Empirical evaluation of open-economy DSGE models using SVARs. An application to exchange rate pass-through.

Ida Wolden Bache (Central Bank of Norway)

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Motivation

- Impulse responses from structural VARs (SVARs) are commonly used as guides when constructing DSGE models

- Choudhri et al. (2005) use SVAR to evaluate and estimate small open economy model with incomplete exchange rate pass-through

- A recent literature has assessed the reliability of the VAR approach using Monte Carlo simulations*
  - Generate artificial data from DSGE model
  - Estimate VAR on artificial data and ask whether the VAR recovers the DSGE model's responses

*Chari et al. (2005), Christiano et al. (2006), Erceg et al. (2005), and Kapetanios et al. (2005)

- This paper extends the literature in two directions
  - Focus on exchange rate pass-through: Dynamic responses of prices to shock to the UIP condition ('risk premium' shock)
  - Examine whether the cointegration properties implied by the DSGE model can be inferred using standard techniques

- Preview of findings
  1. Impulse responses from VAR in first differences are severely biased, even for a high order VAR
  2. Low order vector equilibrium correction model (VEqCM) is a good approximation to the data generating process
  3. Standard cointegration tests have low power to detect the correct cointegration rank and identify the underlying cointegration relations

Structure of talk

1. The model economy

2. Mapping from a DSGE model to a VAR

3. Results of Monte Carlo experiments

4. Conclusions

Model economy

- Small open economy DSGE model with incomplete pass-through based on Choudhri et al. (2005)

Firms

- Production sectors: Nontraded final good, traded intermediate good
- Inputs: Labour, basket of foreign and domestic intermediate goods
- Continua of firms in both sectors; sell differentiated goods under monopolistic competition
- International market segmentation
- Competitive distribution sector uses local labour to distribute imported goods to domestic firms

- Nominal price stickiness modeled using quadratic adjustment cost framework of Rotemberg (1982)

- One fraction of firms engages in producer currency pricing (PCP), one fraction in local currency pricing (LCP)

Households

- Continuum of infinitely-lived households derives utility from leisure and consumption of final good

- Habit persistence in consumption

- Households supply differentiated labour services and set wages subject to quadratic costs of wage adjustment

- Debt elastic interest rate on foreign bonds

Monetary authority

- Sets short-term interest rate according to simple feedback rule

Structural shocks

- Four mark-up shocks + shock to modified UIP condition

Mapping from DSGE model to a VAR

- The model solution has state space representation

$$\begin{aligned} x_{t+1} &= Ax_t + Bw_t \\ y_t &= Cx_t + Dw_t \end{aligned}$$

  - x_t: $n \times 1$ vector of state variables (possibly unobservable)

  - w_t: $m \times 1$ vector of structural shocks $Ew_t = 0, Ew_t w_t' = I$ and $Ew_t w_{t-j} = 0$ for $j \neq 0$

  - y_t: $m \times 1$ vector of observables

- Moving average (MA) representation

$$y_t = Dw_t + \sum_{j=1}^{\infty} CA^{j-1} BL^j w_t,$$

- Econometrician estimates VAR(p)

$$y_t = \sum_{j=1}^{p} A_j y_{t-j} + u_t, \qquad u_t = G\nu_t,\ E\left[u_t u_t'\right] = GG' = \Sigma_u$$

- MA representation

$$y_t = \left(I - \sum_{j=1}^{p} A_j L^j\right)^{-1} G\nu_t$$

- Conditions for impulse responses from VAR and DSGE model to coincide

  1. Invertibility

     – Sufficient condition: Eigenvalues of $A - BD^{-1}C$ strictly less than one in modulus[†]

     – Given invertibility y_t has VAR(∞) representation

$$y_t = \sum_{j=1}^{\infty} C(A - BD^{-1}C)^{j-1} BD^{-1} y_{t-j} + Dw_t,$$

  2. That the infinite order VAR can be approximated by low order VAR

  3. Shocks are correctly identified

  4. Absence of sample estimation bias

[†]Fernández-Villaverde et al. (2005)

Simulation experiments

- Monte-Carlo design:

1. Generate 5000 datasets of lengths $T = \{100,\ 200\}$ from DSGE model

2. Estimate VAR on each artificial dataset

3. Identify UIP shock by using Del Negro & Schorfheide (2004) scheme

$$G = \Gamma_{tr}\Omega^*$$

   where Γ_{tr} is the Choleski factor of $\widehat{\Sigma}_u$, and Ω^* is obtained from

$$\left(\frac{\partial y_t}{\partial w_t}\right)_{DSGE} = D = \Gamma^*_{tr}\Omega^*$$

   where Γ^*_{tr} is lower triangular and $(\Omega^*)'\Omega^* = I$

4. Compute accumulated responses of prices to UIP shock

Results for VAR in first differences

- VAR in first differences:

$$\Delta y_t = A_1 \Delta y_{t-1} + A_2 \Delta y_{t-2} + \ldots + A_p \Delta y_{t-p} + \varepsilon_t$$

  where

$$\Delta y_t' = \{\Delta \ln \overline{P}_t^m, \Delta \ln \overline{P}_t^x, \Delta \ln P_t^y, \Delta \ln P_t^c, \Delta \ln e_t\}.$$

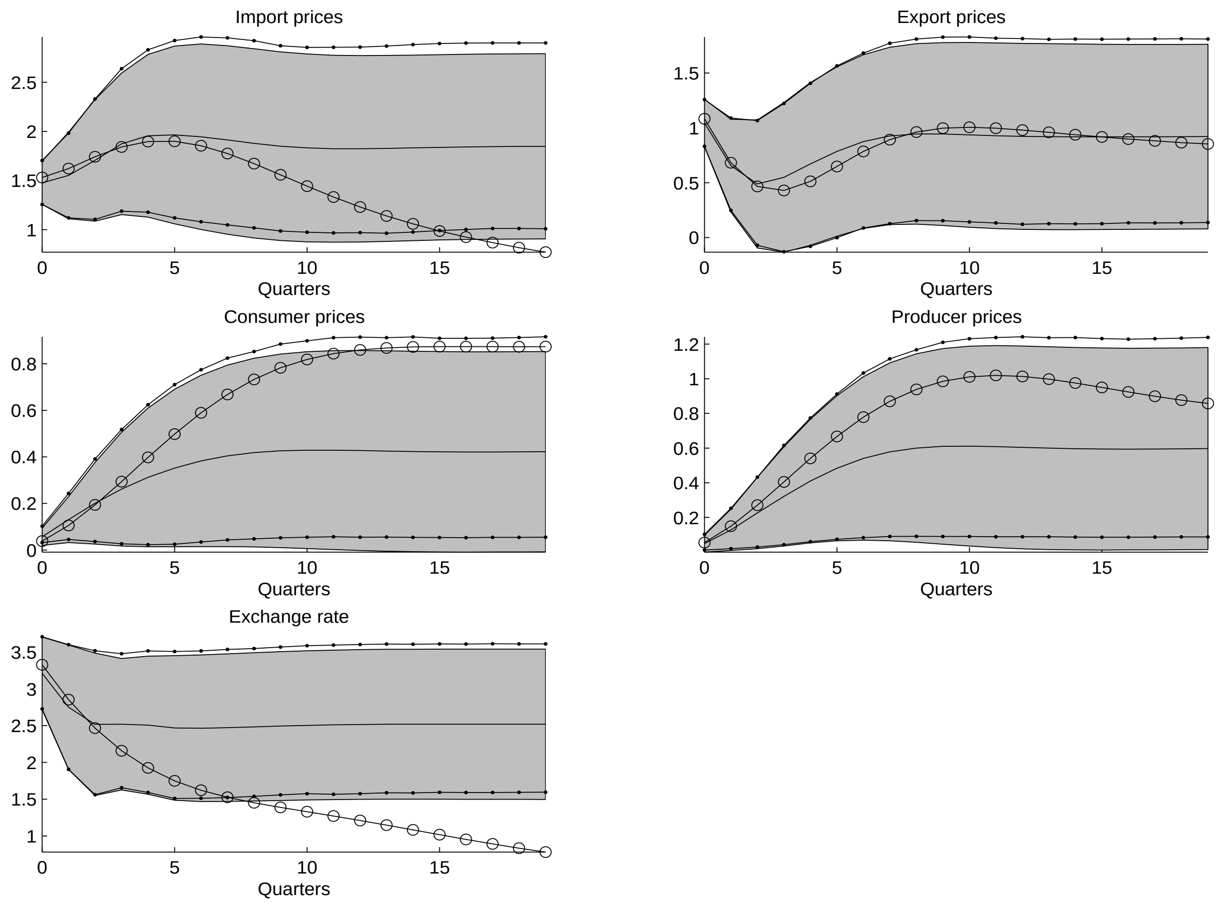

Figure 1: Responses to a one standard deviation UIP shock. In per cent. $T = 100$, $L = 2$.

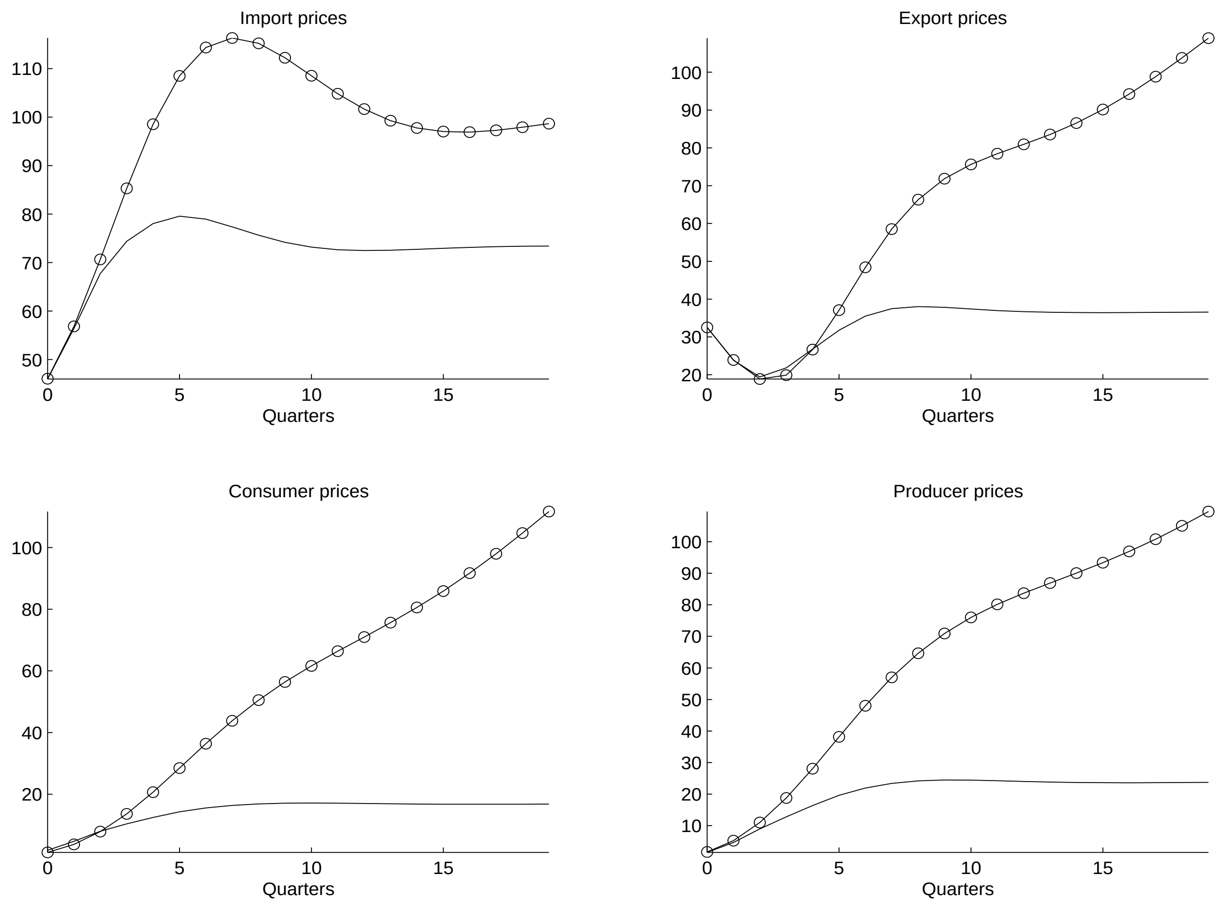

Figure 2: Responses to UIP shock normalised on exchange rate response. In per cent. $T = 100, L = 2$.

- Decomposition of bias

  (i) Bias from approximating VAR(∞) with low order VAR ('specification error')

  (ii) Small sample estimation bias

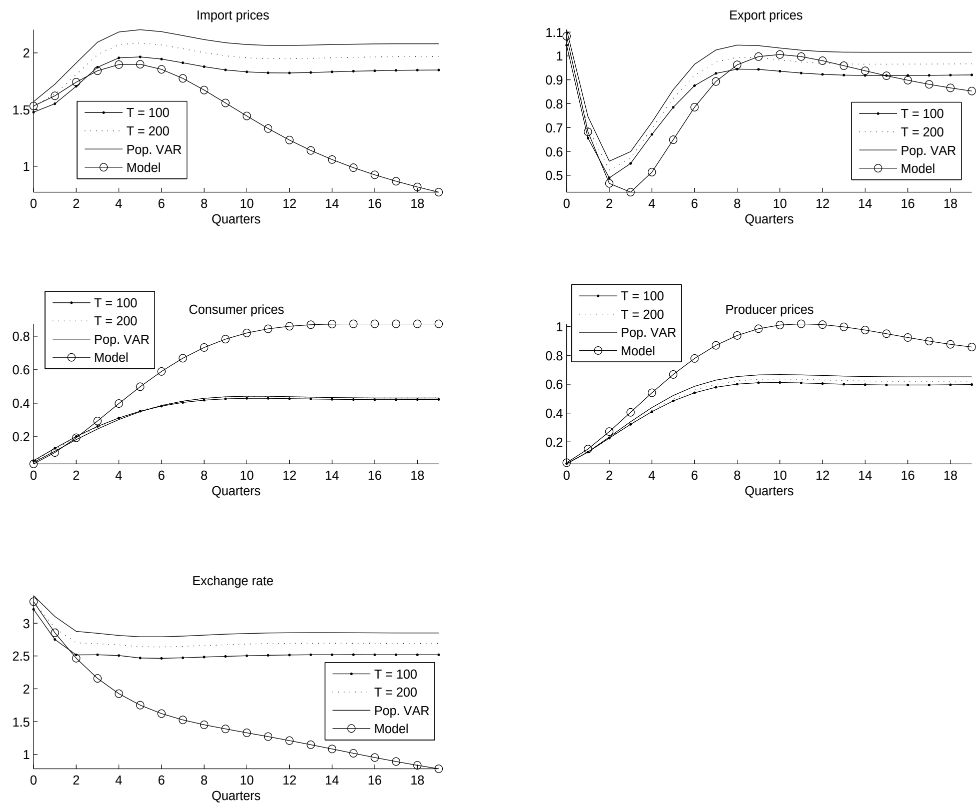

Figure 3: Accumulated responses to one standard deviation UIP shock. In per cent. $L = 2$.

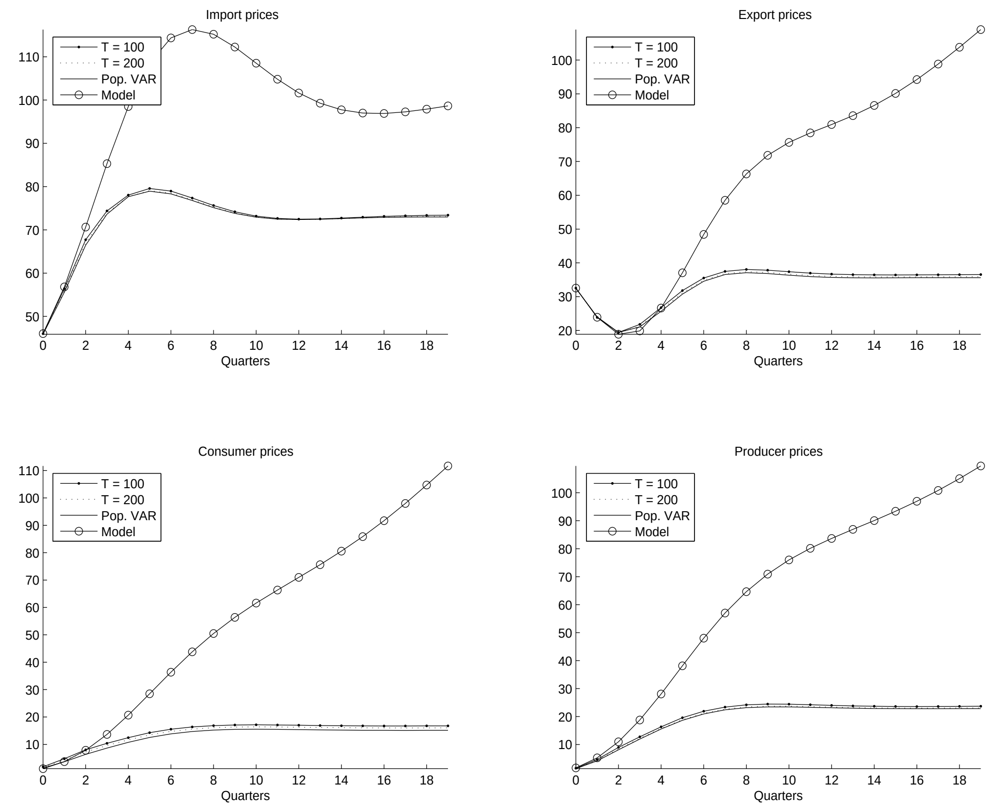

Figure 4: Normalised responses to one standard deviation UIP shock. In per cent. $L = 2$.

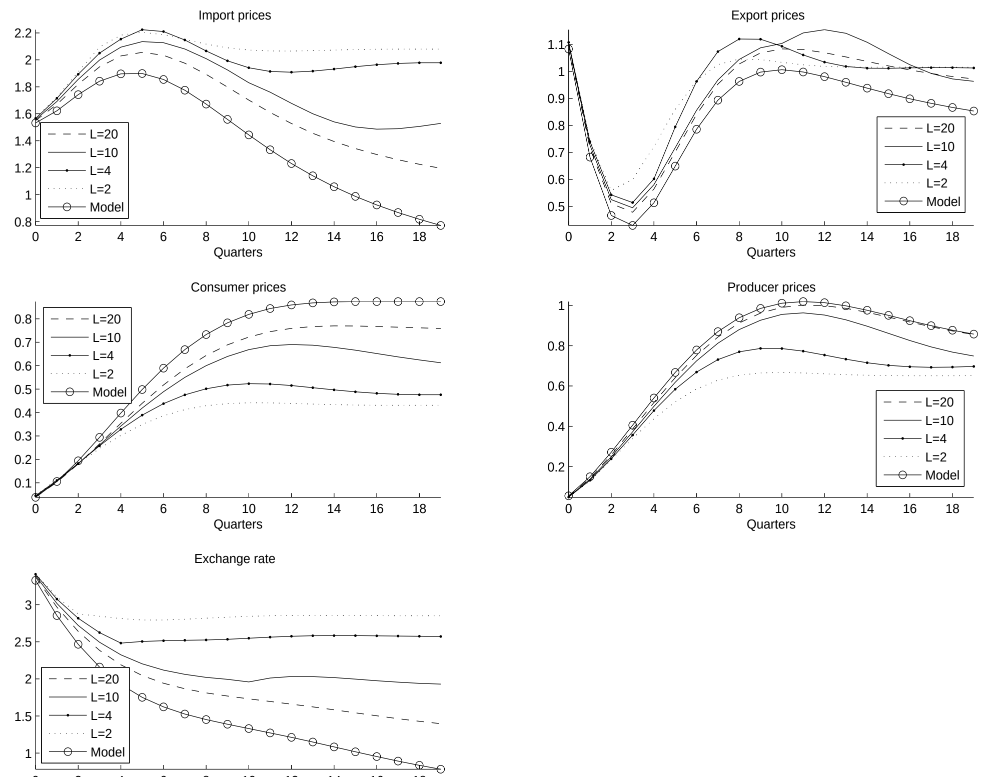

Figure 5: Accumulated responses to one standard deviation UIP shock in population version of VAR for different lag orders. In per cent.

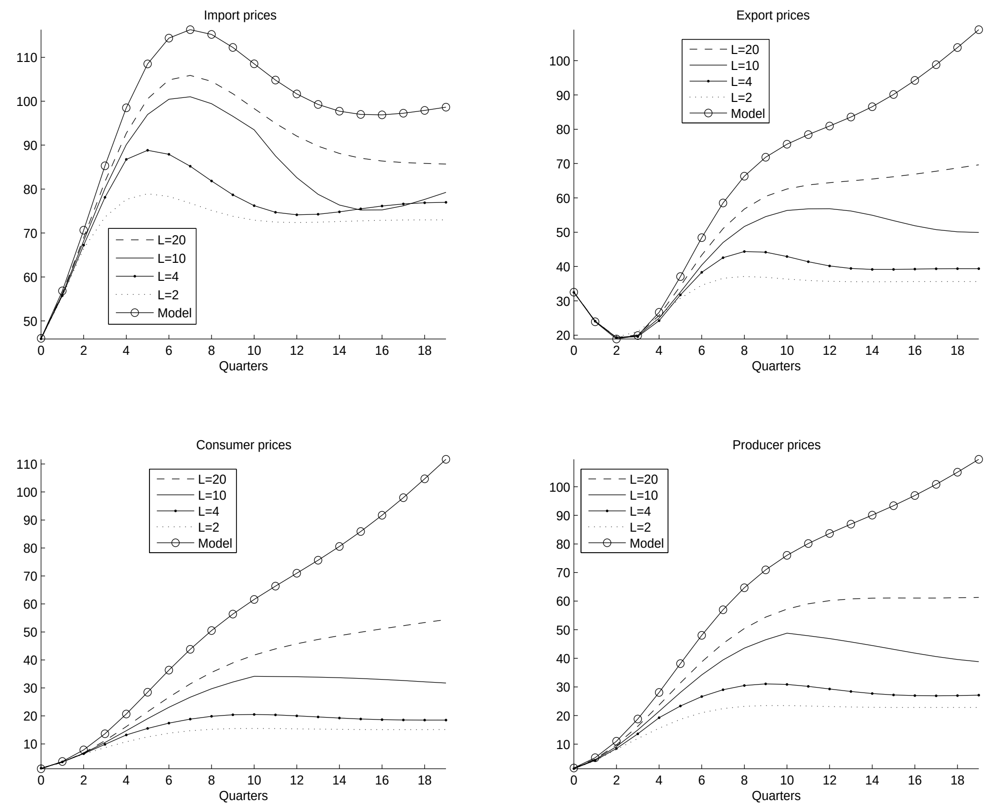

Figure 6: Normalised impulse responses to one standard deviation UIP shock in population version of VAR for different lag orders. In per cent.

Results from VEqCM

- VEqCM

$$\Delta y_t = \alpha\beta' y_{t-1} + A_1^* \Delta y_{t-1} + A_2^* \Delta y_{t-2} + \ldots + A_p^* \Delta y_{t-p} + \varepsilon_t$$

with

$$\beta' y_{t-1} = \left\{ \begin{array}{c} \ln P_{t-1}^m - \ln P_{t-1}^c \\ \ln P_{t-1}^x - \ln P_{t-1}^c \\ \ln P_{t-1}^y - \ln P_{t-1}^c \\ \ln e_{t-1} + \ln P_{t-1}^f - \ln P_{t-1}^c \end{array} \right\}$$

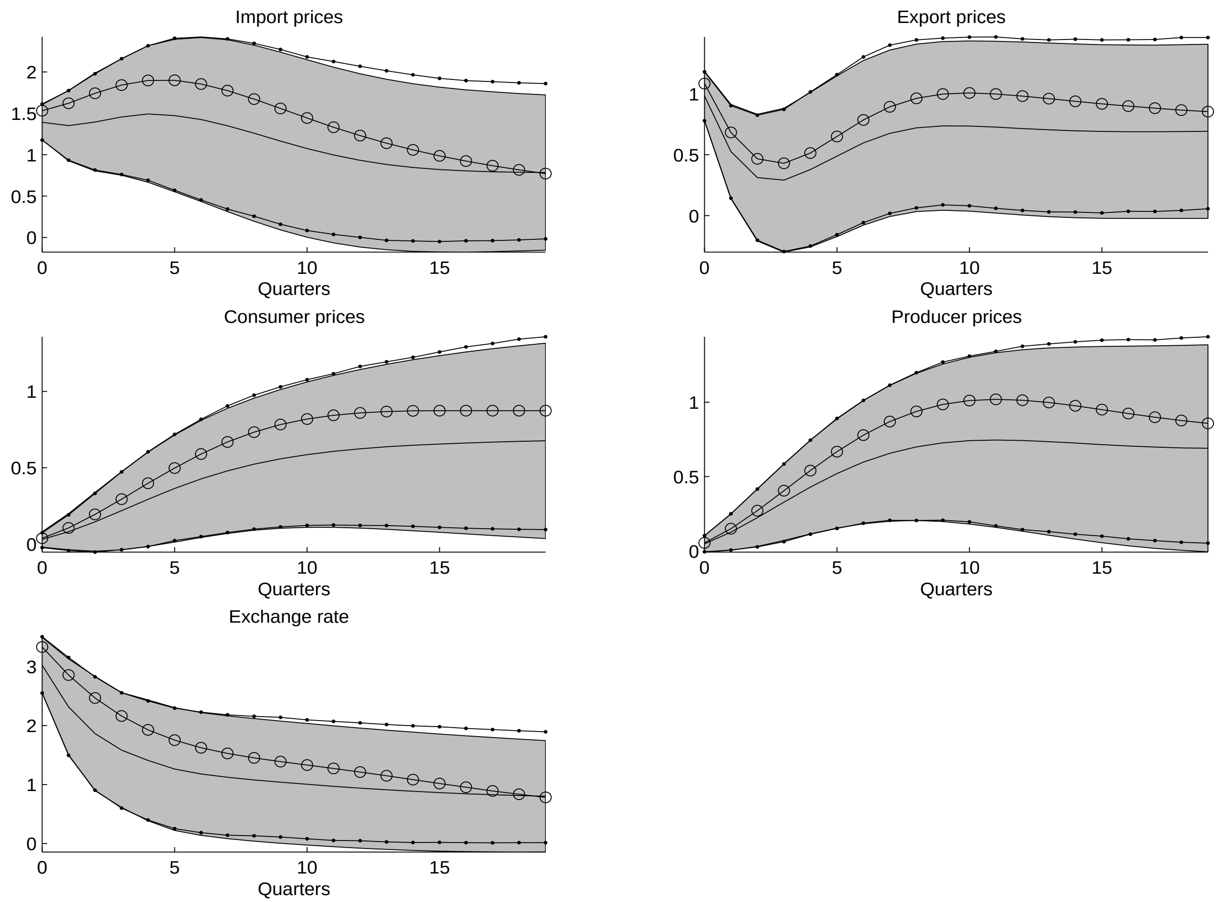

Figure 7: Accumulated responses to one standard deviation UIP shock. In per cent. VEqCM. $T = 100, L = 3$.

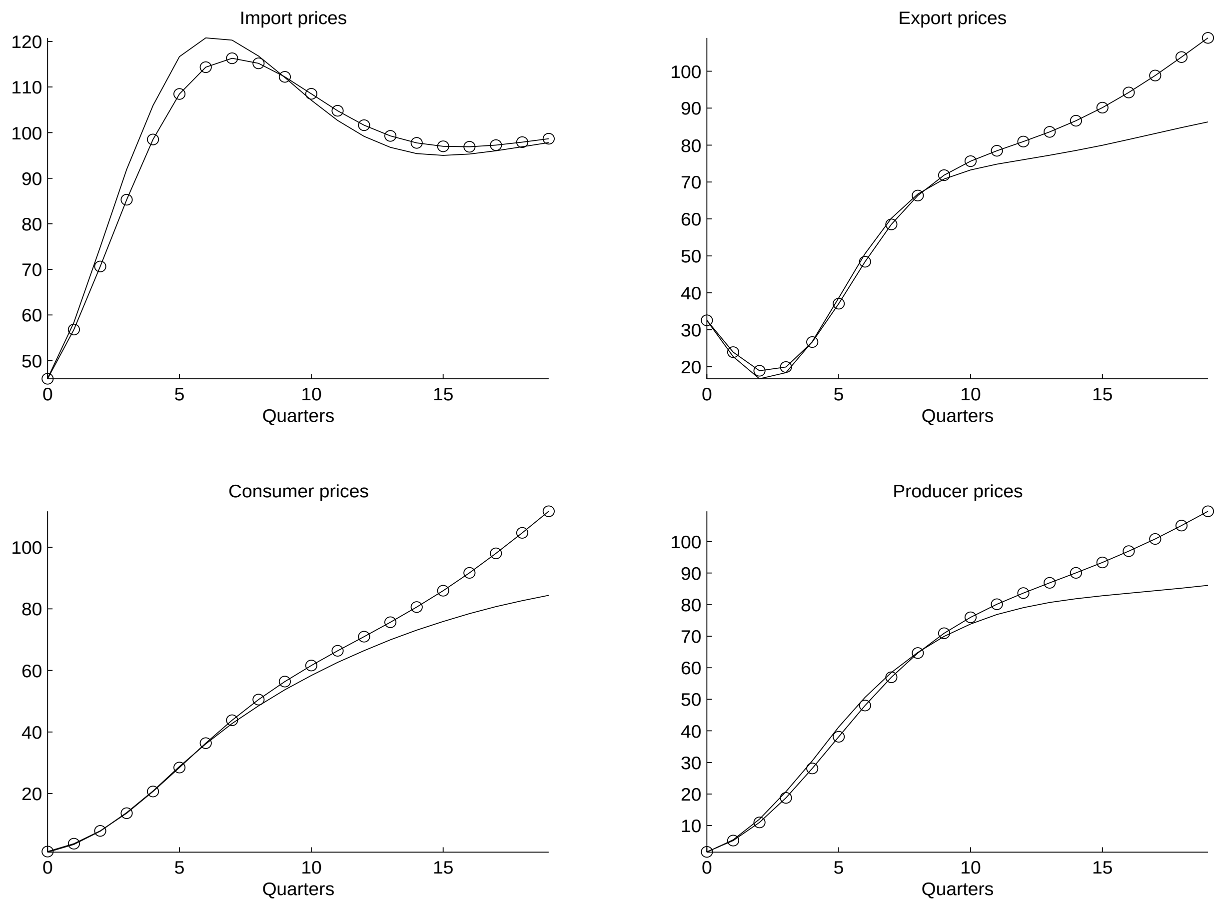

Figure 8: Normalised responses to one standard deviation UIP shock. In per cent. VEqCM. $T = 100, L = 3$.

Cointegration analysis

1. Generate 5000 artificial datasets of lengths $T = \{100, 200\}$ from DSGE model

2. Estimate VAR in levels (unrestricted constant, restricted trend)

3. Determine cointegration rank using trace test for cointegration

4. Test restrictions on cointegration space using LR tests

5. Redo (1)–(4) using population version of VEqCM as the data generating process

Table 1: Frequencies of chosen cointegration rank using Johansen's trace test for different lag orders. Numbers in parenteheses denote the preferred rank when using a small sample correction to the trace test

	$T = 100$					
	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
$L = 3$	17.5 (61.2)	41.3 (29.7)	28.0 (7.2)	10.0 (1.6)	2.7 (0.3)	0.5 (0.0)
$L = 5$	13.7 (84.4)	40.1 (13.9)	32.2 (1.7)	10.5 (0.2)	2.5 (0.0)	0.5 (0.0)

	$T = 200$					
	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
$L = 3$	0.1 (0.5)	2.0 (6.8)	16.1 (28.5)	45.5 (42.0)	30.6 (19.2)	5.7 (3.0)
$L = 5$	1.5 (12.3)	11.9 (32.9)	32.0 (32.7)	37.2 (16.9)	14.7 (4.3)	2.6 (0.1)

Table 2: Rejection frequencies for LR tests of restrictions on cointegration space conditional on r=4 for different lag-order selection criteria. 5% significance level.

$T=100$					
	$\ln\left(\frac{P_t^m}{P_t^c}\right)\sim I(0)$	$\ln\left(\frac{P_t^x}{P_t^c}\right)\sim I(0)$	$\ln\left(\frac{P_t^y}{P_t^c}\right)\sim I(0)$	$\ln\left(\frac{e_t}{P_t^c}\right)+0.005t\sim I(0)$	Joint
$L=3$	22.4	20.2	19.7	21.1	73.4
$L=5$	26.8	25.9	25.7	26.3	88.1

$T=200$					
	$\ln\left(\frac{P_t^m}{P_t^c}\right)\sim I(0)$	$\ln\left(\frac{P_t^x}{P_t^c}\right)\sim I(0)$	$\ln\left(\frac{P_t^y}{P_t^c}\right)\sim I(0)$	$\ln\left(\frac{e_t}{P_t^c}\right)+0.005t\sim I(0)$	Joint
$L=3$	18.2	13.2	14.5	14.3	36.7
$L=5$	22.0	14.3	16.7	17.9	48.0

Concluding remarks

- Are impulse responses from SVARs a useful tool to evaluate DSGE models with incomplete pass-through?

- Depends crucially on the specification of the time series properties of the data:

  - Estimates of exchange rate pass-through obtained from VAR in first differences exhibit systematic downward bias

  - Estimates obtained from VEqCM with cointegration relations implied by the DSGE are more accurate

  - Difficult to infer the correct rank or identify the cointegration relations using standard techniques

Appendix: Is the DSGE model empirically relevant?

- Estimate VAR in first differences on UK data 1980q1–2003q4 with exchange rate first in recursive ordering.

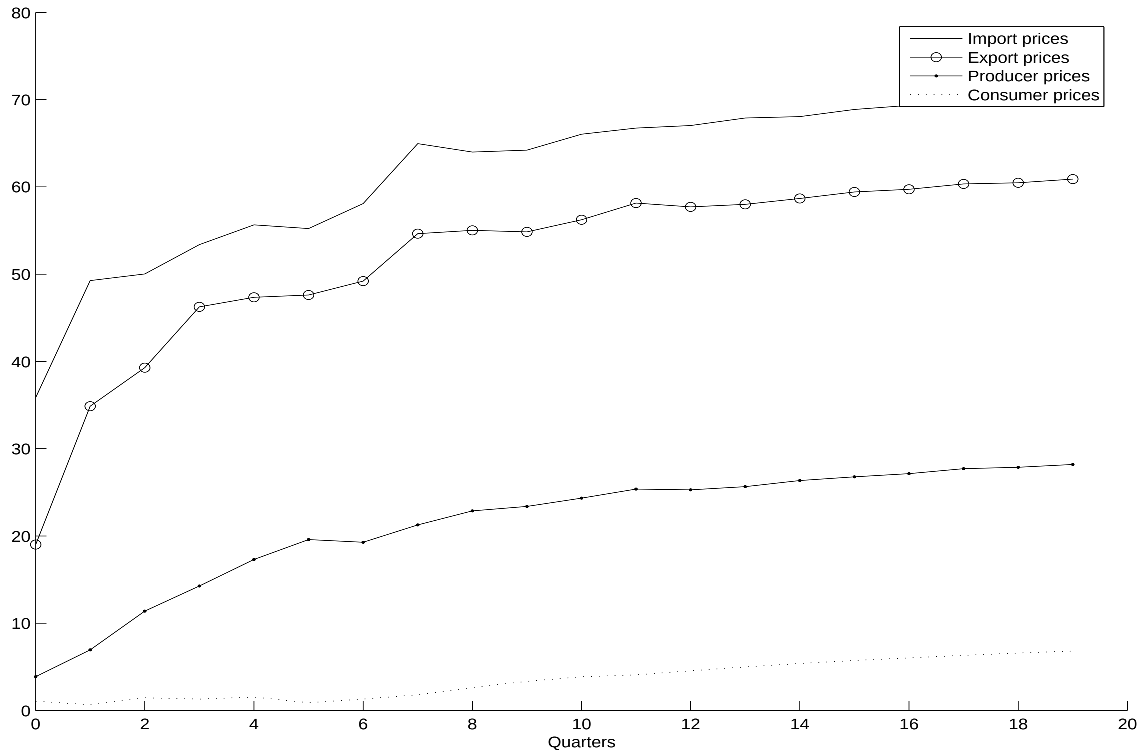

Figure 9: Normalised impulse responses to exchange rate shock. UK data. In per cent

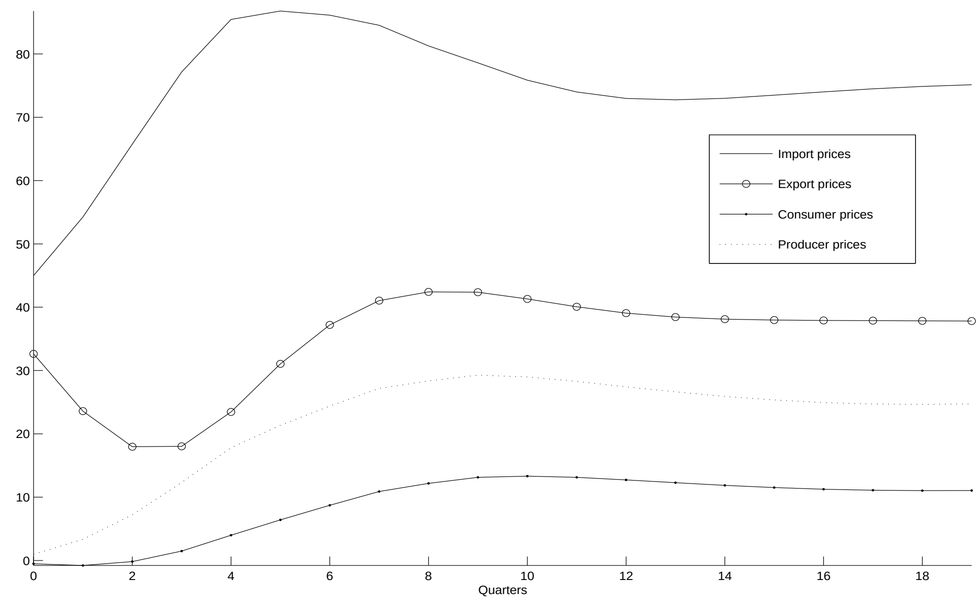

Figure 10: Normalised responses to exchange rate shock. Mean of 5000 datasets from DSGE model using recursive identification scheme. $T = 100$. In per cent

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