International Aspects of the Zero Lower Bound Constraint

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Large negative aggregate demand shocks can drive down an economy's equilibrium real interest rate, and if the central bank is committed to stabilizing inflation, monetary policy may be hampered by the zero lower bound on nominal interest rates –the economy may be in a "liquidity trap." The policy dilemma associated with the zero lower bound has been extensively debated in recent years. Based on the experience of Japan in the 1990's, writers like Krugman (1998), Eggertsson and Woodford (2003; 2005), Jung, Terinishi and Watanabe (2005), Svensson (2003), Auerbach and Obstfeld (2006), among others, explored how monetary policy announcements could be usefully employed even when the authorities have no more room for reducing short-term nominal interest rates. More recently, given the 2008-2009 global recession, a number of authors have explored the options for fiscal stimulus when the economy is stuck in a liquidity trap. Papers by Christiano, Eichenbaum and Rebelo (2009), Eggertsson (2010), Cogan et al. (2009), and Devereux (2010) have investigated the possibility of using government spending expansions and tax cuts when nominal interest rates are at their lower bound. In contrast to the Japanese experience, a key feature of recent history is that the zero lower bound constraint was more of a global phenomenon. Most focused on the problems facing either a closed economy or a small open economy in which policy-makers in the rest of the world were not faced with the analogous constraints. However, when many major countries are

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facing similar constraints on monetary policy, it is not clear how easy or useful it is to follow the conclusions of the previous literature.[1] How does a shock which pushes the economy down to the zero lower bound spread from one country to another? It is possible that the global interaction between countries in goods and assets markets may substantially alter the effects of a given set of policy responses within a liquidity trap? This paper examines the economics of the zero lower bound constraint in a two-country model, where one or both countries experience negative demand shocks that precipitate a liquidity trap. This paper also explores three issues raised by the previous discussion pertinent to the current policy debate. First, how is a liquidity trap propagated across countries? In particular, when taking a negative demand shock in one country, which pushes the unconstrained optimal nominal interest rate below zero, how does this constrain monetary policy in the neighboring country, and how does the answer to this depend on the openness of trade and international assets markets? Secondly, we examine the effectiveness of countercyclical fiscal policy within a liquidity trap. Recent literature has argued that fiscal policy becomes very effective when the monetary authority cannot adjust interest rates. Is there a global public-good aspect to a fiscal expansion? Does fiscal expansion in one country help to alleviate the fall in output in other countries? We explore how this argument holds up in a global framework with separate fiscal responses in an environment where either one or both countries is in a liquidity trap. Finally, we explore the nature of an optimal cooperative fiscal and monetary policy response to a liquidity trap, whether the liquidity trap holds for either one country, or globally.

A key aspect of our investigation is that we allow for variations in the degree to which countries are integrated in trade and financial markets. The model allows trade openness to vary from full trade integration to an environment completely closed to trade. In addition, we allow for a variation between complete asset market integration and financial autarky. Both elements are critical for answering the questions posed above. We find that when trade is fully open and asset markets are complete, then all liquidity traps are global; if the zero lower bound binds in one country, then it will bind generally. But with less than fully open trade, shocks are only transmitted incompletely,

1. Some recent exceptions are Fujiwara et al. (2009), Bodenstein, Erceg and Guerrieri (2009) and Jeanne (2009), and Cook and Devereux (2011, 2013). These papers are discussed further below.

and the country that is the source of the shock will be more likely to hit the zero lower bound. Even with fully open trade, incomplete asset markets also reduce the transmission of shocks, and with financial market autarky, we show that the zero lower bound cannot hold in both countries simultaneously.

A key result from the model is that the transmission of shocks in the zero lower bound is associated with perverse response of relative prices; the worst hit country tends to suffer terms of trade appreciation, rather than depreciation, thus exacerbating the effects of the shock.

We also find that fiscal expansion can be extremely effective in raising economic activity, but that it does so through reducing the terms of trade, and redirecting spending away from trading partners, thus reducing trading partners GDP. Thus, fiscal spending is a "beggar-thy-neighbor" policy in a liquidity trap. This result holds in both a complete and incomplete asset market environment.

Finally, we study an optimal cooperative policy response to the shock that generates the liquidity trap. The optimal response involves a joint policy of fiscal expansion and potential policy rate increases for the least hit country. This surprising result comes from the fact that policy rate increases can ameliorate the perverse response of relative prices to the liquidity-trap shock.

This paper is related to a number of others that have recently examined policy issues in a 'zero lower bound' situation in open economies. Fujiwara, Sudox and Teranishi (2009) examine the optimal monetary problem with commitment in a multi-country situation, but do not examine the determination of fiscal policy, or the transmission of demand shocks across countries. Fujiwara Nakajimaz, Sudo and Teranishi. (2010) extend this framework to look at various types of monetary policy cooperation in a global liquidity trap. Jeanne (2009) examines a "global liquidity trap" in a model of one-period-ahead pricing similar to that of Krugman (1998). Bodenstein, Erceg and Guerrieri (2009) use a fully specific two country DSGE model to examine the international transmission of shocks when one of the countries is in a liquidity trap, but do not focus on optimal monetary policy or fiscal policy choices. Cook and Devereux (2011) explore the effect of fiscal policy cooperation in a global liquidity trap. Cook and Devereux (2013) look at the jointly optimal fiscal and monetary policy problem in an international setting when one of the countries is constrained by the zero lower bound. Devereux and Yetman (2013) examine the role that effects of capital controls play in an environment where the zero-bound constraint is binding.

The rest of the paper is organized as follows. The next section develops the basic model. Section 3 examines the efficient global equilibrium under flexible prices and endogenous fiscal policy determination. Section 4 examines the solution under sticky prices, explores the impact of fiscal policies at the zero lower bound, and discusses the role of international policy spillovers. Section 5 examines the optimal policy-making problem in a global cooperative agreement. Some conclusions are then offered.

1. A Generic Two-Country New Keynesian Model

Take a model of two countries as an example, where in each country, households consume both private and public goods, and choose how much to work given wages and prices. The countries are referred to as "home" and "foreign." The countries are of equal size (with population normalized to unity). Consumption takes place across a range of differentiated goods. Asset markets are complete within countries, but between countries we construct a mechanism which allows for asset market completeness to vary between financial autarky and a full set of security markets. Firms produce private goods, while governments produce public goods and distribute them uniformly to households. Product prices are sticky. This means that demand shocks can have inefficient effects on output and inflation rates. Demand shocks are country specific shocks to household preferences for private goods in the present, relative to the future. When the central bank can freely adjust nominal interest rates, an appropriate monetary policy can completely undo the inefficient response to demand shocks. This would ensure that, in both countries, the adjustment to demand shocks is the same as would take place in a first-best economy. Then the government's optimal fiscal policy would produce the first-best division of output between public and private goods. However, if in one or both countries, the nominal interest rate that would sustain the first-best monetary policy is below zero, then monetary policy is limited by the zero lower bound. In this case, demand shocks do have real effects and generate inefficiencies, both in the response of the economy experiencing the shock, as well as neighboring countries. Most of the analysis of the paper will consist of exploring the nature of international shock transmission as the zero bound. We will also analyze the effects of fiscal policy shocks when the zero-bound constraint is binding.

1.1 Households

Let the utility of a representative home household evaluated from date 0, be defined as:

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left(U(C_t, \xi_t) - V\left(N_t\right) + J(G_t) \right) \tag{1}$$

where U, V, and J represent the utility of the composite home consumption bundle C_t, disutility of labor supply N_t, and utility of the public good G_t, respectively. U and J are differentiable and concave in C and G, while V is differentiable and convex in N. The variable ξ_t represents a preference term, which we label a "demand' shock." It is assumed that $U12 > 0$. An increase in ξ_t is equivalent to a rise in the household's time preference.

Consumption is defined as

$$C_t = \Phi C_{Ht}^{v/2} C_{Ft}^{1-v/2}, \nu \geq 1$$

Where $\Phi = (v/2)^{v/2} \left(1 - (v/2)\right)^{v/2}$, C_H is the consumption of the home country composite good, and C_F is consumption of the foreign composite. The parameter $v \geq 1$ allows for home bias in preferences. Home bias is one of the critical determinants of the degree to which the zero-bound constraint is propagated across countries.

In addition, C_H and C_F are defined over the range of home and foreign differentiated goods with elasticity of substitution θ between goods, so that:

$$C_H = \left[\int_0^1 C_H(i)^{1-\frac{1}{\theta}} di \right]^{\frac{1}{1-\frac{1}{\theta}}}, \; C_F = \left[\int_0^1 C_F(i)^{1-\frac{1}{\theta}} di \right]^{\frac{1}{1-\frac{1}{\theta}}}, \; \theta > 1.$$

Price indices for home and foreign consumption may be written as:

$$P_H = \left[\int_0^1 P_H(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \; P_F = \left[\int_0^1 P_F(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}},$$

while the aggregate (CPI) price index for the home country is $P = P_H^{v/2} P_H^{1-v/2}$.

Demand for the individual differentiated goods and home and foreign composite goods can be derived from these functions in the usual way. Individual firms choose prices given a demand elasticity of θ.

We assume that home government spending falls exclusively on the home composite good, and analogously for the foreign composite good. Government consumption is taken as exogenous by households. The representative home household sells labor services to each of a continuum of home country firms, and receives a nominal wage W_t in return. The household's implicit labor supply is determined by the condition:

$$U_C(C_t, \xi_t)W_t = P_t N'(N_t). \tag{2}$$

We assume that there is a full set of state-contingent securities traded between home and foreign residents. However, we also assume that there is a state-contingent wedge in the security returns across countries that prevents the equalization of marginal utilities of asset returns between households in the two countries. Denote this wedge as Ω_t. Then we have the risk-sharing relationship given by:

$$U_C(C_t, \xi_t) = U_C(C_t^*, \xi_t^*)\frac{S_t P_t^*}{P_t}\Omega_t = U_C(C_t^*, \xi_t^*)T_t^{v-1}\Omega_t, \tag{3}$$

where S_t is the nominal exchange rate (home price of foreign currency), $P_t^* = P_H^{*v/2} P_H^{*1-v/2}$ is the foreign *CPI*, and $T = SP_F^*/P_H$ is the home country terms of trade. Implicit in this condition is the assumption that the law of one price holds in individual goods and home and foreign composite consumption goods (i.e. so that $P_F = SP_F^*$, etc).

Now we assume that the wedge in risk-sharing is governed by the functional relationship:

$$\Omega_t = \left(\frac{P_t C_t}{P_{Ht} Y_{Ht}}\right)^{\frac{\lambda}{1-\lambda}} \tag{4}$$

where Y_{Ht} represents home country GDP, an average of all home firms output. This form can be rationalized by the presence of lump-sum financed taxes that are conditioned on the ratio of consumption to domestic GDP. The specific usefulness of (4) is that it allows us to vary the effective degree of asset market completeness between that of un-restricted cross country risk-sharing (when $\lambda = 0$) to financial autarky (when $\lambda = 1$)[2].

We assume also that households hold domestic nominal government bonds, which pay an interest rate of R_t in all states of the world. Then the Euler equation for nominal bond pricing is given by:

$$\frac{U_C(C_t,\xi_t)}{P_t} = \beta R_{t+1} E_t \frac{U_C(C_{t+1},\xi_{t+1})}{P_{t+1}}. \tag{5}$$

Foreign household's actions can be exactly defined analogously. As we see from the definition of P_t^* given above, the foreign representative household has weight $v/2$, $(1 - v/2)$ on the foreign (home) composite good.

1.2 Firms

Firms use labor to produce individual differentiated goods. In the home country, firm i has the production function:

$$Y_t(i) = N_t(i),$$

The home firm's profits are defined by $\Pi_t(i) = P_{Ht}(i)Y_t(i) - W_t H_t(i)(1 - s_t)$, where s_t is a wage subsidy given to all home firms by the home government, financed with lump-sum taxation. This facilitates approximation of the model around an undistorted steady state.

We assume that each home firm resets its price according to Calvo pricing, where the probability of readjusting its price is $1 - \kappa$ in each period. The home firm sells its product to home and foreign consumers, and the home government, at a common price, facing a demand elasticity of θ. When the firm can adjust its price, it sets the new price, denoted $\tilde{P}_{Ht}(i)$, so as to maximize the present value of profits evaluated using the stochastic discount factor $m_{t+j} = (P_t/U_C(C_t,\varepsilon_t))(U_C(C_{t+j},\xi_{t+j})/P_{t+j})$. This leads to the optimal price setting condition as follows:

2. The form of this risk-sharing wedge is used in Devereux and Yetman (2013). The appeal of (4) is that it allows for intermediate degrees of asset market completeness without adding additional state variables into the model, as would be the case, for instance, if we limited asset trade across countries to that of non-state contingent bonds.

$$\tilde{P}_{Ht}(i) = \frac{\theta}{\theta-1}(1-s_t)\frac{E_t\sum_{j=0}^{\infty} m_{t+j}\kappa^j \dfrac{W_{t+j}}{A_{t+j}} Y_{t+j}(i)}{E_t\sum_{j=0}^{\infty} m_{t+j}\kappa^j Y_{t+j}(i)}. \tag{6}$$

All home firms that can adjust their price, choose the same price. In the aggregate, the price index for the home good then follows the process given by:

$$P_{Ht} = [(1-\kappa)\tilde{P}_{Ht}^{1-\theta} + \kappa P_{Ht-1}^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{7}$$

The behavior of foreign firms and the foreign good price index may be described analogously.

1.3 Fiscal Policy

We assume that governments have access to lump sum taxation. Each government then has the task of choosing both an optimal subsidy for its domestic monopoly producers and the level of public goods spending for its domestic constituents. In addition, we assume that the home government follows the state-contingent security tax policy governed by (4). The optimal profit subsidy is set as $s = 1/\theta$, which offsets the steady state monopoly distortion in price setting. With respect to the size of public spending, in the analysis below, we will focus on a jointly optimal cooperative monetary and fiscal policy set to maximize the sum of home and foreign utility. Government spending will be set as a trade-off between alternative objectives, and will depend on the constraints on monetary policy. If monetary policy is not limited by the zero bound, then government spending is optimally set from a pure public finance perspective. In a liquidity trap however, government spending policies will typically deviate from the optimal public finance rule and will be chosen to satisfy stabilization objectives.

1.4 Monetary Policy

We define monetary policy under three different possible regimes. In the first case, we assume that policy is governed by an arbitrary Taylor rule, which sets the short-term interest rate as a function of domestic inflation, as long as the interest rate is not constrained by the zero bound. If this constraint binds, then the interest rate is zero. In the case where monetary policy is governed by a Taylor rule, we assume the rule given by:

$$R_t^r = (1+\rho_t)(1+\tilde{\pi}_H)\left(\frac{P_{Ht}}{P_{Ht-1}}\frac{1}{1+\tilde{\pi}_H}\right)^{\gamma} \tag{8}$$

where ρ_t represents a desired path for the equilibrium real interest rate, and $\tilde{\pi}_H$ represents a desired path for the home goods inflation rate[3]. We assume that $\gamma > 1$. This rule does not allow for interest rate "smoothing." This simplification allows for simple analytical solutions to the model governed by the Taylor rule, and is not critical for the results.

In some cases, we also assume that monetary policy is set to target the natural interest rate (which is defined below), subject to the zero-bound constraint. Hence, we will assume in this case that the policy interest rate is set such that:

$$\ln(R_t) = \mathrm{Max}(0, \tilde{r}_t) \tag{9}$$

Finally in section 5, we allow for an optimal targeting rule for monetary policy. Interestingly, in general, this will imply a different path of interest rates than that governed in (9). Again, however, this optimal rule must be constrained by the zero bound on nominal interest rates. In all cases, we assume that policy is determined under discretion, so we abstract from the possibility of effective forward guidance in monetary policy (and fiscal policy). In addition, we ignore the possibility of using asset purchases by the central bank to implement monetary policy.

Again, the monetary authority of the foreign country is characterized in an analogous manner.

1.5 Market Clearing

Each home country firm i faces demand for its good from home consumers, foreign consumers and its home government. We can define equilibrium in the market for good i as

3. It is more appropriate to define an interest rate rule over home goods inflation rather than the CPI inflation rate since in the absence of the zero lower bound, the policy-maker would wish to set the former inflation rate to zero.

$$Y_{Ht}(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} \left[\frac{v}{2}\frac{P_t}{P_{Ht}}C_t + (1-\frac{v}{2})\frac{S_t P_t^*}{P_{Ht}}C_t^* + G_t\right],$$

where G_t represents total home government spending on the aggregate home good. Now, aggregating across all home firms, market clearing in the home good is defined as

$$Y_{Ht} = \frac{v}{2}\frac{P_t}{P_{Ht}}C_t + (1-\frac{v}{2})\frac{S_t P_t^*}{P_{Ht}}C_t^* + G_t. \tag{10}$$

Here $Y_{Ht} = V_t^{-1}\int_0^1 Y_{Ht}(i)di$ is aggregate home country output, where we have defined $V_t = \int_0^1 (P_{Ht}(i)/P_{Ht})^{-\theta} di$. It follows that home country employment (employment for the representative home household) is given by $N_t = \int_0^1 N(i)di = A^{-1}Y_{Ht} = V_t$.

In the same manner, we may write the aggregate market clearing condition for the foreign good as

$$Y_{Ft} = \frac{v}{2}\frac{P_t^*}{P_{Ft}^*}C_t^* + \left(1-\frac{v}{2}\right)\frac{P_t}{S_t P_{Ft}^*}C_t^* + G_t^*, \tag{11}$$

and again, we may define foreign employment as $N_t^* = \int_0^1 N_t^*(i) di = A^{*-1}Y_{Ft}V_t^*$, where $V_t^* = \int_0^1 (P_{Ft}^*(i)/F_{Ft}^*)^{-\theta} di$.

An equilibrium in the world economy with positive nominal interest rates may be described by equation (3), and equations (2), (5), (6), (7) and (8) for the home and foreign economy, as well as (10) and (11). For given values of V_t and V_t^*, and given government spending policies, these equations determine an equilibrium sequence for the variables C_t, C_t^*, W_t, W_t^*, S_t, P_{Ht}, P_{Ft}^*, $\tilde{P}_{Ht}$, $\tilde{P}_{Ft}^*$, R_t, R_t^*, and N_t, N_t^*. In a first order approximation of the model, the distribution expressions V_t and V_t^* drop out, so up to a first order approximation, the behavior of all variables is fully determined by the outlined equations. When monetary policy in one or both countries is constrained by the zero lower bound, we have to define equilibrium in a constrained manner, as described below.

2. Equilibrium with Fully Flexible Prices

We first define the equilibrium of the model in a fully flexible price world equilibrium where $\kappa = 0$ in each country. In that case, $P_{Ht}(i) = P_{Ht}$, $P_{Ft}(i) = P_{Ft}$, and $V_t = V_t^* = 1$. In addition (given the presence of optimal subsidies) we have $P_{Ht} = A^{-1}W_t$ and $P^*_{Ft} = A^{-1}W^*_t$.

Letting a circumflex denote values in a flexible price world equilibrium, we may describe the equilibrium by the equations:

$$U_C(\tilde{C}_t,\xi_t)A = \tilde{T}_t^{1-\upsilon/2}V'(\tilde{N}_t), \quad U^*_C(\tilde{C}^*_t,\xi^*_t)A^* = \tilde{T}_t^{-(1-\upsilon/2)}V'(\tilde{N}^*_t) \tag{12}$$

$$U_C(\tilde{C}_t,\xi_t)\tilde{T}_t^{\upsilon-1}\left(\frac{\tilde{T}^{\upsilon/2-1}\tilde{N}}{\tilde{C}}\right)^{\frac{\lambda}{1-\lambda}} = U^*_C(\tilde{C}^*_t,\xi^*_t), \tag{13}$$

$$\tilde{N}_t = \frac{\upsilon}{2}\tilde{T}_t^{1-\upsilon/2}\tilde{C}_t + \left(1-\frac{\upsilon}{2}\right)\tilde{T}_t^{\upsilon/2}\tilde{C}^*_t + \tilde{G}_t, \tag{14}$$

$$\tilde{N}^*_t = \frac{\upsilon}{2}\tilde{T}_t^{-(1-\upsilon/2)}\tilde{C}^*_t + \left(1-\frac{\upsilon}{2}\right)\tilde{T}_t^{-\upsilon/2}\tilde{C}_t + \tilde{G}^*_t \tag{15}$$

This implicitly describes the efficient world equilibrium for consumption, output (or employment), and the terms of trade, conditional on values for home and foreign rates of government spending.

For the moment, we take government spending as exogenously given in both countries. We analyze equations (12) to (15) by taking a linear approximation around the globally efficient steady state. For a given variable X, define $\tilde{x} = \ln(\tilde{X}/\overline{X})$ to be the log difference of the global efficient value from the non-stochastic steady state, except for ε_t (to be defined below), and $\tilde{\pi}_{H+1}$ and $\tilde{r}_t$, which refer respectively to the level of the inflation rate and nominal interest rate. Since the model is symmetric, we have $\overline{T} = 1$ in a steady state. Then, we may express the linear approximation of (12) to (15) as:

$$\sigma\,\tilde{c}_t - \varepsilon_t + \phi\,\tilde{y}_t + \left(1-\frac{\upsilon}{2}\right)\tilde{\tau}_t = 0, \tag{16}$$

$$\sigma\ \tilde{c}_t^* - \varepsilon_t^* + \phi\ \tilde{y}_t^* - \left(1 - \frac{v}{2}\right)\tilde{\tau}_t = 0, \tag{17}$$

$$\tilde{y}_t = c_y\left(\frac{v}{2}\tilde{c}_t + \left(1 - \frac{v}{2}\right)\tilde{c}_t^*\right) + c_y v\left(1 - \frac{v}{2}\right)\tilde{\tau}_t + (1 - c_y)\tilde{g}_t, \tag{18}$$

$$\tilde{y}_t^* = c_y\left(\frac{v}{2}\tilde{c}_t^* + \left(1 - \frac{v}{2}\right)\tilde{c}_t\right) - c_y v\left(1 - \frac{v}{2}\right)\tilde{\tau}_t + (1 - c_y)\tilde{g}_t^*, \tag{19}$$

$$\begin{aligned}&(1-\lambda)\left(\sigma\tilde{c}_t - (\varepsilon_t - \varepsilon_t^*) - \sigma\tilde{c}_t^* - (v-1)\tilde{\tau}_t\right)\\&+\lambda\left(c_y\tilde{c} + (1-c_y)\tilde{g} + (1 - v/2)\tilde{\tau} - \tilde{y}\right) = 0,\end{aligned} \tag{20}$$

Here we have defined $\sigma \equiv -U_{CC}\overline{C}/U_C$ as the inverse of the elasticity of intertemporal substitution in consumption, $\phi \equiv -(V''\overline{H}/V')$ as the elasticity of the marginal disutility of hours worked[4]. The ratio of private consumption to GDP is defined as c_y. Finally, $\varepsilon_t = (U_{C\xi}/U_C)\ln(\xi_t)$ is the measure of a positive demand shock in the home country, with an equivalent definition for the foreign country.

We may solve the system (16) to (20) to obtain the first order solutions for consumption, output and the terms of trade in response to demand shocks. For any variable x, define $x^W = (x + x^*)/2$ as the world *average*, and $x^R = (x + x^*)/2$ as the world *relative* in the variable. Since then $x = x^W + x^R$, we can write home and foreign consumption responses to demand shocks as:

$$\tilde{c}_t = \frac{1}{\phi c_y + \sigma}\varepsilon_t^W + \left[\begin{aligned}&(1-\omega)\left(\frac{1 + c_y\phi v(2-v)}{\sigma + \phi c_y D}\right)\\&+\omega\left(\frac{(v-1)}{\sigma(1-v) + 2 - v + \phi c_y}\right)\end{aligned}\right]\varepsilon_t^R$$

4. In a subsequent section, we also make use of the parameter and $\sigma_g \equiv -(J''\overline{G}/J')$, which is the elasticity of the marginal utility of public goods. We will assume that $\sigma_g = \sigma$.

$$\tilde{c}_t^* = \frac{1}{\phi c_y + \sigma}\varepsilon_t^W - \left[\begin{array}{l} (1-\omega)\left(\dfrac{1 + c_y\phi v(2-v)}{\sigma + \phi c_y D}\right) \\ +\omega\left(\dfrac{(v-1)}{\sigma(1-v) + 2 - v + \phi c_y}\right)\end{array}\right]\varepsilon_t^R$$

where $D \equiv \sigma v\ (2 - v) + (1 - v)^2$, and ω is a function of the risk-sharing weight λ, defined as:

$$\omega(\lambda) = \frac{\lambda c_y(2-v)(2-v+\sigma(v-1)+\phi c_y)}{2(1-\lambda)(\phi c_y D + \sigma) + \lambda c_y(2-v)(2-v+\sigma(v-1)+\phi c_y)}$$

Note that $\omega(0) = 0$, $\omega(1) = 1$, and $\omega'(\lambda) > 0$ for $0 \leq \lambda \leq 1$.

A demand shock raises the efficient flexible-price world consumption level, but the impact on individual country consumption depends on the source of the demand shock, the degree of home bias in preferences, and the openness of international capital markets. A world demand shock will raise the flexible price level of home and foreign consumption equally, but a relative home country demand shock leads consumption to move in opposite directions across countries, and the response will depend upon λ and v. For $\lambda = 0$, there is full risk-sharing, and asset markets will facilitate a rise in home country consumption for $\varepsilon^R > 0$ even for zero home bias in preferences ($v = 1$). But with no capital markets $\lambda = 1$, a rise in ε^R will raise home (and reduce) foreign flexible price consumption only when $v > 1$. The explanation for this latter effect is developed further below when examining the characteristics of the terms of trade under flexible prices.

The impact of demand shocks on flexible price output levels are likewise written as:

$$\tilde{y}_t = \frac{c_y}{c_y\phi + \sigma}\varepsilon_t^W + \left[(1-\omega)\left(\frac{c_y(v-1)}{\sigma + c_y\phi D}\right) + \omega\left(\frac{c_y}{\sigma(v-1) + 2 - v + c_y\phi}\right)\right]\varepsilon_t^R$$

$$\tilde{y}_t^* = \frac{c_y}{c_y\phi + \sigma}\varepsilon_t^W - \left[(1-\omega)\left(\frac{c_y(v-1)}{\sigma + c_y\phi D}\right) + \omega\left(\frac{c_y}{\sigma(v-1) + 2 - v + c_y\phi}\right)\right]\varepsilon_t^R$$

A world demand shock raises equilibrium output in both countries. When there is home bias in preferences, so that $v > 1$, a relative home demand shock tends to raise home output and reduce foreign output when financial markets are complete. Without capital markets, a relative demand shock increases relative home output even for $v = 1$, because without risk-sharing for preference shocks, the increase in relative home time preference directly increases home output through an increase in the supply of labor.

Demand shocks also affect the flexible price efficient response of the terms of trade. We can show that:

$$\frac{\tilde{\tau}_t}{2} = -\left[\frac{(1-\omega)\phi c_y(v-1)}{\sigma + c_y\phi D} - \frac{\omega}{2 - v + c_y\phi + \sigma(v-1)}\right]\varepsilon_t^R$$

The response of the terms of trade to a relative demand shock depends on the degree of capital market integration. When $\lambda = 0$, there is full risk-sharing, and relative output and the terms of trade improve only when $v > 1$. But if $\lambda = 1$, with no capital markets, home relative output increases, and the terms of trade deteriorates. The different response of the terms of trade under the two alternative capital market structures comes from the different types of risk-sharing that occurs. With a full, unhindered set of security markets, cross country insurance allows for the occurrence of preference shocks, so that full risk-sharing does not generally equate consumption levels across countries. Rather, full risk-sharing, by equating the marginal utility of security payoffs across countries, leads to equal responses of labor supply in each country, according to (2). Then, home output will only rise if the rise in demand is tilted towards home goods (i.e. $v > 1$), which in itself tends to raise the terms of trade. In contrast, without capital markets, risk-sharing only takes place indirectly through terms of trade adjustment—relative home output increases due to the shift out in home labor supply, and this leads to an equilibrium fall in the relative home price—i.e. a terms of trade deterioration.

If monetary authorities could adjust nominal interest rates freely to respond to demand shocks, then the flexible-price efficient global equilibrium could be sustained. Following previous literature, we denote the interest rate that would sustain the flexible-price efficient outcome as the "natural interest rate." Denote $\tilde{r}_t$ as the (level of the) net nominal interest rate, and let $\bar{r}_t$ be its steady state value. Then, a log linear approximation of (5) leads to the expressions for the flexible-price equilibrium nominal interest rate in the home country as:

$$\tilde{r}_t = \bar{r} + \sigma E_t(\tilde{c}_{t+1} - \tilde{c}_t) - E_t(\varepsilon_{t+1} - \varepsilon_t) + E_t \tilde{\pi}_{Ht+1} + \left(1 - \frac{v}{2}\right) E_t(\tilde{\tau}_{t+1} - \tilde{\tau}_t) \quad (21)$$

Assume that an efficient monetary policy is to keep the domestic rate of inflation equal to zero. In addition, for now, assume that demand shocks follow an AR(1) process so that $\varepsilon_{t+1} = \mu\varepsilon_t + u_t$ and $\varepsilon^*_{t+1} = \mu\varepsilon^*_t + u^*_t$, where u^*_t and u_t are mean-zero and i.i.d., then the value of $\tilde{r}_t$ when the right hand side is driven by demand shocks alone can be derived as:

$$\tilde{r}_t = \bar{r} + \left(\frac{\phi c_y}{\phi c_y + \sigma} \varepsilon^W_t + \left(\frac{(1-\omega)\phi c_y (v-1)}{\sigma + \phi c_y D} + \frac{\omega \phi c_y}{2 - v + \phi c_y + \sigma(v-1)} \right) \varepsilon^R_t \right) (1-\mu) \quad (22)$$

In a similar manner, the foreign efficient nominal interest rate is:

$$\tilde{r}^*_t = \bar{r} + \left(\frac{\phi c_y}{\phi c_y + \sigma} \varepsilon^W_t - \left(\frac{(1-\omega)\phi c_y (v-1)}{\sigma + \phi c_y D} + \frac{\omega \phi c_y}{2 - v + \phi c_y + \sigma(v-1)} \right) \varepsilon^R_t \right) (1-\mu) \quad (23)$$

Natural interest rates respond to both aggregate and relative demand shocks. An aggregate demand shock raises global marginal utility and raises natural interest rates. A relative demand shock affects natural interest rates in separate ways in the two countries, but this depends upon the degree of capital market openness as well as the degree of home bias in preferences. If securities markets are full (i.e. $\lambda = 0$, so that $\omega = 0$), then a relative demand shock causes a rise (fall) in the home (foreign) natural interest rate, only if $v > 1$. With identical preferences across countries, and full security markets, the natural interest rate is independent of purely relative demand shocks. On the other hand, in the case of no capital markets (i.e. $\lambda = 1$ so that $\omega = 1$), a relative demand shock always raises (lowers) home (foreign) natural real interest rates, independent of whether v exceeds unity.

This discussion has direct bearing on the degree to which the zero-bound constraint will bind across countries in response to time preference shocks (negative demand shocks) emanating from one country. In general, these shocks will have both aggregate and relative components. But if there are full security markets and identical preferences, we see that natural interest rates are always equated across countries. Then, in a monetary policy regime in which authorities in each country attempt to target the natural interest rate, the occurrence of the zero-bound constraint will be perfectly synchronous across countries—all liquidity traps are global. But when there is home bias, or equivalently, when trade is not perfectly open across countries, this is no longer the case. In addition, even with fully open trade, capital market restrictions can also segment countries' natural interest rates so that the zero-bound constraint is not perfectly synchronized across countries when $\lambda > 0$.

Figure 1 illustrates the relationship between (22) and (23) consequent upon a negative shock to ε (the home preference term) as a function of v. In this case, the two expressions (22) and (23) become:[5]

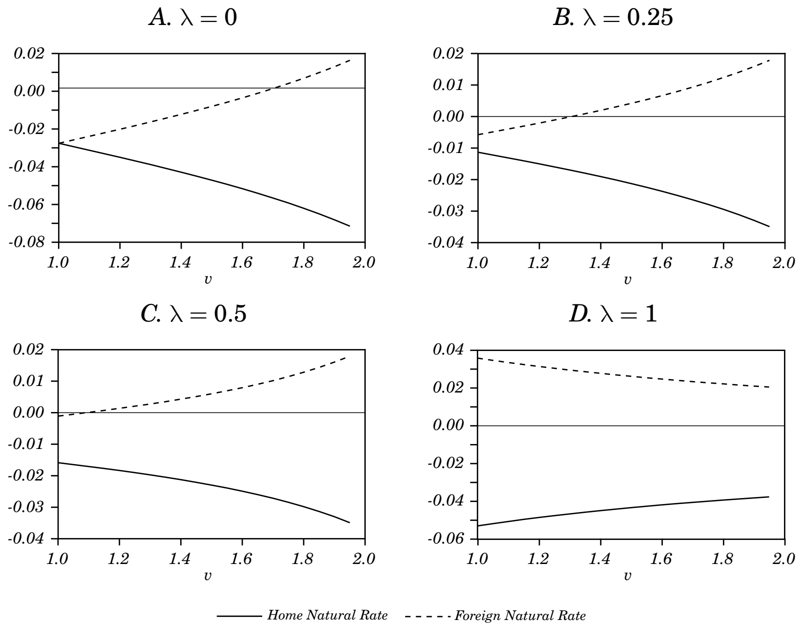
$$\tilde{r}_t = \bar{r} + \phi c_y \left(\frac{1}{\phi c_y + \sigma} + \left(\begin{array}{l} \dfrac{(1-\omega)(v-1)}{\sigma + \phi c_y D} \\ + \dfrac{\omega}{2 - v + \phi c_y + \sigma(v-1)} \end{array} \right) \right) (1-\mu) \frac{\varepsilon_t}{2} \tag{24}$$

and,

$$\tilde{r}_t^* = \bar{r} + \phi c_y \left(\frac{1}{\phi c_y + \sigma} - \left(\begin{array}{l} \dfrac{(1-\omega)(v-1)}{\sigma + \phi c_y D} \\ + \dfrac{\omega}{2 - v + \phi c_y + \sigma(v-1)} \end{array} \right) \right) (1-\mu) \frac{\varepsilon_t}{2}. \tag{25}$$

5. In this figure and the subsequent figures below, we use the following standard calibration of parameters and shocks; $\sigma = 2$, $\phi = 1$, $\beta = 0.99$, $k = 0.05$, $c_y = 0.8$, $\mu = 0.6$, $\bar{r} = 0.02$, and we assume that the shocks ε_t^W and ε_t^R are such that the desired interest rate would be -0.05 in the absence of the zero bound.

Figure 1. Natural Interest Rates



Source: Authors' elaboration.

The first panel of the figure shows the case of $\lambda = 0$, so that security markets are complete. The value of v ranges between 1 and 2. The calibration is such that for $\lambda = 0$ and $v = 1$, both countries fall into a liquidity trap. As v rises above 1, the fall in the home natural real interest rate is greater, while the foreign natural real interest rate falls by less. As v approaches 2, the home natural real interest rate falls by its maximal amount, and there is no affect at all on the foreign natural real interest rate. Two clear implications come from the figure: First, we see that the combination of integrated financial markets and international trade in goods cushions the full impact of savings shocks on the home natural real interest rate[6]. Second,

6. Note that the countries are of equal size. Extending the model to allow for differential size (and home bias in preferences) among countries would be straightforward, but would add notational complexity. In the case of size differences, smaller countries would be more vulnerable to global preference shocks creating a liquidity trap, but more insulated against domestic shocks.

the more open both economies are, the more likely that the liquidity trap will be experienced globally.

The second, third and fourth panels of figure 1 illustrate the relationship between v and natural interest rates as capital markets become more and more restricted. As λ increases, there is a wedge between natural interest rates, even if there is completely open trade ($v = 1$). In the limit, when $\lambda = 1$, as seen in expressions (24) and (25), the natural interest rates are negatively correlated. In this case, a negative home demand shock reduces the home natural rate by more in the open economy (when $v = 1$) than it would in the closed economy ($v = 2$), and the foreign natural interest rate increases above its initial steady state. Figure 1d illustrates an interesting implication of the consequence of capital market restrictions in the face of demand shocks. When $\lambda = 1$, we have financial autarky, so direct financial risk-sharing is eliminated. But the response of the terms of trade indicates that the indirect mechanism for risk sharing, through a terms of trade deterioration, is also prevented. This is because the terms of trade of the country experiencing the shock actually appreciates, as discussed above.

3. Demand Shocks with Sticky Prices and the Zero-bound Constraint

We now turn to the substantial analysis of the paper, exploring the impact of demand shocks in the case where prices are sticky, and the zero-bound constraint may bind. When prices are sticky, we may derive a log-linear approximation of the model in terms of inflation and output gaps in a manner similar to Clarida, Galí and Gertler (2002) and Engel (2010). Taking a linear approximation of (2), (6) and (7) around the zero inflation steady state, we derive the standard forward-looking inflation equation, in terms of "gaps," or deviations from the flexible-price efficient level of each variable. For any variable x, we define $\hat{x} = (X/\tilde{X})\ln$ as the log deviation from the flexible-price efficient level (as before, inflation and nominal interest rates are expressed in levels).

This model has the property of a full dichotomy of the system of equations into world averages and world relative variables. We can write the New Keynesian model for world averages in terms of the world average inflation rate, and world average output gap.

The average system is represented as:

$$\pi_t^W = \frac{k}{c_y}\left((\sigma + c_y\phi)\hat{y}_t^W - \sigma(1-c_y)\hat{g}_t^W\right) + \beta E_t \pi_{t+1}^W \tag{26}$$

$$\rho_t^W + \gamma\pi_t^W = E_t\frac{\sigma}{c_y}(\hat{y}_{t+1}^W - \hat{y}_t^W) + E_t\frac{(1-c_y)}{c_y}(\hat{g}_{t+1}^W - \hat{g}_t^W) + \tilde{r}_{t+1}^W \tag{27}$$

where $k = (1 - \beta\kappa)(1 - \kappa)/\kappa$, and $\tilde{r}_{t+1}^W$ is the world average natural interest rate. Then the relative system is given by:

$$\begin{aligned}\pi_t^R = {} & \frac{k}{c_y}\left(\phi c_y + (1-\omega_1)\sigma_D + \omega_1(2-v+\sigma(v-1))\right)\hat{y}_t^R \\ & -\frac{k(1-c_y)}{c_y}\left((1-\omega_1)\sigma_D + \omega_1(2-v+\sigma(v-1))\right)\hat{g}_t^R + \beta E_t\pi_{t+1}^R\end{aligned} \tag{28}$$

$$\begin{aligned}\rho_t^R + \gamma\pi_t^R = {} & E_t\frac{1}{c_y}\left(\omega_1\sigma_D + (1-\omega_1)(2-v+\sigma(v-1))\right)(\hat{y}_{t+1}^R - \hat{y}_t^R) \\ & -E_t\frac{(1-c_y)}{c_y}\left(\omega_1\sigma_D + (1-\omega_1)(2-v+\sigma(v-1))\right)(\hat{g}_{t+1}^R - \hat{g}_t^R) + \tilde{r}_{t+1}^R\end{aligned} \tag{29}$$

where $\tilde{r}_{t+1}^R$ is the world relative natural interest rate, and we define expressions as follows:

$$\sigma_D \equiv \frac{\sigma}{D}, \text{and } \omega_1(\lambda) = \frac{2D(1-\lambda)}{2D(1-\lambda) + \lambda c_y(2-v)}.$$

Note that the system written in terms of 'world averages' is independent of the degree of trade integration, captured by the home bias parameter v, and of the degree of capital market integration, captured by the parameter λ. The system in terms of 'world relatives' does depend upon both v and λ.

From (26) to (29), we can solve for the values of π_t^W, $\hat{y}_t^W$, π_t^R, and $\hat{y}_t^R$. From these solutions, we can then recover the values of output, terms of trade, and consumption as a function of the movements in natural rates of interest $\tilde{r}_t$ and $\tilde{r}_t^*$ as well as the shocks to government spending gaps $\hat{g}_t$ and $\hat{g}_t^*$.

We note that, so long as it is feasible, a monetary policy rule which ensures that $\rho_t^W = \tilde{r}_{t+1}^W$ and $\rho_t^R = \tilde{r}_{t+1}^R$ can simultaneously ensure zero inflation and zero output gaps in both countries. This rule simply involves adjusting the individual national nominal interest rates to equal their respective natural interest rates in each country. However, if natural interest rates are pushed below zero, this is not feasible. In the analysis below, we focus on situations where the natural interest rates of one or both countries are temporarily below zero. In this case, we immediately cannot achieve zero inflation and zero output gaps for both countries simultaneously.

3.1 International Transmission of Shocks and the Zero Bound, with Full Capital Mobility

Our leading example will focus on the case of a negative demand shock emanating from the home country, so that $\varepsilon_t < 0$ and $\varepsilon_t^* = 0$. As shown above, this will reduce the natural interest rate in the home country, while its effect on the foreign natural interest rate depends upon both the degree of trade openness and the integration of world capital markets. We deal with each of these cases in turn, beginning with the case of fully open capital markets, and focusing on the influencing of varying trade openness on the international transmission of demand shocks at the zero bound, as well as the effectiveness of the fiscal policy response to such shocks. When $\lambda = 0$, it is immediate from (22) and (23), that natural interest rates move in the same direction in response to a demand shock. Furthermore, in the case of fully open trade $v = 1$, natural interest rates are identical, and both countries hit the zero lower bound simultaneously; there is either no liquidity trap, or a global liquidity trap. But more generally, when the shock is large enough, both countries can be in a liquidity trap even when $v > 1$.

While our focus is on the characteristics of the economy when monetary policy is constrained by the zero bound, it is first useful to review the properties of the model under a positive nominal interest rate when policy is governed by a Taylor rule according to (8). Although we have emphasized that in this case, an optimal monetary policy is to adjust nominal interest rates to equal natural interest rates, and thereby eliminating all welfare gaps, we first describe how such adjustment would take place when interest rates are arbitrarily adjusted according to a Taylor rule, and $\gamma > 1$, so that the Taylor principle applies. Since most of the literature in New Keynesian open economy models deals with this case, it serves as a useful contrast to the properties of the model in a liquidity trap.

3.1.1 Demand shocks and fiscal policy with a Taylor rule

Now specialize the Taylor rule (8) to the case where the target real interest rate $\bar{r}$ is constant (i.e. assuming that the monetary and fiscal authorities do not follow a policy of closing all gaps in the economy with positive nominal interest rates). We also assume that the target inflation rate $\tilde{\pi}_{Ht}$ is zero. Thus, in the linearized versions of (8) we set $r_t = \bar{r} + \gamma\pi_H$ and $\tilde{r}_t^* = \bar{r} + \gamma\pi_{Ft}^*$.

As noted above, we focus on a shock to the home country ε. From the above notation, this implies that $\varepsilon_t^W + \varepsilon_t^R = \varepsilon_t$ and $\varepsilon_t^W + \varepsilon_t^R = \varepsilon_t^* = 0$. As noted, we assume that the shock has persistence $0 < \mu < 1$. Then, we may derive the impact of these shocks on home and foreign output as follows:

$$\hat{y}^r_{t(\text{Taylor})} = \left[\frac{1}{(\sigma + \phi c_y)\Delta} + \frac{(v-1)D}{\Delta_1(\sigma + \phi c_y D)}\right]\frac{(1-\mu)(1-\beta\mu)\phi c_y^2 \varepsilon_t}{2} \qquad (30)$$

$$\hat{y}^{*r}_{t(\text{Taylor})} = \left[\frac{1}{(\sigma + \phi c_y)\Delta} - \frac{(v-1)D}{\Delta_1(\sigma + \phi c_y D)}\right]\frac{(1-\mu)(1-\beta\mu)\phi c_y^2 \varepsilon_t}{2} \qquad (31)$$

Here, $\Delta = \sigma(1-\mu)(1-\beta\mu) + k(\gamma_1 - \mu)(\sigma - \phi c_y) > 0$, and $\Delta_1 = \sigma(1-\mu)(1-\beta\mu) + k(\gamma_1 - \mu)(\sigma - \phi D c_y) > 0$.

From (30) and (31), a home country saving shock reduces output (in gap terms) in the home country. The response of the output gap in the foreign country depends on v, as seen below. For v relatively close to unity, the foreign output gap falls also. For $v > 1$, the response of the home output gap is always greater than that of the foreign output gap.

When $v > 1$, the savings shock must also generate some relative price movement across countries. We may compute the response of the terms of trade as follows:

$$\tau^r_{t(\text{Taylor})} = -\frac{\phi(v-1)c_y k\varepsilon_t(\gamma_1 - \mu)}{\Delta_1}$$

In response to a savings shock, the home terms of trade depreciates since $\gamma > \mu$, given that monetary policy satisfies the Taylor principle. Intuitively, the shock leads to a decline in home inflation and a compensating interest-rate cut which facilitates an exchange-rate and terms-of-trade depreciation.

Again, to contrast with their effects in a global liquidity trap, we can examine the effects of fiscal spending shocks in the environment where the Taylor principle applies. Fiscal spending shocks always have differential effects across countries as spending is focused on the home good. Again, take the fiscal spending shock coming from the home country. In addition assume that shocks to the fiscal gap $\hat{g}_t$ are also AR(1) with persistence μ. Then we derive:

$$\hat{y}^{g}_{t(\text{Taylor})} = \frac{1}{2}(1-c_y)\frac{[\Delta_1+\Delta]\Theta}{\Delta\Delta_1}\hat{g}_t \tag{32}$$

$$\hat{y}^{*g}_{t(\text{Taylor})} = \frac{1}{2}(1-c_y)\frac{k\phi c_y(D-1)(\gamma-\mu)\Theta\hat{g}_t}{\Delta\Delta_1}$$

where $\Theta \equiv \sigma(1-\beta\mu)(1-\mu)+k(\gamma-\mu) > 0$. When nominal interest rates are positive, and the Taylor principle applies ($\gamma > 1$), then shocks to the fiscal spending gap will raise the home output gap. Here, and in what follows, we make the empirically relevant assumption that $\sigma > 1$, so that $D \geq 1$. In this case, home spending shocks also raise the foreign output gap so that the international transmission of fiscal policy is positive. But home output always rises more than foreign, since:

$$\hat{y}^{g}_{t(\text{Taylor})} - \hat{y}^{*g}_{t(\text{Taylor})} = (1-c_y)\frac{\Theta}{\Delta_1}\hat{g}_t > 0$$

The response of the terms of trade can be derived as follows:

$$\hat{\tau}^{g}_{t(\text{Taylor})} = -\frac{(1-c_y)k(\gamma-\mu)\phi\hat{g}_t}{\Delta_1} \tag{33}$$

Under a Taylor rule, a domestic fiscal expansion causes a terms-of-trade appreciation, causing a relative price movement, which drives expenditure away from the home good. The impact on home and foreign country consumption is given by:

$$\hat{c}^{g}_{t(\text{Taylor})} = -\frac{(1-c_y)k(\gamma-\mu)\phi\left[\Delta_1+(v-1)\Delta\right]\hat{g}_t}{\Delta\Delta_1} \tag{34}$$

$$\hat{c}^{*g}_{t(\text{Taylor})} = -\frac{(1-c_y)k(\gamma-\mu)\phi\left[\Delta_1-(v-1)\Delta\right]\hat{g}_t}{\Delta\Delta_1}$$

$$= -\frac{(1-c_y)k(\gamma-\mu)\phi(2-v)\left[\Theta+k\phi(\gamma-\mu)c_y(1+v(\sigma-1))\right]\hat{g}_t}{\Delta\Delta_1} \tag{35}$$

Consumption falls in both countries. When $v = 1$, home and foreign consumption fall equally in response to a fiscal gap expansion in either country. More generally, for $v > 1$, consumption falls by more in the home country.

A domestic government spending increase under a Taylor rule increases aggregate demand and domestic output. This raises domestic marginal cost, increasing inflation. The rise in inflation leads to a rise in the domestic nominal and real interest rate. This leads to an exchange-rate and terms-of-trade appreciation for the home country. The fall in aggregate consumption reduces demand for foreign output, but the foreign terms of trade depreciation increases demand for foreign output. For $\sigma > 1$, the second effect dominates, and foreign output increases.

How large is the fiscal multiplier? It is easy to show that both the own country and cross country fiscal multipliers must be less than unity. Define the fiscal spending multiplier as $dY/dG = [(Y/G)(\hat{y}/\hat{g})] = 1/[(1 - cy)(\hat{n}/\hat{g})]$. Since both consumption and the terms of trade fall following a fiscal spending shock, it must be the case that home output rises by less than $(1 - c_y)\hat{g}_t$, as is also evident from (32). Then the government spending multiplier in the economy governed by a Taylor rule is less than unity. Since foreign output rises by less than home, the fiscal spending multiplier for foreign output must also be less than unity.

Finally, note that since the terms of trade response tends to reduce the size of the fiscal multiplier, it should imply that trade openness reduces the effectiveness of fiscal policy in a traditional sense. This is indeed the case, under a Taylor rule. In particular, we can show that

$$\hat{y}^{g}_{t(\text{Taylor,v=2})} - \hat{y}^{g}_{t(\text{Taylor,v=1})} = \text{Sign}(\sigma-1)(\gamma-\mu) > 0$$

Thus, the multiplier in the closed economy ($v = 2$) is higher than that in the fully open economy ($v = 1$). This agrees with textbook intuition about "leakage" effects of fiscal spending shocks in the open economy. In this model, the reasoning is linked to the behavior of the terms of trade. With monetary policy governed by a Taylor rule, a fiscal expansion causes a terms-of-trade appreciation. This dampens the demand effects of fiscal expansion on domestic GDP and reduces the impact relative to that of a closed economy.

3.1.2 Savings shocks and fiscal policy in a liquidity trap

We now look at the same experiment, but assuming that the zero lower bound binds. Again, for the moment, we continue to focus on the case $\lambda = 0$ so that cross-country security trade is complete and unrestricted. As shown above, this is the case where the movement in natural interest rates is positively correlated across countries, for $v \leq 2$. To make a clear comparison to the previous case where the Taylor rule applied, we assume that at time t there is an unanticipated negative ε shock which pushes *both* the home and foreign country natural interest rates below zero. Then we assume further that this shock reverts back to 0 with probability $1-\mu$ in each period henceforth, and then remains at zero thereafter. Therefore, both countries are pushed into a liquidity trap immediately following the shock, and remain in the liquidity trap thereafter with probability μ.

We make the parallel assumption about the fiscal policy shocks. As long as the countries are in a liquidity trap, the fiscal gaps are non-zero. However, once the interest rate is above zero, all fiscal gaps revert back to zero. This allows for a direct comparison to the expressions for the impact of persistent savings and fiscal shocks in the economy operating under a Taylor rule.

Following these assumptions, the impact of a home country savings shock on domestic and foreign output can be derived as

$$\hat{y}^{r}_{t(\mathrm{ZLB})} = \left[\frac{1}{(\sigma+\phi c_y)\Delta_2} + \frac{(v-1)D}{\Delta_3(\sigma+\phi c_y D)}\right]\frac{(1-\mu)(1-\beta\mu)\phi c_y^2 \varepsilon_t}{2} \tag{36}$$

$$\hat{y}^{*r}_{t(\mathrm{ZLB})} = \left[\frac{1}{(\sigma+\phi c_y)\Delta_2} - \frac{(v-1)D}{\Delta_3(\sigma+\phi c_y D)}\right]\frac{(1-\mu)(1-\beta\mu)\phi c_y^2 \varepsilon_t}{2} \tag{37}$$

This notation is an extension of the previous definitions. In particular, $\Delta_2 = \sigma(1-\beta\mu)(1-\mu) - \mu k(\sigma + \phi c_y)$ and $\Delta_3 = \sigma(1-\beta\mu)(1-\mu) - \mu k(\sigma + \phi c_y D)$. In order that the equilibrium be determinate, it is necessary that $\Delta_3 > 0$, which implies that $\Delta_2 > 0$. The condition puts a limit on the degree of persistence of the savings shock that can be accommodated under this analysis. We make this assumption in what follows.[7] Comparing (30) and (36), we can establish that

$$\hat{y}^r_{t(\mathrm{ZLB})} - \hat{y}^r_{t(\mathrm{Taylor})} = \left[\frac{(1-\beta\mu)(1-\mu)\phi k c_y^2 \gamma}{\Delta\Delta_2}\right]\varepsilon_t$$

Since the expression inside the square brackets is positive, then for a negative savings shock, output falls more when the economy is constrained by the zero lower bound than when interest rates are able to adjust. This is a familiar result from Eggertsson and Woodford (2003) and Christiano et al. (2009). Under a Taylor rule, the fall in aggregate demand leads to a fall in inflation, and a compensating fall in the nominal and ex-post real interest rate in each country. When both countries are constrained by the zero bound, the fall in expected inflation leads to a rise in the real interest rate, leading to a further fall in aggregate demand. As long as $\Delta_3 > 0$ (so that $\Delta_2 > 0$), this process converges to a point where output falls such that the desired fall in savings is eliminated.

The international transmission of savings shocks in this case depends upon the degree of trade openness. When $v = 1$, the interest rate linkage across countries ensures that both countries experience the same shock, and thus the output gap moves identically across the two countries. But for a large v close to 2, it is possible that foreign output moves in the opposite direction to home output due to the response of the terms of trade.

How do the terms of trade respond the savings shock? In the previous section, under a Taylor rule, we showed that the terms of

7. When $\mu = 1$, the nominal interest rate is constant (up to a first order), and the equilibrium is indeterminate. With $\mu < 1$, the nominal interest rate will revert to the Taylor rule with probability $1 - \mu$ in the future period. In this case, the condition $\Delta_3 > 0$ ensures that, in response to a negative savings shock, when the nominal interest rate does not respond in the current period, the fall in inflation, by raising the real interest rate, reduces aggregate demand by less than it reduces aggregate supply. In this case there is a unique equilibrium with lower output and lower inflation. If $\Delta_3 < 0$, this stability condition does not apply. The rise in the real interest rate reduces aggregate demand more than it reduces aggregate supply, and an equilibrium with lower output and inflation does not exist. For a discussion, see Eggertsson (2010).

trade depreciated after a negative home country savings shock, so long as $v > 1$. The terms of trade responded in a stabilizing direction. For the case of the zero lower bound, we can show that

$$\tau^{r}_{t(\mathrm{ZLB})} = \frac{\phi(v-1)c_y k \varepsilon_t \mu}{\Delta_3}$$

For a negative ε shock, the terms of trade appreciates. So relative prices move in a direction that exacerbates the fall in demand in the home country. This destabilizing effect of relative prices in a global liquidity trap is emphasized in Cook and Devereux (2013). It is tied to the fact that while nominal interest rates are constrained by the zero bound, there is still arbitrage in bond markets, so a fall in demand in the home country, by reducing inflation in the home country, will raise the home real interest rate, which precipitates a terms-of-trade appreciation.

The impact of shocks to the fiscal gaps on home and foreign output at the zero lower bound can be derived as

$$\hat{y}^{g}_{t(\mathrm{Taylor})} = \frac{1}{2}(1-c_y)\frac{[\Delta_3 + \Delta_2]\Theta_1}{\Delta_2\Delta_3}\hat{g}_t \tag{38}$$

$$\hat{y}^{*g}_{t(\mathrm{Taylor})} = -\frac{1}{2}(1-c_y)\frac{k\phi c_y(D-1)\mu\Theta_1\hat{g}_t}{\Delta_2\Delta_3} \tag{39}$$

where $\Theta_1 = (1 - \beta\mu)(1 - \mu) - k\mu\sigma > 0$. We will compare the multiplier effects of domestic fiscal expansion in (38) with the equivalent under the Taylor rule below. Here, we note that the impact of the fiscal expansion on the foreign country is negative. That is, under a global liquidity trap, fiscal expansion has a beggar-thy-neighbor characteristic, reducing the output of foreign countries.

The derivation of the terms of trade and consumption in the liquidity trap can be expressed simply. The terms of trade responds as:

$$\hat{\tau}^{g}_{t(\mathrm{ZLB})} = \frac{(1-c_y)k\mu\phi\hat{g}_t}{\Delta_3} \tag{40}$$

In contrast to the case with a Taylor rule, a domestic fiscal expansion in a liquidity trap causes a terms of trade depreciation.

Finally, the effect of the fiscal shock on consumption is given by:

$$\hat{c}^{g}_{t(\mathrm{ZLB})} = \frac{(1-c_y)k\mu\phi\left[\Delta_3 + (v-1)\Delta_2\right]\hat{g}_t}{\Delta_2\Delta_3} \tag{41}$$

$$\hat{c}^{*g}_{t(\mathrm{ZLB})} = -\frac{(1-c_y)k\mu\phi\left[\Delta_3 - (v-1)\Delta_2\right]\hat{g}_t}{\Delta_2\Delta_3}$$

$$= -\frac{(1-c_y)k\mu\phi(2-v)\left[\Theta_1 - k\phi\mu_y(1+v(\sigma-1))\right]\hat{g}_t}{\Delta_2\Delta_3} \tag{42}$$

In the liquidity trap, for $v = 1$, a domestic country fiscal expansion leads both home and foreign consumption to rise. Without home bias, they rise by the same amount. But when $v > 1$, home consumption rises by more than foreign consumption and it is possible that foreign consumption falls.

At the zero lower bound, a home country fiscal expansion generates a terms of trade depreciation, which raises demand for the home good. Moreover, since it is easy to show that the preference-weighted growth in consumption of home goods rises, then it immediately follows that the own country fiscal spending multiplier must be greater than unity, since, in response to a rise in home government spending, home output must rise by more than $(1 - c_y)\hat{g}_t$.

Thus, we find that fiscal spending under a liquidity trap has a multiplier above one. But this comes at the expense of a negative cross country multiplier. Intuitively, the impact of an increase in the spending gap in a liquidity trap is to raise expected inflation in the home country relative to the foreign country. This reduces the home country real interest rate, and generates a terms-of-trade depreciation. The terms-of-trade depreciation increases demand for the home good, but reduces demand for the foreign good. Moreover, in this case, the latter effect offsets the impact of a rise in total aggregate consumption, so that foreign output must fall.[8]

In the discussion of the effects of fiscal spending under a Taylor rule, we found that openness reduced the size of the multiplier. How does this contrast to the case where the zero lower bound is binding

8. As we noted above, a similar result is found in Fujiwara and Ueda (2010), for the case $v = 1$ (in our notation).

in both countries? Taking the above fiscal multipliers, and again comparing the case of the fully open with the fully closed economy, we find:

$$\hat{y}^{g}_{t(\mathrm{ZLB},v=2)} - \hat{y}^{g}_{t(\mathrm{ZLB},v=1)} = -\mathrm{Sign}(\sigma - 1)\mu < 0$$

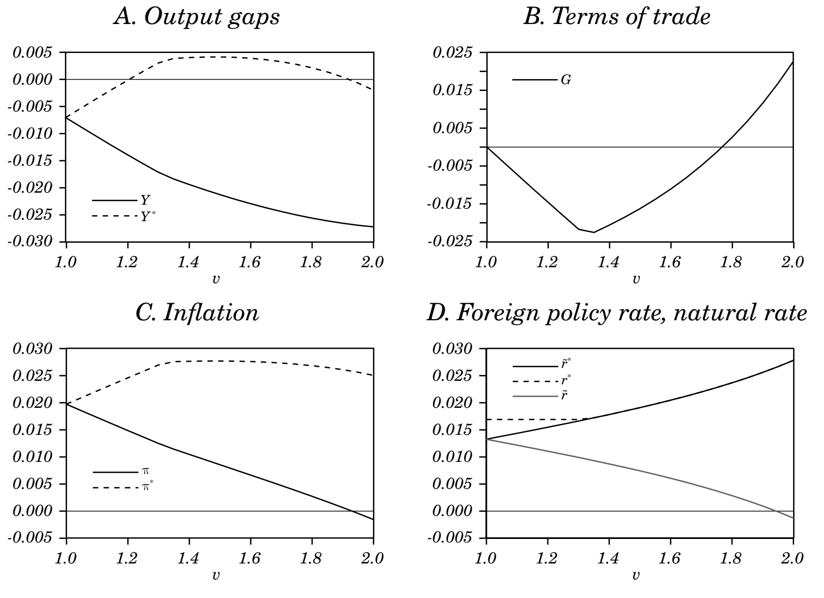
Thus, the fiscal multiplier is unambiguously larger in the open economy than in a closed economy! This is clearly consistent with our result that in the open economy, the fiscal spending shock generates a terms-of-trade deterioration, and crowds in spending from the rest of the world, while causing a fall in foreign output. As a result, paradoxically, for one country on its own, it requires a smaller fiscal expansion to offset a liquidity trap shock when the economy is more open than in a fully closed economy. Fiscal spending shocks in a liquidity trap involve a type of "reverse leakage."

3.1.3 Zero lower bound constraint in one country

So far, we have assumed that both countries are simultaneously constrained by the zero bound, and contrasted this case with one where monetary policy operates actively according to a Taylor rule. However, when less than fully open trade and asymmetric incidence of shocks exist, figure 1 shows that natural interest rates generally differ between countries. For a home country negative demand shock, there is a potentially large range of the v parameter where the foreign natural rate is above zero, and thus, if monetary policy follows the rule in (9), the foreign country will no longer be constrained by the zero bound. The question is, if the foreign country follows such a rule, how will a home country demand shock be transmitted through international goods and capital markets, and how will this in turn affect the response of the home country to the shock?

Figure 2 illustrates the relationship between the parameter v, the home and foreign natural interest rates, the home and foreign output gaps, and the terms of trade, when the foreign monetary authority follows the rule in (9). The figure is drawn such that for $v = 1$, the demand shock is large enough so that the natural interest rate (common across countries) is negative, and so both countries are in a liquidity trap. For $v \leq \bar{v}$, the results are identical to the analysis above, so that the home output gap falls more than the foreign output gap as the home terms of trade appreciates. But

Figure 2. ZLB in One Country

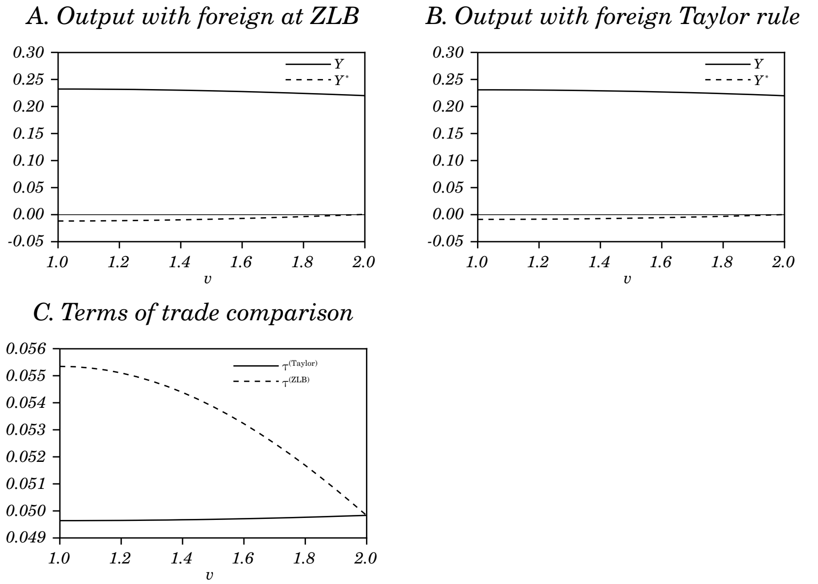


Source: Authors' elaboration.

for $v > \bar{v}$, the foreign country raises its policy rate since its natural interest rate is above zero. This monetary tightening has the effect of reversing the relationship between v and the terms of trade –the terms of trade now tend to depreciate (or strictly speaking, the appreciation is tempered) as v rises, and the foreign country raises its policy rate more and more. As a result, the fall in home country output is reduced. Note also, in this case, that foreign output rises in response to the home negative demand shock for high levels of v driven by the foreign terms of trade deterioration. So the foreign monetary tightening has the effect of reducing inflation and creating a positive output gap in the foreign country.

How do the results for the fiscal policy multiplier change when one country follows an activist monetary policy? For this comparison, we go back to the situation where countries follow a simple Taylor rule for monetary policy if the natural interest rate is positive—in contrast to the policy in (9). Again, we assume that the government spending gap reverts back to zero with probability $1 - \mu$ in each

Figure 3. Government Spending Shock



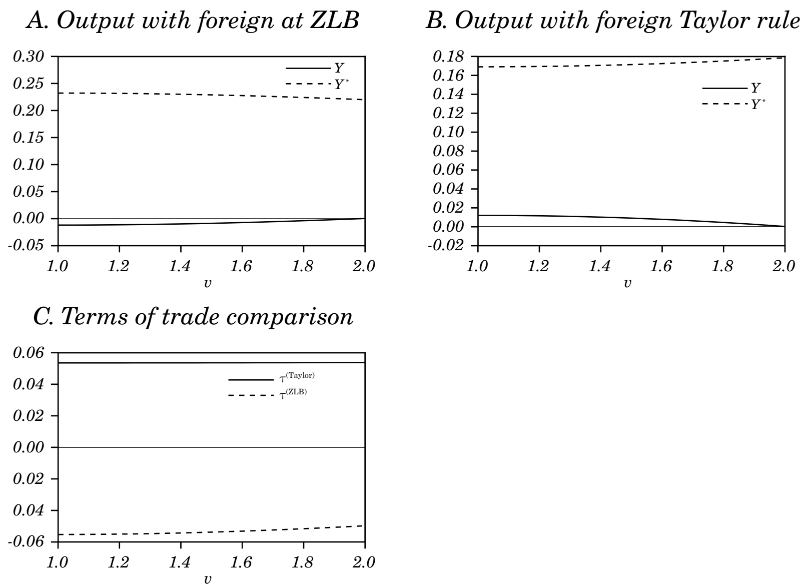
Source: Authors' elaboration.

period. Figure 3 compares the results for the effect of government spending on the home and foreign output gap, and the terms of trade, in the original case where both countries are constrained by the zero bound, and the case where the foreign country is unconstrained and follows a Taylor rule. The own multiplier in the home country is reduced by the endogenous interest rate response of the foreign economy.

Intuitively, this is because the foreign country tends to reduce interest rates, as cross-country fiscal multiplier is still negative (panel B of figure 3). This reduction in foreign interest rates lessens the response of the terms of trade (panel C of figure 3). It is possible to show that the home fiscal spending multiplier is still above unity, but it is clearly smaller than the case with a global liquidity trap.

Finally, figure 4 illustrates the effect of foreign government spending shocks when the foreign country follows an activist Taylor rule, but the home country is constrained by the zero bound on interest rates. Here we find that the spillover of foreign fiscal spending on the home output gap is positive—this is because the foreign spending

Figure 4. Government Spending in Foreign



Source: Authors' elaboration.

shock causes a terms-of-trade appreciation for the foreign economy when it follows a Taylor rule. Hence, we have a situation where spending expansion in the country constrained by the zero bound is highly expansionary with a multiplier above unity, but the spending expansion has a beggar-thy-neighbor impact. But spending shocks in the country unconstrained by the zero bound has a significantly smaller own multiplier, but a positive cross country impact.

3.2 Savings Shocks and the Liquidity Trap without Capital Mobility

A major element of the transmission mechanism for the demand shock in the previous section was the presence of fully open capital markets. Now we turn to the opposite case, where $\lambda = 1$, so that there is no capital mobility, and the transmission of shocks only takes place through goods market trade. In this case, as we saw from above, natural interest rates diverge between countries, even when $v = 1$. In fact, when $\lambda = 1$, natural interest rates move in opposite directions across countries in response to a demand shock emanating from one country alone. It should not therefore be surprising to see that demand shocks generate a negative correlation in output gaps across countries.

To see this, take the case again where neither country is constrained by the zero bound, and the Taylor rule for monetary policy is in effect. Also, to make the algebra more simple, take the $v = 1$ case. Then consequent on a demand shock in the home country, the movement in output gaps is given by

$$\hat{y}^{r}_{t(\text{Taylor})} = \left[\frac{1}{(\sigma+\phi c_y)\Delta} + \frac{1}{(D_1+\phi c_y)(\Delta_4)}\right]\frac{(1-\mu)(1-\beta\mu)\phi c_y^2 \varepsilon_t}{2} \tag{43}$$

$$\hat{y}^{*r}_{t(\text{Taylor})} = \left[\frac{1}{(\sigma+\phi c_y)\Delta} - \frac{1}{(D_1+\phi c_y)(\Delta_4)}\right]\frac{(1-\mu)(1-\beta\mu)\phi c_y^2 \varepsilon_t}{2} \tag{44}$$

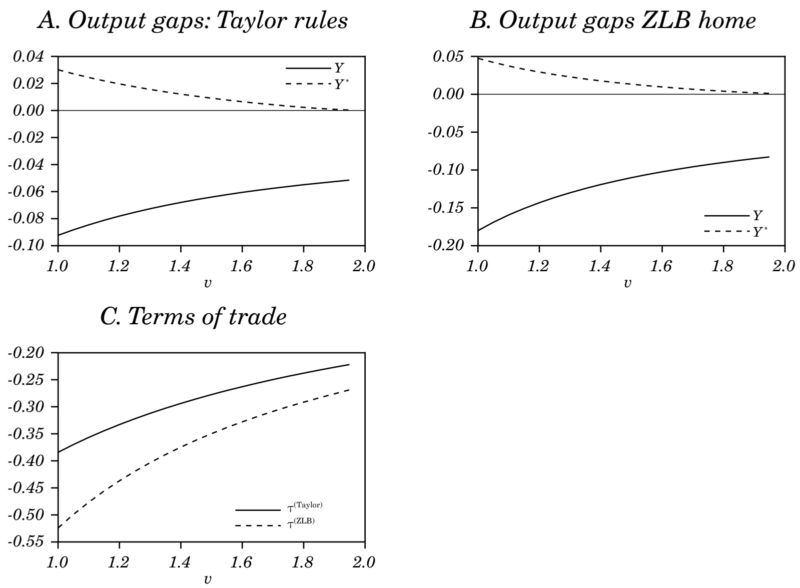
Where $D_1 \equiv 2 - v + \sigma(v-1)$ and $\Delta_4 \equiv D_1(1-\mu)(1-\beta\mu) + \kappa(\gamma_1 - \mu)(D_1 - \phi c_y)$.

When $\sigma > 1$, as is assumed, it is easy to show from (43) and (44) that $y_{t(Taylor)}$ is positive and $\hat{y}^{*}_{t(Taylor)}$ is negative. Thus, a demand shock causes a negative co-movement in output across countries when capital mobility is absent, even if preferences are identical. The key reason is that in the absence of capital mobility, the terms of trade responds in a different direction to a country specific demand shock. Recall that, with capital mobility and monetary policy governed by a Taylor rule, a positive home demand shock caused a rise in domestic inflation, a rise in the real interest rate and a terms-of-trade appreciation. But without capital mobility, the terms of trade is not governed by an interest parity condition across countries. Combining the goods market clearing conditions, and the home budget constraint in the absence of capital mobility, we may show that the terms of trade is determined by the simple condition:

$$\tau = \frac{(y_t - (1-c_y)g_t) - (y_t^* - (1-c_y)g_t^*)}{c_y}.$$

So the terms of trade is determined simply by the ratio of net output responses (net of government spending). A home demand shock that raises home output generates a terms-of-trade deterioration.

Figure 5. No Capital Mobility



Source: Authors' elaboration.

This redirects demand away from foreign goods, and leads to a fall in the foreign output gap.

Now focus on the case of a negative demand shock which leads to a binding zero-bound constraint (in the home country). Since (22) and (23) tell us that natural interest rates move in different directions across countries in this case, it must be that the foreign country is unconstrained by the zero bound. As before, we assume that the foreign country follows the rule in (9). Figure 5 illustrates the results for the movement of home and foreign output gaps as a function of v. Again, we find a negative co-movement across countries, and a terms-of-trade appreciation for the home country. The fall in output in the home country is greater than that when the Taylor rule is operative, for the same reason as before: a negative demand shock tends to persistently reduce inflation, raise real interest rates in the domestic country, and compound the effect of the initial shock. But in a qualitative sense, the international transmission with restricted capital mobility and the zero-bound constraint is similar to that when monetary policy is governed by a Taylor rule. In particular, the movement in relative prices is not reversed by the presence of the zero-bound constraint, as in the case of perfect capital mobility.

Finally, we turn to the evaluation of fiscal multipliers and cross country transmission of fiscal policy in the economy without capital mobility. Again, first, look at the effects of a home country fiscal expansion when both countries follow a Taylor rule. It may be shown the impact of a fiscal spending increase on home and foreign output is given by:

$$\hat{y}^{g}_{t(\text{Taylor})} = \frac{\sigma}{2}(1-c_y)\frac{\left[\Delta_4 + \Delta(2-\upsilon+\sigma(\upsilon-1))\right]\Theta}{\Delta\Delta_4}\hat{g}_t \tag{45}$$

$$\hat{y}^{*g}_{t(\text{Taylor})} = \frac{1}{2}(1-c_y)\frac{\kappa(2-\upsilon)\phi c_y(\sigma-1)(\gamma-\mu)\Theta\hat{g}_t}{\Delta\Delta_4}$$

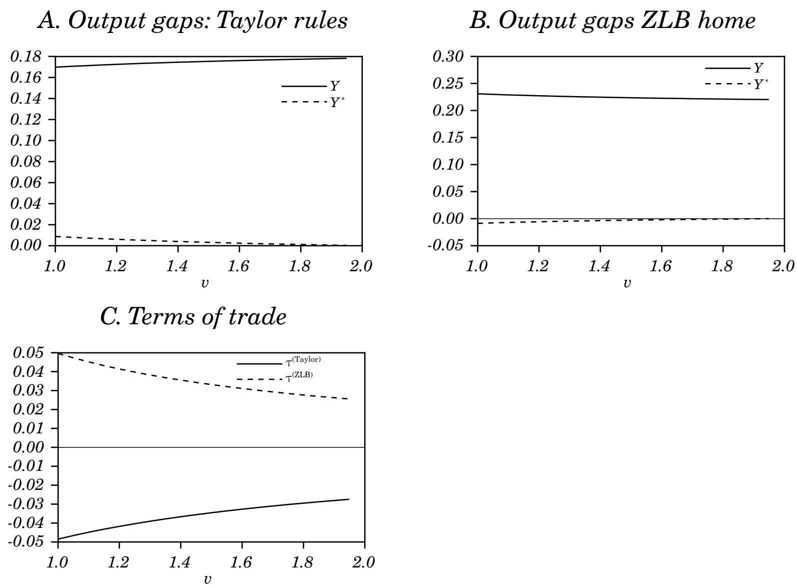
where $\Delta_4 \equiv (2-\upsilon+\sigma(\upsilon-1))\Theta + \kappa c_y \phi(\gamma-\mu)$. As in the case of perfect capital mobility, when monetary policy is operative, and governed by a Taylor rule, the home spending multiplier is positive, and the cross country spending multiplier is positive as well. The latter result is tied again to the terms of trade response. The impact of a home spending shock on the terms of trade is given by

$$\tau_{t(\text{Taylor})} = -\frac{(1-c_y)(\gamma-\mu)\phi\kappa}{\Delta_4}g_t \tag{46}$$

Hence, the home terms of trade appreciates in response to a government spending shock in the home country, as in the case of perfect capital mobility. Since the terms of trade is governed by condition (4.2), this also implies that net output $y_t - (1-c_y)g_t$ falls in response to a government spending shock, so the multiplier in this case is also less than unity.

How does this compare to the case where the home country is constrained by the zero bound? Recall that without capital mobility, both countries cannot be constrained by the zero bound at the same time. So for this case, we look at the response to a home country fiscal spending shock assuming that the foreign country monetary policy is constrained and follows a Taylor rule. Figure 6 illustrates this case, and for completeness, compares the case just examined (with both countries unconstrained) to the present case, where the home country is at the zero bound.

Figure 6. Government Spending, No Capital Mobility



Source: Authors' elaboration.

The results illustrate a notable parallel to the case with full capital mobility. While during normal monetary policy, a fiscal expansion leads to a terms of trade appreciation and a rise in domestic and foreign output, with the home country constrained by the zero bound; the spending shock leads to a terms of trade depreciation, a rise in home output, and a fall in foreign output. Hence, just as in the case of full capital mobility, fiscal spending has a beggar-thy-neighbor effect in a liquidity trap. It has this characteristic due to the reversed effect on relative prices at the zero bound. The fiscal spending shock generates a terms-of-trade depreciation, reducing demand in the foreign economy and reducing foreign output.

Figure 6 shows that, in the liquidity trap, the effect of openness to trade is also perverse without capital mobility. With operative monetary policy, openness reduces the impact of the spending shock on home country output. In contrast, increased openness tends to magnify the impact of the spending shock on output when the home country is constrained by the zero bound. Again, this is because increased openness magnifies the perverse relative price response, increasing demand for the home good and increasing the fiscal spending multiplier.

As before, we can also infer that, because net output $y_t - (1 - c_y)g_t$ is increased in response to the spending expansion, the fiscal spending multiplier is greater than unity in the liquidity trap, without capital mobility.

In the previous section, we argued that the perverse price responses to shocks in the liquidity trap were linked to the interest rate parity condition, which must hold in the environment of perfect capital mobility. Here, there is no such condition. As a result, we did not find a reversal of pricing responses to a demand shock. But in the case of the fiscal spending shock there is still a pricing response reversal. Here, it is tied to the fact that with open trade in goods, the terms of trade is pinned down by relative net outputs, and with a liquidity trap, a spending expansion can increase relative net home output—while outside this situation, the spending expansion causes a fall in relative home net output. Hence, a reversal of the terms of trade response is possible, even in the absence of full capital mobility.

4. Optimal Monetary and Fiscal Policy in a Liquidity Trap

In this section, we explore the determination of optimal policy in face of large preference shocks which push one or both countries against the zero lower bound in interest rates. We do so following the analysis of Jung et al. (2005) or Eggertsson and Woodford (2003), by looking at the optimal policy that minimizes a quadratic approximation to expected utility. In our environment, there are two countries, so the question of optimal policy naturally raises the question of whose welfare is maximized. We follow the recent open economy literature (e.g. Engel (2010), by assuming that policy is set in a cooperative fashion. The determination of optimal policy in a non-cooperative, Nash equilibrium is an important issue, but it raises technical complications which are beyond the scope of this survey[9]. In addition, we assume that policy is set under discretion so that monetary authorities cannot credibly commit to future paths of interest rates.

9. See Cook and Devereux (2013) for an example of a non-cooperative Nash equilibrium in a model where the zero-bound constraint binds.

4.1 Optimal Policy with Complete Capital Mobility

We first explore the determination of optimal monetary and fiscal policy under complete capital mobility, so that $\lambda = 0$. The period welfare function approximation for this case is shown in Cook and Devereux, (2011) to be

$$V_t = V_{(v=1),t} - \frac{1}{2}(\hat{y}_t - \hat{y}_t^*)^2 \Gamma_1 - \frac{1}{2}(\hat{g}_t - \hat{g}_t^*)^2 \Gamma_2 + ((\hat{y}_t - \hat{y}_t^*)(\hat{g}_t - \hat{g}_t^*)\Gamma_3 \quad (47)$$

Where:

$$V_{(v=1),t} = -(\hat{y}_t - \hat{y}_t^*)^2 \frac{(1+\phi c_y)}{2c_y^2} - (\hat{n}_t + \hat{n}_t^*)^2 \frac{(\sigma + \phi c_y)}{2c_y^2} \quad (48)$$

$$-(\hat{g}_t - \hat{g}_t^*)^2 \frac{(1-c_y)((1-c_y)+c_y\sigma)}{2c_y^2} - \frac{(1-c_y)}{2c_y^2}(\sigma(\hat{g}_t + \hat{g}_t^*)^2$$

$$-2((\hat{y}_t - \hat{y}_t^*)(\hat{g}_t - \hat{g}_t^*) + \sigma(\hat{y}_t + \hat{y}_t^*)(\hat{g}_t + \hat{g}_t^*)) - \frac{\theta}{k}\pi_{Ht}^2 - \frac{\theta}{k}\pi_{Ft}^2$$

Where

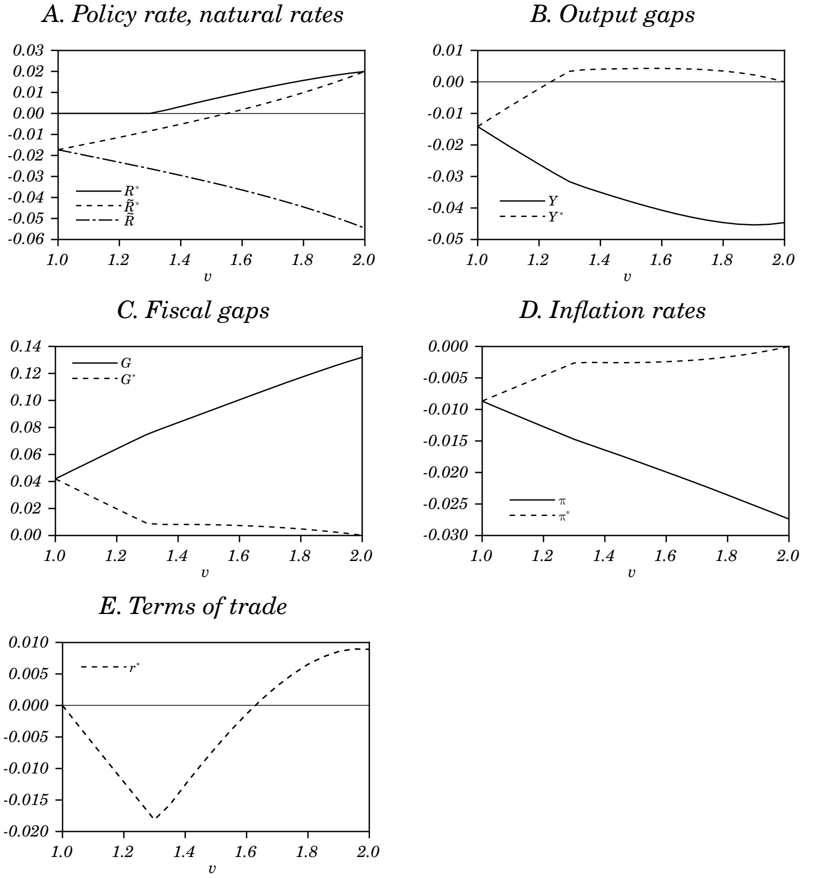
$$\Gamma_1 = \frac{(v-1)^2(\sigma-1)}{D}(1 + \frac{(1-c_y^2)}{c_y^2 D}),$$

$$\Gamma_2 = \frac{(1-c_y)^2(v-1)^2(\sigma-1)}{c_y^2 D^2}(1 + v(v-1)(\sigma-1)(2-v)c_y^2),$$

and $\Gamma_3 = \frac{(1-c_y)(v-1)^2(\sigma-1)}{c_y^2 D^2}(1 + v(v-1)(2-v)(\sigma-1)c_y^2).$

Since the nature of the shocks is such that the response of all macro variables is non-time varying, as long as the shock persists, we can characterize policy in terms of the response of interest rates and fiscal policy gaps following the shock, assuming that policy takes on the persistence characteristics of the shock—i.e. the optimal interest rate and fiscal spending gap response is constant, as long as the shock persists. Figure 7 describes the response of the foreign

Figure 7. Optimal Policy



Source: Authors' elaboration.

country interest rate, the fiscal spending gap, the output gaps, the terms of trade, and inflation rates in the two countries, which represents an optimal policy response to the shock. We assume that the shock emanates solely from the home country, and that the shock is such that the home country is always constrained by the zero bound. It is easy to show that for a large enough shock, the home country will always optimally set its policy interest rate to zero. The figure describes the response of home and foreign variables as a function of the home bias parameter v. Recall that in the case

$v = 1$, and with full capital mobility, the countries have identical natural interest rates so that the policy response must be identical across countries. Hence, with $v = 1$ and $\lambda = 0$, both countries must be constrained by the zero bound, under an optimal policy response. As before, the solutions for policy responses and all other variables are stationary.

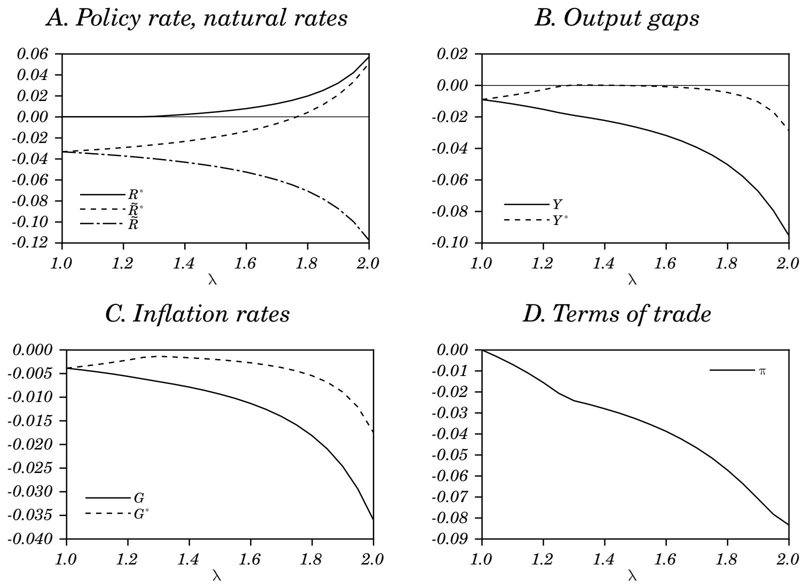
The top left panel of figure 7 illustrates the optimal foreign country policy interest rate as a function of v. For $v \leq \nu_L$, the foreign policy rate is constrained at zero. For $v > \nu_L$, the policy rate is positive, and interestingly, is above the foreign natural interest rate[10]. The top right panel describes the consequent values of home and foreign output. While for $v = 1$, home and foreign output are identical; the output responses diverge sharply as v rises above unity, since, as shown above, this involves a sharp terms of trade appreciation for the home country. As v rises above ν_L, the rise in the foreign policy rate reverses the appreciation of the terms of trade, and the rise in foreign output is restricted, as is the fall in home output. The figure also shows the optimal response of the fiscal spending gap. For no home bias, it is optimal for both home and foreign country fiscal spending gaps to rise. Then, as v rises, the foreign spending gap falls and the home spending gap rises even further. Hence, we see that the optimal policy response to a liquidity trap shock emanating from the home economy may involve a monetary policy tightening in the foreign economy, and a joint policy of fiscal policy expansion in both countries. The final panels of the figure show the response of the terms of trade and inflation rates. The terms of trade sharply appreciates initially in v, but as is clear from the figure, for $v > \nu_L$, the foreign country's monetary policy turns around the terms of trade appreciation. In addition, we find that it is optimal for the foreign country to experience a small deflation, even for $v > \nu_L$.

4.2 Optimal Policy with Limited Capital Mobility

We now turn to the case of $\lambda > 0$, where capital mobility is restricted as described above. Since the results for fiscal policy response are similar to that in the last subsection, we focus only on the optimal discretionary monetary policy response of the foreign

10. See Cook and Devereux (2013) for an explanation of this. It is noteworthy that these results show that, in general, the rule in (9) is not an optimal cooperative rule for monetary policy when one country is constrained by the zero bound.

Figure 8. Optimal Policy, No Capital Mobility



Source: Authors' elaboration.

government in the face of a shock to demand emanating from the home country. In addition, rather than focusing on the impact of the home bias coefficient v, we instead fix v and look at the optimal response to the shock for different values of λ. As before, we focus on a situation where the home country is always constrained by the zero bound, and we look at responses from the foreign country. From the results above, we know that for a higher and higher λ, the impact of the home country shock on natural interest rates and output gaps diverge between countries more and more, so we should anticipate that optimal policy responses would also diverge. This is what we find.

Figure 8 describes the optimal response of interest rates, output gaps, inflation rates, and terms of trade for the two countries as a function of λ. For ease of interpretation, we focus on the case where $v = 1$, so that in the case $\lambda = 1$, all realizations should be the same in each country. From the first panel of figure 8, we see that the optimal policy rate for the foreign country will be zero for $\lambda < \lambda_H$. After this point, the foreign country will raise its policy rate. The second panel of the figure shows that output gaps begin to diverge as λ increases, but the rise in the policy rate at $\lambda = \lambda_H$ limits this output gap. It does so by limiting the terms-of-trade appreciation of the home country, as illustrated in the fourth panel of figure 8.

These results indicate that limited capital mobility may have a large effect on the degree to which demand shocks generate "global liquidity traps," but also have an important implication for the optimal policy responses to such shocks.

5. Conclusions

This paper has explored the international transmission of shocks in an environment where the zero lower bound may be binding on one or more countries. We showed that the nature of transmission sensitively depends on the degree of trade openness and the degree of asset market completeness. When trade is fully open and asset markets are complete, then all liquidity traps are global; if the zero lower bound binds in one country then it will generally bind. But with less than fully open trade, shocks are only incompletely transmitted, and the country which is the source of the shock will be more likely to hit the zero lower bound. Even with fully open trade, incomplete assets markets also reduce the transmission of shocks, and with financial market autarky, we show that the zero lower bound cannot hold in both countries simultaneously.

The paper shows that the transmission of shocks in the zero lower bound is associated with perverse response of relative prices: the worst hit country tends to suffer terms-of-trade appreciation, rather than depreciation, thus exacerbating the effects of the shock. In a liquidity trap, fiscal expansion can be extremely effective in raising economic activity. But it does so through reducing the terms of trade, and redirecting spending away from trading partners, thus reducing trading partners GDP. Thus fiscal spending is a beggar-thy-neighbor policy in a liquidity trap. This result holds both in a complete and incomplete asset market environment.

Finally, we studied an optimal cooperative policy response to the shock which generates the liquidity trap. The optimal response involves a joint policy of fiscal expansion and potential policy rate increases for the least hit country. This surprising result comes from the fact that policy rate increases can ameliorate the perverse response of relative prices to the liquidity trap shock.

References

Auerbach, A. and M. Obstfeld. 2006. "The Role for Open Market Operations in a Liquidity Trap." *American Economic Review* 95(1): 110–37.

Bodenstein, M., C.J. Erceg, and L. Guerrieri. 2009. "The Effect of Foreign Shocks When Interest Rates are at Zero." IFdp 983, Board of Governors of the Federal Reserve System.

Christiano, L., M. Eichenbaum, and S. Rebelo. 2011. When is the Government Spending Multiplier Large? *Journal of Political Economy* 119(1): 78–121.

Clarida, R., J. Galí, and M. Gertler. 2002. "A Simple Framework for International Monetary Policy Analysis." *Journal of Monetary Economics* 49(5): 879–904.

Cogan, J., T. Cwik, J. Taylor, and V. Wieland. 2009. "New Keynesian Versus Old Keynesian Government Spending Multipliers." NBER Working Papers 14782.

Cook, D. and M.B. Devereux. 2011. "Optimal Fiscal Policy in a World Liquidity Trap." *European Economic Review* 55: 443–62.

Cook, D. and M.B. Devereux. 2013. "Sharing the Burden: Monetary and Fiscal Policy Responses to a World Liquidity Trap." *American Economic Journal: Macroeconomics* 55(4): 443–62.

Devereux, M.B. 2010. "Fiscal Deficits, Debt, and Monetary Policy in a Liquidity Trap." Working Paper No.581, Central Bank of Chile.

Devereux, M.B. and J. Yetman. 2013. "Capital Controls, Global Liquidity Traps, and the International Policy Trilemma." *Scandinavian Journal of Economics* 116(1): 158–89.

Engel, C. 2011. "Currency Misalignments and Optimal Monetary Policy: A Reexamination." *American Economic Review* 101(6): 2796–2822.

Eggertsson, G. 2010. "What Fiscal Policy is Effective at Zero Interest Rates?" In NBER *Macroeconomics Annual* 2010 25:59–112.

Eggertsson, G. and M. Woodford. 2003. "The Zero Interest Bound and Optimal Monetary Policy." *Brookings Papers on Economic Activity* 1: 139–211.

Eggertsson, G. and M. Woodford. 2005. "Optimal Monetary and Fiscal Policy in a Liquidity Trap." ISOM Annual.

Fujiwara, I., T. Nakajimaz, N. Sudo, and Y. Teranishi. 2010. "Global Liquidity Trap." Manuscript, Bank of Japan.

Fujiwara, I., N. Sudo, and Y. Teranishi. 2009. "The Zero Lower Bound and Monetary Policy in a Global Economy: A Simple Analytical Investigation." *International Journal of Central Banking* 6(1): 103–33.

Fujiwara, I. and K. Ueda. 2013. "The Fiscal Multiplier and Spillover in a Global Liquidity Trap." *Journal of Economic Dynamics and Control* 37(7): 1264–1283.

Jeanne, O. 2009. "The Global Liquidity Trap." Manuscript.

Krugman, P. 1998. "It's Baaack: Japan's Slump and the Return of the Liquidity Trap." *Brookings Papers on Economic Activity* 2: 137–87.

Jung, T., Y. Terinishi, and T. Watanabe. 2005. "The Zero Bound on Nominal Interest Rates and Optimal Monetary Policy." *Journal of Money Credit and Banking*.

Svensson, L.E.O. 2003. "Escaping from a Liquidity Trap and Deflation: The Foolproof Way and Others." *Journal of Economic Perspectives* 17: 145–66.