{t+1}}{N_{t+1}} - \hat{\omega}_{t+1}h_{t+1} + (1 - s)\frac{\hat{\omega}_{t+1}}{f_{t+1}}\right]\]

\[\hat{w}_th_t = \frac{(1 - \epsilon_t)}{(1 - \rho_t\epsilon_t)} \left[\alpha \frac{\hat{Y}_t}{N_t} + \hat{\omega} \left(1 - s\right) \left(1 - \phi_{t+1} \left(1 - \tau_{w,t+1}\right) + \phi_{t+1} \left(1 - \tau_{w,t+1}\right) \theta_t\right)\right]\]

\[f_t = \min\left(V_t^{\psi}(1 - N_t)^{1-\psi}, 1\right)\]

\[q_t = \min\left(V_t^{\psi}(1 - N_t)^{1-\psi}, 1\right)\]

\[\theta_t = \frac{V_t}{U_t}\]

In order to ensure that the job finding rate and the job filling rate are in \([0, 1]\), we take the minimum between the unconstraint definition of these rates and 1. In accordance with these constraints, the matching function is also redefined.
B Steady state analysis

We then firstly report the complete model

\[(n + s)N = q(\theta)V\]
\[(g + \delta)K = I\]
\[Y = K^{1-\alpha} (ANh)^\alpha\]
\[Y = C + I + C^{col} + \omega V\]
\[1 = \frac{\beta}{1 + g} \left[ r \left(\frac{1 - \tau_k}{1 + \tau_t}\right) + 1 - \delta \right]\]
\[r = (1 - \alpha) \left( \frac{K}{ANh} \right)^{-\alpha}\]
\[\frac{\omega \theta}{f(\theta)} = \beta \left[ \frac{\alpha Y}{N} - wh + (1 - s) \frac{\omega \theta}{f(\theta)} \right]\]
\[wh = \frac{1 - \epsilon}{1 - \rho \epsilon} \left( \frac{\alpha Y}{N} + \omega \theta \right) + \frac{\epsilon}{1 - \rho \epsilon} \frac{1 + \tau_c}{1 - \tau_w} C \left( \Gamma^u + \sigma_t \frac{h^{1+\eta}}{1 + \eta} \right)\]
\[\theta = \frac{V}{U}\]
\[f(\theta) = \Upsilon \theta^{\psi}\]
\[q(\theta) = \frac{Y}{\theta^{1-\psi}}\]
\[N \sigma_t h^n = \alpha \frac{Y}{N} 1 - \tau_w \frac{1}{1 + \tau_c C}\]

We then report for clarity an intermediate step in the substitution. Let’s define the two following values:

\[r = \left[ \frac{1 + g}{\beta} - (1 - \delta) \right] \frac{1 + \tau_{inv}}{1 - \tau_k}\]
\[K = \left( \frac{r}{1 - \alpha} \right)^{-\frac{1}{\alpha}} ANh\]
We can therefore reduce the system of steady state equations to

\[ N(\theta) = \left( \frac{n + s}{T^{\theta \psi}} + 1 \right)^{-1} \]

\[ Y(\theta, h) = \left( \frac{r}{1 - \alpha} \right)^{-\frac{(1-\alpha)}{\alpha}} AN(\theta)h \]

\[ K(\theta, h) = \left( \frac{r}{1 - \alpha} \right)^{-\frac{1}{\alpha}} AN(\theta)h \]

\[ I(\theta, h) = (g + \delta) K(\theta, h) \]

\[ C(\theta, h) = Y(\theta, h) \alpha (1 - \tau_w) \left( \frac{1}{1 + \tau_w} \right) \]

By continuing in substituting, we arrive to the following three equations which represent respectively: the labor supply equation, the combination of the wage equation and the job opening condition and the aggregate market clearing.

\[ \frac{\omega}{\theta^{\psi}} \left[ \frac{1}{\beta} - (1 - s) \right] + \frac{1 - \epsilon}{1 - \rho \epsilon} \omega = \frac{\epsilon}{1 - \rho \epsilon} \left[ (1 - \rho) \alpha \frac{Y(\theta, h)}{N(\theta, h)} \left( \frac{r}{1 - \alpha} \right)^{(1-\alpha)} \right] \]

\[ Y(\theta, h) = C(\theta, h) + I(\theta, h) + \omega(1 - N(\theta)) \]

\[ C h^{1 + \eta} = \alpha \frac{1 - \tau_w}{\sigma_t (1 + \tau_c)} \left( \frac{r}{1 - \alpha} \right)^{-\frac{(1-\alpha)}{\alpha}} A h \]

\[ C + \omega \theta \left( 1 - \left( \frac{n + s}{\theta^{\psi} + 1} \right)^{-1} \right) = A h \left( \frac{r}{1 - \alpha} \right)^{-\frac{(1-\alpha)}{\alpha}} \left( \frac{n + s}{\theta^{\psi} + 1} \right)^{-1} \left( \left( \frac{r}{1 - \alpha} \right) - (g + \delta) \right) \]

C Proofs of proposition

C.1 Proof of proposition 1

Let \( \epsilon_{C|h} = C' h \frac{\partial}{\partial \psi} \) and \( \epsilon_{C|\theta} = C' \frac{\partial}{\partial \theta} \) where \( C(\theta, h) \) is the consumption compatible with the resource constraint. Differentiating the equation (4.3) with respect to \( h \), we obtain

\[ dC = \frac{A h (r - \theta s \left( \frac{r}{1 - \alpha} \right)^{1 - \eta} \theta^{-\eta} (n + s) + 1) dh}{(n + s + 1) (\theta^{\psi} + 1) (1 - \alpha)^{1-\eta}} \]
which is given by \( r = \left( \frac{1 + \tau_{\text{inv}}}{1 - \tau_k} \right) \left( \frac{1 + g}{\beta} - (1 - \delta) \right) \), to obtain that
\[
C_h' \frac{h}{C(\theta, h)} = -A \left[ \delta + g - \left( \frac{1 + \tau_{\text{inv}}}{1 - \tau_k} \right) \left( \frac{1 + g}{\beta} - (1 - \delta) \right) \left( \frac{1}{1 - \alpha} \right) \right] h \frac{1}{C(\theta, h)} > 0
\]
because the term inside parenthesis in the numerator is always negative. 

If we now check the derivative with respect to tightness, we find the following expression:
\[
\frac{\partial C}{\partial \theta} = \omega \left[ \frac{1 + \psi}{n + s} \left( \frac{1}{1 - \alpha} \right) - 1 \right] + \left[ -A \left( \frac{n + s}{1 + \psi} \right) \left( \frac{1}{1 - \alpha} \right) \right] \frac{1}{C(\theta, h)} \frac{\partial C}{\partial \theta}
\]
The second term in square brackets is always positive, so that the overall sign depends on the conditions on the first term in square bracket; we find that a sufficient condition to have an overall positive sign at the numerator is that the first term in square brackets is positive too, which is satisfied if \( \frac{n + s}{1 + \psi} (1 + \psi) > 1 \). Since we know that in steady state \( \frac{L}{n + s} = \frac{N}{1 - N} \), the previous condition reduces to \( \frac{N}{1 - N} (1 + \psi) > 1 \), i.e. \( N > \frac{1}{2 + \psi} \). In the most 'restrictive' case \( (\psi = 1) \), the condition would be satisfied for an employment rate at least equal to 1/3.

### C.2 Proof of proposition 3

Differentiating the equation (4.2) leads to
\[
\left\{ (1 - \psi) \frac{\omega \theta^{1 - \psi}}{1 - (1 - s)} \left[ \frac{1}{\beta} - (1 - \alpha) \right] + \frac{1 - \epsilon}{1 - \rho \epsilon} \omega \theta + \frac{\epsilon}{1 - \rho \epsilon} \Gamma_u \left( \frac{1 + \tau_{\text{inv}}}{1 - \tau_w} \right) C \partial_C/\theta \right\} \frac{d\theta}{h} = \left( \frac{\epsilon}{1 - \rho \epsilon} \right) \left\{ \frac{\eta - \rho (1 + \eta)}{c} A h \left( \frac{r}{1 - \alpha} \right)^{-\left(\frac{1}{1 - \alpha}\right)} \right\} \frac{d\theta}{h} \frac{C(\theta, h)}{h}
\]
With \( \Gamma_u < 0 \) and \( \eta - \rho (1 + \eta) > 0 \), the RHS is positive whereas the sign of the LHS is undermined. Its sign is negative iff
\[
(1 - \psi) \frac{\omega \theta^{1 - \psi}}{1 - (1 - s)} \left[ \frac{1}{\beta} - (1 - \alpha) \right] + \frac{1 - \epsilon}{1 - \rho \epsilon} \omega \theta + \frac{\epsilon}{1 - \rho \epsilon} \Gamma_u \left( \frac{1 + \tau_{\text{inv}}}{1 - \tau_w} \right) C \partial_C/\theta < 0
\]

\( (37) \)This can be seen more easily if we re-arrange it as following
\[
\delta \left( 1 - \frac{1}{1 - \alpha} \left( \frac{1 + \tau_{\text{inv}}}{1 - \tau_k} \right) \right) + \frac{1}{1 - \alpha} \left( \frac{1 + \tau_{\text{inv}}}{1 - \tau_k} \right) \left( 1 - \frac{1}{\beta} \right) + g \left( 1 - \frac{1}{\beta (1 - \alpha)} \left( \frac{1 + \tau_{\text{inv}}}{1 - \tau_k} \right) \right) < 0
\]
If we assume that $\Gamma_u = -\sigma_t^{1+\eta} + e < h$, we then have $\Gamma_u = -\mu \sigma_t^{1+\eta} + \mu < 1$. Using (4.1), we deduce that $\Gamma_u^{(1+\tau_c)}C = -\mu \frac{\alpha}{1+\eta} \frac{Y}{N}$. Hence the previous restriction can be rewritten as follows:

$$(1 - \psi) \frac{\omega \theta^{1-\psi}}{Y} \left[ \frac{1}{\beta} - (1 - s) \right] + \left( \frac{1 - \epsilon}{1 - \rho \epsilon} \right) \omega \theta < \left( \frac{\epsilon}{1 - \rho \epsilon} \right) \mu \frac{\alpha}{1+\eta} \frac{Y}{N} \epsilon \frac{C}{\theta}$$

Which is, when we assume for simplicity that $n \rightarrow 0$ and $\beta \rightarrow 1$

$$\frac{\omega \theta (1 - N)}{Y} \left[ 1 - \psi + \frac{1 - \epsilon}{1 - \rho \epsilon} \frac{N}{1 - \rho \epsilon} \right] < \left( \frac{\epsilon}{1 - \rho \epsilon} \right) \mu \frac{\alpha}{1+\eta} \frac{\alpha}{1-\alpha} (g + \delta) \frac{\frac{r}{1-\alpha}}{1 + \frac{n+\gamma}{Y}}$$

given that

$$\frac{dC}{dh \ C} = Ah \left( \frac{r}{1-\alpha} \right)^{\frac{(1-\alpha)}{\alpha}} \frac{1}{C} \left( \frac{r}{1-\alpha} - (g + \delta) \right)$$

$$\Leftrightarrow \epsilon_C |h = \frac{Y}{C} \left[ \frac{r}{1-\alpha} - (g + \delta) \right] = \frac{Y}{C} \left[ \frac{r}{1-\alpha} - (g + \delta) \right] - \omega \theta (1 - N)$$

Assume that $x = \frac{\omega \theta (1 - N)}{Y} < 1$ is given, we have

$$x \left[ 1 - \psi + \frac{1 - \epsilon}{1 - \rho \epsilon} \frac{N}{1 - \rho \epsilon} \right] < \left( \frac{\epsilon}{1 - \rho \epsilon} \right) \mu \frac{\alpha}{1+\eta} \frac{\alpha}{1-\alpha} (g + \delta) - x$$

where the largest value of the LHS is obtained for $N = 1/2$. Hence a sufficient condition is

$$x \left[ 1 - \psi + \frac{1 - \epsilon}{1 - \rho \epsilon} \right] \left( \frac{1 - \rho \epsilon}{\epsilon} \right) \frac{1 + \eta}{\alpha} \frac{\alpha}{1-\alpha} (g + \delta) - x \frac{\frac{r}{1-\alpha}}{1 + \frac{n+\gamma}{Y}} < \mu$$

### D The role of the subsistence term of consumption $\bar{c}$

In this section we provide a discussion about the role of the term $\bar{c}$: as already highlighted in the main text, the term $\bar{c}$ captures the "exceptionality" of the period between 1960s and 1970s for the economies which undertook a 'reconstruction' period. In order to check the importance of this hypothesis, we consider an alternative version of the model in which $\bar{c}$ is set to zero: the model would require therefore another set of parameters for both countries to be found by matching the same four moments as in Section 4.1.

With this alternative calibration, we show the performance of the model in replicating
the variables of interest for France. As we can see in Figure 22, without the subsistence term \( \tau \) the model underestimates the level of hours till the mid 1980s; the important point is that it predicts a decrease in hours worked, even if with a lower "speed".

We can also compute a simple measure of the 'lost' in the fit of the model by considering the Mean Squared Error (MSE) for hours for the two models: the MSE for hours with the alternative model is 3.35 times higher than that one of the benchmark model (the values are respectively \( 2.8209 \times 10^{-4} \) and \( 9.4368 \times 10^{-4} \)).

\[
\begin{array}{c|cc}
\text{parameter} & \text{US} & \text{FR} \\
\hline
\sigma_{i} & 4.8 & 4.8 \\
\Gamma^{u} & -0.09 & -0.09 \\
\Upsilon & 0.58 & 0.34 \\
\tau & 0 & 0 \\
\end{array}
\]
Figure 22: The French economy

Hours per worker

Employment rate

Total hours

I/Y