

Forecasting Canadian Time Series with the New-Keynesian Model

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Background: Parsimony and Forecasting

- Estimated DGSE models are restricted VARs (economic structure imposing the restrictions)
- Clements and Hendry's (1998) *Forecasting Time Series* : a parsimonious (restricted) model might be better at forecasting
 - more so if T is small
 - more so if h is high
 - trading off squared inconsistency (how wrong are the restrictions) versus estimation uncertainty
 - no general results

Question

*What are the out-of-sample forecasting properties of estimated DSGE
(New Keynesian) models?*

Comparable, better, or worse than those of unrestricted VAR counterparts?

This Paper

- Employs a standard New Keynesian (NK) Model
- Using Canadian aggregate data, estimates the model on a series of rolling subsamples
- After each estimation, computes out-of-sample forecasts one- to eight-quarter-ahead horizon at each step
- Compares NK model's resulting forecasts with those from benchmark (unrestricted VARs, or BVARs) using econometric tests of forecasting accuracy

Results

- Forecasting accuracy of the NK model compares favorably to that of benchmarks
 - relatively better for output, interest rates, real money balances
 - relatively (but not significantly) worse for inflation
 - NK model's relative performance improves as forecasting horizon increases
 - statistical tests suggest some of these differences are significant
- *key message*: imposing the restrictions implied by the NK model on a four-variable VAR does not (at least) diminish out-of-sample forecasting accuracy

Literature

- Ireland (2004), Dolar and Moran (2002): RBC model, ML estimation
→ Model (mostly) better than unrestricted VARs. ‘
- Smets and Wouters (2003,2004), Del Negro and Schorfheide (2004): NK model with Bayesian estimation
→ Model equivalent or better than benchmark VARs
- Boivin and Gianonni (2005): NK model where model variables are factors
→ Model equivalent or better than benchmark VARs

Rest of the Talk

- Overview of the Model
- Estimation
- Forecasting Experiments
- Concluding remarks and future work

The New Keynesian Model

- Closed Economy
- Infinitely lived, representative household maximizing lifetime utility
- A final good producer operating in perfect competition
- Continuum of intermediate-good producers operating in monopolistic competition, 'sticky' changes in nominal prices (à la Calvo and Yun)
- Taylor-type monetary Policy Rule followed by monetary authorities

Estimate the Model

- Write solution as State Space System :

$$\hat{s}_t = \Phi_1 \hat{s}_{t-1} + \Phi_2 \epsilon_t \quad (\text{state eq.})$$

$$\hat{d}_t = \Phi_3 \hat{s}_t \quad (\text{observation eq.})$$

- use available data on four variables; i.e. $d_t = [y_t, \pi_t, R_t, m_t]$.
- maximize likelihood (*Matlab*, simplex algorithm)

Estimate the Model, cont.

- Data used:
 - y_t = final domestic demand, in per-capita terms
 - π_t = increases in GDP implicit price deflator
 - R_t = Three Month Treasury Bill Rate
 - m_t = M2 divided by implicit price deflator, per-capita
- Data show upward (y_t and m_t) trends...
 - incorporate productivity growth in model to address trends in y_t and m_t ($\rightarrow \eta$)
 - use moving windows of data to reduce importance of earlier periods \checkmark

Forecasting Experiment

- Follow Ireland (2001, 2004), Stock and Watson (1999, 2002)
- Repeatedly estimate NK model *and* VAR benchmark
- Forecast out of sample from estimated NK model and VAR benchmark

Forecasting Experiment, cont.

Estimate	Forecast k periods ahead				
	$k = 1$	$k = 2$	$k = 3$	\dots	$k = 8$
1981 : 3 \longrightarrow 1995 : 4	1996 : 1	1996 : 2	1996 : 3	\dots	1997 : 4
1981 : 3 \longrightarrow 1996 : 1	1996 : 2	1996 : 3	1996 : 4	\dots	1998 : 1
1981 : 3 \longrightarrow 1996 : 2	1996 : 3	1996 : 4	1997 : 1	\dots	1998 : 2
1981 : 3 \longrightarrow 1996 : 3	1996 : 4	1997 : 1	1997 : 2	\dots	1998 : 3
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
1981 : 3 \longrightarrow 2003 : 3	2003 : 3	2003 : 4	2004 : 1	— — —	— — —
1981 : 3 \longrightarrow 2003 : 4	2003 : 4	2004 : 1	— — —	— — —	— — —
1981 : 3 \longrightarrow 2004 : 1	2004 : 1	— — —	— — —	— — —	— — —

Figure 5: Actual Data and Forecasts from the NK Model

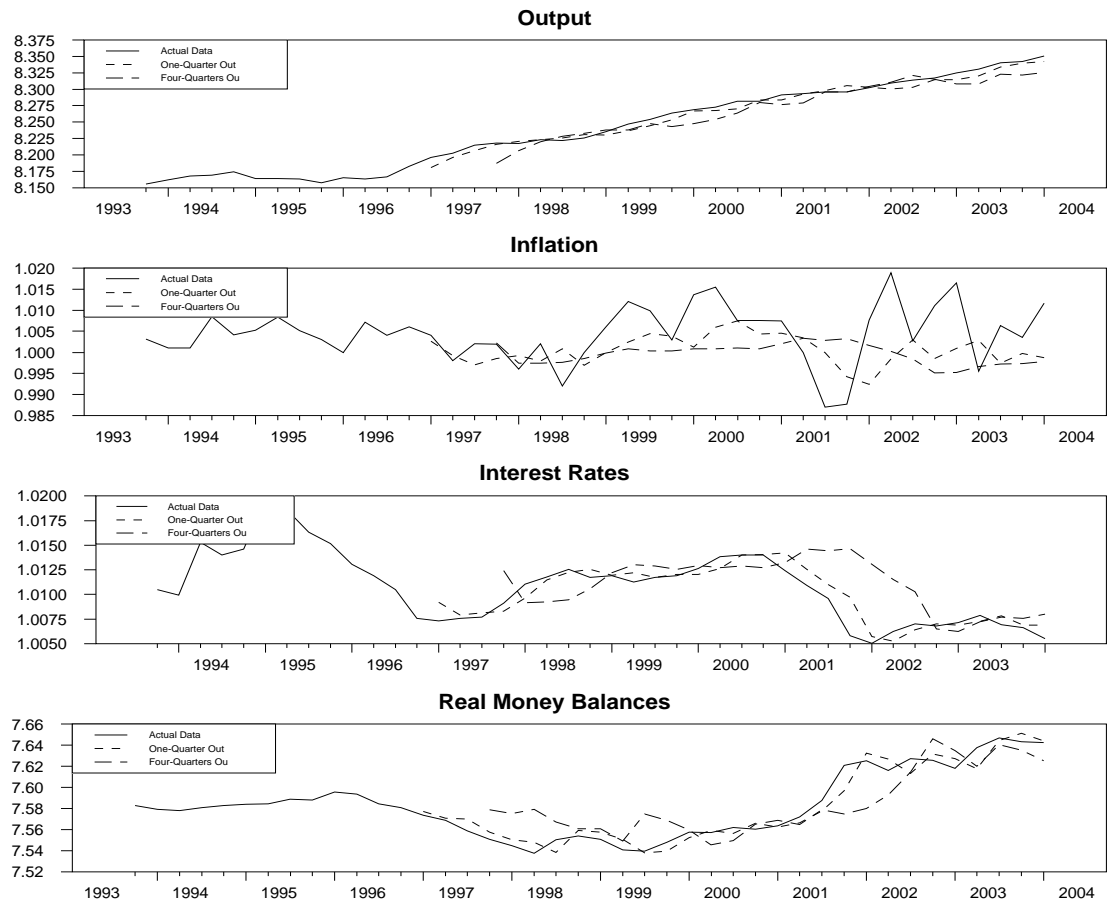


Figure 6: Forecast Errors, One Quarter Ahead

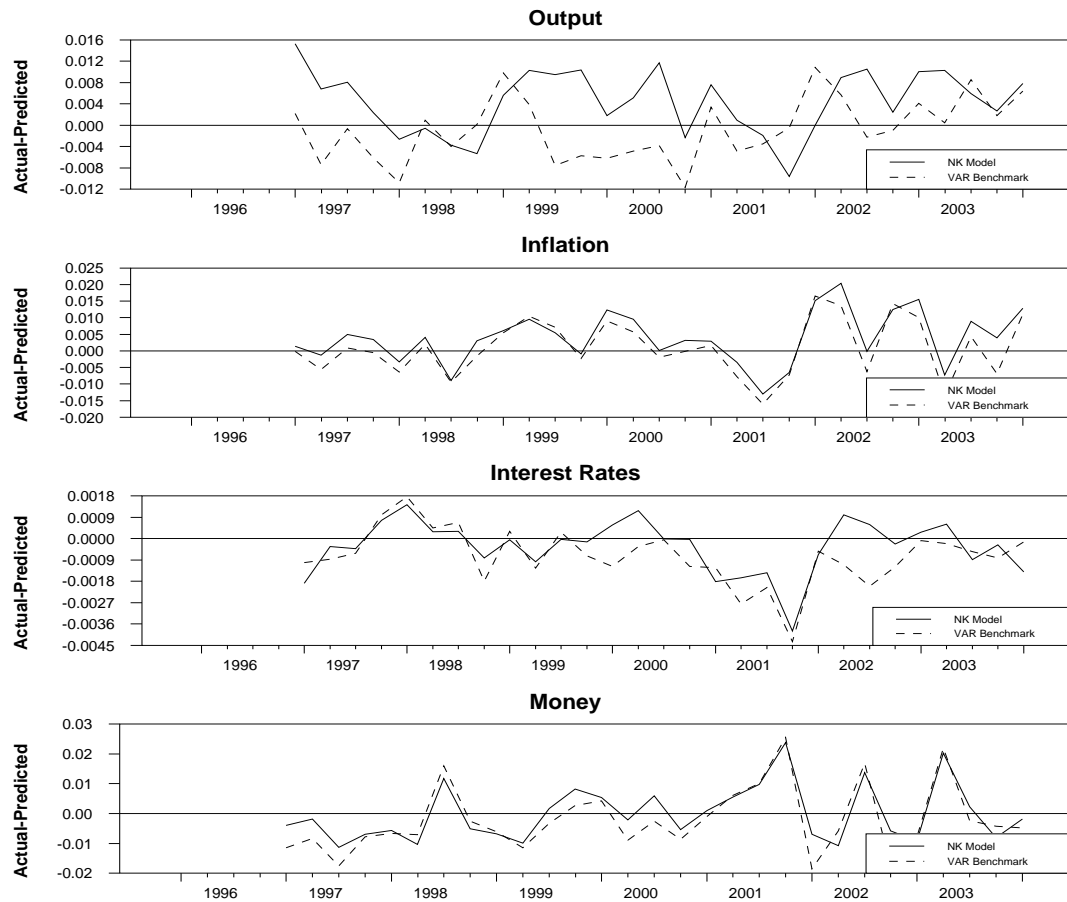


Figure 7: Forecast Errors, Four Quarters Ahead

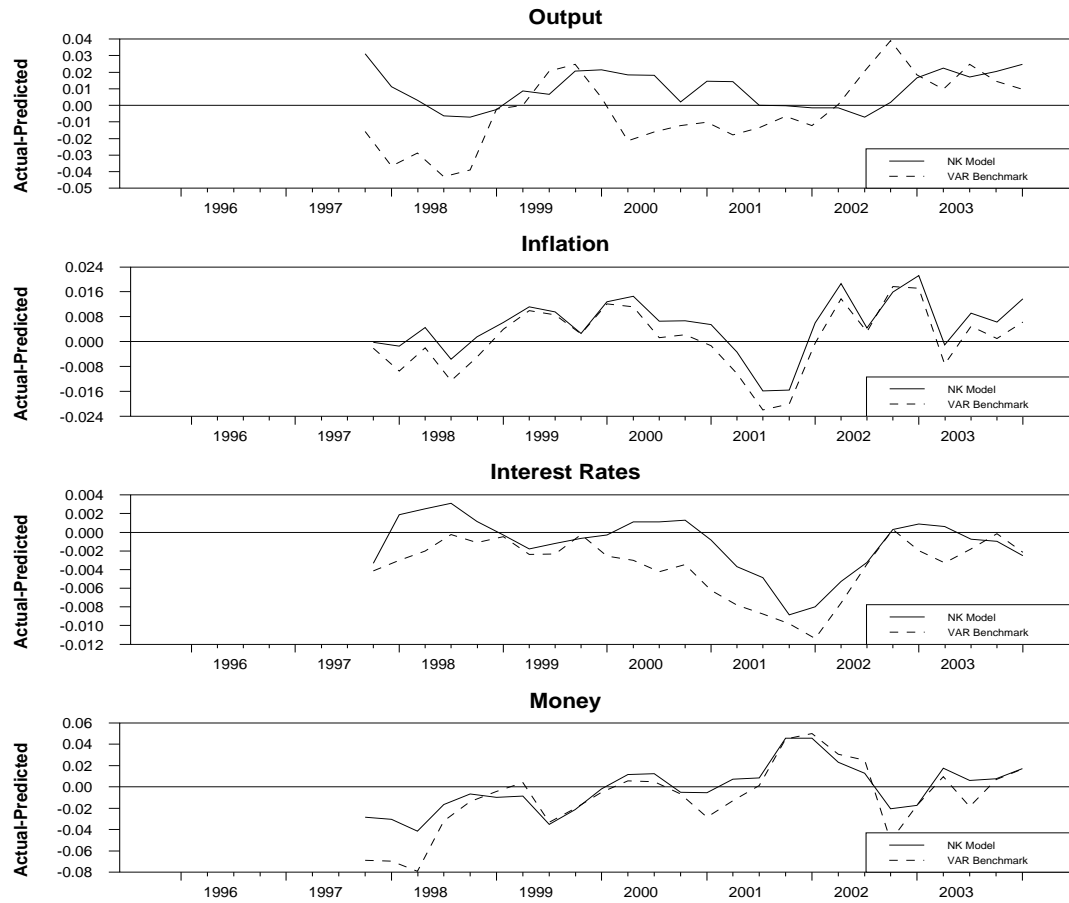
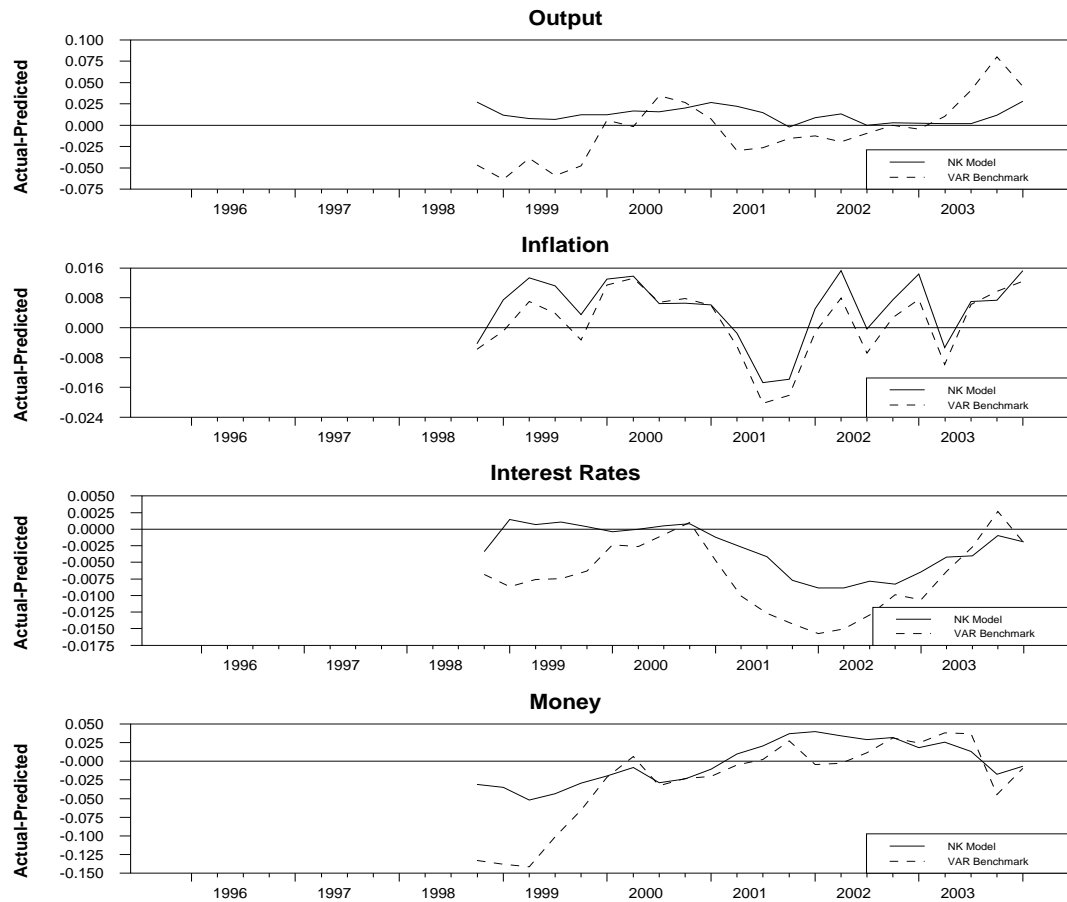


Figure 8: Forecast Errors, Eight Quarters Ahead



Comparison: Relative Mean Squared Error(MSE) of the NK model

- MSE of the Benchmark: $(\sum_{t=1}^N e_t^B)/N$ with e_t^B are forecasting errors from benchmark
- MSE of the NK model: $(\sum_{t=1}^N e_t^M)/N$
- Report MSE(NK model) / MSE(VAR Benchmark)

Table 5: Testing for Equal Forecasting Accuracy: NK Model and VAR(2) Benchmark (1996Q1 to 2004Q1)

Variable	Relative MSE (NK model)	DM stat.	HLN stat. (<i>p</i> -value)
<i>Forecasting One Period Ahead</i>			
Output	1.73	-2.18(0.03)	-2.14(0.04)
Inflation	1.21	-0.51(0.61)	-0.50(0.62)
Interest Rate	0.73	1.57(0.12)	1.54(0.13)
Money	0.67	2.29(0.02)	2.25(0.03)
<i>Forecasting Four Periods Ahead</i>			
Output	0.57	0.70(0.48)	0.61(0.55)
Inflation	1.28	-0.46(0.64)	-0.40(0.69)
Interest Rate	0.53	2.72(0.00)	2.36(0.03)
Money	0.30	1.87(0.06)	1.62(0.12)
<i>Forecasting Eight Periods Ahead</i>			
Output	0.26	1.20(0.23)	0.79(0.44)
Inflation	1.32	-0.81(0.42)	-0.54(0.60)
Interest Rate	0.40	NA(NA)	NA(NA) ₁₇
Money	0.13	1.24(0.22)	0.82(0.42)

Comparison (II): Diebold and Mariano(1995)'s test of equal forecast accuracy

- $\{e_t^M\}_{t=1}^T$: forecasting errors from NK model
- $\{e_t^B\}_{t=1}^T$: forecasting errors from VAR benchmark
- $H_0 : E[l_t \equiv (e_t^B)^2 - (e_t^M)^2] = 0$
- test statistic: $\bar{l}/\hat{\sigma}(\bar{l})$ where $\bar{l} = T^{-1} \sum_{t=1}^T l_t$

Table 6: Forecast Encompassing: Does the NK model provide *any* information not contained in the VAR Benchmark?

Variable	DM stat. (<i>p</i> -value)	HLN stat. (<i>p</i> -value)
<i>Forecasting One Period Ahead</i>		
Output	2.94 (0.00)	2.88 (0.00)
Inflation	0.54 (0.30)	0.53 (0.30)
Interest Rate	3.42 (0.00)	3.36 (0.00)
Money	2.40 (0.01)	2.36 (0.01)
<i>Forecasting Four Periods Ahead</i>		
Output	1.74 (0.04)	1.51 (0.07)
Inflation	0.60 (0.28)	0.52 (0.31)
Interest Rate	6.00 (0.00)	5.18 (0.00)
Money	1.90 (0.03)	1.56 (0.04)
<i>Forecasting Eight Periods Ahead</i>		
Output	1.29 (0.10)	0.85 (0.20)
Inflation	0.61 (0.27)	0.40 (0.35)
Interest Rate	NA (NA)	NA (NA)
Money	1.39 (0.08)	0.91 (0.19)

Comparison (III): Forecast Encompassing Test (Harvey et al., 1998)

- $e_t^B = y_t - \hat{y}_t^B$: forecasting errors from the VAR benchmark
- Run the regression $e_t^B = \gamma(e_t^B - e_t^M) + \epsilon_t$
- test $H_0 : \gamma = 0$; errors from the NK model contain no information that helps explain (reduce) errors of the VAR benchmark.
- test statistic: variant of the one from Diebold and Mariano (1995)

Table 7: Testing for Equal Forecasting Accuracy: NK Model and BVAR(2) Benchmark (1997Q1 to 2004Q1)

Variable	Relative MSE (NK)	DM stat. (<i>p</i> -val.)	HLN stat. (<i>p</i> -val.)
<i>Forecasting One Period Ahead</i>			
Output	2.31	-2.44 (0.02)	-2.40 (0.02)
Inflation	1.01	-0.04 (0.97)	-0.04 (0.97)
Interest Rate	0.90	0.70 (0.49)	0.68 (0.50)
Money	0.78	1.64 (0.10)	1.61 (0.12)
<i>Forecasting Four Periods Ahead</i>			
Output	0.69	0.69 (0.49)	0.60 (0.58)
Inflation	0.94	0.32 (0.75)	0.28 (0.78)
Interest Rate	0.65	1.24 (0.22)	1.07 (0.30)
Money	0.56	1.07 (0.29)	0.93 (0.36)
<i>Forecasting Eight Periods Ahead</i>			
Output	0.24	2.17 (0.03)	1.43 (0.17)
Inflation	1.06	-0.42 (0.68)	-0.27 (0.79)
Interest Rate	0.38	7.80 (0.00)	5.14 (0.00) ₂₁
Money	0.30	1.44 (0.15)	0.95 (0.35)

Conclusion

- Using Canadian data, we repeatedly estimate a standard NK model on subsamples of increasing length
- At each step, we forecast out-of-sample one to eight quarters ahead
- Using recent statistical tests, we compare the computed forecasts to those arising from an unrestricted VAR
- The results presented suggest that
 - ◇ the NK model forecast's accuracy is comparable to that of VAR benchmark, possibly better for output, worse for inflation
 - ◇ combining both forecasts may improve accuracy

Future Work

- More involved versions of the New Keynesian Model
 - trend in inflation target
 - hybrid NK Phillips curve
 - wage indexation à la Calvo
 - transform data by differencing rather than linear detrending (technological progress $I(1)$)