Modelling a Housing and Mortgage Crisis

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Current crisis has centered on:

- Borrower defaults on mortgages
- Knock-on effects of that on banks' own credit standing
- Tightened conditions for lending to new (mortgage) borrowers

Standard DSGE models cannot account for the above since:

- They assume away the possibility of default altogether
- Do not incorporate an optimizing heterogeneous banking sector and an interbank market
- Money is not modeled explicitly
- Default is an incomplete markets phenomenon -not every eventuality can be hedged- and trade plays a non-trivial role in the value of the collateral on mortgages

Shubik & Wilson, 1977, Dubey, Geanakoplos & Shubik, 2005

- Endogenous default under the existence of default penalties
- Argument: Default is an equilibrium phenomenon and default penalties should be intermediate to improve welfare

Geanakoplos, 2003, Geanakoplos and Zame, 1995, Chichilnisky, Heal & Tsomocos, 1995

- Collateral equilibrium
- The possibility of default limits available liquidity. If the potential default draws nearer, a liquidity crisis may ensue, causing a crash in asset prices

Tsomocos, 2003, Goodhart, Sunirand & Tsomocos, 2004,2006

- Optimizing banks
- Explicit modelling of money
- Capital requirement
- Liquidity shortage and financial fragility
- Initial shocks affecting a single institution can be amplified and spread across the banking system via the interbank and equity markets

Key features of Goodhart *et al.* 2004, 2006

- Money is introduced by a cash in advance constraint
- Heterogeneous households and banks
- A Central Bank which can inject extra money into the system through open market operations (OMOs)
- A Financial Supervisory Agency, which can set both liquidity and capital minimum requirements and imposes penalties on failures to meet such requirements and on defaults
- Individual bank borrowers are assigned , by history or by informational constraints, to borrow from a single bank (i.e. restricted participation)
- The amount of loans households repay is a choice variable - continuous (endogenous) default

Contribution of our paper

The main purpose is to model the market for mortgages and to examine the implications of default in bank lending and of a housing market crisis

To do so we extend Goodhart *et al.* 2004, 2006 in the following ways:

1. **Introduction of another good into the economy which is durable and gives utility in every period**

   The utility of consuming this good resembles the utility from buying a house

2. **We introduce a new agent λ who is only "born" in period two**

   The motivation behind this is that the healthy functioning of the housing market generally depends on the existence of first time buyers

3. We allow for **short-term loan markets** operating within each period to provide credit to first time buyers

4. **Explicit modelling of a market for mortgages**

   -Agents pledge as collateral the amount of housing they purchase and they default on their mortgage when the value of the collateral is less than the amount they have to repay (Geanakoplos, 2003, Geanakoplos and Zame, 1995)

   -When they default the bank seizes the amount of housing pledged as collateral

   -In this sense default is highly discontinuous as consumers do not choose the exact amount they want to default, but only decide on whether to default or not

5. Since we are not considering wider asset markets, **we exclude capital requirements for banks from our analysis**

Agents

- Four households $(\alpha, \beta, \phi, \lambda)$

  α and β are endowed with goods in the first period and in every state $s \in S$ tomorrow

  ϕ is endowed with housing only in the initial period

  λ is endowed with goods in every state $s \in S$ tomorrow

- Two banks (γ,δ)

  Households α and ϕ are affiliated with bank γ, while β and λ with bank δ

- The Central Bank acts in the interbank market at t=0 by providing liquidity M^{CB} and in the short-term loan markets at t=1 by providing liquidity $M^{CB}_{\gamma s}$ and $M^{CB}_{\delta s}$ in those markets organized by banks γ and δ respectively

- The set of all states is given by $s^* \in S^* = \{0\} \cup S$

Time-line

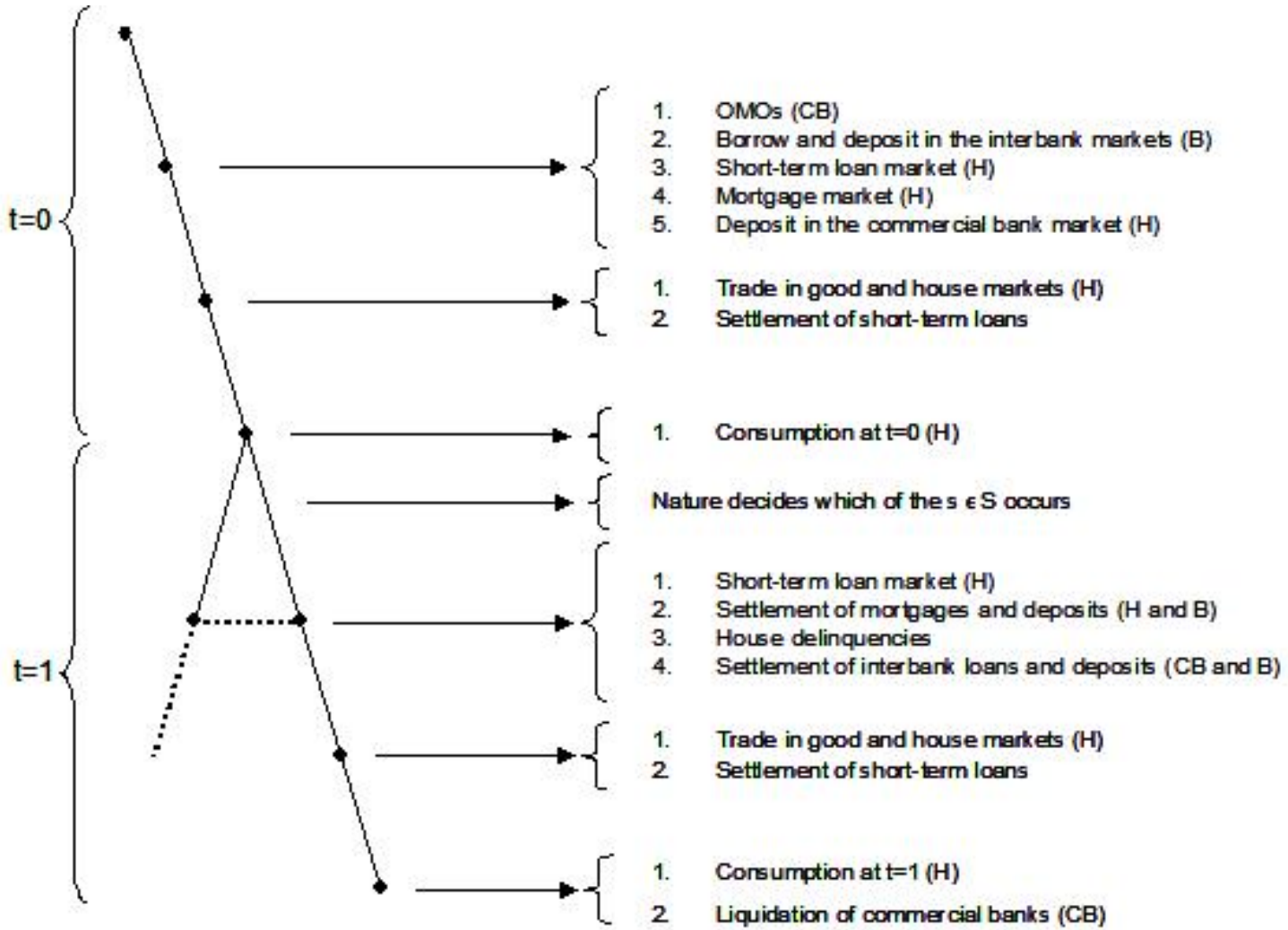

Figure: CB: Central Bank, B: Commercial Banks, H: Households

A note on collateral

- Agent $h \in (\alpha, \beta)$ enters a mortgage contract to purchase housing
- He pledges as collateral the amount of housing he purchases
- He honours his mortgage when the value of the collateral is greater than the amount he has to repay
- In essence he repays the min (value of collateral, mortgage amount)
- We denote by $S_1^h \subset S$ the set of states that agent h does not default on his mortgage, i.e. $S_1^h = \{s \in S : \text{ value of collateral} \geq \text{mortgage amount}\}$

Household $h \in (\alpha, \beta)$ optimization problem

Objective function

$$\max_{q_{s*1}, b_{s*2}, \mu_{s*}, \overline{\mu}} \Pi^h = u(e^h_{01} - q^h_{01}) + u\left(\frac{b^h_{02}}{p_{02}}\right) + \sum_{s \in S} \theta_s u(e^h_{s1} - q^h_{s1}) +$$
$$+ \sum_{s \in S^1} \theta_s u\left(\frac{b^h_{02}}{p_{02}} + \frac{b^h_{s2}}{p_{s2}}\right) + \sum_{s \notin S^1} \theta_s u\left(\frac{b^h_{s2}}{p_{s2}}\right)$$

Notation

- b^h_{s*2} $\equiv$ fiat money in the housing market in s^*
- $\overline{\mu}^h$ $\equiv$ mortgage amount that $h \in H$ takes out
- $\overline{r}^k$ $\equiv$ mortgage rate offered by bank k
- p_{s*1} $\equiv$ price of the good in s^*
- e^h_{s*1} $\equiv$ endowment of goods of $h \in H$ in s^*
- q^h_{s*1} $\equiv$ amount of goods offered for sale by $h \in H$ in s^*
- μ^h_{s*} $\equiv$ short-term borrowing by $h \in H$ in s^*
- r^k_{s*} $\equiv$ short-term rate offered by bank k in s^*
- p_{s*2} $\equiv$ price of housing in s^*
- m^h_{s*} $\equiv$ monetary endowment of $h \in H$ in s^*

Household $h \in (\alpha, \beta)$ optimization problem

Budget constraints

$$s.t. \quad b^h_{02} \leq \frac{\mu^h_0}{1 + r^k_0} + \frac{\overline{\mu}^h}{1 + \bar{r}^k} + m^h_0 \qquad (1)$$

(*expenditure* for housing at t $= 0$) $\leq$ (amount borrowed short $-$ term at t $= 0$) $+$ (mortgage amount) $+$ (initial private monetary endowment)

$$\mu^h_0 \leq p_{01} q^h_{01} \qquad (2)$$

(*short* $-$ *term* loan repayment) $\leq$ (good sales at t $= 0$)

$$b^h_{s2} + \overline{\mu}^h \leq \frac{\mu^h_s}{1 + r^k_s} + m^h_s, \text{ for } \; s \in S^h_1 \qquad (3)$$

(*expenditure* for housing in the second period , state s $\in S^h_1$) $+$ (mortgage repayment) $\leq$ (amount borrowed short $-$ term) $+$ (private monetary endowment in s $\in S^h_1$)

$$\mu^h_s \leq p_{s1} q^h_{s1}, \text{ for } \; s \in S^h_1 \qquad (4)$$

(*short* $-$ *term* loan repayment) $\leq$ (good sales in s $\in S^h_1$)

$$b^h_{s2} \leq \frac{\mu^h_s}{1 + r^k_s} + m^h_s, \text{ for } \; s \notin S^h_1 \qquad (5)$$

(*expenditure* for housing in the second period , state s $\notin S^h_1$) $\leq$ (amount borrowed short $-$ term) $+$ (private monetary endowment in s $\notin S^h_1$)

$$\mu^h_s \leq p_{s1} q^h_{s1}, \text{ for } \; s \notin S^h_1 \qquad (6)$$

(*short* $-$ *term* loan repayment) $\leq$ (good sales in s $\notin S^h_1$)

$$q^h_{s^*1} \leq e^h_{s^*1}, \text{ for } \; s^* \in S^* \qquad (7)$$

Household $\phi's$ optimization problem

Objective function

$$\max_{q_{s^*2}, b_{s^*1}, \overline{d}, \mu_{s^*}} \Pi^{\phi} = u\left(\frac{b^{\phi}_{01}}{p_{01}}\right) + u(e^{\phi}_{02} - q^{\phi}_{02}) + \sum_{s \in S} \theta_s u\left(\frac{b^{\phi}_{s1}}{p_{s1}}\right) + \sum_{s^*} \theta_s u\left(e^{\phi}_{02} - q^{\phi}_{02} - q^{\phi}_{s2}\right)$$

Notation

- $b^{\phi}_{s^*1} \equiv$ fiat money in the goods market in s^*
- $\overline{d}^{\phi} \equiv$ deposit amount for ϕ
- $\overline{r}_d \equiv$ deposit rate
- $e^{\phi}_{02} \equiv$ endowment of housing of ϕ at t=0
- $q^{\phi}_{s^*2} \equiv$ amount of housing offered for sale by ϕ in s^*
- $\mu^{\phi}_{s^*} \equiv$ short-term borrowing by ϕ in s^*
- $r^{\gamma}_{s^*} \equiv$ short-term rate offered by bank γ in s^*
- $m^{\phi}_{s^*} \equiv$ monetary endowment of ϕ in s^*

Household $\phi's$ optimization problem

Budget constraints

$$s.t. \quad b_{01}^{\phi} + \overline{d}^{\phi} \leq \frac{\mu_0^{\phi}}{1 + r_0^{\gamma}} + m_0^{\phi} \qquad (8)$$

(*expenditure* for goods) + (interperiod deposits) $\leq$ (amount borrowed short − term) + (private monetary endowment at t = 0)

$$\mu_0^{\phi} \leq p_{02} q_{02}^{\phi} \qquad (9)$$

(*short* − *term* loan repayment) $\leq$ (housing sales at t = 0)

$$q_{02}^{\phi} \leq e_{02}^{\phi} \qquad (10)$$

(*quantity* of housing sold at t = 0) $\leq$ (endowment of housing at t = 0)

$$b_{s1}^{\phi} \leq \frac{\mu_s^{\phi}}{1 + r_s^{\gamma}} + \overline{d}^{\phi}(1 + \overline{r}_d) + m_s^{\phi} \qquad (11)$$

(*expenditure* for goods) $\leq$ (amount borrowed short − term) + (deposits and interest payment) + (private monetary endowment in s)

$$\mu_s^{\phi} \leq p_{s2} q_{s2}^{\phi} \qquad (12)$$

(*short* − *term* loan repayment) $\leq$ (housing sales in s)

$$q_{s2}^{\phi} \leq e_{02}^{\phi} - q_{02}^{\phi} \qquad (13)$$

(*quantity* of housing sold in s) $\leq$ (endowment of housing at t = 0)− (quantity of housing sold at t = 0)

Household $\lambda's$ optimization problem

$$\max_{q_{s1}, b_{s2}, \mu_s} \Pi^{\lambda} = \sum_{s \in S} \theta_s u\left(e_{s1}^{\lambda} - q_{s1}^{\lambda}\right) + \sum_{s \in S} \theta_s u\left(\frac{b_{s2}^{\lambda}}{p_{s2}}\right)$$

$$s.t. \quad b_{s2}^{\lambda} \leq \frac{\mu_s^{\lambda}}{1 + r_s^{\delta}} + m_s^{\lambda} \qquad (14)$$

(*expenditure* for housing) $\leq$ (amount borrowed short $-$ term) $+$ (private monetary endowment in s)

$$\mu_s^{\lambda} \leq p_{s1} q_{s1}^{\lambda} \qquad (15)$$

(*short* $-$ *term* loan repayment) $\leq$ (good sales in s)

$$q_{s1}^{h} \leq e_{s1}^{h} \qquad (16)$$

(*quantity* of goods sold in s) $\leq$ (endowment of goods in s)

$b_{s2}^{\lambda} \equiv$ fiat money in the housing market in s^*

$\mu_s^{\lambda} \equiv$ short-term borrowing by λ in s

$e_{s1}^{\lambda} \equiv$ endowment of goods of λ in s

$q_{s1}^{\lambda} \equiv$ amount of goods offered for sale by λ in s

$r_s^{\delta} \equiv$ short-term rate offered by bank δ in s

$m_s^{\lambda} \equiv$ monetary endowment of λ in s

Bank $\gamma's$ optimization problem

$$\max_{\pi_s, m_{s*}, \overline{m}, \mu_l, \overline{\mu}_d} \Pi^{\gamma} = \sum_{s \in S} \theta_s \left(\pi_s^{\gamma} - c^{\gamma} (\pi_s^{\gamma})^2 \right)$$

$$s.t. \quad m_0^{\gamma} + \overline{m}^{\gamma} \leq \frac{\mu_l^{\gamma}}{1+\rho} + \frac{\overline{\mu}_d^{\gamma}}{1+\overline{r}_d} + e_0^{\gamma} \qquad (17)$$

$$(short - term \text{ lending}) + (\text{mortgage extension}) \leq (\text{interbank loans}) + (\text{consumer deposits}) + (\text{initial capital endowment at t} = 0)$$

$$m_s^{\gamma} + \overline{\mu}_d^{\gamma} + \mu_l^{\gamma} \leq \overline{m}^{\gamma}(1+\overline{r}_s^{\gamma}) + m_0^{\gamma}(1+r_0^{\gamma}) + e_s^{\gamma} \qquad (18)$$

$$(short - term \text{ lending}) + (\text{deposit repayment}) + (\text{interbank loan repayment} \leq \text{effective mortgage repayment}) + (\text{first period short} - \text{term loan repayment}) + (\text{capital endowment in s} \in \text{S})$$

$$\pi_s^{\gamma} = m_s^{\gamma}(1+r_s^{\gamma}) \qquad (19)$$

$$(profits) = (\text{short} - \text{term loans repayment s} \in \text{S})$$

- $\pi_s^{\gamma} \equiv$ bank $\gamma's$ profits at state s
- $m_{s*}^{\gamma} \equiv$ short-term loan extension by bank γ at state s
- $\overline{\mu}_d^{\gamma} \equiv$ amount borrowed from consumers in the form of deposits by bank γ
- $r_{s*}^{\gamma} \equiv$ short-term rate offered by bank γ in s^*
- $e_{s*}^{\gamma} \equiv$ capital endowment of bank γ at state s^*
- $\overline{m}^{\gamma} \equiv$ mortgage extension by bank γ
- $\mu_l^{\gamma} \equiv$ interbank borrowing by bank γ
- $\overline{r}_s^{\gamma} \equiv$ effective repayment rate on the mortgage at state s
- $\rho \equiv$ interbank rate

Bank δ's optimization problem

$$\max_{\pi_s, m_{s*}, \overline{m}, d_I} \Pi^{\delta} = \sum_{s \in S} \theta_s \left(\pi_s^{\delta} - c^{\delta} (\pi_s^{\delta})^2 \right)$$

$$s.t. \quad m_0^{\delta} + \overline{m}^{\delta} + d_I^{\delta} \leq e_0^{\delta} \qquad (20)$$

$$(short - term \text{ lending}) + (\text{mortgage extension}) + (\text{interbank deposits}) \leq (\text{initial capital endowment at t} = 0)$$

$$m_s^{\delta} \leq \overline{m}^{\delta}(1 + \bar{r}_s^{\delta}) + m_0^{\delta}(1 + r_0^{\delta}) + d_I^{\delta}(1 + \rho) + e_s^{\delta} \qquad (21)$$

$$(short - term \text{ lending}) \leq (\text{effective mortgage repayment}) + (\text{first period short} - \text{term loan repayment}) + (\text{interbank deposits and interest payment}) + (\text{capital endowment in s} \in \text{S})$$

$$\pi_s^{\delta} = m_s^{\delta}(1 + r_s^{\delta}) \qquad (22)$$

$$(profits) = (\text{short} - \text{term loans repayment s} \in \text{S})$$

- $\pi_s^{\delta} \equiv$ bank δ's profits at state s
- $m_{s*}^{\delta} \equiv$ short-term loan extension by bank δ at state s
- $\bar{r}_s^{\delta} \equiv$ effective repayment rate on the mortgage at state s
- $e_{s*}^{\delta} \equiv$ capital endowment of bank δ at state s^*
- $\overline{m}^{\delta} \equiv$ mortgage extension by bank δ
- $d_I^{\delta} \equiv$ interbank deposits by bank δ
- $r_{s*}^{\delta} \equiv$ short-term rate offered by bank δ in s^*

Market Clearing Conditions

Goods market

$$p_{01} = \frac{b_{01}^{\phi}}{q_{01}^{\alpha}+q_{01}^{\beta}}$$

$$p_{s1} = \frac{b_{s1}^{\phi}}{q_{s1}^{\alpha}+q_{s1}^{\beta}+q_{s1}^{\lambda}}$$

Housing Market

$$p_{02} = \frac{b_{02}^{\alpha}+b_{02}^{\beta}}{q_{02}^{\phi}}$$

$$p_{s2} = \frac{b_{s2}^{\alpha}+b_{s2}^{\beta}+b_{s2}^{\lambda}}{q_{s2}^{\phi}} \text{ for } s \in S_1^{\alpha} \cap S_1^{\beta}$$

$$p_{s2} = \frac{b_{s2}^{\alpha}+b_{s2}^{\beta}+b_{s2}^{\lambda}}{q_{s2}^{\phi}+\frac{b_{02}^{\alpha}}{p_{02}}} \text{ for } s \in S_1^{\beta} \setminus S_1^{\alpha} \cap S_1^{\beta}$$

$$p_{s2} = \frac{b_{s2}^{\alpha}+b_{s2}^{\beta}+b_{s2}^{\lambda}}{q_{s2}^{\phi}+\frac{b_{02}^{\beta}}{p_{02}}} \text{ for } s \in S_1^{\alpha} \setminus S_1^{\alpha} \cap S_1^{\beta}$$

$$p_{s2} = \frac{b_{s2}^{\alpha}+b_{s2}^{\beta}+b_{s2}^{\lambda}+b_{s2}^{\phi}}{\frac{b_{02}^{\alpha}}{p_{02}}+\frac{b_{02}^{\beta}}{p_{02}}} \text{ for } s \notin S_1^{\alpha} \cup S_1^{\beta}$$

Mortgage market

$$1+\bar{r}^k = \frac{\overline{\mu}^h}{\overline{m}^k}$$

$$1+\bar{r}_s^k = 1+\bar{r}^k \text{ for } s \in S_1^h$$

$$1+\bar{r}_s^k = (1+\bar{r}^k)\frac{b_{02}^h}{\overline{\mu}^h}\frac{p_{s2}}{p_{02}} \text{ for } s \notin S_1^h$$

Interbank Market

$$1+\rho = \frac{\mu_I^{\gamma}}{d_I^{\delta}+M^{CB}}$$

Short-term Loan Market

$$1+r_0^{\gamma} = \frac{\mu_0^{\alpha}+\mu_0^{\phi}}{m_0^{\gamma}}$$

$$1+r_s^{\gamma} = \frac{\mu_s^{\alpha}+\mu_s^{\phi}}{m_s^{\gamma}+M_{\gamma s}^{CB}}$$

$$1+r_0^{\delta} = \frac{\mu_0^{\beta}}{m_0^{\delta}}$$

$$1+r_s^{\delta} = \frac{\mu_s^{\beta}+\mu_s^{\lambda}}{m_s^{\delta}+M_{\delta s}^{CB}}$$

Consumer Deposit Market

$$1+\bar{r}_d = \frac{\overline{\mu}_d^{\gamma}}{\overline{d}^{\phi}}$$

A note on effective returns on mortgages

In the second period the bank receives the repayment on the mortgage it extended

- Full repayment for $s \in S_1^h$
- Partial elsewhere since the value of the collateral is less than the amount of the mortgage

The effective return on the mortgage is $\frac{\min(\text{ value of collateral, mortgage amount })}{\textit{initial credit extension}}$ or

$1 + \bar{r}_s^k = \frac{\min\left(\frac{b_{02}^h}{p_{02}} p_{s2}, \bar{\mu}^h\right)}{\bar{m}^k}$, where $k = \gamma$ for $h = \alpha$ and $k = \delta$ for $h = \beta$

From market clearing we get:

- $1 + \bar{r}_s^k = 1 + \bar{r}^k$ for $s \in S_1^h$
- $1 + \bar{r}_s^k = (1 + \bar{r}^k) \frac{b_{02}^h}{\bar{\mu}^h} \frac{p_{s2}}{p_{02}}$ for $s \notin S_1^h$

Definition of Equilibrium

We say that $(\sigma^{\alpha}, \sigma^{\beta}, \sigma^{\phi}, \sigma^{\lambda}; p_{s^*1}, p_{s^*2}, r^{\gamma}_{s^*}, r^{\delta}_{s^*}, \overline{r}^{\gamma}, \overline{r}^{\gamma}_{s}, \overline{r}^{\delta}, \overline{r}^{\delta}_{s}, \rho)$ is a **monetary equilibrium with commercial banks, collateral and default** iff:

(i)

$$(a)\ \sigma^{\mathrm{n}} \in \mathrm{Argmax}_{\sigma^{\mathrm{n}} \in \mathrm{B}^{\mathrm{n}}(\eta)} \mathrm{P}^{\mathrm{n}}(\chi^{\mathrm{n}}),\ \mathrm{n} \in \{\alpha, \beta, \phi, \lambda\}$$

$$(b)\ \sigma^{\mathrm{k}} \in \mathrm{Argmax}_{\sigma^{\mathrm{k}} \in \mathrm{B}^{\mathrm{t}}(\eta)} \mathrm{P}^{\mathrm{k}}(\chi^{\mathrm{k}}),\ \mathrm{k} \in \{\gamma, \delta\}$$

(ii) All markets clear.

Initial Equilibrium

Selection of exogenous variables

- Our model is general enough to allow for the examination of a wide variety of shocks, which can lead to financial instability
- In our simulation the shocks that we model in the second period of our two period model can be categorised as supply shocks, in which the endowment of our agents declines greatly in the case of adverse shock
- We have chosen the exogenous parameters in our model in such a way as to be able to illustrate a housing and mortgage crisis
- Three different scenarios which can occur in the second period. State 1 occurs with the highest probability (0.9) and state 2 is more probable than state 3 (0.075 and 0.025 respectively)

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Selection of exogenous variables

- State 1 is the good period in which neither borrower defaults. In state 2 one of the two agents, Mr β, defaults on his mortgage debt, but the other does not. In state 3 both default
- Agent α is richer in endowments of the good in the first period, whereas agent β is relatively richer in the second state in the second period
- Bank γ has less initial capital than bank δ, while it has more capital in the second period
- The capital of both banks in the second period can be interpreted as outside banking profits or capital injections obtained in the second period and will play a crucial role in the comparative statics we perform

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Table: Exogenous variables

Coefficient of risk aversion	Endowment Goods	Houses	Money	Capital	Others
$c^{\alpha} = 1.3$	$e_{01}^{\alpha} = 11$	$e_{02}^{\phi} = 5.5$	$m_0^{\alpha} = 0.1$	$e_0^{\gamma} = 4$	$M^{CB} = 65$
$c^{\beta} = 1.3$	$e_{11}^{\alpha} = 10$		$m_1^{\alpha} = 0.1$	$e_1^{\gamma} = 0.7$	$M_{\gamma 1}^{CB} = 10.9$
$c^{\phi} = 1.3$	$e_{21}^{\alpha} = 10$		$m_2^{\alpha} = 4.4$	$e_2^{\gamma} = 0.7$	$M_{\gamma 2}^{CB} = 8$
$c^{\lambda} = 1.3$	$e_{31}^{\alpha} = 0.7$		$m_3^{\alpha} = 0.1$	$e_3^{\gamma} = 0.7$	$M_{\gamma 3}^{CB} = 0.5$
$c^{\gamma} = 0.005$	$e_{01}^{\beta} = 2$		$m_0^{\beta} = 5.8$	$e_0^{\delta} = 13$	$M_{\delta 1}^{CB} = 2.4$
$c^{\delta} = 0.005$	$e_{11}^{\beta} = 7$		$m_1^{\beta} = 0.1$	$e_1^{\delta} = 1$	$M_{\delta 2}^{CB} = 0.8$
	$e_{21}^{\beta} = 3$		$m_2^{\beta} = 0.1$	$e_2^{\delta} = 1$	$M_{\delta 3}^{CB} = 0.5$
	$e_{31}^{\beta} = 0.1$		$m_3^{\beta} = 0.1$	$e_3^{\delta} = 1$	$\theta_1 = 0.90$
	$e_{11}^{\lambda} = 4$		$m_0^{\phi} = 0.1$		$\theta_2 = 0.075$
	$e_{21}^{\lambda} = 4$		$m_1^{\phi} = 0.1$		$\theta_3 = 0.025$
	$e_{31}^{\lambda} = 3$		$m_2^{\phi} = 0.1$		
			$m_3^{\phi} = 0.1$		
			$m_1^{\lambda} = 0.1$		
			$m_2^{\lambda} = 0.1$		
			$m_3^{\lambda} = 0.1$		

Discussion of Initial Equilibrium

- The level of default on the mortgages depends on the relative-second period-differential between the value of houses that each agent has bought and the mortgage amount they have to repay
- Agent α, who is richer in the first period, needs to take a comparatively lower loan to value mortgage for the amount of houses he wants to purchase than agent β, since he can finance the purchase through the sale of goods in the first period
- The effective return to the lending bank on the mortgages in state 3 when both agents default will be higher for α than β
- Since α does not default in state 2, the result is a lower interest rate on the mortgage for α than for β

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- The level of default on the mortgages depends on the relative-second period-differential between the value of houses that each agent has bought and the mortgage amount they have to repay
- Agent α, who is richer in the first period, needs to take a comparatively lower loan to value mortgage for the amount of houses he wants to purchase than agent β, since he can finance the purchase through the sale of goods in the first period
- The effective return to the lending bank on the mortgages in state 3 when both agents default will be higher for α than β
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- The prices of the good and of the house move in a reverse way in the second period
- The good is relatively more expensive in state 2 than 1 and in state 3 than 2. The opposite holds for the price of the house
- Agents default on their mortgages when the value of the house is low
- This happens when the endowments of goods are low, an adverse supply shock, since agents will not have enough income to allocate to the housing market
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Discussion of Initial Equilibrium

STATE 1

- α and β do not default on their mortgages, and then find themselves with more houses than they want, they sell some of the amount they bought in the previous period
- House prices are high relative to goods prices, and utility maximization leads α and β to switch out of housing into goods
- Endowments of goods are high and there is a strong demand for housing from agent λ a first time buyer who enters the economy in the second period
- Agent ϕ also finds it profitable to sell some of the housing he is left with at those prices

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STATE 2

- β defaults on his mortgage and essentially loses his house, he finds himself in a situation when he still wants to purchase some housing
- Although the supply of houses due to delinquencies is high, his demand in combination with $\alpha's$ and $\lambda's$ prevents the price of houses from collapsing
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STATE 3

- α and β become extremely poor in their endowments of goods in that state, so their demand for houses is infinitely low
- The housing market should collapse and agent λ, who is the only one endowed with a sufficient amount of the scarce good, would enjoy the services from the purchase of housing at a very low price
- This is avoided for two reasons:
  1. Agent ϕ finds it profitable to purchase back some of the houses he sold in the first period
  2. In state 3 banks are short of liquidity$\rightarrow$ Agent λ can only find credit at an extremely high interest rate, which prevents him from enjoying the full benefits of the falling housing market

     Note: This is not the case for ϕ as he has money at hand from depositing in the first period

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Discussion of Initial Equilibrium

Table: Initial Equilibrium

Prices	Interest rates	Loans/Deposits *Households*		Loans/Deposits *Banks*	Repayment rates on mortgages	Goods	Houses
$p_{01} = 4.10$	$\overline{r}^{\gamma} = 0.079$	$\overline{\mu}^{\alpha} = 12.33$	$\mu_1^{\lambda} = 6.23$	$\overline{m}^{\gamma} = 11.43$	$v_1^{\alpha} = 100\%$	$q_{01}^{\alpha} = 7.20$	$b_{02}^{\alpha} = 49.78$
$p_{11} = 2.60$	$\overline{r}^{\delta} = 0.123$	$\mu_0^{\alpha} = 29.47$	$\mu_2^{\lambda} = 6.97$	$\overline{\mu}_d^{\gamma} = 25.50$	$v_2^{\alpha} = 100\%$	$q_{11}^{\alpha} = 4.42$	$q_{12}^{\alpha} = 0.09$
$p_{21} = 3.10$	$r_0^{\gamma} = 0.043$	$\mu_1^{\alpha} = 12.80$	$\mu_3^{\lambda} = 18.46$	$\overline{\mu}_l^{\gamma} = 69.07$	$v_3^{\alpha} = 62\%$	$q_{21}^{\alpha} = 4.48$	$b_{22}^{\alpha} = 5.12$
$p_{31} = 12.31$	$r_1^{\gamma} = 0.047$	$\mu_2^{\alpha} = 13.65$		$m_0^{\gamma} = 83.24$	$v_1^{\beta} = 100\%$	$q_{31}^{\alpha} = 0.35$	$b_{32}^{\alpha} = 1.24$
$p_{02} = 26.12$	$r_2^{\gamma} = 0.047$	$\mu_3^{\alpha} = 4.35$		$m_1^{\gamma} = 5.28$	$v_2^{\beta} = 62\%$	$q_{01}^{\beta} = 0.29$	$b_{02}^{\beta} = 17.58$
$p_{12} = 15.17$	$r_3^{\gamma} = 2.83$	$\overline{\mu}^{\beta} = 11.96$		$m_2^{\gamma} = 5.28$	$v_3^{\beta} = 28\%$	$q_{11}^{\beta} = 4.54$	$q_{12}^{\beta} = 0.04$
$p_{22} = 10.96$	$r_0^{\delta} = 0.043$	$\mu_0^{\beta} = 1.18$		$m_3^{\gamma} = 0.64$		$q_{21}^{\beta} = 1.71$	$b_{22}^{\beta} = 5.08$
$p_{32} = 5.04$	$r_1^{\delta} = 0.048$	$\mu_1^{\beta} = 12.43$		$\overline{m}^{\delta} = 10.65$		$q_{31}^{\beta} = 0.04$	$b_{32}^{\beta} = 0.29$
	$r_2^{\delta} = 0.049$	$\mu_2^{\beta} = 5.22$		$d_l^{\delta} = 1.22$		$b_{01}^{\phi} = 30.64$	$q_{02}^{\phi} = 2.20$
	$r_3^{\delta} = 1.58$	$\mu_3^{\beta} = 0.50$		$m_0^{\delta} = 1.13$		$b_{11}^{\phi} = 29.55$	$q_{12}^{\phi} = 0.27$
	$\overline{r}_d = 0.043$	$\overline{d}^{\phi} = 24.44$		$m_1^{\delta} = 15.41$		$b_{21}^{\phi} = 25.84$	$q_{22}^{\phi} = 0.03$
	$\rho = 0.043$	$\mu_0^{\phi} = 57.36$		$m_2^{\delta} = 10.82$		$b_{31}^{\phi} = 23.30$	$b_{32}^{\phi} = 2.30$
		$\mu_1^{\phi} = 4.14$		$m_3^{\delta} = 6.84$		$q_{11}^{\lambda} = 2.40$	$b_{12}^{\lambda} = 6.05$
		$\mu_2^{\phi} = 0.25$				$q_{21}^{\lambda} = 2.29$	$b_{22}^{\lambda} = 6.75$
						$q_{31}^{\lambda} = 1.50$	$b_{32}^{\lambda} = 7.25$

Herein we concentrate on four comparative statics that we find interesting, i.e.

1. A decrease in the money supply in the initial period (i.e. contractionary monetary policy)
2. An increased desire to take on risk by banks, (as occurred in 2003-6, and leads to adverse shocks having a stronger effect on the system)
3. An expected intervention by the authorities to provide liquidity assistance in very bad states
4. A combination of the first two simultaneously, (partly to examine how non-linear are the resulting effects)

Other comparative statics we have performed include a decrease in the liquidity in the short-term loan markets in the last period, a decrease in banks' initial capital, a decrease in banks' capital in the last period, a change in agents' expectations regarding the occurrence of each state of the world and a production shock in the goods market. The results can be found in our working paper.

The analysis involves the following principles, which derive from the model structure:

1. The determination of interest rates (liquidity structure of interest rates)
2. Quantity theory of money proposition
3. Fisher effect

Contractionary monetary policy

Table: Summary of directional effects

p_{01}	-	$\overline{m}^{\gamma}$	-	μ_1^{β}	-
p_{11}	-	$\overline{m}^{\delta}$	+	μ_2^{β}	-
p_{21}	-	m_0^{γ}	-	μ_3^{β}	-
p_{31}	-	m_1^{γ}	+	$\overline{d}^{\phi}$	-
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p_{22}	-	m_0^{δ}	-	μ_2^{ϕ}	+
p_{32}	-	m_1^{δ}	+	μ_1^{λ}	-
$\overline{r}^{\gamma}$	+	m_2^{δ}	+	μ_2^{λ}	-
$\overline{r}_3^{\gamma}$	-	m_3^{δ}	-	μ_3^{λ}	-
$\overline{r}^{\delta}$	+	$\overline{\mu}_d^{\gamma}$	-	U^{α}	-
$\overline{r}_2^{\delta}$	-	$\overline{\mu}_l^{\gamma}$	-	U^{β}	+
$\overline{r}_3^{\delta}$	-	d_l^{δ}	+	U^{ϕ}	-
ρ,$\overline{r}_d$, r_0^{γ}, r_0^{δ}	+	$\overline{\mu}^{\alpha}$	-	U^{λ}	+
r_1^{γ}	-	$\overline{\mu}^{\beta}$	+	$\gamma's$ profits	-
r_2^{γ}	+	μ_0^{α}	-	$\delta's$ profits	-
r_3^{γ}	-	μ_1^{α}	-		
r_1^{δ}	-	μ_2^{α}	-		
r_2^{δ}	-	μ_3^{α}	-		
r_3^{δ}	-	μ_0^{β}	-		

Contractionary monetary policy

- Decrease in the money supply (M^{CB}) in the interbank market in the initial period

  $\longrightarrow$ interbank rate goes up

  $\longrightarrow$ bank γ borrows less from the interbank market and therefore to reduce its supply of short-term loans and mortgages to Mr. α and Mr. ϕ

  $\longrightarrow$ the corresponding lending rates r_0^{γ} and $\overline{r}^{\gamma}$ go up

  $\longrightarrow$ Mr. α reduces his short-term and mortgage borrowing

- β faces stricter credit conditions in the short-term

  $\longrightarrow$ bank δ reallocates its portfolio and supplies slightly more mortgages to him

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Contractionary monetary policy

- Higher interest rate$\longrightarrow$trade becomes less efficient and quantities of goods and houses traded in the initial period go down and so do prices
- Low price of housing and increased mortgage extension by bank δ

  $\longrightarrow \beta$ demands more mortgages

  $\longrightarrow$higher mortgage rate ($\overline{r}^{\delta}$)
- Less money chases the same amount of goods

  $\longrightarrow$lower prices in the last period (**Quantity Theory of Money**)

  $\longrightarrow$lower effective returns on the mortgages

  $\longrightarrow$higher level of default given the higher mortgage rates and the increase in mortgage extension

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Contractionary monetary policy

- **Banking Profits**
  Expected banking profits go down, since the higher mortgage rates are not enough to outweigh the negative effect of increase default on mortgages
- **Households' welfare**
  -Mr. α is worse-off: He borrows less since he is affiliated with the poor bank
  -Mr. β is better-off: He undertakes a bigger mortgage to take advantage of the falling housing prices in the initial period and benefits from the falling short-term loan rates in the last period
  -Mr. λ is better-off: Credit is cheaper for him due to falling short-term loan rates in the last period
  -Mr. $\phi's$ welfare is almost unaffected
- **Financial Fragility**
  According to the Goodhart-Tsomocos financial stability measure, lower banking profits and increased default lead to higher financial instability due to contractinary monetary policy

Contractionary monetary policy

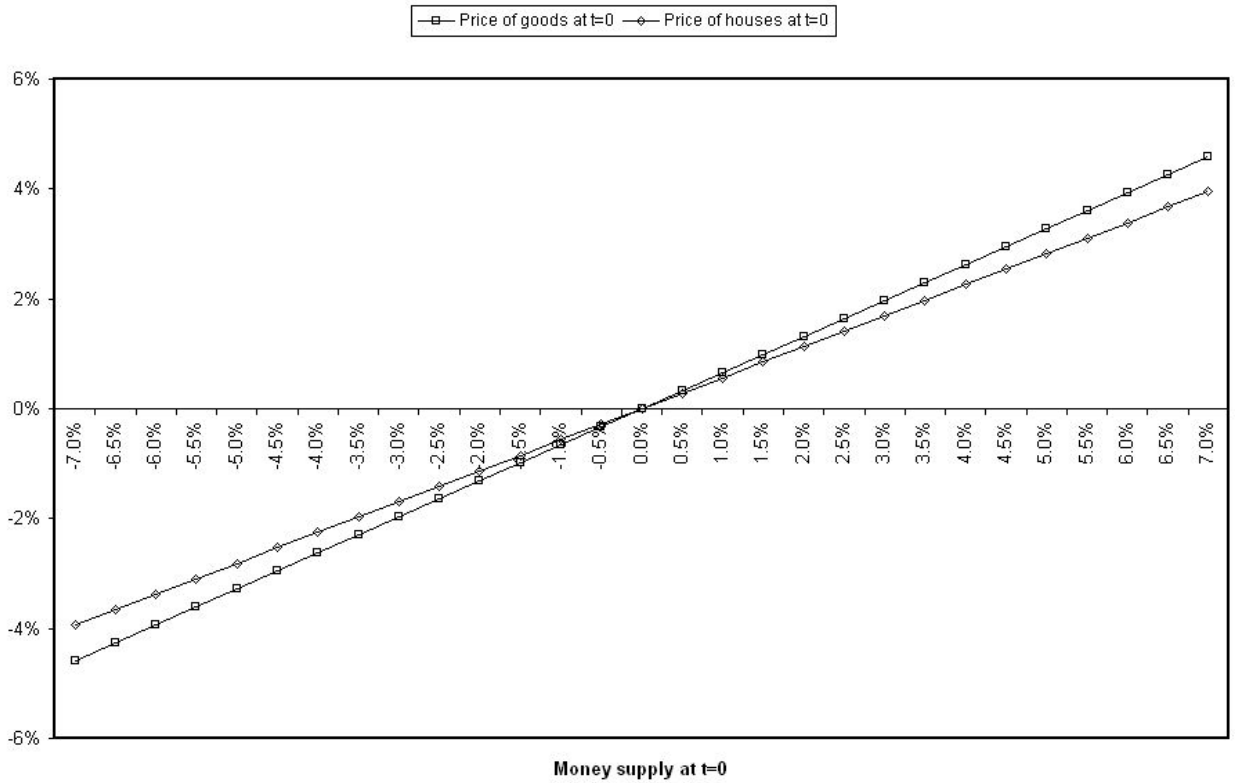

Figure: Housing and goods prices vs money supply

Contractionary monetary policy

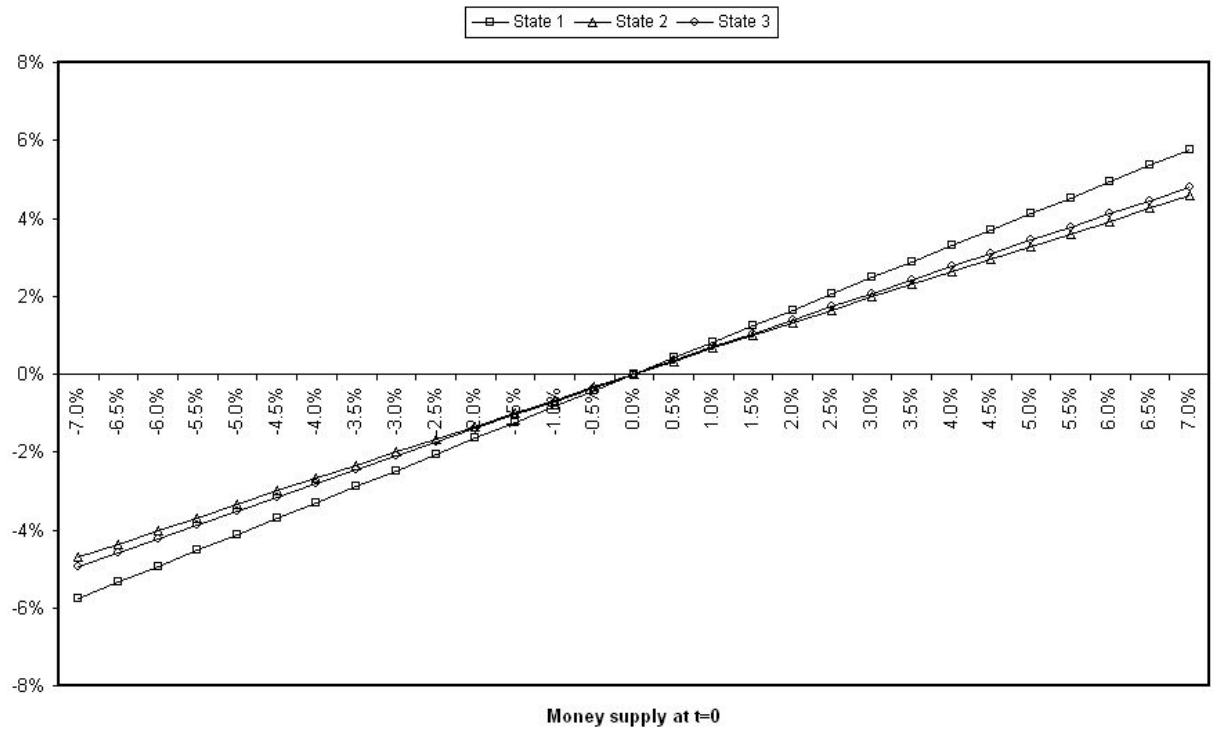

Figure: Short-term interest rates by bank δ vs money supply

Liquidity assistance to banks in the very bad state of the world

Table: Summary of directional effects

p_{01}	-	$\overline{m}^{\gamma}$	+	μ_1^{β}	-
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Liquidity assistance to banks in the very bad state of the world

Increase in the capital of banks in the third state of the world
$\longrightarrow$price increase in goods and housing at that state
$\longrightarrow$higher effective return for both banks in the very bad state
$\longrightarrow$banks will increase their extension of mortgages at the initial period
$\longrightarrow$mortgage rates go down and the demand for mortgages goes up
$\longrightarrow$bank δ will switch from interbank deposits towards mortgages
$\longrightarrow$interest rates in the short loan market go up in the initial period

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Liquidity assistance to banks in the very bad state of the world

- Effective returns on the mortgages go up and the overall default in the very bad state goes down
- However, both banks will sustain a drop in their expected profitability

  Reasons:

  1. Rates on mortgages to which they switch their portfolios go down
  2. Bank γ has to pay a higher interest for the money it borrows from depositors and the interbank market
  3. Bank δ does not fully take advantage of the higher interbank rate, since it reallocates its portfolio towards mortgages. In addition, the effective return on the mortgages it extends goes down in the second state

- Agents α, β and λ are better-off since the first two benefit from the lower mortgage rates and all three from the lower short-term rates in the last period, which translates into cheaper credit. Mr. ϕ is worse-off mainly due to the fact that liquidity assistance happened in the state that he had relatively more wealth than the other households, since he receives the money he deposited in the initial period

Banks become less risk-averse

Table: Summary of directional effects

p_{01}	-	$\overline{m}^{\gamma}$	+	μ_1^{β}	-
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r_2^{δ}	+	μ_3^{α}	+		
r_3^{δ}	+	μ_0^{β}	-		

Banks become less risk-averse

- Banks' first response will be to switch to riskier investments →extension of mortgages goes up and short-term lending goes down
- Bank δ also reduces its interbank deposits, which results in bank γ having less funds to extend credit
- Mr. α takes advantage of the lower mortgage rates and demands more mortgages
- He also reduces his sales of goods in the initial period, since he can finance his housing purchases with more mortgages, and the transaction cost for selling his goods (short-term interest rate) has gone up due to banks' funds reallocation
- Mr. β reduces his sales of goods and short-term lending

Banks become less risk-averse

- The mortgage rate falls more for Mr. β than for Mr. α is that he is affiliated with bank δ which has more funds to allocate to mortgages
- Although demand for housing goes up, its price in the initial period goes down **Reason:**the initial supply of goods onto the market by Mr. α and Mr. β has fallen and Mr. ϕ has to sell more of his housing endowment to fund his purchase of goods
- Mr. $\phi's$ disposable income falls and he allocates less money into the goods markets forcing their initial price to go down as well
- Lower housing prices and higher mortgage extension $\rightarrow$lower effective returns on mortgages because of higher absolute default in the economy in the bad states
- Depending on the severity of the reduction in risk-aversion and its initial level, aggregate default may increase a lot

Banks become less risk-averse

- **Banking Profits**
  Expected banking profits go up, since banks enjoy higher profits in expectation due to increased mortgage extension. However, banking profits is the bad states of the world will go down
- **Households' welfare**
  -Mr. α is worse-off: He faces a higher interest rate for short-term loans in the initial period, which is his main source for funding his housing purchases
  -Mr. β is better-off: will benefit from the lower mortgage rates and enjoy an increase in his utility, since he is poorly endowed in the initial period
  -Mr. λ is worse-off: Short-term interest rates in the last period go up in the presence of higher aggregate default
  -Mr. ϕ is better-off: The price of housing in the initial period decreases less than the price of goods
- **Financial Fragility**
  According to the Goodhart-Tsomocos financial stability measure, lower banking profits (in the bad states of the world) and increased default lead to higher financial instability due to a decrease in banks' risk-aversion

Banks become less risk-averse

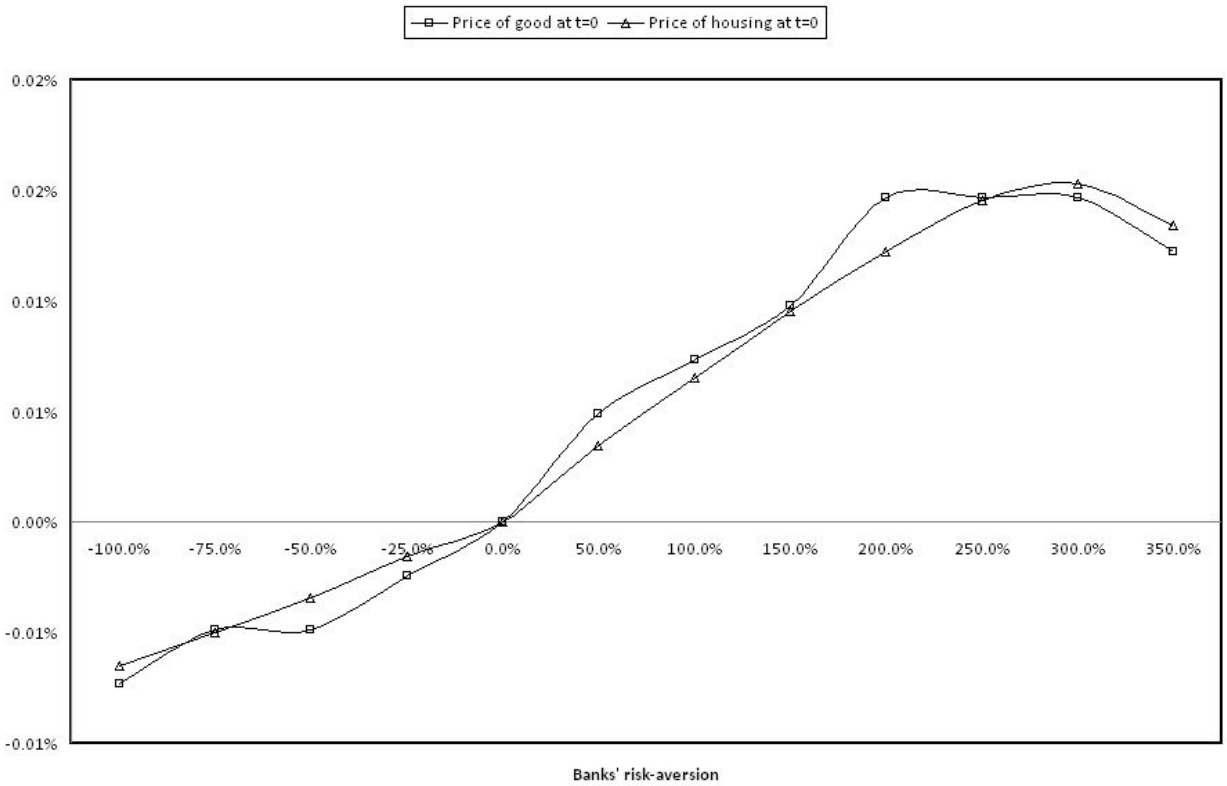

Figure: Housing and goods prices at t=0 vs banks' risk-aversion

Banks become less risk-averse

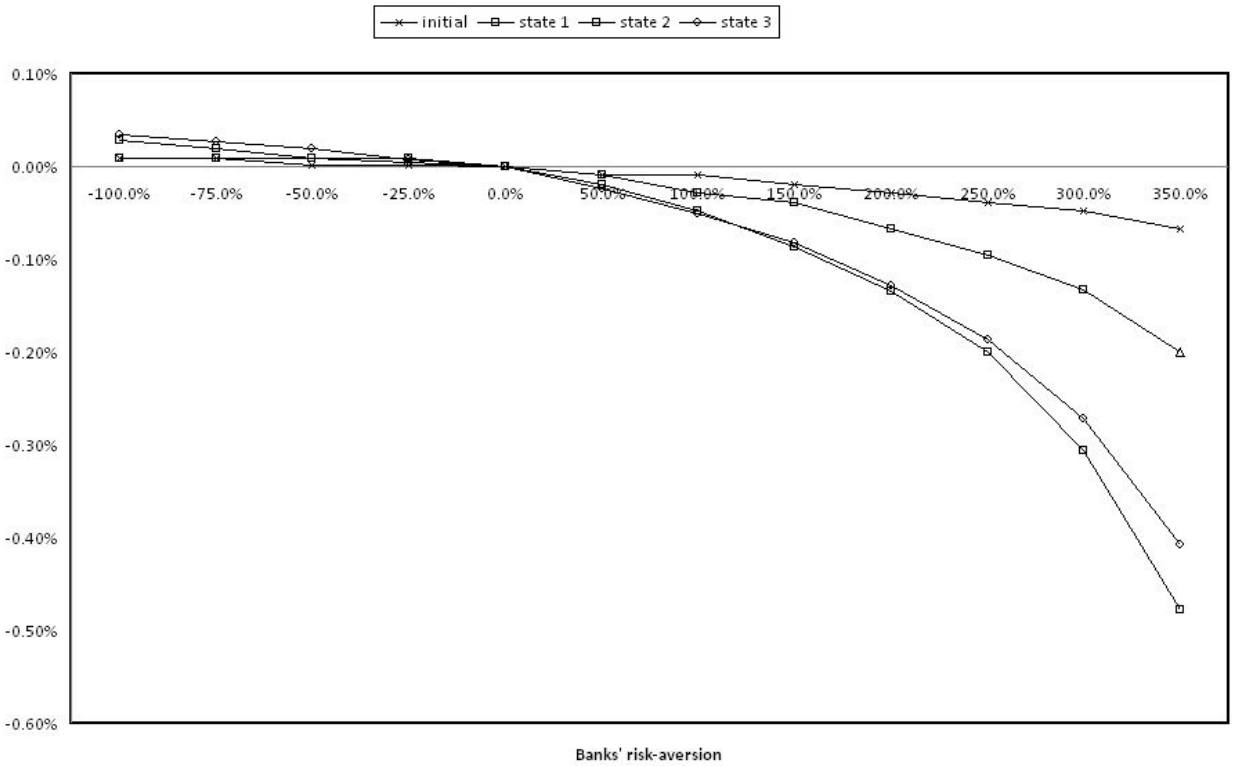

Figure: Short-term interest rates by bank δ vs banks' risk-aversion

Compound comparative static

Table: Summary of directional effects

p_{01}	-	$\overline{m}^{\gamma}$	-	μ_1^{β}	-
p_{11}	-	$\overline{m}^{\delta}$	+	μ_2^{β}	-
p_{21}	-	m_0^{γ}	-	μ_3^{β}	-
p_{31}	-	m_1^{γ}	+	$\overline{d}^{\phi}$	-
p_{02}	-	m_2^{γ}	+	μ_0^{ϕ}	-
p_{12}	-	m_3^{γ}	$\pm$	μ_1^{ϕ}	+
p_{22}	-	m_0^{δ}	-	μ_2^{ϕ}	+
p_{32}	-	m_1^{δ}	+	μ_1^{λ}	-
$\overline{r}^{\gamma}$	+	m_2^{δ}	+	μ_2^{λ}	-
$\overline{r}_3^{\gamma}$	-	m_3^{δ}	-	μ_3^{λ}	-
$\overline{r}^{\delta}$	+	$\overline{\mu}_d^{\gamma}$	-	U^{α}	-
$\overline{r}_2^{\delta}$	-	$\overline{\mu}_l^{\gamma}$	-	U^{β}	+
$\overline{r}_3^{\delta}$	-	d_l^{δ}	+	U^{ϕ}	-
ρ,$\overline{r}_d$, r_0^{γ}, r_0^{δ}	+	$\overline{\mu}^{\alpha}$	-	U^{λ}	+
r_1^{γ}	-	$\overline{\mu}^{\beta}$	+	$\gamma's$ profits	+
r_2^{γ}	+	μ_0^{α}	-	$\delta's$ profits	+
r_3^{γ}	$\pm$	μ_1^{α}	-		
r_1^{δ}	+	μ_2^{α}	-		
r_2^{δ}	-	μ_3^{α}	-		
r_3^{δ}	-	μ_0^{β}	-		

Compound comparative static

- Contractionary monetary policy and a decrease in banks' risk-aversion simultaneously
- Reduction in the money supply$\rightarrow$first order effect pushing the interbank rate up
  Bank δ increases its interbank lending and reduces its mortgage extension
- The reduction in risk-aversion will moderate this pressure
  The trade-off between these two effects will determine whether bank δ will extend more mortgages or not
  In our simulation we find that mortgage extension by bank δ increases
- The reduction is more severe for bank γ, since it is more dependent on monetary injections

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- Mortgage rates go up, since the demand does not decrease much due to the higher cost of short-term borrowing
- Prices of goods and housing in all periods and states go down, as predicted by the Quantity Theory of Money
- The pressure is greater due to lower risk-aversion
- The result is lower expected returns on mortgages, which translate into higher defaults in conjunction with the fact that mortgage rates were higher to start with
- Second order effect outweighed by the increased default due to a lower money supply
- **Banking Profits**
  On the one hand lower money supply and increased default put downward pressure on expected profits and on the other lower risk-aversion pushes them up. In our simulation the latter prevails

Compound comparative static

Non-linearities in default

- Default increases disproportionally when contractionary monetary policy is combined with a higher appetite for risk by banks
- When these adverse shocks occur at the same time, expected repayment on mortgages falls more than the aggregate change when they happen independently

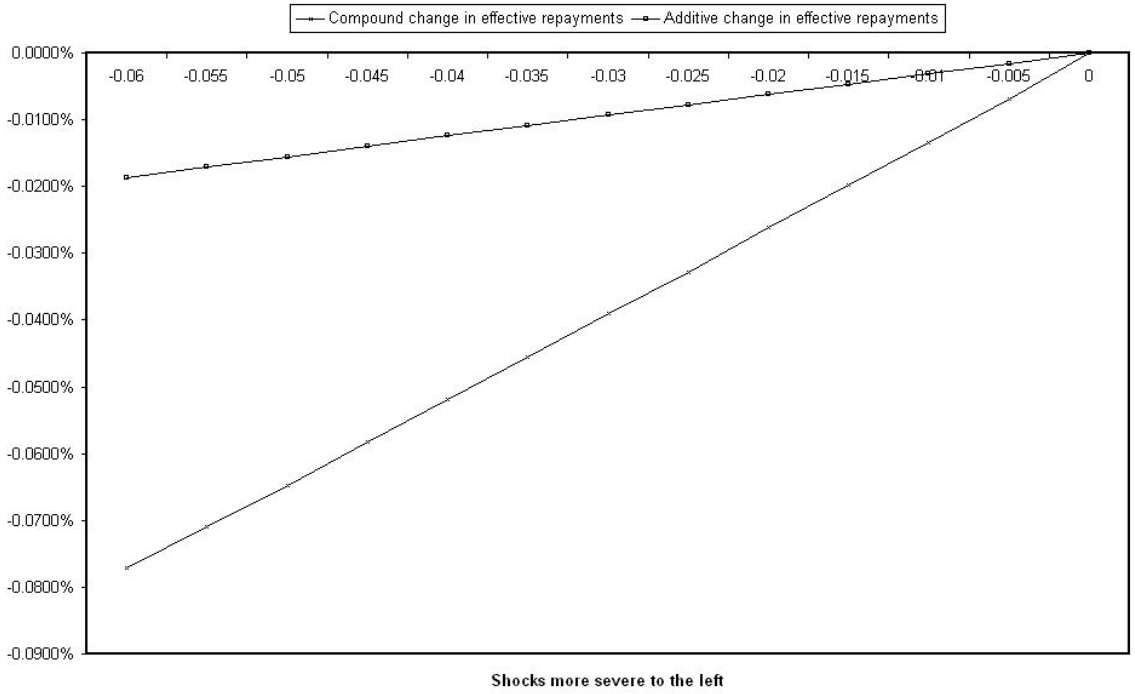

Figure: Nonlinear effects on mortgage repayment vs compound decrease in money supply and in banks' risk-aversion

Compound comparative static

- **Households' welfare**
  -Mr. α is worse-off: Stricter credit environment affects poorly capitalized banks more severely & the initial price of goods falls more that of housing affecting negatively the purchasing power of Mr. α
  -Mr. β is better-off: (a) Benefit from the falling price of housing in the initial period via entering a mortgage contract
  (b) Housing prices in state 1 fall more than goods' prices (fig. 13) due to the fact that Mr. ϕ decreased its deposits in the initial period and increases his sales of housing in that state to finance the purchase of goods
  (c) The lower demand for money by Mr. β in the last period (partially due to lower prices and higher defaults) and the well-capitalized position of bank δ put downward pressure on the short-term interest rates at the states in which agents default
  -Mr. λ is better-off: Credit is cheaper for him due to falling short-term loan rates in the last period
  -Mr. ϕ is worse-off: Stricter credit environment affects poorly capitalized banks more severely
- **Financial Fragility**
  According to the Goodhart-Tsomocos financial stability measure, lower banking profits (in the bad states of the world) and increased default lead to higher financial instability due to contractionary monetary policy and a decrease in banks' risk-aversion . The financial system will be more fragile if these two shocks happen together

Compound comparative static

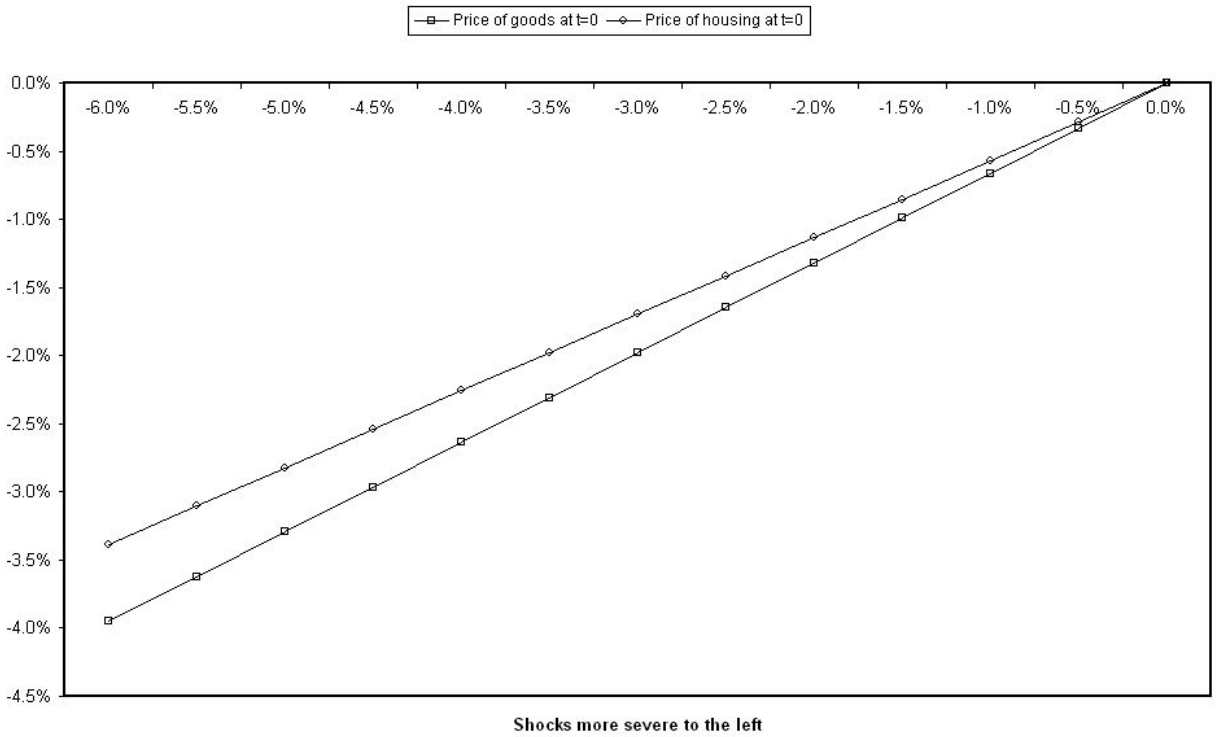

Figure: Housing and goods prices at t=0 vs compound decrease in money supply and in banks' risk-aversion

Compound comparative static

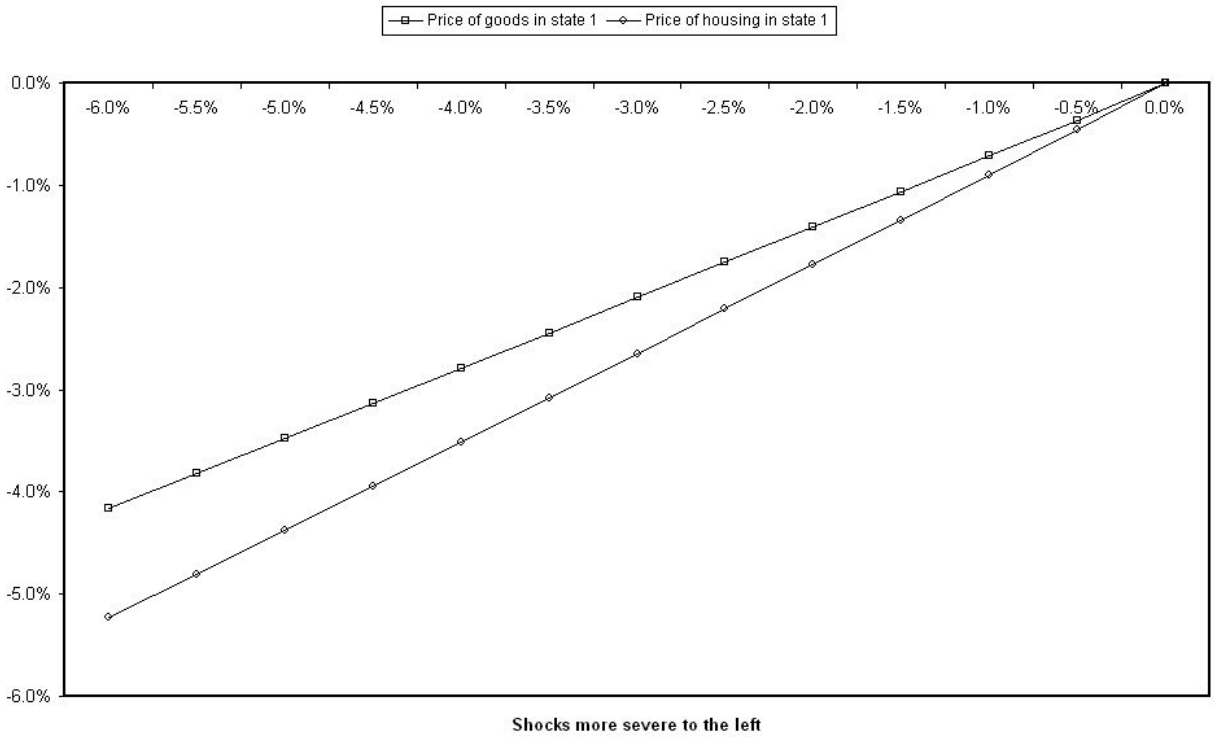

Figure: Housing and goods prices in state 1 vs compound decrease in money supply and in banks' risk-aversion

To study the current financial crisis one needs a model with at least:

- Fiat money & monetary policy
- Optimizing banks
- A durable good
- Endogenous default and collateral

We provide such a model and find that both contractionary monetary policy before uncertainty is resolved and higher appetite for risk by banks increase financial instability. If the two are combined then the effect on financial instability is more severe