A Network Model of Super-Systemic Crises

Prasanna Gai and Sujit Kapadia

Australian National University
Bank of England
November 2008

Are Financial Systems Shock Absorbers or Amplifiers?

- Have financial innovation and integration made financial systems safer or "fattened" the tail of the distribution of potential losses?
- Advanced country financial systems seem to have dealt well with some recent shocks, but not others.
- The spread of contagion depends crucially on
  - The pattern of interconnectedness between FIs
  - Asset price interactions

Contributions of Paper

- Nest both world views in one framework. Show that financial systems exhibit a ***robust-yet-fragile*** tendency.

- Develop a model of contagion in financial networks that can deal with ***complexity*** and identifies both probability and potential impact of shocks to the financial system.

- A vehicle for examining how the expansion of credit risk transfer has influenced the nature of contagion

The AIG "Network"

Existing Literature (1)

- Theoretical literature (Allen and Gale, 2000; Freixas, Parigi and Rochet, 2000; Castiglionesi and Navarro, 2007)
  - based on small, stylised networks with rigid structures;
  - generally does not distinguish the probability of contagious default from its potential impact.

- Empirical literature (see Upper, 2007, for a survey) uses actual or estimated data on interbank lending to simulate the effects of bank failures on the system.
  - Extent to which reported exposures reflect true linkages is unclear.
  - Difficult to model global contagion.
  - Difficult to use this approach to analyse the effects of changing network structure on contagion risk.

Existing Literature (2)

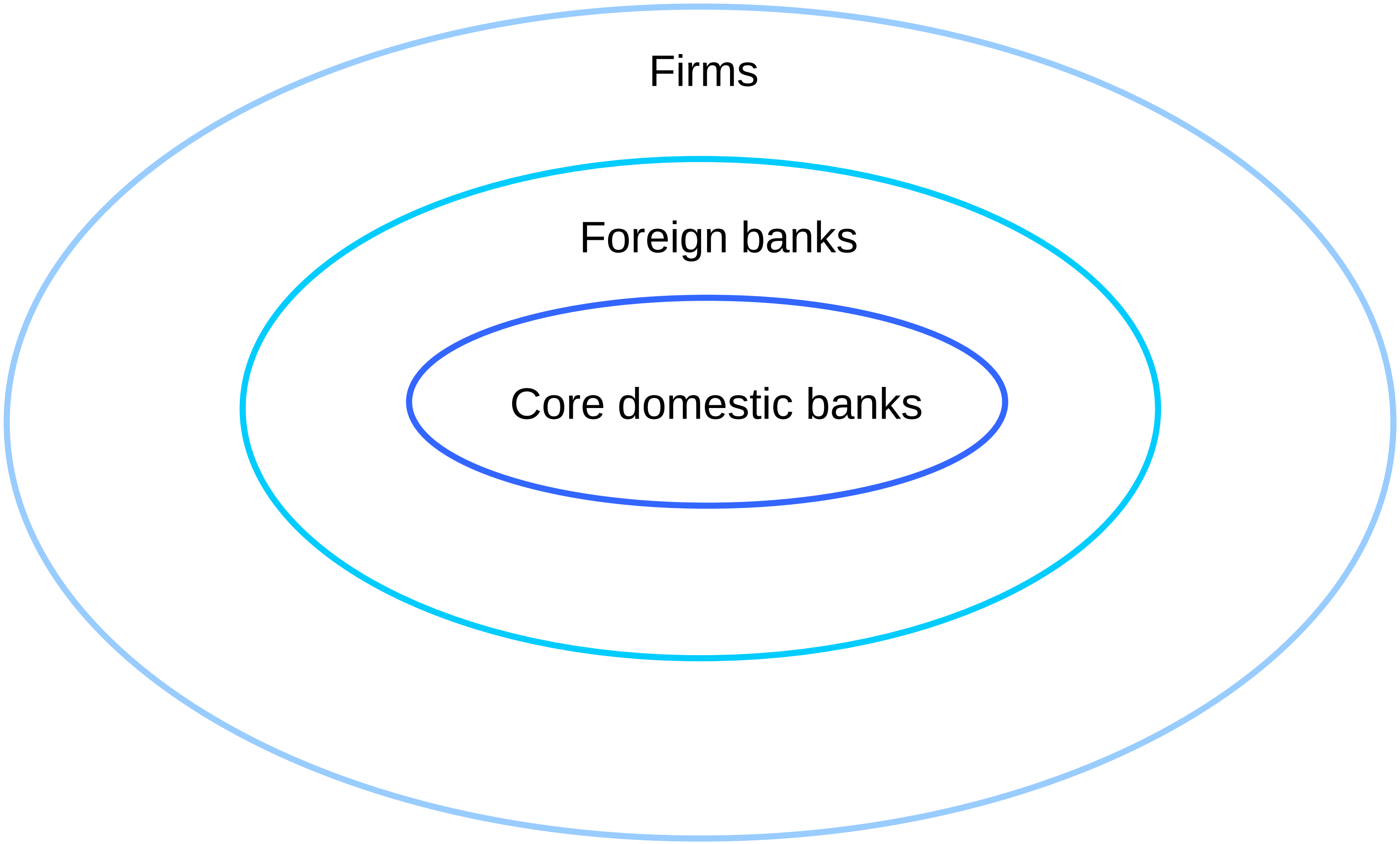
- Network literature (e.g. Watts, 2002; Newman, 2003) explores percolation processes taking place on networks.
  - Techniques developed to explore how shocks can be transmitted along the links in a network.
  - But models are abstract and not applied to financial systems.

- This paper applies network techniques to the modelling of interbank contagion.

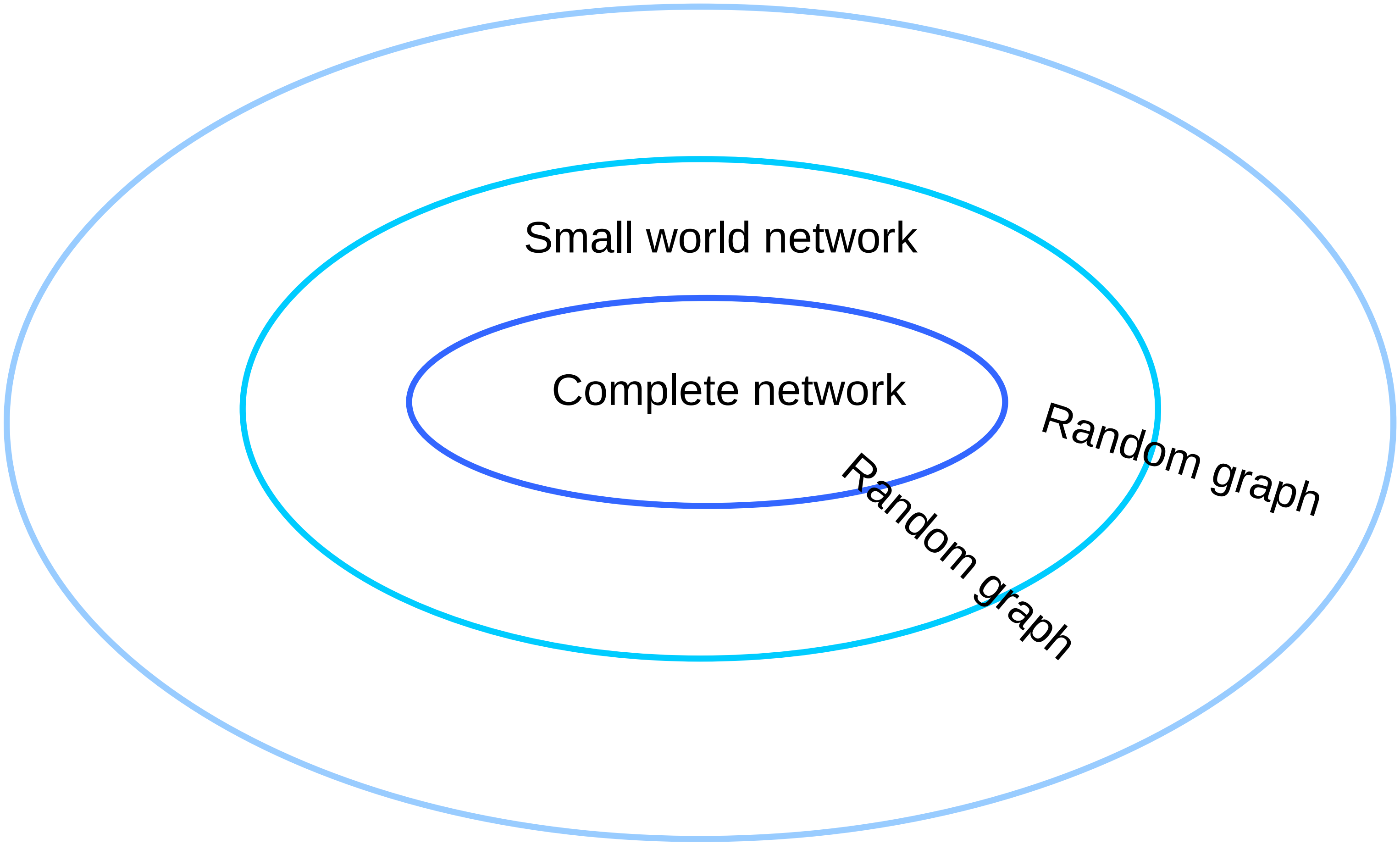
Properties of Complex Networks

- "Six Degrees of Separation"
  - Typical message passes through just 6 people between randomly chosen initial and final individuals
  - Highlights the "small world" nature of many systems
- Connections between nodes often "fat-tailed"
  - Small fraction of highly connected "hubs"
  - Model via generalised random graphs
- "Tipping point" behaviour or phase transitions

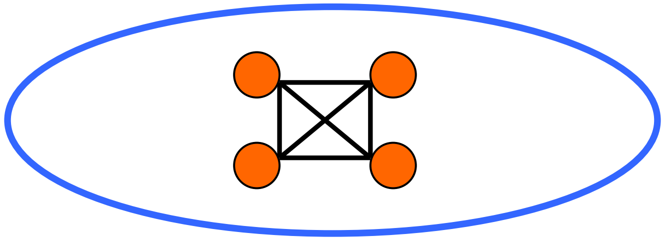
The financial system



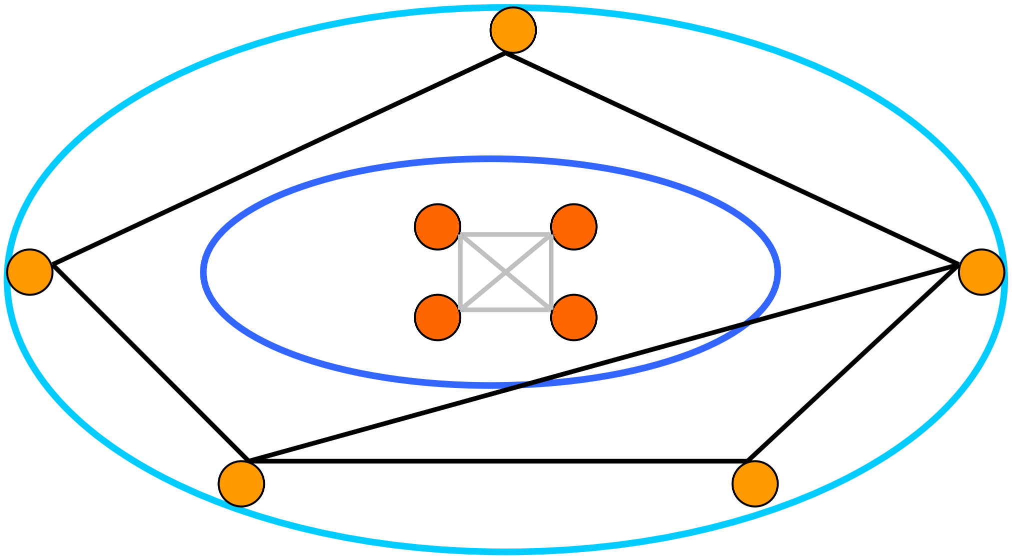
The financial system



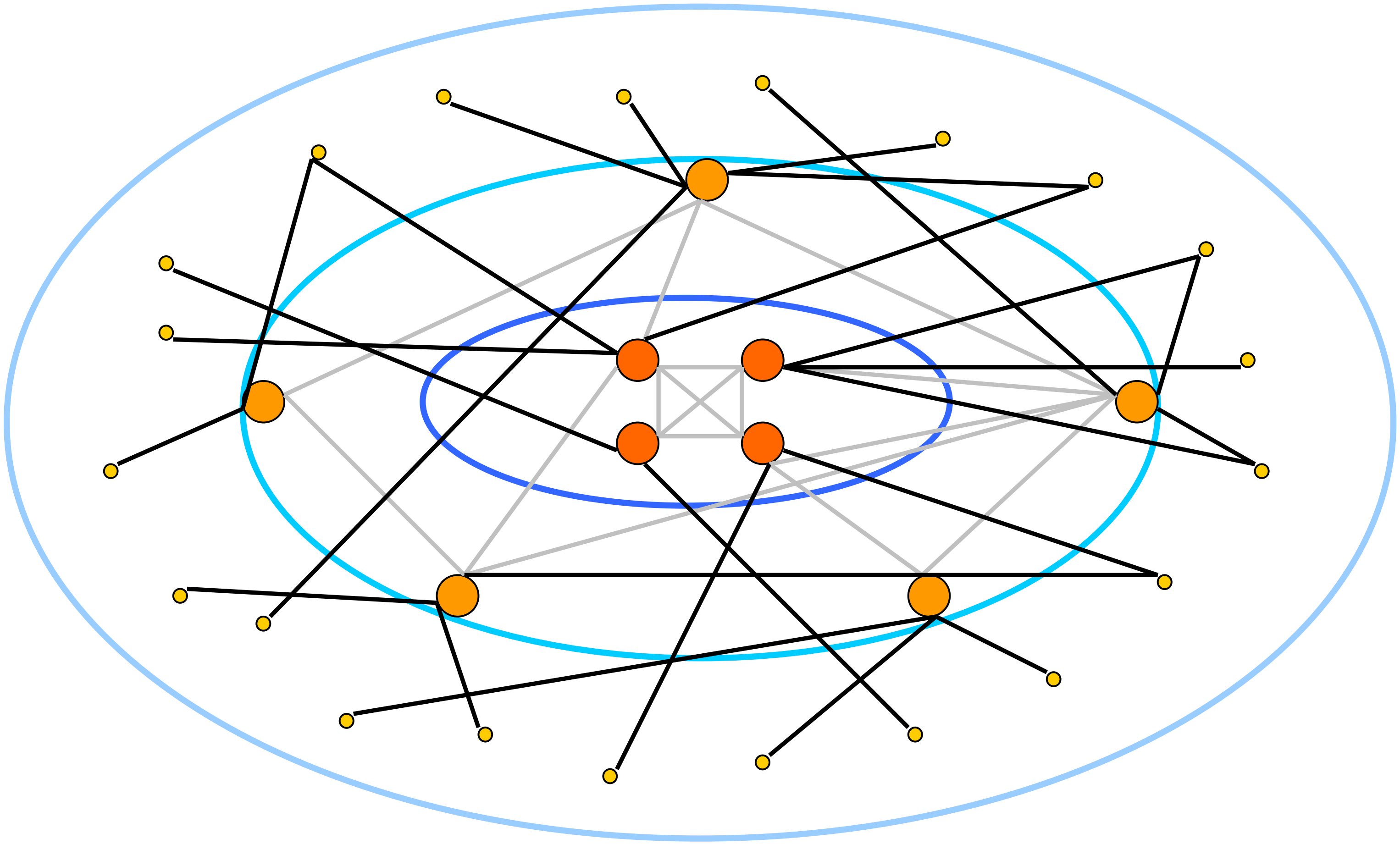
The financial system



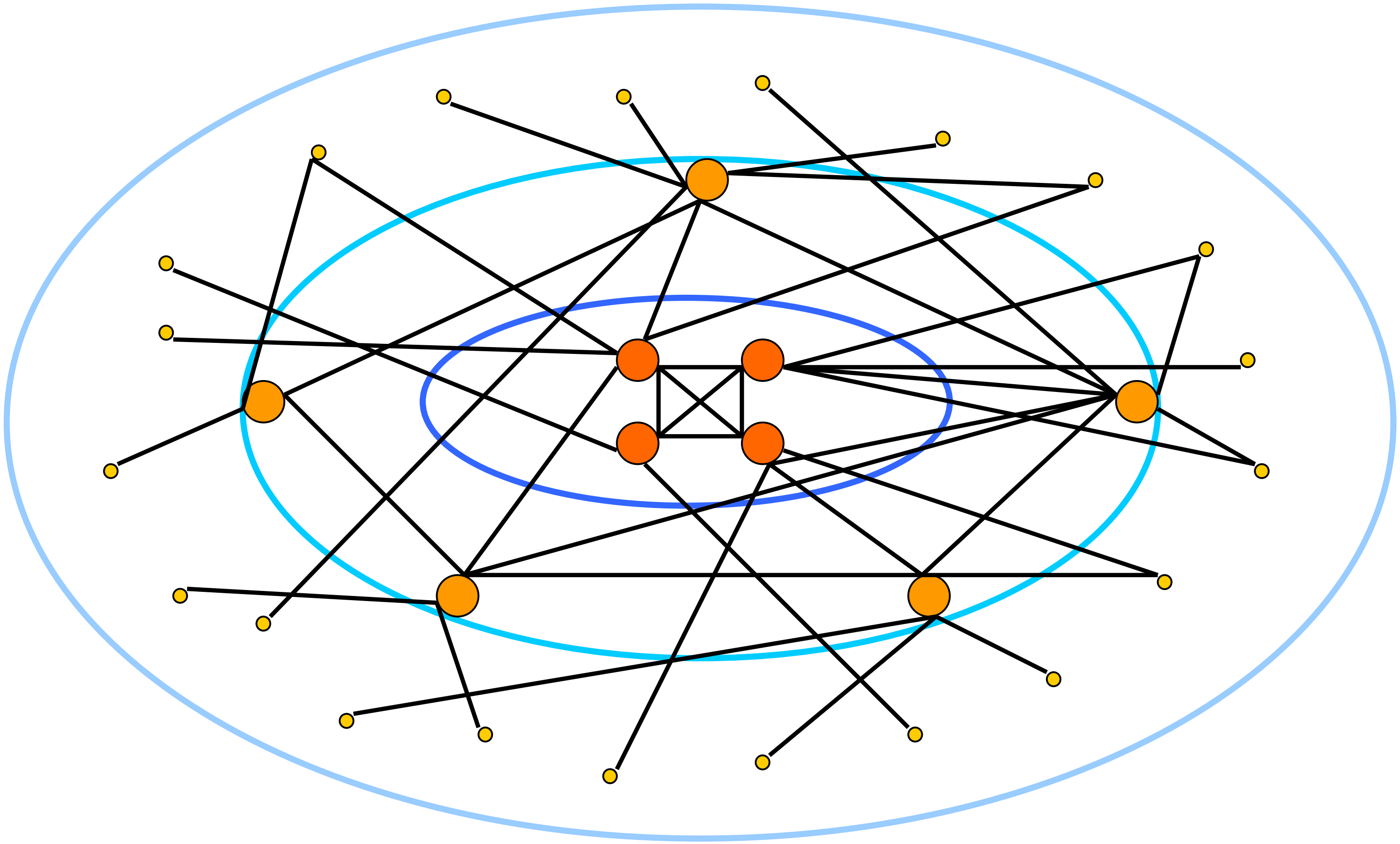
The financial system



The financial system



The financial system



Basic Intuition - 1

- Financial systems can be “robust-yet-fragile”
  - Greater connectivity allows losses to be absorbed and dispersed across a vast number of players.
  - So the probability of contagion declines.
  - But – conditional on a failure – connectivity increases the chances that surviving institutions are exposed to vulnerable second parties.
  - So the impact of a crisis is potentially larger

Basic Intuition – 2

- Example: spread of an idea.
  - A person who hears an idea spreads it with independent probability, *r*, to each of his friends.
  - A well-connected person can spread the idea to many.
  - But they are also more likely to hear of the idea in the first place, since they have many friends to hear the idea from.
  - So connectivity (degree) enters twice. A person with degree 10 is 10x10=100 times more effective at spreading an idea than someone with degree 1.

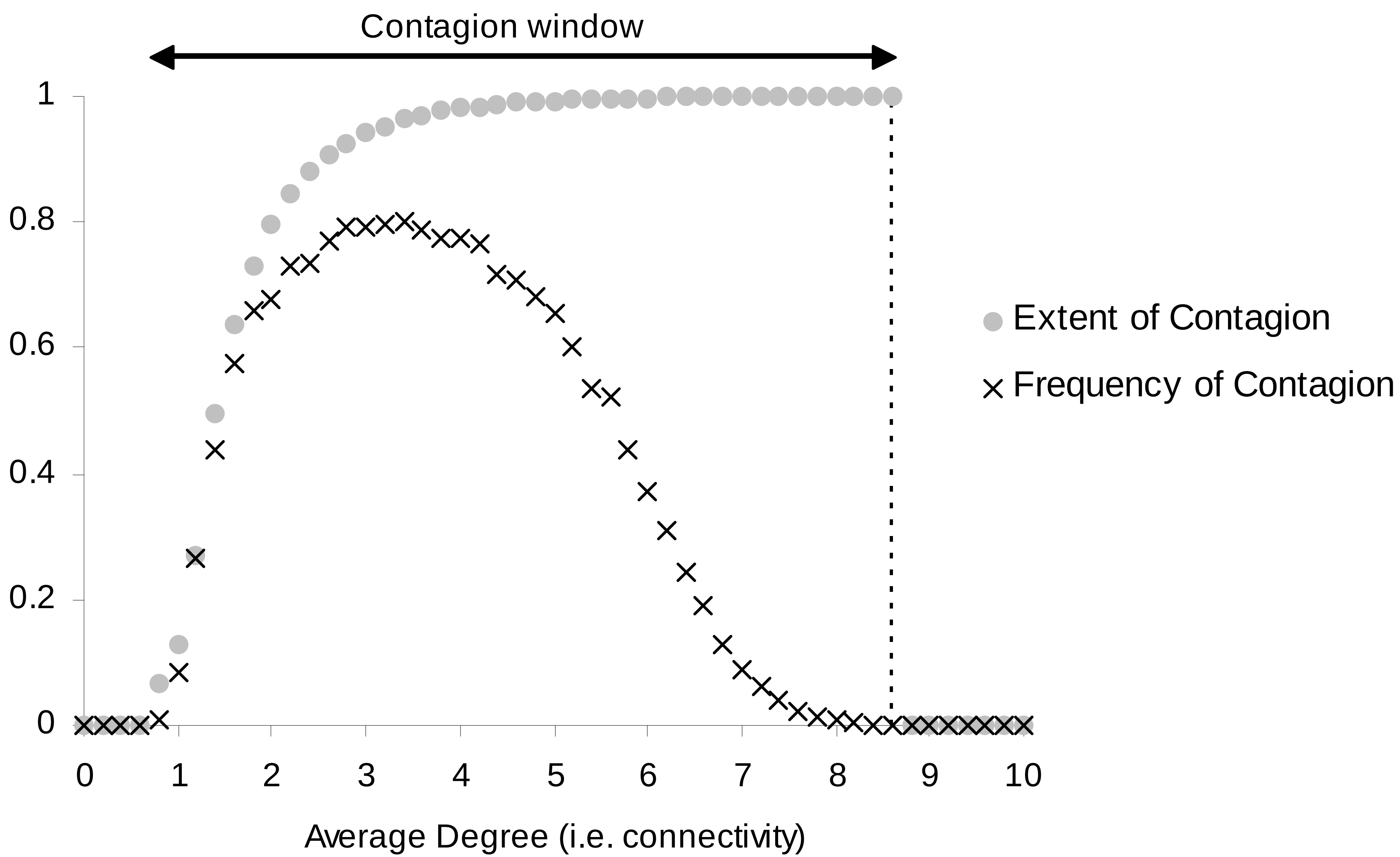
Basic Intuition – 3

- If the degree is k, then I have k-1 friends to spread the idea to. And the expected number who hear it is $r(k\text{-}1)$.
- Performing this weighted average over all nodes means that the average number of people a person passes an idea to is

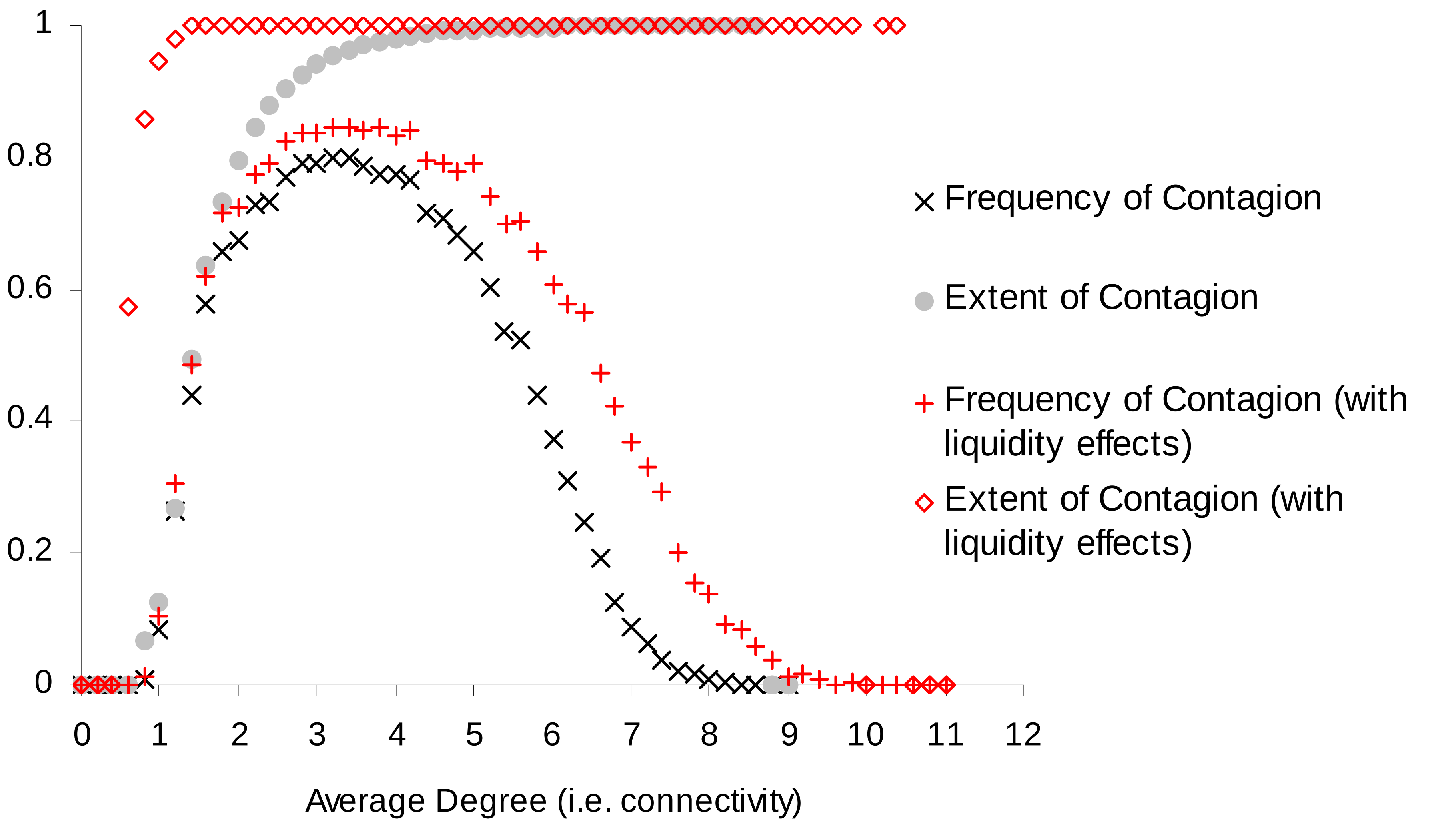
$$R = r \cdot \frac{\sum_i k_i (k_i - 1)}{\sum_i k_i} = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} \cdot r$$

- If $R<1$, idea dies. $R>1$, idea takes off exponentially

The Benchmark Case



Liquidity Effects and Contagion



Credit Derivatives

- Banks have been net buyers of credit protection – this has increased their exposure to financial counter-parties while decreasing links to corporates.

- Sellers of credit protection (insurance firms) have become part of the network through their activities and are exposed to corporate firms.

- Greater use of credit derivatives implies banks' interbank exposures are increasing in the number of incoming links

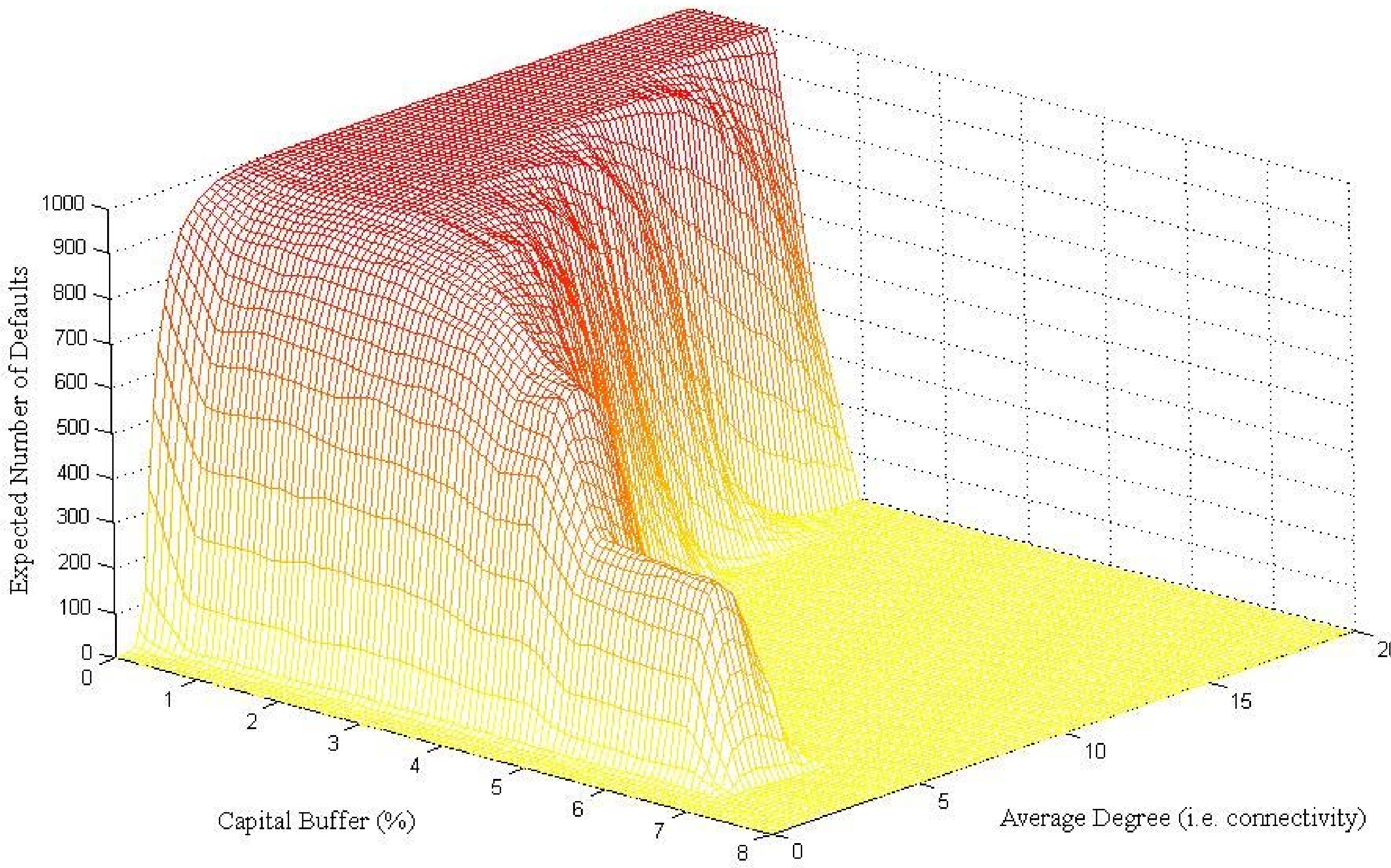
- Assume stock of assets in economy constant and total capital remains unchanged
  - The number of institutions in the network increases
  - System slightly less well capitalised.

- Spread of contagion increases, but probability declines.
- With 5 links, contagion only affects 5% of the system. Increase to 20 leads to system breakdown.

Average Degree (z)	*Share of Interbank Assets (%)*	*Share of Retail Assets (%)*	*Total Number of Banks*	*Capital Buffer (%)*	*Frequency of Contagion (%)*	*Extent of Contagion (% of system)*
2	6.6%	93.4%	100	4%	7.8%	3.8%
5	10.9%	89.1%	105	3.82%	6.2%	5.4%
10	17.2%	82.8%	113	3.55%	2.1%	35.4%
15	23.0%	77.0%	121	3.30%	0.9%	67.8%
20	28.5%	71.5%	131	3.06%	0.9%	89.1%
25	33.9%	66.1%	141	2.83%	0.2%	100%

Table 1: Credit Derivative Simulation

Connectivity, Capital Buffers and the Expected Number of Defaults



Conclusion

Summary of the Paper and its Main Results

- Developed a general model of contagion in financial networks.
  - Model applies broadly to systems of agents linked together by their financial claims on each other, including through interbank markets and payment systems.

- Showed that while greater connectivity may reduce the probability of contagion, it could also increase its severity should problems occur.
  - Speaks to the growing financial integration of recent times.

- Adverse aggregate shocks and liquidity risk amplify the likelihood and scale of contagion.

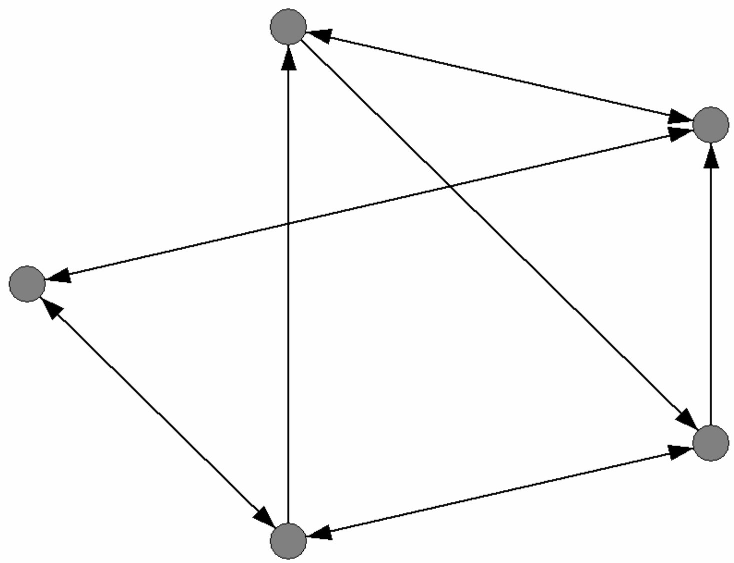
Applications and Extensions

- Large Exposure Limits
- Modelling Payment Systems
- Funding Contagion
- International Contagion

The Model

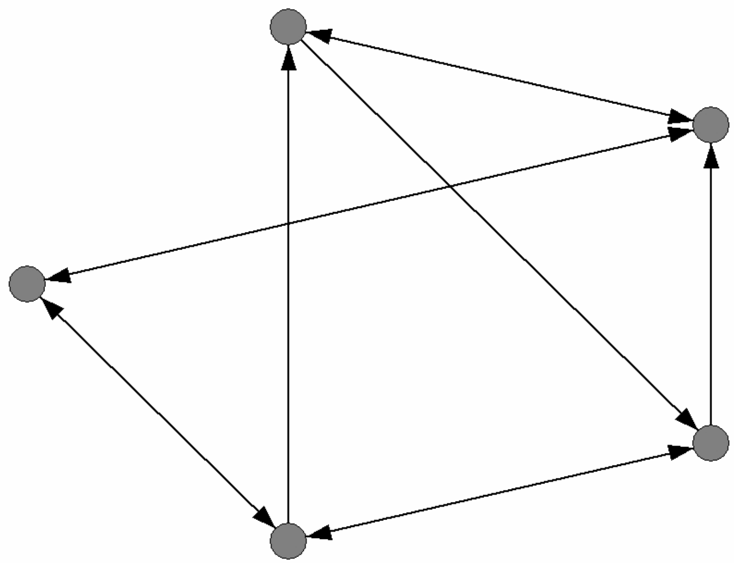
Network Structure

- Network has *n* financial intermediaries ('banks') linked together by their claims on each other.
- Each bank represents a *node*; interbank exposures define *links*.
- Links are *directed*.
  - Incoming links = interbank assets
  - Outgoing links = interbank liabilities



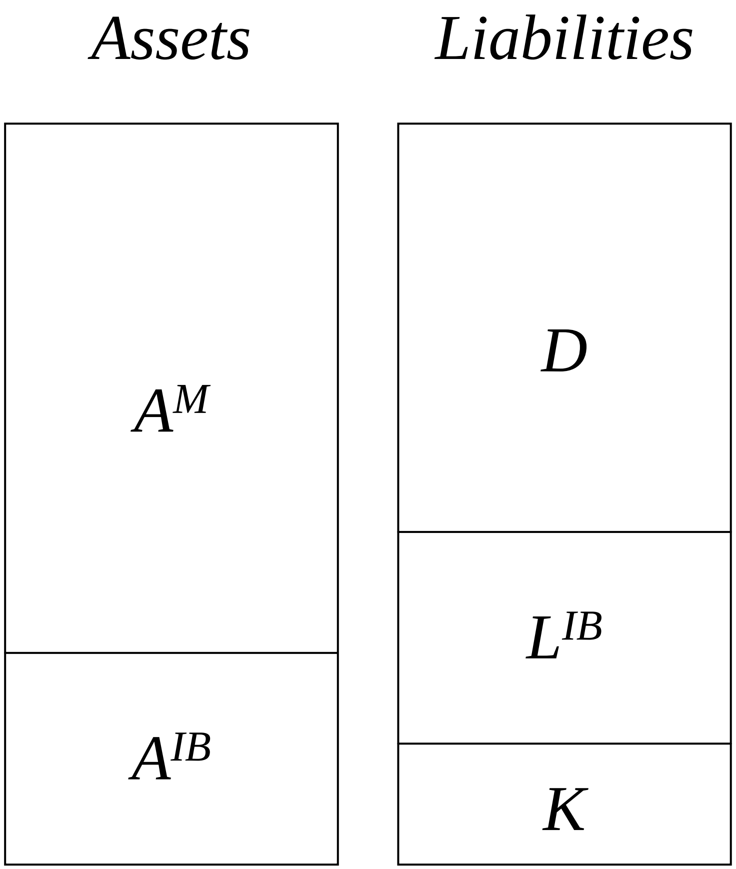
Network Structure

- *In-degree*: number of links that point into the node.
- *Out-degree*: number of links that point out of a node.
- Average in-degree = average out-degree. Denote this *average degree*, *z*.



- Joint distribution of in- and out-degree governs the potential for the spread of shocks through the network.
- Degree distribution is arbitrary in the theoretical model.

The Balance Sheet and the Solvency Condition



- Total assets of each bank are normalised to unity.
- Total interbank asset position of every bank is evenly distributed over each of its incoming links
- Zero recovery assumption.

Solvency Condition:

$$(1-\phi)\,A_i^{IB} + qA_i^M - L_i^{IB} - D_i > 0$$

$$\phi < \frac{K_i - (1-q)\,A_i^M}{A_i^{IB}}, \text{ for } A_i^{IB} \neq 0$$

‘Vulnerable’ Banks

- Suppose that all banks are initially solvent except for a single institution which defaults.
- Assume that banks have in-degree *j* (i.e. they have *j* incoming links). Then linked banks lose $1/j_i$ of their interbank assets.

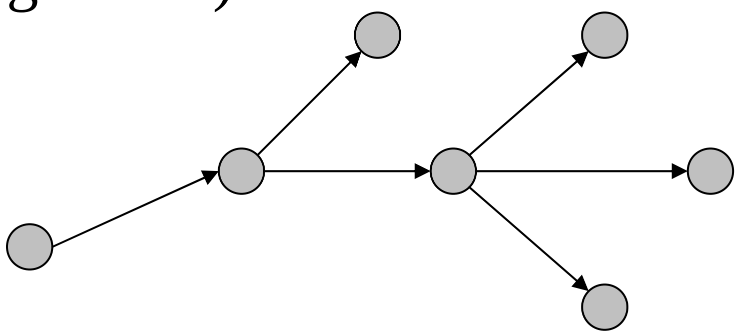
- So, the probability that a bank with in-degree *j* is *vulnerable* to the default of one of its neighbours is:

$$v_j = P\left[\frac{K_i - (1-q)\,A_i^M}{A_i^{IB}} < \frac{1}{j}\right] \quad \forall\, j \geq 1$$

- Define banks not exposed in this sense as *safe*.

The Transmission of Shocks (1)

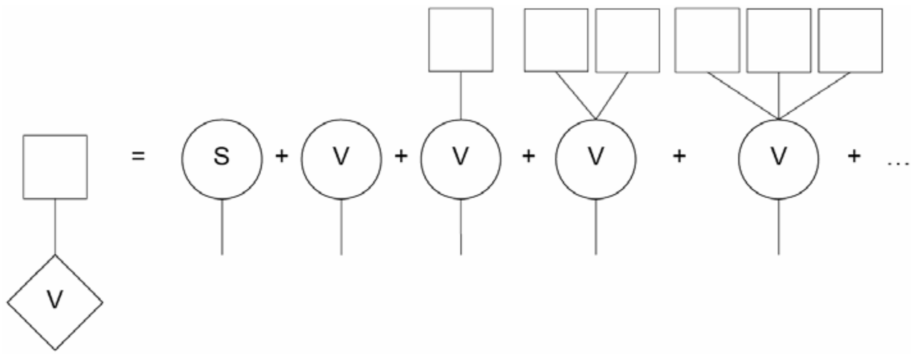
- In sufficiently large networks, for contagion to spread beyond vulnerable first neighbours, those neighbours must have outgoing links to other vulnerable banks.
- Going via outgoing links, we are interested in the second neighbours of vulnerable banks (i.e. the neighbours of vulnerable neighbours).



- Let each bank have k outgoing links (recall that each bank has j incoming links).
- Joint distribution of in- and out-degree is p_{jk}.

The Transmission of Shocks (2)

- Now consider the cluster of vulnerable banks which can be reached following an initial default.



- Let $H_1(y)$ be the generating function for the probability of reaching an outgoing cluster of a given size following an initial default. Then:

$$H_1(y) = \Pr[reach\ safe\ bank] + \Pr[arrive\ at\ vulnerable\ bank]$$

$$+ \Pr[arrive\ at\ vulnerable\ bank\ with\ clusters].$$

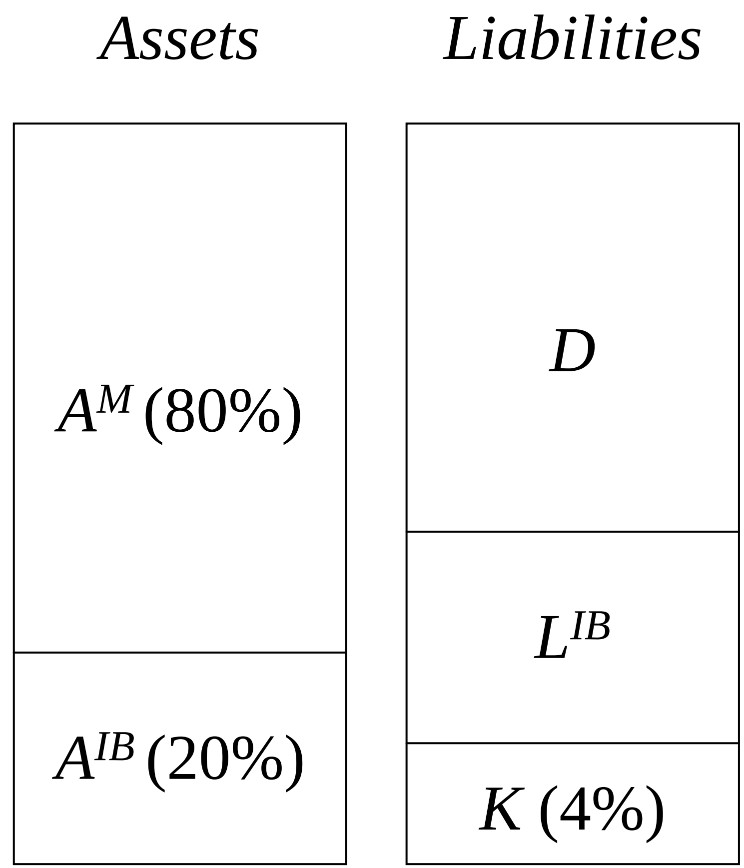
$$= \Pr[reach\ safe\ bank] + y \sum_{j,k} v_j \cdot q_{jk} \cdot [H_1(y)]^k$$

Numerical Simulations

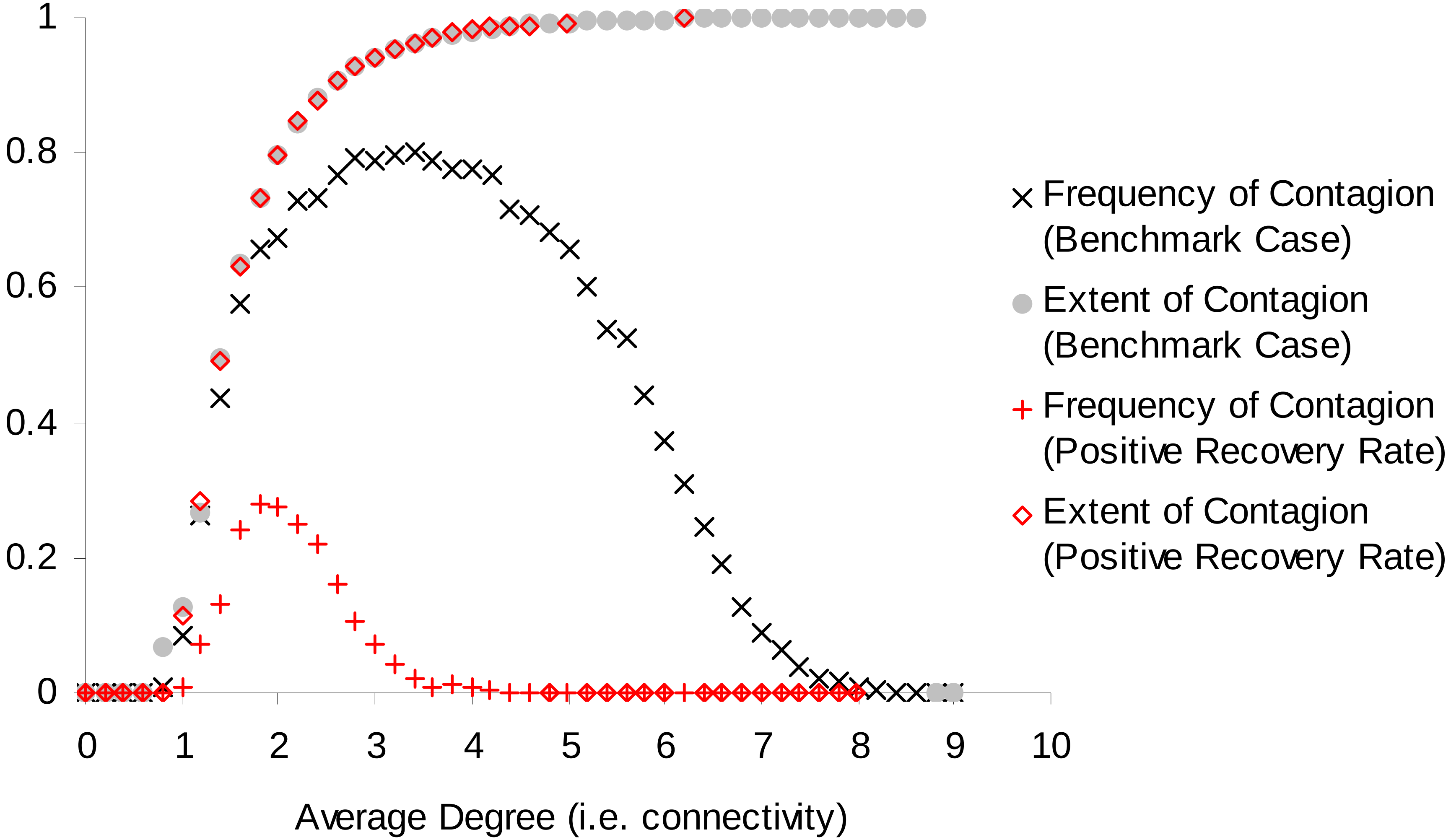
Methodology

- Assume a (Poisson) random graph. Allow *z* to vary in all experiments.
- Network of 1,000 banks.
- Identical balance sheets.

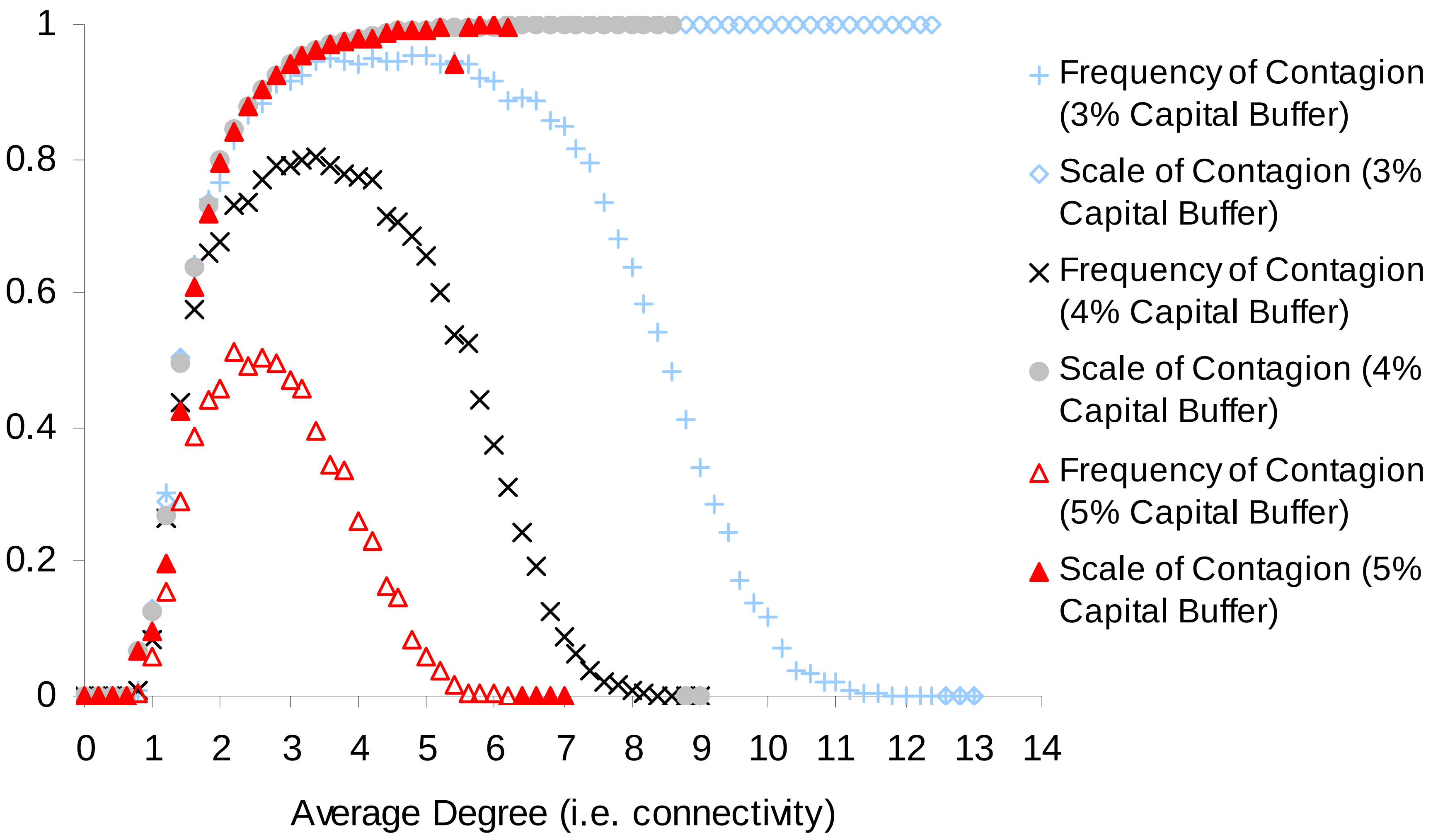
- Draw 1,000 realisations of the network for each value of *z*.
  - In each draw, we shock one bank at random, wiping out all of its external assets.
  - Solve iteratively until no new banks are pushed into default.
- When calculating probability and conditional spread of contagion, only count episodes in which over 5% of banks default.



Relaxing the Zero Recovery Assumption



Varying the Capital Buffer



Liquidity Risk

Modelling Liquidity Risk

- Following Schnabel and Shin (2004) and Cifuentes, Ferruci and Shin (2005), we assume that the price of the illiquid asset is given by:

$$q = e^{-\alpha x}$$

where $x > 0$ is the fraction of system (illiquid) assets which have been sold onto the market.

- Introduces a second source of contagion into the model.
  - But liquidity risk only materialises upon default.

Modelling Liquidity Risk

- Calibrate α so that the asset price falls by 10% when one-tenth of system assets have been sold.

- Integrate the pricing equation into our numerical simulations.
  - When a bank fails, all of its external assets are sold onto the market, reducing the asset price.
  - The external assets of all other banks are then marked-to-market to reflect the new asset price.

Reserve Slides