# A STICKY-INFORMATION GENERAL EQUILIBRIUM MODEL FOR POLICY ANALYSIS

## Ricardo Reis
*Columbia University*

Following on Keynes's desire that economists be as useful as dentists, Lucas (1980) argues that this would amount to the following: "Our task, as I see it, is to write a FORTRAN program that will accept specific economic policy rules as 'input' and will generate as 'output' statistics describing the operating characteristics of time series we care about, which are predicted to result from these policies." Starting with Kydland and Prescott (1982), and with Rotemberg and Woodford (1997) in the context of monetary policy, the computer program that Lucas asked for has taken the form of dynamic stochastic general equilibrium (DSGE) models.[1] This paper follows the seminal work of Taylor (1979) in using one of these models to ask a series of hypothetical monetary policy questions.

However, the initial versions of monetary DSGE models suffer from one problem: they imply a rapid adjustment of many macroeconomic variables to shocks, while in the data, these responses tend to be gradual and delayed. The predictions of the standard classical model regarding investment, consumption, real wages, or inflation lack stickiness, to use the term coined by Sims (1998) and Mankiw and Reis (2006). The most popular approach for addressing this disconnect between theory and data follows the

influential work of Christiano, Eichenbaum, and Evans (2005) by adding many rigidities that stand in the way of adjustment: sticky but indexed prices in goods markets, adjustment costs in investment markets, habits in consumption markets, and sticky but indexed wages in labor markets.

This paper contributes to the literature by providing an alternative DSGE model of business cycles and monetary policy. The only source of rigidity is inattention in all markets by agents who choose to only update their information sporadically in order to save on the fixed costs of acquiring, absorbing, and processing information (Reis, 2006a, 2006b). Information is sticky because different agents update their information at different dates, so they only gradually learn of news. I call it the sticky information in general equilibrium, or SIGE, model. Mankiw and Reis (2006, 2007) provided a first glimpse of SIGE, and this paper presents the model and its solution in full. I then proceed to estimate it for the United States after 1986 and the euro area after 1993 and to conduct a few policy experiments.

The paper is organized as follows. Section 1 presents the model and discusses its current limitations. Section 2 log-linearizes the model to arrive at a set of reduced-form relations that characterize the equilibrium. Section 3 describes an algorithm to compute a solution and derives formulas to calculate the key inputs into estimation (the likelihood function) and policy analysis (a social welfare function). Section 4 reviews the literature on estimating models with sticky information and describes the approach taken in this paper. Section 5 presents the estimation results for the United States and the euro area, while section 6 examines the sensitivity of the estimates. Section 7 answers a few policy questions, and section 8 concludes.

## 1. THE SIGE MODEL

The SIGE model belongs to the wide class of general-equilibrium models with monopolistic competition that have become the workhorse for the study of monetary policy (surveyed in Woodford, 2003b). There are three sets of markets where agents meet every period: markets for different varieties of goods, where monopolistic firms sell varieties of goods to households; a market for savings, where households trade bonds and interest rates change to balance borrowing and lending; and markets for labor, where monopolistic households sell varieties of labor to firms. I present each of these markets in turn, before describing the assumptions on information and attention.

## 1.1 The Goods Market

On the buying side, there is a continuum of shoppers indexed by $j$ that consume a continuum of varieties of goods in the unit interval indexed by $i$, denoted by $C_{t,j}(i)$. A bundle of these varieties of goods yields utility according to a Dixit-Stiglitz function with a time-varying and random elasticity of substitution $\tilde{v}_t$. Each good trades at price $P_{t,i}$ and the problem of a shopper with $Z_{t,j}$ to spend that observes current prices is

$$\max_{\left\{C_{t,j}(i)\right\}_{i \in [0,1]}} C_{t,j} = \left[ \int_0^1 C_{t,j}(i)^{\frac{\tilde{v}_t}{\tilde{v}_t - 1}} di \right]^{\frac{\tilde{v}_t - 1}{\tilde{v}_t}}, \tag{1}$$

subject to $\int_0^1 P_{t,i} C_{t,j}(i) < Z_{t,j}$. $\tag{2}$

The solution to this problem is $C_{t,j}(i) = C_{t,j} \left( P_{t,i} / P_t \right)^{-\tilde{v}_t}$, where the price index is defined as

$$P_t = \left( \int_0^1 P_{t,i}^{1-\tilde{v}_t} di \right)^{1/(1-\tilde{v}_t)}$$

and implies that, conditional on the optimal choices of the shopper, $Z_{t,j} = P_t C_{t,j}$. Integrating over the continuum of shoppers gives the total demand for variety $i$:

$$\int_0^1 C_{t,j}(i) dj = \left( P_{t,i} / P_t \right)^{-\tilde{v}_t} \int_0^1 C_{t,j} dj. \tag{3}$$

On the selling side of the market, there is a monopolistic firm for each variety of the good. Each of these firms, indexed by $i$, operates a technology that uses labor $N_{t,i}$ at cost $W_t$ to produce good $i$ under diminishing returns to scale with $\beta \in (0, 1)$ and a common technology shock $A_t$. The firm's sales department is in charge of setting the price $P_{t,i}$ and selling the output $Y_{t,i}$ to maximize real after-tax profits subject to the technology and the demand for the good:

$$\max_{P_{t,i}} E_t^{(i)} \left[ \frac{\left(1 - \tau_p \right) P_{t,i} Y_{t,i}}{P_t} - \frac{W_t N_{t,i}}{P_t} \right], \tag{4}$$

subject to $Y_{t,i} = A_t\, N\beta_{t,i}$,                                                                    (5)

$$Y_{t,i} = G_t \int_0^1 C_{t,j}(i)dj.$$                                                                     (6)

The $E_t^{(i)}(.)$ expectations operator of the sales department of firm $i$ depends on its information, which I will discuss later. The government intervenes in two ways in the actions of the firm: collecting a fixed sales tax, $\tau_p$, and buying a time-varying and random share, $1 - 1/G_t$, of the goods in the market. These governmental purchases are wasted, and I refer to them broadly as aggregate demand shocks. Aggregate output is $Y_t = \int_0^1 Y_{t,i} di$.[2]

After some rearranging, the first-order condition from this problem becomes

$$P_{t,i} = \frac{E_t^{(i)}\left[(1-\tau_p)\tilde{v}_t W_t N_{t,i} / P_t\right]}{E_t^{(i)}\left[(\tilde{v}_t - 1)\beta Y_{t,i} / P_t\right]}.$$                                    (7)

If the firm observed all the variables on the right-hand side, this condition would state that the nominal price charged, $P_{t,i}$, is equal to a markup, $(1-\tau_p)\,\tilde{v}_t/(\tilde{v}_t-1)$, stemming from taxes and the ability to exploit an elastic demand curve, over nominal marginal costs, which equal the cost of an extra unit of labor, $W_t$, divided by its marginal product, $\beta Y_{t,i} / N_{t,i}$.

## 1.2 The Bond Market

In this market, saver-planners meet each other to trade one-period bonds. Their aim is to maximize the expected discounted utility from consumption:

$$E_t^{(j)} \sum_{t=0}^\infty \xi^t \left( \frac{C_{t,j}^{1-1/\theta}}{1-1/\theta} \right),$$                               (8)

2. Defining aggregate output as $Y_t = \left( \int_0^1 Y_{t,i}^{(1-1/\tilde{v}_t)} di \right)^{\tilde{v}_t/(\tilde{v}_t - 1)}$ leads to the same results, up to a first-order log-linear approximation.

where $\xi$ is the discount factor and $\theta$ is the intertemporal elasticity of substitution. They have an intertemporal budget constraint:

$$M_{t+1,j} = \Pi_{t+1}\left[M_{t,j} - C_{t,j} + \frac{(1-\tau_w)W_{t,j}L_{t,j}}{P_t} + T_{t,j}\right]. \tag{9}$$

The saver-planner $j$ enters the period with real wealth $M_{t,j}$, uses some of it to consume, earns labor income at the wage rate $W_{t,j}$ after paying a fixed labor income tax $\tau_w$, and receives a lump-sum transfer $T_{t,j}$. The transfer $T_{t,j}$ includes lump-sum taxes, profits and losses from firms, and payments from an insurance contract that all households signed at date 0 that ensures that every period they are all left with the same wealth. Savings accumulate at the real interest rate $\Pi_{t+1}$, although, in equilibrium, bonds are in zero net supply, so savings integrate to zero over all consumers.

The dynamic program that characterizes the saver-planner's problem is messy, so it is covered in the appendix. If $j = 0$ denotes the saver-planner that forms expectations rationally based on up-to-date information, so $E_t^{(0)} = E_t$, then the optimality conditions are

$$C_{t,0}^{-1/\theta} = \xi E_t\left(\Pi_{t+1}C_{t+1,0}^{-1/\theta}\right), \tag{10}$$

$$C_{t,j}^{-1/\theta} = E^{(j)}\left(C_{t,0}^{-1/\theta}\right). \tag{11}$$

The first equation is the standard Euler equation for a well-informed agent. It states that the marginal utility of consuming today is equal to the expected discounted marginal utility of consuming tomorrow times the return on savings. The second equation notes that agents who are not so well informed set their marginal utility of consumption to what they expect it would be with full information.

The monetary policymaker intervenes in this market by supplying reserves at an interest rate. Because these reserves are substitutable with the bonds that consumers trade among themselves, the central bank can target a value for the nominal interest rate, $i_t \equiv \log[E_t(\Pi_{t+1}P_{t+1}/P_t)]$, standing ready to issue as many reserves as necessary to ensure it. Alternatively, one could introduce money directly as an additive term in the agents' utility function and then have the central bank control the money supply to target an interest rate (see Woodford, 1998, for an elaboration of

this point). The nominal interest rate follows some policy rule subject to exogenous monetary shocks $\varepsilon_t$. To fix ideas, and because it will be the policy rule used in the estimation, consider a Taylor rule:

$$i_t = \phi_y \log\left(\frac{Y_t}{Y_t^c}\right) + \phi_p \log\left(\frac{P_t}{P_{t-1}}\right) - \varepsilon_t, \tag{12}$$

where $Y_t^c$ is the level of output in the classical or attentive equilibrium (sometimes called the natural output level).

## 1.3 The Labor Market

This market features workers on the selling side and firms on the buying side. The firms, indexed by $i$, have a purchasing department hiring a continuum of varieties of labor indexed by $k$ in the amount $N_{t,i}(k)$ at price $W_{t,k}$ and combining them into the labor input $N_{t,i}$ according to a Dixit-Stiglitz function with a random and time-varying elasticity of substitution $\tilde{\gamma}_t$. The purchasing department's problem is to solve the following problem, given current wages and a total desired amount of inputs $N_{t,i}$:

$$\min_{\left\{N_{t,i}(k)\right\}_{k\in[0,1]}} \int_0^1 W_{t,k} N_{t,i}(k)dk, \tag{13}$$

$$\text{subject to } \left[\int_0^1 N_{t,i}(k)^{\frac{\tilde{\gamma}_t}{\tilde{\gamma}_t-1}} dk\right]^{\frac{\tilde{\gamma}_t-1}{\tilde{\gamma}_t}} = N_{t,i},$$

The solution to this problem is $N_{t,i}(k) = N_{t,i}\left(W_{t,k}/W_t\right)$, where $W_t N_{t,i} = \int_0^1 W_{t,k} N_{t,i}(k)dk$ for a static wage index $W_t = \left(\int_0^1 W_{t,k}^{1-\tilde{\gamma}_t} dk\right)^{1/(1-\tilde{\gamma}_t)}$.

Aggregating over all firms gives the total demand for labor variety $k$:

$$\int_0^1 N_{t,i}(k)di = \left(\frac{W_{t,k}}{W_t}\right)^{-\tilde{\gamma}_t} \int_0^1 N_{t,i}di. \tag{14}$$

Each worker is a monopolistic supplier of a variety of labor. The workers' aim is to minimize their expected discounted disutility of labor:

$$E_t^{(k)} \sum_{t=0}^{\infty} \xi^t \left( \frac{\kappa L_{t,k}^{1+1/\psi}}{1+1/\psi} \right), \tag{15}$$

where $\xi$ is the discount factor and $\psi$ is the Frisch elasticity of labor supply. They face the same intertemporal budget constraint as the consumers in equation (9), and they also take into account the demand for their good $L_{t,k} = \int_0^1 N_{t,i}(k)di$ and equation (14). Aggregate labor employed is $L_t = \int L_{t,k} dk$.[3] The optimality conditions are

$$\frac{\tilde{\gamma}_t}{\tilde{\gamma}_t - 1} \times \frac{L_{t,0}^{1/\psi} P_t}{W_{t,0}} = \xi E_t \left( \Pi_{t+1} \times \frac{\tilde{\gamma}_{t+1}}{\tilde{\gamma}_{t+1} - 1} \times \frac{L_{t+1,0}^{1/\psi} P_{t+1}}{W_{t+1,0}} \right); \tag{16}$$

$$W_{t,k} = \frac{E_t^{(k)} \left[ (1 - \tau_w) \kappa \tilde{\gamma}_t L_{t,k}^{1/\psi} \right]}{E_t^{(k)} \left[ \tilde{\gamma}_t L_{t,k} L_{t,0}^{1/\psi - 1} / W_{t,0} \right]}. \tag{17}$$

The first condition is the standard intertemporal labor supply Euler equation for a well-informed worker. If $\tilde{\gamma}_t$ is fixed, the equation states that the marginal disutility of supplying labor today ($L_{t,0}^{1/\psi}$) divided by the real wage ($W_{t,0}/P_t$) equates the discounted marginal disutility tomorrow ($L_{t+1,0}^{1/\psi}$) divided by the real wage tomorrow ($W_{t+1,0}/P_{t+1}$) times the real interest rate. With time-varying $\tilde{\gamma}_t$, the Euler equation takes into account the change in the markup that the monopolistic worker wants to charge. The second condition is the counterpart to condition (11) in the consumer problem—for the fully-informed case $E_t^{(k)} = E_t$, it simply states that $W_{t,k} = W_{t,0}$.

## 1.4 Information, Agents, and Attention

Uncertainty in this economy arises because every period there is a different realization of the random variables characterizing

---

3. As with output, defining aggregate labor as $L_t = \left( \int_0^1 L_{t,k}^{(1-1/\tilde{\gamma}_t)} dk \right)^{\tilde{\gamma}_t/(\tilde{\gamma}_t - 1)}$ leads to the same results up to a log-linear approximation.

productivity ($A_t$), aggregate demand ($G_t$), price and wage markups ($\tilde{\nu}_t$ and $\tilde{\gamma}_t$), and monetary policy ($\varepsilon_t$).

If all agents are fully informed, then the model described above is a standard classical model. While the discussion presented consumers (shoppers and saver-planners) and workers separately, they are all members of one household with period preferences

$$U\left(C_{t,j}, L_{t,k}\right) = \frac{C_{t,j}^{1-1/\theta}}{1-1/\theta} - \frac{\kappa L_{t,k}^{1+1/\psi}}{1+1/\psi}, \tag{18}$$

and with $j = k$ since there is common information. The decisions on the consumption of each variety, total consumption, and the wage to charge, are all made with rational expectations using all available information. Likewise, if the two departments of the firm share their information, they can be thought of as a single decisionmaker.

The SIGE model introduces only one new assumption relative to this classical benchmark: while the expectations of each agent are formed rationally, they do not necessarily use all available information. More concretely, it assumes that there are fixed costs of acquiring, absorbing, and processing information, so that agents optimally choose to only update their information sporadically (Reis, 2006a, 2006b). This inattentiveness is present in all of the markets—by the planner-savers in the savings market, by the sales departments of firms in the goods markets, and by the workers in the labor markets. Separating consumers from workers allows them to potentially update their information at different frequencies. In this case, while they share a household, in the sense of a common objective (equation 18) and a common budget constraint (equation 9), they do not necessarily need to share information. When workers update their information, they also learn about what the consumers have been doing, and vice versa for consumers when they update.

While inattentiveness occurs in all markets, not all agents in this economy are inattentive. In the goods market, the model assumes that the consumer is separated into two units: the saver-planner who updates information infrequently and the shopper who knows about the expenditure plan of the saver and observes the relative prices of the different goods. This assumption is not implausible: while the choice of how much to spend in total and how much to save requires solving an intertemporal optimization problem and making forecasts into the infinite future, choosing the relative proportion of each good to buy requires only seeing goods' prices. The main reason to make

this assumption, though, is a current limitation in our knowledge. If the monopolistic firms in the goods' market faced inattentive shoppers, they would want to exploit them to raise profits, but the shoppers would then take this into account in choosing how often to be inattentive. The equilibrium of this game has not yet been fully studied, and assuming that shoppers are attentive avoids it entirely. The same argument leads to separating the firm into an inattentive sales-production team and an attentive purchasing department.

Within the inattentiveness model, the SIGE model adds an extra restriction: that the stochastic process for the expected costs of planning is such that the distribution of inattentiveness for consumers, workers, and firms is exponential. Reis (2006b) establishes the strict conditions under which this will hold for the firms' problem. Under these conditions, for a linearized homoskedastic economy, the optimal rate of arrival of information is fixed so that it can be treated as a parameter (bearing in mind that it maps into the monetary cost of updating information). Therefore, every period, a fraction of planner-savers $\delta$ updates its information, so there are $\delta$ agents who have current information, $\delta(1 - \delta)$ that have one-period-old information, $\delta(1 - \delta)^2$ with two-period-old information, and so on. Because agents only differ on the date at which they last updated, we can group them and let $j$ denote how long ago the planner last updated. Likewise, a share $\lambda$ of firms and $\omega$ of workers update their information every period, so they can be grouped into groups $i$ of size $\lambda(1 - \lambda)^i$ and groups $k$ of size $\omega(1 - \omega)^k$, according to how long it has been since they last updated.

The inattentive equilibrium is defined as follows: the set of aggregate variables $\{Y_t, L_t\}$, the output of each variety $\{Y_{t,i}\}$, the labor of each variety $\{L_{t,j}\}$, the prices of each good $\{P_{t,i}\}$, wages $\{W_{t,i}\}$, and interest rates $\{i_t\}$, such that consumers, workers, and firms behave optimally (as described above), all markets clear, and monetary policy follows a rule like equation (12), with $P_{-1} = 0$ for all dates $t$ from 0 to infinity as a function of the exogenous paths for technology $\{A_t\}$, monetary policy shocks $\{\varepsilon_t\}$, aggregate demand $\{G_t\}$, goods' substitutability $\{\tilde{v}_t\}$, and labor substitutability $\{\tilde{\gamma}_t\}$. The classical equilibrium is the equilibrium when $\delta = \lambda = \omega = 1$, so that all agents are attentive.

## 1.5 Missing Work on the Micro-Foundations of the Model

In the tradition of Kydland and Prescott (1982) and Rotemberg and Woodford (1997), the SIGE model presented above makes a

few simplifying assumptions, some of which are more common and others perhaps more unusual. Each of these presents an opportunity for future work to improve the model. I now discuss a few that seem particularly promising.

First, the model lacks investment and capital accumulation. Whether this absence significantly affects the dynamics of the other variables in this class of models is open to debate (Woodford, 2005; Sveen and Weinke, 2005), but modelling investment has the benefit of extending the model to explain one more macroeconomic variable. The SIGE model omits investment because the behavior of inattentive investors accumulating capital has not yet been studied, whereas there is previous work on the micro-foundations and implications of inattentiveness on the part of consumers (Reis, 2006a), price-seting firms (Mankiw and Reis, 2002; Reis, 2006b), and workers (Mankiw and Reis 2003). Gabaix and Laibson (2002) and Abel, Eberly, and Panageas (2007) study financial investment decisions with inattentiveness, but the step from this work to studying physical investment and capital accumulation remains to be taken.

Second, the model lacks international trade and exchange rates. The reason for this omission is the same as for investment: the models of inattentive behavior in international markets are still missing. Progress in this area will likely come soon, as Bachetta and van Wincoop (2006) have already filled some of the gap. Once this is completed, one can build an open economy SIGE to use for economies other than the United States or the euro area.

Third, the model lacks wealth heterogeneity since it assumes a complete insurance contract with which households fully diversify their risks. Most business cycle models make this assumption because it makes them more tractable by collapsing the wealth distribution to a single point. Relaxing this assumption and numerically computing the equilibria should not be difficult, but it has not yet been undertaken.

With regard to the micro-foundations of inattentiveness, the model assumes that when agents pay the cost to obtain new information, they can observe everything. While there is an explicit fixed cost of information, the variable cost is zero. This assumption is useful because it allows the model to emphasize the decision of when and how often to pay attention, which can then be studied in detail. It can easily be relaxed to allow people to observe only some things but not everything when they update (see, for example, Carroll and Slacalek, 2006). A harder extension would be to also consider the decision of how

much to pay attention, by letting people pick which pieces of news to look at when they update. Mackowiack and Wiederholt (2007) have made promising progress in this area, following Sims (1998), but the models are still not at the point where they can be put in general equilibrium and taken to the data.

One implication of removing the assumption that updating agents learn everything, is that there is no longer common knowledge in the economy. This leads to a new source of strategic interactions between agents who have different information and know that no one knows everything. Woodford (2003a), Hellwig (2002), Amato and Shin (2006), Morris and Shin (2006), and Adam (2007) all study some of the implications of this behavior, and recent work by Lorenzoni (2008) moves toward turning these insights into a business cycle model that could be taken to the data. Hellwig and Veldkamp (2008) study another source of strategic interaction, namely, whether agents coordinate their attention times. These extra ingredients promise to enrich future models of inattentiveness.

The SIGE model ignores another source of strategic interaction. The model assumes that consumers have inattentive planners and attentive shoppers, while firms have inattentive sales departments and attentive purchasing departments. Consequently, monopolists only face attentive agents in every market. This is important because if a monopolist sells its product to some buyers that are inattentive, then it will want to exploit their inattentiveness to raise its profits (Gabaix and Laibson, 2006). These inattentive buyers would take into account this extra cost of being inattentive and alter their choices of when to update their information and how to act when uninformed. The equilibrium of this game has not, to my knowledge, been fully studied.

Overall, the SIGE model ignores many features that could lead to new and interesting insights. They were omitted mainly because they are not sufficiently understood to put them into the full DSGE setup used in this paper.

## 2. THE REDUCED-FORM LOG-LINEAR EQUILIBRIUM

The appendix describes how to log-linearize the equilibrium conditions around the Pareto-optimal steady state, where all the random variables are equal to their mean and the tax rates ensure that markups are zero. This gives a set of reduced-form relations characterizing the equilibrium of the log-linearized values of key aggregate variables (denoted with small letters and a $t$ subscript),

as a function of parameters and steady-state values (in small letters but no subscript).

First, summing the production function for the individual firms gives an aggregate relation between output $(y_t)$, productivity $(a_t)$, and labor $(l_t)$ with decreasing returns to scale at rate $\beta$:

$$y_t = \alpha_t + \beta l_t. \tag{19}$$

Second, the equilibrium in the goods market leads to a Phillips curve (or aggregate supply) linking the price level $(p_t)$ to marginal costs and desired markups. Real marginal costs rise with real wages $(w_t - p_t)$, since these are the cost of inputs; they rise with output $(y_t)$, as a result of decreasing returns to scale; and they fall with productivity $(a_t)$. Desired markups are lower the higher the elasticity of substitution across goods' varieties $(\nu_t)$, where $\nu$ is the steady-state elasticity of substitution for goods:

$$p_t = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i \, E_{t-i} \left\{ p_t + \frac{\beta(w_t - p_t) + (1-\beta)y_t - a_t}{\beta + \nu(1-\beta)} - \frac{\beta \nu_t}{(\nu-1)[\beta + \nu(1-\beta)]} \right\}. \tag{20}$$

Since only a fraction $\lambda$ of firms update their information and set their plans, current shocks only have an immediate impact of $\lambda$ on prices.

Third, the equilibrium in the bond market leads to an IS curve (or aggregate demand) relating output to three variables: a measure of wealth, namely, $y_\infty^c = \lim_{i \to \infty} E_t(y_{t+i})$, since higher expected future output stimulates current spending; the long real interest rate, defined as $R_t = E_t \sum_{j=0}^{\infty} (i_{t+j} - \Delta p_{t+1+j})$, since higher expected interest rates encourage postponing consumption; and shocks to government spending $(g_t)$, since these subtract from consumption:

$$y_t = \delta \sum_{j=0}^{\infty} (1-\delta)^j \, E_{t-j} \left( y_\infty^c - \theta R_t \right) + g_t, \tag{21}$$

Every period, only a randomly drawn share $\delta$ of consumers update their plan, so the larger the value of $\delta$, the more consumption responds to shocks as they occur.

Fourth, equilibrium in the labor market leads to a wage curve (or labor supply) according to which current wages ($w_t$) are higher with higher prices, since workers care about real wages; with higher expected real wages, since these push up the demand for a worker's variety of labor; with higher employment, since the marginal disutility of working rises; with higher wealth, since leisure is a normal good; with lower interest rates, since the return on savings is lower and the incentive to work to save is thus also lower; and with a lower elasticity of substitution across labor varieties, since desired markups are then higher:

$$
w_t = \omega \sum_{k=0}^{\infty} (1-\omega)^k E_{t-k} \left[ \begin{array}{c} p_t + \dfrac{\gamma(w_t - p_t)}{\gamma + \psi} + \dfrac{l_t}{\gamma + \psi} \\[2ex] + \dfrac{\psi(y_\infty^c - \theta R_t)}{\theta(\gamma + \psi)} - \dfrac{\psi \gamma_t}{(\gamma + \psi)(\gamma - 1)} \end{array} \right].
\tag{22}
$$

The fraction of up-to-date workers is $\omega$, with the remaining workers setting their wage to what they expected would be optimal when they last updated.

Finally, the policy rule gives the last reduced-form equilibrium relation. In the case of the Taylor rule, this relation is

$$
i_t = \phi_p \Delta p_t + \phi_y (y_t - y_t^c) - \varepsilon_t.
\tag{23}
$$

These five equations give the equilibrium values for inflation, nominal interest rates, output growth, employment, and real wage growth, $x_t = \{\Delta p_t, i_t, \Delta y_t, l_t, \Delta(w_t - p_t)\}$, as a function of the five exogenous shocks to aggregate productivity growth, aggregate demand, goods markups, labor markups, and monetary policy, $s_t = \{\Delta a_t, g_t, \nu_t, \gamma_t, \varepsilon_t\}$. I assume that each of these shocks follows an independent stationary stochastic process with (potentially infinite) moving-average representation. This assumption allows for a very general representation of the shocks hitting the economy. One implication is that there is a stochastic trend in the economy driven by productivity, which seems consistent with the data.

## 3. SOLVING FOR THE EQUILIBRIUM

I first solve for the equilibrium when all are attentive and then solve for the inattentive equilibrium under different policy rules.

Finally, I derive expressions for the likelihood and social welfare functions.

## 3.1 The Classical Equilibrium

In the classical equilibrium, all the agents are attentive, and simple algebra shows that output:

$$y_t^c = a_t + \Xi\left(g_t + \frac{\gamma_t}{\gamma - 1} + \frac{\nu_t}{\nu - 1}\right), \tag{24}$$

where $\Xi = \beta\psi/(1 + \psi)$, under the assumption that $\theta = 1$. Assuming that the elasticity of intertemporal substitution equals one implies that output moves one-to-one with the nonstationary productivity shocks, while hours worked, $l_t^c = (y_t^c - a_t)/\beta$, are stationary, as seems to be the case in the data.

In the classical equilibrium, output rises with each of the four real shocks, but it is independent of monetary policy shocks and the monetary policy rule. There are no nominal rigidities in this classical economy, so the classical dichotomy holds, with real variables being independent of monetary shocks.

Finally, it is important to note that this classical equilibrium is not necessarily optimal. The definition of a Pareto optimum is not obvious when there are changes in preferences. However, if the shocks to the preferences lead to an inefficiency relative to their steady-state values, then the optimal output is $y_t^o = a_t + \Xi g_t$, so shocks to the markups leads to inefficient fluctuations even if all agents are attentive.

## 3.2 The Inattentiveness Equilibrium

The solution of the inattentiveness equilibrium is a little more involved. One useful piece of notation is to write each variable in terms of its moving-average representation. For instance, for the generic shock $s \in S = \{\Delta a, g, \nu, \gamma, \varepsilon\}$, Wold's theorem implies that there is a representation $s_t = \sum_{n=0}^{\infty} \hat{s}_n e_{t-n}^s$, where the $e_t^s$ are independent zero-mean random variables. For the endogenous variables that depend on all five shocks, $y_t^c = \sum_s \sum_n \Xi(s)s_n$, where the new coefficients $\Xi(s)$ follow easily from equation (24) and the definitions of $\Xi$ and $\hat{s}_n$. Another useful piece of notation is to denote the share of people that have updated after $n$ periods by $\Lambda_n = \lambda\sum_{i=0}^{n}(1 - \lambda)^i$, $\Delta_n = \delta\sum_{i=0}^{n}(1 - \delta)^i$, and $\Omega_n = \omega\sum_{i=0}^{n}(1 - \omega)^i$.

The first result gives the first key step in the algorithm to solve the model:

*Proposition 1.* Writing the solution for the price level as $p_t = \sum_{s\in S}\sum_{n=0}^{\infty}\hat{p}_n(s)e_{t-n}^s$, where $\hat{p}_n(s)$ is a scalar measuring the impact of shock $s$ at lag $n$, and likewise for output with $\hat{y}_n(s)$, then, regardless of the policy rule,

$$\hat{y}_n(s) = \Psi_n \hat{p}_n(s) + \Upsilon_n(s)\hat{s}_n, \tag{25}$$

where

$$\Psi_n^{den} = (1-\beta)(\gamma+\psi)\theta\Delta_n + \Omega_n\left\{\theta\Delta_n\left[1-\gamma(1-\beta)\right]+\psi\beta\right\}, \tag{26}$$

$$\Psi_n = \frac{\theta\Delta_n\left\{\left[\psi+\gamma(1-\Omega_n)\right]\left[\dfrac{\beta+\nu(1-\beta)}{\Lambda_n}-\nu(1-\beta)\right]-\beta\psi\Omega_n\right\}}{\Psi_n^{den}}, \tag{27}$$

and

$$\Upsilon_n(s) = \begin{cases} \theta\Delta_n\left(\gamma+\psi+\Omega_n-\gamma\Omega_n\right)a_n/\Psi_n^{den} & \text{for } s = a \\ \beta\psi\Omega_n g_n/\Psi_n^{den} & \text{for } s = g \\ 3\theta\psi\Omega_n\Delta_n\gamma_n/\Psi_n^{den}\left(\gamma-1\right) & \text{for } s = \gamma \\ 3\theta\Delta_n\left(\psi+\gamma-\gamma\Omega_n\right)\nu_n/\Psi_n^{den}\left(\nu-1\right) & \text{for } s = \nu \\ 0 & \text{for } s = \varepsilon. \end{cases} \tag{28}$$

The proof of this (and all other results) is in the appendix. It implies that given a solution for prices, one can easily compute the solution for output. A closely associated result is the following:

*Proposition 2.* The moving-average coefficients for the short-term real interest, wages, and hours worked as a function of those for prices and output are,

$$\hat{r}_n(s) = \frac{\hat{y}_{n+1}(s)}{\theta\Delta_{n+1}} - \frac{\hat{y}_n}{\theta\Delta_n} + \begin{cases} \dfrac{\hat{s}_n}{\theta\Delta_n} - \dfrac{\hat{s}_{n+1}}{\theta\Delta_{n+1}} & \text{for } s = g \\ 0 & \text{for } s = a,\gamma,\nu,\varepsilon \end{cases} \tag{29}$$

$$(\hat{w}_n - \hat{p}_n)(s) = \left[1 + \nu\left(\frac{1}{\beta} - 1\right)\right]\left(\frac{1}{\Lambda_n} - 1\right)\hat{p}_n(s)$$

$$+\left(1 - \frac{1}{\beta}\right)\hat{y}_n(s) + \begin{cases} \dfrac{\hat{s}_n}{\beta} & \text{for } s = a \\[2mm] \dfrac{\hat{s}_n}{\nu - 1} & \text{for } s = \nu \\[2mm] 0 & \text{for } s = g, \gamma, \varepsilon \end{cases} \tag{30}$$

$$\hat{l}_n(s) = \frac{\hat{y}_n(s)}{\beta} - \begin{cases} \dfrac{\hat{s}_n}{\beta} & \text{for } s = a \\[2mm] 0 & \text{for } s = g, \gamma, \nu, \varepsilon \end{cases} \tag{31}$$

With these two propositions and a solution for prices, we have the equilibrium values of all the real variables independently of the monetary policy rule. We can therefore focus on solving for prices alone.

If the policy rule is the one proposed by Taylor, then using the Fisher equation, $i_t = r_t + E_t(\Delta p_{t+1})$, and the results in the previous two propositions leads to the solution for the price level: [4]

*Proposition 3.* If the policy rule is a Taylor rule, $i_t = \phi_p \Delta p_t + \phi_y(y_t - y_t^n) - \varepsilon_t$, the undetermined coefficients for the price level satisfy the second-order difference equation:

$$A_{n+1}\hat{p}_{n+1}(s) - B_n\hat{p}_n(s) + C_{n-1}\hat{p}_{n-1}(s) = D_n(s)\hat{s}_n \text{ for } n = 0,1,2,... \tag{32}$$

where

$$A_n = 1 + \Psi_n / \theta\Delta_n, \; B_n = A_n + \phi_p + \phi_y\Psi_n, \text{ and } C_n = \phi_p, \tag{33}$$

and where

$$D_n(s) = \begin{cases} \dfrac{\Upsilon_n(s)}{\theta\Delta_n} - \dfrac{\Upsilon_{n+1}(s)\hat{s}_{n+1}}{\theta\Delta_{n+1}\hat{s}_n} + \phi_y\left[\Upsilon_n(s) - \Xi(s)\right] & \text{for } s = a, \gamma, \nu \\[3mm] \dfrac{[\Upsilon_n(s) - 1]}{\theta\Delta_n} - \dfrac{[\Upsilon_{n+1}(s) - 1]\hat{s}_{n+1}}{\theta\Delta_{n+1}\hat{s}_n} + \phi_y\left[\Upsilon_n(s) - \Xi(s)\right] & \text{for } s = g \\[3mm] -1 & \text{for } s = \varepsilon \end{cases} \tag{34}$$

4. Mankiw and Reis (2007) present an initial version of this result, limited to AR(1) shocks.

Solving the difference equation requires two boundary conditions. As the time from the shock goes to infinity, all agents become aware of it, so the effect of the shock on the inattentive equilibrium is the same as that in the attentive equilibrium. Since the price level converges to a constant (nonzero for the technology shocks and zero for the other shocks) regardless of the shock, one boundary condition is $\lim_{n \to \infty} (\hat{p}_n - \hat{p}_{n+1}) = 0$. The other boundary condition is $\hat{p}_{-1} = 0$.

I solve the difference equations by writing, separately for each shock, a system of $N + 1$ equations for the $N + 1$ undetermined coefficients from $\hat{p}_0(s)$ to $\hat{p}_N(s)$ :

$$
\begin{pmatrix}
-B_0 & A_1 & \dots & 0 & 0 & 0 \\
C_0 & -B_1 & \dots & 0 & 0 & 0 \\
\dots & \dots & \dots & \dots & \dots & \dots \\
0 & 0 & \dots & -B_{N-2} & A_{N-1} & 0 \\
0 & 0 & \dots & C_{N-2} & -B_{N-1} & A_N \\
0 & 0 & \dots & 0 & 1 & -1
\end{pmatrix}
\begin{pmatrix}
\hat{p}_0(s) \\
\hat{p}_1(s) \\
\dots \\
\hat{p}_{N-2}(s) \\
\hat{p}_{N-1}(s) \\
\hat{p}_N(s)
\end{pmatrix}
=
\begin{pmatrix}
D_0(s) \\
D_1(s) \\
\dots \\
D_{N-2}(s) \\
D_{N-1}(s) \\
0
\end{pmatrix}. \tag{35}
$$

Because the system has a special tri-diagonal structure, it is numerically easy to solve. I have set $N$ at either 100, 500, or 1,000. In almost all cases, both the ignored terms of order above $N$ and the change in the first 100 coefficients as $N$ changed were negligible.

Because the goal of this paper is to provide a model that can be used to study monetary policy, it is important to consider alternative policy rules to the Taylor rule. The main alternative to interest-rate rules are targeting rules (Svensson, 2003). Ball, Mankiw, and Reis (2005) show that if only firms are inattentive, an elastic price standard is optimal:

*Proposition 4.* If policy follows an elastic price-level standard, $p_t = K_t - \phi(y_t - y_t^o)$, the undetermined coefficients for the price level are as follows:

$$
p_n(s) = \frac{\phi \left[ \tilde{\Xi}(s) - \Upsilon_n(s) \right] s_n}{1 + \phi \Psi_n} \text{ for } n = 0,1,2,\dots \tag{36}
$$

where $\tilde{\Xi}(s) = \Xi(s)$ for $s = a, g$; and $\tilde{\Xi}(s) = 0$ for $s = \gamma, \nu$.

The literature contains many alternative policy rules, and the appendix presents a few more and their corresponding solution. Together with the results in this section, this should provide sufficient evidence that despite the infinite number of expectations going

backward and the lack of a recursive representation for the endogenous variables, the SIGE model is still easy to solve.[5]

## 3.3 The Likelihood and Welfare Functions

The key input in likelihood-based estimation is the likelihood function. Letting $\mathbf{x}_t$ denote the $5 \times 1$ column vector with the endogenous variables of the model and $\mathbf{e}_t$ denote the column vector with the 5 exogenous shocks, the solution in propositions 1 to 4 can be expressed as a set of $5 \times 5$ matrices $\Phi_n$, such that $\mathbf{x}_t = \sum_{n=1}^{N} \Phi_n \mathbf{e}_{t-n}$. The data consists of time-series on $\mathbf{x}_t$ from $t = 1$ to $t = T$ for the endogenous variables, which can be stacked in a $5T \times 1$ vector $\mathbf{X}$, and the unknown parameters can be collected in the vector $\theta$. The likelihood function is then denoted by $L(\mathbf{X}|\theta)$.

I assume that the five zero-mean shocks $e_t^s$ are normally distributed with variances $\sigma_s^2$. The vector $\mathbf{e}_t$ therefore follows a multivariate normal distribution with diagonal covariance matrix $\boldsymbol{\Sigma}$. The notation $\mathbf{I}_N$ denotes an identity matrix of size $N$ and $\otimes$ for the Kronecker product of two matrices. Since the model is linear, $\mathbf{X}$ follows a multivariate normal distribution. This leads to the next proposition, taken from Mankiw and Reis (2007):

*Proposition 5.* Let $\boldsymbol{\Omega}$ be the $5T \times 5N$ matrix,

$$
\begin{pmatrix}
\Phi_0 & \Phi_1 & \Phi_2 & \dots & \dots & \dots & \Phi_{N-3} & \Phi_{N-2} & \Phi_{N-1} \\
0 & \Phi_0 & \Phi_1 & \dots & \dots & \dots & \Phi_{N-4} & \Phi_{N-3} & \Phi_{N-2} \\
0 & 0 & \Phi_0 & \dots & \dots & \dots & \Phi_{N-5} & \Phi_{N-4} & \Phi_{N-3} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \dots & 0 & \Phi_0 & \Phi_1 & \dots & \Phi_{N-T-1} & \Phi_{N-T}
\end{pmatrix};
\tag{37}
$$

the likelihood function is then

$$
L(\mathbf{X}|\theta) = -2.5T \ln(2\pi) - 0.5 \ln \left| \boldsymbol{\Omega}(\mathbf{I}_N \otimes \boldsymbol{\Sigma})\boldsymbol{\Omega}' \right| - 0.5 \mathbf{X}' \left( \boldsymbol{\Omega}(\mathbf{I}_N \otimes \boldsymbol{\Sigma})\boldsymbol{\Omega}' \right)^{-1} \mathbf{X}.
$$

Mankiw and Reis (2007) note that the large $5T \times 5T$ matrix $\boldsymbol{\Omega}(\mathbf{I}_N \otimes \boldsymbol{\Sigma})\boldsymbol{\Omega}'$ can be inverted either with a Choleski decomposition

---

5. Building on some of these results, Meyer-Gohde (2007) combines this approach with others in the literature to provide a unified user-friendly algorithm that can solve most DSGE models with forward and lagged expectations without requiring almost any algebra on the part of the user (unlike the propositions above). His set of programs holds the promise of further advancing this literature.

or by choosing $N = T$ to re-express the problem in terms of a system of linear equations. Either way, one can evaluate the log-likelihood function quickly and reliably.

A natural way to compare the performance of different policy rules is to compute the utility of the agents in the model. I focus on the unconditional expectation of a utilitarian measure of social welfare:

$$E\left[(1-\xi)\sum_{t=0}^{\infty}\xi^t \int\int U\left(C_{t,j},L_{t,k}\right)djdk\right]. \tag{38}$$

Because the model assumes that all households are ex ante identical and there are complete insurance markets, it is natural to assume that all households get the same weight in the integral. Moreover, because one wants a rule that performs well across circumstances, it makes sense to take the ex ante perspective provided by the unconditional expectation that integrates over all possible initial conditions. The appendix proves the following result:

*Proposition 6.* An approximate formula for the welfare benefits in percentage units of steady-state consumption of a policy $\theta^{(1)}$ starting from a policy $\theta^{(0)}$ are

$$\exp\left\{0.5\beta\left(1+\frac{1}{\psi}\right)\left[W\left(\theta^{(1)}\right)-W\left(\theta^{(0)}\right)\right]\right\}, \tag{39}$$

where

$$W\left(\theta\right)=-\sum_{s\in S}\sum_{n=0}^{\infty}\left[(1-\Omega_n)\varsigma_n\left(s\right)^2+\Omega_n\zeta_n\left(s\right)^2\right]\sigma_s^2, \tag{40}$$

$$\varsigma_n\left(s\right)=\hat{l}_n\left(s\right)+\gamma\hat{w}_n\left(s\right), \text{ for all } s, \tag{41}$$

and

$$\frac{\left(\gamma+\psi\right)\varsigma_n\left(s\right)}{\gamma\psi}=\frac{\hat{l}_n\left(s\right)}{\gamma}+\left(\hat{w}-\hat{p}\right)_n\left(s\right)+\frac{\hat{y}_n\left(s\right)}{\Delta_n}+\begin{cases}0 & \text{for } s=\varepsilon,a,\nu \\ \dfrac{\hat{s}_n}{\gamma-1} & \text{for } s=\gamma \\ -\dfrac{\hat{s}_n}{\Delta_n} & \text{for } s=g\end{cases} \tag{42}$$

Combining this result with those in propositions 1 to 4, it is easy to evaluate this expression and compare the performance of different policy rules.

## 4. ESTIMATING STICKY INFORMATION

Taking sticky information models to the data has been an active field of research. One approach is to look for direct evidence of inattentiveness using microeconomic data. Carroll (2003) uses surveys of inflation expectations to show that the public's forecasts lag the forecasts made by professionals.[6] Mankiw, Reis, and Wolfers (2004) show that the disagreement in the inflation expectations in the survey data have properties consistent with sticky information.[7] Reis (2006a) and Carroll and Slacalek (2006) interpret some of the literature on the sensitivity and smoothness of microeconomic consumption data in the light of sticky information, and Klenow and Willis (2007) and Knotek (2006) find slow dissemination of information in the microeconomic data on prices. For the most part, this literature supports the assumption of sticky information, and the associated estimates of the information-updating rates are consistent.

A second approach is to estimate Phillips curves assuming sticky information on the part of price setters only.[8] These limited-information approaches typically use data on inflation, output, marginal costs, and expectations to estimate simpler versions of equation (20), and the results are typically good or mixed. One interesting finding that comes out of many of these studies is that the main source of discrepancy between the model and the data is not the inattentiveness or the slow dissemination of information, but the assumption that, conditional on their information sets, agents form expectations rationally.

This paper takes a third approach, of estimating the model using full-information techniques that exploit the restrictions imposed by general equilibrium. The few papers that attempt this exercise typically find either mixed or poor fits between the model and the data.[9] Mankiw and Reis (2006) explain the contrast between the

6. See also Dopke and others (2008) and Nunes (2006).
7. Also focussing on disagreement, see Gorodnichenko (2006), Branch (2007), and Rich and Tracy (2006).
8. See Khan and Zhu (2006), Dopke and others (2006), Korenok (2005), Pickering (2004), Coibion (2007), and Molinari (2007).
9. See Trabandt (2007), Andrés, Nelson, and López-Salido (2005), Kiley (2007), Laforte (2007), Korenok and Swanson (2005, 2007), and Paustian and Pytlarczyk (2006).

negative results in some of these papers and the mostly positive results found by the other two approaches. They note that the papers in this literature assume inattentiveness only in price setting, while assuming that the other agents in the model are fully attentive. To fit the data, however, stickiness should be pervasive, and for the internal coherence of the model, inattentiveness should apply to all decisions. By assuming attentive consumer and workers, the general-equilibrium restrictions imposed in these papers are misspecified.

Allowing for pervasive stickiness, I take a Bayesian approach to deal with the uncertainty, starting with a prior joint probability density $p(\theta)$ and using the likelihood function $L(\mathbf{X}|\theta)$ to obtain the posterior density of the parameters $p(\theta|\mathbf{X})$. This is done numerically, using Markov chain Monte Carlo simulations.[10]

The prior density $p(\theta)$ follows the convention in the DSGE literature (for example, An and Schorfheide, 2007), including assuming that the shocks $s_t$ follow first order autoregressive, or AR(1), processes with coefficients $\rho_s$ and innovation standard deviations $\sigma_s$. There are twenty parameters in the model: $\theta = \{\theta, \psi, \nu, \gamma, \beta, \rho_{\Delta a}, \sigma_{\Delta a}, \rho_\varepsilon, \sigma_\varepsilon, \rho_g, \sigma_g, \rho_\nu, \sigma_\nu, \rho_\gamma, \sigma_\gamma, \phi_p, \phi_y, \delta, \omega, \lambda\}$. Table 1 shows the moments of the prior densities.

Four of the parameters have a tight prior with zero variance: $\theta$, which is set to one to ensure stationary hours worked; $\beta$, which equals two-thirds to match the labor share in the data; and $\rho_{\Delta a}$ and $\sigma_{\Delta a}$, since a series for productivity growth follows from the data on output and employment in equation (19), so we can recover these parameters by a simple least-squares regression.[11]

Each of the remaining sixteen parameters is treated independently and is assigned a particular distribution (gamma, beta, or uniform) with a relatively large variance. The mean elasticity of labor supply, $\psi$, is 2 and the elasticities of substitution across goods and labor varieties, $\nu$ and $\gamma$, are set at 11, in line with the typical assumptions in the literature. The mean $\rho_s$ for the four shocks other than productivity are set to 0.9, so that the half-life of the shocks is approximately six quarters and the $\sigma_s$ are set to 0.5, which lies between the two values estimated for $\sigma_{\Delta a}$.[12] The

---

10. The exact algorithm is described in the appendix.

11. The values for $\rho_{\Delta a}$ and $\sigma_{\Delta a}$ are 0.03 and 0.51, respectively, for the United States and 0.66 and 0.28 for the euro area.

12. For the markups, the value for the standard deviation is multiplied by ten and the elasticities of substitution are multiplied by minus one, to counteract the multiplier that is visible in equations (20) and (22).

**Table 1. Prior Distribution**

| Parameter | Density[a] | Mean | Standard deviation | Percentile | | |
|---|---|---|---|---|---|---|
| | | | | 2.5 | 50.0 | 97.5 |
| Preferences | | | | | | |
| $\nu$ | $1+G$ | 11 | 3.16 | 5.80 | 10.67 | 18.08 |
| $\gamma$ | $1+G$ | 11 | 3.16 | 5.80 | 10.67 | 18.08 |
| $\psi$ | $G$ | 2 | 2.00 | 0.05 | 1.39 | 7.38 |
| Nonpolicy shocks | | | | | | |
| $\rho_g$ | $B$ | 0.90 | 0.20 | 0.23 | 0.99 | 1.00 |
| $\sigma_g$ | $G^{-1/2}$ | 0.50 | 0.24 | 0.21 | 0.39 | 1.02 |
| $\rho_\nu$ | $B$ | 0.90 | 0.20 | 0.23 | 0.99 | 1.00 |
| $\sigma_\nu$ | $G^{-1/2}$ | 0.50 | 0.24 | 0.21 | 0.39 | 1.02 |
| $\rho_\gamma$ | $B$ | 0.90 | 0.20 | 0.23 | 0.99 | 1.00 |
| $\sigma_\gamma$ | $G^{-1/2}$ | 0.50 | 0.24 | 0.21 | 0.39 | 1.02 |
| Monetary policy | | | | | | |
| $\phi_p$ | $1+G$ | 1.24 | 0.25 | 1.00 | 1.16 | 1.92 |
| $\phi_y$ | $G$ | 0.33 | 0.25 | 0.03 | 0.27 | 0.97 |
| $\rho_\varepsilon$ | $B$ | 0.90 | 0.22 | 0.23 | 0.99 | 1.00 |
| $\sigma_\varepsilon$ | $G^{-1/2}$ | 0.50 | 0.11 | 0.21 | 0.39 | 1.02 |
| Inattentiveness | | | | | | |
| $\delta$ | $U$ | 0.50 | 0.29 | 0.03 | 0.50 | 0.98 |
| $\omega$ | $U$ | 0.50 | 0.29 | 0.03 | 0.50 | 0.98 |
| $\lambda$ | $U$ | 0.50 | 0.29 | 0.03 | 0.50 | 0.98 |

Source: Author's calculations.
a. The densities are the gamma ($G$), beta ($B$) and uniform ($U$).

monetary policy parameters are set at $\phi_p = 1.24$ and $\phi_y = 0.33$, which are the values estimated by Rudebusch (2002) on U.S. data. Finally, the inattentiveness parameters $\delta$, $\omega$, and $\lambda$ have a flat prior in the unit interval.

As for the data, I use quarterly observations for two large economies: the United States from 1986:3 to 2006:1 and the euro area from 1993:4 to 2005:4. I chose these countries because they are closer to the closed-economy approximation in the model. The starting dates coincide with the start of Alan Greenspan's term as chairman of the Federal Open Market Committee (FOMC) in the United States and with the signing of the Maastricht treaty that created the European Union and started the coordination of monetary policy towards the euro, so they are consistent with assuming a stable monetary policy rule. They come after the "great moderation" in economic activity, consistent with assuming constant variances of the shocks.

The data for the United States are seasonally adjusted, refer to the nonfarm business sector, and comprise observations on growth in real output per capita, growth in total real compensation per hour, hours worked per capita, and inflation. All series are de-meaned; they use the implicit nonfarm business price deflator for the price level and for deflating nominal values; and growth rates refer to the change in the natural logarithm. The nominal interest rate is the effective Federal funds rate. The data for the euro area are the area-wide quarterly dataset that combines data from each country's national accounts to build consistent pseudo-aggregates for the whole region. Inflation is the change in the log of the GDP deflator, output growth is the change in log real GDP, and wages are measured using total compensation. To obtain variables per capita, I use an interpolated euro area population series. The hours data are detrended using a linear trend.

## 5. Estimates of the Model

I discuss the estimates for the two regions separately.

### 5.1 The United States

Table 2 displays summary statistics of the posterior distribution of the parameters. The posterior moments for the elasticities of

**Table 2. Posterior Distribution for the United States[a]**

| Parameter | Mean | Standard deviation | Percentile | | |
|---|---|---|---|---|---|
| | | | 2.5 | 50.0 | 97.5 |
| **Preferences** | | | | | |
| $\nu$ | 10.09 | 2.67 | 5.83 | 9.75 | 15.93 |
| $\gamma$ | 9.09 | 2.64 | 4.74 | 8.83 | 14.63 |
| $\psi$ | 5.15 | 2.52 | 1.18 | 4.94 | 10.95 |
| **Nonpolicy shocks** | | | | | |
| $\rho_g$ | 0.99 | 0.01 | 0.98 | 1.00 | 1.00 |
| $\sigma_g$ | 0.83 | 0.16 | 0.59 | 0.81 | 1.23 |
| $\rho_\nu$ | 0.28 | 0.10 | 0.08 | 0.28 | 0.48 |
| $\sigma_\nu$ | 0.11 | 0.06 | 0.03 | 0.09 | 0.26 |
| $\rho_\gamma$ | 0.86 | 0.08 | 0.71 | 0.85 | 1.00 |
| $\sigma_\gamma$ | 0.12 | 0.06 | 0.05 | 0.11 | 0.27 |
| **Monetary policy** | | | | | |
| $\phi_p$ | 1.17 | 0.16 | 1.01 | 1.12 | 1.60 |
| $\phi_y$ | 0.06 | 0.03 | 0.01 | 0.06 | 0.14 |
| $\rho_\varepsilon$ | 0.29 | 0.12 | 0.07 | 0.30 | 0.52 |
| $\sigma_\varepsilon$ | 0.44 | 0.09 | 0.30 | 0.43 | 0.65 |
| **Inattentiveness** | | | | | |
| $\delta$ | 0.08 | 0.03 | 0.03 | 0.08 | 0.16 |
| $\omega$ | 0.74 | 0.17 | 0.34 | 0.78 | 0.98 |
| $\lambda$ | 0.52 | 0.17 | 0.28 | 0.48 | 0.94 |

Source: Author's calculations.
a. All numbers are based on 450,000 draws from the posterior.

substitution across varieties are close to the prior assumptions from the literature. The elasticity of labor supply is quite large, but still in line with typical assumptions in the business cycle literature. As for the shocks, the aggregate demand disturbances are very persistent and quite volatile, so one can already guess that they are playing an important part in the volatility of the economy.

The more interesting estimates are those of the inattentiveness parameters, on which the prior had less information. Firms are estimated to be inattentive for six months, on average, which is slightly more attentive than what was found in the studies described in the previous section. Consumers are very inattentive, updating their information once every three years, on average. This is not too shocking considering that fixed costs of planning of less than $100 per household can easily generate this length of inattentiveness. Moreover, between 20 percent and 50 percent of the U.S. population lives hand-to-mouth, which is equivalent to being inattentive forever (Reis, 2006a).

The more surprising estimate in the table is the inattention of workers, who update their information very often, on average once every four months. One possible explanation for this result is that the data series used for wages measured total compensation, a large fraction of which is accounted for by nonwage payments. It is conceivable that the many dimensions of an employee's compensation may actually be updated to include new information quite often, even if the wage component of this compensation is not. Preliminary calculations using a wage series find more inattentive workers, and workers are also more inattentive in the euro area, where nonwage compensation is less important.

Figure 1 shows the impulse responses of four variables (namely, inflation, nominal interest rates, hours worked, and the output gap) to one-standard-deviation impulses to the five shocks. The most surprising finding is perhaps the quick response of inflation to monetary policy shocks. The conventional wisdom from studies using postwar U.S. data is that this response should be delayed and hump shaped. As recent studies have shown, however, inflation responds much faster to monetary policy after 1980, which some researchers attribute to changes in monetary policy.[13] From the perspective of the SIGE model, inflation responds quickly to monetary policy because monetary policy shocks are quite short-lived. When policy changes, the SIGE model predicts a

13. See Boivin and Giannoni (2006) and the references therein.

change in the dynamics of the model that matches the data, surviving the Lucas critique in a way that pricing models that always produce a hump shape do not.

## Figure 1. Impulse Response Functions to the Five Shocks: United States
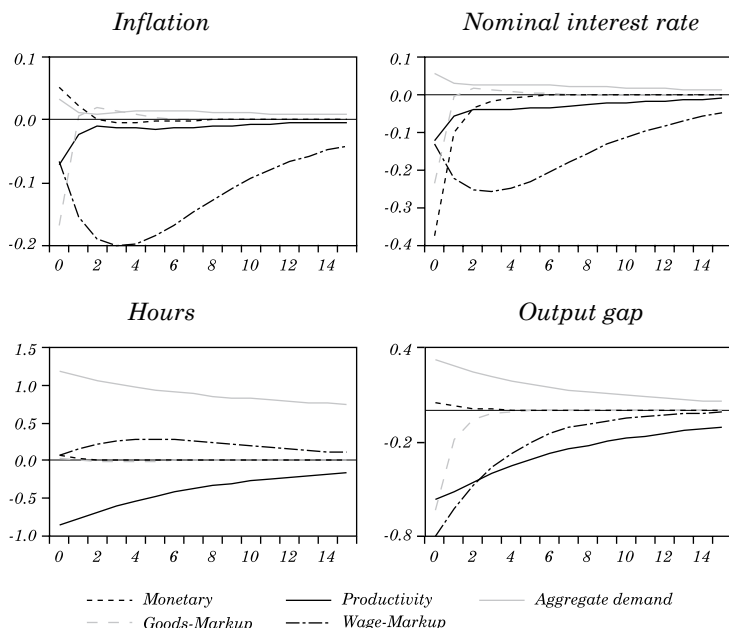


Source: Author's calculations.

Table 3 presents the predicted variance decompositions at different horizons. Monetary policy shocks play a small role in the variance of most macroeconomic variables in the United States after 1986, with the exception of the nominal interest rate and wages. Productivity shocks are important for real wages at all horizons and for hours worked at short horizons, while aggregate demand shocks explain much of the variability of output growth and hours worked.[14] Finally, inflation is significantly driven by the markup shocks.

14. Of all the model's shocks, these aggregate demand shocks are closest to the shocks to the marginal rate of substitution between consumption and leisure that Hall (1997) argues account for most of the U.S. business cycle.

**Table 3. Variance Decompositions for the United States[a]**
In percentage points

| Variable | Shock | | | | |
|---|---|---|---|---|---|
| | *Monetary* | *Aggregate productivity* | *Aggregate demand* | *Goods markup* | *Labor markup* |
| *A. Contribution to unconditional variance* | | | | | |
| Inflation | 1 (0, 35) | 7 (1, 35) | 3 (0, 58) | 11 (1, 58) | 68 (7, 92) |
| Interest rate | 24 (1, 69) | 6 (1, 30) | 3 (0, 44) | 8 (2, 28) | 49 (8, 84) |
| Output growth | 0 (0, 3) | 4 (1, 11) | 94 (83, 98) | 0 (0, 1) | 1 (0, 5) |
| Hours worked | 0 (0, 0) | 4 (1, 21) | 94 (26, 98) | 0 (0, 0) | 1 (0, 70) |
| Wage growth | 12 (1, 30) | 45 (20, 77) | 6 (1, 18) | 8 (3, 22) | 24 (0, 59) |
| *B. Contribution to one-quarter-ahead, one-year-ahead, and four-year-ahead variance* | | | | | |
| Inflation | 7, 3, 1 | 12, 7, 7 | 3, 2, 2 | 64, 24, 12 | 8, 57, 71 |
| Interest rate | 63, 40, 26 | 6, 6, 6 | 1, 2, 2 | 21, 12, 9 | 5, 31, 49 |
| Output growth | 0, 0, 0 | 1, 3, 4 | 98, 95, 94 | 0, 0, 0 | 0, 1, 1 |
| Hours worked | 0, 0, 0 | 35, 32, 21 | 64, 67, 75 | 0, 0, 0 | 0, 1, 3 |
| Wage growth | 11, 12, 12 | 47, 46, 45 | 6, 6, 6 | 6, 9, 8 | 26, 23, 24 |

Source: Author's calculations.
a. Panel A reports the median of 10,000 parameter draws from the posterior distribution; the numbers in parentheses are the 2.5 and 97.5 percentiles. Panel B reports medians from the same number of draws. Rows will not add up to 100, since the medians are cell by cell.

## 5.2 The Euro Area

Table 4 shows moments from the posterior distribution for the euro area. Relative to the U.S. estimates, there are two differences. First, the estimated average markups are larger for the euro area than for the United States. Second, the elasticity of labor supply is somewhat smaller, although it is still large compared with typical estimates based on microeconomic data. The inattentiveness of European firms is similar to that of American firms, while consumers are more attentive and workers less attentive. This brings the two members of the household in line, with both updating every nine to fifteen months, on average.

**Table 4. Posterior Distribution for the Euro Area[a]**

| | | Standard | Percentile | | |
| | | | 2.5 | 50.0 | 97.5 |
| Parameter | Mean | deviation | 2.5 | 50.0 | 97.5 |
|---|---|---|---|---|---|
| Preferences | | | | | |
| $\nu$ | 8.16 | 1.31 | 5.94 | 7.98 | 10.80 |
| $\gamma$ | 7.11 | 0.75 | 5.49 | 7.26 | 8.34 |
| $\psi$ | 2.70 | 0.43 | 1.92 | 2.74 | 3.46 |
| Nonpolicy shocks | | | | | |
| $\rho_g$ | 0.99 | 0.01 | 0.95 | 0.99 | 1.00 |
| $\sigma_g$ | 0.37 | 0.10 | 0.22 | 0.35 | 0.62 |
| $\rho_\nu$ | 0.70 | 0.21 | 0.31 | 0.67 | 0.98 |
| $\sigma_\nu$ | 0.08 | 0.05 | 0.03 | 0.07 | 0.20 |
| $\rho_\gamma$ | 0.37 | 0.15 | 0.09 | 0.39 | 0.62 |
| $\sigma_\gamma$ | 0.19 | 0.09 | 0.08 | 0.17 | 0.41 |
| Monetary policy | | | | | |
| $\phi_p$ | 1.06 | 0.10 | 1.00 | 1.01 | 1.35 |
| $\phi_y$ | 0.07 | 0.02 | 0.01 | 0.05 | 0.24 |
| $\rho_\varepsilon$ | 0.51 | 0.11 | 0.27 | 0.54 | 0.66 |
| $\sigma_\varepsilon$ | 0.46 | 0.12 | 0.30 | 0.44 | 0.75 |
| Inattentiveness | | | | | |
| $\delta$ | 0.21 | 0.11 | 0.10 | 0.17 | 0.52 |
| $\omega$ | 0.31 | 0.18 | 0.15 | 0.26 | 0.93 |
| $\lambda$ | 0.58 | 0.15 | 0.26 | 0.62 | 0.79 |

Source: Author's calculations.
a. All numbers are based on 450,000 draws from the posterior.

Figure 2 shows the impulse responses to shocks in the euro area. The response of inflation to a monetary shock is now slightly hump shaped, but it peaks just two quarters after the shock. Moreover, the response of all variables to a monetary shock is more delayed than in the United States.

## Figure 2. Impulse Response Functions to the Five Shocks: Euro Area
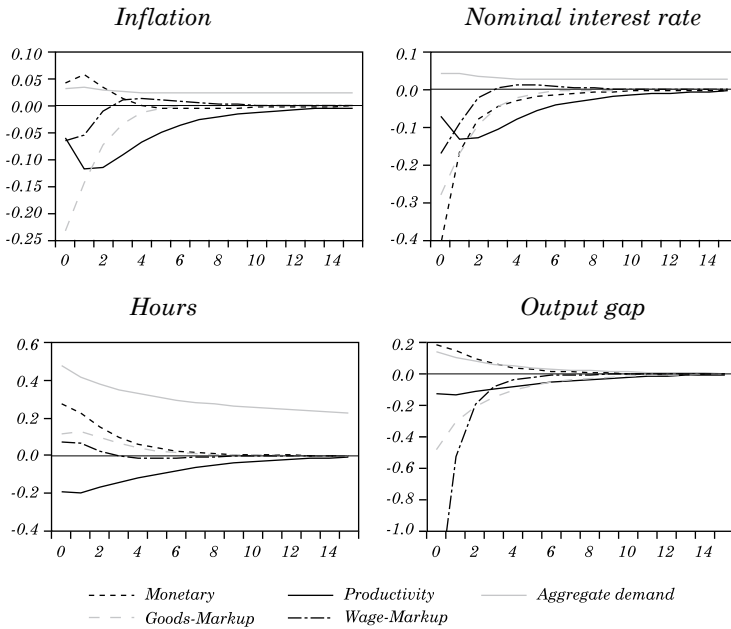


Source: Author's calculations.

As was the case for the United States, a positive productivity shocks raises total output but lowers hours worked and the output gap on impact, consistent with the evidence in Galí (2004). Because many firms initially do not know about the shock, they do not raise their output as much as they would with full information. Likewise, an increase in the elasticities of substitution (that is, a positive markup shock) raises hours worked and output, but leads to a negative output gap, because the expansion is smaller than would be the case with full information. Aggregate demand shocks boost inflation and the output gap and thus raise nominal interest rates, via the Taylor rule.

Table 5 has the variance decompositions for the euro area. Monetary policy shocks play a significantly larger role in explaining the variability of output growth and hours worked than they did in the United States, while productivity shocks are also more important drivers of output and inflation. Aggregate demand shocks are still important in explaining output and hours worked, as are markup shocks for inflation.

## 6. ROBUSTNESS OF THE ESTIMATES

This section summarizes the impact of several changes to the specification choices on the posterior estimates. Starting with the priors, I attempted a few variations from the baseline in table 1. Because fully characterizing the posterior distributions is computationally time consuming, I focused only on their modes. The three experiments were as follows: raising the prior mean for the elasticity of labor supply from 2 to 4; lowering the prior mean correlation of the shocks from 0.9 to 0.5; and setting the prior standard deviation of the shocks equal to $\sigma_{\Delta a}$ in each region, rather than to the 0.5 in-between value. Each of these changes had a negligible difference in the mode of the posterior distribution.

With regard to the policy rule, an alternative to the Taylor rule in equation (23) with serially correlated shocks is an inertial rule:

$$i_t = \phi_p \Delta p_t + \phi_y \left( y_t - y_t^c \right) + \rho_i i_{t-1} - \varepsilon_t, \tag{43}$$

where the $\varepsilon_t$ are serially uncorrelated. I estimated this alternative model and obtained a mean posterior estimate for $\rho_i$ of 0.25 for the United States and 0.16 for the euro area. In terms of overall fit to the data, the results are mixed. For the United States, the marginal density for the inertial rule is higher, whereas for the euro area, the Taylor rule with correlated shocks dominates.

In terms of the data, the main issue to address is a clear upward trend in hours worked in the euro area, associated with the slow decline in European unemployment. In the main results, I dealt with it by removing a linear trend from the data. Using a Hodrick-Prescott (HP) filter led to the same results. There is no trend in the U.S. data, so detrending it with the HP filter or even not detrending it at all led to almost indistinguishable data series.

Finally, for the sample periods, Mankiw and Reis (2007) estimate a subset of the parameters using postwar U. S. data.

**Table 5. Variance Decompositions for the Euro Area**[a]
In percentage points

| Variable | Shock | | | | |
| --- | --- | --- | --- | --- | --- |
| | Monetary | Aggregate productivity | Aggregate demand | Goods markup | Labor markup |
| *A. Contribution to unconditional variance* | | | | | |
| Inflation | 2 | 15 | 12 | 40 | 2 |
| | (0, 18) | (1, 52) | (0, 85) | (5, 91) | (0, 46) |
| Interest rate | 30 | 10 | 7 | 22 | 4 |
| | (8, 78) | (1, 26) | (0, 71) | (4, 78) | (0, 37) |
| Output growth | 16 | 25 | 50 | 3 | 1 |
| | (4, 42) | (11, 42) | (20, 79) | (1, 16) | (0, 12) |
| Hours worked | 2 | 5 | 84 | 1 | 0 |
| | (0, 17) | (0, 27) | (28, 99) | (0, 64) | (0, 2) |
| Wage growth | 2 | 31 | 1 | 36 | 20 |
| | (0, 22) | (10, 53) | (0, 5) | (4, 71) | (3, 80) |
| *B. Contribution to one-quarter-ahead, one-year-ahead, and four-year-ahead variance* | | | | | |
| Inflation | 3, 4, 3 | 8, 31, 26 | 2, 3, 4 | 78, 53, 48 | 5, 4, 3 |
| Interest rate | 60, 46, 40 | 3, 15, 14 | 1, 1, 2 | 25, 25, 25 | 7, 6, 5 |
| Output growth | 18, 16, 16 | 12, 24, 26 | 63, 51, 49 | 3, 3, 3 | 1, 1, 1 |
| Hours worked | 17, 13, 6 | 14, 18, 13 | 60, 59, 69 | 3, 2, 1 | 1, 0, 0 |
| Wage growth | 2, 2, 2 | 13, 31, 31 | 1, 1, 1 | 47, 34, 36 | 32, 21, 20 |

Source: Author's calculations.
a. Panel A reports the median of 10,000 parameter draws from the posterior distribution; the numbers in parentheses are the 2.5 and 97.5 percentiles. Panel B reports medians from the same number of draws. Rows will not add up to 100, since the medians are cell by cell.

Relative to the results in table 2, they find that workers and consumers update their information every five to six quarters, on average, which is close to the euro area estimates in this paper. They also find much more persistent and volatile monetary policy shocks, such that monetary shocks account for a large share of the volatility of the macroeconomic series. One conjecture for what is behind this discrepancy is that including the high inflation of the 1970s in the sample requires large monetary policy shocks that play a large role in the business cycle.

## 7. POLICY QUESTIONS

To begin applying the two estimated models to policy analysis, I explore some questions about monetary policy.

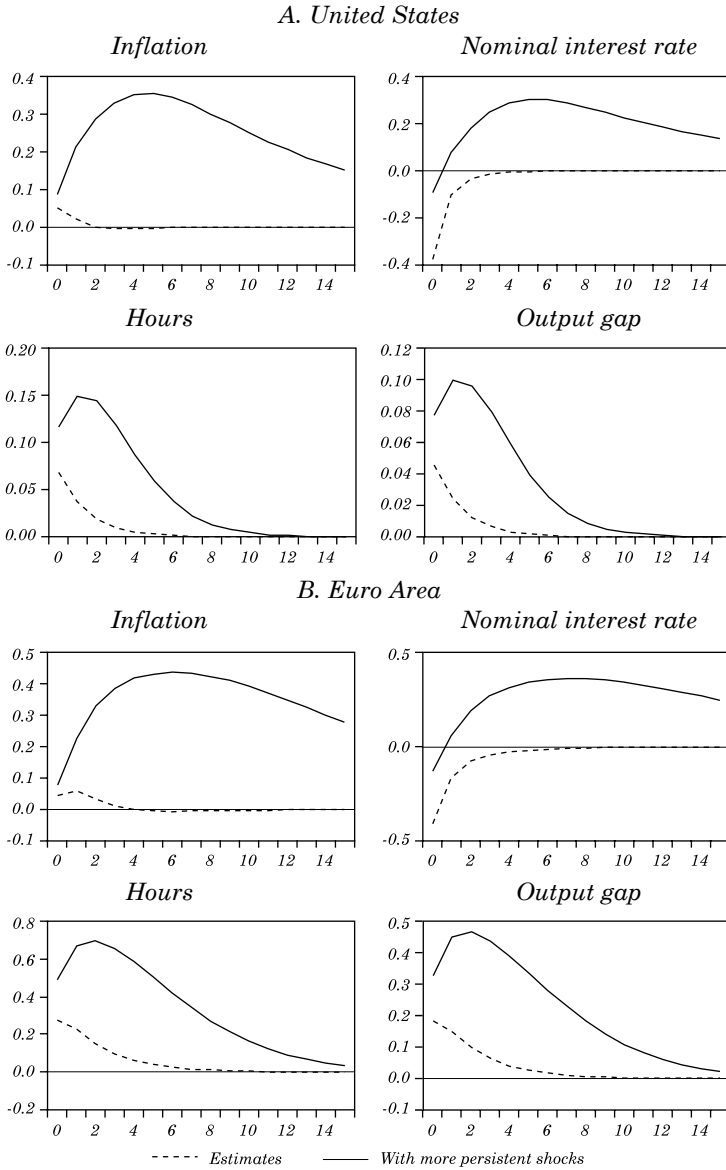## 7.1 What Rule Has Best Described Policy?

An extensive literature, starting with Taylor (1993), documents that the policy rule in equation (23) provides a good description of policy in the United States and a reasonable description of policy in the euro area. Within this common rule, there is room for differences between the two regions in the parameters of the rule.

According to the estimates in tables 2 and 4, monetary policy has been quite similar in the United States post-1986 and in the euro area post-1993, especially in only modestly responding to real activity. The estimates of $\phi_p$ and $\phi_y$ are somewhat lower than the typical result in the literature, but the more surprising posterior mean is the low persistence of monetary policy shocks, especially in the United States.

As noted in section 5, the estimated quick response of most macroeconomic variables to monetary policy shocks is linked to these low estimates of persistence. Figure 3 backs this claim by comparing the impulse responses in the status quo with the responses to raising the persistence of monetary shocks from the posterior means to the prior mean of 0.9. This change reestablishes the conventional delayed hump-shaped responses found in the literature on the post-war United States (Christiano, Eichenbaum, and Evans, 1999).[15]

_____

15. Coibion (2006) first pointed out the role of the persistence of interest rate shocks in delivering hump shapes.

**Figure 3. Impulse Response Functions to a More Persistent Monetary Shock**

### A. United States

Inflation

Nominal interest rate

Hours

Output gap

### B. Euro Area

Inflation

Nominal interest rate

Hours

Output gap

- - - - Estimates  ——— With more persistent shocks

## 7.2 What Is the Role of Policy Announcements?

The past decade has seen an increasing emphasis on transparency in central banking. Part of the argument for transparency is that if the central bank acts predictably, it will reduce confusion and mistakes on the part of private decisionmakers. According to this point of view, if policy shocks must take place, then they should be announced in advance and clearly communicated to the general public. In the context of the SIGE model, this calls for announcing monetary policy shocks a few quarters in advance, so that a large fraction of agents have time to learn of the event in the interim between announcement and action.
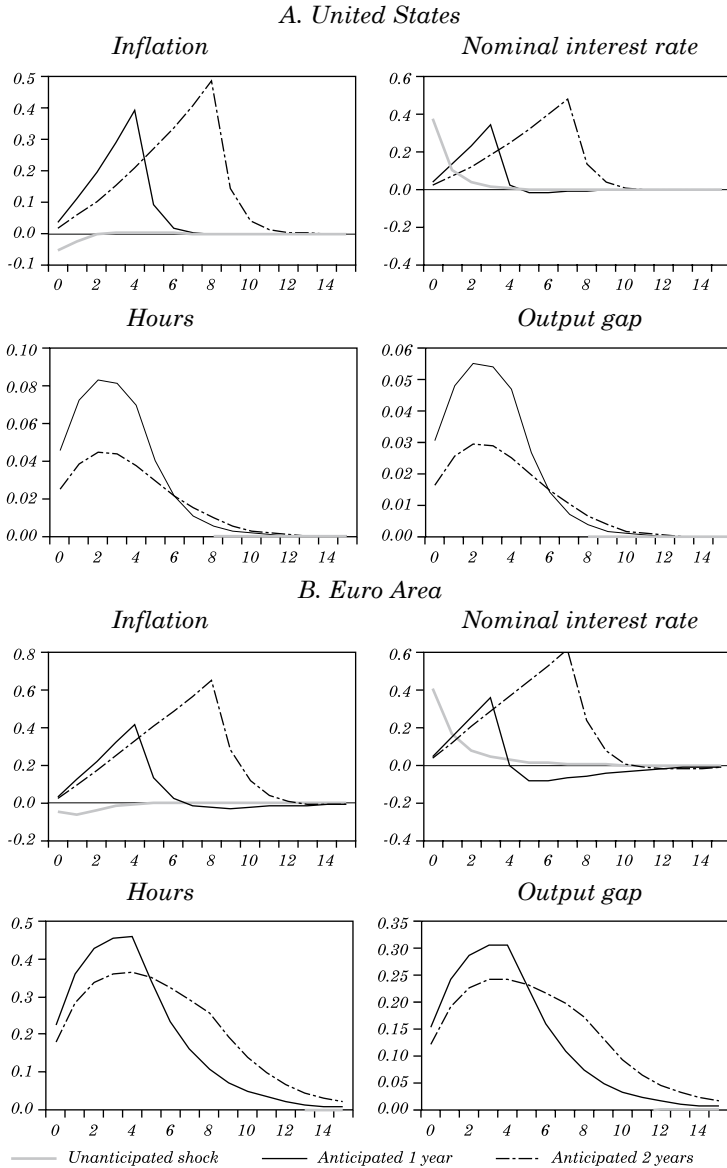
Figure 4 shows the results of announcing a monetary policy shock one or two years ahead in the United States and the euro area. The exercise here consists of learning at date $t = 0$ the value of the monetary shock to occur at dates $t = 4$ or $t = 8$. The announcement is therefore still a shock in the sense of a deviation from the policy rule. The figure reveals that inflation and nominal interest rates move even before the shock materializes because forward-looking agents react instantly to the news of a future shock. The agents that update their information learn about the shocks before it happens and adjust their actions in response. In both regions, announcements lower the initial impact of monetary policy shocks on hours worked and the output gap, while significantly increasing the overall impact on inflation.

## 7.3 What Is the Result of Having Interest Rates Move Gradually?

As described by Bernanke (2004), the FOMC tends to change interest rates gradually. Academic arguments in favor of such actions typically involve financial stability, the gradual revelation of news, or the desire to move long-term interest rates. Woodford (2003c) notes that in forward-looking models like SIGE, gradualism involves combining policy responses with announcements of future policy changes.
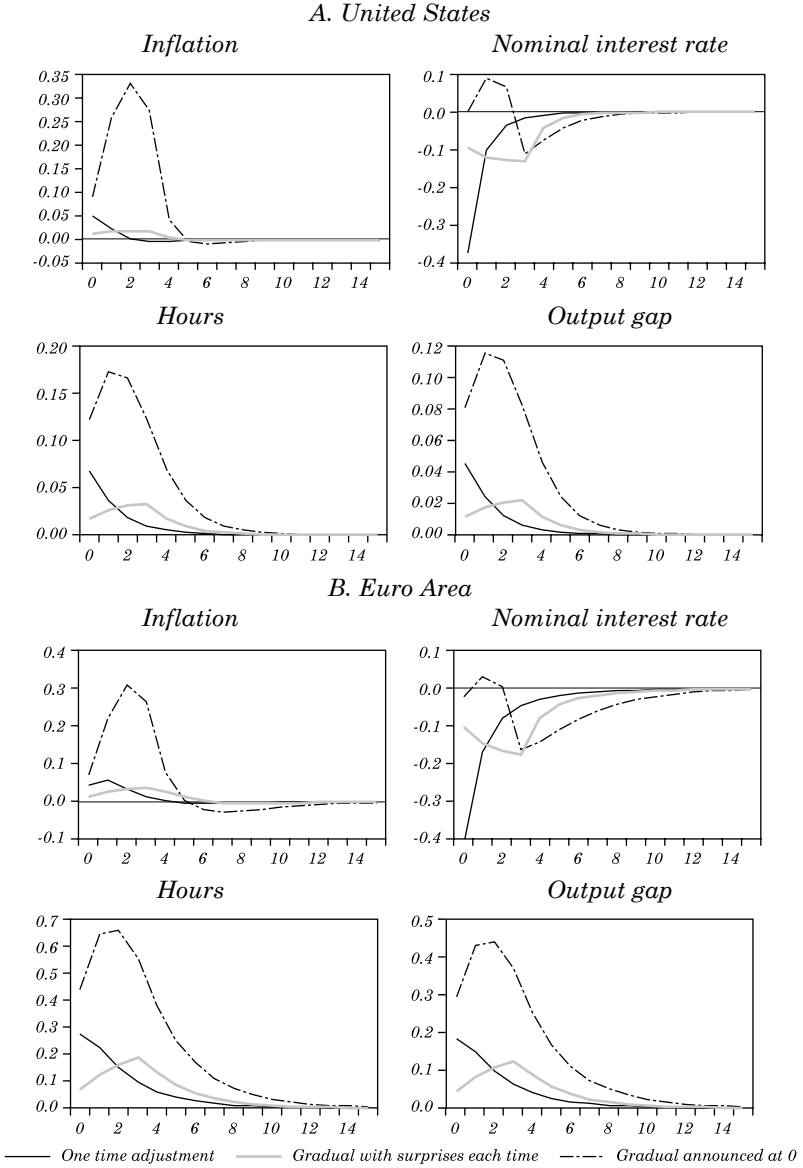
Figure 5 compares three different patterns of shocks for the two regions. In the first case, there is a one-standard-deviation shock to interest rates at date 0. In the second case, there are four consecutive shocks, each of size $\sigma_\varepsilon/4$ and each coming as a surprise to the agents. In the third scenario, the sequence of four shocks is announced at date 0. The results indicate that an anticipated gradual cut in interest rates has a much stronger impact than an expected cut of the same size. If the gradual cut is unexpected, however, the impact is actually smaller. Therefore, gradual policy changes can be quite effective according to the SIGE model, but only if they are announced and credible.

# Figure 4. Impulse Response Functions to Policy Announcements

## A. United States

### Inflation



### Nominal interest rate



### Hours



### Output gap



## B. Euro Area

### Inflation



### Nominal interest rate



### Hours



### Output gap



*Unanticipated shock* ——— *Anticipated 1 year* ——— *Anticipated 2 years*

Source: Author's calculations.

# Figure 5. Impulse Response Functions to Gradual Movements in Policy

## A. United States

### Inflation



### Nominal interest rate



### Hours



### Output gap



## B. Euro Area

### Inflation



### Nominal interest rate



### Hours



### Output gap



——— One time adjustment ——— Gradual with surprises each time ——— Gradual announced at 0

Source: Author's calculations.

## 7.4 How Would Taylor's Proposal Compare?

Taylor (1993) originally suggested that the interest rate responses to inflation and output should be 1.5 and 0.5, respectively. Figure 6 compares this rule with the one estimated here for the impulse responses of inflation and hours worked to productivity and aggregate demand shocks. For both shocks and both regions, Taylor's more aggressive policy rule leads to a smaller response in the output gap to the shock. The unconditional variance of hours worked would fall by 1.3 percent (2.7 percent) if the United States (euro area) moved to this rule, and welfare would be 4 (6) basis points of steady-state consumption higher.

## 7.5 How Does a Price-Level Target Compare?

Ball, Mankiw, and Reis (2005) show that in an economy with inattentive firms, the optimal policy is an "elastic price standard" that keeps the price level close to a deterministic target $K_t$, allowing for deviations of the price level from the target in response to deviations of output from the Pareto-optimal level:
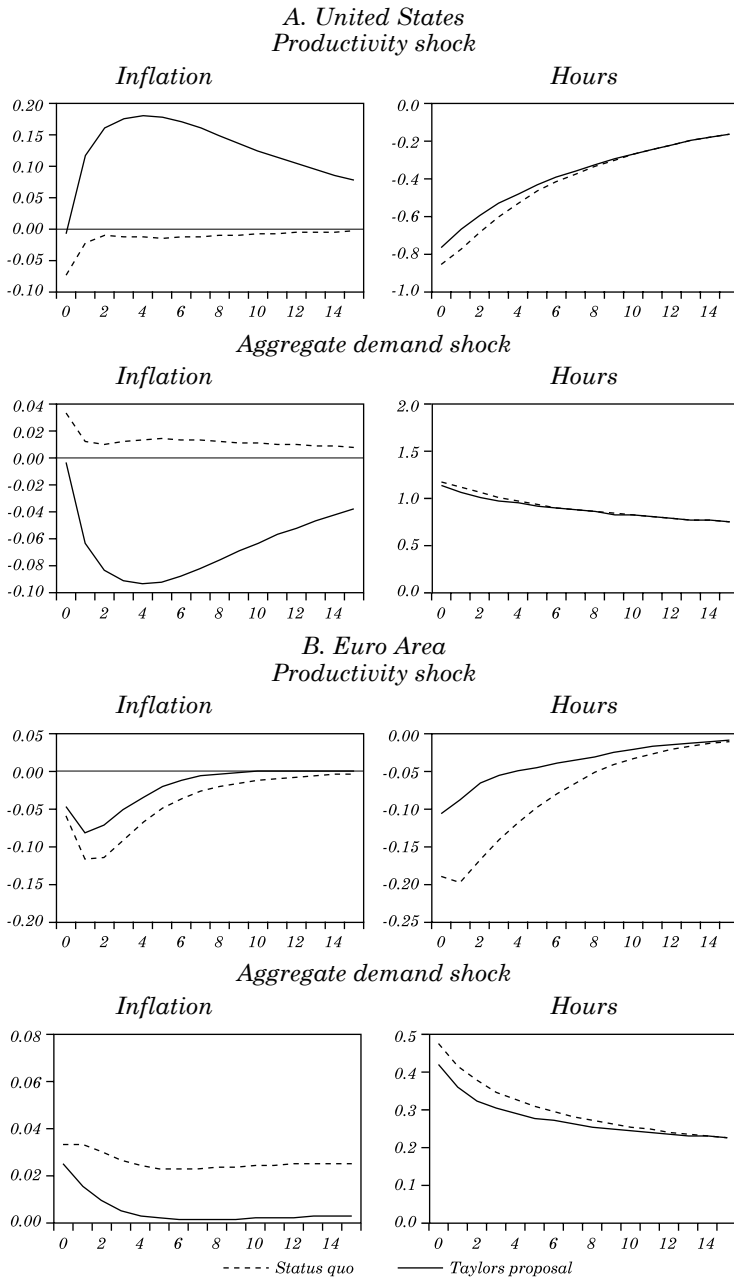
$$p_t = K_t - \phi\left(y_t - y_t^o\right). \tag{44}$$

Under this rule, positive deviations of inflation from the target are not bygones, but must be accompanied by future negative deviations to revert the price level back to target.

Figure 7 shows the impulse responses to productivity and aggregate demand shocks of having a strict rule with $\phi = 0$. In the United States, fully stabilizing inflation has little impact on the response of hours worked. The response of hours worked to the markup shocks (not reported) becomes significantly more pronounced, though, so the rule has a negative effect on welfare of 4 basis points on impact. For the euro area, the welfare loss from this rule would be a substantial 17 basis points.

Figure 8 graphs the responses to an elastic rule, where $\phi$ is set following the guidelines of Ball, Mankiw, and Reis (2005).[16] The $\phi$ for the United States is 0.12, while that for the euro area is 3.08. Both lead to a slight loss in welfare relative to the Taylor rule with the estimated coefficients.
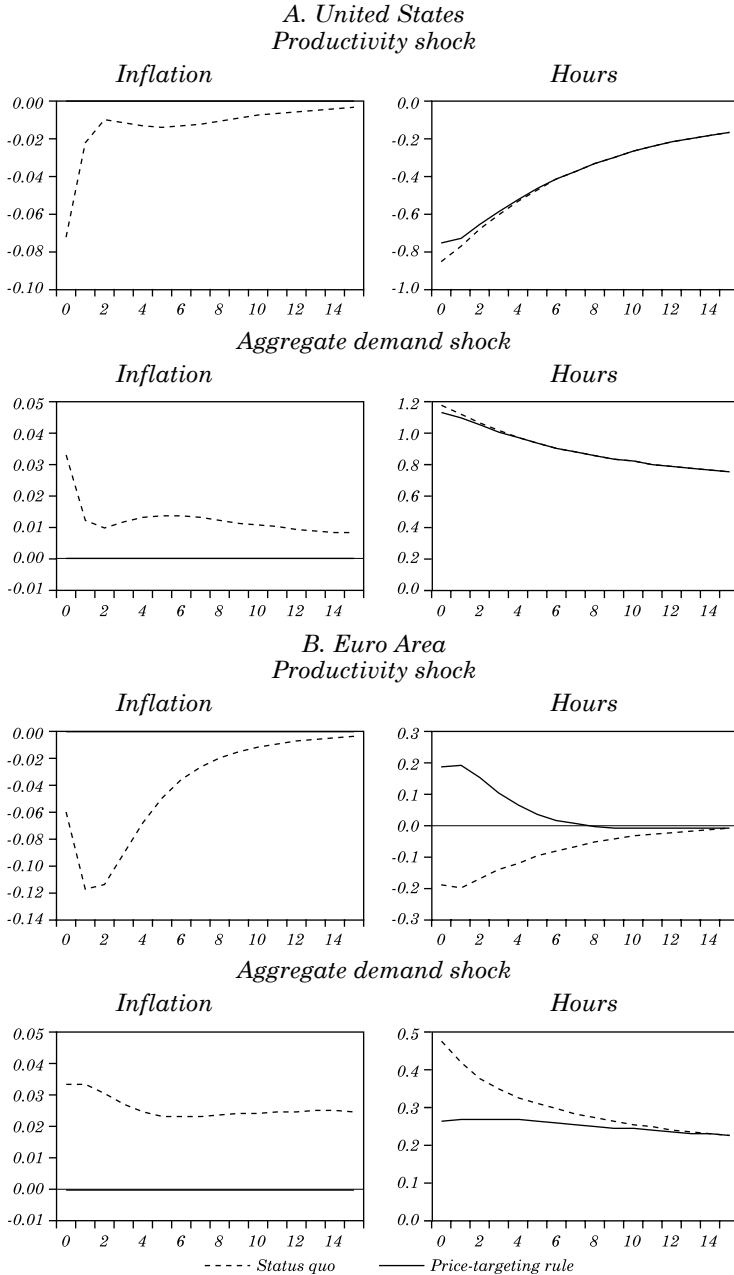
16. More concretely, Ball, Mankiw, and Reis (2005) show that the optimal $\phi$ is the inverse of the product of $(1 + \psi)/(1 + \psi\nu)$ and the relative weight of relative-price distortions and output-gap fluctuations in the policymaker's objective function. I approximate this relative weight by the ratio of the change in the volatility of the output gap and the change in the volatillity of inflation, both in response to a one-basis-point increase in the standard deviation of all shocks.

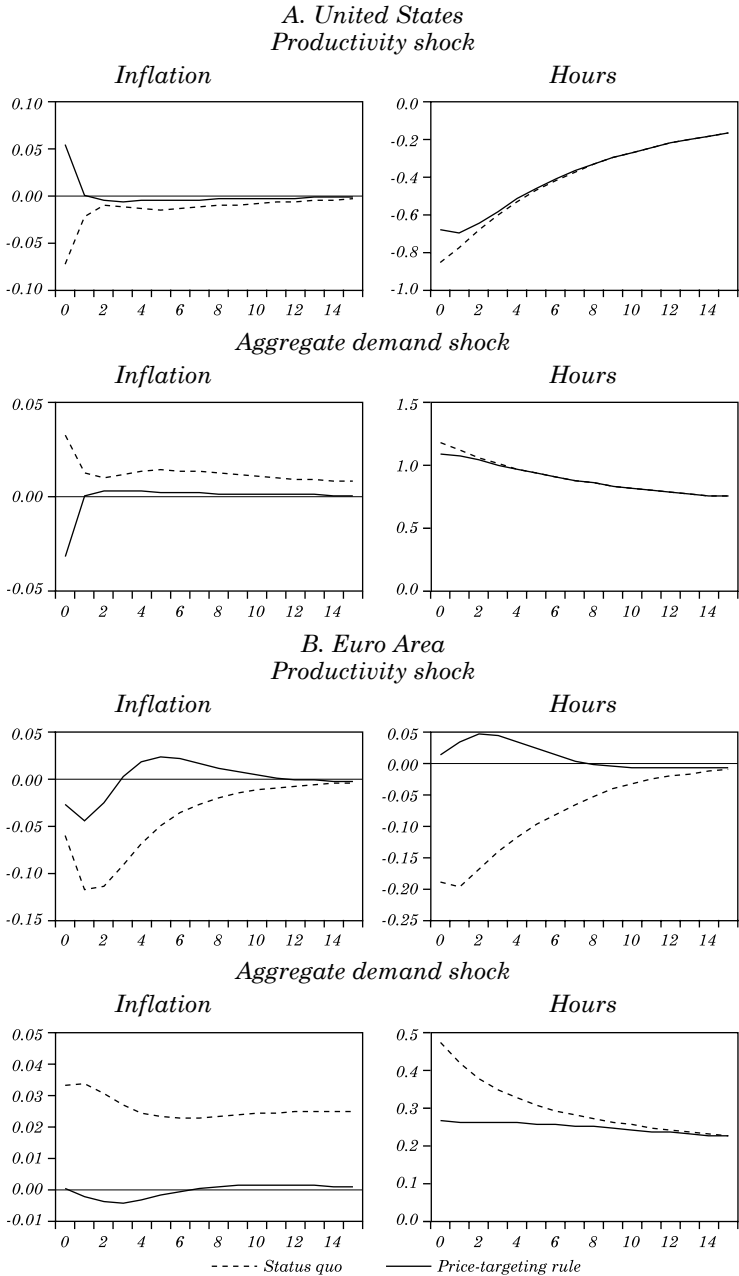# Figure 6. Impulse Response Functions with a Taylor Rule

### A. United States
### Productivity shock



### Aggregate demand shock



### B. Euro Area
### Productivity shock



### Aggregate demand shock



Source: Author's calculations.

# Figure 7. Impulse Response Functions with a Strict Price-Level Target

### A. United States
#### Productivity shock

Inflation

Hours

### Aggregate demand shock

Inflation

Hours

### B. Euro Area
#### Productivity shock

Inflation

Hours

### Aggregate demand shock

Inflation

Hours

- - - - - Status quo ———— Price-targeting rule

Source: Author's calculations.

# Figure 8. Impulse Response Functions with an Elastic Price-Level Target

## A. United States
### Productivity shock



## Aggregate demand shock



## B. Euro Area
### Productivity shock



## Aggregate demand shock



*- - - - Status quo*          *———— Price-targeting rule*

Source: Author's calculations.

## 8. CONCLUSION

The aim of this paper was to build one particular model of the macroeconomy that can be used to give systematic policy advice. The two guiding principles behind the construction of the model were, first, that inattentiveness is a feature of behavior that affects all markets and decisions and, second, that it is the only feature that leads to a deviation from an otherwise classical equilibrium. In reality, many frictions are probably at play, but insisting on a single friction allows one to explore how far inattentiveness alone affects macroeconomic dynamics and policy, while staying within a coherent theoretical framework where in which all details are explicitly stated.

Many of the details of the model, as well as the way in which the parameters were picked, may be open to debate, and there is room for disagreement on how well the model fits the data. I have tried throughout the paper to highlight the theoretical gaps in the model, the different views on how to set its parameters, and the ways in which it succeeded and failed at explaining the data. In the model's defense, it did not seem to perform noticeably worse than some popular alternatives, like the models in Christiano, Eichenbaum, and Evans (2005), Levin, Onatski, Williams and Williams (2006), or Smets and Wouters (2003, 2007).

While the model's performance is probably still far from the level of success one should demand to confidently give precise policy recommendations, the exercise did provide some policy lessons. First, the persistence of monetary policy shocks has been low, and this is a crucial determinant of the speed at which inflation and output respond to these shocks. Second, announcements and gradualism, through their effects on the expectations of forward-looking agents, can have a large impact on the effects of monetary policy. Third, Taylor's suggested policy rule parameters would lead to better outcomes than the status quo, while an elastic price standard has a disappointing performance when inattentiveness is pervasive.

APPENDIX
## A1. Inattentive Actions

Planner-savers, who every period face a probability $\delta$ of revising their plans, have a value function $V(M_t)$ conditional on date $t$ being a planning date. They choose a plan for current and future consumption all the way into infinity $\{C_{t+l,l}\}_{l=0}^{\infty}$ since with a vanishingly small probability they may never update again:

$$V\left(M_t\right) = \max_{\{C_{t+l,l}\}} \left\{ \begin{array}{l} \displaystyle\sum_{l=0}^{\infty} \xi^l \left(1-\delta\right)^l \frac{C_{t+l,l}^{1-1/\theta}}{1-1/\theta} \\ +\xi\delta\displaystyle\sum_{l=0}^{\infty} \xi^l \left(1-\delta\right)^l E_t\left[V\left(M_{t+1+l}\right)\right] \end{array} \right\}, \tag{45}$$

subject to the sequence of budget constraints in equation (9) and a no-Ponzi condition.

The optimality conditions are

$$\xi^l \left(1-\delta\right)^l C_{t+l,l}^{-1/\theta} = \xi\delta\sum_{k=l}^{\infty} \xi^k \left(1-\delta\right)^k E_t\left[V'\left(M_{t+1+k}\right)\bar{\Pi}_{t+l,t+1+k}\right] \tag{46}$$

and

$$V'\left(M_t\right) = \xi\delta\sum_{l=0}^{\infty} \xi^l \left(1-\delta\right)^l E_t\left[V'\left(M_{t+1+l}\right)\bar{\Pi}_{t,t+1+l}\right], \tag{47}$$

where

$$\bar{\Pi}_{t+l,t+1+k} = \prod_{z=t+l}^{t+k} \Pi_{z+1}$$

is the the compound return between $t + l$ and $t + 1 + k$ for $k > l$. Now, for $l = 0$, the right-hand side of equation (46) is the same as the right-hand side of equation (47). Therefore, $C_{t,0}^{-1/\theta} = V'\left(M_t\right)$, or the marginal utility of an extra unit of consumption equals the marginal value of an extra unit of wealth. Using this result to replace the $V'(M_{t+1+l})$ terms in equation (47) and writing the equation recursively gives the Euler equation in equation (10). The second Euler equation in equation (11) then follows.

The workers face a similar problem:

$$\hat{V}(M_t) = \max_{\{W_{t+l,l}\}} \left\{ \begin{array}{l} -\sum_{l=0}^{\infty} \xi^l \left(1-\omega\right)^l \kappa E_t \left(\dfrac{L_{t+l,l}^{1+1/\psi}+1}{1+1/\psi}\right) \\[2ex] +\xi\omega\sum_{l=0}^{\infty} \xi^l \left(1-\omega\right)^l E_t \left[\hat{V}\left(M_{t+1+l}\right)\right] \end{array} \right\},$$ (48)

subject to the sequence of budget constraints in equation (9), a no-Ponzi scheme condition, and the demand for the variety of labor $j$ in equation (14), which each worker supplies monopolistically. The optimality conditions are

$$\frac{\xi^l \left(1-\omega\right)^l \kappa E_t \left(\tilde{\gamma}_{t+l} L_{t+l,l}^{1+1/\psi}\right)\left(1-\tau_w\right)}{W_{t+l,l}} =$$

$$\xi\omega\sum_{k=l}^{\infty} \xi^k \left(1-\omega\right)^k E_t \left[\frac{V'(M_{t+1+k})\bar{\Pi}_{t+l,t+1+k}\left(\tilde{\gamma}_{t+l}-1\right)L_{t+l,l}}{P_{t+l}}\right]$$ (49)

and

$$\hat{V}'\left(M_t\right) = \xi\omega\sum_{k=0}^{\infty} \xi^k \left(1-\omega\right)^k E_t \left[\hat{V}'\left(M_{t+1+k}\right)\bar{\Pi}_{t,t+1+k}\right].$$ (50)

Now, as in the consumer problem, combining equation (49) for $l = 0$ with equation (50) leads to the following conclusion:

$$\frac{\hat{V}_t'(M_t)W_{t,0}}{P_t} = \frac{\left(1-\tau_w\right)\tilde{\gamma}_t\kappa L_{t,0}^{1/\psi}}{\tilde{\gamma}_t-1}.$$ (51)

This expression shows that $\psi$ is the Frisch elasticity of labor supply for attentive agents and that the marginal disutility of working is equated to the real wage rate times the marginal value of wealth times a markup taking into account the elasticity of demand for the good. Using it in the optimality condition leads to the two Euler equations in equations (16) and (17).

## A2. The Log-Linear Equilibrium for the Full Model

At the nonstochastic steady state, the five exogenous processes are constant. Using the conditions defining the optimum, it follows that output is $Y = AL^\beta$, consumption is $C = Y/G$, and labor is

$$\kappa L^{1+1/\psi} = \frac{\beta G(\nu - 1)(\gamma - 1)}{(1 - \tau_w)(1 - \tau_p)\nu\gamma}. \tag{52}$$

I log-linearize the equilibrium conditions around this point. Small caps denote the log-deviations of the respective large-cap variable from the steady state, with the exceptions of the following: $\upsilon_t$ and $\gamma_t$, which are the log-deviations of $\tilde{\nu}_t$ and $\tilde{\gamma}_t$; $r_t$, which is the log-deviation of the short rate $E_t[\Pi_{t+1}]$; and $R_t$, which is the log-deviation of the long rate $\lim_{k\to\infty} E_t[\bar{\Pi}_{t,t+1+k}]$.

Starting with the goods market, log-linearizing the demand for good $j$ by combining equations (3) and (6) gives

$$y_{t,i} = y_t - \nu\ (p_{t,i} - p_t). \tag{53}$$

The production function (5) and the firm's optimality condition (7) become

$$y_{t,i} = \alpha_t + \beta l_{t,i} \tag{54}$$

and

$$p_{t,i} = E_{t-i}\left[p_t + \frac{\beta(w_t - p_t) + (1 - \beta)y_t - a_t - \nu_t\beta/(\nu - 1)}{\beta + \nu(1 - \beta)}\right]. \tag{55}$$

Turning to the bond market, the consumer's Euler equations in equations (10) and (11) become

$$c_{t,0} = E_t(c_{t+1,0} - \theta r_t) \tag{56}$$

and

$$c_{t,j} = E_{t-j}(c_{t,0}) \tag{57}$$

Next, in the labor market, the demand for a labor variety in equation (14), together with the market clearing condition in this market, leads to:

$$l_{t,k} = l_t - \gamma\left(w_{t,k} - w_t\right), \tag{58}$$

and the optimality conditions in the workers' problem become

$$w_{t,0} - p_t - \frac{l_{t,0}}{\psi} + \frac{\gamma_t}{\gamma-1} = E_t\left[-r_t + w_{t+1,0} - p_{t+1} - \frac{l_{t+1,0}}{\psi} + \frac{\gamma_{t+1}}{\gamma-1}\right] \tag{59}$$

and

$$w_{t,k} = E_{t-k}(w_{t,0}) \tag{60}$$

Finally, the static price indices and aggregate quantity are

$$p_t = \lambda\sum_{i=0}^{\infty}\left(1-\lambda\right)^i p_{t,i}, \tag{61}$$

$$w_t = \omega\sum_{k=0}^{\infty}\left(1-\omega\right)^k w_{t,k}, \tag{62}$$

and

$$y_t = g_t + \delta\sum_{j=0}^{\infty}\left(1-\delta\right)^j c_{t,j}. \tag{63}$$

These eleven equations over time characterize the equilibrium solution for the set of twelve variables ($y_{t,i}$, $y_t$, $c_{t,0}$, $c_{t,j}$, $l_{t,0}$, $l_{t,k}$, $l_t$, $w_{t,k}$, $w_t$, $p_t$, $p_{t,i}$, $r_t$) as a function of the five exogenous processes ($\Delta a_t$, $g_t$, $\gamma_t$, $\nu_t$, $\varepsilon_t$). There is one equation missing, namely, the policy rule in equation (23).

## A3. The Reduced-Form Aggregate Relations

Integrating equation (54) over $i$ gives the aggregate production function in equation (19).

For the Phillips curve, starting with equation (61), replace $y_{t,j}$ using equation (53) and $p_{t,i}$ using equation (55). Rearrange to obtain equation (20).

Moving to the IS curve, iterate equation (56) forward and take the limit as time goes to infinity. Then, the facts that there is complete insurance and that eventually all agents become aware of the shocks imply that $\lim_{\tau \to \infty} E_t\left(c_{t+\tau,0}\right) = \lim_{\tau \to \infty} E_t\left(y_{t+\tau}\right) \equiv y_t^\infty$. Using the definition of the long rate $R_t$ and replacing for $c_{t,0}$ in equations (57) and (63) gives an expression for output. Using the fact that $\lim_{\tau \to \infty} E_t\left[g_{t+\tau}\right] = 0$ gives the IS curve in equation (21).

Finally, for the wage curve, take very similar steps as in the IS curve: iterate equation (59) forward and use the solution to replace $w_{t,0}$ in equation (60). Combining the $w_{t,j}$ in the aggregator for $w_t$ in equation (62) and replacing out $l_{t,j}$ using equation (58) gives the wage curve in equation (22).

## A4. Proof of Propositions 1 and 2

Take the case of $s = a$. By a method of undetermined coefficients, equations (19) through (22) imply[17]

$$\hat{y}_n = \hat{a}_n + \beta \hat{l}_n ; \tag{64}$$

$$\hat{p}_n = \Lambda_n \left[ \hat{p}_n + \frac{\beta \hat{w}_n + (1-\beta)\hat{y}_n - \hat{a}_n - \beta \nu_n / (\nu - 1)}{\beta + \nu(1-\beta)} \right]; \tag{65}$$

$$\hat{r}_n = \frac{\hat{y}_{n+1}}{\theta \Delta_{n+1}} - \frac{\hat{y}_n}{\theta \Delta_n}; \tag{66}$$

$$(\gamma + \psi)\hat{w}_n = \Omega_n \left[ (\psi + \gamma)\hat{p}_n + \gamma(\hat{w}_n - \hat{p}_n) + \hat{l}_n + \frac{\psi \hat{y}_n}{\theta \Delta_n} \right]. \tag{67}$$

Rearranging the first three equations immediately proves proposition 2. Using the first two expressions to replace $\hat{l}_n$ and $\hat{w}_n$ in the fourth expression proves proposition 1. The case of the other four shocks follows along the same lines.

---

17. I have omitted the ($s$) arguments to save space.

## A5. Proof of Proposition 3

Taking again the case $s = a$, combining the Taylor rule with the Fisher equation, and again omitting the $(s)$ arguments, the undetermined coefficients are

$$\hat{r}_n + \hat{p}_{n+1} - \hat{p}_n = \phi_p(\hat{p}_n - \hat{p}_{n-1}) + \phi_y\,(\hat{y}_n - \Xi_n\hat{s}_n).$$

Using the results in propositions 1 and 2 to replace $\hat{r}_n$ and $\hat{y}_n$ and rearranging delivers the proposition. The other cases are similar.

## A6. Proof of Proposition 4

Since the $K_t$ is known to all agents, real variables are neutral with respect to it, and it only induces a deterministic component in prices. Focusing on the stochastic component, in terms of moving-average coefficients, the policy rule implies that

$$\hat{p}_n = \phi\,(\hat{y}_n - \tilde{\Xi}_n\hat{s}_n).$$

Using the expression in proposition 1 to replace $\hat{y}_n$ delivers the result.

## A7. Solutions for Other Interest Rate Rules

The proofs for the case of these rules follow along the same lines as propositions 3 and 4 so they are omitted. First, consider alternative interest rate rules:

*Proposition 7.* If policy follows the interest rate rules below, the undetermined coefficients for the price level satisfy the following second-order difference equation:

$$A_{n+1}\hat{p}_{n+1}(s) - B_n\hat{p}_n(s) + C_{n-1}\hat{p}_{n-1}(s) = D_n(s) \quad \text{for } n = 0,1,2,... \tag{68}$$

with $A_n = 1 + \Psi_n\,/\,\theta\Delta_n$ and $D_n(\varepsilon) = -1$ for all cases. The remaining coefficients are as follows:

—For the employment rule, $i_t = \phi_p\Delta p_t + \phi_y l_t$,

$$B_n = A_n + \phi_p + \frac{\phi_y\Psi_n}{\beta}, \quad C_n = \phi_p \tag{69}$$

and

$$D_n(s) = \begin{cases} \dfrac{\Upsilon_n(s)}{\theta\Delta_n} - \dfrac{\Upsilon_{n+1}(s)\hat{s}_{n+1}}{\theta\Delta_{n+1}\hat{s}_n} + \dfrac{\phi_y[\Upsilon_n(s)-1]}{\beta} & \text{for } s = a \\[3mm] \dfrac{(\Upsilon_n(s)-1)}{\theta\Delta_n} - \dfrac{[\Upsilon_{n+1}(s)-1]\hat{s}_{n+1}}{\theta\Delta_{n+1}\hat{s}_n} + \dfrac{\phi_y\Upsilon_n(s)}{\beta} & \text{for } s = g \\[3mm] \dfrac{\Upsilon_n(s)}{\theta\Delta_n} - \dfrac{\Upsilon_{n+1}(s)\hat{s}_{n+1}}{\theta\Delta_{n+1}\hat{s}_n} + \dfrac{\phi_y\Upsilon_n(s)}{\beta} & \text{for } s = \gamma, \nu \end{cases} \tag{70}$$

—For the speed-limit rule, $i_t = \phi_p\Delta p_t + \phi_y\Delta(y_t - y_t^c)$,

$$B_n = A_n + \phi_p + \phi_y\,\Psi_n, \; C_n = \phi_p + \phi_y\,\Psi_n \tag{71}$$

and

$$D_n(s) = \begin{cases} \dfrac{\Upsilon_n(s)}{\theta\Delta_n} - \dfrac{\Upsilon_{n+1}(s)\hat{s}_{n+1}}{\theta\Delta_{n+1}\hat{s}_n} \\[2mm] \quad +\phi_y\left\{\Upsilon_n(s) - \Xi(s) - \dfrac{[\Upsilon_{n-1}(s) - \Xi(s)]\hat{s}_{n-1}}{\hat{s}_n}\right\} & \text{for } s = a,\gamma,\nu \\[5mm] \dfrac{[\Upsilon_n(s)-1]}{\theta\Delta_n} - \dfrac{[\Upsilon_{n+1}(s)-1]\hat{s}_{n+1}}{\theta\Delta_{n+1}\hat{s}_n} \\[2mm] \quad +\phi_y\left\{\Upsilon_n(s) - \Xi(s) - \dfrac{[\Upsilon_{n-1}(s) - \Xi(s)]\hat{s}_{n-1}}{\hat{s}_n}\right\} & \text{for } s = g \end{cases} \tag{72}$$

—For the inertial rule, $i_t = (1 - \phi_i)[\phi_p\Delta p_t + \phi_y(y_t - y_t^c)] + \phi_i i_{t-1}$,

$$B_0 = A_0 + (1-\phi_i)\phi_p + (1-\phi_i)\phi_y\Upsilon_0(s), \tag{73}$$

$$B_n = A_n(1+\phi_i) + (1-\phi_i)\phi_p + (1-\phi_i)\phi_y\Upsilon_0(s), \quad n \geq 1, \tag{74}$$

$$C_n = (1-\phi_i)\phi_p + \phi_i A_n, \tag{75}$$

and

$$
D_n(s) = \begin{cases}
\dfrac{\Upsilon_n(s)(1-\phi_i)}{\theta\Delta_n} - \dfrac{\Upsilon_{n+1}(s)\hat{s}_{n+1}}{\theta\Delta_{n+1}\hat{s}_n} - \dfrac{\phi_i\Upsilon_{n-1}(s)\hat{s}_{n-1}}{\theta\Delta_{n-1}\hat{s}_n} \\
\qquad +\phi_y(1-\phi_i)(\Upsilon_n(s)-\Xi(s)) & \text{for } s=a,\gamma,\nu \\[2ex]
\dfrac{[\Upsilon_n(s)-1](1-\phi_i)}{\theta\Delta_n} - \dfrac{[\Upsilon_{n+1}(s)-1]\hat{s}_{n+1}}{\theta\Delta_{n+1}\hat{s}_n} - \dfrac{\phi_i[\Upsilon_{n-1}(s)-1]\hat{s}_{n-1}}{\theta\Delta_{n-1}\hat{s}_n} \\
\qquad +\phi_y(1-\phi_i)[\Upsilon_n(s)-\Xi(s)] & \text{for } s=g
\end{cases}
\tag{76}
$$

—For the wage-inflation rule $i_t = \phi_p\Delta w_t + \phi_y(y_t - y_t^c)$,

$$
B_n = A_n + \phi_p\left[\left[1+\nu\left(\frac{1}{\beta}-1\right)\right]\left(\frac{1}{\Lambda_n}-1\right) + \left(1-\frac{1}{\beta}\right)\Psi_n\right] + \phi_y\Psi_n,
$$

$$
C_n = \phi_p\left[\left[1+\nu\left(\frac{1}{\beta}-1\right)\right]\left(\frac{1}{\Lambda_n}-1\right) + \left(1-\frac{1}{\beta}\right)\Psi_n\right]
\tag{77}
$$

and

$$
D_n(s) = \begin{cases}
\dfrac{\Upsilon_n(s)}{\theta\Delta_n} - \dfrac{\Upsilon_{n+1}(s)\hat{s}_{n+1}}{\theta\Delta_{n+1}\hat{s}_n} + \phi_y[\Upsilon_n(s)-\Xi(s)] + \phi_p\left(1-\dfrac{1}{\beta}\right)\left[\Upsilon_n(s)-\dfrac{\Upsilon_{n-1}(s)\hat{s}_{n-1}}{\hat{s}_n}\right] & \text{for } s=a \\[2ex]
\dfrac{[\Upsilon_n(s)-1]}{\theta\Delta_n} - \dfrac{[\Upsilon_{n+1}(s)-1]\hat{s}_{n+1}}{\theta\Delta_{n+1}\hat{s}_n} + \phi_y[\Upsilon_n(s)-\Xi(s)] \\
\qquad +\phi_p\left(1-\dfrac{1}{\beta}\right)\left[\Upsilon_n(s)-\dfrac{\Upsilon_{n-1}(s)\hat{s}_{n-1}}{\hat{s}_n}\right] & \text{for } s=g \\[2ex]
\dfrac{\Upsilon_n(s)}{\theta\Delta_n} - \dfrac{\Upsilon_{n+1}(s)\hat{s}_{n+1}}{\theta\Delta_{n+1}\hat{s}_n} + \phi_y[\Upsilon_n(s)-\Xi(s)] + \phi_p\left(1-\dfrac{1}{\beta}\right)\left[\Upsilon_n(s)-\dfrac{\Upsilon_{n-1}(s)\hat{s}_{n-1}}{\hat{s}_n}\right] & \text{for } s=\gamma \\[2ex]
\dfrac{\Upsilon_n(s)}{\theta\Delta_n} - \dfrac{\Upsilon_{n+1}(s)\hat{s}_{n+1}}{\theta\Delta_{n+1}\hat{s}_n} + \phi_y[\Upsilon_n(s)-\Xi(s)] + \phi_p\left(1-\dfrac{1}{\beta}\right)\left[\Upsilon_n(s)-\dfrac{\Upsilon_{n-1}(s)\hat{s}_{n-1}}{\hat{s}_n}\right] \\
\qquad +\dfrac{\phi_p}{\nu-1}\left(1-\dfrac{\hat{s}_{n-1}}{\hat{s}_n}\right) & \text{for } s=\nu
\end{cases}
\tag{78}
$$

Finally, consider alternative price-targeting rules,

*Proposition 8.* If the policy rule follows other price-level standards, the undetermined coefficients for the price level are as follows:

—With an employment rule, $p_t = K_t - \phi l_t$,

$$\hat{p}_n(s) = \begin{cases} \dfrac{\phi\left[1 - \Upsilon_n(s)\right]\hat{s}_n}{\beta + \phi\Psi_n} & \text{for } s = a \\[3mm] \dfrac{-\phi\Upsilon_n(s)\hat{s}_n}{\beta + \phi\Psi_n} & \text{for } s = g,\gamma,\nu \end{cases} \tag{79}$$

for n = 0, 1, 2,....

—With a speed-limit rule, $p_t = K_t - \phi\Delta(y_t - y_t^o)$,

$$(1 + \phi\Psi_n)\hat{p}_n(s) - \phi\Psi_{n-1}\hat{p}_{n-1}(s) = \phi\begin{Bmatrix} \left[\Xi(s) - \Upsilon_n(s)\right]s_n \\ -\left[\Xi(s) - \Upsilon_{n-1}(s)\right]s_{n-1} \end{Bmatrix}, \tag{80}$$

for $n = 0, 1, 2,...$ and with $\hat{p}_{-1}(s) = 0$.

—With an inertial rule, $p_t = K_t - \phi(y_t - y_t^o) + \phi_p p_{t-1}$,

$$(1 + \phi\Psi_n)\hat{p}_n(s) - \phi_p\hat{p}_{n-1}(s) = \phi[\Xi(s) - \Upsilon_n(s)]s_n \tag{81}$$

for n = 0, 1, 2,... and with $\hat{p}_{-1}(s) = 0$.

—With a wage-targeting rule, $w_n = K_n - \phi(y_t - y_t^o)$,

$$\hat{p}_n(s) = \frac{\left\{\phi\left[\tilde{\Xi}(s) - \Upsilon_n(s)\right] - \left(\dfrac{1}{\beta} - 1\right)\Upsilon_n(s)\right\}\hat{s}_n - \begin{cases} \dfrac{\hat{s}_n}{\beta} & \text{for } s = a \\[2mm] \dfrac{\hat{s}_n}{\nu - 1} & \text{for } s = \nu \\[2mm] 0 & \text{for } s = g,\gamma \end{cases}}{1 + \left[1 + \nu\left(\dfrac{1}{\beta} - 1\right)\right]\left(\dfrac{1}{\Lambda_n} - 1\right) + \left(1 - \dfrac{1}{\beta}\right)\Psi_n + \phi\Psi_n} \tag{82}$$

for n = 0, 1, 2,....

## A8. Proof of Proposition 5

Since $\mathbf{X}_t$ is a sum of multivariate normal distributions it is also multivariate normal. Its mean is a column vector of zeros, and its variance-covariance matrix is $\mathbf{\Omega}(\mathbf{I}_N \otimes \mathbf{\Sigma})\mathbf{\Omega}'$. Using the formula for the density of a multivariate normal, the result in the proposition follows immediately.

## A9. Proof of Proposition 6

Taking the unconditional expectation through the arguments of expression (38), the goal is to maximize the folowing expression:

$$\int_0^1 \left\{ E\left[\ln\left(C_{t,j}\right)\right] - \frac{\kappa E\left(L_{t,j}^{1+1/\psi}\right)}{1+1/\psi} \right\} dj. \tag{83}$$

With the definition of the log-linearized values, $c_{t,j} = \ln(C_{t,j}) - \ln(C)$ and $l_{t,j} = \ln(L_{t,j}) - \ln(L)$, this becomes

$$\ln\left(C\right) + \int_0^1 \left[ E\left(c_{t,j}\right) - \frac{\kappa L^{1+1/\psi} E\left(e^{(1+1/\psi)l_{t,j}}\right)}{1+1/\psi} \right] dj. \tag{84}$$

Recall that the model assumed that the tax on prices exactly offsets the monopoly distortion in the goods market: $1 - \tau_p = \nu/(\nu - 1)$; the tax on wages exactly offsets the monopoly distortion in the goods market: $1 - \tau_w = \gamma/(\gamma - 1)$; and the distortion from government spending is, on average, zero: $G = 1$. In this case, the nonstochastic steady state is an efficient equilibrium without uncertainty. These assumptions lead to focusing monetary policy on the task of stabilizing economic activity (Woodford, 2003b). From equation (52), they imply that $\kappa L^{1+1/\psi} = \beta$.

In the log-linear solution of the model, both $c_{t,j}$ and $l_{t,j}$ are normal variables with a zero mean. Therefore, social welfare is

$$\ln\left(C\right) - \frac{\beta}{1+1/\psi} \int_0^1 \exp\left[0.5\left(1+\frac{1}{\psi}\right)^2 \text{var}\left(l_{t,j}\right)\right] dj. \tag{85}$$

Because $l_{t,j}$ is a normal variable, var($l_{t,j}$) is a linear function of the variance of the exogenous shocks. These are small in the data, so approximating $\exp[\text{var}(l_{t,j})]$ by $1 + \text{var}(l_{t,j})$ involves little numerical error. Social welfare then becomes:

$$\ln(C) + \beta\left(1 + \frac{1}{\psi}\right) - 0.5\beta\left(1 + \frac{1}{\psi}\right)\int_0^1 \text{var}\left(l_{t,j}\right)dj. \tag{86}$$

Using the distribution of workers according to when they last updated, this becomes

$$\ln(C) + \beta\left(1 + \frac{1}{\psi}\right) - 0.5\beta\left(1 + \frac{1}{\psi}\right)\omega\sum_{j=0}^{\infty}(1-\omega)^j\,\text{var}\left(l_{t,j}\right) \tag{87}$$

Next, combining equation (58) with equations (59) and (60) to replace $w_{t,0}$ gives the following expressions:

$$l_{t,j} = l_t - \gamma\left(w_{t,j} - w_t\right) \tag{88}$$

and

$$w_{t,j} = E_{t-j}\left(p_t + \frac{l_{t,j}}{\psi} - \frac{\gamma_t}{\gamma-1} - R_t + y_n^{\infty}\right). \tag{89}$$

Using a method of undetermined coefficients, make the guess that $l_{t,j} = \sum_{s\in S}\left[\sum_{n=0}^{j-1}\varsigma_n(s) + \sum_{n=j}^{\infty}\zeta_n(s)\right]e_{t-n}^s$ and solve to find the expressions in equations (41) and (42). From this, it follows that

$$\text{var}\left(l_{t,j}\right) = \sum_{s\in S}\left[\sum_{n=0}^{j-1}\varsigma_n(s)^2 + \sum_{n=j}^{\infty}\zeta_n(s)^2\right]\sigma^2(s). \tag{90}$$

Finally, some grouping shows that

$$\omega\sum_{j=0}^{\infty}(1-\omega)^j\left[\sum_{n=0}^{j-1}\varsigma_n(s)^2 + \sum_{n=j}^{\infty}\zeta_n(s)^2\right] = \sum_{n=0}^{\infty}\begin{bmatrix}(1-\Omega_n)\varsigma_n(s)^2\\ +\Omega_n\zeta_n(s)^2\end{bmatrix}, \tag{91}$$

where $\Omega_n = \omega \sum_{i=0}^{n} (1-\omega)^i$. Ignoring the terms that are invariant to policy changes, the social welfare function then becomes the expression in equation (40). To evaluate the welfare benefit in percentage units of steady-state consumption of a policy that implies $\theta^{(1)}$ starting from another that implies $\theta^{(0)}$, use equation (87) to obtain equation (39).

### A10. MCMC Algorithm

I used a Metropolis-Hastings algorithm to draw from the posterior. In the first step, I looked for the mode of the posterior distribution by using line-search and Newton-Raphson algorithms starting from twenty different points on the parameter space (chosen from previous estimates of similar models and from drawing randomly from either the prior or a uniform on the parameter space). In the second step, I used a mixture of normal approximations around the highest local maxima found, to obtain an approximation of the posterior. This is then used as the proposal function for the Metropolis-Hastings algorithm. In the third step, I took a few sequences of 2,000 draws, scaling the variance-covariance matrix of the proposal function by different values, until the acceptance rates of the Metropolis-Hastings algorithm are 10–20 percent.

In the fourth step, I took 5 independent sequences of 200,000 draws, discarding the first 100,000. Inspecting the 500,000 mixed draws made clear that the algorithm was far from converging, and that the normal approximation of the posterior was poor. I therefore revised the proposal function to a normal distribution with a variance-covariance matrix equal to the scaled estimate of the variance-covariance matrix of the existing 500,000 draws.

In the fifth step, I took five independent sequences of 1,000,000 draws, discarding the first 100,000 draws and keeping only every tenth draw to save on memory space. The Brooks-Gelman scale reduction factors and the plots of the between-chain and within-chain variances indicated that the results were satisfactory in terms of convergence, so I proceeded to mix them to obtain the final 450,000 draws of the posterior, which are used in all the tables.

## References

Abel, A.B., J.C. Eberly and S. Panageas. 2007. "Optimal Inattention to the Stock Market." *American Economic Review* 97(2): 244–49.

Adam, K. 2007. "Optimal Monetary Policy with Imperfect Common Knowledge." *Journal of Monetary Economics* 54(2): 276–301.

Amato, J.D. and H.S. Shin. 2006. "Imperfect Common Knowledge and the Information Value of Prices." *Economic Theory* 27: 213–41.

An, S. and F. Schorfheide. 2007. "Bayesian Analysis of DSGE Models." *Econometric Reviews* 26(2–4): 113–72.

Andrés, J., E. Nelson and D. López-Salido. 2005. "Sticky-Price Models and the Natural Rate Hypothesis." *Journal of Monetary Economics* 52(5), 1025–53.

Bacchetta, P. and E. van Wincoop. 2006. "Incomplete Information Processing: A Solution to the Forward Discount Puzzle." Working paper 2006-35. Federal Reserve Bank of San Francisco.

Ball, L., N.G. Mankiw and R. Reis. 2005. "Monetary Policy for Inattentive Economies." *Journal of Monetary Economics* 52(4): 703–25.

Bayoumi, T. 2004. "GEM: A New International Macroeconomic Model." Occasional paper 239. Washington: International Monetary Fund.

Bernanke, B. 2004. "Gradualism." Remarks at an economics luncheon cosponsored by the Federal Reserve Bank of San Francisco (Seattle Branch) and the University of Washington. Seattle, Washington, 20 May 2004.

Boivin, J. and M. Giannoni. 2006. "Has Monetary Policy Become More Effective?" *Review of Economics and Statistics* 88(3): 445–62.

Branch, W.A. 2007. "Sticky Information and Model Uncertainty in Survey Data on Inflation Expectations." *Journal of Economic Dynamics and Control* 31(1): 245–76.

Carroll, C.D. 2003. "Macroeconomic Expectations of Households and Professional Forecasters." *Quarterly Journal of Economics* 118: 269–98.

Carroll, C.D. and J. Slacalek. 2006. "Sticky Expectations and Consumption Dynamics." Johns Hopkins University.

Christiano, L.J., M. Eichenbaum and C.L. Evans. 1999. "Monetary Policy Shocks: What Have We Learned and to What End?" In *Handbook of Macroeconomics,* edited by J. Taylor and M. Woodford. Elsevier.

————. 2005. "Nominal Rigidities and the Dynamic Effects of an Shock to Monetary Policy." *Journal of Political Economy* 113(1): 1–45.

Coibion, O. 2006. "Inflation Inertia in Sticky Information Models." *Contributions to Macroeconomics* 6(1): 1374–74.

————. 2007. "Testing the Sticky Information Phillips Curve." Working paper 61. College of William and Mary.

Dopke, J., J. Dovern, U. Fritsche and J. Slacalek. 2006. "Sticky Information Phillips Curves: European Evidence." Discussion paper 615. DIW Berlin.

————. 2008. "The Dynamics of European Inflation Expectations." *Topics in Macroeconomics* 8(1): 1540.

Erceg, C.J., L. Guerrieri and C. Gust. 2006. "SIGMA: A New Open Economy Model for Policy Analysis." *International Journal of Central Banking* 2(1): 1–50.

Gabaix, X. and D. Laibson. 2002. "The 6D Bias and the Equity Premium Puzzle." In *NBER Macroeconomics Annual 2001*, edited by B. Bernanke and K. Rogoff, 257–312. MIT Press.

————. 2006. "Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets." *Quarterly Journal of Economics* 121(2): 505–40.

Galí, J. 2004. "On the Role of Technology Shocks as a Source of Business Cycles: Some New Evidence." *Journal of the European Economic Association* 2(2–3): 372–80.

Gorodnichenko, Y. 2006. "Monetary Policy and Forecast Dispersion: A Test of the Sticky Information Model." University of Michigan.

Hall, R.E. 1997. "Macroeconomic Fluctuations and the Allocation of Time." *Journal of Labor Economics* 15(1): S223–S250.

Hellwig, C. 2002. "Public Announcements, Adjustment Delays, and the Business Cycle." Economics online papers 208. University of California at Los Angeles.

Hellwig, C. and L. Veldkamp. 2008. "Knowing What Others Know: Coordination Motives in Information Acquisition." *Review of Economic Studies* (forthcoming).

Khan, H. and Z. Zhu. 2006. "Estimates of the Sticky-Information Phillips Curve for the United States." *Journal of Money, Credit, and Banking* 38(1): 195–207.

Kiley, M.T. 2007. "A Quantitative Comparison of Sticky-Price and Sticky-Information Models of Price Setting." *Journal of Money, Credit, and Banking* 39(s1): 101–25.

Klenow, P.J. and J.L. Willis. 2007. "Sticky Information and Sticky Prices." *Journal of Monetary Economics* 54(sup. 1): 79–99.

Knotek, E.S. 2006. "A Tale of Two Rigidities: Sticky Prices in a Sticky-Information Environment." Research working paper 06-15. Federal Reserve Bank of Kansas City.

Korenok, O. 2005. "Empirical Comparison of Sticky Price and Sticky Information Models." Working paper 501. Virginia Commonwealth University, School of Business.

Korenok, O. and N.R. Swanson. 2005. "The Incremental Predictive Information Associated with Using Theoretical New Keynesian DSGE Models vs. Simple Linear Econometric Models." *Oxford Bulletin of Economics and Statistics*, 67(1): 905–30.

————. 2007. "How Sticky Is Sticky Enough? A Distributional and Impulse Response Analysis of New Keynesian DSGE Models." *Journal of Money, Credit, and Banking* 39(6): 1481–508.

Kydland, F.E. and E.C. Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica* 50(6): 1345–70.

Laforte, J.P. 2007. "Pricing Models: A Bayesian DSGE approach for the US Economy." *Journal of Money, Credit, and Banking* 39(s1): 127-154.

Levin, A.T., A. Onatski, J.C. Williams and N. Williams. 2006. "Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models." In *NBER Macroeconomics Annual 2005*, edited by M. Gertler and K. Rogoff. MIT Press.

Lorenzoni, G. 2008. "A Theory of Demand Shocks." Massachusetts Institute of Technology.

Lucas, R.E. Jr. 1980. "Methods and Problems in Business Cycle Theory." *Journal of Money, Credit, and Banking* 12(4): 696–715.

Mackowiak, B. and M. Wiederholt. 2007. "Optimal Sticky Prices under Rational Inattention." Northwestern University.

Mankiw, N.G. and R. Reis. 2002. "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve." *Quarterly Journal of Economics* 117(4): 1295–328.

————. 2003. "Sticky Information: A Model of Monetary Non-neutrality and Structural Slumps." In *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, edited by P. Aghion, R. Frydman, J. E. Stiglitz, and M. Woodford. Princeton University Press.

————. 2006. "Pervasive Stickiness." *American Economic Review* 96(2): 164–69.

————. 2007. "Sticky Information in General Equilibrium." *Journal of the European Economic Association* 2(2–3): 603–13.

Mankiw, N.G., R. Reis and J. Wolfers. 2004. "Disagreement about Inflation Expectations." In *NBER Macroeconomics Annual 2003*, edited by M. Gertler and K. Rogoff, 209–48. MIT Press.

Meyer-Gohde, A. 2007. "Solving Linear Rational Expectations Models with Lagged Expectations Quickly and Easily." SFB-649 discussion paper 2007-069. Berlin: Humboldt University.

Molinari, B. 2007. "The Role of Sticky Information in Inflation Dynamics: Estimates and Findings." Temi di Ricerca 50. Rome: Ente Einaudi.

Morris, S. and H.S. Shin. 2006. "Inertia of Forward-Looking Expectations." *American Economic Review* 96(2): 152–57.

Nunes, R.C. 2006. "On the Epidemiological Microfoundations of Sticky Information." Universitat Pompeu Fabra.

Paustian, M. and E. Pytlarczyk. 2006. "Sticky Contracts or Sticky Information? Evidence from an Estimated Euro Area DSGE Model." Bowling Green State University.

Pickering, A. 2004. "Sticky Information and the Phillips Curve: A Tale of Two Forecasts." University of Bristol.

Reis, R. 2006a. "Inattentive Consumers." *Journal of Monetary Economics* 53(8): 1761–800.

————. 2006b. "Inattentive Producers." *Review of Economic Studies* 73(3): 793–821.

Rich, R.J. and J.S. Tracy. 2006. "The Relationship between Expected Inflation, Disagreement, and Uncertainty: Evidence from Matched Point and Density Forecasts." Staff report 253. Federal Reserve Bank of New York.

Rotemberg, J. and M. Woodford. 1997. "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy." *NBER Macroeconomics Annual 1997*, edited by B. S. Bernanke and J. J. Rotemberg, 297–346. MIT Press.

Rudebusch, G.D. 2002. "Term Structure Evidence on Interest Rate Smoothing and Monetary Policy Inertia." *Journal of Monetary Economics* 49(6): 1161–87.

Sims, C.A. 1998. "Stickiness." *Carnegie-Rochester Conference Series on Public Policy* 49(1): 317–56.

Smets, F. and R. Wouters. 2003. "An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area." *Journal of the European Economic Association* 1(5): 1123–75.

————. 2007. "Shocks and Frictions in U.S. Business Cycles: A Bayesian DSGE Approach." *American Economic Review* 97(3): 586–606.

Sveen, T. and L. Weinke. 2005. "New Perspectives on Capital, Sticky Prices, and the Taylor Principle." *Journal of Economic Theory* 123(1): 21–39.

Svensson, L.E.O. 2003. "What Is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules." *Journal of Economic Literature* 41: 426–77.

Taylor, J.B. 1979. "Estimation and Control of a Macroeconomic Model with Rational Expectations." *Econometrica* 47(5): 1267–86.

————. 1993. "Discretion versus Policy Rules in Practice." *Carnegie-Rochester Conference Series in Public Policy* 39: 195–214.

Trabandt, M. 2007. "Sticky Information vs. Sticky Prices: A Horse Race in a DSGE Framework." Working paper 1369. Kiel Institute for the World Economy.

Woodford, M. 1998. "Doing without Money: Controlling Inflation in a Post-Monetary World." *Review of Economic Dynamics* 1(1): 173–219.

————. 2003a. "Imperfect Common Knowledge and the Effects of Monetary Policy." In *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, edited by P. Aghion, R. Frydman, J. E. Stiglitz, and M. Woodford. Princeton University Press.

————. 2003b. *Interest and Prices*. Princeton University Press.

————. 2003c. "Optimal Interest-Rate Smoothing." *Review of Economic Studies* 70(4): 861–86.

————. 2005. "Firm-Specific Capital and the New Keynesian Phillips Curve." *International Journal of Central Banking* 1(2): 1–46.