

An Estimated Stochastic General Equilibrium Model with Partial Dollarization: A Bayesian Approach*

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Abstract

This paper develops and estimates a dynamic stochastic general equilibrium New Keynesian model of a small open economy with partial dollarization. We use Bayesian techniques and Peruvian data to evaluate two forms of dollarization: currency substitution (CS) and price dollarization (PD). Our empirical results are as follows. First, we find that the two forms of partial dollarization are important to explain the Peruvian data. Second, models with both forms of dollarization dominate models without dollarization. Third, a counterfactual exercise shows that by eliminating both forms of partial dollarization the response of both output and consumption to a monetary policy shock doubles, making the interest rate channel of monetary policy more effective. Forth, based on the variance decomposition of the preferred model (with CS and PD), we find that demand type shocks explain almost all the fluctuation in CPI inflation, being the monetary shock the most important (39 percent). Remarkably, foreign disturbances account for 34 percent of output fluctuations.

Keywords: Bayesian Estimation, DSGE, Partial Dollarization

JEL Classification: F31;F32;F41; C11.

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1 Introduction

A distinct feature of economies with history of high inflation is the persistent partial use of a foreign currency by domestic agents. This characteristic, known in the literature as partial dollarization, is present in many former high-inflation economies even after several years of achieving low and stable inflation. Partial dollarization is defined as the partial replacement of the domestic currency by a foreign currency (i.e US dollars) in its basic functions. In this context, partial dollarization can be classified in three types: a) Transaction dollarization, also known as currency substitution. In this case dollars are accepted as medium of payment, b) price dollarization. In this case prices are indexed to changes in the exchange rate and, c) financial dollarization. In this case dollars are preferred as store of value.

Recently, a series of papers have used dynamic stochastic general equilibrium models (DSGE) to address the implications of different forms of partial dollarization for the transmission mechanism of monetary policy. DSGE models are useful to study this issue since they make explicit the operating mechanisms of dollarization. Moreover, this type of models are suitable for policy-simulation exercises because they are robust to the Lucas's critique. Cespedes et al (2005) and Gertler et. al. (2006) study the implications of financial dollarization by introducing the financial accelerator mechanism into a small open economy model. Their main finding is that a policy of fixed exchange rates can exacerbate financial distress because it obliges monetary authorities to raise domestic interest rates during such contractionary episodes. Castillo (2006), Felices and Tuesta (2006), and Batini, et. al. (2006) analyze the role of transaction dollarization in DSGE models. They show how is more difficult, for a central bank, to stabilize both inflation and output gap under this environment¹. Regarding price dollarization, Castillo and Montoro (2004) and Ize and Parrado (2005) analyze the link between price dollarization and financial dollarization. More recently, Castillo (2006b) studies the effects of price dollarization in economies with both

¹Castillo (2006) and Felices and Tuesta (2006) build up small open economy with currency substitution showing the limitations of the central bank in stabilizing inflation and the output gap in this environment. Battini et. al. (2006) analyze the rational expectations determinacy under interest rate monetary policy rules in economies with CS. They find that conditions for determinacy of the rational expectation equilibrium are more difficult to meet when CS is present.

sector specific shocks and sticky prices.

In spite of the importance of partial dollarization for emerging economies, the literature has not yet provided empirical assessments of the implications of different types of partial dollarization. Thus, this paper tries to contribute in this line by *estimating* and *modeling* a DSGE small open economy with different forms of partial dollarization². At this first stage of the project we focus our estimation on the first two forms of partial dollarization³: Currency Substitution (CS) and Price Dollarization (PD). The distinction between CS and PD is important since they affect the economy through different channels. Several questions emerge once these forms of partial dollarization are considered. How do CS and PD affect the dynamics of the small open economy?. Are the quantitative effects of CS and PD in terms of output and inflation fluctuations important?. Our model strategy shows that CS amplifies the effect of foreign interest rate over both consumption and output, hence it weakens the interest rate channel of monetary policy. Instead, PD mainly affects inflation dynamics. In particular, it makes domestic inflation more sensitive to the nominal exchange rate depreciation because prices are sticky in the foreign currency.

On the methodological front, we use Bayesian techniques with data for Peru to estimate and to compare different models that feature CS, PD, and both. This technique is appealing for two reasons. First, by combining prior information with the likelihood function provided by the data, Bayesian techniques allows us to characterize the posterior distribution of the parameters related with dollarization, and at the same time to take into account the uncertainty nearby other model's coefficients. Second, Bayesian evaluation allows a consistent comparison among models taking advantage of the general equilibrium approach⁴. This comparison is one of the main objectives of the paper, and the Bayesian methodology is perfectly suited for it⁵. Yet, we do not base our analysis solely in model comparison but rather complement this approach by

²We are not aware of any formal work comparing DSGE models with different forms of partial dollarization. Tovar (2005, 2006) and Elekdag et al.(2006) have recently evaluated the role of balance sheet effects for emerging market countries.

³We are currently working on a version that includes also financial dollarization.

⁴Estimation of reduced-forms or partial equilibrium models might suffer from serious identification problems.

⁵Fernández-Villaverde and Rubio-Ramirez (2004) show that, even in the case of misspecified models, Bayesian estimation and model comparison are consistent.

contrasting the implied second moments among models.

Our model is generalization of a standard DSGE for a small open economy as in Sutherland (2001) and Galí and Monacelli (2005) . In addition to aforementioned dollarization mechanisms, we depart from their framework by considering incomplete markets, slow adjustment of real wages, as in Blanchard and Galí (2005), intermediate degree of pass-through as in Lubik and Schoerfeide (2005), and external habits in consumption and capital adjustment costs, as in Christiano, Eichenbaum and Evans (2001).

The main insights of the paper are the following. First, adding CS and PD dollarization to a standard small open economy model improves the fit of the Peruvian data. This result holds because: first, introducing CS reduces the output response to domestic interest rate and therefore, it better fits consumption volatility, and second, introducing PD adds an additional Phillips curve that generates endogenous persistence in the economy and also increases the sensitivity of inflation to exchange rate movements.

Second, regarding the estimated parameters, we find that the interest rate feedback-rule of the central bank assigns a large weight to both CPI inflation and the exchange rate depreciation, and a very low reaction to output growth rates. The estimation also indicates a very small degree of interest rate smoothing for the Peruvian economy, and a relatively low degree of price stickiness in both the domestic and imported sector, compared to developed economies. We also find that real frictions such as habit formation and real wage rigidities are relevant in the Peruvian data.

Third, the estimated volatility of structural shocks are larger than those estimated for developed economies. Fourth, variance decomposition analysis based on the model with both CS and PD indicates a major role of demand-type shocks, in particular monetary shocks, in explaining CPI inflation. The same analysis shows that supply shocks explain most of output fluctuations. However, none of the models is very successful at matching simultaneously unconditional volatilities of macro aggregates and international relative prices. Finally, the inclusion of a Purchasing Power Parity shock to the preferred model improves the fit, making the estimated volatility of the endogenous variables closer to that of the data.

Recently it has been an outburst of papers that estimate DSGE models for closed and open economies using Bayesian methods. Our estimation follows previous studies that use this technique. For instance, Smets and Wouters (2004) and Rabanal and Rubio (2005) have performed Bayesian estimation for closed economies. Adolfson et al. (2005), Justiniano and Preston (2006), and Lubik and Schorfheide (2006) estimate small open economy models, whereas Lubik and Schorfheide (2005), Batini et al. (2005), de Walque, Smets and Wouters (2005) and Rabanal and Tuesta (2006) estimate two-country models using U.S and Euro area data. This paper differs from previous papers in the literature mainly because it applies Bayesian methods to estimate the effects of different forms of partial dollarization in a small open economy using Peruvian data.

The remainder of the paper proceeds as follows. Section 2 lays out the benchmark model. Section 3 discusses the main extensions namely CS and PD. Section 4 provides the data and the estimation methodology. In section 5, we report the results on estimated parameters, model comparison, and variance decomposition of the preferred model. Section 6 presents some robustness exercises. Finally, section 7 concludes.

2 The Model

We depart from a two-country model following the work of Sutherland (2001). We model the small open economy by taking the size of one these economies close to zero⁶. Unlike the previous author we add capital accumulation and several frictions in order to fit better the data. Thus, the benchmark set up considers households who consume final goods, supply labor to intermediate goods producers and save using one period nominal zero coupon bonds in both domestic and foreign currency. Firms produce intermediate goods, final goods, and capital goods. Investors decide how much new capital to accumulate. International trade consists on exports and imports of final goods.

Additionally, we allow for a series of nominal and real rigidities necessary to capture the

⁶De Paoli (2006), Castillo (2006), and Felices and Tuesta (2006) adopt the same strategy to analyze issues on monetary policy in small open economies. Galí and Monacelli (2005) model a small open economy by considering the world economy is determined by a continuum of small open economies.

dynamics of the data. More precisely, the model exhibits the following features: external habits in consumption, adjustment costs in capital accumulation, slow adjustment in real wages to capture real distortions in the labor market, nominal rigidities in the form of staggered price setting in the final goods production sector, price indexation, and incomplete pass-through from exchange rate to imported good prices as in Monacelli (2003).

The benchmark set up is later extended by introducing two different forms of dollarization: CS and PD. We model CS by considering that both domestic and foreign real money balances generate utility to domestic agents. Since both currencies can be used as medium of payment, the optimal composition of household money holdings generates a link between the foreign nominal interest rate and consumption and labor supply decisions. We introduce PD by assuming that a subset of domestic firms pre-set prices in a foreign currency (i.e dollars).

The model contains 8 shocks. One permanent technology shock that has a unit root, and seven AR(1) stationary shocks, technology, domestic inflation mark-up, intermediate imported mark-up, monetary, preference, foreign monetary policy and UIP. We further perform sensitivity analysis by adding a PPP shock (purchasing power parity shock) to the preferred model.

2.1 Preferences

The world economy is populated by a continuum of household of mass 1, where a fraction n of them is allocated in the home economy, whereas the remaining one in the foreign economy. Each household j at the home economy enjoys utility from the consumption of a basket of final goods, C_t^j and receives disutility from working. The households preferences are represented by the following utility function:

$$U_t = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left[U \left(C_t^j, \xi_t \right) - V \left(L_t^j \right) \right] \right], \quad (2.1)$$

where E_0 denotes the conditional expectation on the information set at date $t = 0$, and β is the intertemporal discount factor, with $0 < \beta < 1$. C_t^j and L_t^j denote the level of consumption and labor supply of agent j at period t . We choose the following particular functional form for the

utility function,

$$U(C_t^j) = \log \left[\xi_t (C_t^j - hC_{t-1}) \right], V(L_t^j) = \frac{(L_t^j)^{1+\eta}}{1+\eta} \quad (2.2)$$

Also, preferences on consumption exhibit external habit formation. The level of the marginal utility of consumption is decreasing not on C_t^j , but on the difference between this variable and the aggregate consumption level of the previous period, C_{t-1} , where $h \in [0, 1]$ denotes the importance of the habit stock. Thus, households enjoy more utility as their consumption level increases with respect to their habits. The presence of habit formation on New Keynesian models has been reported to help capturing the dynamic response of output to monetary policy shocks and in generating persistence on consumption⁷. The disutility that labor generates is captured by a isoelastic function, where $\eta > 0$ is the inverse elasticity of labor supply with respect to the real wage. Finally ξ_t is a domestic preference shock that follows an $AR(1)$ process in logs

$$\log \xi_t = \rho_\xi \log \xi_{t-1} + \mu_t^\xi$$

The consumption basket of final goods is a composite of domestic and foreign goods, aggregated using the following consumption index

$$C_t \equiv \left[\gamma^{1/\varepsilon_H} (C_t^H)^{\frac{\varepsilon_H-1}{\varepsilon_H}} + (1-\gamma)^{1/\varepsilon_H} (C_t^M)^{\frac{\varepsilon_H-1}{\varepsilon_H}} \right]^{\frac{\varepsilon_H}{\varepsilon_H-1}}, \quad (2.3)$$

where ε_H is the elasticity of substitution between domestic (C_t^H) and foreign (C_t^M) goods, and γ is the share of domestically produced goods in the consumption basket of the small open economy. In turn, C_t^H and C_t^M are indexes of consumption across the continuum of differentiated goods produced in home country and imported from abroad, respectively. These consumption indices

⁷Also, models with external habit formation have proven to be useful in accounting for asset prices empirical regularities. For instance Campbell and Cochrane (1999) show that introducing a time-varying subsistence level to a basic isoelastic power utility function allow to solve for a series of puzzles related to asset prices such as: the equity premium puzzle, countercyclical risk premium and forecastability of excess of stocks

are defined as follows:

$$C_t^H \equiv \left[\frac{1}{n} \int_0^n c_t^H(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}, C_t^M \equiv \left[\frac{1}{1-n} \int_n^1 c_t^M(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where $\varepsilon > 1$ is the elasticity of substitution across goods produced within the home economy, denoted by $c_t^H(z)$ and within the foreign economy, $c_t^M(z)$, respectively. The household optimal demands for home and foreign consumption goods are given by:

$$C_t^H(z) = \frac{1}{n} \gamma \left(\frac{P_t^H(z)}{P_t^H} \right)^{-\varepsilon} \left(\frac{P_t^H}{P_t} \right)^{-\varepsilon H} C_t, \quad (2.4)$$

$$C_t^M(z) = \frac{1}{1-n} (1-\gamma) \left(\frac{P_t^M(z)}{P_t^M} \right)^{-\varepsilon} \left(\frac{P_t^M}{P_t} \right)^{-\varepsilon H} C_t \quad (2.5)$$

This set of demand functions is obtained by minimizing the total expenditure generated in the consumption of C_t . Notice that the consumption of each type of good, $C_t^H(z)$, is increasing in the consumption level, C_t , and decreasing on their corresponding relative prices, $\left(\frac{P_t^H(z)}{P_t^H} \right)$. Also, it is easy to show that the consumer price index, under these preference assumptions, is determined by the following condition,

$$P_t \equiv \left[\gamma (P_t^H)^{1-\varepsilon H} + (1-\gamma) (P_t^M)^{1-\varepsilon H} \right]^{\frac{1}{1-\varepsilon H}}$$

where P_t^H and P_t^M denote the price level of the home produced and imported goods, respectively. Each of these price indexes is defined as follows,

$$P_t^H \equiv \left[\frac{1}{n} \int_0^n P_t^H(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}}, P_t^M \equiv \left[\frac{1}{1-n} \int_n^1 P_t^M(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}}$$

where $P_t^H(z)$ and $P_t^M(z)$ represent the prices expressed in domestic currency of the variety z of home and imported consumption goods, respectively.

2.2 Asset Market Structure and Households First Order Conditions

For modeling simplicity, we choose an incomplete assets market structure with two risk-free one-period nominal bonds denominated in domestic and foreign currency respectively, with the

particular feature that it is costly to trade foreign bonds for domestic households. This assumption allows us to achieve stationarity for both the path of consumption and the foreign debt⁸. Under this asset market structure, the budget constraint of the domestic households (j) in units of home currency is given by:

$$\frac{B_t^j}{1+i_t} + \frac{S_t B_t^{j*}}{(1+i_t^*) \Psi_B \left(B_t^* \frac{S_t}{P_t} \right)} \leq B_{t-1}^j + S_t B_{t-1}^{j*} + W_t L_t^j - P_t C_t^j + \Gamma_t^j + T_t^j$$

where W_t is the nominal wage, S_t the nominal exchange rate, i_t the domestic nominal interest rate, i_t^* the foreign nominal interest rate and Γ_t^j are nominal profits distributed from all the firms in the economy to the home consumer. We assume that each household holds a fraction $\frac{1}{n}$ of all firms in the economy and that there is no trade in firms' shares⁹. B_t^j is the home household's holding of the risk free domestic nominal bond. B_t^{j*} is the home household's holding of the foreign risk-free nominal bond expressed in foreign currency. The function $\Psi_B(\cdot)$ represents the real cost associated of trading foreign bonds, which depends on the aggregate stock of foreign bonds in real terms¹⁰. For simplicity, we place this friction only in the domestic economy by further assuming that foreign households can only allocate their wealth in foreign currency denominated bonds. The conditions characterizing the optimal allocation of domestic and foreign consumption and the holdings of real bonds are given by the following two equations:

$$\begin{aligned} U_C(C_t) &= (1+i_t) \beta E_t \left\{ U_C(C_{t+1}) \frac{P_t}{P_{t+1}} \right\} \\ U_C(C_t) &= (1+i_t^*) \Psi_B \left(B_{F,t} \frac{S_t}{P_t} \right) \beta E_t \left\{ \frac{S_{t+1} P_t}{S_t P_{t+1}} U_C(C_{t+1}) \right\} \end{aligned} \quad (2.6)$$

Equation (2.6) corresponds to the Euler equation that determines the optimal path of consumption for households at the home economy by equalizing the marginal benefits of savings to its

⁸We follow Benigno (2001), Schmitt-Grohe and Uribe (2001) and Kollmann (2002) who develop open-economy models introducing the same cost to achieve stationarity. Heathcote and Perri (2001) also make a similar assumption in a two-country RBC model.

⁹This assumption allows us to work with the aggregate economy as a representative agent model economy. Otherwise, we should have to keep track of the wealth position of each household in the economy.

¹⁰As Benigno, P.(2001) points out, some restrictions on $\Psi(\cdot)$ are necessary: $\Psi(0) = 1$; assumes the value 1 only if $B_t^* = 0$; differentiable; and decreasing in the neighborhood of zero.

corresponding marginal costs. On the other hand, the second equation represents the holdings by a home household of foreign bonds. From these conditions we are able to derive the uncovered interest parity, which links the depreciation of the nominal exchange rate to the net foreign asset position and both the domestic and foreign nominal interest rates,

$$\frac{(1 + i_t)}{(1 + i_t^*)} = \frac{\Psi_B \left(B_{F,t} \frac{S_t}{P_t} \right) E_t \left\{ \frac{S_{t+1} P_t}{S_t P_{t+1}} U_C (C_{t+1}) \right\} (UIP_t)}{E_t \left\{ U_C (C_{t+1}) \frac{P_t}{P_{t+1}} \right\}} \quad (2.7)$$

Furthermore, to capture possible deviations of the UIP condition, we introduce a UIP shock, UIP_t , which follows a first order autoregressive process,

$$UIP_t = (UIP_{t-1})^{\rho_{uip}} \exp \left(\mu_t^{uip} \right)$$

where, $\mu_t^{uip} \sim iid \left(0, \sigma_{uip}^2 \right)$. The first order conditions that determine the supply of labor are characterized by the following two equations,

$$MRS_t = \frac{V_L(L_t)}{U_C(C_t)} \quad (2.8)$$

$$WP_t = (WP_{t-1})^{\lambda_{wp}} MRS_t^{1-\lambda_{wp}} \quad (2.9)$$

where $WP_t \equiv \frac{W_t}{P_t}$ denotes real wages. Equation (2.8) determines the marginal rate of substitution between consumption and working hours. In a competitive equilibrium labor market, MRS_t should be equal to the real wage. Nevertheless, in order to capture frictions in the labor market, which are very likely to be present in the peruvian data, we depart from the standard efficiency condition for the labor market by assuming that real wages adjust slowly in response to changes in the marginal rate of substitution (see Blanchard and Galí (2005)). In (2.9), λ_{wp} measures the degree of persistence of real wages and also index the degree of real frictions in the labor market¹¹.

¹¹Notice that in this case, it is possible to obtain the unemployment rate as an equilibrium variable by comparing the level of hours employed when, $\lambda_{wp} = 0$ with those where real rigidities are present, thus we can define: $ur_t = \frac{L_{\lambda_{wp},t}}{L_t}$, where ur_t represent the unemployment rate, L_t equilibrium hours when equation(2.8) holds, finally, $L_{\lambda_{wp},t}$ represents the level of hours when real rigidities at the labor market are present.

2.3 Terms of trade and Real Exchange Rate

The real exchange rate and the terms of trade are key relative prices in an open economy. To ease the exposition of the model, in this section we define and derive the relationship between these two variables. First, we denote by S_t the nominal exchange rate, which is defined as the price of the foreign currency in terms of the domestic one. The real exchange rate and the terms of trade are defined as follows:

$$RER_t = \frac{S_t P_t^*}{P_t}, TOT_t = \frac{P_t^H}{P_t^M}$$

Our definition of TOT is the relative price between export and import prices¹². The gross inflation rate of the consumer price index, $\Pi_t \equiv \frac{P_t}{P_{t-1}}$, is obtained by transforming the the consumer price index identity

$$(\Pi_t)^{1-\varepsilon_H} \equiv \left[\gamma (\Pi_t^H T_{t-1}^H)^{1-\varepsilon_H} + (1-\gamma) (\Pi_t^M T_{t-1}^M)^{1-\varepsilon_H} \right] \quad (2.10)$$

where we define the following relative prices, $\Pi_t^H \equiv \frac{P_t^H}{P_{t-1}^H}$, $\Pi_t^M \equiv \frac{P_t^M}{P_{t-1}^M}$, $T_t^H \equiv \frac{P_t^H}{P_t}$ and $T_t^F \equiv \frac{P_t^F}{P_t}$. Furthermore, the relative prices of domestic and foreign goods with respect to the consumer price index are related to TOT_t by the following identities,

$$(T_t^H)^{\varepsilon_H-1} \equiv \left[\gamma + (1-\gamma) (TOT_t)^{\varepsilon_H-1} \right] \quad (2.11)$$

$$(T_t^M)^{\varepsilon_H-1} \equiv \left[\gamma (TOT_t)^{1-\varepsilon_H} + (1-\gamma) \right] \quad (2.12)$$

Finally, the dynamics of the terms of trade evolves as follows,

$$TOT_t = TOT_{t-1} \frac{\Pi_t^H}{\Pi_t^M} \quad (2.13)$$

2.4 Firms

Five types of firms operate in the home economy: a) final goods producers, b) intermediate goods producers, c) unfinished capital producers, d) investors and e) distributors of imported goods.

¹²It is worth noting that our definition of the terms of trade is the inverse to the traditional definition in standard open economy literature.

Intermediate producers combine capital goods and labor to produce a wholesale intermediate good. These firms operate under perfect competition. Also, they rent both labor from households and capital from investors in a perfectly competitive market. They sell the wholesale good to the final goods producers, which in turn use this good to produce a continuum of differentiated consumption goods. Final good producers operate in an environment of monopolistic competition. On the other hand, capital goods are produced using final consumption goods and the previous period stock of capital. Production of this type of good is subject to convex adjustment cost, which generates a slow adjustment of investment.

2.4.1 Intermediate goods Producers

There exists a continuum of mass n of firms that produce intermediate goods using capital and labor. These firms operate in a perfectly competitive market and use a constant returns to scale technology of the following form,

$$Y_t^H = Z_t^H (K_{t-1}^H)^{\alpha_H} (X_t L_t^H)^{(1-\alpha_H)} \quad (2.14)$$

where, $0 < \alpha_H < 1$, represents the share of capital in production, K_{t-1}^H the stock of capital goods rented for production at the end of period $t - 1$, from investors, L_t^H is the amount of labor demanded from households, Z_t^H a transitory productivity shock, and X_t a permanent labor augmenting productivity shock. The two technology shocks evolve as follows,

$$Z_t^H = (Z_{t-1}^H)^{\rho_H} \exp(\mu_t^{Z^H})$$

and

$$X_t = (1 + g)X_{t-1} \exp(\mu_t^x)$$

Both shocks are assumed to follow a identical and independent distribution with mean zero and finite variance. $\mu_t^i \sim iid(0, \sigma_i^2)$. Firms choose their labor and capital demands by maximizing the present discounted value of their profits. The corresponding first order conditions of this

problem are given by,

$$WP_t = MC_t^H T_t^H (1 - \alpha_H) \frac{Y_t^H}{L_t^H} \quad (2.15)$$

$$R_t^H = MC_t^H T_t^H \alpha_H \frac{Y_t^H}{K_{t-1}^H} \quad (2.16)$$

Where, MC_t^H represent real marginal costs in terms of home prices. Equations (2.15), (2.16) determine the demand for production factors, labor and capital by equalizing the marginal product of each factor to their corresponding relative price.

2.4.2 Capital Good Firms

These firms use final goods in combination with previous period stock of capital, K_{t-1}^H to produce new capital, using the following technology

$$K_t^H = \Psi \left(\frac{INV_t^H}{K_{t-1}^H} \right) K_{t-1}^H + (1 - \delta) K_{t-1}^H \quad (2.17)$$

where, δ represents the depreciation rate, Ψ a concave function, defined as follows,

$$\Psi \left(\frac{INV_t^H}{K_{t-1}^H} \right) = \frac{INV_t^H}{K_{t-1}^H} - \frac{\psi_H}{2} \left(\frac{INV_t^H}{K_{t-1}^H} - \frac{\delta + g}{1 + g} \right)^2$$

where, $\psi_H > 0$ measures how costly is to adjust the stock of capital. From the first order conditions of the profit maximization problem we obtain the following optimal condition for investment,

$$Q_t^H \Psi' \left(\frac{INV_t^H}{K_{t-1}^H} \right) = 1 \quad (2.18)$$

where Q_t^H denotes the relative price of capital goods with respect to final consumption goods. Condition (2.18) is the standard Tobin's q, by which, the optimal level of investment is determined by equalizing the market value of the stock of capital to its replacement cost. To keep tractability of the model, we assume that capital goods use a composite of final goods to produce capital, which is identical to that of consumption goods, therefore, the consumer price index and the investment price index are exactly the same. Furthermore, in this case, investment generates

a induced demand for domestic and foreign produced goods, which are determined according the following functions,

$$INV_t^{H,d} = \gamma (T_t^H)^{-\varepsilon_H} INV_t^H \quad (2.19)$$

$$INV_t^{M,d} = (1 - \gamma) (T_t^M)^{-\varepsilon_H} INV_t^H \quad (2.20)$$

2.4.3 Investors

Investors are firms dedicated to invest on capital goods acting as an intermediary on behalf of households. The optimal condition that determines the level of new capital goods is given by,

$$E_t \left[R_{t+1}^{KH} \beta \frac{U_{C,t+1}}{U_{C,t}} \right] = 1 \quad (2.21)$$

where:

$$R_{t+1}^{KH} = \frac{1}{Q_t^H} [R_{t+1}^H + (1 - \delta) Q_{t+1}^H] \quad (2.22)$$

Equation (2.21) shows that the optimal level of investment in capital goods is achieved by making the real return of investing in capital, R_{t+1}^{KH} goods orthogonal to the stochastic discount factor of households. Where, R_{t+1}^{KH} is composed of two factors, the rental payment obtained from intermediate good producers, R_{t+1}^H and the gains for increases in the price of capital goods net of depreciation, Q_{t+1}^H .

2.4.4 Price Setting of Final Good Producers

Final goods producers purchase intermediate goods and transform them into differentiated final consumption goods. These firms operate in monopolistic competitive market, where each firm faces a downward sloping demand function, given below. Furthermore, we assume that at each period t final goods producers face an exogenous probability of changing prices given by $(1 - \theta^H)$. Following Calvo (1983) and Yun (1996), we assume that this probability is independent of the price level chosen by the firm in previous periods and on the last time the firm changed its price. Additionally, we assume that firm's prices adjust automatically to previous period inflation by

a factor, $\lambda_\pi \in [0, 1]$. Therefore, conditioned on a fixed price, the demand of firm (z) is given by,

$$Y_{t+k}^H(z) = \left(\frac{\tilde{P}_t^H(z)}{P_{t+k}^H} \Upsilon_{t+k}^H \right)^{-\varepsilon} Y_{t+k}^H$$

where Υ_{t+k}^H and F_{t+k}^H are defined as follows:

$$\Upsilon_{t+k}^H = (F_{t+k}^H)^{1-\lambda_\pi} \left(\frac{\Pi_t^H}{\Pi_{t+k}^H} \right)^{\lambda_\pi}, \quad F_{t+k}^H = \frac{P_t^H}{P_{t+k}^H}$$

and Y_{t+k}^H represents the aggregate level of domestic output, defined latter in section 2.6.1. The previous auxiliary variables allow to write the present discounted value of firm z profits function as follows, :

$$E_t \left[\sum_{k=0}^{\infty} (\beta\theta^H)^k \Lambda_{t+k} \left(\left(\frac{\tilde{P}_t^H(z)}{P_t^H} \Upsilon_{t+k}^H - MC_{t+k}^H(z) \right) Y_{t+k}^H(z) \right) \right]$$

Each firm z choose $\tilde{P}_t^H(z)$ to maximize the above equation. The first order condition of this problem, can be written in the following way,

$$E_t \left[\sum_{k=0}^{\infty} (\beta\theta^H)^k \Lambda_{t+k} \left(\left(\frac{\tilde{P}_t^H(z)}{P_t^H} \Upsilon_{t+k}^H - MUP_t MC_{t+k}^H(z) \right) (\Upsilon_{t+k}^H)^{-\theta} Y_{t+k}^H \right) \right] = 0$$

where MUP_t represents a mark-up shock that evolves as follows,

$$MUP_t = (1 + \mu) (MUP_{t-1})^{\rho_{MUP}} \exp(\mu_t^{MUP})$$

Following Benigno and Woodford (2004), the previous first order condition can be written in a recursive way using two auxiliary variables, V_t^D and V_t^N , defined as follows:

$$\left(\frac{\tilde{P}_t^H(z)}{P_{t+k}^H} \right) = (\Pi_t^H)^{-\lambda_\pi} \frac{V_t^N}{V_t^D}$$

where,

$$V_t^D = Y_t^H U_{C,t} (\Pi_{t-1}^H)^{-\lambda_\pi(1-\varepsilon)} + \theta^H \beta E_t \left[(\Pi_{t+1}^H)^{(1-\lambda_\pi)(\varepsilon-1)} V_{t+1}^D \right] \quad (2.23)$$

$$V_t^N = MUP_t Y_t^H U_{C,t} MC_t^H (\Pi_t^H)^{\lambda_\pi \varepsilon} + \theta^H \beta E_t \left[(\Pi_{t+1}^H)^{\varepsilon(1-\lambda_\pi)} V_{t+1}^N \right] \quad (2.24)$$

Also, since at each period t only a fraction $(1 - \theta^H)$ of firms change prices, and the remaining firms only update their prices according to past inflation rates, the gross rate of domestic inflation is determined by the following condition,

$$\theta^H \left(\Pi_t^H (\Pi_{t-1}^H)^{-\lambda_\pi} \right)^{\varepsilon-1} = 1 - (1 - \theta^H) \left(\frac{V_t^N}{V_t^D} (\Pi_t^H)^{-\lambda_\pi} \right)^{1-\varepsilon} \quad (2.25)$$

2.4.5 Retailers of Imported Goods

These firms operate in a monopolistic competitive sector. They buy an homogenous good in the world market and they differentiated them into final imported goods $Y_t^F(z)$. In order to generate incomplete pass-through, we assume firms operating in this sector also face an exogenous probability of changing prices, $1 - \theta^M$, which is independent on the last time the firm set prices and on the previous price level. Thus, a typical firm choose an optimal price $P_{M,t}^o(z)$ to maximize the present discounted value of its expect flow of profits, given by:

$$E_t \left[\sum_{k=0}^{\infty} (\theta^M \beta)^k \left(\lambda_{t+k} \left(\frac{P_t^{M,o}(z)}{P_{t+k}^M} - \frac{S_t P_{t+k}^{M*}}{P_{t+k}^M} \right) \tilde{Y}_{H,t+k}(z) \right) \right]$$

let's denote by Ψ_{t+k} the inverse of the cumulative inflation of imported prices as follows:

$$\Psi_{t+k} = \frac{P_t^M}{P_{t+k}^M}$$

by $\tilde{Y}_{M,t+k}(z)$ the demand of intermediate good z , given that its price has kept fixed at $P_{M,t}^o(z)$:

$$\tilde{Y}_{t+k}^M(z) = \left(\frac{P_t^{M,o}(z)}{P_t^M} \right)^{-\epsilon} \Psi_{t+k}^{-\epsilon} Y_{t+k}^M$$

and by $LOP_{t+k} = \frac{S_t P_{M,t+k}^*}{P_{M,t+k}}$. The first order condition that solve the problem of these firms is given by:

$$E_t \left[\sum_{k=0}^{\infty} (\theta^M \beta)^k \left(C_{t+k}^{-\sigma} \left(\frac{P_t^{M,o}(z)}{P_t^M} \Psi_{t+k} - MUP_t^M LOP_{t+k} \right) \tilde{Y}_{t+k}^M(z) \right) \right] = 0$$

$$MUP_t^M = (MUP_{t-1}^M)^{\rho_{MUP^M}} \exp \left(\mu_t^{MUP^M} \right)$$

where MUP_t^M represents the time varying mark-up shock . Similarly to the case of the Phillips curve for the home goods sector, it is possible to write first order condition using two auxiliary variables, $V_t^{N,M}$ and, $V_t^{D,M}$, which in turn are defined as follows,

$$V_t^{N,M} = MUP_t^M \Lambda_t LOP_t C_t^M + \theta^M \beta (\Pi_{t+1}^M)^\epsilon V_{t+1}^{N,M} \quad (2.26)$$

$$V_t^{D,M} = \Lambda_t C_t^M + \theta^M \beta (\Pi_{t+1}^M)^{\epsilon-1} V_{t+1}^{D,M} \quad (2.27)$$

Finally, since only a fraction $(1 - \theta^M)$ are allowed to change prices during each period, the dynamics of inflation of imported goods is determined by the following non-linear equation,

$$\theta^M (\Pi_t^M)^{\epsilon-1} = 1 - (1 - \theta^M) \left(\frac{V_t^{N,M}}{V_t^{D,M}} \right)^{1-\epsilon} \quad (2.28)$$

where, the LOP_t evolves according the following law of motion,

$$\frac{LOP_t}{LOP_{t-1}} = DS_t \frac{\Pi_t^{M*}}{\Pi_t^M} \quad (2.29)$$

and where

$$MUP_t^M = (1 + \mu) (MUP_{t-1}^M)^{\rho_{MUP^M}} \exp \left(\mu_t^{MUP^M} \right)$$

2.5 Monetary Policy

The central bank implements monetary policy by setting the nominal interest rate according a Taylor Type feedback-rule that depends on CPI inflation, the depreciation of the nominal exchange rate and output growth. We consider as well some degree of nominal interest rate smoothing¹³. The generic form of the interest rate rule that the central bank uses is given by,

$$(1 + i_t) = (1 + i_{t-1})^{\varphi_i} \left[\left(\frac{\Pi_t}{\Pi} \right)^{\varphi_\pi} \left(\frac{DS_t}{DS} \right)^{\varphi_s} \left(\frac{Y_t}{Y_{t-1}} \right)^{\varphi_y} \right]^{1-\varphi_i} MON_t \quad (2.30)$$

where, $\varphi_i, \varphi_y, \varphi_s > 0$ and $\varphi_\pi > 1$. i_t denotes the short-term nominal interest rate, Π_t, DS_t the gross rate of domestic inflation, CPI inflation and depreciation rate of the nominal exchange rate,

¹³As Woodford (2003) shows interest rate smoothing might reflect an optimal behavior for the central bank when there exists transaction frictions.

and $\frac{Y_t}{Y_{t-1}}$ the growth rate of aggregate output. We assume that monetary policy shocks follow an $AR(1)$ process of the following form

$$MON_t = (MON_{t-1})^{\rho_{MON}} \exp(\mu_t^{MON})$$

Hence, we do not restrict the process of monetary surprise to be *iid*.

2.6 Market Clearing

The aggregate resource constraint of the economy is given by the following condition,

$$\frac{NFD_t}{(1+i_t^*) \left(1 + \frac{1}{2}\psi_B (NFD_t - NFD)^2\right)} = \frac{NFD_{t-1}DS_t}{\Pi_t} + T_t^H Y_t^H - (C_t + INV_t^H) \quad (2.31)$$

This equation determines the dynamics of net foreign assets, NFD_t , as a function of the current account surplus, $Y_t^H - (C_t + INV_t^H)$ and the flow of interest payments generated by NFD_t , where $\frac{B_t^* S_t}{P_t} = NFD_t$. Also, from the equilibrium between the supply and demand for domestic goods we have that,

$$Y_t^H = C_t^H + INV_t^{H,d} + C_t^{H*} \quad (2.32)$$

2.6.1 The Small Open Economy and the Rest of the World

The aggregate demand for the final good z is obtained by adding up the demand for this good of all agents at both the home and foreign economy. Since final goods are used for private consumption, investment and exports, we have that:

$$Y_t^H(z) = \int_0^n C_t^{H,i}(z) d(i) + \int_n^1 C_t^{H,i^*}(z) d(i) + \int_0^n INV_t^{H,i}(z) d(i)$$

The law of one price holds in this economy for domestic goods, thus for a particular good z , we have that: $P_t^H(z) = S_t P_t^{H*}(z)$, consequently, the aggregate demand for home intermediated good z can be written as follows:

$$Y_t^H(z) = \left(\frac{P_t^H(z)}{P_t^H}\right)^{-\varepsilon} (T_t^H)^{-\varepsilon_H} \left(\gamma ABS_t + \frac{(1-\gamma^*)(1-n)}{n} RER_t^{\varepsilon_H} C_t^*\right)$$

where,

$$ABS_t = C_t + INV_t^H \quad (2.33)$$

Following Sutherland (2001), we parameterize the participation of foreign goods both in the consumption basket of home goods and in the consumption basket of foreign households, $1 - \gamma$ and γ^* , respectively, as follows: $(1 - \gamma) = (1 - n)(1 - \gamma_H)$ and $(1 - \gamma^*) = n(1 - \gamma_H)$ where n represents the size of the home economy, and $(1 - \gamma_H)$ its degree of openness.

This particular parametrization implies that as the economy becomes more open, the fraction of imported goods in the consumption basket of domestic households increases, whereas as the economy becomes larger, this fraction falls. The parametrization defined previously allows us to obtain the SOE as the limiting case of the two country economy model, making the size of domestic economy to approach towards zero, $n \rightarrow 0$. In this case we have that $(1 - \gamma) \rightarrow (1 - \gamma_H)$ and $\gamma^* \rightarrow 1$. Therefore, in the limiting case the foreign economy does not use any home produced intermediated good for production of foreign final goods, and the demand condition for domestic goods can be re-written as follows,

$$Y_t^H(z) = \left(\frac{P_t^H(z)}{P_t^H} \right)^{-\varepsilon} (T_t^H)^{-\varepsilon_H} (\gamma_H ABS_t + (1 - \gamma_H) RER_t^{\varepsilon_H} Y_t^*)$$

In order to save notation, we denote by

$$Y_t^H = (T_t^H)^{-\varepsilon_H} (\gamma_H ABS_t + (1 - \gamma_H) RER_t^{\varepsilon_H} Y_t^*) \quad (2.34)$$

thus the demand facing individual intermediate goods producing firms can be simply expressed as:

$$Y_t^H(z) = \left(\frac{P_t^H(z)}{P_t^H} \right)^{-\varepsilon} Y_t^H$$

Given the small open economy assumption we can express equation the consumer price index in the following way,

$$1 = \gamma_H (T_t^H)^{1-\varepsilon_H} + (1 - \gamma_H) RER_t^{1-\varepsilon_H} \quad (2.35)$$

The above equation would imply a link between the real exchange rate and the terms of trade.

Notice, that the SOE assumption implies that $\gamma \mapsto \gamma_H$. The rest of the equations should be modified accordingly.

The SOE assumption allows us to treat the rest of the world (RoW) as a standard closed economy. The RoW structure includes a IS-type equation, a New Keynesian Phillips Curve and contemporaneous interest rate rule. We further assume an exogenous monetary policy shock. Hence, given the exogenous foreign monetary shock mon_t^* , foreign output (Y_t^*), foreign inflation (Π_t^*) and the foreign nominal interest rate ($1 + i_t^*$) are determined in the RoW sub-system.

2.7 The Log-linear Dynamics

Equation (2.4) - (2.36) and the system of equations of the foreign economy (that solve for $Y_t^*, \Pi_t^*, (1 + i_t^*)$), are sufficient to determine the dynamic rational expectation equilibrium of our benchmark SOE. We take a log-linear approximation of these equations around a deterministic steady-state with zero inflation derived in appendix A. Variables in log linear deviations from the steady-state are denoted by lower case letters, $z = \log(\frac{Z_t}{\bar{Z}})$. Furthermore, we normalize all real variables by the permanent productivity shock to induce stationarity. Normalized variables are denoted using variables with tilde, i.e, $\tilde{Z}_t = \frac{Z_t}{X_t}$.

The full log-linear version of the benchmark model is detailed in Appendix B

3 Extensions: Partial Dollarization

We consider two extensions to the benchmark economy. Each of them incorporates one particular form of partial dollarization. The first one corresponds to CS, which is defined as the partial replacement of the domestic currency in its function of medium of payment. The second one, PD, which occurs when the domestic currency is partially substituted as unit of account.

3.1 Currency Substitution

We follow Felices and Tuesta (2006) in modelling CS. We modify the benchmark utility function by including both domestic and foreign currency $\frac{M_t^j}{P_t}$ and $\frac{D_t^j S_t}{P_t}$ as composites of aggregate

consumption. Hence, the new utility function adopts the following form:

$$U(C_t^j) = \log \left\{ \xi_t \left[b (C_t^j - hC_{t-1})^{\frac{\omega-1}{\omega}} + (1-b) Z_{t+i}^j \frac{\omega-1}{\omega} \right]^{\frac{\omega}{\omega-1}} \right\}, V(L_t^j) = \frac{(L_t^j)^{1+\eta}}{1+\eta} \quad (3.1)$$

where Z_{t+i}^j is a money aggregate defined as

$$Z_{t+i}^j = \left(\frac{M_{t+i}^j}{P_{t+i}} \right)^{1-\delta^{cs}} \left(\frac{D_{t+i}^j S_{t+i}}{P_{t+i}} \right)^{\delta^{cs}} \quad (3.2)$$

where $\omega > 0$ captures the degree of complementary between consumption and the overall money aggregate. When $\omega > 1$, the marginal utility of consumption is decreasing in real money balances $U_{CZ}^j < 0$. Therefore, higher interest rates along with the associated reduction in real balance holdings, increase the marginal utility of consumption, hence the overall money aggregate and consumption are substitutes. On the other hand, when $0 < \omega < 1$, the marginal utility of consumption is increasing in real money balances $U_{CZ}^j > 0$ and therefore the overall money aggregate and consumption are complements¹⁴. Parameter $0 < b < 1$ is the weight of consumption in the consumption-money aggregate; and $0 < \delta^{cs} < 1$ denotes the preference for foreign currency within the overall money aggregate.

Under this new formulation the marginal utility of consumption can be expressed in terms not only of consumption but also of both foreign and domestic interest rates and their relative weights are sensitive to the ratio of foreign currency in the total money aggregates. The marginal utility of consumption with CS adopts the following log-linear form

$$u_{ct}^{CS} = u_{ct} + \Lambda [(1 - \delta^{cs}) i_t + \delta^{cs} i_t^*] \quad (3.3)$$

where $\Lambda \equiv \beta(\omega - 1)(1 - b)$. where u_{ct} corresponds to the marginal utility of consumption of the benchmark economy defined in appendix B. The previous equation reveals the fragility of monetary policy in a partially dollarized environment. Note that the higher the degree of CS, higher δ^{cs} , the greater the effect of foreign interest rates over the marginal utility of consumption.

¹⁴See Woodford (2003) chapter 2 for a brief discussion related to the consequences of nonseparable utility function and price determination.

3.2 Price dollarization

We introduce price dollarization (PD) by exogenously assuming that a subset of firms that produce home goods set their prices in foreign currency¹⁵. We further assume that prices in foreign currency are also sticky. The derivation of this new Phillips curve follows exactly the same steps as the once described in section 2.6.2. In order to save space we only present the log-linear expressions that characterize the model with PD. The log-linear expressions read as follows:

$$\pi_{Ht} = \left(1 - \delta^{pd}\right) \pi_{s,t} + \delta^{pd} (\pi_{d,t} + ds_t) \quad (3.4)$$

$$\pi_{s,t} - \lambda_{\pi_s} \pi_{s,t-1} = \beta (E_t \pi_{s,t+1} - \lambda_{\pi_s} \pi_{s,t}) + \kappa_H mc_t + \kappa_H \delta^{pd} rpd_t \quad (3.5)$$

$$\pi_{d,t} - \lambda_{\pi_d} \pi_{d,t-1} = \beta (E_t \pi_{d,t+1} - \lambda_{\pi_d} \pi_{d,t}) + \kappa_{PD} mc_t - \kappa_{PD} \left(1 - \delta^{pd}\right) rpd_t \quad (3.6)$$

$$\Delta rpd_t = ds_t + \pi_{d,t} - \pi_{s,t} \quad (3.7)$$

The dynamics of domestic inflation is determined by three endogenous variables: the inflation of goods that arises from firms that set prices in domestic currency (soles), $\pi_{s,t}$, the inflation of goods that comes from firms that set prices in foreign currency (dollars), $\pi_{d,t}$, and the relative price between soles and dollars defined by $rpd_t \equiv p_{s,t} - s_t - p_{d,t}$.

where λ_{π_s} and λ_{π_d} indicates the degrees of price indexation for each type of firms, $\kappa_{PD} = \frac{(1-\theta_{PD})(1-\beta\theta_{PD})}{\theta_{PD}}$ is the slope of the Phillips Curve with respect to marginal costs for the case in which firms set prices in foreign currency, and ds_t denotes the change in the nominal exchange rate.

Notice that to the extent that firms setting price in dollars face nominal rigidities, nominal prices in domestic currency differs from those set in foreign currency, hence $p_{s,t} \neq s_t p_{d,t}$. Variable Δrpd_t could be interpreted as a form of deviations from the law of one price within the the country.

Overall, the main implications of PD are that, on the one hand, it increases the sensitivity of domestic inflation, π_{Ht} to the depreciation of the nominal exchange rate, and on the other hand,

¹⁵See Castillo(2006b) for a model in which price dollarization arises endogenously.

it adds endogenous persistence to inflation and international relative prices.

3.3 Financial Dollarization (FD)

To be written,,,,

4 Estimation

In this section, we describe the data for the Peruvian economy. We also explain the Bayesian methodology used to estimate the parameters of each model, and to compare the different versions of the DSGE model with dollarization. We estimate the model using 8 observable variables and as we mentioned we consider 8 shocks.

4.1 Data

The data is obtained from the Banco Central de Reserva del Perú's Statistics Department and belongs to the set of the main aggregate information the central bank uses in order to take monetary policy decisions. We use quarterly series for real GDP, real private consumption, real private investment, CPI index, interbanking nominal interest rate. We use an index of real wages which is obtained from surveys to companies of more than 10 employers. The real exchange rate measure corresponds to the multilateral real exchange rate index. Finally, the terms of trade is measured by the ratio of export prices and import prices indexes.

Our sample period goes from 1992:2 to 2006:1, at quarterly frequency. To compute output, consumption, investment and wages growth rates, and inflation, we take natural logs and first differences of output, consumption, investment, wages, and the CPI index, respectively¹⁶. We also take natural logs and first differences of both the real exchange rate and the terms of trade. We have adjusted the raw data for two series. Both CPI inflation and the short term nominal interest rate show a downward trend and structural breaks in their deterministic component throughout the sample period¹⁷. Hence, we transform both series into stationary variables by

¹⁶Data on output, consumption and investment have been previously seasonally adjusted with the X-12 method.

¹⁷See Humala (2006) and Castillo, Humala and Tuesta (2006) for a discussion of this issue.

fitting a linear trend with breaks in both the slope and the levels. Finally, we de-mean the data prior the estimation.

Figure 1 depicts the evolution of the observable series throughout the sample period. Table 1 displays some relevant statistics of the series¹⁸. In terms of volatility, consumption growth rate is as volatile as output growth. Investment growth rate exhibits the largest volatility being 2.76 times more volatile than output growth. Wages growth rate is also more volatile than output (2 times). Interestingly, the real exchange rate is only 1.5 times more volatile than output growth which is a low value compared to the large real exchange rate volatility observed in developed economies (over 4 times). Terms of trade and inflation exhibit low volatility with respect to output while the nominal interest rate is more volatile than output growth. All real variables exhibit a relative large persistence with values between 0.62 and 0.84, whereas nominal variables are less persistence. Regarding contemporaneous correlations with output, the Peruvian data exhibits some distinct features. Both CPI inflation and the real exchange rate are countercyclical, whereas the nominal interest rate and terms of trade are procyclical.

4.2 Bayesian Estimation of the Model's Parameters

According to Bayes' rule, the posterior distribution of the parameters is proportional to the product of the prior distribution of the parameters and the likelihood function of the data. An appealing feature of the Bayesian approach is that additional information about the model's parameters (i.e. micro-data evidence, features of the first moments of the data) can be introduced via the prior distribution. To implement the Bayesian estimation method, we need to evaluate numerically the prior and the likelihood function. First, the posterior mode and Hessian matrix are evaluated at the mode which is computed by the optimization algorithm suggested by Christopher Sims (csminwel). The likelihood function is evaluated using the state-space representation of the law of motion of the model and the Kalman filter. Second, we use the Metropolis-Hastings algorithm to obtain random draws from the posterior distribution, from which we obtain the

¹⁸For a detailed analysis of the stylized facts of the peruvian economy for a broader period see Castillo, Montoro and Tuesta (2006).

relevant moments of the posterior distribution of the parameters.¹⁹

Let Ψ denote the vector of parameters that describe preferences, technology, the monetary policy rules, and the shocks in the small open economy. The vector of observable variables consists of

$$x_t = \{\Delta c_t, \Delta y_t, \Delta inv_t, \Delta wp_t, \Delta rer_t, \Delta tot_t, i_t, \pi_t\}'$$

In the model, the assumption of a technology shock with a unit root makes real variables to be stationary in first differences. Hence, we use real wages, private consumption, private investment and output growth rates, which are stationary in the data and in the model. We first-difference the real exchange rate and the terms of trade, while inflation and the nominal interest rate enter in levels²⁰. We express all variables as deviations from their sample mean. We denote by $L(\{x_t\}_{t=1}^T | \Psi)$ the likelihood function of $\{x_t\}_{t=1}^T$

4.2.1 Priors

Table 2 shows the prior distributions for the model's parameters that we denote by $\Pi(\Psi)$. For the estimation, we decide to fix some parameters, which reflect more or less their average historical values. The steady-state growth rate of the economy, $(1 + g)$ is set equal to 1, which implies that the growth rate of GDP is about 4 percent per year. In order to match a real interest rate in the steady state of about 4 percent per year, we set the discount factor to $\beta = 0.99$. The share of domestic consumed goods in aggregate consumption, γ_H , is set equal to 0.4 which represents the sample average of domestic consumed goods over tradable consumption. Following previous studies for the Peruvian economy we set the share of capital in the production function α_H equal to 0.6²¹. The depreciation rate parameter is set to $\delta = 0.025$ which implies an annual depreciation rate of 10%. We set the elasticity of substitution across differentiated goods equal to 6 which implies a steady state mark-up over the marginal cost in each sector (domestic and distributor) of 15%. The debt/GDP ratio, γ_B , is set equal to 0.4 which represents the average debt/GDP ratio

¹⁹See the Appendix for some details on the estimation. Lubik and Schorfheide (2005, 2006) also provide useful details on the estimation procedure.

²⁰Hence, we avoid the discussion on which detrending method (linear, quadratic or HP-filter) to use.

²¹Carranza, Liliana et al. (2005) estimate this parameter for the peruvian economy in a range of 0.5-0.7.

for the sample period. Finally, the parameters of the Taylor rule and price rigidity for the U.S. are calibrated as in previous studies, $\varphi_{\pi^*} = 1.5$, $\varphi_{y^*} = 0.5$ and $\theta^* = 0.66$ ²². For the model with CS we set parameters $\omega = 2$ and $b = 0.17$ based on previous studies for the Peruvian economy. (see Felices and Tuesta 2006).

For the remainder of parameters, inverse gamma distributions are used as priors when non-negativity constraints are necessary, and beta priors for fractions or probabilities. Normal distributions are used when more informative priors seem to be necessary.

The prior mean for the elasticity of substitution between tradable goods, ε_H , is set equal to 1.5 (with standard deviation equal to 0.5) which is a standard value used in open economy models, see for example Chari Kehoe and McGrattan (2002)²³. The parameter ψ_B , that measures the elasticity of the domestic interest rate with respect to the net debt position is assumed to have an inverse gamma distribution with mean 0.01 and standard deviation 0.05. We assume a tight prior for this parameter so that the estimated parameter do not distort the business cycle properties of the model²⁴.

Following previous studies for both closed and open economies we use a beta distribution for the coefficient of habit formation, h , centered at 0.7 with standard deviation of 0.1 (see Smets and Wouters 2004). For the inverse of the elasticity of the labor supply with respect to real wages, η , we assume an inverse gamma distribution with mean 1.0 and standard deviation of 0.3, which is a conventional value in the literature. The prior on the adjustment cost parameter for investment, ψ_K , is set around 1 with standard error 0.5.

Parameters measuring the degree of price stickiness in both sectors, are assumed to have the same mean value of 0.66 with standard deviation of 0.1. Price indexation in the home producer goods sector and the degree of real rigidity are assumed to have mean value of 0.5 with standard

²²See Rabanal and Rubio-Ramirez (2006) for example.

²³The elasticity of substitution between home and foreign goods is a source of controversy. Trade studies typically find values for the elasticity of import demand to respect to price (relative to the overall domestic consumption basket) in the neighborhood of 5 to 6, see Treffer and Lai (1999). Most of the NOEM models consider values of 1 for this elasticity which implies Cobb-Douglas preferences in aggregate consumption. Rabanal and Tuesta (2006), in an estimated two-country model, have found values for this elasticity, conditional on the asset market structure (complete and incomplete markets), between 0 and 1.

²⁴Selaive and Tuesta (2003) find values around 0.007 and 0.003 for OECD countries. This exogenous cost is only useful to make the net foreign debt position stationary.

deviation of 0.1. In all the above cases, beta distributions are assumed.

For the coefficients of the interest rate rule, we center their prior distribution to those values suggested by *MPT* model (Modelo de Proyección Trimestral) of the Central Reserve Bank of Perú. Hence, φ_π has a prior mean of 1.5 with a standard deviation of 0.25, φ_y has a prior mean of 0.5 with a standard deviation of 0.1 and the reaction to exchange rate movements, φ_e , has a prior mean of 0.5 with a standard deviation of 0.1. We also truncate the prior distributions of the Taylor rule coefficients such that the models deliver a unique, stable solution. The lagged interest rate coefficient is assumed to have a beta distribution with mean 0.5 and standard deviation equal to 0.2

Regarding the priors for the shocks of the model, we use uniform distributions for the autoregressive coefficients of the seven *AR*(1) shocks. We truncate the upper bound of the distribution to 0.96, because we want to examine how far can the models go in endogenously replicating persistence. Furthermore, we are agnostic about the source of business cycle fluctuations and that is why we adopt uninformative inverse gamma distributions on the standard deviations of all shocks, hence we set the mean to 0.4 and the standard deviation of 0.3.

Finally, for the transaction and price dollarization parameters, δ^T and δ^P , respectively, we adopt beta distributions centered at 0.5 with standard deviation equal to 0.1. These priors are consistent with the degree of CS and PD observed in the Peruvian data²⁵. For the estimation under PD, we constraint the parameters λ_{π_s} and λ_{π_d} and $\theta_H = \theta_{PD}$.

4.2.2 Drawing from the Posterior and Model Comparison

We implement the Metropolis-Hastings algorithm to draw from the posterior. The results are based on 250,000 draws from the posterior distribution²⁶. The definition of the marginal likelihood

²⁵See Armas, Battini and Tuesta (2006) for measures of these two forms of partial dollarization for the peruvian economy.

²⁶As is standard in the Markov Chain Monte Carlo methods, the initial 20 percent of draws was discarded, and the variance-covariance matrix of the perturbation term in the algorithm was adjusted such that the acceptance rate lies between 25 and 35 percent.

for each model is as follows:

$$L(\{x_t\}_{t=1}^T | m) = \int_{\psi \in \Psi} L(\{x_t\}_{t=1}^T | \Psi, m) \Pi(\Psi, m) d\psi$$

The marginal likelihood averages all possible likelihoods across the parameter space, using the prior as a weight. Multiple integration is required to compute the marginal likelihood, making the exact calculation impossible. We use a technique known as modified harmonic mean to estimate it.²⁷

Then, for two different models (A and B), the posterior odds ratio is

$$\frac{P(A | \{x_t\}_{t=1}^T)}{P(B | \{x_t\}_{t=1}^T)} = \frac{\Pr(A) L(\{x_t\}_{t=1}^T | \text{model} = A)}{\Pr(B) L(\{x_t\}_{t=1}^T | \text{model} = B)}.$$

If there are $m \in M$ competing models, and one does not have strong views on which model is the best one (i.e. $Pr(A) = Pr(B) = 1/M$) the posterior odds ratio equals the Bayes factor (i.e. the ratio of marginal likelihoods).

5 Results

We present our results in the following way. First, we discuss the posterior estimates obtained for the economy without dollarization vis-à-vis the two alternative specifications with dollarization. Second, we perform a model comparison by evaluating the marginal likelihood for each model. Third, we compute the standard deviations and correlations of each model at the mode posterior values. Fourth, we discuss the dynamics of our preferred model by analyzing the importance of the structural shocks in terms of the dynamics of the main macro-variables. Fifth, we perform counterfactual exercises by analyzing impulse response functions.

5.1 Parameters Estimates

In this subsection we report the main findings of the paper regarding the estimated parameter values of the model. Table 5 reports the posterior mode, and 5 and 95 percentile of the posterior

²⁷See Fernández-Villaverde and Rubio-Ramírez (2004) for computational details.

distribution of the parameters.

5.1.1 Partial Dollarization

The estimation results show that PD is slightly higher than CS. The 5 and 95 percentile for the first parameter are 0.33 and 0.66, whereas for the second parameter the same percentiles reach 0.35 and 0.66, respectively. Importantly, the posterior mode estimation is robust to different assumptions about the prior of both parameters²⁸.

5.1.2 Reaction Function of the Central Bank

The estimated reaction function of the central bank shows strong responses to both inflation and the depreciation of the nominal exchange rate. In particular, the posterior mode of φ_π is between 1.5 and 2.02, being the CS model the one that delivers the largest response of the to current inflation, $\varphi_\pi = 2.02$. In contrast, the model with only PD indicates a much lower estimate for this parameter, $\varphi_\pi = 1.52$. The opposite happens for φ_s , which takes its lowest value, 0.78, for the model with CS, and its highest value, 0.95 for the benchmark model.

Remarkably, this result indicates that the reaction of the central bank to the depreciation rate aims, at least partially, to offset the effects of both CS and PD on the volatility of consumption and inflation. Recall that PD increases the sensitivity of inflation to the depreciation of the nominal exchange rate, therefore, a central bank that tries to reduce the volatility of inflation should react more strongly to the nominal exchange rate in an economy with PD.

Also, the mode estimated value for φ_y is around 0.1 and its lowest 5 percentile is around zero, evidence of a weak response of the central bank to the growth rate of output. This result is robust across different model specifications and it can be rationalized, as table 7 shows, by noticing that almost 95 percent of the variance of output growth rate is explained by real shocks.

Contrary to our prior, the degree of interest rate smoothing reported by the posterior mode of φ_i is very small, close to zero. However, estimations using a shorter sample period, which

²⁸To gauge robustness of our baseline estimates, we allow for an even looser prior on δ^{CS} and δ^{PD} . We consider an alternative specification that is centered at 0.5 with standard deviation of 0.3. Estimations are available upon request.

are presented in the section devoted to the robustness analysis, shows a much higher degree of interest rate smoothing.

5.1.3 Price Stickiness

In comparison to develop economies, the results presented in table 7 indicate that in Peruvian economy the degrees of price stickiness both in the domestic and imported sector are relatively low. For example, Smets and Wouters (2003) report for the Euro area, values of θ^H around 0.9, and De Walque, Smets and Wouters (2005), 0.7 for the USA. Our estimations indicates that in Peru the posterior mode of θ^H is between 0.34 and 0.52 and the one of θ^M lies between 0.43 and 0.45, respectively. These parameter values imply that, on average, firms change prices every 2 quarters. Moreover, the estimated values of θ^H tend to be lower in those models that consider some form of dollarization, in particular in models with PD.

Also, domestic inflation exhibits some form of indexation. The mode of parameter, λ_{π_H} that captures how much domestic inflation depends on lagged inflation, is estimated around 0.4, moreover, this value does not change significantly across different model specifications. The previous estimated parameters imply a slope of the Phillips curve in terms of marginal costs of 0.52.

5.1.4 Labor Market Distortions

As we explained in section 2.1, we introduce real rigidities in the labor market to capture distortions very likely to be present in the Peruvian economy. Thus, the estimated posterior mode of λ_{wp} , for the model with CS and PD, is around 0.42, with a 5 and 95 percentiles of 0.43 and 0.7, respectively. Similar values are found using the rest of the models. The inverse of the elasticity of labor supply, η , is estimated to be between 2. and 6 across different models. This value is in the range reported by studies using microeconomics data for developed economies.

5.1.5 Structural shocks

Looking at the benchmark economy, the results show that the most persistent shocks are those corresponding to the UIP, and the mark-up in the imported sector shocks. Thus, the first order autoregressive coefficient is around 0.96 for both shocks. The transitory productivity, the mark up and the preference shock also exhibit some degree of persistence, being the mode value of their autocorrelation coefficients of 0.84, 0.7 and 0.85, respectively. The only shock that is not persistent is the monetary policy shock. In terms of volatilities, the mark up, the foreign and the preferences shocks are the ones with the highest volatility.

5.2 Model Comparison

The last row of Table 5 shows the marginal likelihood of the four alternative models. While both, a model with either CS or PD improve the fit with respect to the benchmark model (typical SOE), the model that ranks highest is the one that includes both forms of dollarization. Hence, since the Bayes factor clearly favors CS_PD over the CS , the PD and the benchmark models, the data favors model with both transaction and price dollarization over models with either one type of dollarization or none.

The differences are some how important using the bayesian model comparison language as suggested by Kass and Raftery (1995). For example, the log marginal likelihood difference between the CS model and the benchmark model is 6.69, suggesting that in order to choose the benchmark model over the CS model, we need a prior probability over the benchmark model $8 * 10^2 (= \exp(6.69))$ times larger than our prior probability over the CS model after observing the data. Therefore, we conclude that currency substitution model outperforms the baseline model. The inclusion of PD does not improve the marginal likelihood that much (the difference is about 4 with respect to the benchmark case), but still implies strong evidence supporting the PD model compared with the benchmark.

How much both transaction and price dollarization add to the benchmark model?. For instance the difference between the log-marginals of the model with both extensions (CS and PD)

and the benchmark model is about 9.3. This means that we would need a prior that favors the latter over the former by a factor of $11 * 10^3 (= \exp(9.3))$ in order to accept it after observing the data. Note however, that the log-marginal differences between the *CS* model and the preferred model is 2.61, which means that there is not that strong evidence favoring the latter model.

5.3 Second Moments

The previous analysis gives some evidence that the model with both extension, namely currency substitution and price dollarization, fits better the data. In this subsection we present some selected second moments in order to understand why the model with more features does rank first in terms of the Bayes factor comparison. The model implied statistics are constructed using the posterior mean parameter estimates by simulating 10,000 series of length 10000 and dropping the first 1000 observations. Table 6 presents some selected second moments implied by our estimations and are compared with those in the actual data.

Regarding the standard deviations, we find that all models generate relative high volatility for almost all variables. The implied volatility of all models more than double the volatility of the actual data. Perhaps the only exception is that all models get relative closer at matching CPI inflation volatility. Although, as it will be show later, adding a PPP shock to the preferred model helps fitting the data in this dimension.

Extending the benchmark model by allowing for CS, allows the model to encompass better the volatility of all variables with one exception, the nominal interest rate. This result follows from the smaller size of the foreign interest rate shock in the CS's model, as discussed above. In particular, the model with CS, by endogenously capturing the effect of foreign interest rates into the marginal utility of consumption, induces a minor role of foreign interest rate shocks in explaining endogenous macro volatility, hence smaller volatilities in the rest of the variables are predicted. However, the reduction of volatility is obtained at the cost of larger nominal interest rate volatility. The previous result is consistent with the limitations that a central bank might face under this environment, since the larger the degree of currency substitution the larger the movements in the interest rate in order to stabilize inflation and output (see equation 3.3). Adding

PD to the benchmark economy does not change the results significantly. Standard deviations and autocorrelations are broadly the same with respect to the model with CS.

Remarkably, the model that combines both extensions fits better the data in terms of volatility. In particular, the CS and PD model better matches the volatility of both international prices and inflation, getting closer to the data in this dimension. Hence, this is why the model with both extensions rank best using the Bayes factor.

Panel 2 of table 6 reports the autocorrelations of some selected variables, which have been previously detrended with the HP filter. It is immediate that all models match reasonable good the autocorrelation of output and inflation. In addition, all models under predict the autocorrelation of investment, nominal interest rates and the real exchange rate, and instead they over predict the autocorrelation of both consumption and the terms of trade. It is also worth noting that the preferred model gets closer to the data in terms of the real exchange rate persistence. This result is generated, basically, by the endogenous persistence induced by price dollarization, hence, the additional Phillips curve that arises from PD seems to be supported by the data.

Panel 3 of table 6 reports the cross-correlation of the observed variables vis-a-vis output. All variables, except CPI inflation and the nominal interest rates, were previously HP filtered. Overall, models perform quite well in generating a *positive* consumption-output and investment-output correlations. Yet, the consumption-output correlations are very closed to the those of the data while the investment-output correlation under predicts the one observed in the data. The main shortcoming of all models is the difficulty at matching the cross-correlation of output with international relative prices, the real exchange rate and the terms of trade. In fact, all models predict counterfactual correlations between output vis-a-vis the real exchange rate on the one hand and the terms of trade on the other hand. It seems that the model adds too stringent cross-equation restrictions that are not supported by the data. We believe that the previous finding can be overcome by introducing non tradable goods in the model, which might enrich the real exchange rate dynamics and provide additional cross-equation restrictions helping to fit the data²⁹.

²⁹Cristadoro et al. (2006) have estimated a two country model with U.S -Euro area data in which non tradable

In our model the larger than one estimated value of the elasticity of substitution between tradable goods ($\varepsilon_H = 1.45$) induces a positive expenditure switching effect generating a negative correlation between the terms of trade and output which is at odds with the Peruvian data (0.61).

Overall, Table 6 shows that each model matches a particular moment of the data better than the others. The advantage of the Bayesian approach to model comparison is that it is a likelihood-based method: all the implications of each model for fitting the data are contained in the likelihood function. Our results show that the best model according to the marginal likelihood criterion seems to deliver the best fit to most features of the data.

5.4 Variance Decomposition of the Preferred Model

In this subsection, we investigate what is the importance of the different shocks for explaining the main macro variables. We perform this exercise only for our “preferred” model, which is the model with both *CS* and *PD*. Table 7 reports the contribution of each shock to the standard deviation of the observable variables in the model.

The first observation to highlight from the output variance decomposition is the dominant influence of the permanent technology shock (around 48 percent). For instance, most of the unexpected output fluctuations are mainly explained by domestic supply shocks (64 percent), that is the mark-up shock to the domestic prices, stationary productivity shock and the permanent technology shock. From the rest of the shocks associated to demand innovations, the one that explain the most of output unexpected fluctuation is the foreign interest rate shock (33 percent). In contrast, the forecast errors of both consumption and investment are mainly explained by demand-type shocks (96 and 87 percent, respectively). These shocks include *UIP*, preferences, monetary and foreign interest rate shocks. .

The variance decomposition of the domestic nominal interest rate is broadly explained by external shocks. Thus, foreign interest rate shocks and *UIP* shocks together explain 98 percent of domestic interest rate. This finding highlights the important role of foreign factors, in particular, goods along with nominal rigidities are key elements to get closer to the data in explaining the real exchange rate dynamics.

external financial markets, over the Peruvian economy. Yet, as it will be shown later the relevance of the foreign interest rate will become negligible once a PPP shock is considered.

Interestingly, the shock that explains most of *CPI* inflation variance is the monetary policy shock. It explains 38.9 percent of its variance. The second largest component is the *UIP* shock which explains 25.1 percent of the variance of the *CPI* inflation, and the third largest component is the foreign interest rate shock which explains 21.81 percent. Interestingly, the estimated model shows that *CPI* fluctuations are fairly unaffected with either mark-up and domestic productivity shocks (3.7 and 2.3 percent, respectively).

The variance decomposition of the real exchange rate is mainly explained by the foreign interest rate shocks (45.65 percent). Mark-up shock in the distributor sector and the permanent technology shock have had some importance (14.70 and 15.18 percent).

Interestingly, our results show that foreign interest rate shocks have some importance at explaining the dynamics of domestic variables in our small open economy. Hence, unlike Justiniano and Preston (2006) we find that the Peruvian economy can account for a meaningful role of foreign disturbances³⁰

5.4.1 Posterior Impulse Response Analysis

To understand how dollarization affects the transmission mechanisms we report the impulse response functions of the preferred model vis-a-vis a counterfactual model in which both forms of dollarization are taken out of the preferred model. We perform the analysis for both monetary policy shock and foreign interest rate shocks.

The standard error of the monetary shock is reasonable big 2.19 percent and as we have shown in the previous section it explains great part of CPI inflation. The effects of monetary policy shocks are as expected. Output and consumption decline in response to domestic contractionary policy in both models. Note that, the preferred model without dollarization displays a greater reaction on these two aggregate variables. Hence, our estimation predicts that monetary policy

³⁰In contrast, Justiniano and Preston (2006) find that foreign shocks (U.S. economy shocks) can account for at most one percent of the variation in Canadian output, inflation and interest rates.

would be more effective if dollarization is absent. However, the impact of the monetary policy shock on inflation and the depreciation rate is similar in both models: inflation rate falls and the nominal exchange rate appreciates as expected.

It is worth noting that even so there is an impact effect of the monetary policy shock over aggregate variables, it dyes out very rapidly. The reason of this results is explained, first by the estimated zero coefficient for the autoregressive coefficient of the monetary policy shock and second by the almost null estimated parameter of smooth adjustment in the monetary policy reaction function.

Now let's turn to analyze the impulse responses conditional to a foreign interest rate shock. Note that an unanticipated increase in the foreign interest rate has a larger impact on consumption and output when dollarization is present. Again, this result highlights the potential difficulties that a central bank would face in a partial dollarized environment when foreign interest rate are in place. In addition, this shock depreciates the exchange rate and as expected increases -by the pass-through effect- both home and total inflation . The impulse response also shows that the foreign interest rate shock has a slightly lower impact on inflation when dollarization is not present compared to the CS and PD economy.

6 Robustness Analysis

6.1 PPP Shocks

Even so we have found evidence that favors the model with both type of dollarization, the model (CS and PD) does not perform well in matching some second moments like the volatility of real variables and the co-movement between international relative prices and output. As De Walque et al. (2005), Lubik and Schorfheide (2005) and Rabanal and Tuesta (2006) pointed out, estimated open economies model predict a tension between the fit of domestic aggregate variables and international relative prices. Some authors have suggested some form of PPP shocks as a resolution³¹.

³¹These type of shocks (PPP or UIP shocks) only helps to better fit individual equations. They do not appear anywhere else in the model, hence they do not imply any additional cross-equation restriction.

In our economies we have not considered the role of non tradable goods. In the Peruvian economy nontradable goods accounts for about 2/3 of the real exchange rate volatility and they represent around 50 percent of the aggregate consumption bundle.³² In order to capture this potential model misspecification in the simplest way we append a shock to the traditional equation of the real exchange rate dynamics

$$rer_t = -\gamma_H tot_t + lop_t + ppp_t$$

where ppp_t denotes the PPP shock. This shock might be capturing an additional channel through which we can induce deviations from PPP. In order to factor the importance of this shock we re-estimate the preferred model assuming the same prior for this shock as that considered for the rest of the shocks ($ppp_t = \rho_{ppp} ppp_{t-1} + \mu_t^{ppp}$).

Results of the estimated parameters are reported in table 8. Remarkably, this shock significantly improves the fit of the preferred model. Parameters estimates change in important dimensions. First the autoregressive coefficient of the monetary policy shock increases from 0.01 to 0.65. Likewise, the smoothness parameter in the policy reaction function of the bank increases from 0.03 to 0.22. Changes in those parameters might induce a larger effect of monetary policy over aggregate variables. The parameter that captures PD decreases from 0.47 to 0.39 whereas the one that measures the degree of CS increases from 0.40 to 0.56. Finally, prices in the domestic good sector become significant more flexible (θ_H decreases from 0.52 to 0.27) whereas prices in the imported distributor sector become stickier (θ_H goes up from 0.45 to 0.82).

More interestingly the PPP shock has a relative large standard deviation (around 6.04%) that compensates other shocks volatility. Indeed, we observe an important reduction in the estimated standard deviations of preference shocks with respect to the preferred model without PPP shock. Finally, the overall fit of the model increases as evidence of the reduction of the marginal likelihood.

³²This broad figure was calculated by comparing the volatility of non tradable productivity volatility with respect to the real exchange rate volatility at annual frequencies. We have used the data constructed by Ferreyra and Salas (2006).

6.1.1 Second Moments

Table 9 reports some selected moments comparing the preferred model with and without PPP shock. It shows that, in terms of both standard deviations and autocorrelations, the PPP model performs better than the one without this shock. For instance, the volatility of almost all variables moves closer to the data. Furthermore, the autocorrelation coefficient of the nominal interest rate increases from 0.09 in a model without PPP shock to 0.46 in a model with PPP, getting very close to the value of 0.5 observed in the data.

6.1.2 Variance decomposition

The variance decomposition of the preferred model with PPP shock is reported in Table 10. Compared to the model without a PPP shock the variance decomposition changes in many grounds. The first observation is that PPP shocks mitigate all the effect of foreign interest rate shock over domestic business cycles. Indeed, the effect of foreign interest rate over aggregate variables and international relative prices become almost negligible.

Monetary policy shocks still explain most of CPI inflation (58 percent). Finally, unlike the model without PPP shock, most of the variances of both the RER and the TOT are explained by the PPP shock.

6.2 Shorter Sample Period (1995-2006)

A simple inspection of Figure 1 shows that volatility of real variables during the two first years of the sample were relatively high as a consequence of the post stabilization period. In this subsection we briefly discuss the results of reducing our sample period to starting in 1995:01, disregarding the period 1992-1994.

The main differences with respect to the preferred model are. First, the parameter on the reaction of the Taylor rule to both CPI inflation and output growth are estimated to be larger (from 1.94 to 2.20 and from 0.09 to 0.22, respectively). Second, the autoregressive coefficient of monetary policy shock increases significantly (from 0.01 to 0.31) although not that much as in the case of the preferred model with a PPP shock. Third, both the elasticity of substitution

between tradable goods, ε_H , and the inverse of the elasticity of labor supply, η , increase. In both cases the values almost double in the shorter sample. The large value of η is consistent with the high relative volatility of real wages with respect to the low volatility of output. The remaining parameters of the model do not change significantly.

In terms of second moments the implied standard deviations of the observable variables decrease with respect to the preferred model, getting closer to the one observed in the data (see Table 9).

7 Conclusions

This paper has developed and estimated a small open economy with partial dollarization using Bayesian techniques. We provide estimation of models with two forms of dollarization: transaction and price dollarization. We perform the estimation using data of the Peruvian economy which is, by far, one of the most highly dollarized economies among emerging market countries that target inflation.

Our results favor a structure that includes both forms of partial dollarization. Since the inclusion of CS and PD increase the marginal likelihood of the model, there is strong evidence that both mechanisms of dollarization are supported with the data. Also, the contrafactual exercise shows that eliminating dollarization would increase the effectiveness of monetary policy in stabilizing both output and consumption.

Our model can be easily extended to include other types of dollarization, such as financial dollarization (FD). FD can be included in two forms: first, through the financial accelerator, which would capture balance sheets effects, and second, by assuming that firms use foreign currency to finance working capital. The latter form will capture a direct supply effect of dollarization through the marginal costs of firms. Indeed, the inclusion of this third type of dollarization would give a complete framework to study the effects of monetary policy in a dollarized economy. Furthermore, given improvement in the fit of the data with PPP shocks, extending the model by including non tradable goods is an avenue to pursue.

Finally, our empirical estimates could be used as starting point for policy analysis at the Bank of Peru. Since this is the first DSGE estimated with Peruvian data, the estimated parameters give a benchmark on the importance of the different mechanisms involved in the dynamics of Peruvian variables..

References

- [1] Adolfson, M., S. Laseen, J. Lindé and M. Villani, 2004, "Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through," Sveriges Riskbank Working Paper No.179
- [2] Armas, A. N. Batini and V. Tuesta, 2006, " Is Inflation Targeting Possible in a Partial Dollarized Economy", mimeo, Banco Central de Reserva del Perú.
- [3] Batini, N., A. Justiniano, P. Levine, and J. Pearlman, 2005, "Model Uncertainty and the Gains from Coordinating Monetary Rules," paper presented at the International Research Forum on Monetary Policy: Third Conference, European Central Bank.
- [4] Batini, N., P. Levine, and J. Pearlman, 2006, "Optimal Exchange Rate Stabilization in a Dollarized Economy with Inflation Targets", mimeo presented at 12th international conference on computing in economics and finance
- [5] Blanchard O and J Galí, 2006, "Real Wage Rigidities and the New Keynesian Model" Forthcoming in the *Journal of Money, Credit, and Banking*.
- [6] Benigno, P., 2001, "Price Stability with Imperfect Financial Integration," Manuscript, New York University.
- [7] Benigno, P. and M. Woodford, 2005," "Inflation Stabilization and Welfare: The Case of a Distorted Steady State". *Journal of the European Economic Association*, 3(6): 1-52.
- [8] Campbell, J. and J. Cochrane, 1999, " By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behaviour". *Journal of Political Economy*, Vol 107, No 2, pp205-pp251.
- [9] Castillo, P., 2006a, "Monetary Policy under Currency Substitution". mimeo, London School of Economics y Banco Central de Reserva del Perú

- [10] Castillo, P., 2006b, " Optimal Monetary Policy and Endogenous Price Dollarization ". mimeo, London School of Economics y Banco Central de Reserva del Perú
- [11] Castillo, P and C Montoro, 2004, " Income Distribution and Endogenous Dollarization" mimeo, London School of Economics y Banco Central de Reserva del Perú.
- [12] Castillo, P, C. Montoro and V. Tuesta, 2006, " Hechos estilizados de la Economía Peruana". Working paper DT N° 2006-005 Banco Central de Reserva del Perú.
- [13] Castillo, P., Humala, A and V. Tuesta, 2006, " Monetary Policy, Regime Shifts and Inflation Uncertainty in Peru (1949-2006), mimeo Banco Central de Reserva del Perú.
- [14] Calvo, G., 1983, "Staggered Prices in a Utility Maximizing Framework". *Journal of Monetary Economics*, 12, 383-398.
- [15] Céspedes, L. F., R. Chang and A. Velasco, 2004, "Balance Sheets and Exchange Rate Policy", *American Economic Review*, 94(4), 1183-1193.
- [16] Christiano L, M Eichenbaum and C Evans, 2005, " Nominal Rigidities and the Dynamics Effects of a Shock to Monetary Policy", *Journal of Political Economy* 113(1), 1-45.
- [17] Cristadore, R, A Gerli, S. Neri, M.Pisani, 2006, " The dynamics of the real exchange rate: a Bayesian DSGE approach". Mimeo Bank of Italy.
- [18] De Paoli, B., 2004, "Monetary Policy and Welfare in a Small Open Economy", CEP Discussion Paper N°639
- [19] Del Negro, M., F. Schorfheide, F., Smets and R. Wouters, 2004, "On the Fit and Forecasting Performance of New-Keynesian Models," Department of Economics, University of Pennsylvania.
- [20] De Walque G., F. Smets and R. Wouters, 2005, " An Open Economy DSGE Model Linking the Euro Area and the US Economy", mimeo, National Bank of Belgium

- [21] Felices, G. y V. Tuesta, 2006, "Monetary Policy in a Dual Currency Environment", mimeo, Banco Central de Reserva del Perú
- [22] Elekdag S., A. Justiniano and I. Tchakarov, 2006, "An Estimated Small Open Economy Model of the Financial Accelerator" *IMF Staff Papers*, vol 53, #2.
- [23] Fernández-Villaverde, J., and J.F. Rubio-Ramírez, 2004, "Comparing Dynamic Equilibrium Models to Data: A Bayesian Approach," *Journal of Econometrics*, Vol. 123, pp. 153-187.
- [24] Galí, J. and T. Monacelli, 2005. " Monetary Policy and Exchange Rate Volatility in a Small Open Economy". *Review of Economic Studies*, No. 72, pp. 707-734
- [25] Humala, A., 2006, "Expectativas de depreciación y diferencial de tasas de interés: ¿hay regímenes cambiantes? El caso de Perú" Documento de Trabajo DT N° 2006-002, Banco Central de Reserva del Perú.
- [26] Justiniano, A., and B. Preston, 2006, "Can Structural Small Open Economy Models Account for the Influence of Foreign Disturbances", Manuscript, Columbia University and Board of Governors of the Federal Reserve System.
- [27] Heathcote, J., and F. Perri, 2002, "Financial Autarky and International Business Cycles," *Journal of Monetary Economics*, Vol. 49/3, pp. 601-627
- [28] Ize A. and E. Parrado, 2002, " Dollarization, Monetary Policy and the Pass-through" IMF Working paper 188.
- [29] Kass, R. and A. Raftery, 1995, "Bayes Factors," *Journal of the American Statistical Association*, Vol. 90, No. 430, pp. 773-795.
- [30] Kollmann, R., 2002, "Monetary Policy Rules in the Open Economy: Effect on Welfare and Business Cycles," *Journal of Monetary Economics* 49, pp. 989-1015
- [31] Llosa, G., V. Tuesta and M. Vega, 2005, " A BVAR Forecasting Model for Peruvian Inflation", Documento de Trabajo 2005-07, Banco Central de Reserva del Perú.

- [32] Luque, J. and M. Vega, 2003, "Usando un modelo semi-estructural de pequeña escala para hacer proyecciones: Algunas consideraciones " *Revista de Estudios Económicos N° 10. Banco Central de Reserva del Perú.*
- [33] Lubik, T., and F. Schorfheide, 2005, "A Bayesian Look at New Open Economy Macroeconomics," *NBER Macroeconomics Annual 2005*, in press.
- [34] Monacelli, T. , 2003, "Monetary Policy in a Low Pass-through Environment", European Central Bank, working paper No. 227.
- [35] Rabanal, P. and J.F. Rubio-Ramírez, 2005, "Comparing New Keynesian Models of the Business Cycle: A Bayesian Approach," *Journal of Monetary Economics, Vol. 52, No. 6, pp.1151-1166 .*
- [36] Rabanal, P. and V. Tuesta, 2005, " Euro-Dollar Real Exchange Rate Dynamics in an Estimated Two-Country Model: What is Important and What is Not? mimeo, International Monetary Fund.
- [37] Sutherland, A., 2002, " International Monetary Policy Coordination and Financial Market Integration". International Finance Discussion Papers, Board of Governors of the Federal Reserve System. N° 751.
- [38] Selaive, J. and V. Tuesta, 2003, "Net Foreign Assets and Imperfect Financial Integration: An Empirical Approach," Central Bank of Chile Working Paper No. 252.
- [39] Smets, F. and R. Wouters, 2003, "An Estimated Stochastic Dynamic General Equilibrium Model for the Euro Area," *Journal of the European Economic Association, Vol. 1, No. 5, pp. 1123-1175.*
- [40] Schmitt-Grohe, S., and M. Uribe ,2003, "Closing Small Open Economy Models," *Journal of International Economics, 61, pp 163-185.*
- [41] Trefler, D., and H. Lai, 1999, "The Gains from Trade: Standard Errors with CES Monopolistic Competition Model," mimeo University of Toronto.

- [42] Tovar, C., 2005, "The Mechanism of Devaluations and the Output Response in a DSGE Model: How Relevant is the Balance Sheet Effect?", BIS Working Paper # 192.
- [43] Tovar, C. , 2006, "An Analysis of Devaluation and Output Dynamics in Latin American Using an Estimated DSGE Model", mimeo.
- [44] Woodford, M, 2003, *Interest and Prices*, Princeton University Press.

A Solving the steady-state

We know that non-stationary variables grow in the balanced growth path at rate g , we normalize this variables by the level of technology. We denote these variables with tilde, such that $\tilde{Z}_t = \frac{Z_t}{X_t}$ is stationary. Also, we denote variables without time subscripts as variables in the steady state.

Replace the functional form of the marginal utility in the Euler equation:

$$\frac{\xi_t}{\tilde{C}_t X_t - h\tilde{C}_{t-1} X_{t-1}} = (1 + i_t) \beta E_t \left\{ \frac{\xi_{t+1}}{\tilde{C}_{t+1} X_{t+1} - h\tilde{C}_t X_t} \right\}$$

divide both sides in the numerator by X_t and evaluate it in the steady state, we obtain the interest rate in steady state:

$$(1 + i) = \frac{1 + g}{\beta} \quad (\text{A.1})$$

Similarly, under the assumption that trading frictions in asset markets are zero in steady-state, thus, $\phi(NFD) = 1$, equation (2.2) implies, that $DS = 1$. From the Phillips Curve in the home produced goods sector, equation (2.3) we have,

$$MC^H = \frac{1}{\mu} = \frac{\varepsilon}{\varepsilon - 1} \quad (\text{A.2})$$

From equations (2.28), (2.29) we have that,

$$(1 + i) = R^{KH} = \frac{1 + g}{\beta} \quad (\text{A.3})$$

$$R^H = \frac{1 + g}{\beta} - (1 - \delta) = \delta + \left(\frac{1 + g - \beta}{\beta} \right) \quad (\text{A.4})$$

Similarly using equations (2.24) and (2.25), we obtain from the law of motion of capital:

$$\widetilde{INV}^H = \frac{\delta + g}{1 + g} \tilde{K}^H \quad (\text{A.5})$$

From the equation. (2.22), the marginal productivity of capital, we obtain the capital - output ratio:

$$\frac{\tilde{K}^H}{\tilde{Y}^H} = \alpha^H (1 + g) T^H \frac{MC^H}{R^H}$$

after replacing out the steady state values of T^H , MC^H and R^H , the above equation can be re-written as ³³:

$$\frac{\widetilde{K}^H}{\widetilde{Y}^H} = \frac{\alpha^H (1+g)}{\mu \left[\delta + \left(\frac{1+g-\beta}{\beta} \right) \right]} \quad (\text{A.6})$$

Plugging equation (A.6) in equation (A.5), we obtain the investment-output ration:

$$\frac{\widetilde{INV}^H}{\widetilde{Y}^H} = \frac{(\delta+g)\alpha^H}{\mu \left[\frac{1+g}{\beta} - (1-\delta) \right]} \quad (\text{A.7})$$

We take as given net foreign asset-output ratio:

$$\frac{\widetilde{NFD}}{\widetilde{Y}^H} = \gamma_B \quad (\text{A.8})$$

From the aggregate resource constraint, the net exports-output ratio is:

$$\frac{\widetilde{NX}}{\widetilde{Y}^H} = -\gamma_B \frac{1-\beta}{1+g} \quad (\text{A.9})$$

the absorption-output ratio is:

$$\frac{\widetilde{ABS}}{\widetilde{Y}^H} = 1 - \frac{\widetilde{NX}}{\widetilde{Y}^H} \quad (\text{A.10})$$

and the consumption-output ratio is:

$$\frac{\widetilde{C}}{\widetilde{Y}^H} = \frac{\widetilde{ABS}}{\widetilde{Y}^H} - \frac{\widetilde{INV}^H}{\widetilde{Y}^H} \quad (\text{A.11})$$

The steady state of the rest of the variables are a function of these ratios. Calculations are straightforward and to save space we do not report them.

³³We calibrate the levels of the domestic and foreign productivity such that all the relative prices are equal to one in steady state. That is: $RER = TOT = T^{H,T} = T^{M,T} = 1$

B Log-linear system, benchmark model without dollarisation

This appendix summarize the log-linear equations of the benchmark economy. We take a log-linear approximation of the equations of the model around a deterministic steady-state with zero inflation defined in the previous appendix . Variables in log linear deviations from the steady-state are denoted by lower case letters, $z = \log(\frac{Z_t}{Z})$. We normalize all real variables by the level of technology to make them stationary. Normalized variables are denoted with tilde, i.e, $\tilde{Z}_t = \frac{Z_t}{X_t}$.

The benchmark economy contains 47 equations for 40 endogenous variables and 7 exogenous shocks. We include 4 more equations when we add both CS and PD to the benchmark model.

We divide the system of equations as follows: a) households and firms optimal allocation decisions, b) monetary policy rule, c) market clearing conditions, d) the equilibrium for the foreign economy, e) the exogenous process of shocks and, f) observable variables.

B.1 Households

B.1.1 First order conditions

The euler equation for the representative consumer is

$$\widetilde{u}_{ct} = (i_t - E_t\pi_{t+1}) + E_t\widetilde{u}_{ct+1} \quad (\text{B.1})$$

The uncovered interest rate parity condition

$$(i_i - E_t\pi_{t+1}) - (i_t^* - E_t\pi_t^{M*}) = E_t\Delta rer_{t+1} - \psi_B nfd_t + uip_t \quad (\text{B.2})$$

where uip_t denotes the shock to the uncovered interest rate parity condition, $nfd_t = (NFD_t - \widetilde{NFD}) / \widetilde{Y}^H$ and $NFD_t = \frac{B_t^* S_t}{P_t}$.

The marginal rate of substitution is equal to

$$\widetilde{mrs}_t = v_{lt} - u_{ct} \quad (\text{B.3})$$

where v_{lt} is the marginal desutility of labor:

$$v_{lt} = \eta l_t \quad (\text{B.4})$$

and u_{ct} is the marginal utility of consumption:

$$u_{ct} = - \left(\left[\frac{1+g}{1+g-h} \right] \tilde{c}_t - \frac{h}{(1+g-h)} \tilde{c}_{t-1} - \frac{h}{(1+g-h)} \mu_t^x \right) + \xi_t \quad (\text{B.5})$$

where $\mu_t^x = \Delta x_t$ is the unit root shock.

Real wages evolve according to the following equation:

$$\tilde{w}p_t = \lambda_{wp} (\tilde{w}p_{t-1} - \mu_t^x) + (1 - \lambda_{wp}) \tilde{m}r s_t \quad (\text{B.6})$$

B.1.2 Consumption Demands

The domestic demands for home produced and imported goods are:

$$c_t^H = -\varepsilon_H t_t^H + \tilde{c}_t \quad (\text{B.7})$$

$$c_t^M = -\varepsilon_H t_t^M + \tilde{c}_t \quad (\text{B.8})$$

and the foreign demand for domestic goods is:

$$c_t^{H*} = -\varepsilon_H (t_t^H - rer_t) + y_t^* \quad (\text{B.9})$$

B.1.3 Price Indexes and Relative Prices:

The price indexes are defined from the optimal allocation of consumption across goods. Total inflation is given by:

$$\pi_t = \gamma_H \pi_t^H + (1 - \gamma_H) \pi_t^M \quad (\text{B.10})$$

We have defined the terms of trade (TOT) as follows: $TOT = \frac{S_t P_t^{H*}}{P_t^M} = \frac{P_t^H}{P_t^M}$. Hence, relative prices of domestic goods/tradable goods and imported goods/tradable goods are:

$$t_t^H = (1 - \gamma_H) tot_t \quad (\text{B.11})$$

$$\gamma_H t_t^H + (1 - \gamma_H) t_t^{M,T} = 0 \quad (\text{B.12})$$

The evolution of the real exchange rate, the terms of trade and deviations to the law of one price are given by:

$$rer_t = -\gamma_H tot_t + lop_t \quad (\text{B.13})$$

$$tot_t = tot_{t-1} + \pi_t^H - ds_t - \pi_t^{M*} \quad (\text{B.14})$$

$$lop_t = lop_{t-1} + \pi_t^M - ds_t - \pi_t^{M*} \quad (\text{B.15})$$

B.2 Firms

B.2.1 Intermediate goods Firms

The production function for the intermediate goods firm:

$$\tilde{y}_t^H = (1 - \alpha_H) l_t^H + \alpha_H (\tilde{k}_{t-1}^H - \mu_t^x) + z_t^H \quad (\text{B.16})$$

The first order conditions for the firm equalize the marginal productivities to the rental price of labor and capital:

$$\tilde{w}p_t = mc_t^H + t_t^H + \tilde{y}_t^H - l_t^H \quad (\text{B.17})$$

$$r_t^H = mc_t^H + t_t^H + \tilde{y}_t^H - \tilde{k}_{t-1}^H + \mu_t^x \quad (\text{B.18})$$

B.2.2 Capital Goods firms

New capital is produced using the following technology:

$$\tilde{k}_t^H = \frac{(1 - \delta)}{1 + g} \tilde{k}_{t-1}^H + \left(1 - \frac{(1 - \delta)}{1 + g}\right) \tilde{inv}_t^H - \frac{(1 - \delta)}{1 + g} \mu_t^x \quad (\text{B.19})$$

Optimal investment made by the firms that produce unfinished capital goods satisfies the following optimality condition:

$$q_t^H = \psi_K (\tilde{inv}_t^H - \tilde{k}_{t-1}^H + \mu_t^x) \quad (\text{B.20})$$

where $\psi_K = \frac{\Psi_K''(INV^H/K^H)}{\Psi_K'(INV^H/K^H)} \frac{INV^H}{K^H}$ is the adjustment costs elasticity and q_t^H is the relative price of capital goods with respect to final goods.

B.2.3 Investors

The optimal conditions that determines the level of new capital goods are given by:

$$\begin{aligned} 0 &= -E_t r_{t+1}^{KH} + E_t \widetilde{u}_{ct+1} - \widetilde{u}_{ct} \\ (q_t^H + r_{t+1}^{KH}) &= \left[(1 - (1 - \delta) \beta) r_{t+1}^H + (1 - \delta) \beta q_{t+1}^H - \beta \psi_K \left(\widetilde{inv}_{t+1}^H - \widetilde{k}_t^H \right) \right] \end{aligned}$$

plugging the above equations into the euler equation we get

$$E_t \left[(1 - (1 - \delta) \beta) r_{t+1}^H + (1 - \delta) \beta q_{t+1}^H - \beta \psi_K \left(\widetilde{inv}_{t+1}^H - \widetilde{k}_t^H \right) \right] - q_t^H = i_t - E_t \pi_{t+1} \quad (\text{B.21})$$

B.2.4 Final goods producers (retailers)

Phillips curve for the home produced goods:

$$\pi_t^H - \lambda_H \pi_{t-1}^H = \kappa_H (mc_t^H + mup_t) + \beta E_t (\pi_{t+1}^H - \lambda_H \pi_t^H) \quad (\text{B.22})$$

where $\kappa_H \equiv \frac{(1-\theta_H)}{\theta_H} (1 - \theta_H \beta)$.

B.2.5 Distributors of imported goods

Similarly, aggregating the optimal price setting in the importing sector

$$\pi_t^M = \kappa_M (lop_t + mup_t^M) + \beta E_t \pi_{t+1}^M \quad (\text{B.23})$$

where $\kappa_M \equiv \frac{(1-\theta_M)}{\theta_M} (1 - \theta_M \beta)$

B.3 Policy Rule

The policy rule followed by the monetary authority is:

$$i_t = \varphi_i i_{t-1} + (1 - \varphi_i) \left[\varphi_\pi E_t \pi_t + \varphi_y \Delta y_t + \varphi_s ds_t \right] + mon_t \quad (\text{B.24})$$

B.4 Market clearing

Absortion is defined by the sum of consumption and investment:

$$abs_t = \frac{\widetilde{C}}{\widetilde{ABS}} \widetilde{c}_t + \frac{\widetilde{INV}}{\widetilde{ABS}} \widetilde{inv}_t \quad (\text{B.25})$$

Aggregating the resources constraint of the economy, we obtain an equation for the net foreign asset accumulation:

$$(1 + \gamma_B \psi_B) \beta n f d_t = \gamma_B (\beta i_t^* - \pi_t + d s_t) + \frac{1}{1+g} n f d_{t-1} + \widetilde{y}_t^H + t_t^H - \frac{\widetilde{ABS}}{\widetilde{Y}^H} abs_t \quad (\text{B.26})$$

The demand for domestic produced good is given by:

$$\widetilde{y}_t^H = \gamma_H \frac{\widetilde{C}}{\widetilde{Y}^H} c_t^H + (1 - \gamma_H) \frac{\widetilde{C}}{\widetilde{Y}^H} c_t^{H*} + \gamma_H \frac{\widetilde{INV}^H}{\widetilde{Y}^H} inv_t^{H,d} \quad (\text{B.27})$$

The demand for investment in home produced and imported goods:

$$inv_t^{H,d} = -\varepsilon_H t_t^H + \widetilde{inv}_t^H \quad (\text{B.28})$$

$$inv_t^{M,d} = -\varepsilon_H t_t^M + \widetilde{inv}_t^H \quad (\text{B.29})$$

Net exports are defined as:

$$\frac{NX}{Y^H} n x_t = y_t + t_t^H - \frac{ABS}{Y^H} abs_t \quad (\text{B.30})$$

B.5 Foreign Economy

The aggregate demand, the Phillips Curve and the monetary policy rule for the foreign economy are the following:

$$E_t (y_{t+1}^* - y_t^*) = (i_t^* - E_t \pi_{t+1}^{M*}) \quad (\text{B.31})$$

$$\pi_t^{M*} = E_t \pi_{t+1}^{M*} + \kappa^* y_t^* \quad (\text{B.32})$$

$$i_t^* = \varphi_{\pi^*} \pi_t^{M*} + \varphi_{y^*} y_t^* + mon_t^* \quad (\text{B.33})$$

where $\kappa^* = \frac{(1-\theta^*)}{\theta^*} (1 - \theta^* \beta)$.

B.6 Exogenous shocks

Preferences shock:

$$\xi_t = \rho_\xi \xi_{t-1} + \mu_t^\xi \quad (\text{B.34})$$

Domestic productivity shock:

$$z_t^H = \rho_{Z^H} Z_{t-1}^H + \mu_t^{Z^H} \quad (\text{B.35})$$

Domestic interest rate shock:

$$mon_t = \rho_{MON} mon_{t-1} + \mu_t^{MON} \quad (\text{B.36})$$

Mark-up shock:

$$mup_t = \rho_{MUP} mup_{t-1} + \mu_t^{MUP} \quad (\text{B.37})$$

Imported sector mark-up shock

$$mup_t^M = \rho_{MUP^M} mup_{t-1}^M + \mu_t^{MUP^M} \quad (\text{B.38})$$

Uncovered interest rate parity shock:

$$uip_t = \rho_{uip} uip_{t-1} + \mu_t^{uip} \quad (\text{B.39})$$

Foreign interest rate shock

$$mon_t^* = \rho_{MON^*} mon_{t-1}^* + \mu_t^{MON^*} \quad (\text{B.40})$$

B.7 Observable variables:

We use the following 8 observable variables for the estimation:

$$\{\Delta c_t, \Delta rer_t, \Delta y_t, \Delta inv_t, \Delta wp_t, \Delta tot_t, i_t, \pi_t\}$$

The corresponding transformed equations that describe the dynamics of the previous variables are given by,

$$\tilde{c}_t - \tilde{c}_{t-1} = \Delta c_t - \mu_t^x \quad (\text{B.41})$$

$$\widetilde{inv}_t - \widetilde{inv}_{t-1} = \Delta inv_t - \mu_t^x \quad (\text{B.42})$$

$$\tilde{y}_t - \tilde{y}_{t-1} = \Delta y_t - \mu_t^x \quad (\text{B.43})$$

$$\widetilde{wp}_t - \widetilde{wp}_{t-1} = \Delta wp_t - \mu_t^x \quad (\text{B.44})$$

$$\Delta rer_t = rer_t - rer_{t-1} \quad (\text{B.45})$$

$$\Delta tot_t = tot_t - tot_{t-1} \quad (\text{B.46})$$

C The Likelihood Function and The Metropolis-Hastings Algorithm

The law of motion and the likelihood function

Let Ψ denote the vector of parameters that describe preferences, technology, the monetary policy rules, and the shocks in the small open economy model, d_t be the vector of all endogenous variables (state and forward looking), z_t the vector of exogenous variables (i.e. shocks), and ϵ_t the vector of innovations. x_t is the vector of the nine observable variables that will enter the likelihood function. The system of equilibrium conditions and the process for the exogenous shocks can be written as a second-order difference equation

$$\begin{aligned} A(\Psi)E_t d_{t+1} &= B(\Psi)d_t + C(\Psi)d_{t-1} + D(\Psi)z_t, \\ z_t &= N(\Psi)z_{t-1} + \epsilon_t, \quad E(\epsilon_t \epsilon_t') = \Sigma(\Psi). \end{aligned}$$

We use standard solution methods for linear models with rational expectations to write the law of motion in state-space form. The transition and measurement equations are:

$$\begin{aligned} d_t &= F(\Psi)d_{t-1} + G(\Psi)z_t, \\ z_t &= N(\Psi)z_{t-1} + \epsilon_t, \quad E(\epsilon_t \epsilon_t') = \Sigma(\Psi). \end{aligned}$$

and

$$x_t = H d_t$$

Let $y_t = [d_t', z_t']'$ be the vector of all variables, endogenous and exogenous. The evolution of the system can be rewritten as

$$y_t = \tilde{A}y_{t-1} + \tilde{B}\xi_t \text{ where } E(\xi_t \xi_t') = I, \tilde{B} = \tilde{C}\Sigma^{1/2}, \text{ and } \epsilon_t = \Sigma^{1/2}\xi_t$$

and

$$x_t = \tilde{D}y_t$$

The $\tilde{A}, \tilde{B}, \tilde{C}$ and \tilde{D} matrices are functions of F, G, N , and Σ . The matrix \tilde{D} contains zeros

everywhere, and a one in each row to select the variable of interest from the vector of all variables y_t . We can evaluate the likelihood function of the observable data conditional on the parameters $L(\{x_t\}_{t=1}^T | \Psi)$, by applying the Kalman filter recursively as follows

Define the prediction error as

$$v_t = x_t - x_{t|t-1} = x_t - \tilde{D}y_{t|t-1}$$

whose mean squared error is

$$K_t = \tilde{D}P_{t|t-1}\tilde{D}'$$

where $x_{t|t-1}$ is the conditional expectation of the vector of observed variables using information up to $t - 1$, and

$$P_{t|t-1} = E[(y_t - y_{t|t-1})(y_t - y_{t|t-1})']$$

The updating equations are:

$$y_t = y_{t|t-1} + P_{t|t-1}\tilde{D}'K_t^{-1}v_t \text{ and } P_t = P_{t|t-1} - P_{t|t-1}\tilde{D}'K_t^{-1}\tilde{D}P_{t|t-1}$$

And the prediction equations are:

$$y_{t+1|t} = \tilde{A}y_t, \text{ and } P_{t+1|t} = \tilde{A}P_t\tilde{A}' + \tilde{C}\Sigma\tilde{C}'$$

Then, the log-likelihood function is equal to

$$L_t = -\frac{1}{2} \sum_{t=1}^T \{n \log(2\pi) + \log[\det(K_t)] + v_t'K_t^{-1}v_t\}.$$

where n is the size of the vector of observable variables x . Note that the log-likelihood function has to be computed recursively. To initialize the filter, we set $y_0 = x_0 = 0$, and we set P_0 as the solution to the nonlinear system of equations.

Drawing from the Posterior

To obtain a random draw of size N from the posterior distribution, a random walk Markov Chain using the Metropolis-Hastings algorithm is generated. The algorithm is implemented as follows:

1. Start with an initial value (Ψ^0). From that value, evaluate the product $L(\{x_t\}_{t=1}^T | \Psi^0)\Pi(\Psi^0)$

2. For each i :

$$\begin{cases} \Psi^i = \Psi^{i-1} \text{ with probability } 1 - R \\ \Psi^i = \Psi^{i,*} \text{ with probability } R \end{cases}$$

where $\Psi^{i,*} = \Psi^{i-1} + v^i$, v^i follows a multivariate Normal distribution, and

$$R = \min\left\{1, \frac{L(\{x_t\}_{t=1}^T | \Psi)\Pi(\Psi^{i,*})}{L(\{x_t\}_{t=1}^T | \Psi^{i-1})\Pi(\Psi^{i-1})}\right\}$$

The idea for this algorithm is that, regardless of the starting value, more draws will be accepted from the regions of the parameter space where the posterior density is high. At the same time, areas of the posterior support with low density (the tails of the distribution) are less represented, but will eventually be visited. The variance-covariance matrix of v^i is proportional to the inverse Hessian of the posterior mode and the constant of proportionality is specified such that the random draw has some desirable time series properties.

In all cases, the acceptance rates were between 25 and 35 percent, and the autocorrelation functions of the parameters decay fairly fast. First, we find the posterior mode using standard optimization algorithms to be used as initial value. Then, we generate a chain of 250,000 draws.

Table 1: Selected moments of the data

Variables	Standard deviation	Autocorrelation	Cross-correlation with output
Output (y)	0.81	0.73	1.00
Consumption (c)	0.85	0.71	0.78
Investment (inv)	2.26	0.84	0.81
Interest Rate (i)	2.14	0.50	0.15
Total Inflation (π)	0.62	0.33	-0.14
RER (rer)	1.21	0.71	-0.25
TOT (tot)	1.46	0.62	0.61
Real Wages (wp)	1.66	0.62	0.35

Note: the standard deviations are calculated to the variables in log10 difference. The autocorrelations and cross-correlations are calculated applying the HP filter to the log10 of the variables. Output, consumption and Investment are seasonally adjusted with ARIMA X12, and the domestic interest rate and total inflation are adjusted by a deterministic trend

Table 2: Prior Distributions of the Model's Parameters

Parameter	symbol	distribution	mean	std.dev
Habit formation	h	Beta	0.70	0.10
Inverse labor supply elasticity	η	InvGamma	1.00	0.30
Elasticity of substitution, exporting/imported good	ε_H	Normal	1.50	0.50
Capital adjustment	ψ_K	Normal	1.00	0.50
Risk premium adjustment	ψ_B	InvGamma	0.01	0.05
Probability of not adjusting prices, domestic goods	θ_H	Beta	0.66	0.10
Probability of not adjusting prices, imported goods	θ_M	Beta	0.66	0.10
Degree of price indexation	λ_P	Beta	0.50	0.10
Degree of wage rigidity	λ_W	Beta	0.50	0.10
Taylor rule: inflation	φ_π	Normal	1.25	0.25
Taylor rule: output	φ_y	Normal	0.25	0.10
Taylor rule: depreciation rate	φ_s	Normal	0.50	0.10
Taylor rule: smoothing	φ_i	Beta	0.50	0.20
Currency substitution ratio	δ^{cs}	Beta	0.50	0.10
Price dollarisation ratio	δ^{pd}	Beta	0.50	0.10
AR coefficient shocks x	ρ_x	Beta	0.50	0.20
Std. Deviation shocks x	σ_x	InvGamma	0.40	0.30

Table 3: Calibrated Parameters

Parameter	symbol	value
Discount factor	β	0.99
Long-run growth rate	g	0.005
Share of capital in production function	α_H	0.60
Elasticity of substitution, same type of goods	ε	6.00
Share of domestic goods in consumption basket	γ_H	0.40
Depreciation rate	δ	0.025
Net foreign deposits/output steady state ratio	γ_B	-0.40
Sustituibility between consumption/ Z	ω	2.00
Share of Z in utility	b	0.17

Table 4: Implied steady state relationships

Ratio	symbol	value
Consumption - output ratio	C/Y	0.560
Investment - output ratio	INV/Y	0.436
Net exports - output ratio	NX/Y	0.004
Absortion - output ratio	ABS/Y	0.996

Table 5: Posterior Distributions

Coefficient	BENCHMARK			CS			PD			CS and PD		
	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
h	0.80	0.92	0.97	0.91	0.91	0.98	0.75	0.89	0.97	0.89	0.90	0.98
η	2.15	2.14	5.03	2.93	2.38	6.05	2.94	1.99	4.81	1.99	2.15	4.56
ε_H	1.19	1.57	2.31	1.71	1.46	2.15	1.46	1.55	2.26	1.45	1.44	2.17
ψ_K	0.63	0.79	1.26	0.88	0.79	1.28	0.62	0.76	1.22	0.77	0.74	1.21
ψ_B	0.00	0.00	0.01	0.00	0.00	0.01	0.01	0.00	0.01	0.00	0.00	0.01
θ_H	0.52	0.28	0.52	0.43	0.27	0.52	0.34	0.27	0.54	0.52	0.30	0.56
θ_M	0.48	0.35	0.51	0.43	0.37	0.54	0.43	0.36	0.54	0.45	0.37	0.54
λ_P	0.38	0.27	0.61	0.41	0.28	0.59	0.40	0.26	0.59	0.40	0.26	0.58
λ_W	0.49	0.45	0.72	0.52	0.46	0.73	0.49	0.45	0.73	0.42	0.43	0.70
φ_π	1.51	1.73	2.43	2.05	1.72	2.39	1.52	1.74	2.43	1.94	1.75	2.45
φ_y	0.13	0.01	0.33	0.14	-0.01	0.31	0.09	-0.02	0.30	0.09	-0.03	0.29
φ_s	0.95	0.62	0.91	0.78	0.62	0.90	0.92	0.61	0.90	0.84	0.61	0.90
φ_i	0.01	0.01	0.09	0.01	0.01	0.09	0.00	0.01	0.07	0.03	0.01	0.10
δ^{cs}	0.00	0.00	0.00	0.46	0.33	0.66	0.00	0.00	0.00	0.40	0.33	0.66
δ^{pd}	0.00	0.00	0.00	0.00	0.00	0.00	0.44	0.32	0.63	0.47	0.35	0.66
ρ_ξ	0.84	0.32	0.89	0.74	0.24	0.81	0.88	0.38	0.91	0.66	0.24	0.76
ρ_{ZH}	0.85	0.76	0.95	0.91	0.79	0.97	0.87	0.77	0.95	0.91	0.80	0.96
ρ_{MON}	0.06	0.02	0.18	0.09	0.02	0.20	0.05	0.02	0.19	0.01	0.02	0.19
ρ_{MUP}	0.70	0.63	0.84	0.73	0.61	0.84	0.79	0.59	0.85	0.68	0.56	0.84
ρ_{UIP}	0.96	0.95	1.00	0.96	0.97	1.00	0.96	0.97	0.99	0.96	0.96	0.99
ρ_{MUP^M}	0.96	0.94	0.95	0.89	0.75	0.97	0.95	0.74	0.97	0.90	0.74	0.96
ρ_{i^*}	0.10	0.09	0.30	0.18	0.12	0.33	0.08	0.09	0.31	0.21	0.11	0.32
σ_ξ	5.54	11.40	26.39	10.89	8.87	28.32	4.89	9.42	22.81	8.98	7.72	27.88
σ_{ZH}	0.84	0.72	1.02	0.75	0.68	0.95	0.75	0.73	1.05	0.82	0.68	0.99
σ_{MON}	2.09	1.78	2.55	2.24	1.76	2.53	2.08	1.80	2.56	2.19	1.75	2.54
σ_{MUP}	6.20	3.58	6.87	5.29	3.49	6.91	4.15	3.65	7.93	6.82	3.80	7.90
σ_{UIP}	0.32	0.27	0.47	0.40	0.28	0.46	0.37	0.28	0.45	0.38	0.28	0.47
σ_x	2.74	2.78	4.09	3.16	2.75	3.98	2.89	2.76	3.99	2.96	2.73	3.92
σ_{MUP^M}	1.64	1.24	1.84	1.51	1.26	2.06	1.62	1.26	2.06	1.53	1.27	2.04
σ_{i^*}	6.40	5.36	7.56	6.38	5.32	7.48	6.81	5.25	7.42	6.08	5.26	7.52
Marg dens.	-940.30			-933.61			-936.11			-931.00		

Table 6: Selected Second Moments of the Models

Standard deviation (in percent)								
Model	Δy	Δc	Δinv	i	π	Δrer	Δtot	Δwp
Data	0.81	0.85	2.26	2.14	0.62	1.21	1.46	1.66
Benchmark	2.12	1.66	6.88	2.95	1.04	2.68	2.89	6.47
CS	1.79	1.64	5.56	3.02	1.02	2.30	2.47	4.83
PD	1.77	1.81	7.05	3.16	1.16	2.68	3.02	6.01
CS and PD	2.27	1.56	6.55	3.01	0.98	2.05	2.41	6.59
Autocorrelations								
Model	y	c	inv	i	π	rer	tot	wp
Data	0.73	0.71	0.84	0.50	0.33	0.71	0.62	0.62
Benchmark	0.70	0.82	0.38	-0.02	0.36	0.57	0.79	0.49
CS	0.74	0.81	0.55	0.11	0.32	0.54	0.76	0.65
PD	0.72	0.82	0.42	-0.01	0.28	0.52	0.73	0.44
CS and PD	0.70	0.80	0.47	0.09	0.23	0.65	0.81	0.53
Cross-correlation with output								
Model	y	c	inv	i	π	rer	tot	wp
Data	1.00	0.78	0.81	0.15	-0.14	-0.25	0.61	0.35
Benchmark	1.00	0.65	0.45	-0.24	-0.06	0.44	-0.68	0.43
CS	1.00	0.73	0.42	-0.10	-0.07	0.49	-0.74	0.45
PD	1.00	0.68	0.36	-0.14	-0.08	0.42	-0.67	0.42
CS and PD	1.00	0.70	0.46	-0.27	-0.17	0.46	-0.71	0.48

Note: all model-based second moments are computed by simulation the model at the posterior mean.

Autocorrelations and cross correlations of real variables come from simulating the model 10000 times with 10000 periods at the posterior mean and dropping the first 1000 observations and applying the HP filter. The standard deviation of real variables are the theoretical standard deviations for the variables in differences.

Table 7: Contributions of the shocks to the variance
(Model with currency substitution and price dollarisation)

SHOCK	Δy	Δc	Δinv	i	π	Δrer	Δtot	Δwp
<i>DEMAND SHOCKS:</i>								
Preferences	0.13	57.12	0.09	0.19	0.20	0.07	0.22	0.04
Domestic interest rate	0.43	0.15	1.08	0.55	38.90	6.38	1.69	1.04
<i>SUPPLY SHOCKS:</i>								
Domestic productivity	6.70	1.60	0.63	0.29	2.35	4.38	9.90	2.00
Mark-up	9.71	0.24	4.38	0.32	3.68	4.69	8.49	22.42
Imported sector mark-up	0.03	0.45	0.63	0.30	4.39	14.70	8.90	0.28
Unit root	47.98	2.24	7.17	0.74	3.60	15.18	35.10	18.26
<i>EXTERNAL SHOCKS:</i>								
UIP	2.20	22.40	30.55	19.78	25.08	8.95	19.33	12.23
Foreign interest rate	32.83	15.82	55.48	77.82	21.81	45.65	16.37	43.73

Note: all model-based second moments are computed by simulation the model at the posterior mean. The variance of real variables are the theoretical variance for the variables in differences.

Table 8: Posterior Distributions - Extensions

Coefficient	CS and PD			With PPP shocks			Data from 1995		
	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
h	0.89	0.90	0.98	0.75	0.68	0.90	0.94	0.92	0.99
η	1.99	2.15	4.56	2.98	1.98	4.64	4.62	2.70	10.64
ε_H	1.45	1.44	2.17	1.35	1.07	1.68	3.01	2.55	3.58
ψ_K	0.77	0.75	1.21	0.44	0.34	0.60	1.03	0.88	1.53
ψ_B	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.01
θ_H	0.52	0.30	0.56	0.27	0.18	0.36	0.41	0.27	0.51
θ_M	0.45	0.37	0.54	0.82	0.78	0.87	0.56	0.48	0.68
λ_P	0.40	0.26	0.58	0.43	0.27	0.59	0.44	0.29	0.61
λ_W	0.42	0.43	0.70	0.42	0.30	0.58	0.53	0.37	0.71
φ_π	1.94	1.75	2.45	1.61	1.27	2.00	2.20	1.89	2.53
φ_y	0.09	-0.03	0.29	0.16	0.01	0.34	0.22	0.07	0.39
φ_s	0.84	0.61	0.90	0.71	0.59	0.84	0.85	0.70	0.99
φ_i	0.03	0.01	0.10	0.22	0.06	0.39	0.03	0.01	0.08
δ^{cs}	0.40	0.33	0.66	0.56	0.38	0.69	0.51	0.34	0.66
δ^{pd}	0.47	0.35	0.66	0.39	0.23	0.54	0.42	0.28	0.59
ρ_ξ	0.66	0.24	0.76	0.74	0.41	0.90	0.62	0.38	0.88
ρ_{ZH}	0.91	0.80	0.96	0.92	0.84	0.97	0.89	0.79	0.96
ρ_{MON}	0.01	0.02	0.19	0.65	0.41	0.79	0.31	0.12	0.51
ρ_{MUP}	0.68	0.56	0.84	0.79	0.68	0.88	0.80	0.69	0.90
ρ_{UIP}	0.96	0.96	1.00	0.95	0.91	0.98	0.96	0.97	1.00
ρ_{MUPM}	0.90	0.74	0.96	0.61	0.44	0.74	0.73	0.57	0.86
ρ_{i^*}	0.22	0.11	0.32	0.51	0.20	0.84	0.21	0.10	0.33
ρ_{PPP}	0.00	0.00	0.00	0.23	0.08	0.36	0.00	0.00	0.00
σ_ξ	8.98	7.72	27.88	4.02	2.75	7.85	8.29	5.18	20.49
σ_{ZH}	0.82	0.68	0.99	0.73	0.61	0.90	0.75	0.62	0.94
σ_{MON}	2.19	1.75	2.54	1.37	0.90	2.11	1.34	1.07	1.72
σ_{MUP}	6.82	3.80	7.90	4.13	3.15	5.58	2.75	1.74	3.81
σ_{UIP}	0.38	0.29	0.47	0.21	0.17	0.29	0.38	0.27	0.45
σ_x	2.96	2.73	3.92	1.97	1.60	2.49	2.26	1.87	2.73
σ_{MUPM}	1.53	1.27	2.05	10.92	6.16	19.11	1.37	1.04	2.04
σ_{i^*}	6.09	5.26	7.52	0.25	0.14	0.55	5.97	4.83	7.08
σ_{PPP}	0.00	0.00	0.00	6.04	4.68	7.92	0.00	0.00	0.00
Marginal Density	-931.00			-929.10			-638.67		

Table 9: Selected Second Moments of the Models - Extensions

Standard deviation (in percent)								
Model	Δy	Δc	Δinv	i	π	Δrer	Δtot	Δwp
Data	0.81	0.85	2.26	2.14	0.62	1.21	1.46	1.66
CS & PD	2.27	1.56	6.55	3.01	0.98	2.05	2.41	6.59
CS & PD (data from 1995)	1.26	0.96	4.69	2.95	0.81	1.61	1.14	3.96
CS & PD (with PPP shocks)	1.18	1.31	3.52	1.13	1.29	1.96	2.11	3.36
Autocorrelations								
Model	y	<i>c</i>	<i>inv</i>	i	π	<i>rer</i>	<i>tot</i>	<i>wp</i>
Data	0.73	0.71	0.84	0.50	0.33	0.71	0.62	0.62
CS & PD	0.70	0.80	0.47	0.09	0.23	0.65	0.81	0.53
CS & PD (data from 1995)	0.74	0.78	0.56	0.13	0.42	0.48	0.76	0.51
CS & PD (with PPP shocks)	0.75	0.82	0.75	0.46	0.44	0.77	0.85	0.74
Cross-correlation with output								
Model	y	<i>c</i>	<i>inv</i>	i	π	<i>rer</i>	<i>tot</i>	<i>wp</i>
Data	1.00	0.78	0.81	0.15	-0.14	-0.25	0.61	0.35
CS & PD	1.00	0.70	0.46	-0.27	-0.17	0.46	-0.71	0.48
CS & PD (data from 1995)	1.00	0.77	0.36	-0.07	-0.06	0.35	-0.72	0.47
CS & PD (with PPP shocks)	1.00	0.71	0.41	-0.16	-0.14	0.50	-0.49	0.45

Note: all model-based second moments are computed by simulation the model at the posterior mean.

Autocorrelations and cross correlations of real variables come from simulating the model 10000 times with 10000 periods at the posterior mean and dropping the first 1000 observations and applying the HP filter. The standard deviation of real variables are the theoretical standard deviations for the variables in differences.

Table 10: Contributions of the shocks to the variance
(Model with currency substitution, price dollarisation and PPP shocks)

SHOCK	Δy	Δc	Δinv	i	π	Δrer	Δtot	$\Delta \widetilde{wp}$
<i>DEMAND SHOCKS</i>								
Preferences	0.11	56.52	0.23	0.16	0.09	0.15	0.10	0.01
Domestic interest rate	0.29	2.26	3.88	10.10	57.68	11.88	16.02	1.95
<i>SUPPLY SHOCKS:</i>								
Domestic productivity	19.92	4.36	2.54	0.35	2.73	3.71	5.72	0.46
Mark-up	10.41	0.12	6.51	0.04	0.62	0.47	1.19	23.92
Imported sector mark-up	0.23	3.74	4.32	1.19	3.28	12.81	18.21	1.36
Unit root	32.37	1.52	1.99	0.96	3.48	11.53	10.85	5.03
<i>EXTERNAL SHOCKS:</i>								
Foreign interest rate	0.02	0.06	0.10	0.05	0.03	0.17	0.07	0.05
UIP	0.58	10.68	11.81	5.11	4.00	6.25	3.69	1.80
Purchase power parity	36.08	20.74	68.60	82.06	28.08	53.03	44.15	65.43

Note: all model-based second moments are computed by simulation the model at the posterior mean. The variance of real variables are the theoretical variance for the variables in differences.