Imperfect Knowledge and the Pitfalls of Optimal Control Monetary Policy

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Sixty years ago, Milton Friedman questioned the usefulness of the optimal control approach because of policymakers' imperfect knowledge of the economy and favored instead a simple rule approach to monetary policy (1947, 1948). These are still live issues, despite the development of powerful techniques to derive and analyze optimal control policies, which central banks use in their large-scale models (see Svensson and Woodford, 2003; Woodford, 2003; Giannoni and Woodford, 2005; Svensson and Tetlow, 2005). Although the optimal control approach provides valuable insights, it also presents problems. In particular, because it assumes a single correctly specified reference model, it ignores important sources of uncertainty about the economy that monetary policymakers face. Robust control methods of the type analyzed by Hansen and Sargent (2007) extend the standard optimal control approach to allow for unspecified model uncertainty; however, these methods are designed for relatively modest deviations from the reference model.[1] In practice, policymakers

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1. Svensson and Williams (2007) propose a method to compute optimal policy under model uncertainty using a Markov-switching framework. Computing optimal policies under model uncertainty with this method is extremely computationally intensive, and its application to real-world problems remains infeasible.

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are concerned with more fundamental sources of model uncertainty, and the robustness of monetary policy strategies to uncertainty is generally viewed as important (McCallum, 1988; Taylor, 1993). Thus, a key question is whether our understanding of the macroeconomic environment has improved enough to make the optimal control approach to monetary policy preferable to well-designed simple rules.

Relatively little research to date explores the robustness properties of optimal control policies to moderate or large degrees of model misspecification.[2] Gianonni and Woodford (2005) show that optimal control policies are robust to misspecification of the shock processes as long as the central bank forecasts are optimal. In contrast, Levin and Williams (2003) show that optimal control policies can perform very poorly if the structural equations of the central bank's reference model are badly misspecified. Orphanides and Williams (2008) examine the robustness of optimal control policies if the reference model misspecifies the way private agents form expectations. That paper finds that if private agents are uncertain of the true model and form expectations based on an estimated forecasting model, then optimal control policies designed under the assumption of rational expectations can perform poorly. The paper also shows that optimal control policies can be made more robust to this type of model uncertainty by placing less weight on stabilizing economic activity and interest rates in the central bank objective used in deriving the optimal control policy.

This paper extends the analysis in Orphanides and Williams (2008) to include uncertainty about the natural rates of interest and unemployment. We allow for exogenous time variation in the natural rates of interest and unemployment that the central bank may measure with error. There is considerable evidence of significant time variation in these natural rates and the difficulties of their real-time estimation (see, for example, Staiger, Stock, and Watson, 1997; Laubach, 2001; Orphanides and Williams, 2002; Laubach and Williams, 2003).[3] We assume that the central bank has a good understanding of the process describing the evolution of these natural rates, but may not observe

2. In contrast, there has been considerable research on the robustness of simple monetary policy rules to model uncertainty, including Taylor (1999), Levin, Wieland, and Williams (1999, 2003), Orphanides and Williams (2002, 2007), and Brock, Durlauf, and West (2007).

3. The natural rate of output is prone to considerable real-time mismeasurement, causing problems for monetary policy similar to the mismeasurement of the natural rate of unemployment, as discussed in Orphanides and others (2000), Orphanides and van Norden (2002), and Cukierman and Lippi (2005).

them directly, in which case it must estimate the natural rates using available data. We consider both the case in which the central bank uses the optimal statistical filter—the Kalman filter in the model of this paper—to estimate the natural rates, and the case in which the central bank's estimate of the key gain parameter of the filter is misspecified. Laubach and Williams (2003) and Clark and Kozicki (2005) document the imprecision in estimates of the gain parameter in the Kalman filter, making uncertainty about this key parameter a real-world problem for central bank estimates of natural rates.

We find that the optimal control policy derived assuming rational expectations and known natural rates performs relatively poorly in our estimated model of the U.S. economy when agents have imperfect knowledge of the structure of the economy, but instead must learn and the central bank must estimate movements in natural rates. The key shortcoming of the optimal control policy derived under the assumption of perfect knowledge is that it is overly fine-tuned to the assumptions in the benchmark model. As a result, the optimal control policy works extremely well when private and central bank knowledge are perfect. When agents learn, however, and the central bank may make mistakes due to misperceptions of natural rates, expectations can deviate from those implied under perfect knowledge, and the finely-tuned optimal control policy can go awry. In particular, by implicitly assuming that inflation expectations are always well anchored, the optimal control policy responds insufficiently strongly to movements in inflation, which results in excessive variability of inflation.

We then seek to construct policies that take advantage of the optimal control approach, but are robust to the forms of imperfect knowledge that we study.[4] Specifically, following the approach in Orphanides and Williams (2008), we look for weights in the central bank objective function such that an optimal control policy derived using these "biased" weights performs well under imperfect knowledge about the structure of the economy. We find that optimal policies derived assuming much lower weights on stabilizing economic activity and interest rates than in the true central bank objective perform well in the presence of both private agent learning and natural rate uncertainty. Relative to our earlier results, the incorporation of natural

4. An alternative approach, followed by Gaspar, Smets, and Vestin (2006), is to derive optimal monetary policy under learning. Because the model with learning is nonlinear, they apply dynamic programming techniques that are infeasible for the type of models studied in this paper and used in central banks for monetary policy analysis.

rate uncertainty further reduces the optimal weights on economic activity and interest rates in the objective function used in deriving optimal policies that are robust to imperfect knowledge.

Finally, we compare the performance of optimal control policies to two types of simple monetary policy rules that have been found to be robust to various types of model uncertainty in the literature. The first is a forward-looking version of a Taylor-type policy rule, similar to the rule that Levin, Wieland, and Williams (2003) found to perform very well in a number of estimated rational expectations models of the U.S. economy. The second is the rule proposed by Orphanides and Williams (2007), which differs from the first rule in that policy responds to the change in the measure of economic activity, rather than the level. This type of rule has been shown to be robust to mismeasurement of natural rates in the economy (Orphanides and Williams, 2002, 2007) and to perform very well in a counterfactual analysis of monetary policy in 1996–2003 (Tetlow, 2006). Under rational expectations, these rules perform somewhat worse than the optimal control policy.

The two simple monetary policy rules perform very well under learning and with natural rate mismeasurement. These rules clearly outperform the optimal control policy when knowledge is imperfect and generally perform about as well as the optimal control policies derived to be robust to imperfect knowledge by using a biased objective function. The relatively small advantage that the optimal control policy has over these robust rules when the model is correctly specified implies that the "insurance" payment required to gain the sizable robustness benefits found here is quite small.

The remainder of the paper is organized as follows. Section 1 describes the model and its estimation. Section 2 describes the central bank objective and the optimal control policy. Section 3 describes the models of expectation formation and the simulation methods. Section 4 examines the performance of the optimal control policy under imperfect knowledge. Section 5 analyzes the optimal weights in the central bank objective that yield robust optimal control policies that perform well under imperfect knowledge. Section 6 compares the performance of the simple rules to optimal control policies. Section 7 concludes.

1. An Estimated Model of the U.S. Economy

Our analysis is conducted using an estimated quarterly model of the U.S. economy. The basic structure of the model is the same as in Orphanides and Williams (2008), but it is extended to incorporate time variation in the natural rates of interest and unemployment. The model consists of equations that describe the dynamic behavior of the unemployment rate and the inflation rate and equations describing the natural rates of interest and unemployment and the shocks. To close the model, the short-term interest rate is set by the central bank, as described in the next section.

1.1 The Model

The IS curve equation is motivated by the Euler equation for consumption with adjustment costs or habit:

$$u_t = \phi_u u^e_{t+1} + (1-\phi_u)u_{t-1} + \alpha_u(i^e_t - \pi^e_{t+1} - r^*_t) + v_t; \tag{1}$$

$$v_t = \rho_v\, v_{t-1} + e_{v,t}, \qquad e_v \sim N\left(0, \sigma^2_{e_v}\right). \tag{2}$$

We specify the IS equation in terms of the unemployment rate rather than output to facilitate the estimation of the equation using real-time data. This equation relates the unemployment rate, u_t, to the unemployment rate expected in the next period, one lag of the unemployment rate, and the difference between the expected ex ante real interest rate (equal to the difference between the nominal short-term interest rate, i_t, and the expected inflation rate in the following period, π_{t+1}) and the natural rate of interest, r_t^*. The unemployment rate is subject to a shock, v_t, that is assumed to follow a first-order autoregressive, or AR(1), process with innovation variance $\sigma^2_{e_v}$. The AR(1) specification for the shocks is based on the evidence of serial correlation in the residuals of the estimated unemployment equation, as discussed below.

The Phillips curve equation is motivated by the New-Keynesian Phillips curve with indexation:

$$\pi_t = \phi_\pi \pi^e_{t+1} + (1-\phi_\pi)\pi_{t-1} + \alpha_\pi\left(u_t - u^*_t\right) + e_{\pi,t}, \qquad e_\pi \sim N\left(0, \sigma^2_{e_\pi}\right). \tag{3}$$

It relates inflation, π_t, (measured as the annualized percent change in the GNP or GDP price index, depending on the period) during quarter t to lagged inflation, expected future inflation, denoted by π^e_{t+1}, and the difference between the unemployment rate, u_t, and the natural rate of unemployment, u_t^*, in the current quarter. The parameter ϕ_π measures the importance of expected inflation on the determination of inflation, while $(1 - \phi_\pi)$ captures the effects of inflation indexation. The mark-up shock, $e_{\pi,t}$, is assumed to be a white noise disturbance with variance $\sigma^2_{e_\pi}$.

We model the low-frequency behavior of the natural rates of unemployment and interest as exogenous AR(1) processes independent of all other variables:

$$u_t^* = (1 - \rho_{u^*})\bar{u}^* + \rho_{r^*} u_{t-1}^* + e_{u^*,t}, \qquad e_{u^*} \sim N\left(0, \sigma^2_{e_u^*}\right); \tag{4}$$

$$r_t^* = (1 - \rho_{r^*})\bar{r}^* + \rho_{u^*} r_{t-1}^* + e_{r^*,t}, \qquad e_{r^*} \sim N\left(0, \sigma^2_{e_r^*}\right). \tag{5}$$

We assume these processes are stationary based on the finding using the standard augmented Dickey-Fuller (ADF) test that one can reject the null of nonstationarity of both the unemployment rate and real federal funds rate over 1950–2003 at the 5 percent level. The unconditional mean values of the natural rates are irrelevant to the policy analysis, so we set them both to zero.[5]

1.2 Model Estimation and Calibration

The details of the estimation method for the inflation and unemployment rate equations are described in Orphanides and Williams (2008). The estimation results are reported below, with standard errors indicated in parentheses.

Unrestricted estimation of the IS curve equation yields a point estimate for ϕ_u of 0.39, with a standard error of 0.15. This estimate is below the lower bound of 0.5 implied by theory; however, the null hypothesis of a value of 0.5 is not rejected by the data.[6] We therefore impose $\phi_u = 0.5$ in estimating the remaining parameters of

5. Because we ignore the zero lower bound on nominal interest rates, as well as any other potential source of nonlinear behavior in the structural model, the unconditional means of variables are irrelevant. Inclusion of the zero bound would severely complicate the analysis and is left for future work.

6. This finding is consistent with the results reported by Giannoni and Woodford (2005), who find, in a similar model, that the corresponding coefficient is constrained to be at its theoretical lower bound.

the equation. The estimated equation also includes a constant term (not shown) that provides an estimate of the natural real interest rate, which is assumed to be constant for the purpose of estimating this equation.

$$u_t = 0.5u^e_{t+1} + 0.5u_{t-1} + \underset{(0.022)}{0.056}(\tilde{r}^e_t - r^*) + v_t, \tag{6}$$

$$v_t = \underset{(0.085)}{0.513}v_{t-1} + e_{v,t}, \qquad \hat{\sigma}_{e_v} = 0.30, \tag{7}$$

$$\pi_t = 0.5\pi^e_{t+1} + 0.5\pi_{t-1} - \underset{(0.087)}{0.294}(u_t - u^*_t) + e_{\pi,t}, \qquad \hat{\sigma}_{e_\pi} = 1.35. \tag{8}$$

Unrestricted estimation of the Phillips curve equation yields a point estimate for ϕ_π of 0.51, just barely above the lower bound implied by theory.[7] For symmetry with our treatment of the IS curve, we impose $\phi_\pi = 0.5$ and estimate the remaining parameters using ordinary least squares (OLS). The estimated residuals for this equation show no signs of serial correlation in the price equation (Durbin-Watson = 2.09), consistent with the assumption of the model.

There is considerable uncertainty regarding the magnitude and persistence of low-frequency fluctuations in the natural rates of unemployment and interest (see Staiger, Stock, and Watson, 1997; Laubach, 2001; Orphanides and Williams, 2002; Laubach and Williams, 2003; Clark and Kozicki, 2005.) We do not estimate a model of natural rates; instead, we calibrate the parameters of the AR(1) processes based on estimates found elsewhere in the literature. To capture the highly persistent movements in natural rates, we set the autocorrelation parameters, ρ_{u^*} and ρ_{r^*}, to 0.99. In our benchmark calibration, we set the innovation standard deviation of the natural rate of unemployment to 0.07 and that of the natural rate of interest to 0.085. These values imply an unconditional standard deviation of the natural rate of unemployment (interest) of 0.50 (0.60), in the low end of the range of standard deviations of smoothed estimates of these natural rates suggested by various estimation methods. We also consider an alternative calibration in which the standard deviations of the natural rate innovations are twice as large, consistent with the upper end of the range of estimates of natural rate variation.

7. For comparison, Giannoni and Woodford (2005) find that the corresponding coefficient is constrained to be at its theoretical lower bound of 0.5.

2. Optimal Control Monetary Policy

In this section, we describe the optimal control monetary policy. The policy instrument is the nominal short-term interest rate. We assume that the central bank observes all variables from previous periods when making the current-period policy decision. We further assume that the central bank has access to a commitment technology; that is, we study policy under commitment.

The central bank's objective is to minimize a loss equal to the weighted sum of the unconditional variances of the inflation rate, the difference between the unemployment rate and the natural rate of unemployment, and the first-difference of the nominal federal funds rate:

$$L = \text{var}(\pi - \pi^*) + \lambda\, \text{var}(u - u^*) + \nu\, \text{var}(\Delta(i)), \tag{9}$$

where var(x) denotes the unconditional variance of variable x. We assume an inflation target of zero percent. As a benchmark for our analysis, we assume $\lambda = 4$ and $\nu = 1$. Based on an Okun's Law relationship, the variance of the unemployment gap is about one-quarter that of the output gap, so this choice of λ corresponds to equal weights on inflation and output gap variability.

The optimal control monetary policy is that which minimizes the loss subject to the equations describing the economy. We construct the optimal control policy, as is typical in the literature, assuming that the policymaker knows the true parameters of the structural model and assumes all agents use rational expectations and the central bank knows the natural rates of unemployment and interest.[8] For the optimal control policy, as well as the simple monetary policy rules described below, we use lagged information in the determination of the interest rate, reflecting the lag in data releases. The optimal control policy is described by a set of equations representing the first-order optimality condition for policy and the behavior of the Lagrange multipliers associated with the constraints on the optimization problem implied by the structural equations of the model economy.

Because we are interested in describing the setting of interest rates in a potentially misspecified model, it is useful to represent the

8. See, for example, Sargent's (2007) description of the optimal policy approach.

optimal control policy in an equation that relates the policy instrument to macroeconomic variables, rather than in terms of Lagrange multipliers that depend on the model. There are infinitely many such representations. In the following, we focus on one representation of the optimal control (OC) policy. In the OC policy, the current interest rate depends on three lags of the following variables: the inflation rate, the difference between the unemployment rate and the central bank's estimate of the natural rate of unemployment, and the difference between the nominal interest rate and the estimate of the natural rate of interest. The OC representation yields a determinate rational expectations equilibrium. We find that including three lags of these variables is sufficient to very closely mimic the optimal control outcome assuming the central bank observes natural rates.

2.1 Central Bank Estimation of Natural Rates

As noted above, we compute the OC policy assuming the central bank observes the true values of the natural rates of interest and unemployment. In our policy evaluation exercises, we consider the possibility that the central bank must estimate natural rates in real time. In such cases, we assume that the central bank knows the true structure of the model, including the model parameters (and the unconditional means of the natural rates), and observes all other variables including private forecasts, but does not observe the shocks directly. Given our model, the Kalman filter is the optimal method for estimating the natural rates, and we assume that the central bank uses the appropriate specification of the Kalman filters to estimate natural rates. These assumptions represent a best case for the central bank with respect to its ability to estimate natural rates. In other work, we examine the implications of model uncertainty regarding the data-generating processes for natural rates (Orphanides and Williams, 2005, 2007).

The central bank's real-time estimate of the natural rate of unemployment, $\hat{u}_t^*$, is given by

$$\hat{u}_t^* = a_1 \hat{u}_{t-1}^* + a_2 \left(u_t^* - \frac{e_{\pi,t}}{\alpha_\pi} \right), \tag{10}$$

where a_1 and a_2 are the Kalman gain parameters and the term within the parentheses is the current-period shock to inflation, which incorporates the effects of the transitory inflation disturbance and the deviation of the natural rate of unemployment from its unconditional mean, scaled in units of the unemployment rate. The central bank only observes this surprise and not the decomposition into its two components.

The central bank estimate of the natural rate of interest, $\hat{r}_t^*$, is given by

$$\hat{r}_t^* = b_1 \hat{r}_{t-1}^* + b_2 \left(r_t^* - \frac{v_t}{\alpha_u} \right) + b_3 \left(r_{t-1}^* - \frac{v_{t-1}}{\alpha_u} \right), \tag{11}$$

where the first term in parentheses is the current-period unemployment rate shock and the final term is the lagged shock. The final term appears in the equation due to the assumption of an AR(1) process for the shocks to the unemployment rate equation.

The optimal values of the gain parameters depend on the variances of the four shocks. In our policy evaluation exercises, we consider alternative assumptions regarding the parameter values that the central bank uses in implementing the Kalman filters. In one case, we assume that the central bank uses the optimal values implied by the variances in our baseline calibration of the model. These values are as follows: $a_1 = 0.982$, $a_2 = 0.008$, $b_1 = 0.987$, $b_2 = 0.006$, and $b_3 = -0.003$. As noted above, there is a great deal of uncertainty regarding the values of the gain parameters, and real-world estimates tend to be very imprecise. We therefore examine two cases in which the central bank uses incorrect gain parameters. In one, the central bank assumes that the natural rates are constant, so the gain parameters are zero. In the other, we assume that the central bank uses the appropriate gain parameters for our baseline model calibration, but in fact the standard deviations of the natural rate shocks are twice as large as in the baseline calibration.

3. Expectations and Simulation Methods

As noted above, we are interested in studying the performance of the optimal control monetary policy derived under a misspecified model of expectations formation. We assume that private agents and, in some cases, the central bank, form expectations using an estimated reduced-form forecasting model. Specifically, following Orphanides and Williams (2005), we posit that private agents engage in perpetual learning, that is, they reestimate their forecasting model using a constant-gain least squares algorithm that weights recent data more heavily than past data.[9] This approach to modeling learning allows for the possible presence of time variation in the economy, including the natural rates of interest and unemployment. It also implies that agents' estimates are always subject to sampling variation, in that the estimates do not eventually converge to fixed values.

We assume that private agents forecast inflation, the unemployment rate, and the short-term interest rate using an unrestricted vector autoregression model (VAR) containing three lags of these three variables and a constant. We further assume that private agents do not observe or estimate the natural rates of unemployment and interest directly in forming expectations. The effects of time variation in natural rates on forecasts are reflected in the forecasting VAR by the lags of the interest rate, inflation rate, and unemployment rate. First, variants of VARs are commonly used in real-world macroeconomic forecasting, making this a reasonable choice on realism grounds. Second, the rational expectations equilibrium of our model with known natural rates is very well approximated by a VAR of this form. As discussed in Orphanides and Williams (2008), this VAR forecasting model provides accurate forecasts in model simulations.

At the end of each period, agents update their estimates of their forecasting model using data through the current period. To fix notation, let $\mathbf{Y}_t$ denote the 1×3 vector consisting of the inflation rate, the unemployment rate, and the interest rate, each measured at time t: $\mathbf{Y}_t = (\pi_t, u_t, i_t)$. Let $\mathbf{X}_t$ be the 10×1 vector of regressors in the forecast model: $\mathbf{X}_t = (1, \pi_{t-1}, u_{t-1}, i_{t-1}, \ldots, \pi_{t-3}, u_{t-3}, i_{t-3})$. Let $\mathbf{c}_t$ be the 10×3 vector of coefficients of the forecasting model. Using data through period t, the coefficients of the forecasting model can be written in recursive form:

$$\mathbf{c}_t = \mathbf{c}_{t-1} + \kappa \mathbf{R}_t^{-1} \mathbf{X}_t (\mathbf{Y}_t - \mathbf{X}_t' \mathbf{c}_{t-1}), \tag{12}$$

$$\mathbf{R}_t = \mathbf{R}_{t-1} + \kappa (\mathbf{X}_t \mathbf{X}_t' - \mathbf{R}_{t-1}), \tag{13}$$

where κ is the gain. Agents construct the multi-period forecasts that appear in the inflation and unemployment equations in the model using the estimated VAR.

9. See Sargent (1999), Cogley and Sargent (2002), and Evans and Honkapohja (2001) for related treatments of learning.

For some specifications of the VAR, the matrix $\mathbf{R}_t$ may not be full rank. To circumvent this problem, in each period of the model simulations, we check the rank of $\mathbf{R}_t$. If it is less than full rank, we assume that agents apply a standard Ridge regression (Hoerl and Kennard, 1970), where $\mathbf{R}_t$ is replaced by $\mathbf{R}_t + 0.00001^* \mathbf{I}(10)$, where $\mathbf{I}(10)$ is a 10×10 identity matrix.

3.1 Calibrating the Learning Rate

A key parameter in the learning model is the private agent updating parameter, κ. Estimates of this parameter tend to be imprecise and sensitive to model specification, but they generally lie between 0.00 and 0.04.[10] We take 0.02 to be a reasonable benchmark value for κ, a value that implies that the mean age of the weighted sample is about the same as for standard least squares with a sample of twenty-five years. Given the uncertainty about this parameter, we report results for values of κ between 0.01 (equivalent in mean sample age to a sample of about fifty years) and 0.03 (equivalent in mean sample age to a sample of about sixteen years).

3.2 Simulation Methods

In the case of rational expectations with constant and known natural rates, we compute model unconditional moments numerically as described in Levin, Wieland, and Williams (1999). In the case of learning, we compute approximations of the unconditional moments using stochastic simulations of the model.

For the stochastic simulations, we initialize all model variables to their respective steady-state values, which we assume to be zero. The initial conditions of $\mathbf{C}$ and $\mathbf{R}$ are set to the steady-state values implied by the forecasting perceived law of motion (PLM) in the rational expectations equilibrium with known natural rates. Each period, innovations are generated from independent Gaussian distributions with variances reported above. The private agent's forecasting model is updated each period and a new set of forecasts computed, as are the central bank's natural rate estimates. We simulate the model for 44,000 periods and discard the first 4,000 periods to eliminate the effects of initial conditions. We compute the unconditional moments from the remaining 40,000 periods (10,000 years) of simulated data.

10. See Sheridan (2003), Orphanides and Williams (2005), Branch and Evans (2006), and Milani (2007).

Learning introduces nonlinear dynamics into the model that may cause the model to display explosive behavior in a simulation. In simulations where the model is beginning to display signs of explosive behavior, we follow Marcet and Sargent (1989) and stipulate modifications to the model that curtail the explosive behavior. One potential source of explosive behavior is that the forecasting model itself may become explosive. We take the view that in practice private forecasters reject explosive models. Therefore, in each period of the simulation, we compute the maximum root of the forecasting VAR (excluding the constants). If this root falls below the critical value of 1, the forecast model is updated as described above; if not, we assume that the forecast model is not updated and the matrices **C** and **R** are held at their respective values from the previous period.[11] This constraint is encountered relatively rarely with the policies analyzed in this paper.

This constraint on the forecasting model is insufficient to ensure that the model economy does not exhibit explosive behavior in all simulations. We therefore impose a second condition that eliminates explosive behavior. In particular, the inflation rate, the nominal interest rate, and the unemployment gap are not allowed to exceed (in absolute value) six times their respective unconditional standard deviations (computed under the assumption of rational expectations and known natural rates) from their respective steady-state values. This constraint on the model is invoked extremely rarely in the simulations.

4. Performance of the Optimal Control Policy

In this section, we examine the performance of the optimal control policy derived under the assumption of rational expectations and known natural rates to deviations from this reference model. We start by considering the case in which private agents learn and natural rates are known by the central bank. We then turn to the case of natural rate uncertainty.

4.1 Known Natural Rates

The OC policy, derived for $\lambda = 4$ and $\nu = 1$, is given by the following equation:

11. We chose this critical value so that the test would have a small effect on model simulation behavior while eliminating explosive behavior in the forecasting model.

$$
\begin{aligned}
i_t = {} & 1.13(i_{t-1} - \hat{r}^*_{t-1}) + 0.02(i_{t-2} - \hat{r}^*_{t-1}) - 0.26(i_{t-3} - \hat{r}^*_{t-1}) + 0.18\pi_{t-1} \\
& +0.03\pi_{t-2} + 0.04\pi_{t-3} - 2.48(u_{t-1} - \hat{u}^*_{t-1}) + 2.03(u_{t-2} - \hat{u}^*_{t-1}) \\
& -0.34(u_{t-3} - \hat{u}^*_{t-1}).
\end{aligned} \tag{14}
$$

The first line of table 1 reports the outcomes for the OC policy under rational expectations and known natural rates. These outcomes serve as a benchmark against which the results under imperfect knowledge can be compared. The OC policy is characterized by a high degree of policy inertia, as measured by the sum of the coefficients on the lagged interest rates of 0.89. The sum of the coefficients on lagged inflation equals 0.25 and that on the lagged differences between the unemployment rates equals –0.89. As discussed in Orphanides and Williams (2008), the optimal control policy is characterized by a muted interest rate response to deviations of inflation from target. Following a shock to inflation, the OC policy only gradually brings inflation back to target and thus restrains the magnitude of deviations of unemployment from its natural rate and that of changes in the interest rate.

Table 1. Performance of Alternative Monetary Policies under Rational Expectations and Known Natural Rates[a]

	Standard deviation			*Loss*
Policy	π	$u - u^*$	Δi	L
Optimal control	1.83	0.68	1.20	6.64
Levin, Wieland, and Williams (2003)	1.87	0.70	1.24	6.98
Orphanides and Williams (2008)	1.83	0.73	1.39	7.45

Source: Authors' calculations.
a. The policies are derived for $\lambda = 4$ and $\nu = 1$.

Macroeconomic performance under the OC policy deteriorates under private agent learning, with the magnitude in fluctuations in all three objective variables increasing in the updating rate, κ. The upper panel of table 2 reports the results when private agents learn assuming constant natural rates. These results are very similar to those reported in Orphanides and Williams (2008), where natural rates are assumed to be constant and known. Thus, the incorporation of known time-varying natural rates does not have notable additional implications for the design of optimal monetary policy under imperfect knowledge. With learning, agents are never certain of the structure of the economy or the behavior of the central bank. As discussed in Orphanides and Williams (2005), particularly large shocks or a "bad run" of one-sided shocks can be misinterpreted by agents as reflecting a monetary policy regime that places less weight on inflation stabilization or has a different long-run inflation target than is actually the case. This confusion adds persistent noise to the economy, which worsens macroeconomic performance relative to the rational expectations benchmark.

Table 2. Performance of OC Policy under Learning and Time-Varying Natural Rates[a]

	Standard deviation			*Loss*
κ	π	$u - u^*$	Δi	L
Known natural rates				
0.01	2.28	0.80	1.33	9.52
0.02	2.77	0.93	1.55	13.59
0.03	3.23	1.09	1.80	18.46
Natural rate estimates with optimal Kalman filters				
0.01	2.26	0.88	1.33	9.99
0.02	3.16	1.10	1.82	17.79
0.03	3.59	1.23	1.99	22.94
Natural rates assumed constant				
0.01	2.81	0.92	1.44	13.39
0.02	3.68	1.12	1.82	21.89
0.03	4.11	1.25	2.09	27.53

Source: Authors' calculations.
a. The policies are derived for $\lambda = 4$ and $\nu = 1$.

4.2 Estimated Natural Rates

We now analyze the performance of the OC policy designed assuming rational expectations and known natural rates when private agents learn and natural rates are not directly observable. The middle section of table 2 reports the results assuming that the central bank uses the optimal Kalman filters to estimate both natural rates. As noted above, this case assumes that the central bank has precise knowledge of the structure of the IS and Phillips curve equations, observes private expectations that appear in those equations, and knows the covariance matrix of the shocks (which is used in determining the coefficients of the Kalman filter).

If expectations are close to the rational expectations benchmark and the central bank efficiently estimates natural rates, then natural rate uncertainty by itself has little additional effect on macroeconomic performance under the OC policy. For example, in the case of $\kappa = 0.01$, the standard deviations of inflation and the first difference of interest rates are about the same whether natural rates are known or optimally estimated. Not surprisingly, the standard deviation of the difference between the unemployment rate and its natural rate is somewhat higher if natural rates are not directly observed, since in that case the central bank will sometimes aim for the "wrong" unemployment rate target. These errors do not spill over into increased variability of other variables, however.

If the learning rate is 0.02 or above, the interaction of natural rate misperceptions and learning leads to a much greater deterioration of macroeconomic performance. Natural rate misperceptions introduce serially correlated errors into monetary policy. When agents are learning, these policy errors interfere with the public's understanding of the monetary policy rule. As a result, the variability of all three target variables increases. If the central bank uses the incorrect gains in the Kalman filters, macroeconomic performance worsens even further. The effects of using the wrong Kalman gains are illustrated in the lower panel of table 1. In this example, the central bank incorrectly assumes Kalman gains of zero in estimating natural rates (that is, it assumes that the variances of the shocks to the natural rates are zero). The resulting outcomes under the OC policy are significantly worse for all three learning rates shown in the table. The deterioration in performance is primarily due to a rise in the variability of inflation. Evidently, the combination of private agent learning and policy mistakes associated with poor measurement of natural rates significantly worsens the anchoring of inflation expectations and the stabilization of inflation.

5. Robust Optimal Control Policies

The preceding analysis shows that the optimal control policy derived under rational expectations and known natural rates may not be robust to imperfect knowledge. We now consider an approach to deriving policies that take advantage of the optimal control methodology but are robust to imperfect knowledge. Specifically, following Orphanides and Williams (2008), we search for the "biased" central bank loss function for which the implied OC policy derived with rational expectations and known natural rates performs best under imperfect knowledge for the true social loss function. This approach applies existing methods of computing optimal policies under rational expectations and is therefore feasible in practice.

For a given value of κ and assumptions regarding natural rates and natural rate measurement, we search for the values of $\tilde{\lambda}$ and $\tilde{\nu}$ such that the OC policy derived using the loss,

$$\tilde{L} = \text{var}(\pi - \pi^*) + \tilde{\lambda}\,\text{var}(u - u^*) + \tilde{\nu}\,\text{var}(\Delta(i)),$$

minimizes the true social loss, which we assume to be given by the benchmark values of $\lambda = 4$ and $\nu = 1$.[12] We use a grid search to find the optimal weights (up to one decimal place) for the biased central bank loss and refer to the resulting policy as the robust optimal control (ROC) policy.

5.1 Known Natural Rates

With known natural rates, the optimal weights for the central bank loss on unemployment and interest rate variability are significantly smaller than the true weights in the social loss, and this downward bias is increasing in the learning rate κ. The results from this exercise are reported in the upper panel of table 3, which considers the same set of assumptions regarding natural rate measurement as in table 2. For comparison, the losses under the OC policy, denoted L^*, are reported in the final column of the table. The results with known natural rates are similar to that in Orphanides and Williams (2008), where natural rates are assumed to be constant. The presence of learning makes it optimal to assign the central bank a loss that places much greater relative weight on inflation stabilization than the true social loss—that is, to employ a conservative central banker, in the terminology of Rogoff (1985).[13] The ROC policies yield significantly lower losses than the OC policy.

12. This approach can be generalized to allow the inclusion of additional variables in the loss function. We leave this to future research.

13. Orphanides and Williams (2005), using a very simple theoretical model, similarly find that a central bank loss function biased toward stabilizing inflation (relative to output) is optimal when private agents learn.

Table 3. Optimal Weights in Central Bank Loss under Imperfect Knowledge[a]

	Weights		*Standard deviation*			*Loss*	
κ	$\tilde{\lambda}$	$\tilde{\upsilon}$	π	$u - u^*$	Δi	$\tilde{L}$	L^*
Known natural rates							
0.01	1.4	0.3	1.95	0.80	1.55	8.76	9.52
0.02	1.2	0.2	2.14	0.89	1.85	11.13	13.59
0.03	0.2	0.1	2.03	1.04	2.20	13.31	18.46
Natural rate estimates with optimal Kalman filters							
0.01	1.2	0.4	1.95	0.91	1.45	9.29	9.99
0.02	0.5	0.1	2.06	1.03	1.80	11.70	17.79
0.03	0.4	0.2	2.10	1.06	2.23	13.85	22.94
Natural rates assumed constant							
0.01	0.5	0.2	1.97	0.91	1.64	9.90	13.39
0.02	0.5	0.2	2.22	0.98	1.78	11.97	21.89
0.03	0.2	0.1	2.18	1.07	2.21	14.18	27.53

Source: Authors' calculations.

a. The policies are derived for $\lambda = 4$ and $\nu = 1$. $\tilde{L}$ denotes the loss under the OC policy under the optimized values of $\tilde{\lambda}$ and $\tilde{\upsilon}$, evaluated using $\lambda = 4$ and $\nu = 1$. L^* denotes the loss under the OC policy derived for $\lambda = 4$ and $\nu = 1$.

5.2 Estimated Natural Rates

With estimated natural rates, the optimal weights for the central bank loss on unemployment and interest rate variability are generally smaller than in the case of known natural rates. Thus, the combination of learning and natural rate mismeasurement strengthens the case for placing much greater relative weight on inflation stabilization than the true social loss. For example, in the case of $\kappa = 0.02$ and optimally estimated natural rates, the optimal central bank objective weights are about one-half as large as in the case of known natural rates. In that case, the ROC policy for $\kappa = 0.02$ and optimally estimated natural rates is given by the following equation:

$$\begin{aligned} i_t = {} & 1.11(i_{t-1} - \hat{r}^*_{t-1}) - 0.12(i_{t-2} - \hat{r}^*_{t-1}) - 0.15(i_{t-3} - \hat{r}^*_{t-1}) + 0.51\pi_{t-1} \\ & + 0.28\pi_{t-2} + 0.00\pi_{t-3} - 3.32(u_{t-1} - \hat{u}^*_{t-1}) + 2.40(u_{t-2} - \hat{u}^*_{t-1}) \\ & - 0.43(u_{t-3} - \hat{u}^*_{t-1}). \end{aligned} \tag{15}$$

This ROC policy is characterized by a much larger direct response to the inflation rate than the OC policy derived for the benchmark loss (and reported in equation 14), reflecting the greater relative weight on inflation stabilization for the biased central bank loss. The ROC policy responds somewhat more to lags of the difference between the unemployment rate and the perceived natural rate of interest, with a sum of coefficients of –1.35 (versus –0.89 in the OC policy). It also exhibits less intrinsic policy inertia, with the sum of the coefficients on the lagged interest rate of 0.84 (versus 0.89 in the OC policy), reflecting the much smaller weight on interest rate variability underlying the ROC policy.

When the central bank incorrectly assumes that natural rates are constant, the optimal weights for the central bank loss on unemployment and on interest rate variability are at most one-fifth as large as the true values. The differences in the losses under the OC and ROC policies are much larger than in the case of known natural rates. The central bank loss under imperfect knowledge tends to be relatively insensitive to small differences in the weights used in deriving the robust optimal policies. As a result, the precise choice of optimal weights is not crucial. What is crucial is that the weights on unemployment and the change in interest rates are small relative to the weight on inflation.

5.3 Greater Natural Rate Variability

Thus far, we have assumed a relatively low degree of natural rate variability. We now explore the implications of more variable natural rates, consistent with some estimates in the literature.[14] In the following discussion, we assume that the standard deviation of the natural rate innovations is twice that assumed in our benchmark calibration. The results for these experiments are reported in table 4. The final column of the table reports the loss, denoted L^*, under the standard OC policy derived assuming rational expectations and known natural rates with the benchmark calibration of innovation variances.

If the central bank is assumed to observe the true values of the natural rates, then the greater degree of natural rate variability does not significantly affect the optimal choices of weights in the objective function used to derive the ROC policy. Comparing the upper panels of tables 3 and 4 shows that the optimal values of $\tilde{\lambda}$ and $\tilde{\nu}$ are similar for the two calibrations of natural rate variability. The losses associated with the OC policy are much larger when natural rates are more variable. In contrast, the losses under the appropriate ROC policies are not that different in the two cases.

If, however, the central bank underestimates the degree of natural rate variability in estimating natural rates, the optimal values of $\tilde{\lambda}$ and $\tilde{\nu}$ are very small, implying that the central bank should focus almost entirely on inflation stabilization in deriving optimal control policies. The lower panel of the table reports outcomes for the case in which the central bank uses the Kalman filter gains appropriate for the benchmark calibration of natural rate variability, but in fact the natural rates are twice as variable (in terms of standard deviations). In this case, the OC policy performs very badly, and the benefits of following the ROC policy rather than the OC are dramatic.

6. Simple Rules

We now compare the performance of two alternative monetary policies that have been recommended in the literature for being robust to various forms of model uncertainty to the optimal control policies

14. The case of zero variability of natural rates is analyzed in Orphanides and Williams (2008).

Table 4. Optimal Weights in Central Bank Loss under Imperfect Knowledge and High Natural Rate Variability[a]

	Weights		*Standard deviation*			*Loss*	
κ	$\tilde{\lambda}$	$\tilde{\upsilon}$	π	$u - u^*$	Δi	$\tilde{L}$	L^*
Known natural rates							
0.01	1.1	0.3	1.96	0.85	1.53	9.06	10.26
0.02	0.9	0.2	2.16	0.94	1.81	11.49	15.38
0.03	0.4	0.1	2.13	1.05	2.22	13.89	20.93
Natural rate estimates with baseline Kalman filters							
0.01	0.1	0.2	2.14	1.12	1.66	12.31	24.18
0.02	0.1	0.1	2.18	1.18	2.11	14.74	36.34
0.03	0.1	0.1	2.32	1.25	2.27	16.81	53.36

Source: Authors' calculations.
a. $\tilde{L}$ denotes the loss under the OC policy under the optimized values of $\tilde{\lambda}$ and $\tilde{\upsilon}$, evaluated using $\lambda = 4$ and $\nu = 1$. L^* denotes the loss under the OC policy derived for $\lambda = 4$ and $\nu = 1$.

analyzed above. The first rule is a version of the forecast-based policy rule proposed by Levin, Wieland, and Williams (2003). According to this rule, the short-term interest rate is determined as follows:

$$i_t = i_{t-1} + \theta_\pi(\bar{\pi}^e_{t+3} - \pi^*) + \theta_u(u_{t-1} - \hat{u}^*_{t-1}), \tag{16}$$

where $\bar{\pi}^e_{t+3}$ is the forecast of the four-quarter change in the price level and u^* is the natural rate of unemployment which we take to be constant and known. Because this policy rule characterizes policy in terms of the first difference of the interest rate, it does not rely on estimates of the natural rate of interest, as does the standard Taylor rule (1993). The second rule we consider is proposed by Orphanides and Williams (2007) for its robustness properties in the face of natural rate uncertainty:

$$i_t = i_{t-1} + \theta_\pi(\bar{\pi}^e_{t+3} - \pi^*) + \theta_{\Delta u}(u_{t-1} - u_{t-2}). \tag{17}$$

A key feature of this policy is the absence of any measures of natural rates in the determination of policy.[15]

We choose the parameters of these simple rules to minimize the loss under rational expectations and constant natural rates using a hill-climbing routine.[16] The resulting optimized Levin-Wieland-Williams rule is given by

$$i_t = i_{t-1} + 1.05(\bar{\pi}^e_{t+3} - \pi^*) - 1.39(u_{t-1} - \hat{u}^*_{t-1}). \tag{18}$$

The optimized Orphanides-Williams rule is given by

$$i_t = i_{t-1} + 1.74(\bar{\pi}^e_{t+3} - \pi^*) - 1.19(u_{t-1} - u_{t-2}). \tag{19}$$

15. This policy rule is related to the elastic price standard proposed by Hall (1984), whereby the central bank aims to maintain a stipulated relationship between the forecast of the unemployment rate and the price level. It is also closely related to the first difference of a modified Taylor-type policy rule in which the forecast of the price level is substituted for the forecast of the inflation rate.

16. If we allow for time-varying natural rates that are known by all agents, the optimized parameters of the Levin-Wieland-Williams and Orphanides-Williams rules under rational expectations are nearly unchanged. The relative performance of the different policies is also unaffected.

In the following, we refer to these specific parameterizations of these two rules simply as the Levin-Wieland-Williams and Orphanides-Williams rules.[17]

The lower part of table 1 reports the outcomes for the Levin-Wieland-Williams rule and the Orphanides-Williams rule under rational expectations and known natural rates. Under rational expectations and known natural rates, the OC policy yields a modestly lower loss than the Levin-Wieland-Williams and Orphanides-Williams rules, which is consistent with the findings in Williams (2003) and Levin and Williams (2003) about the relative performance of simple rules for other models.

In contrast to the OC policy, the Levin-Wieland-Williams and Orphanides-Williams rules perform very well under imperfect knowledge. Table 5 compares the performance of these rules to that of the OC policy derived under the true central bank loss and the ROC policies. (Because the Orphanides-Williams rule does not respond to natural rate estimates, outcomes are invariant to the assumption regarding central bank natural rate estimation.) In all cases reported in the table, the Levin-Wieland-Williams rule performs as well as or better than the OC policy, with the performance advantage larger the higher the learning rate and the greater the degree of natural rate misperceptions. As discussed in detail in Orphanides and Williams (2008), the Levin-Wieland-Williams rule consistently brings inflation back to target quickly following a shock to inflation, and it contains the response of inflation to the unemployment shock. The Orphanides-Williams rule does even better than the Levin-Wieland-Williams rule at containing the inflation responses to shocks, but at the cost of greater variability in the difference between the unemployment rate and its natural rate and the change in the interest rate. Consequently, the Levin-Wieland-Williams rule performs somewhat better than the Orphanides-Williams rule in terms of the stipulated central bank loss for all the cases that we consider here.

The outcomes under the Levin-Wieland-Williams and Orphanides-Williams rules are generally similar to those under the ROC policies. The first column of table 5 reports the losses under the ROC policies (repeated from table 3). The Levin-Wieland-Williams rule does slightly worse than the ROC policy in the cases closest to the perfect knowledge benchmark (that is, a low κ and modest natural rate misperceptions) and performs better as the degree of model misspecification increases.

17. These are the same rules analyzed in Orphanides and Williams (2008).

Table 5. Performance of Simple Rules under Imperfect Knowledge[a]

	OC policy		Levin-Wieland-Williams rule				Orphanides-Williams rule			
	Loss		Standard deviation			Loss	Standard deviation			Loss
κ	$\tilde{L}$	L^*	π	$u - u^*$	Δi	L	π	$u - u^*$	Δi	L
Known natural rates										
0.01	8.76	9.52	1.94	0.86	1.39	8.68	1.93	0.98	1.60	10.16
0.02	11.13	13.59	2.00	0.99	1.57	10.39	1.99	1.08	1.78	11.84
0.03	13.31	18.46	2.09	1.10	1.80	12.43	2.09	1.18	2.03	14.03
Natural rate estimates with optimal Kalman filters										
0.01	9.29	9.99	1.94	0.93	1.41	9.17	1.93	0.98	1.60	10.16
0.02	11.70	17.79	2.00	1.03	1.57	10.65	1.99	1.08	1.78	11.84
0.03	13.85	22.94	2.07	1.12	1.78	12.47	2.09	1.18	2.03	14.03
Natural rates assumed constant										
0.01	9.90	13.39	2.03	0.94	1.44	9.72	1.93	0.98	1.60	10.16
0.02	11.97	21.89	2.08	1.01	1.60	10.98	1.99	1.08	1.78	11.84
0.03	14.18	27.53	2.17	1.12	1.86	13.18	2.09	1.18	2.03	14.03

Source: Authors' calculations.

a. The policies are derived for $\lambda = 4$ and $\nu = 1$. $\tilde{L}$ denotes the loss under the OC policy under the optimized values of $\tilde{\lambda}$ and $\tilde{\upsilon}$, evaluated using $\lambda = 4$ and $\nu = 1$. L^* denotes the loss under the OC policy derived for $\lambda = 4$ and $\nu = 1$.

The Orphanides-Williams policy performs about the same or slightly worse than the ROC policies, except in the case of known natural rates, when the ROC policy performs much better. Evidently, the extra fine-tuning in the ROC policy compared to the simple rules is of little value in an environment characterized by learning and natural rate misperceptions. The results are qualitatively similar with greater natural rate variability, as seen in table 6. In this case, however, if the central bank uses the Kalman gains based on the benchmark calibration, the Orphanides-Williams rule outperforms the ROC policies.

The strong performance of the Levin-Wieland-Williams and Orphanides-Williams rules in the presence of natural rate mismeasurement reflects the fact that these rules do not rely on natural rate estimates as much as the OC policy. Indeed, the Orphanides-Williams rule does not respond to natural rates at all, while the Levin-Wieland-Williams rule responds only to estimates of the natural rate of unemployment. Importantly, these rules respond aggressively to movements in inflation. In the case of the Levin-Wieland-Williams rule, policy errors stemming from misperceptions of the natural rate of unemployment cause some deterioration in macroeconomic performance, but the consequences of these errors are limited by the countervailing effect of the strong response to resulting deviations of inflation from target.

7. Conclusion

Current methods of deriving optimal control policies ignore important sources of model uncertainty. This paper has examined the robustness of optimal control policies to uncertainty regarding the formation of expectations and natural rates and analyzed monetary policy strategies designed to be robust to these sources of imperfect knowledge. Our analysis shows that standard approaches to optimal policy yield policies that are not robust to imperfect knowledge. More positively, this analysis helps us identify and highlight key features of policies that are robust to these sources of model uncertainty.

The main finding is that a reorientation of policy toward stabilizing inflation relative to economic activity and interest rates is crucial for good economic performance in the presence of imperfect knowledge. Indeed, focusing on price stability in this manner is the policy that should be pursued even when the central bank cares greatly about stabilizing economic activity and interest rates. Although following

Table 6. Performance of Simple Rules under Imperfect Knowledge and High Natural Rate Variability[a]

	OC policy		*Levin-Wieland-Williams rule*				*Orphanides-Williams rule*			
	Loss		*Standard deviation*			*Loss*	*Standard deviation*			*Loss*
κ	$\tilde{L}$	L^*	π	$u - u^*$	Δi	L	π	$u - u^*$	Δi	L
Known natural rates										
0.01	9.06	10.26	1.94	0.90	1.42	9.00	1.99	1.17	1.65	12.17
0.02	11.49	15.38	2.00	1.03	1.57	10.73	2.06	1.28	1.86	14.23
0.03	13.89	20.93	2.09	1.18	1.81	13.23	2.13	1.36	2.07	16.19
Natural rate estimates with baseline Kalman filters										
0.01	12.31	24.18	2.09	1.12	1.48	11.57	1.99	1.17	1.65	12.17
0.02	14.74	36.34	2.14	1.20	1.64	12.99	2.06	1.28	1.86	14.23
0.03	16.81	53.36	2.24	1.33	1.91	15.70	2.13	1.36	2.07	16.19

Source: Authors' calculations.

a. $\tilde{L}$ denotes the loss under the OC policy under the optimized values of $\tilde{\lambda}$ and $\tilde{\upsilon}$, evaluated using $\lambda = 4$ and $\nu = 1$. L^* denotes the loss under the OC policy derived for $\lambda = 4$ and $\nu = 1$.

policies that place greater weight on economic stability may appear desirable in an environment of perfect knowledge, doing so is counterproductive and leads to greater instability when knowledge is imperfect. Moreover, in an environment of imperfect knowledge, well-designed robust simple rules perform about as well as optimal control policies designed to be robust to imperfect knowledge. This raises further doubts about the wisdom of relying on the optimal control approach in lieu of simple rules for policy design. Given the many other sources of model uncertainty, further research should be directed at analyzing robust monetary policy with a full array of sources of model uncertainty.

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