SOURCES OF UNCERTAINTY IN CONDUCTING MONETARY POLICY IN CHILE

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Monetary policy is made in an environment of substantial uncertainty. Consequently, academic researchers have sought to formally demonstrate the implications of uncertainty, as well as the ways in which central banks can manage it. The theoretical literature on uncertainty distinguishes between three types: additive uncertainty refers to central banks' lack of knowledge on the shocks the economy will face in the future; multiplicative uncertainty represents the lack of knowledge, or the erroneous knowledge, on one or more parameters of the behavioral model of the economy; and data uncertainty is associated with the fact that the information used by the central bank at the time policy decisions are made could be incorrect or could incompletely reflect the actual state of the economy. The objective of this paper is to review the quantitative relevance of these three types of uncertainty in the Central Bank of Chile's monetary policy. The paper is divided into two parts: the first covers the problem of data uncertainty and focuses on the output gap estimates for the full-fledged inflation-targeting period (1999 onward); the second centers on additive and multiplicative uncertainty for the period 1990–2006, with a special emphasis on the period after 1999.

Our analysis of data uncertainty focuses on the output gap because of its importance in forecasting inflation and because only preliminary figures for real output (real-time data) are available when monetary decisions are made. Also, the estimation of the output trend (part of the output gap) depends on statistical filters applied to output series, which contain these preliminary figures. For our exercise, we use several well-known univariate filters: the Hodrick-Prescott (HP) filter, the Baxter-King (BK) filter, the Christiano-Fitzgerald (CF) filter, the quadratic trend, and the Clark method based on the unobserved components model. To analyze their reliability and statistical accuracy with real-time data, we follow the methodology proposed by Orphanides and van Norden (1999). We find that revisions of the output gap in the case of Chile are important and persistent, and that correlations between the final data output gap and the real-time data output gap are relatively low. Nonetheless, the Clark method produces the best results, implying that caution should be used when evaluating the business cycle with real-time data and that using popular filters, like HP, could be misleading.

To evaluate the empirical importance of additive and multiplicative uncertainty, we use the methodology proposed by Zhang and Semmler (2005) to estimate behavioral equations for the Chilean economy with time-varying parameters and shocks with state-dependent variance (two states), which follow a first-order Markov process. To estimate behavioral equations, we use a slightly modified version of the forward-looking specification of Svensson (2000) and Al-Eyd and Karasulu (2008) for the equations that govern the behavior of a small open economy—namely, aggregate demand, the Phillips curve, and the real uncovered interest parity condition. We also use a technique from Kim (1993) to decompose total uncertainty, measured through the conditional variance of the forecast error, into two components: that associated with multiplicative uncertainty and that associated with additive uncertainty. We find that for all the behavioral equations of the economy, the uncertainty of shocks (that is, additive uncertainty) has been the most important factor in explaining total uncertainty. Moreover, the estimations support the hypothesis of state-dependent variances, as well as the hypothesis that these states could be considered periods of high and low volatility in the shocks. Also, total uncertainty of both the output gap and the inflation rate has declined over time, and the period of greatest stability coincides with the establishment of the full-fledge inflation-targeting framework for the conduct of monetary policy.[1]

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1. This period also coincides with the establishment of the structural surplus rule for the conduct of fiscal policy and with a highly stable international context.

The paper is organized as follows. Section 1 reviews the literature on the types of uncertainty faced by central banks, the implications for the conduct of monetary policy, and the way uncertainty is usually modeled empirically. In section 2, we analyze the quantitative relevance of data uncertainty, focusing on the output gap estimates. Section 3 explores the importance of additive and multiplicative uncertainty in the models typically used to study the effects of monetary policy. Finally, concluding remarks are presented in section 4.

1. MONETARY POLICY AND UNCERTAINTY

In the last few years, academic researchers have become increasingly interested in formally demonstrating how central banks can deal with uncertainty (Schellekens, 2002; Feldstein, 2003). Some papers study the distinct types of uncertainty faced by central banks, which introduce important challenges in the modeling of monetary policy, and their implications for the behavior of the monetary authority. This group of studies includes Isard, Laxton, and Eliasson (1999), Martin and Salmon (1999), Svensson (1999), Wieland (2000), Meyer, Swanson, and Wieland (2001), Tetlow and von zur Muehlen (2001), Giannoni (2002), Orphanides and Williams (2002), and Söderström (2002). Other papers propose different strategies for managing uncertainty, such as robust monetary policy rules and learning mechanisms. Examples include Craine (1979), Holly and Hughes Hallett (1989), Basar and Salomon (1990), Bertocchi and Spagat (1993), Balvers and Cosimano (1994), Sargent (1998), Onatski and Stock (2002), and Wieland (2000).

Feldstein (2003) argues that central banks typically face four types of uncertainty: uncertainty about the current and future states of the economy, uncertainty about how the economy operates, uncertainty of individuals about their personal futures, and uncertainty about the impact of potential future monetary policies. However, the most common classification defines three types of uncertainty: additive uncertainty, multiplicative uncertainty, and data uncertainty.[2] Additive uncertainty represents the component of a forecast error associated with the outcome of an exogenous variable in the system (the regression model error). This type of uncertainty captures central

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2. Another type of uncertainty considered in the literature, but not analyzed in this paper, is uncertainty about the probability distributions over possible events, known as Knightian uncertainty.

banks' lack of knowledge on the future shocks that the economy will face (Zhang and Semmler, 2005; de Grauwe and Senegas, 2006). Multiplicative (or parameter) uncertainty, in turn, represents the lack of knowledge, or the erroneous knowledge, of one or more parameters of the behavioral model of the economy (and its agents). Hall and others (1999) claim that this type of uncertainty can occur for several reasons, including the stochastic nature of the parameters, measurement errors in the data used to estimate the model, and structural changes. The distinction between additive and multiplicative uncertainty is based on the assumption that the true behavioral model of the economy is known. The limitation of this assumption is that total uncertainty, which could also result from misspecification of the model, is underestimated, so the results of any efforts to quantify this uncertainty using a particular specification of the behavioral model of the economy should be undertaken with caution.[3] Finally, data uncertainty is associated with the fact that the information used by the central bank at the time policy decisions are made could be incorrect or could incompletely reflect the actual state of the economy (Orphanides and van Norden, 1999). When these types of uncertainty are combined, they weigh heavily on policymakers (Rudebush, 2001). If policymakers have no knowledge of the actual state of the economy (regardless of whether the uncertainty lies in the data or in the behavior of the economy), they must base their decisions on expected outcomes. This could generate dilemmas in the adoption of an adequate policy if the outcome is not clear (for example, whether the central banks should react more aggressively or more passively).

Phillips (1954) and Theil (1964) were the first to introduce the idea of additive uncertainty, and their contributions have led to the expansion of the literature in this area. Phillips (1954), in studying whether the stabilization policy recommendations of simple models based on multipliers are appropriate and under what conditions this might be the case, showed that in a system that is automatically regulated (with flexible prices and interest rates), monetary policy could be a suitable instrument for stabilizing the economy, or at least for maintaining the economic system close to its desired values.

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3. Although part of the existing literature defines multiplicative uncertainty as the lack of knowledge on the parameters and on the model, the distinction between the two is important from a practical point of view. If the distinction is not made, it is not possible to separate the concepts of additive and multiplicative uncertainty, given that any specification error affects both the regression error and the magnitude of the parameters (bias).

Monetary policy should also be able to deal with all but the most severe shocks. Theil (1964) expanded on Phillips (1954) by assuming that the policymakers choose their policy by maximizing a quadratic expected utility. He found that in a world where there is only uncertainty in shocks, policymakers could conduct their policy as if there were total certainty regarding the possible outcomes of the economy. This result is known as certainty equivalence and has important implications for monetary policy.

The period in which Philllips and Theil were working was marked by a high degree of confidence in econometric modeling, such that any error could be eliminated in the estimation of structural models, except that associated with additive uncertainty. However, the principle of certainty equivalence is valid only under certain conditions, specifically those pertaining in a linear-quadratic world. The policy implications could therefore differ depending on the assumptions adopted regarding the behavior of the central bank (that is, its loss function). Walsh (2004) finds that optimal monetary policy rules, derived from a quadratic loss function for the central bank, are robust under this type of uncertainty and do not require that the monetary authority change its rule in the presence of shocks. However, simple Taylor reaction functions can generate important increases in the central bank's loss function depending on whether, based on particular situations, they require changes in the central bank's behavior. Sack (2000) estimates and simulates a vector autoregression (VAR) model for the U.S. economy under different assumptions. He finds that if the only source of uncertainty is additive, the U.S. Federal Reserve should behave more aggressively than it does in practice. He argues that other types of uncertainty, such as multiplicative, generate greater gradualism in the Federal Reserve's monetary policy.

Holt (1962) was the first to analyze multiplicative uncertainty (uncertainty in the parameters). While exploring linear decision rules for stabilization and growth, he shows that policymakers are only able to apply an active stabilization policy when they can adequately anticipate the implications of the policies they adopt. Otherwise, they would contribute more to the instability of the economic system than to its stability. If the way in which the economy reacts is uncertain—that is, if the parameters of the behavioral model of the economy are uncertain—then the performance of monetary policy could be seriously affected. The certainty equivalence principle is not valid in this context, and the central bank should consider this type of uncertainty when making policy decisions.

Brainard (1967) uses a quadratic utility function for the policymaker, similar to that of Theil (1964), to study the effect of uncertainty in shocks and parameters. He finds that the certainty equivalence principle is valid if the only source of uncertainty is associated with shocks. However, when the economy's reaction to policy actions is unknown (that is, when the model feedback parameters are uncertain), the central bank's behavior is seriously affected and it becomes optimal to respond more cautiously to changes in the economic system. This result has important practical implications for the conduct of monetary policy, since it indicates that it could be optimal for policymakers not to expect to completely eliminate the gap between the observed objective variable and its target value, in a given period. This could be interpreted as a justification for a gradual monetary policy. Although Brainard's (1967) result is quite intuitive and is widely discussed in the literature (see Blinder, 1998), it cannot be generalized. Papers such as Martin and Salmon (1999) and Sack (2000) may provide some empirical validity to Brainard's (1967) work, but other studies show that the results depend on the model specification.[4] For example, Söderström (2002) shows that when the coefficients of the lagged variables in the model are subject to uncertainty, the optimal policy could be for the central bank to react more aggressively.[5]

The study of data uncertainty is relatively new in the literature on monetary policy. Academics and policymakers have only recently invested resources in studying the properties of real-time data and the implications for policy decisions (Bernhardsen and others, 2005). Croushore and Stark (2001) were the first to construct a database that provides a snapshot of the macroeconomic data available at any given point of time in the past, with the objective of showing the implications of forecasting with revised and real-time data. In their database, they refer to the data for a particular date as vintage and the collection of the vintages as the real-time data set. This methodology has been used in various empirical applications, which primarily focus on developed countries. Examples of studies exploring the implications of real-time

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4. Both Martin and Salmon (1999) and Sack (2000) estimate a VAR model, the former for England and the latter for the United States. They show that incorporating multiplicative uncertainty in the model could explain the preference for gradualism in the central bank's behavior.

5. Other examples in support of the argument that multiplicative uncertainty does not necessarily lead the central bank to behave more cautiously can be found in Giannoni (2002) and Gonzalez and Rodriguez (2004).

data for monetary policy include Orphanides and van Norden (1999) and Orphanides (2001).[6] This literature highlights that the moment at which the data are obtained, their availability and their reliability for the empirical evaluation of policy rules are crucial for monetary policy performance, since they condition the decisions of the policymakers (Ghysels, Swanson, and Callan, 2002). In this regard, Rudebush (2001) and Bernhardsen and others (2005) argue that the new information that central banks obtain between two policy meetings does not justify drastic changes in the policy instrument, which can lead to very slow responses to particular economic events.

One of the variables that summarize the actual state of the economy—and that is thus crucial for monetary policy decisions—is the output gap. If potential output measures are not reliable, policy decisions may fail to react to the true economic conditions and may instead reflect measurement error. Orphanides and van Norden (1999) argue that the output gap is associated with important components of uncertainty, since central banks typically face at least three types of problems when evaluating the business cycle with real-time data. First, output data are revised continuously. Second, different methods of estimating potential output generally provide different results. When trend output is used as a proxy, different filtering procedures also yield a variety of outcomes; this problem is particularly critical with the end-of-sample estimates that are precisely the most relevant for policy decisions.[7] Third, a future evaluation of output data may indicate that the economy has experienced a structural change, which might not have been revealed by real-time data.

To illustrate these concepts, we consider the following economic model based on Zhang and Semmler (2005), which is standard in the literature of optimal rules of monetary policy:

$$\min_{\{\mathbf{u}_t\}_0^\infty} E_0 \sum_{t=0}^\infty \rho^t L(\mathbf{x}_t, \mathbf{u}_t), \tag{1}$$

subject to

$$\mathbf{x}_{t+1} = f(\mathbf{x}_t, \mathbf{u}_t, \varepsilon_t), \tag{2}$$

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6. For an excellent literature review for the case of the United States, see Kozicki (2004).

7. Kuttner (1994) and St-Amant and van Norden (1998) find substantial differences in the estimation of trends using final output data and different estimation methods.

where ρ is the discount factor bounded between 0 and 1, $L(\mathbf{x}_t, \mathbf{u}_t)$ is the loss function of an economic agent (in this case, the central bank), $\mathbf{x}_t$ is the vector of state variables, $\mathbf{u}_t$ is the vector of control variables (the policy instrument), ε_t is the vector of shocks, and E_0 is the mathematical expectation operator based on the initial values of the state variables. This kind of model represents the basic framework of monetary policy analysis and control used by Clarida, Galí, and Gertler (1999), Svensson (1997, 1999), and Beck and Wieland (2002), where the constraints in equation (2) are the Phillips curve, the IS curve, and the interest rate parity condition (Svensson, 2000).

Given the state equations in (2), the central bank's problem consists in deriving a path for its instrument (the control variable $\mathbf{u}_t$) that satisfies equation (1). The question that arises, however, is whether the state equations can be correctly specified with time series estimates. Given the previous discussion, the answer to this question is no, since these equations can be subject to a high degree of uncertainty caused by shocks (ε_t), as well as to parameter uncertainty and data uncertainty. This is particularly important since the optimal monetary policy rule is derived from the solution of the previous problem.[8] This rule therefore depends on the parameters of the state equations. If the model parameters are uncertain, the estimated "optimal" monetary policy rule could be unreliable.

The brief literature review presented in this section shows that the different types of uncertainty (namely, additive, multiplicative, and data uncertainty) have important and different implications for the conduct of monetary policy. When the economy is faced with additive uncertainty, or uncertainty about the shocks it will face, the central bank could potentially behave as if it has total certainty about the results of its policy; this is known as the certainty equivalence principle. This result, however, depends on the type of assumptions adopted regarding the behavior of the central bank (its preferences) and the structure of the economy, since this principle is only valid in a linear-quadratic world and depends on whether the monetary authority behaves optimally. With regard to multiplicative uncertainty, or uncertainty in the parameters, the fact that the central bank does not know how the economy reacts to its policies would, in principle, justify a preference for a more gradual monetary policy. There is no consensus on this result, however, and the literature shows that the assumptions that are adopted in a particular model can lead to

8. See, for example, Svensson (1999).

different implications, including a preference for a more aggressive policy response. Finally, data uncertainty arises when the data are unknown at the moment policy decisions are made, when they contain measurement errors (resulting from previous revisions), or when they are unobservable. Policy decisions are seriously conditioned to the available information. Nevertheless, sudden changes in policy when a new set of information becomes known may not be justified, since the actual information could present an erroneous notion of the actual state of the economy. The literature has sought monetary policy rules that are immune to this type of uncertainty, for example, by using output growth rates or unemployment level rates rather than the gap with respect to their natural values.

2. Data Uncertainty: The Output Gap

To analyze the quantitative relevance of data uncertainty in the case of Chile, we focus on the output gap—defined as the difference between actual (measured) gross domestic product (GDP) and its trend—for the period from 2000 to 2006. We chose this period for two reasons: (1) the availability of historical data taken from the output series publications at each moment in time; and (2) this is the period in which the Central Bank of Chile adopted a full-fledged inflation-targeting scheme to conduct its monetary policy. We use real-time data (that is, data available to the Central Bank at the time policy decisions were made) and various well-known methods to estimate the output trend. For each method, we analyze the behavior of the end-of-sample output gap estimates, which are relevant for policy decisions, as well as the revisions of these estimates across time. We present the statistical properties of the revisions and verify the reliability of the estimates for each method.

The section is divided into two subsections. The first describes the methodological issues related to the construction of the output gap with real-time data and the detrending methods; the second presents the estimation results and their implications.

2.1 Methodological Issues

Monetary policy decisions are typically based on real-time data, which are classified as preliminary data (Bernhardsen and others, 2005). This is also true, to a lesser degree, of very old historical data. The preliminary nature of the data calls for it to be in constant

revision. As suggested by the Central Bank of Chile[9], the data revision is motivated by factors such as the inclusion of new basic information (resulting from new sources of information or the improvement of these sources); the recalculation of the estimates (that is, revisions attributed to new estimates);[10] methodological improvements (reflecting changes in statistical methods, concepts, definitions, or classification); and error correction (either in the basic sources or in the calculations). One of the variables that encompasses the actual state of the economy and that is key for monetary policy decisions is the output gap. At the time policy decisions are made, this variable is estimated using preliminary output data, so it is necessary to assess the degree of reliability of these estimates.[11] For this assessment, we use real-time data to replicate the available information for the policymakers at every point in time. We thus simulate the actual environment of the monetary policy setting process (Ghlysels, Swanson, and Callan, 2002).

To analyze the reliability and statistical accuracy of the output gap estimates commonly used in the literature, we follow the methodology proposed by Orphanides and van Norden (1999). This consists of measuring, at each point in time, the degree to which the output gap estimates vary when the data are revised using different output gap estimation methods. This allows us to capture the effects caused by data revisions and the misspecification of statistical models used to estimate the output trend. The advantage of this methodological approach is that it does not require a priori assumptions on the true structure of the economy or on the process that generated the observed output time series. This approach has certain limitations: the analysis of the data revisions is based on a comparison of the output level observed at the end of the sample at every point in time with the "final" output, but there could still be measurement errors.

Orphanides and van Norden (1999) base their approach on two key definitions: the final and the real-time estimates of the output gap. The final estimate of the output gap is simply the difference between the last available vintage of output data and its trend (obtained via a detrending method). The real-time estimate, in turn, is a time series consisting of the last observed estimate of the output gap constructed

9. *Monetary Policy Report*, September 2004.
10.The recalculation of the estimates refers to the updating of either seasonal factors or the base period used in the constant price estimates.
11. If the output gap measures are not reliable, it could be advantageous, in some situations, for the central bank to base their monetary policy decisions on information on output growth (Orphanides and others, 2000; Bernhardsen and others, 2005).

as the difference between the observed output for each point in time (each vintage) and its trend. The real-time estimate for each period t contains all the revisions available up to that period and represents the estimate that the central bank may have calculated at the time policy decisions were made. Formally, assuming that we have access to the observed output series published at each point in time during N periods, we would have a matrix of the form $(\mathbf{y}^1, \mathbf{y}^2,..., \mathbf{y}^N)$, where each $\mathbf{y}^i$ (with $i = 1,..., N$) is a column vector that contains the output time series and where each column is an observation (row) shorter than the one that follows it.[12] If $f^{dt}(\cdot)$ is a function that detrends the time series $\mathbf{y}$, the final estimate of the output gap is given by

$$\text{GAP}^{final} = \ln(\mathbf{y}^N) - \ln[f^{dt}(\mathbf{y}^N)]. \tag{3}$$

If we then define the function $l(\cdot)$ as one that extracts the last real observation of the column vector $\mathbf{y}^i$, we have the real-time estimate of the output gap:

$$\text{GAP}^{real\text{-}time} = \ln[\, l(\mathbf{y}^1),\, l(\mathbf{y}^2),\, ...,\, l(\mathbf{y}^N)\,]$$
$$- \ln\{\, l[f^{dt}(\mathbf{y}^1)],\, l[f^{dt}(\mathbf{y}^2)],\, ...,\, l[f^{dt}(\mathbf{y}^N)]\,\}'. \tag{4}$$

The difference between the final output gaps and the real-time output gaps represents the total revision of the estimates at each point in time. The statistical properties of these series of revisions inform our evaluation of the reliability and accuracy of the output gap estimates. For the estimates drawn from equations (3) and (4), it is necessary to define the function $f^{dt}(\cdot)$ (the detrending method), given that in practice neither the true potential output of the economy nor its data-generating process are known. This is important since these methods generally provide quite different results. In the case of Chile, Gallego and Johnson (2001) find that the set of methods used to estimate the trend component of output provide a wide range of estimates. The method chosen thus constitutes a source of uncertainty in addition to the revisions in the data.

A detrending method decomposes real output y_t (measured in logarithms) into two components: trend (y_t^T) and cycle (y_t^C), such that $y_t = y_t^T + y_t^C$. We consider five alternative univariate methods that

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12. In the matrix $(\mathbf{y}^1, \mathbf{y}^2,..., \mathbf{y}^N)$, we consider the missing observations as imaginary numbers.

are widely used in the literature: the Hodrick-Prescott filter; the
Baxter-King filter; the Christiano-Fitzgerald filter; the quadratic
trend; and Clark's method based on the unobservable components
model.[13] Table 1 summarizes these methods and the models on
which they are based. We focus only on univariate techniques
of detrending, since the use of multivariate techniques requires
the compilation of information on the data that is not revised (in
real time) for each possible regressor in the model. Hence, the
conclusions that are derived from the analysis correspond only to
the evaluation of the univariate filters used here and cannot be
applied to other alternative methods such as those used by the
Central Bank of Chile and in some other papers for Chile (see
Gredig, 2007; Fuentes, Gredig, and Larraín, 2007).[14]

The Hodrick-Prescott (HP) filter is one of the most popular
detrending methods. It is based on choosing the trend that
minimizes the variance of the cyclical component of the series, and
it is subject to penalization for variations in the second difference
of the cyclical growth component (Hodrick and Prescott, 1997).
Both the Baxter-King (BK) filter and the Christiano-Fitzgerald
(CF) filter are based on smoothing the series through the use of
weighted moving averages. The fundamental difference between the
two, for the case of symmetric filters as considered in this paper,
lies in the choice of the objective function that defines the weights
(Baxter and King, 1999; Christiano and Fitzgerald, 2003). Moreover,
the Christiano-Fitzgerald filter imposes the restriction that the
filter weights add up to zero when unit roots are considered. The
quadratic trend, in turn, is a method of deterministic components
that assumes that the behavior of the trend series is triggered
by a second-order polynomial. This method is thus flexible at the
moment of detecting slow trend changes.[15] Finally, the unobserved
components model allows us to specify the data-generating processes
for the output time series and use these to identify the trend and

13. See Orphanides and van Norden (1999) for an extensive revision of the
detrending methods and its principal advantages and disadvantages. See Gallego and
Johnson (2001) for an interesting compilation of the use of the five methods in different
central banks.

14. The approach currently used by the Central Bank of Chile to estimate the
output gap is based on the production function.

15. Its simplicity has made it quite valuable for empirical applications related
to monetary policy (for example, Clarida, Galí, and Gertler, 1998), but its use has
generated much controversy based on the argument that better modeling of output
requires stochastic components in the model.

Table 1. Alternative Methods of Calculating the Output Trend

Method	*Calculation*
Hodrick-Prescott ($\lambda = 1{,}600$)	$y_t^T = \arg\min \sum_{t=1}^{T}\left[\left(y_t - y_t^T\right)^2 + \lambda\left(\Delta^2 y_{t+1}^T\right)\right]$
Baxter-King (6, 32)[a]	$y_t^T = \sum_{c=1}^{q+1}\omega^{BK}(1,c)y_{t+1-c} + \sum_{c=2}^{q+1}\omega^{BK}(1,c)y_{t+c-1}$ $t = q+1,...,n-q$
Christiano-Fitzgerald (6, 32, 1, 0, 0)[b]	$y_t^T = \sum_{c=1}^{q+1}\omega^{CF}(1,c)y_{t+1-c} + \sum_{c=2}^{q+1}\omega^{CF}(1,c)y_{t+c-1}$ $t = q+1,...,n-q$
Quadratic trend	$y_t = \alpha + \beta t + \gamma t^2 + y_t^C$
Clark (unobserved components)	$y_t = y_t^T + y_t^C$ $y_t^T = g_{t-1} + y_{t-1}^T + \nu_t$ $g_t = g_{t-1} + \omega_t$ $y_t^C = \delta_1 y_{t-1}^C + \delta_2 y_{t-2}^C + e_t$

Source: Authors' calculations.
a. The numbers 6 and 32 represent the minimum and maximum of the desired oscillation period for quarterly data.
b. The numbers 6 and 32 have the same interpretation as in the Baxter-King filter. The numbers 1 ,0, 0 represent the existence of unit roots, without drift and symmetric filter, respectively.

cyclical components. Clark (1987) proposes a model in which he assumes that the trend component follows a random walk process with drift and the cyclical component follows an AR(2) process. The main advantage of this type of model is that it allows a richer short-term dynamic specification for the model.

2.2 Results

The output data observed at each point in time were constructed using data compiled from the monthly publications (bulletins) of the Central Bank of Chile. We constructed an output series for each new statistical entry in which a new output record was published,

incorporating the revisions of the data published before.[16] For the quantitative evaluation of uncertainty in the output gap estimates, we consider the period between the first quarter of 2000 and the last quarter of 2006, although the output gap estimates were calculated based on information since 1986.[17] Hence, the first time series we use covers the period between the first quarter of 1986 and the first quarter of 2000. The series that follows contains an additional quarter not included in the previous series, and this occurs successively until the last series, which comprises the complete period from the first quarter of 1986 to the last quarter of 2006. Each output series was seasonally adjusted using the X-12-ARIMA procedure employed by the Central Bank of Chile. The series thus reflect both the revisions and the reestimation of seasonal factors. Finally, the series published in the last quarter of 2006 is our final output series, although we are aware that this series contains data that will be revised in the future.

The compilation of the information described above produced a total of twenty-eight output series for each point in time. We apply the five detrending methods to each of these estimates to calculate the output gap. Following the methodology applied by Orphanides and van Norden (1999), our final estimates are the output gap for the last available series and our real-time estimates are the series constructed with the last observation of each of the output gaps estimated with the twenty-eight series. Figures 1 and 2 illustrate these estimates using final and real-time data.

As the figures show, most of the estimations generated by the different detrending methods behave similarly in terms of their trajectories. This is true for both the estimations using final data and those using real-time data. The only exception is the estimation of the output gap based on the quadratic trend. Despite the comovements observed in the different series, however, the magnitude of the changes varies considerably among methods. The different methods also produce a wide range of output gap estimates. The average difference between the highest and lowest estimates is 6 percent with final data and 12 percent with real-time data. The order of magnitude of these differences

16. In some cases, the revisions were observed for one or two quarters back, while in others, such as the periods with base changes, the revisions were performed on the full series. The Central Bank revised the national accounts and changed the base year on two separate occasions during the sample period. The first time was in the fourth quarter of 2001, when the base year changed from 1986 to 1996 prices, and the second time was in the last quarter of 2006, when the base year changed to 2003. (The vertical dotted lines in figures 1 to 3 show these changes.)

17. For a statistical filter to produce reasonable results, we need at least one complete cycle in the series, which implies that long time series are necessary.

Figure 1. Output Gap Estimates for the Chilean Economy with Final Data

Percent
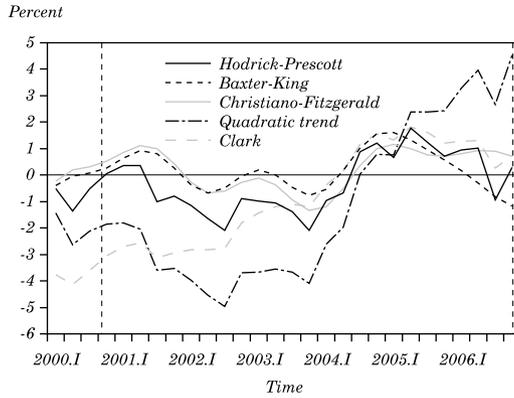
Time

Source: Authors' calculations.

Figure 2. Output Gap Estimates for the Chilean Economy with Real-Time Data

Percent

Time
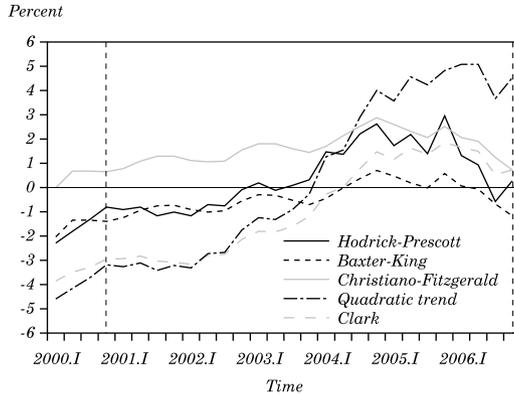
Source: Authors' calculations.

is considerable since they are much greater than the difference between the highest and the lowest points of the business cycle within the period considered (approximately 5 percent for both types of data and for a majority of filters). The average dispersion between methods is also important, reaching 2.3 percent with final data and 4.3 percent with real-time data. In addition, the estimations using final data tend to be clustered between the fourth quarter of 2004 and the third quarter of

2005. These estimates remain relatively close toward the end of the period of analysis, with the exception of the output gap based on the quadratic trend. This latter pattern is not observed with real-time estimates. To provide a qualitative idea of the importance of data revision, figure 3 shows the difference between the estimates with final data and those with real-time data for the five detrending methods. This difference represents the total revision in the output gap.

Figure 3. Total Revisions in the Output Gap for the Chilean Economy
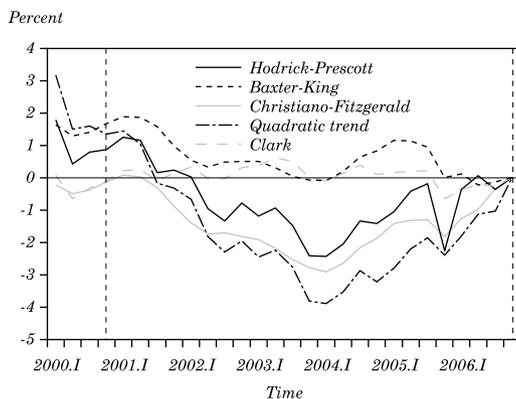
Source: Authors' calculations.

The figure reveals that the magnitude of the revisions is also important and differs substantially among the filters used, with an average dispersion of revisions among different measures of 2.8 percent. The most extreme cases are observed in early 2004, when revisions of the HP, CF, and quadratic trend methods were the most important in the entire sample. This is due to the fact that these filters do not adequately capture the turning point of the output gap in that period (see figures 1 and 2), and it suggests that real-time estimates were imprecise. This is not the case for the BK and Clark methods, which present practically null revisions at that same point in time. Rather, the most important revisions for these last two filters are seen at the beginning of the sample. To better understand the differences between the estimates with final data and those with real-time data, we present descriptive statistics of the output gap estimates and of the revisions for the five filters in tables 2 and 3, respectively. Figure 4 shows the time behavior of all these estimates.

Table 2. Descriptive Statistics of the Output Gap Measures calculated with Final and Real-Time Data

Filter and data	Mean	Absolute value	Standard deviation	Minimum	Maximum	Correlation
Hodrick-Prescott						
Final estimates	−0.003	0.010	0.011	−0.021	0.018	1.000
Real-time estimates	0.002	0.012	0.014	−0.023	0.030	0.611
Baxter-King						
Final estimates	0.002	0.006	0.007	−0.012	0.016	1.000
Real-time estimates	−0.005	0.007	0.007	−0.020	0.007	0.561
Christiano-Fitzgerald						
Final estimates	0.002	0.007	0.008	−0.013	0.012	1.000
Real-time estimates	0.015	0.015	0.007	0.000	0.029	0.203
Quadratic trend						
Final estimates	−0.012	0.028	0.029	−0.050	0.045	1.000
Real-time estimates	0.001	0.031	0.035	−0.046	0.051	0.841
Clark						
Final estimates	−0.010	0.019	0.020	−0.041	0.018	1.000
Real-time estimates	−0.011	0.020	0.020	−0.039	0.019	0.988

Source: Authors' calculations.

Table 3. Descriptive Statistics of the Total Revisions in the Output Gap

Filter	Mean	Absolute value	Standard deviation	Minimum	Maximum	AR(1)
Hodrick-Prescott	−0.005	0.010	0.011	−0.024	0.018	0.700
Baxter-King	0.007	0.007	0.007	−0.002	0.019	0.875
Christiano-Fitzgerald	−0.013	0.013	0.009	−0.029	0.001	0.939
Quadratic trend	−0.013	0.020	0.019	−0.039	0.032	0.842
Clark	0.000	0.002	0.003	−0.006	0.006	0.473

Source: Authors' calculations.

Figure 4. Estimation of the Output Gap and Total Revisions using Final and Real-Time Data for the Five Alternative Filters

A. Hodrick- Prescott

Percent

B. Baxter-King

Percent

C. Christiano-Fitzgerald

Percent

D. Quadratic trend

Percent

E. Clark

Percent

——— Final estimates
- - - - Real-time estimates
— · — Total revision
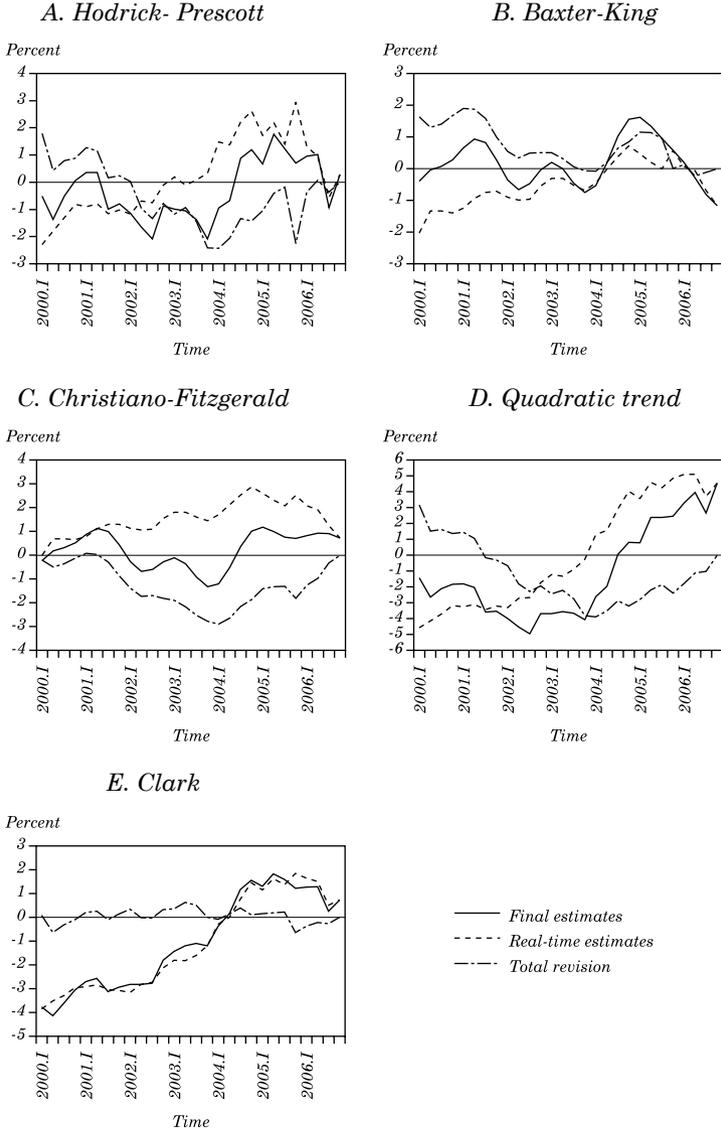
Comparing the results in tables 2 and 3 shows that, on average, total revisions are similar to or greater than the output gap estimates for all filters used.[18] Something similar occurs with the average gap in absolute value. This confirms the previous discussion, since the revisions are always significant in magnitude regardless of whether the economy is in a recession or is expanding. With respect to the minimum and maximum points of the cycle, the estimations with final and real-time data coincide with the minimum values of the gap only in the case of the Clark method (see figure 4, panel E), while the estimations coincide with the maximum values for the BK filters, the quadratic trend, and the Clark method (see figure 4, panels B, D, and E). This suggests that most of the methods fail to identify the magnitude of the recessive periods. The last column of table 2 shows the correlation coefficients between final data estimates and real-time data estimates for each filter. The highest correlations are observed for the Clark and the quadratic trend methods (over 0.8), while the CF and BK filters produce the lowest correlations. Another important element is the degree of persistence of the revisions, since as the revisions persist over time, the discrepancies between the final and real-time estimates tend to remain or disappear slowly over time. The last column of table 3 reports the estimated first-order autocorrelation coefficients for total revisions, which indicate that these revisions are highly persistent, with the exception of the Clark filter.

We have yet to address the issue of whether the output gap measures constructed with real-time data are reliable.[19] Since the different methods vary substantially with respect to the size of the cyclical component, it is more convenient to compare the reliability of the real-time estimates using independent scale measures. Table 4 presents the reliability measures used by Orphanides and van Norden (1999). In the first column, we present the correlation between final and real-time series for each detrending method. The other three indicators in table 4 measure in different ways the relative importance of the revisions (the ideal value for these three indicators is zero). The N/S indicator is the ratio of the standard deviation of the revision to that of the final estimate of the output gap and approximates the

18. This result is qualitatively similar to that found by Orphanides and van Norden (1999) for the U.S. economy.
19. We define reliability in terms of quantifying the difference between the final estimates and the real-time estimates. Our measures thus do not indicate anything regarding the reliability of each method as a tool for the estimation of the true output gap (Bernhardsen and others, 2005).

noise-to-signal ratio. The OPSING indicator shows the frequency with which the real-time estimates of the output gap reveal a different sign than the final estimates. Finally, the XSIZE indicator shows the frequency with which the absolute value of the revision exceeds the absolute value of the final estimates of the output gap. The Clark and the quadratic trend methods reveal smaller noise levels and smaller frequencies in observations with errors in the sign and with significant size in the revision. The CF filter performs the worst under these reliability measures.

Table 4. Descriptive Statistics of the Reliability Measures for Alternative Different Filters[a]

Filter	*Correlation*	*N/S*	*OPSIGN*	*XSIZE*
Hodrick-Prescott	0.611	1.055	0.286	0.500
Baxter-King	0.560	0.902	0.321	0.536
Christiano-Fitzgerald	0.203	1.229	0.393	0.750
Quadratic trend	0.841	0.650	0.071	0.214
Clark	0.988	0.156	0.000	0.036

Source: Authors' calculations.
a. The first column presents the correlation between the final and real-time series for each detrending method. The N/S indicator is the ratio of the standard deviation of the revision to the standard deviation of the final estimate of the output gap; it approximates the noise-to-signal ratio. The OPSING indicator shows the frequency with which the real-time estimates of the output gap reveal a different sign than the final estimates. The XSIZE indicator shows the frequency with which the absolute value of the revision exceeds the absolute value of the final estimates of the output gap.

In sum, the above results show that, in general, revisions of the output gap seem to be important and persistent for the period considered and that the correlations between the final and real-time estimates of the output gap are relatively low. Nonetheless, the Clark method provides the most favorable statistics. The analysis also reveals that the Clark method is the most reliable with real-time data.[20] Comparing our results with those of Orphanides and van Norden (1999) for the U.S. economy, we find that the different reliability measures generally produce similar values. These results

20. As a robustness test, we also calculated the reliability measures in real time using output gap estimations with unadjusted and seasonally adjusted data through the seasonal dummy variables. Our conclusions do not change (for details, see appendix A). This exercise was done to verify whether the reestimation of the seasonal factors, which is not present in the unadjusted data and is constant when we use seasonal dummy variables, substantially influences our results.

imply that caution should be used when assessing the level of the real-time estimates of the output gap, at least with the methodologies used here. Additionally, our results should be considered a lower bound for measurement errors that could be present in the output gap estimates, because comparisons are made with respect to a measure of the final output gap that could contain unrevised data.

3. ADDITIVE AND MULTIPLICATIVE UNCERTAINTY

To focus on the empirical importance of additive and multiplicative uncertainty, we use data for the 1990 to 2006 period but with emphasis on the 1999–2006 subsample, the full-fledged inflation-targeting period. We adopt a slightly modified version of the forward-looking specification of Svensson (2000) and Al-Eyd and Karasulu (2008) to estimate the behavioral equations of a small open economy, as is the case of Chile (namely, aggregate demand, the Phillips curve, and the real uncovered interest parity condition). As in Zhang and Semmler (2005), we do not include a monetary policy rule in this specification, given that the paper's objective is to analyze the primary sources of uncertainty faced by the Central Bank, which is associated with the structure and behavior of the economy.[21] To capture the sources of uncertainty, we estimate the model with time-varying parameters, assuming that shocks have state-dependent variances (two states) and that their behavior follows a first-order Markov process. This strategy allows us to decompose the conditional variance of the forecast error into two components: one associated with the parameters (multiplicative uncertainty) and one with the shocks in the model (additive uncertainty).

3.1 Methodological Issues

The existing literature on additive and multiplicative uncertainty typically uses models that explicitly consider the stochastic volatility potentially present in the errors (heteroskedasticity) and time-varying parameters (Zhang and Semmler, 2005). The papers that explicitly address parameter uncertainty include Cogley and Sargent (2002), who study the inflation dynamics of the United States in the postwar period using a Bayesian VAR with time-varying parameters. Another

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21. Moreover, the optimal monetary policy feedback parameters will depend on the structure and behavior of the economy.

example is Semmler, Greiner, and Zhang (2005), who estimate the Phillips curve and a monetary policy Taylor rule for the euro area, also with time-varying parameters. Both works find substantial changes in the model parameters. However, even though the evidence encountered when using models with time-varying parameters points to the existence of important degrees of uncertainty, this analysis cannot be separated from additive uncertainty in the modeling process. When additive uncertainty is not considered, volatility in the parameters could be exaggerated when it is indeed captured (Sims, 2002). Sims and Zha (2006), who study regime changes in the dynamics of the U.S. economy, find evidence in favor of stable model dynamics but unstable variance of the disturbances. In response, Cogley and Sargent (2005) modify their original model considering time-varying parameters and stochastic volatility; they also find the existence of regime changes. More recent examples of the estimation of Taylor rules with time-varying parameters and stochastic volatility can be found in Kim and Nelson (2006) and Zampolli (2006).

To incorporate both additive and multiplicative uncertainty, we follow the approach used by Zhang and Semmler (2005). We use a model with time-varying parameters and shocks characterized by state-dependent variance. In contrast to Cogley and Sargent (2005), who assume that the variance of the shocks changes each period, we assume that the variance has only two states (high and low) and follows a Markov process, as in Sims and Zha (2006).[22] This specification, besides having the advantage of dealing with both types of uncertainty in the same model, allows the decomposition of the variance of the forecast error into two components: one associated with additive uncertainty and one with multiplicative uncertainty (Kim, 1993).

The specification we use for the behavioral equations of the economy is a slightly modified version of the specification of Svensson (2000) and Al-Eyd and Karasulu (2008); it is a neo-Keynesian version for a small open economy comprising the IS curve (aggregate demand), the short-run supply curve (Phillip's curve), and the real uncovered interest parity condition (UIP). We diverge from these authors however, in allowing deviations of the UIP because of imperfections in the capital markets, capital controls, speculative bubbles, and so forth. As is usual in the modern dynamic stochastic general equilibrium (DSGE) literature, the deviations in the UIP are modeled by introducing a

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22. These authors assume that the variance of the regression errors follows a Markov process with three states.

backward-looking component in the original specification of Svensson (2000) and Al-Eyd and Karasulu (2008). The behavioral equations of the economy can thus be written as

$$y_t = \theta_{1,t}\, y_{t-1} + \theta_{2,t}\, E_t[y_{t+1}] + \theta_{3,t}\, r_{t-1} + \theta_{4,t}\, q_{t-1} + \varepsilon_t^d, \tag{5}$$

$$\pi_t = \phi_{1,t}\, \pi_{t-1} + \phi_{2,t}\, E_t[\pi_{t+1}] + \phi_{3,t}\, y_{t-1} + \phi_{4,t}\, q_t + \varepsilon_t^s, \tag{6}$$

and

$$q_t = \gamma_{1,t}\, E_t[q_{t+1}] + \gamma_{2,t}\, (r_t - r_t^f) + \gamma_{3,t}\, q_{t-1} + \upsilon_t, \tag{7}$$

where y_t represents the real output gap, π_t is the inflation rate, r_t is the short-term real interest rate, q_t is the real exchange rate, and r_t^f is the foreign real interest rate, observed in period t. The terms $E_t[y_{t+1}]$, $E_t[\pi_{t+1}]$ and $E_t[q_{t+1}]$ represent the expectations for period $t + 1$ of the output gap, the inflation rate, and the real exchange rate, respectively, conditional on the information available at period t (E_t is the expectations operator). The terms ε_t^d, ε_t^s and υ_t are shocks with state-dependent variances. The first two are aggregate demand and supply shocks, respectively, and the third is associated with the exchange market. As described by Al-Eyd and Karasulu (2008), this last disturbance term could be interpreted as a risk premium that captures the effects of unobservables, such as the exchange market sentiments. Finally, $\theta_{i,t}$ (with $i = 1, 2, 3, 4$), $\phi_{i,t}$ (with $i = 1, 2, 3, 4$), and $\gamma_{i,t}$ (with $i = 1, 2, 3$) are the time-varying parameters.

Two interesting observations can be made about this specification. First, the explicit inclusion of the exchange rate in the modeling process is relevant for an economy such as Chile, whose Central Bank uses inflation targeting as a monetary policy framework. Relative to the closed economy models, the specification introduces an important additional transmission channel of monetary policy and incorporates the external shock effect on the domestic economy. Second, the specification incorporates both forward-looking and backward-looking terms (hybrid model), for which there is empirical backing at least in the case of the Phillips curve (Caputo, Liendo, and Medina, 2006, and Céspedes, Ochoa, and Soto, 2005). Forward-looking terms can be justified by appealing to sticky price models of the Calvo (1983) type, whose wage-setting (or price-setting) mechanism is built in for a share of Chilean labor contracts.

The inclusion of forward-looking components, however, introduces the problem of how the components are measured or approximated, a choice that can have important implications for estimation properties (namely, consistency). The literature proposes various ways to deal with these variables and the most appropriate estimation techniques in each case. An obvious option is to use ex post data, that is, to approximate the expectation variables with their respective observed future values. While this option is operationally simple, it generates an endogeneity bias in the estimation of the model parameters, which leads to inconsistent estimates (Kim and Nelson, 2006).[23]

Galí and Gertler (1999), Roberts (2001), and Galí, Gertler, and López-Salido (2005) propose a methodology to address the endogeneity problem using ex post data for the forward-looking component of the model and instrumentalizing expectations through generalized method of moments (GMM) estimation. The use of GMM techniques to estimate the Phillips curve and the forward-looking Taylor rules is very common in the literature.[24] Along these lines, Kim (2004, 2006) proposes the application of instrumental variables for the estimation with endogenous regressors, using time-varying parameter models and regime changes. This methodological proposal solves the endogeneity problem by applying the Kalman filter in a two-stage Heckman (1976) estimation.[25] The specification of the behavioral equations in equations (5) to (7) can be rewritten in a state-space form under Kim's (2004, 2006) methodology as follows:

$$
\begin{aligned}
x_t &= \mathbf{w}_t'\boldsymbol{\beta}_{1,t} + \mathbf{v}_t'\boldsymbol{\beta}_{2,t} + \varepsilon_t, & \varepsilon_t &\sim \mathrm{N}\,(0,\,\sigma^2_{\varepsilon,S_t}); \\
\boldsymbol{\beta}_t &= \boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t, & \boldsymbol{\eta}_t &\sim \mathrm{N}\,(\mathbf{0},\,\mathbf{Q}_\eta); \\
\mathbf{v}_t &= \mathbf{Z}_t'\boldsymbol{\delta}_t + \boldsymbol{\xi}_t, & \boldsymbol{\xi}_t &\sim \mathrm{N}\,(\mathbf{0},\,\mathbf{Q}_\xi); & (8) \\
\boldsymbol{\delta}_t &= \boldsymbol{\delta}_{t-1} + \boldsymbol{\kappa}_t, & \boldsymbol{\kappa}_t &\sim \mathrm{N}\,(\mathbf{0},\,\mathbf{Q}_\kappa); \\
\sigma^2_{\varepsilon,S_t} &= \sigma^2_{\varepsilon,0} + (\sigma^2_{\varepsilon,1} - \sigma^2_{\varepsilon,0})S_t, & \sigma^2_{\varepsilon,1} &> \sigma^2_{\varepsilon,0};
\end{aligned}
$$

23. This is relevant because one of our objectives is precisely to study parameter uncertainty. Another straight-forward option is to use data from expectation surveys to construct a proxy variable of expectations (Roberts, 1995). This alternative has two potential problems: the first is associated with the availability of long time series for the estimation; the second is survey measurement error.

24. Several papers apply this methodology to Chile, including Céspedes, Ochoa, and Soto (2005), who estimates a hybrid Phillips curve, and Corbo (2002), who estimates a reaction function for the Central Bank.

25. Kim and Nelson (2006) use this methodology to estimate a forward-looking Taylor rule with ex post data for the United States.

where x_t represents a vector of state variables (y_t, π_t, and q_t for aggregate demand, the Phillip's curve, and the UIP, respectively), $\mathbf{w}_t$ is the vector of explanatory variables that are assumed to be exogenous or predetermined (y_{t-1}, r_{t-1}, and q_{t-1} for aggregate demand, π_{t-1}, y_{t-1}, and q_t for the Phillip's curve, and $r_t - r_t^f$ and q_{t-1} for the UIP), $\mathbf{v}_t$ is a vector of endogenous explanatory variables, which are correlated with the model errors ε_t (y_{t+1}, π_{t+1}, and q_{t+1}, respectively), $\mathbf{Z}_t$ is a vector of instrumental variables, $\boldsymbol{\beta}_t = (\boldsymbol{\beta}_{1,t}, \boldsymbol{\beta}_{2,t})'$ and $\boldsymbol{\delta}_t$ are vectors of time-varying parameters, $\boldsymbol{\eta}_t$, $\boldsymbol{\xi}_t$, and $\boldsymbol{\kappa}_t$ are Gaussian errors with a matrix of variances $\mathbf{Q}_i$ (with $i = \boldsymbol{\eta}, \boldsymbol{\xi}, \boldsymbol{\kappa}$), and S_t is an unobservable indicator variable that is equal to one in the high-volatility state and zero otherwise. We assume that the variance of errors ε_t present two states with transition probabilities that follow a Markov process and that are expressed as $\Pr[S_t = 1 \mid S_{t-1} = 1] = p$ and $\Pr[S_t = 0 \mid S_{t-1} = 0] = q$.

Kim (2006) proposes specifying the endogeneity in the model assuming that the correlation between the error term, ε_t, and the standardized forecast error associated with the endogenous variables, ξ_t^* (that is, the prediction error associated with the rational expectations of the agents), is constant and equal to ρ. On the other hand, in an earlier work that considers state-dependent variance of the errors, Kim (2004) suggests that this correlation will also be state dependent. The model error can thus be rewritten as

$$\varepsilon_t = \xi_t^{*\prime} \rho_{S_t} \sigma_{\varepsilon,S_t} + \sqrt{1 - \rho_{S_t}{}' \rho_{S_t}}\, \sigma_{\varepsilon,S_t} \omega_t,$$

with $\omega_t \sim N(0, 1)$. Using this last expression we can write the first equation of model (8) as

$$x_t = \mathbf{w}_t{}' \boldsymbol{\beta}_{1,t} + \mathbf{v}_t{}' \boldsymbol{\beta}_{2,t} + \xi_t^{*\prime} \rho_{S_t} \sigma_{\varepsilon,S_t} + \sqrt{1 - \rho_{S_t}{}' \rho_{S_t}}\, \sigma_{\varepsilon,S_t} \omega_t, \qquad (9)$$

with $\omega_t \sim N(0, 1)$, where $\rho_{S_t} = \rho_0 + (\rho_1 - \rho_0)S_t$ and S_t is the same indicator variable defined above. In this last equation, the model error is independent of $\mathbf{v}_t$ and ξ_t^*. Hence, the estimation generates parameters that are consistent. For the estimation, Kim (2004, 2006) proposes the following two-stage procedure. The first stage consists in estimating a model that instrumentalizes the endogenous variables using the maximum log-likelihood method based on the error forecast and the conventional Kalman filter. That is,

$$\mathbf{v}_t = \mathbf{Z}_t'\boldsymbol{\delta}_t + \boldsymbol{\xi}_t, \qquad \boldsymbol{\xi}_t \sim \mathrm{N}\,(\mathbf{0}, \mathbf{Q}_\xi);$$

$$\boldsymbol{\delta}_t = \boldsymbol{\delta}_{t\text{-}1} + \boldsymbol{\kappa}_t, \qquad \boldsymbol{\kappa}_t \sim \mathrm{N}\,(\mathbf{0}, \mathbf{Q}_\kappa).$$

(10)

The standardized forecast error of $\mathbf{v}_t$ is then calculated as

$$\xi_t^* = \mathbf{Q}_{\xi,t|t-1}^{-1/2}\left(\mathbf{v}_t - \mathbf{Z}_t'\boldsymbol{\delta}_{t|t-1}\right),$$

for all $t = 1, 2,\dots, T$. The second stage consists in using the calculated forecast error to estimate the following model using maximum log-likelihood techniques that combine the Kalman filter and the expectation-maximization (EM) algorithm proposed by Hamilton (1989, 1990):[26]

$$x_t = \mathbf{w}_t'\boldsymbol{\beta}_{1,t} + \mathbf{v}_t'\boldsymbol{\beta}_{2,t} + \xi_t^{*\prime}\rho_{S_t}\sigma_{\varepsilon,S_t} + \sqrt{1-\rho_{S_t}'\rho_{S_t}}\,\sigma_{\varepsilon,S_t}\omega_t, \quad \omega_t \sim \mathrm{N}\,(0,1);$$

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t\text{-}1} + \boldsymbol{\eta}_t, \qquad\qquad\qquad\qquad\qquad\qquad \boldsymbol{\eta}_t \sim \mathrm{N}\,(\mathbf{0}, \mathbf{Q}_\eta);$$

(11)

$$\sigma_{\varepsilon,S_t}^2 = \sigma_{\varepsilon,0}^2 + (\sigma_{\varepsilon,1}^2 - \sigma_{\varepsilon,0}^2)S_t, \qquad\qquad\qquad \sigma_{\varepsilon,1}^2 > \sigma_{\varepsilon,0}^2;$$

$$\rho_{S_t} = \rho_0 + (\rho_1 - \rho_0)S_t.$$

Finally, Kim (1993) suggests a procedure, using specification (8), to decompose the conditional variance of the forecast error (f) into two components: f^1, the conditional variance resulting from changes (lack of knowledge) in the model parameters (that is, multiplicative uncertainty) and f^2, the conditional variance given the heteroskedasticity in the error term (additive uncertainty).[27]

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26. The estimation algorithm is presented in appendixes B, C, and D. A potential limitation of this methodology for estimating the behavioral equations of the economy is that Kim (2004, 2006) assumes that the shocks associated with each equation are independent of each other, and he thus does not take advantage of the information contained in the possible correlations (that is, common states). In other words, the methodology permits the estimation of each equation separately, so the different states of the shocks will not necessarily coincide for the three equations. Zhang and Semmler (2005) find very different occurrence probabilities for each state of the shocks depending on whether they are dealing with aggregate demand or the Phillips curve, indicating that the states in the model do not coincide in the same time period.

27. In this paper, Kim (1993) identifies the sources of uncertainty and their importance associated with the process of monetary creation in the United States.

In this procedure, Kim exploits the informational structure of the model related to the probability distributions in the different states. The conditional variance stemming from multiplicative uncertainty depends on the state in a previous period, while the conditional variance from additive uncertainty depends on the state in the current period. This decomposition is quite useful since it gives us the percentage of the total variance of the forecast error that is caused by each source of uncertainty. Formally,[28]

$$f_t = f_t^1 + f_t^2$$

$$
\rightarrow
\begin{cases}
f_t^1 = (\mathbf{w}_{t-1}, \mathbf{v}_{t-1}) \left\{ \displaystyle\sum_{i=0}^{1} \Pr\!\left[S_t = i \mid \psi_{t-1}\right] \right. \\[2em]
\qquad \left. \left[\mathbf{P}_{t|t-1}^i + \left(\tilde{\boldsymbol{\beta}}_{t|t-1} - \boldsymbol{\beta}_{t|t-1}^i \right) \left(\tilde{\boldsymbol{\beta}}_{t|t-1} - \boldsymbol{\beta}_{t|t-1}^i \right)' \right] \right\} (\mathbf{w}_{t-1}, \mathbf{v}_{t-1})' \\[2em]
f_t^2 = \sigma_{\varepsilon,S_t}^2 = \sigma_{\varepsilon,0}^2 + (\sigma_{\varepsilon,1}^2 - \sigma_{\varepsilon,0}^2) \Pr\!\left[S_t = 1 \mid \psi_{t-1}\right]
\end{cases}
\qquad (12)
$$

where

$$\tilde{\boldsymbol{\beta}}_{t|t-1} = \sum_{i=0}^{1} \Pr\!\left[S_t = i \mid \psi_{t-1}\right] \boldsymbol{\beta}_{t|t-1}^i$$

and where $\mathbf{P}_{t|t-1}^i$ is the variance-covariance matrix of $\boldsymbol{\beta}_{t|t-1}^i$ in state i.

3.2 Results

To estimate equation (8), we use quarterly data for the period beginning in the first quarter of 1990 and ending in the last quarter of 2006. The output gap, y_t, is the difference between the observed GDP and its trend, calculated using the HP filter. We use the HP filter because it is one of the most commonly used filters in the literature and it thus allows us to compare our results with those of other papers that estimate behavioral equations for Chile. Although the Clark

28. For details on the formal derivation of the decomposition of the conditional variance of the forecast error, see Kim and Nelson (1999).

filter behaves best with real-time data, according to the results in the previous section, this does not imply that it is the best filter to estimate the "true" output trend. Additionally, our measure of the output gap is "final" output, based on an output series that ends in 2006. Thus, the uncertainty associated with data revisions is not included in the types of uncertainty analyzed in this section.[29] The quarterly inflation rate, π_t, is measured as the quarterly variation of the underlying consumer price index excluding regulated prices and prices of fuel and of some perishable goods such as fruits and vegetables (CPIX). As in Céspedes, Ochoa, and Soto (2005), we use the CPI variation instead of the implicit deflator variation of the GDP since the latter is measured with considerable noise in the case of Chile and is strongly influenced by variations in the terms of trade. Also, the Central Bank's inflation target is expressed in terms of CPI variation. In the case of the real exchange rate, q_t, we chose the bilateral exchange rate index with the United States. Finally, the foreign and domestic short-term interest rates, r_t and r_t^f, are defined as the monetary policy rates of Chile and the United States, respectively. All the previous data were obtained from the Central Bank's database. Table 5 shows the parameters estimated using Heckman's two-stage procedure detailed in Kim (2004, 2006).[30] The parameters presented in this table are not structural parameters of the model.

We would like to highlight two interesting results. The first is that variances of shocks confirm that there are two states in the three behavioral equations: a high-volatility state and a low-volatility state. For the aggregate demand estimations, the variance of shocks in the high-volatility state is substantially greater than in the low-volatility state (0.48 versus 0.05). The difference between these variances for the Phillips curve is just as large (0.54 and 0.03 in the high and low-volatility states, respectively). We obtain similar results in the case of the UIP (3.75 versus 2.45), although the magnitude of the difference is not as large as in the previous two cases. All the variances are statistically significant, with the exception of the variance associated with the high-volatility state of the Phillips curve. Finally, while the variances of shocks for the UIP do not differ significantly, the size of the variances is considerable compared with those found for aggregate

29. The way detrending is done may affect the estimations, so we run a robustness analysis below.

30. In applying the Kalman filter in the evaluation of the likelihood function, we eliminated twelve observations at the beginning of the sample owing to the presence of nonstationary time series in the model; see Kim and Nelson (1999).

Table 5. Estimation of the Behavioral Equations

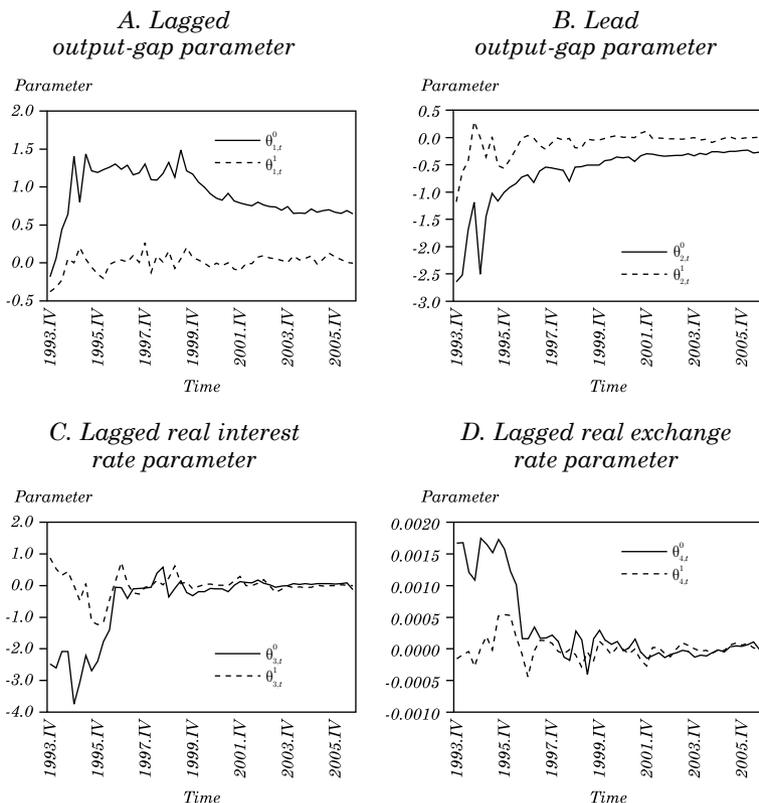| Aggregate demand			Phillips curve			Real uncovered interest parity		
Parameter	Estimated value	Standard deviation	Parameter	Estimated value	Standard deviation	Parameter	Estimated value	Standard deviation
p	0.6571	0.5267	p	0.6639	1.5101	p	0.9479	3.2325
q	0.6586	0.0644	q	0.8475	0.0501	q	1.0000	0.0001
$\sigma_{\eta_1}^{\theta}$	0.0697	0.2565	$\sigma_{\eta_1}^{\phi}$	2.4407	1.0338	$\sigma_{\eta_1}^{\gamma}$	0.0000	0.0026
$\sigma_{\eta_2}^{\theta}$	0.0797	0.2441	$\sigma_{\eta_2}^{\phi}$	1.2700	0.8449	$\sigma_{\eta_2}^{\gamma}$	0.0007	2.3801
$\sigma_{\eta_3}^{\theta}$	0.2942	0.2540	$\sigma_{\eta_3}^{\phi}$	0.0000	0.0001	$\sigma_{\eta_3}^{\gamma}$	0.0000	0.0029
$\sigma_{\eta_4}^{\theta}$	0.0002	0.0002	$\sigma_{\eta_4}^{\phi}$	1.6518	0.9554			
$\sigma_{\varepsilon,0}$	0.0570	0.0098	$\sigma_{\varepsilon,0}$	0.0329	0.0084	$\sigma_{\upsilon,0}$	2.4467	0.2295
$\sigma_{\varepsilon,1}$	0.4806	0.2347	$\sigma_{\varepsilon,1}$	0.5497	1.2718	$\sigma_{\upsilon,1}$	3.7539	0.1850
ρ_0	0.5123	0.1594	ρ_0	0.0010	0.2473	ρ_0	0.4924	0.2057
ρ_1	0.6324	0.1892	ρ_1	0.4705	0.1446	ρ_1	1.0000	0.2750
Log-likelihood	−64.026			−80.389			−109.64	

Source: Authors' calculations.

demand and the Phillips curve. The second interesting result is the existing correlation between the shocks of the behavioral equations and the errors in the economic agents' expectations, which also vary substantially with the states. In particular, the results suggest that agents tend to commit crucial errors in their forecasts in high-volatility states of the shocks. This fact is particularly true for the Phillips curve, where such correlation varies between 0.001 and 0.470 for both states, and for the real uncovered interest parity condition (0.49 versus 1.00). In the case of aggregate demand, there is also an important correlation in the high-volatility state. Nonetheless, the difference between the correlations of the two states is less evident than in the previous two cases. Also, the correlation coefficients are highly significant for all cases except the one associated with the low-volatility state of the shocks in the Phillips curve.

Figures 5 to 7 show the behavior over time of the structural parameters of the equations estimated in table 5. There are two series in each figure, which correspond to the relevant values of the parameters in each possible state of shocks in the model (that is, high volatility and low volatility). In the case of the aggregate demand parameters (figure 5), there are two clearly defined periods. The first period, which ends in 1999, is marked by high instability and substantial differences between the parameters of the two states associated with the demand shocks. During this period, the average probability that the economy was in a high-volatility state was 0.82, and the macroeconomic context was characterized by a substantial range of variation in the annual GDP growth rate (from 15 percent to below 6 percent) and by high inflation rates. The second period (from 1999 onward) saw a significant reduction in instability, as well as in the differences of the parameters with respect to the state of the shocks, with the exception of the parameter associated with the output gap's degree of persistence. The average probability that the economy was in a high-volatility state was only 0.10. These results suggest that the multiplicative uncertainty associated with aggregate demand tends to decline over time. Also, the output gap's degree of persistence ($\theta_{1,t}$) and its response to changes in relative prices ($\theta_{4,t}$) have declined over time, while the contrary has occurred with the degree of response to expectations ($\theta_{2,t}$) and the monetary policy interest rate ($\theta_{3,t}$). This is consistent with the logic of the inflation-targeting framework.[31]

31. In 1999, the full-fledged inflation-targeting framework was established.

Figure 5. Time-Varying Parameters Estimated for the Aggregate Demand

*A. Lagged
output-gap parameter*

*B. Lead
output-gap parameter*

*C. Lagged real interest
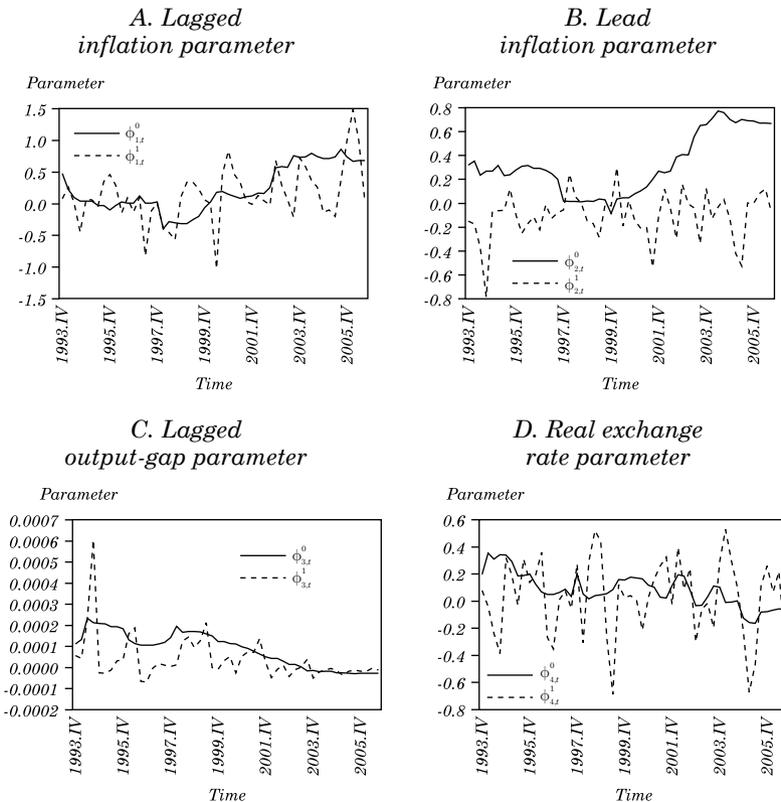rate parameter*

*D. Lagged real exchange
rate parameter*

Source: Authors' calculations.

The parameters of the Phillips curve show a significant dependence on the state of the supply shocks (see figure 6). In periods of high volatility, the parameters tend to show high instability, while they are much more stable in periods of low volatility. Unlike the results of the aggregate demand parameters, this dependence was maintained throughout the entire period. The state of shocks is thus key to explaining greater or lower degrees of uncertainty in the Phillips curve parameters. A high-volatility state of shocks prevailed throughout most of the 1990s (with an average probability of 0.9), so the relevant parameters in that period were those of the high-volatility state. In

the most recent period (1999 onward), the average probability was only 0.06. Figure 6 also reveals that when the economy experiences a relatively calm period with respect to the supply shocks, persistence of the inflation rate ($\phi_{1,t}$) and the importance of expectations in the determination of the inflation rate ($\phi_{2,t}$) are greater. This happens toward the end of the period of analysis. The trend is lower in the case of the response of inflation to the business cycle ($\phi_{3,t}$) and to variations in the real exchange rate ($\phi_{4,t}$). When the supply shocks are highly volatile, however, there is no definite trend for the Phillips curve parameters.
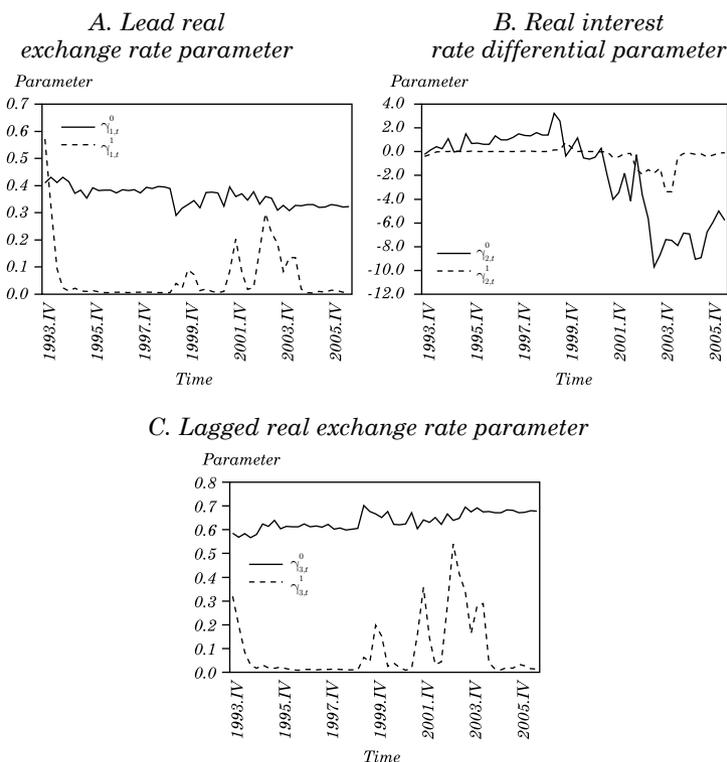
Figure 6. Time-Varying Parameters Estimated for the Phillips Curve

A. Lagged inflation parameter

B. Lead inflation parameter

C. Lagged output-gap parameter

D. Real exchange rate parameter

Source: Authors' calculations.

Finally, parameters associated with the UIP show substantial differences depending on the state of shocks (see figure 7). There is no defined trend in any of the cases. Moreover, in the entire period of analysis, the UIP parameters are more stable in the low-volatility state than in the high-volatility state. In this latter state there are two defined periods: one covering the decade of the 1990s, during which the parameters showed greater stability, and another from 2000 onward, in which the parameters increased their variability and magnitude substantially, in comparison with the first period. This change could be explained by the adoption of a completely flexible exchange rate scheme in 1999. Also, the estimations suggest that the economy was experiencing a high-volatility state of shocks in the entire period, since the occurrence probability of this state did not fall below 0.7 at any time.

Figure 7. Time-Varying Parameters Estimated for the Real Uncovered Interest Parity

A. Lead real exchange rate parameter

B. Real interest rate differential parameter

C. Lagged real exchange rate parameter

Source: Authors' calculations.

Based on the estimated parameters presented in table 5, we calculated the decomposition of the conditional variance of the forecast error. Figure 8 shows the decomposition for the set of equations associated with aggregate demand. Total uncertainty in the output gap (aggregate demand) equation has been relatively high throughout the entire period (with the output gap measured as the percentage deviation of output with respect to its trend). The forecast error variance was 0.021, on average, of which 87.6 percent was explained by uncertainty in the demand shocks and 12.4 percent by instabilities in the model parameters (see table 6). Total uncertainty registered significant spikes (almost twice the average) in the mid-1990s and in 1998–99. After 2000, however, total uncertainty declined by a little over 30 percent relative to the average observed between 1993 and 1999. We obtained similar results with the contributions of additive and multiplicative uncertainty to total uncertainty. While parameter instability contributed approximately 15 percent to total uncertainty throughout the 1990s, this contribution decreased to less than 10 percent in the period after 2000.

Figure 8. Decomposition of the Conditional Variance of the Forecast Error of the Output Gap
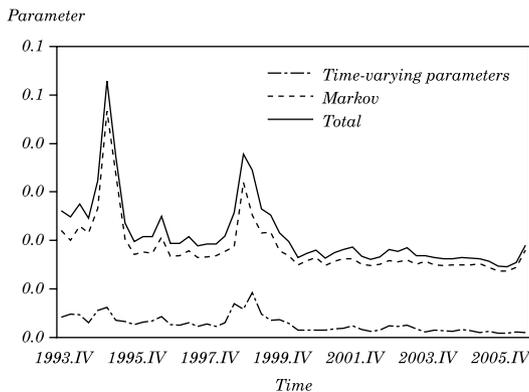
Source: Authors' calculations.

The decomposition of the conditional variance of the forecast error for the inflation rate equation (that is, the Phillips curve) is shown in figure 9. Results in this case are similar to those found for the output gap with respect to magnitude and behavior

Table 6. Decomposition of the Conditional Variance of the Forecast Error of the Output Gap

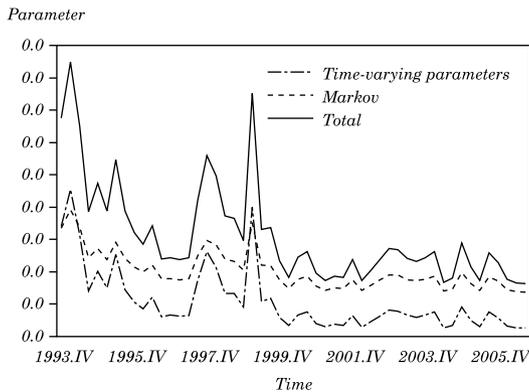|	Conditional variance of the forecast error			Percentage	
Period	Time-varying parameters	Markov	Total	Time-varying parameters	Markov
1993–99	0.00407	0.02173	0.02580	15.7	84.3
2000–06	0.00160	0.01535	0.01696	9.3	90.7
Total sample	0.00279	0.01842	0.02121	12.4	87.6

Source: Authors' calculations.

Table 7. Decomposition of the Conditional Variance of the Forecast Error of the Inflation Rate

	Conditional variance of the forecast error			Percentage	
Period	Time-varying parameters	Markov	Total	Time-varying parameters	Markov
1993–99	0.00852	0.01235	0.02087	37.5	62.5
2000–06	0.00260	0.00818	0.01078	23.2	76.8
Total sample	0.00545	0.01019	0.01563	30.1	69.9

Source: Authors' calculations.

(principally for the decade of the 90s). Total uncertainty associated with the inflation rate was 0.015, on average, for the entire period, of which 69.9 percent is explained by uncertainty in the supply shocks and 30.1 percent by parameter instability (see table 7). As in the case of the output gap, the two recurrent periods of high uncertainty are in the mid-1990s and 1998–99, when uncertainty reached levels more than twice the observed average for the entire period. Although additive uncertainty explains the largest share of total uncertainty for the whole period, the contribution pattern is briefly reverted during Asian crisis, when parameter uncertainty is most relevant. Total inflation uncertainty decreased over time, as in the case of the output gap, while the contribution of additive uncertainty increased with time.

Figure 9. Decomposition of the Conditional Variance of the Forecast Error of the Inflation Rate

Source: Authors' calculations.

Finally, figure 10 presents the decomposition of the conditional variance of the forecast error associated with the real exchange rate equation. Total uncertainty, measured by the variance, was quite important throughout the period (approximately 4.1, on average) and is basically explained (92 percent) by uncertainty in the UIP shocks or uncertainty in the risk premium that captures the effects of the unobservables of the exchange market sentiments. Total uncertainty does not follow a defined pattern over time (see table 8).

Figure 10. Decomposition of the Conditional Variance of the Forecast Error of the Real Exchange Rate

Parameter

Time
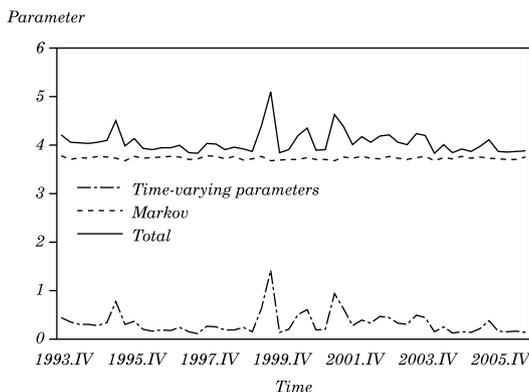
Source: Authors' calculations.

In sum, overall uncertainty is dominated by additive uncertainty in all three sets of equations (namely, the output gap, inflation, and the real exchange rate). Moreover, the results of estimating the behavioral equations (aggregate demand and aggregate supply) suggest that the variance of shocks is state dependent and that such states could be considered as high-volatility periods and low-volatility periods in the shocks. For these two sets of equations, total uncertainty has consistently declined in the current decade, resulting in a rather long period of stability (so far) that coincides with the establishment of a full-fledged inflation-targeting framework for the conduct of the Chilean monetary policy and an explicit rule for setting fiscal policy. In the 1990s, in contrast, total uncertainty increased substantially in the output gap and the inflation rate, with a clear division into the two states in the variance of shocks. This also indicates that during these periods the Chilean economy experienced a high-volatility state of shocks. Finally, uncertainty in the real exchange rate is basically explained by the exchange market shocks, and it has not decreased over time like inflation and the output gap.

We use bootstrapping to verify whether the differences between the variance of the forecast error due to additive uncertainty and that due to multiplicative uncertainty are statistically significant and whether the assumption of Gaussian errors in the estimation

Table 8. Decomposition of the Conditional Variance of the Forecast Error of the Real Exchange Rate

Period	Conditional variance of the forecast error			Percentage	
	Time-varying parameters	Markov	Total	Time-varying parameters	Markov
1993–99	0.32816	3.73569	4.06385	8.8	92.3
2000–06	0.32701	3.72663	4.05364	8.8	92.1
Total sample	0.32756	3.73099	4.05855	8.8	92.2

Source: Authors' calculations.

introduces significant biases.[32] The most important findings of this exercise can be summarized as follows (for details on the results see appendix E): first, while the average bootstrap estimates differ from estimates based on the assumption of Gaussian errors, the bias does not seem to be important in magnitude; and second, the bootstrap estimations confirm the observed trends in total uncertainty (figures 8 to 10), as well as the statistical significance of the differences in the decomposition of the variance.

To conclude this subsection, we present a robustness analysis for the decomposition of the forecast error variance. In section 2, we found evidence of important differences in the estimation of the output gap when we tested five output detrending methods. Given that the aggregate demand and the Phillips curve equations contemplate an output gap measure for their estimation, measurement errors in the estimation of this variable will be part of the additive and multiplicative uncertainty without any possibility of discrimination.[33] Tables 9 and 10 show the results of the decomposition of uncertainty into its two sources, additive and multiplicative, for these two equations and for each of the five filters used in section 2. The first row of both tables shows the decomposition presented in the analysis of this subsection, where the gap was calculated using the HP filter; this represents our benchmark. In the case of the output gap (table 9), total uncertainty is generally quite similar for all filters, and differences arise in the contribution of each type of uncertainty to total uncertainty, as expected. However, all the detrending methods maintain additive uncertainty as the most important source of uncertainty (with a contribution ranging from a minimum of 84.7 percent with the BK filter and a maximum of 90.0 percent with the Clark filter). With respect to the inflation rate (table 10), the difference among filters can be observed in both the estimation of total uncertainty and the contributions of each type of uncertainty to the total. In the former

32. Our bootstrap resampling followed the methodologies of Stoffer and Wall (1991) and Psaradakis (1998) for state-space models using the Kalman filter and for the sampling of errors with Markov regime changes, respectively.

33. When the measurement error is associated with the dependent variable, as in the case of aggregate demand, the estimated parameters will still be unbiased and consistent. The measurement error will be captured by the regression error. When the measurement error is associated with one or more independent variables, as is the case with the Phillips curve, the parameters will be biased and inconsistent. Although the measurement error has different effects depending on the type of variable on which it operates, this could have implications for the decomposition of uncertainty (through the error or the magnitude of the parameters).

Table 9. Robustness Analysis for the Decomposition of the Conditional Variance of the Forecast Error of the Output Gap

Filter	Conditional variance of the forecast error			Percentage	
	Time-varying parameters	Markov	Total	Time-varying parameters	Markov
Hodrick-Prescott	0.00279	0.01842	0.02121	13.2	86.8
Baxter-King	0.00314	0.01734	0.02048	15.3	84.7
Christiano-Fitzgerald	0.00304	0.01733	0.02037	14.9	85.1
Quadratic trend	0.00287	0.01901	0.02189	13.1	86.9
Clark	0.00200	0.01803	0.02003	10.0	90.0

Source: Authors' calculations.

Table 10. Robustness Analysis for the Decomposition of the Conditional Variance of the Forecast Error of the Inflation Rate

| Filter | Conditional variance of the forecast error | | | Percentage | |
	Time-varying parameters	Markov	Total	Time-varying parameters	Markov
Hodrick-Prescott	0.00545	0.01019	0.01563	34.8	65.2
Baxter-King	0.00385	0.00988	0.01374	28.0	72.0
Christiano-Fitzgerald	0.00393	0.01006	0.01398	28.1	71.9
Quadratic trend	0.00761	0.01514	0.02274	33.4	66.6
Clark	0.00504	0.01397	0.01901	26.5	73.5

Source: Authors' calculations.

case, the estimations are in the range of 0.01374 and 0.02274 with the BK filter and the quadratic trend, respectively, while the contribution of additive uncertainty varies from 66.6 percent with the BK filter to 73.5 percent with the Clark filter. In this case, additive uncertainty again explains total uncertainty of inflation, regardless of the method used to estimate the output gap. These results strengthen our earlier conclusions regarding the importance of additive uncertainty for the Chilean economy.

4. FINAL REMARKS

Current macroeconomic policy in Chile is world-class. The Central Bank of Chile has been operating within a full-fledged inflation-targeting framework since 1999–2000, while fiscal policy has been bounded by an explicit budget rule since 2001 that eliminates procyclical influences. As a result, inflation has remained within the target range most of the time, and economic activity has grown steadily between 2 and 6 percent annually (with no recessions or booms whatsoever). This stable period appears in our findings in the sense that overall uncertainty concerning monetary policy declined in the first seven years of the current decade. Moreover, uncertainty attributed to shocks has played a greater role, while uncertainty linked to unstable parameters has diminished, in the case of both inflation and the output gap, as could be expected. However, the prominence of additive uncertainty characterizes the entire period, including both the tranquil current decade and the more volatile 1990s. This means that investigating the (stochastic) nature of shocks affecting the Chilean economy should be high on the research agenda of the Central Bank.

The full-fledged inflation-targeting scheme applied since 1999 incorporated a floating exchange rate and no explicit or implicit target for the exchange rate (as was loosely the case during most of the 1990s). This important policy innovation has left the exchange rate as the main adjustment variable—a sort of fuse. This feature shows in our results: parameters in the exchange rate equation are less stable in the current decade than they were in the 1990s.

Our findings assume that there is no model uncertainty, so the only uncertainties relevant for the conduct of monetary policy are those in the shocks and parameters. Our results must therefore be interpreted with caution. To analyze uncertainty in the model, we could estimate the behavioral equations of the economy using the

methodology presented in this paper but different specifications. This approach could be used to verify whether the decomposition of the uncertainty found here holds.[34] We leave this exercise pending for future research.

Finally, results on uncertainty about the quality and completeness of output gap data indicate that using the Hodrick-Prescott filter based on real-time data could be misleading. The Central Bank of Chile should thus consider a wide spectrum of filters for detrending real activity data.[35] More importantly, an ample menu of proxy variables should be employed to check the economy's temperature when making monetary policy decisions. The literature suggests that monetary policy rules that consider, for example, output growth rates or unemployment level rates (as opposed to the output gap) are more "immune" to this type of uncertainty.

34. This exercise was done only with the UIP under two specifications: the original equation of Svensson (2000) and Al-Eyd and Karasulu (2008) and the equation that includes the backward-looking term to allow deviations from the parity (presented here). We found that although the behavior of the parameters and the magnitude of total uncertainty change significantly, the decomposition of the uncertainty is not altered (additive uncertainty is maintained as the principal factor of uncertainty).

35. It should also use some alternative methodologies for estimating potential output, as it currently does.

APPENDIX A

Robustness Test for the Reliability of Real-Time Estimates using Seasonally Unadjusted Data and Seasonal Dummies

The tables presented in this appendix provide additional details on the results obtained in the estimation of the output gap with real-time data using seasonally unadjusted data and seasonally adjusted data through seasonal dummy variables.

Table A1. Descriptive Statistics of the Total Revisions in the Output Gap Using Seasonally Unadjusted Data

Filter	Mean	Standard deviation	Minimum	Maximum	AR(1)
Hodrick-Prescott	−0.005	0.015	−0.036	0.031	0.331
Baxter-King	0.006	0.007	−0.008	0.023	0.722
Christiano-Fitzgerald	−0.013	0.009	−0.029	0.005	0.836
Quadratic trend	−0.011	0.021	−0.050	0.033	0.676
Clark	0.001	0.006	−0.014	0.010	0.023

Source: Authors' calculations.

Table A2. Descriptive Statistics of the Reliability Measures for the Alternative Distinct Filters Using Seasonally Unadjusted Data[a]

Filter	Correlation	N/S	OPSIGN	XSIZE
Hodrick-Prescott	0.773	0.754	0.286	0.536
Baxter-King	0.529	0.958	0.286	0.464
Christiano-Fitzgerald	0.244	1.290	0.393	0.821
Quadratic trend	0.846	0.642	0.179	0.393
Clark	0.963	0.290	0.036	0.107

Source: Authors' calculations.
a. The first column presents the correlation between the final and real-time series for each detrending method. The N/S indicator is the ratio of the standard deviation of the revision to the standard deviation of the final estimate of the output gap; it approximates the noise-to-signal ratio. The OPSING indicator shows the frequency with which the real-time estimates of the output gap reveal a different sign than the final estimates. The XSIZE indicator shows the frequency with which the absolute value of the revision exceeds the absolute value of the final estimates of the output gap.

Table A3. Descriptive Statistics of the Total Revisions in the Output Gap Using Seasonal Dummies

Filter	Mean	Standard deviation	Minimum	Maximum	AR(1)
Hodrick-Prescott	0.002	0.017	−0.034	0.031	0.260
Baxter-King	0.008	0.007	−0.002	0.019	0.874
Christiano-Fitzgerald	−0.011	0.010	−0.029	0.002	0.942
Quadratic trend	−0.004	0.024	−0.051	0.046	0.521
Clark	0.005	0.007	−0.013	0.017	−0.063

Source: Authors' calculations.

Table A4. Descriptive Statistics of the Reliability Measures for the Alternative Filters Using Seasonal Dummies[a]

Filter	Correlation	N/S	OPSIGN	XSIZE
Hodrick-Prescott	0.413	1.044	0.321	0.429
Baxter-King	0.646	0.772	0.321	0.500
Christiano-Fitzgerald	0.312	1.031	0.357	0.571
Quadratic trend	0.745	0.771	0.179	0.321
Clark	0.932	0.367	0.071	0.214

Source: Authors' calculations.
a. The first column presents the correlation between the final and real-time series for each detrending method. The N/S indicator is the ratio of the standard deviation of the revision to the standard deviation of the final estimate of the output gap; it approximates the noise-to-signal ratio. The OPSING indicator shows the frequency with which the real-time estimates of the output gap reveal a different sign than the final estimates. The XSIZE indicator shows the frequency with which the absolute value of the revision exceeds the absolute value of the final estimates of the output gap.

APPENDIX B

Estimation Based on the Kalman Filter and the EM Algorithm

Our estimation approach follows the two-stage procedure proposed by Kim (2004, 2006). The first stage, described in the main text, consists in estimating a model that instrumentalizes the endogenous variables using the maximum log-likelihood method based on the error forecast and the conventional Kalman filter. The second stage is based on maximum log-likelihood techniques that combine the Kalman filter and the EM algorithm proposed by Hamilton (1989, 1990). The latter estimation is defined by the following series of equations (Kim and Nelson, 1999):

Kalman Filter

$$\beta_{t|t-1}^{(i,j)}, \mathbf{P}_{t|t-1}^{(i,j)}, \tau_{t|t-1}^{(i,j)}, f_{t|t-1}^{(i,j)}, \mathbf{H}_{t|t-1}^{(i,j)}.$$

Hamilton's EM Algorithm

$$\Pr\left[S_t, S_{t-1} \mid \psi_{t-1}\right] = \Pr\left[S_t, S_{t-1}\right]\Pr\left[S_{t-1} \mid \psi_{t-1}\right];$$

$$f(x_t \mid \psi_{t-1}) = \sum_{S_t}\sum_{S_{t-1}} f(x_t \mid S_t, S_{t-1}, \psi_{t-1})\Pr\left[S_t, S_{t-1} \mid \psi_{t-1}\right];$$

$$l(\theta) = l(\theta) + \ln\left[f(x_t \mid \psi_{t-1})\right];$$

$$\Pr\left[S_t, S_{t-1} \mid \psi_{t-1}\right] = \frac{f(x_t, S_t, S_{t-1}, \psi_{t-1})}{f(x_t \mid \psi_{t-1})}$$

$$= \frac{f(x_t \mid S_t, S_{t-1}, \psi_{t-1})\Pr\left[S_t, S_{t-1} \mid \psi_{t-1}\right]}{f(x_t \mid \psi_{t-1})}$$

$$\Pr\left[S_t \mid \psi_t\right] = \sum_{S_{t-1}}\Pr\left[S_t, S_{t-1} \mid \psi_t\right].$$

Approximations

$$\beta_{t|t}^{j} = \frac{\sum_{i=1}^{2} \Pr\left[S_{t-1} = i, S_{t} = j \mid \psi_{t}\right]\beta_{t|t}^{(i,j)}}{\Pr\left[S_{t} = j \mid \psi_{t}\right]};$$

$$\mathbf{P}_{t|t}^{j} = \frac{\sum_{i=1}^{2} \Pr\left[S_{t-1} = i, S_{t} = j \mid \psi_{t}\right]\left[\mathbf{P}_{t|t}^{(i,j)} + \left(\beta_{t|t}^{j} - \beta_{t|t}^{(i,j)}\right)\left(\beta_{t|t}^{j} - \beta_{t|t}^{(i,j)}\right)'\right]}{\Pr\left[S_{t} = j \mid \psi_{t}\right]}.$$

Log-likelihood function

$$l(\theta) = \sum_{t=1}^{T} \ln\left[f(x_{t} \mid \psi_{t-1})\right].$$

APPENDIX C
Kalman Filter with Endogenous Regressors

Kim (2006) uses the following series of equations to describe the Kalman filter with endogenous regressors.

$$\beta_{t|t-1} = E\left(\beta_t \mid \mathbf{w}_t, \mathbf{v}_t, \xi_t^*, \psi_{t-1}\right) = \beta_{t-1|t-1};$$

$$\mathbf{P}_{t|t-1} = \operatorname{var}\left(\beta_t \mid \mathbf{w}_t, \mathbf{v}_t, \xi_t^*, \psi_{t-1}\right) = \mathbf{P}_{t-1|t-1} + \mathbf{Q}_\eta;$$

$$\tau_{t|t-1} = x_t - E\left(x_t \mid \mathbf{w}_t, \mathbf{v}_t, \xi_t^*, \psi_{t-1}\right) = x_t - (\mathbf{w}_t, \mathbf{v}_t)'\beta_{t-1|t-1} - \xi_t^{*'}\rho\sigma_\varepsilon;$$

$$\mathbf{H}_{t|t-1} = \operatorname{var}\left(x_t \mid \mathbf{w}_t, \mathbf{v}_t, \xi_t^*, \psi_{t-1}\right) = (\mathbf{w}_t, \mathbf{v}_t)'\mathbf{P}_{t|t-1}(\mathbf{w}_t, \mathbf{v}_t) + (1 - \rho'\rho)\sigma_\varepsilon^2;$$

$$\beta_{t|t} = E\left(\beta_t \mid \mathbf{w}_t, \mathbf{v}_t, \xi_t^*, \psi_{t-1}\right) = \beta_{t|t-1} + \mathbf{P}_{t|t-1}(\mathbf{w}_t, \mathbf{v}_t)\mathbf{H}_{t|t-1}^{-1}\tau_{t|t-1};$$

$$\mathbf{P}_{t|t} = \operatorname{var}\left(\beta_t \mid \mathbf{w}_t, \mathbf{v}_t, \xi_t^*, \psi_{t-1}\right) = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}(\mathbf{w}_t, \mathbf{v}_t)\mathbf{H}_{t|t-1}^{-1}(\mathbf{w}_t, \mathbf{v}_t)'\mathbf{P}_{t|t-1}.$$

APPENDIX D
Log-Likelihood Function

The log-likelihood function defined by Kim and Nelson (1999) is as follows:

$$f(x_t \mid \psi_{t-1}) = \sum_{i=1}^{2} \sum_{j=1}^{2} f(x_t, S_t = i, S_{t-1} = j \mid \psi_{t-1})$$

$$= \sum_{i=1}^{2} \sum_{j=1}^{2} f(x_t \mid S_t = i, S_{t-1} = j \mid \psi_{t-1}) \Pr\!\left[S_t = i, S_{t-1} = j \mid \psi_{t-1}\right],$$

where

$$f(x_t \mid S_t = i, S_{t-1} = j, \psi_{t-1}) = (2\pi)^{-\frac{N}{2}} \mid f_{t\mid t-1}^{(i,j)} \mid^{-\frac{1}{2}} \exp\left\{-\frac{1}{2} \tau_{t\mid t-1}^{(i,j)} {}' f_{t\mid t-1}^{(i,j)^{-1}} \tau_{t\mid t-1}^{(i,j)}\right\}.$$

APPENDIX E

Bootstrap of the Decomposition of the Conditional Variance of the Forecast Error

Table E1 presents the results obtained from the bootstrap of the decomposition of the conditional variance of the forecast error for the three models (mean estimation and 95 percent confidence intervals). The table also shows, for comparison purposes, the previous results found under the assumption of Gaussian errors in the estimation. The bootstrap resampling followed the methodologies of Stoffer and Wall (1991) for state-space models that use the Kalman filter and Psaradakis (1998) for the sampling of errors with Markov regime changes.

Table E1. Bootstrap Decomposition of the Conditional Variance of the Forecast Error

Variable and period	Gaussian maximum likelihood			Bootstrap					
	Time-varying parameters	Markov	Total	Time-varying parameters		Markov		Total	
				Mean	0.95 confidence interval	Mean	0.95 confidence interval	Mean	0.95 confidence interval
A. Output gap									
1993–95	0.00424	0.02566	0.02990	0.00585	[0.00572, 0.00598]	0.05667	[0.05542, 0.05790]	0.06251	[0.06119, 0.06384]
1996–98	0.00353	0.01881	0.02234	0.00548	[0.00533, 0.00564]	0.02330	[0.02264, 0.02401]	0.02878	[0.02796, 0.02961]
1999–2006	0.00208	0.01616	0.01824	0.00193	[0.00188, 0.00197]	0.01807	[0.01749, 0.01870]	0.02000	[0.01938, 0.02066]
Full sample	0.00279	0.01842	0.02121	0.00342	[0.00334, 0.00351]	0.02596	[0.02524, 0.02671]	0.02938	[0.02860, 0.03020]
B. Inflation rate									
1993–95	0.01172	0.01428	0.02599	0.01204	[0.01199, 0.01207]	0.06555	[0.02062, 0.15871]	0.07758	[0.03267, 0.18638]
1996–98	0.00612	0.01099	0.01711	0.00588	[0.00586, 0.00590]	0.04010	[0.01541, 0.09914]	0.04598	[0.02129, 0.09725]
1999–2006	0.00337	0.00869	0.01205	0.00289	[0.00288, 0.00291]	0.02381	[0.01130, 0.04950]	0.02670	[0.01420, 0.05276]
Full sample	0.00545	0.01019	0.01563	0.00516	[0.00514, 0.00518]	0.03479	[0.01386, 0.07986]	0.03996	[0.01903, 0.08616]
C. Real exchange rate									
1993–95	0.54577	10.54444	11.09022	0.81944	[0.77777, 0.86192]	9.21296	[9.02628, 9.40670]	10.03222	[9.83458, 10.24011]
1996–98	0.37324	10.46275	10.83625	0.74982	[0.70561, 0.79519]	9.20454	[9.01600, 9.39305]	9.95438	[9.75733, 10.16217]
1999–2006	0.56164	10.58610	11.14777	0.91215	[0.86287, 0.96336]	9.19028	[8.99863, 9.38625]	10.10250	[9.89786, 10.31116]
Full sample	0.51542	10.55042	11.06592	0.85864	[0.81185, 0.90699]	9.19750	[9.00743, 9.39136]	10.05616	[9.85448, 10.26448]

Source: Authors' calculations.

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