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On the Response of Inflation and Monetary Policy to an Immigration Shock*

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Abstract

An immigration shock has an ambiguous effect on inflation. On one hand, aggregate consumption increases with a suddenly larger population; this "demand channel" creates inflationary pressures. On the other hand, the labor market becomes more slack as immigrants search for jobs, containing wage growth; this "labor supply channel" creates disinflationary pressures. The response of an inflationtargeting central bank to an immigration shock is, therefore, not obvious. We study these competingchannels in a New Keynesian model of a small open economy with search frictions in the labor market. Our simulations are designed to characterize the possible response of inflation and monetary policy in Chile, a small open emerging country that has experienced a substantial immigration flow in recent years.

Resumen

Una ola inmigratoria tiene un efecto ambiguo sobre la inflación. Por un lado, el consumo agregado aumenta con el crecimiento de la población; este "canal de demanda" genera presiones inflacionarias. Por otro lado, las holguras en el mercado laboral aumentan mientras los inmigrantes buscan empleo; este "canal de oferta laboral" genera presiones desinflacionarias. Por lo tanto, la respuesta de un banco central con metas de inflación a una ola inmigratoria no es obvia. En este trabajo estudiamos estos canales en un modelo Neo-Keynesiano de economía pequeña y abierta con fricciones de búsqueda en el mercado laboral. Nuestras simulaciones están diseñadas para caracterizar la posible respuesta de la inflación y la política monetaria en Chile, una economía emergente pequeña y abierta que ha recibido una ola inmigratoria importante en años recientes.

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1 Introduction

An immigration shock has an ambiguous effect on inflation. On one hand, aggregate consumption increases with a suddenly larger population; this "demand channel" creates inflationary pressures. On the other hand, the labor market becomes more slack as immigrants search for jobs, containing wage growth; this "labor supply channel" creates disinflationary pressures.

We study the demand and labor supply channels, their effects on inflation, and the response of monetary policy, in a DSGE model with three key features: (i) search frictions in the labor market, (ii) nominal rigidities, and (iii) a small open economy structure. The search-and-matching specification allows us to analyze the effects of immigration on the extensive margin of labor supply (employment and unemployment) of immigrants and natives. In particular, we model immigrants that search for jobs upon arrival. We use a New Keynesian model with standard Calvo-style nominal rigidities that allows us to study the inflationary effects of immigration, and the response of an inflation-targeting central bank. Finally, the small open economy structure is useful for modeling immigrants that send part of their income as remittances to their home countries, as well as for studying the effects of the immigration shock on variables such as the exchange rate.

The model is estimated on data from Chile, a small open emerging country that has experienced a substantial immigration inflow in recent years. Our simulations, therefore, characterize the possible outcomes of an ongoing phenomenon from a general equilibrium perspective. We expect our framework to be of special interest to those concerned with the formulation of monetary policy, which requires an understanding of the driving forces of inflation.

The literature on the macroeconomic effects of immigration is scarce, and very few papers study, even secondarily, the response of inflation and monetary policy. Burriel, Fernández-Villaverde, and Rubio-Ramírez (2010) build a small open economy model for business-cycle analysis in Spain that includes shocks to population growth in order to consider the substantial immigration flow that Spain received in the late 1990s and early 2000s. Although their model has a standard labor market specification (not one with search frictions), the demand and labor supply channels are operative. A shock to population growth is expansionary in their model, but generates a decline in inflation and the interest rate, which a central bank sets following a Taylor rule.

The vector autoregressive approach due to Furlanetto and Robstad (2019), who study the macroeconomic effects of immigration in Norway, finds a small positive effect on inflation, though the response of monetary policy is not investigated. The authors argue the increase in inflation is due to the exchange rate depreciating in response to the shock, which puts upward pressure on the price of imported goods, rather than to an increase in the price of domestic goods. Furlanetto and Robstad (2019) conjecture the exchange rate depreciation might be due to the remittances immigrants send to their home countries. Bentolila, Dolado, and Jimeno (2008) argue immigration may have contributed to the flattening of the Phillips curve in Spain, and suggest that immigration may reduce inflation, but it is unclear whether they consider the demand channel in their analysis, and do not discuss the response of monetary policy to an immigration shock.

Some papers study the effect of immigration on the prices of many goods and services. Lach (2007) finds that the inflow of migrants to Israel from the former Soviet Union generated a decrease in prices.

Interestingly, the author argues that this is due to the increase in aggregate demand. Higher aggregate demand may lead to a decrease in prices if immigrants have higher price elasticities and lower search costs than natives. In our framework, the demand channel associated to the inflow of migrants puts upward pressure on aggregate measures of inflation. Only the labor supply channel puts downward pressure on prices. Cortes (2008) finds that low-skilled immigration to the US generates a decrease in the prices of nontraded goods and services that are intensive in immigrant labor, which is consistent with a labor supply effect that contains wages and prices. Finally, Frattini (2008) finds that, in the UK, immigration generates a decrease in the price of services that are intensive in low-skilled labor, which is consistent with the labor supply channel, and an increase in the price of "low-value grocery goods," which is consistent with the demand channel.

There are papers that study the macroeconomic effects of immigration that are, in principle, capable of reaching meaningful conclusions on its impact on inflation and the interest rate, but they ignore these variables, focusing instead on others such as GDP. See, for example, Kiguchi and Mountford (2017), and Stähler (2017).

Finally, a few papers study the macroeconomic effects of immigration using real DSGE models, and so are not equipped to study its impact on inflation and monetary policy. See, for example, Liu (2010), Smith and Thoenissen (2019), and Mandelman and Zlate (2012).

Several articles in the literature previously mentioned model differences in the skill level of immigrant and native workers. While this is an important feature of immigration in many countries, such as the U.S., where migrants are mostly low-skilled, it is less relevant in Chile, where immigrant and native workers display a similar educational level (see Aldunate, Contreras, de la Huerta, and Tapia, 2019b). It is possible, however, even if immigrants and natives have similar education and skills, that immigrants experience a transitory period of underemployment as they adapt to the local labor market, i.e., a period in which they cannot fully exercise the productivity associated to their skill level. One of our simulations explores this possibility in reduced form, even though our model does not include household heterogeneity.[1]

The paper is structured as follows. The next section presents evidence that supports our modeling approach to study the inflationary effects of an immigration flow. Section 3 describes our DSGE model, which is an extension of the Central Bank of Chile's large-scale model for policy analysis and forecasting, including our parameterization strategy. We analyze the effects of an immigration shock on inflation, the interest rate, and other variables in section 4, and conclude in section 5. The appendix contains the full set of the model's equilibrium conditions and its steady state.

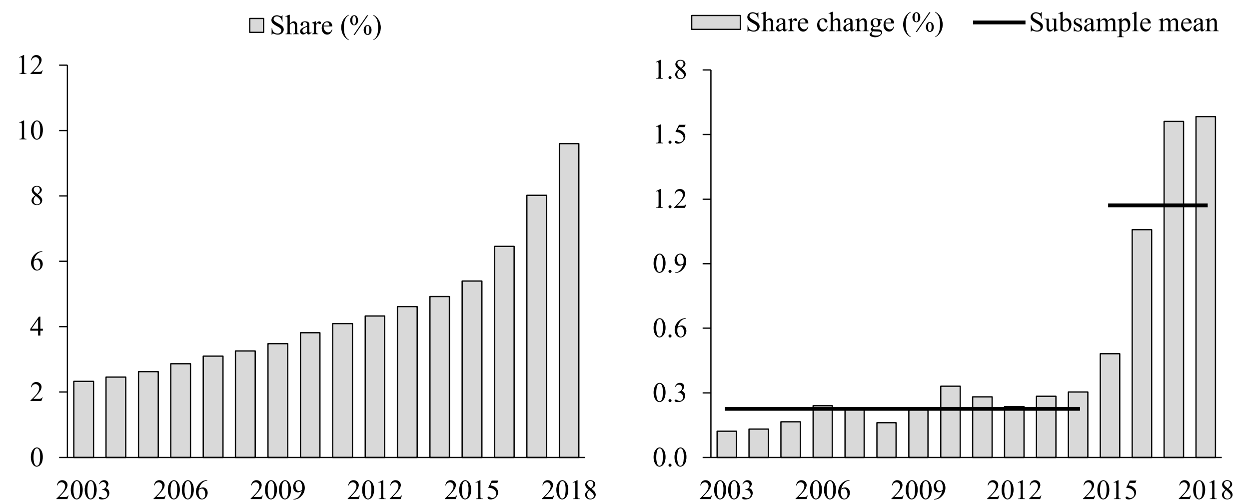
2 Motivating Evidence

The key features of our simulations are motivated by evidence on the immigration flow that Chile has received in recent years. Figure 1 shows the extent of this immigration wave. Between 2014 and 2018, the share of immigrants in the total labor force almost doubled, jumping from 4.9% to 9.6%. This evidence motivates us to study the effects of an immigration shock that is *exogenous* from the perspective of Chile, since it is unlikely that anything that happened there could account for such an inflow of migrants.

[1] Arias and Guerra-Salas (2019) study the medium- and long-term effects of immigration in Chile in a real overlapping-generations framework, with households of different skill levels and an informal sector. In their paper, immigrants experience a transitory underemployment spell.

Economic growth, for instance, was sluggish in Chile during this time. Instead, it is likely that this immigration flow is related to the economic and social crisis in Venezuela,[2] which is exogenous from the perspective of Chile.[3]

Figure 1: Share of Immigrants in Chile's Labor Force



Source: National Statistics Institute.

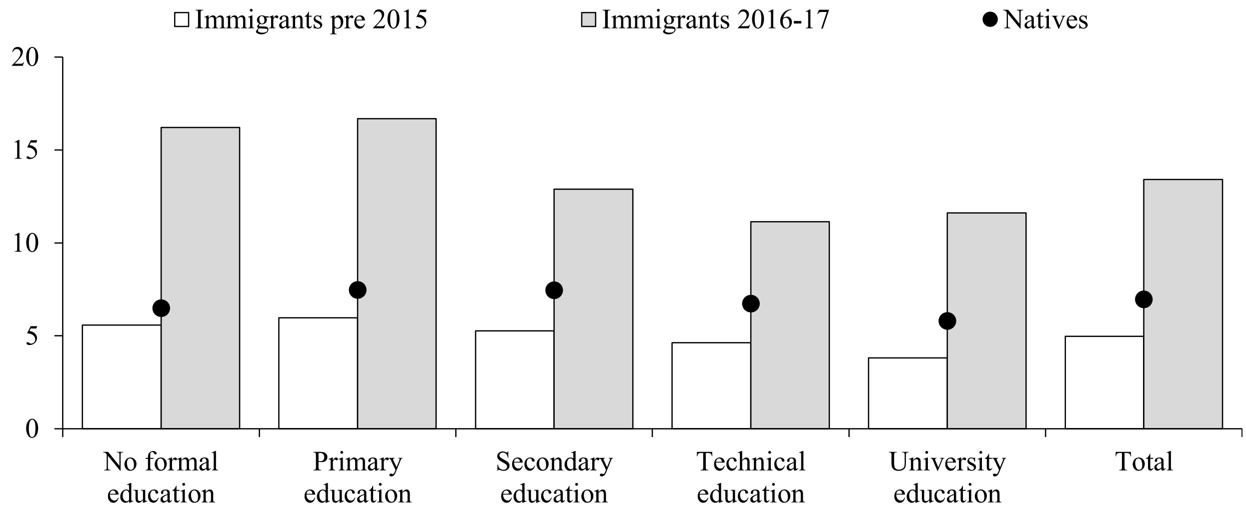
Throughout the paper, we assume that immigrants arrive as unemployed members of the labor force that search for jobs. This assumption leads to a transitory increase in the unemployment rate following an immigration shock. We also assume that immigrants are generally more efficient than natives at finding a job, although the results we report below are not sensitive to this assumption. These two assumptions allow us to square the following two facts: a) the unemployment rate of immigrants increases temporarily following an immigration flow, and b) in the long-run, immigrants have a lower unemployment rate than natives. The first of these facts suggests most immigrants do not arrive with a contract. The second fact may reflect that immigrants cannot afford to conduct lengthy job searches because they do not have savings and/or family support, and/or because they need to send remittances to their home countries. Figure 2 shows evidence consistent with the fact that immigrants arrive as unemployed members of the labor force. Using data from the 2017 Census, it shows that the unemployment rate among recently arrived immigrants (those that arrived after 2016) is substantially higher than that among immigrants that arrived prior to that year, regardless of their education. The figure also shows that the average unemployment rate among natives is about 2 percentage points higher than that among immigrants that arrived prior to the immigration wave. Since the high unemployment rate among recently arrived immigrants is likely to be transitory, we interpret this evidence as suggesting that immigrants have a lower unemployment rate than natives in the long run.[4]

[2]McAuliffe and Khadria (2019) estimate that, by mid-2019, political and economic turmoil has resulted in four million displaced Venezuelans worldwide.

[3]Mandelman and Zlate (2012) study the business cycle effects of endogenous decisions on migration and remittances in a two-country DSGE model calibrated to the U.S. and Mexico.

[4]Another reason immigrants might face lower unemployment rates than natives in the long run is that, by searching for jobs in a wider geographical area, they become more efficient in finding available jobs. Bentolila, Blanchard, Calmfors, de la Dehesa, and Layard (1990) show a negative correlation between workers mobility and equilibrium unemployment, while Basso, D'Amuri, and Peri (2019) find empirical evidence on higher mobility of immigrants in the Euro area and the U.S.

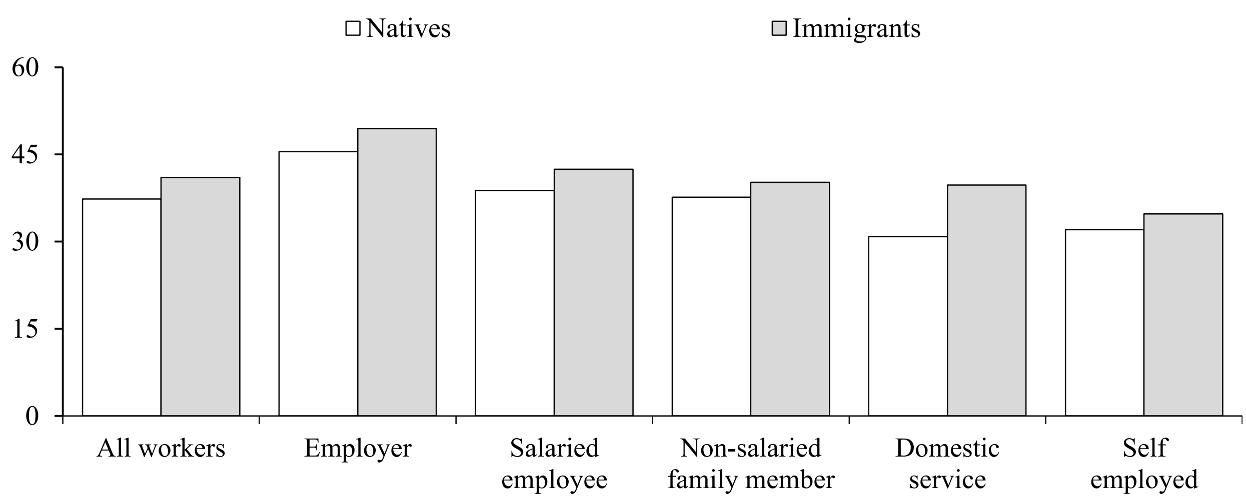
Figure 2: Unemployment Rate of Immigrants and Natives



Source: Aldunate et al. (2019b), based on data from the 2017 census.

In some of our simulations, we model immigrants that get lower disutility from labor than natives, to capture a scenario in which they are willing to accept jobs for lower wages or, put another way, work longer hours for a given wage. Figure 3 shows that immigrants work for longer hours than natives across all occupational categories.

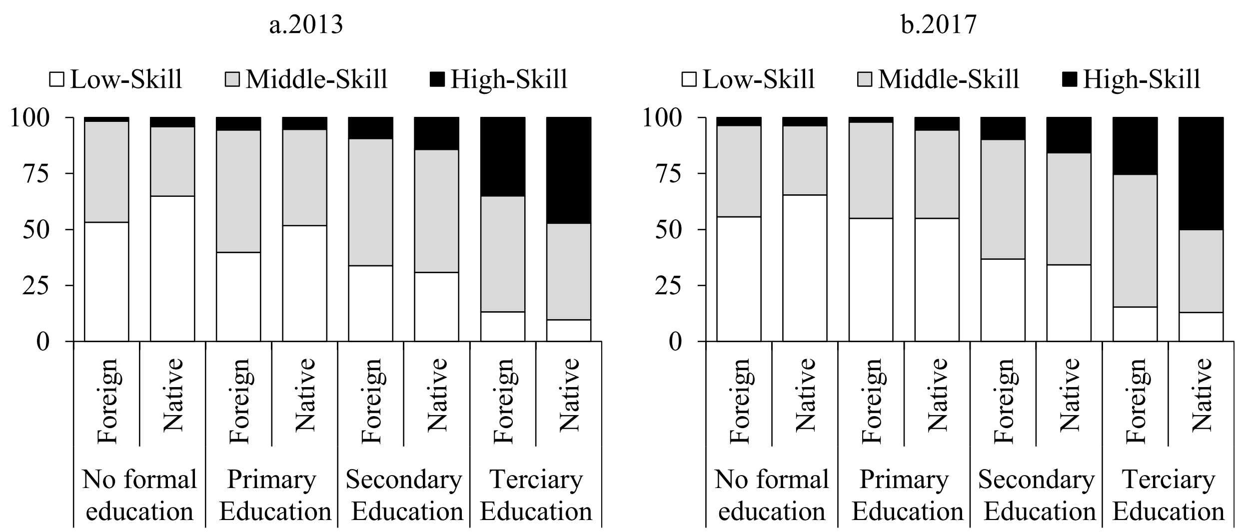
Figure 3: Hours Worked of Immigrants and Natives



Source: Calculations based on data from the National Statistics Institute's employment survey; average between 2017:M12 and 2018:M8.

Finally, in some of our simulations, immigration generates a transitory decline in average labor productivity. Even though immigrants and natives seem to have similar skills and latent productivity, as suggested by the evidence on education levels (see Aldunate et al., 2019b), it takes time for newly arrived workers to find jobs that match their qualifications. The evidence seems to suggest they experience an underemployment spell, a period during which they cannot fully exercise the productivity associated to their skill level. Figure 4 shows employment by education level and skill level of the occupation before and during the immigration flow.[5] In 2013 (left panel), before the immigration flow, immigrants and natives with tertiary education held a proportion of high-skill jobs of 35% and 47%, respectively. In 2017 (right panel), during the immigration flow, immigrants with tertiary education held a substantially lower proportion of high-skill jobs than natives: 25% versus 50%. During the immigration flow, therefore, immigrants with education comparable to that of natives have jobs that require lower skills. We interpret this evidence as reflecting a transitory phenomenon related to the adaptation of immigrants to the labor market, rather than a permanent phenomenon related, perhaps, to the education quality of immigrants.[6]

Figure 4: Employment by education level and skill level of occupation



Source: Aldunate et al. (2019b). The underlying data come from the CASEN survey. The classification of occupations by skill level follows Lagakos, Moll, Porzio, Qian, and Schoellman (2018). Individuals with primary education have 1–8 years of schooling, those with secondary education have 9–12 years of schooling, and those with tertiary education have more than 13 years of schooling.

The next section describes a DSGE model that incorporates these features of the data.

3 The DSGE Model

We extend Xmas, the Central Bank of Chile's large-scale dynamic stochastic general equilibrium (DSGE) model (see García, Guarda, Kirchner, and Tranamil, 2019), to allow for exogenous variation in the size of the population. Xmas is a small open economy model with nominal and real rigidities, and search frictions in the labor market, among other features. Following Burriel et al. (2010), we model immigration shocks as exogenous changes to the size of the representative household. Immigrants arrive as unemployed and must search for jobs. In some simulations, they send a fraction of their income as remittances to

[5] This evidence comes from the CASEN survey, which is conducted every two years. Since the recent immigration flow began around 2015, data from 2013 are prior to it, whereas data from 2017 are the only available after the immigration wave began.

[6] Arias and Guerra-Salas (2019) also use this evidence to support the assumption of an underemployment spell for immigrants in Chile during this period. Dustmann, Schönberg, and Stuhler (2016) discuss evidence of a similar transitory adjustment for the U.S. Canova and Ravn (2000) model the integration of East Germany to the west as an inflow of migrants of permanently lower skills, despite having similar years of education.

their home country. In this section, we present only the features of the extended model that are crucial for our analysis. The appendix contains the complete set of equilibrium conditions and the steady state.

The model also features Ricardian and non-Ricardian (hand-to-mouth) households subject to involuntary unemployment, there is habit formation in consumption, adjustment costs in investment, firms face a Calvo-pricing problem, there is imperfect exchange rate pass-through into import prices due to local currency price stickiness, and the economy also exports a commodity good. The small open economy is buffeted by a range of domestic and foreign shocks.

3.1 Households

The extension of the model is focused on the households and their participation in the labor market. The model is populated by a continuum of infinitely lived households of two types: non-Ricardian (NR) and Ricardian (R), with mass $L_t\omega$ and $L_t(1-\omega)$, respectively, where $\omega \in (0,1)$ is the share of non-Ricardian households, and L_t denotes the size of the population or labor force, which is given by the exogenous process $L_t = L_{t-1}^{\rho_L}\mu^L$, with $\log\mu^L \sim N(0,\sigma_{\mu L})$.[7,8] Household members are either employed (N_t) or unemployed (U_t) in period t, with $n_t = N_t/L_t$ being the employment rate, i.e., the share of members currently employed, and $u_t = U_t/L_t$ the unemployment rate. Each type of household has identical preferences. Each period t, utility is a function of per capita consumption services $\hat{C}_t^s$, and the number of hours worked by the household's employed members h_t.[9] Expected discounted utility of a representative household of type $j \in \{R, NR\}$ is then given by:

$$E_t \sum_{s=0}^{\infty} \beta^s L_{t+s}\varrho_{t+s}\left[\frac{1}{1-\sigma}\left(\hat{C}_{t+s}^{s,j}\right)^{1-\sigma} - n_{t+s}\Xi_{t+s}^j\right], \qquad j \in \{R, NR\}, \tag{1}$$

where E_t is the expectation conditional on period t information, $\beta \in (0,1)$ is the discount factor, ϱ_t is an exogenous preference shock, and $\sigma > 0$ is the inverse intertemporal elasticity of substitution. $\Xi_t^j = \Theta_t^j\kappa_t(A_{t-1}^H)^{1-\sigma}\frac{h_t^{1+\phi}}{1+\phi}$ denotes the disutility of work of an employed household member, where A_t^H is a non-stationary technology index for home goods that is needed here to maintain a balanced growth path, $\phi \geq 0$ is the inverse Frisch elasticity of labor supply, and, following Galí, Smets, and Wouters (2012), Θ_t^j is an endogenous preference shifter that regulates the strength of the wealth effect on labor supply.[10] The variable κ_t captures exogenous shifts on the average household members's labor dissutility, and has two components: one is an exogenous disutility shock (common to all households), and another, active in some of our simulations, is designed to capture a willingness of immigrants to accept lower wages than natives, which would be the case if they experienced less disutility from working. κ_t is then given by:

$$\kappa_t = \kappa_t^x + \kappa_t^M, \qquad \kappa_t^M = \rho_\kappa\kappa_{t-1}^M - \alpha_\kappa(L_t^M/L_t - L_{t-1}^M/L_{t-1}), \tag{2}$$

[7] We abstract from the decision to participate in the labor market, so we use the terms "population" and "labor force" interchangeably.

[8] Shocks to L_t are quasi-permanent, since the persistence parameter ρ_L is nearly one, though not exactly one to maintain stationarity.

[9] Throughout the document we denote per capita variables with a hat ($\hat{X}_t = X_t/L_t$).

[10] This endogenous shifter is designed so that a parameter in the $[0,1]$ range, which is estimated, governs the strength of the wealth effect. In one extreme, preferences are of the CRRA type. In the other extreme, the wealth effect disappears, as in the formulation of preferences due to Greenwood, Hercowitz, and Huffman (1988).

where κ_t^x follows an exogenous process, and $\left(L_t^M/L_t\right)$ is the share of immigrants L_t^M in the total population L_t, so that an increase in the migrant share reduces the average disutility from work in the economy. The specification for the law of motion of κ_t^M makes use of the assumption that the native component of the population is constant and that all movements of the labor force are due to immigration.

Per capita consumption services $\hat{C}_t^{s,j}$ are a constant elasticity of substitution (CES) bundle of the household's per capita consumption purchases $\hat{C}_t^j$, and the government's per capita consumption purchases $\hat{C}_t^G = \frac{C_t^G}{L_t}$.[11] Additionally, the household forms habits with respect to private consumption purchases:[12]

$$\hat{C}_t^{s,j} \equiv \hat{C}^{s,j}(\hat{C}_t^j, \check{\hat{C}}_{t-1}^j, \hat{C}_t^G) = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\hat{C}_t^j - \varsigma \check{\hat{C}}_{t-1}^j\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + o_{\hat{C}}^{\frac{1}{\eta_{\hat{C}}}} \left(\hat{C}_t^G\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}}\right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}},$$

where $o_{\hat{C}}$ denotes the share of government consumption goods in the CES bundle, $\eta_{\hat{C}}$ denotes the elasticity of substitution between private and public per capita consumption purchases, and $\check{\hat{C}}_{t-1}^j$ denotes average per capita consumption across households of type j (with $\hat{C}_t^j = \check{\hat{C}}_t^j$ in equilibrium), which each household takes as given.

3.1.1 Ricardian Households

Only Ricardian households save and borrow by purchasing domestic-currency-denominated bonds (B_t^R) and by trading foreign-currency bonds (B_t^{R*}) with foreign agents, both being non-state contingent assets. They also purchase an investment good (I_t^R), which determines their stock of physical capital for next period (K_t^R), and receive dividends (D_t^R) from the ownership of domestic firms, as well as net rents REN_t^{R*} from abroad (see below). They pay a tax rate τ^D on dividends and τ^K on capital income. Additionally, unemployed members receive an amount UB_t of unemployment benefits.

Let r_t, r_t^* and r_t^K denote the gross real returns on B_{t-1}^R, B_{t-1}^{R*} and the services from capital $K_t^{S,R}$, respectively, and let rer_t be the real exchange rate (i.e., the price of foreign consumption goods in terms of domestic consumption goods). We allow for the distinction between capital services ($K_t^{S,R}$), which are used in the production of goods, and physical units of capital (K_{t-1}^R), which are owned by the households and follow a law of motion governed by the investment and depreciation rates. Capital services are defined as the productive potential of the available physical capital stock for a given utilization rate $\bar{u}_t$ chosen by the households, and in per capita terms given by:

$$\hat{K}_t^{S,R} = \bar{u}_t \frac{\hat{K}_{t-1}^R}{\gamma_t^L}, \tag{3}$$

where $\gamma_t^L \equiv L_t/L_{t-1}$ is the gross rate of population growth. We follow Christiano, Trabandt, and Walentin (2011) and introduce $\phi_{\bar{u}}(\bar{u}_t)\hat{K}_{t-1}$, the investment goods used for private capital maintenance, as a part of total private investment, alongside the investment goods used for increasing the household's physical capital. These maintenance costs are deducted from capital taxation, and their functional form follows García-Cicco, Kirchner, and Justel (2015):

[11] Note that this formulation assumes that government consumption goods are rival.

[12] The way in which government consumptions contributes to private utility is similar to the formulation of preferences in Coenen, Straub, and Trabandt (2013).

$$\phi_{\bar{u}}(\bar{u}_t) = \frac{r^K}{\Phi_{\bar{u}}} \left(e^{\Phi_{\bar{u}}(\bar{u}_t - 1)} - 1 \right), \tag{4}$$

where the parameter $\Phi_{\bar{u}} \equiv \phi''_{\bar{u}}(1)/\phi'_{\bar{u}}(1) > 0$ governs the importance of these utilization costs. The per capita physical capital stock evolves according to the law of motion:

$$\hat{K}_t^R = (1-\delta)\frac{\hat{K}_{t-1}^R}{\gamma_t^L} + \left[1 - \phi_I\left(I_t^R / I_{t-1}^R\right)\right] \varpi_t \hat{I}_t^R, \tag{5}$$

with depreciation rate $\delta \in (0,1]$, where ϖ_t is an investment shock that captures changes in the efficiency of the investment process (see Justiniano, Primiceri, and Tambalotti, 2011), $\hat{I}_t^R$ denotes capital-augmenting per capita investment expenditures, and $\phi_I\left(I_t^R/I_{t-1}^R\right) \equiv (\Phi_I/2)\left(I_t^R/I_{t-1}^R - a\right)^2$ are convex investment adjustment costs with elasticity $\Phi_I = \phi''_I(a) \geq 0$.

The period-by-period per capita budget constraint of the representative Ricardian household is then given by:

$$\begin{aligned}
\left(\hat{B}_t^R + rer_t \hat{B}_t^{R*}\right) - \left(\frac{\hat{B}_{t-1}^R}{\gamma_t^L} + rer_t \frac{\hat{B}_{t-1}^{R*}}{\gamma_t^L}\right) \quad = \quad & rer_t R\hat{E}N_t^{R*} + T\hat{R}_t^R + (1-\tau_t^L) W_t h_t n_t + (1-n_t)\, UB_t \\
& + (r_t - 1)\frac{\hat{B}_{t-1}^R}{\gamma_t^L} + (r_t^* - 1)\, rer_t \frac{\hat{B}_{t-1}^{R*}}{\gamma_t^L} + (1-\tau_t^D)\hat{D}_t^R \\
& + \frac{\hat{K}_{t-1}^R}{\gamma_t^L}\left[r_t^K \bar{u}_t \left(1 - \tau_t^K\right) + \tau_t^K p_t^I \left(\delta + \phi_{\bar{u}}(\bar{u}_t)\right)\right] \\
& - (1+\tau_t^C)\hat{C}_t^R - p_t^I \left(\hat{I}_t^R + \frac{\hat{K}_{t-1}^R}{\gamma_t^L}\phi_{\bar{u}}(\bar{u}_t)\right) - \hat{T}_t^R.
\end{aligned} \tag{6}$$

Per capita net rents from abroad $R\hat{E}N_t^{R*}$ have a positive component due to ownership of firms abroad that evolves exogenously, and a negative component due to remittances that immigrants send to their home countries: $R\hat{E}N_t^{R*} = \overline{ren}^{R*}\xi_t^{ren}/(L_t/L) - \lambda_R (W_t h_t n_t / rer_t)(N_t^M/N_t)$, where $\overline{ren}^{R*} \geq 0$, ξ_t^{ren} is an exogenous process that affects received rents, and $\lambda_R \geq 0$ is the fraction of labor income that employed immigrants (with share N_t^M/N_t) send to their home countries.

The household chooses C_t^R, I_t^R, K_t^R, B_t^R, B_t^{R*}, and $\bar{u}_t$ to maximize (1) subject to (3)-(6), taking r_t, r_t^*, r_t^K, rer_t, T_t^R, REN_t^{R*}, TR_t^R, D_t^R and $\check{C}_t^R$ as given.

The nominal interest rates are implicitly defined as

$$\begin{aligned}
r_t &= R_{t-1}(\pi_t)^{-1}, \\
\pi_t &= \left(\frac{P_t}{P_{t-1}}\right)\frac{1+\tau_t^C}{1+\tau_{t-1}^C} \\
r_t^* &= R_{t-1}^* \xi_{t-1} (\pi_t^*)^{-1}, \\
\pi_t^* &= \frac{P_t^*}{P_{t-1}^*},
\end{aligned}$$

where π_t and π_t^* denote the gross inflation rates of the domestic and foreign consumption-based price indices, after tax in the domestic case. A debt elastic country premium (ξ_t) is given by :

$$\xi_t = \bar{\xi} \exp \left[-\psi \left(\frac{rer_t B_t^*}{p_t^Y Y_t} - \frac{rerb^*}{p^Y y} \right) + \frac{\zeta_t^O - \zeta^O}{\zeta^O} + \frac{\zeta_t^U - \zeta^U}{\zeta^U} \right], \qquad \psi > 0, \qquad \bar{\xi} \geq 1,$$

where ζ_t^O and ζ_t^U are observed and unobserved exogenous shocks to the country premium, respectively, and ψ denotes the elasticity of the premium to the country's net asset position (see Adolfson, Laséen, Lindé, and Villani, 2008; Schmitt-Grohé and Uribe, 2003). The foreign nominal interest rate R_t^* evolves exogenously, whereas the domestic central bank sets R_t.

3.1.2 Non-Ricardian Households

The subset of households that don't have access to asset markets face the following per capita budget constraint:

$$(1 + \tau_t^C)\hat{C}_t^{NR} = (1 - \tau_t^L) W_t h_t n_t + (1 - n_t)\, UB_t + \hat{TR}_t^{NR} - \hat{T}_t^{NR} - \lambda_R \frac{W_t h_t n_t}{rer_t} \frac{N_t^M}{N_t}, \tag{7}$$

where the last term refers to remittances sent by immigrant workers. Thus, non-Ricardian households solve a much simpler period-by-period problem.

3.2 Labor Market

Following Kirchner and Tranamil (2016) and Guerra-Salas, Kirchner, and Tranamil (2018), the labor market is modeled with search and matching frictions as in Mortensen and Pissarides (1994), allowing for both exogenous and endogenous separations, as in Cooley and Quadrini (1999), and den Haan, Ramey, and Watson (2000).

By assumption, Ricardian and non-Ricardian workers have the same productivity, though newly arrived immigrants temporarily have lower productivity than natives in the simulations that allow for a reduced-form underemployment spell. As in Boscá, Domenech, and Ferri (2011), a labor union negotiates a unique labor contract for everyone, considering average productivity. This implies that firms, during contract negotiations, can't differenciate between different kinds of workers—Ricardian and non-Ricardian, natives and immigrants—and thus all workers receive the same wages and work the same number of hours.

The aggregate matching function follows a standard Cobb-Douglas specification, augmented with a factor that allows for immigrants to have a higher efficiency at finding jobs: $\mathcal{M}_\mathrm{t} = m_t v_t^{1-\mu} U_t^{\mu}$, where U_t is the total number of unemployed workers searching for a job, v_t is the number of vacancies posted by firms, μ is the match elasticity parameter, and m_t is the average match efficiency at time t, which is given by $m_t = \overline{m}(U_t^L + \lambda_M U_t^M)/U_t$, where U_t^L and U_t^M are native and immigrant searchers, respectively, and $\lambda_M \geq 1$ governs the efficiency of immigrant searchers relative their native counterparts. At the beginning of each period, a fraction ρ_t^x of employment relationships is assumed to terminate exogenously. After exogenous separations, and before production starts, the surviving workers may separate endogenously at the rate ρ_t^n. This occurs if the worker's operating cost $\widetilde{c}_t$ is greater than an endogenously determined threshold $\overline{c}_t$. The operating cost is assumed to be a random variable which is i.i.d across workers and time with c.d.f. F, which implies that $\rho_t^n = P(\widetilde{c}_t > \overline{c}_t) = 1 - F(\overline{c}_t)$. We further assume that matches formed in the current period become productive in the following period, so the evolution of aggregate employment is given by $n_t L_t = N_t = (1-\rho_t)\left[N_{t-1} + \mathcal{M}_{\text{t-1}}\right]$, where ρ_t is the total separation rate, given by $\rho_t^x + (1-\rho_t^x)\rho_t^n$. Individuals are either employed or unemployed, so $L_t = N_t + U_t$.[13]

The disaggregation of unemployed workers searching for jobs, as well as other variables, among natives and immigrants, is possible by assuming that shocks that raise the labor force L_t above its steady state level are due to immigration. The native labor force is, thus, given by $L_t^L = L(1-\omega_M)$, where L is the steady-state labor force, and ω_M is the steady-state share of immigrants. The immigrant labor force is then $L_t^M = L_t - L_t^L$. The unconditional probability that a searching worker is matched to a new job is $s_t = \mathcal{M}_{\text{t}}/U_t$, whereas the probability that a native or immigrant searching worker finds a job are given respectively by $s_t^L = s_t \overline{m}/m_t$ and $s_t^M = \lambda_M s_t \overline{m}/m_t$. The number of native unemployed workers evolves according to the following law of motion: $U_t^L = \left[1 - s_t^L(1-\rho_t)\right] U_{t-1}^L + \rho_t N_{t-1}^L$, i.e., currently unemployed native workers are those that had a job in the previous period but were separated, plus those that remain unemployed, where $N_t^L = L_t^L - U_t^L$ denotes the number of employed natives. Finally, the number of employed immigrants is given by $N_t^M = N_t - N_t^L$.

The probability that a firm fills a vacancy is $e_t = \mathcal{M}_{\text{t}}/v_t$. The number of vacancies posted, as well as the job termination threshold $\overline{c}_t$ is optimally determined by profit maximizing firms. The wage earned by employed members, as well as their labor effort (hours worked), is the outcome of a bargaining process between firms and a union that represents the households.

3.3 Wholesale Domestic Goods

We will not elaborate on the production side of the model, which is standard in the small open economy New Keynesian literature, but we note that one of our simulations considers the possibility that newly arrived immigrants experience a transitory underemployment spell—a period during which they cannot fully exercise the productivity associated to their skill level. We model this, in reduced form, as a transitory decline in average labor productivity. Only wholesale domestic goods, which are produced in a competitive environment, employ labor. Specifically, the production of these goods requires oil as an intermediate input, and a composite of capital and labor services given by:

$$Y_t^{\tilde{Z}} = \left(\tilde{K}_t\right)^{\alpha} \left(A_t^H z_t^M N_t h_t\right)^{1-\alpha}, \tag{8}$$

where $\tilde{K}_t$ is a bundle of private and public capital services, and, as previously mentioned, A_t^H is a non-stationary labor-augmenting technology index. The variable z_t^M adjusts for lower average labor productivity in the case immigrants experience an underemployment spell. It is given by:

$$log(z_t^M) = \rho_{ZM} log(z_{t-1}^M) - \alpha_{ZM}\left(\frac{N_t^M}{N_t} - \frac{N_{t-1}^M}{N_{t-1}}\right), \tag{9}$$

where N_t^M/N_t is the share of immigrants in employment, and ρ_{ZM} and α_{ZM} govern, respectively, the length and strength of the average productivity drop following an immigration shock. When this share increases, average labor productivity in the economy declines.

[13] Notice that we are implicitly assuming that new labor force members are added to the unemployment pool: $U_t = U_{t-1} + (L_t - L_{t-1}) + \rho_t N_{t-1} - (1-\rho_t)\mathcal{M}_{\text{t-1}}$

3.4 Monetary Policy

Monetary policy is carried out according to a Taylor rule of the form:

$$R_t = (R_{t-1})^{\rho_R} \left[\overline{R}_t \left(\frac{\widetilde{\pi}_t}{\bar{\pi}_t} \right)^{\alpha_\pi} \left(\frac{Y_t^D / Y_{t-1}^D}{a_{t-1}} \right)^{\alpha_y} \right]^{1-\rho_R} \exp(\varepsilon_t^R), \qquad \rho_R \in (0,1), \qquad \alpha_\pi > 1, \qquad \alpha_y \geq 0,$$

where Y_t^D is GDP as measured by demand, which excludes operation, vacancy, and capital utilization costs, ε_t^R is an AR(1) exogenous process that captures deviations from the rule, and $\widetilde{\pi}_t$ is the inflation rate monitored by the central bank, which is an average between the present and expected total and core inflation rates:

$$\widetilde{\pi}_t = \left[\left(\pi_t^Z \right)^{\alpha_{\pi Z}} \left(\pi_t \right)^{1-\alpha_{\pi Z}} \right]^{1-\alpha_{\pi E}} \left[\left(E_t \pi_{t+4}^Z \right)^{\alpha_{\pi Z}} \left(E_t \pi_{t+4} \right)^{1-\alpha_{\pi Z}} \right]^{\alpha_{\pi E}},$$

where $\pi_t^Z = \frac{P_t^Z}{P_{t-1}^Z} \left(\frac{1+\tau_t^C}{1+\tau_{t-1}^C} \right)$ is the after-tax core inflation rate, $\alpha_{\pi Z} \in (0,1)$ governs the importance of core inflation relative to headline inflation, and $\alpha_{\pi E} \in (0,1)$ governs the importance of expected relative to current inflation.

3.5 Calibration and Estimation

Our general parameterization strategy closely follows that of Xmas—a subset of parameters are calibrated, either to match first moments in the data, or following the literature, and the rest of parameters are estimated using Bayesian techniques. We comment on the parameters related to our analysis of an immigration shock. For a detailed description of the parameterization of Xmas, see García et al. (2019).

Table 1 shows the parameters we introduce to study an immigration shock. The steady-state share of immigrants in the labor force ω_M is calibrated to match the average of 7.2% observed in 2015–18. Immigrants are 35% more efficient than natives at finding jobs ($\lambda_M = 1.35$), so that their unemployment rate is 2 percentage points (pp) lower in steady state, following evidence from the 2017 population census (see figure 2). In the simulations that allow for remittances, we assume immigrants send 17% of their income every period ($\lambda_R = 0.169$).[14] For the simulations in which immigrants get lower disutility from working, we set the persistence parameter ρ_κ to 0.9, and the elasticity of total labor disutility to the share of immigrants in the labor force α_κ to 20. With these values, an immigration shock generates an additional cumulative decline in nominal wage growth of 0.5pp, a year after the shock, relative to the decline in the baseline simulation, which ignores the possibility that immigrants accept lower wages than natives. Finally, the parameters that govern the decline in average labor productivity following an immigration shock, ρ_{ZM} and α_{ZM}, in the simulations that aim to capture a transitory underemployment spell, are calibrated to replicate a scenario in which all newly immigrants must climb a job ladder "from the bottom." These parameters are set to 0.9814 and 0.5711 respectively, leading to a persistent decline of about 0.5pp in average labor productivity following an immigration shock. These benchmark productivity dynamics are based on the work of Albagli, García, Guarda, Naudón, and Tapia (2020), who build a model with job ladders and on-the-job-search, calibrated for the Chilean economy, in which workers that lose their job and are unemployed before finding another one, must climb the job ladder once again, pushing

[14] This value is calculated for 2018 and is consistent with balance of payments data from the Central Bank of Chile, mean income data from the Supplementary Income Survey, and estimates of employed immigrants from Aldunate et al. (2019b).

average labor productivity downwards.

Table 1: Selected Calibrated Parameters

Parameter	Description	Value	Source/Target
ω_M	Immig. share of labor force in s.s.	0.072	Average (2015–18)
λ_M	Relative match efficiency of immig.	1.35	2pp diff. in s.s. unemp. rates
λ_R	Remittances: fraction of income	0.169	2018 estimate; see text.
ρ_κ	Lower disutility of work: persistence	0.9	0.5pp add'l ↓ in nom. wage growth
α_κ	Lower disutility of work: elasticity	20	
ρ_{ZM}	Lower avg. labor prod.: persistence	0.9814	0.5pp add'l ↓ in avg. labor prod.
α_{ZM}	Lower avg. labor prod.: elasticity	0.5711	

Note: The table shows the parameters related to the simulated immigration wave, including the adaptation of immigrants to the labor market. See García et al. (2019) for a complete list of calibrated parameters and targeted steady state values in Xmas, as well as details on its estimated parameters.

To estimate the model with the extension that allows for exogenous changes in the labor force, we add an additional variable to the set of observables used in the estimation of Xmas—the share of immigrants in the labor force, which we obtain from Aldunate, Bullano, Canales, Contreras, Fernández, Fornero, García, García, Pena, Tapia, and Zúniga (2019a).[15,16] Since the share of immigrants in the labor force increases substantially only in the very end of the sample, the estimated parameters are similar to those estimated by García et al. (2019) for Xmas.[17]

4 Results

This section discusses the effects of an immigration shock, focusing on the response of inflation and the monetary policy rate, under different assumptions on the behavior of immigrants and their integration to the labor market. We begin with a baseline setting in which immigrants arrive as unemployed members of the labor force that search for jobs, but a) do not experience any underemployment spell, b) do not send part of their income as remittances to their home countries, and c) are not willing to work for a lower wage than natives. Subsequently, we study how the baseline results change when each of these features is included in the analysis. Finally, we study the effects of immigration when we combine all these features.

Figure 5 shows, on the baseline parameterization, the effects of an immigration shock that permanently increases the size of the labor force (L denotes the labor force in the figure) by 1%.[18] In this case, the demand channel dominates the labor supply channel, leading to an increase in inflation ($4\pi^Z$ denotes annualized quarterly core inflation) of 0.2 percentage points (pp), on an annualized basis, on impact. The permanent expansion of the labor force generates a gradual but also permanent expansion of output (YR

[15] We thank the authors for sharing these data with us.

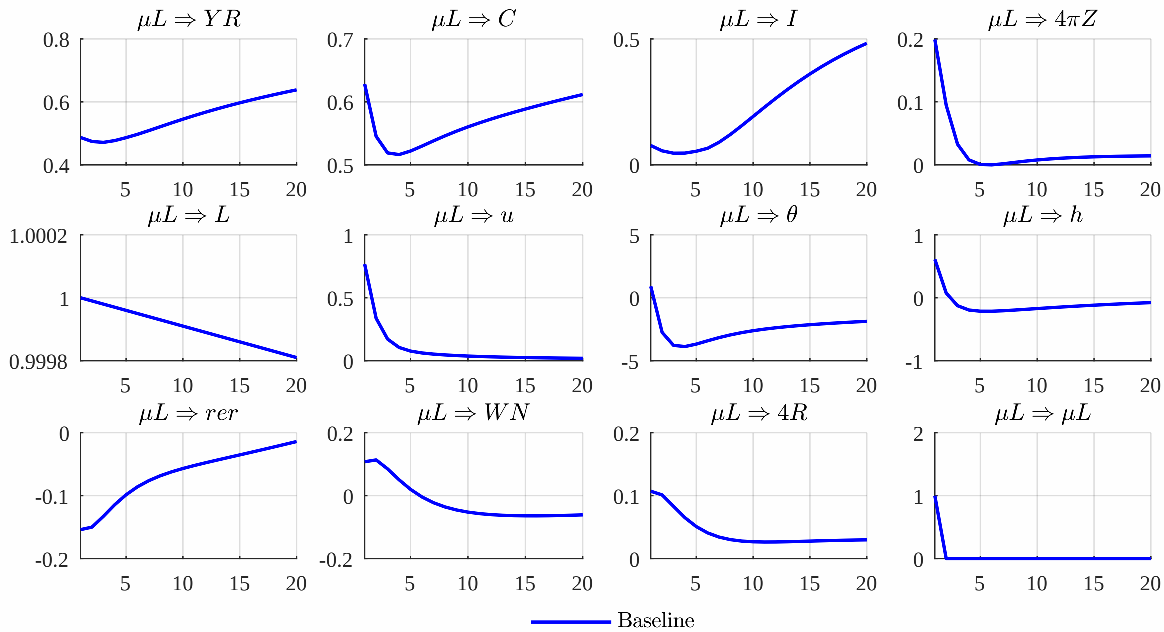
[16] During estimation, this variable is observed without measurement error, and calibrating the standard deviation of the shock to the labor force $\sigma_{\mu L}$ to 0.01.

[17] Of course, the Kalman smoother infers substantial immigration shocks to the labor force towards the end of the sample.

[18] The increase in the labor force is quasi-permanent in the sense that we model an immigration shock with a persistence parameter of nearly one.

denotes non-mining output), consumption (C) and investment (I). The labor market becomes more slack (θ denotes labor market tightness, i.e., the ratio of vacancies to unemployment), since immigrants arrive as unemployed members of the labor force that immediately search for jobs. The slackness in the labor market contains the growth of wages (WN denotes the hourly nominal wage) and prices. But the overall effect on inflation is positive, so under a Taylor rule, the central bank responds by raising the interest rate ($4R$ denotes the monetary policy rate on an annual basis) by about 10 basis points.

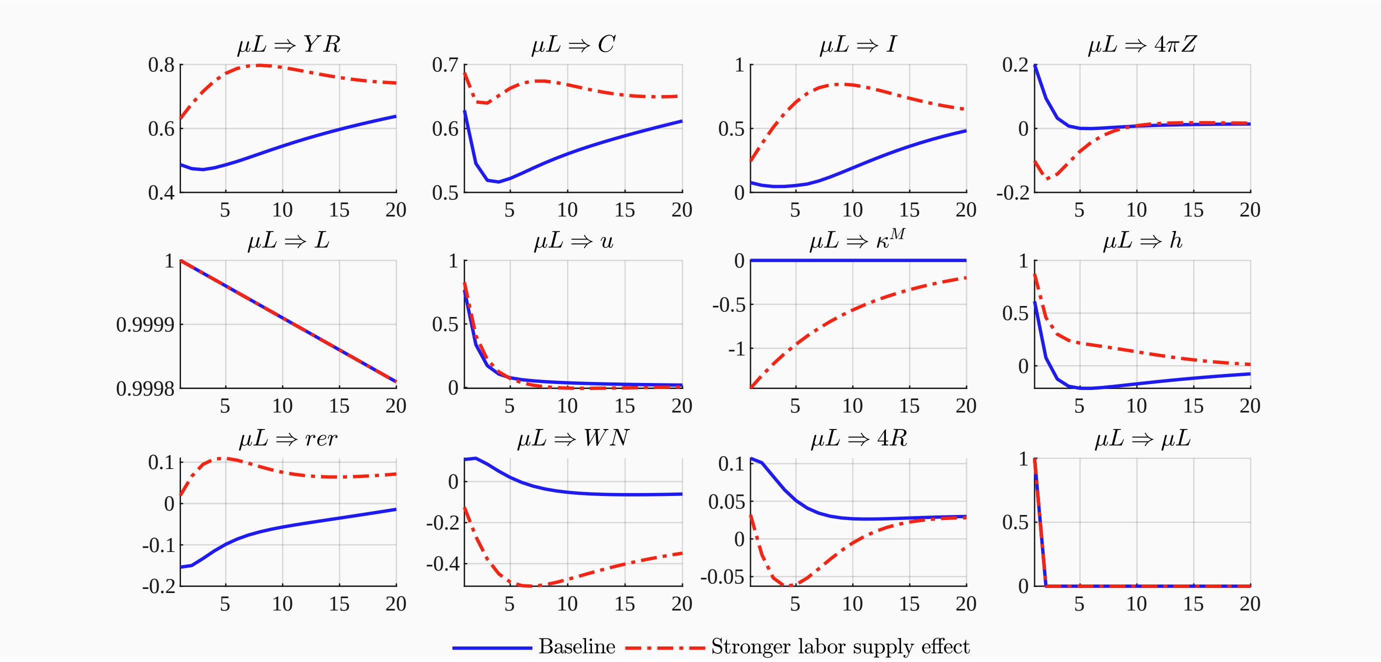
Figure 5: Baseline Effects of an Immigration Shock



Note: Impulse responses to a shock (μL) that increases the size of the labor force (L) permanently by 1%. Responses of the following variables are expressed as percent deviations from steady state: non-mining output (YR), consumption (C), investment (I), labor market tightness (θ), hours worked (h), the real exchange rate (rer), and the hourly nominal wage (WN). $4\pi^Z$ denotes annualized quarterly core inflation, $4R$ the interest rate on an annual basis, and u the unemployment rate, all in percentage points and as deviations from steady state. Horizontal axes show quarters.

One way in which the labor supply channel may dominate the demand channel is if immigrants were willing to accept lower wages than natives or, put another way, to work longer hours at the going wage. This behavior is plausible, for example, if immigrants bring little savings and need to get a job quickly. Figure 6 compares the baseline results to this scenario of a stronger labor supply effect. As we mentioned previously, we model this stronger labor supply effect as a transitory reduction in the disutility from work experienced by immigrants (see the graph in the second row and third column of the figure). Hours worked in the economy (h), therefore, are higher throughout the 20-quarter horizon. This higher labor supply leads to a persistently lower wage. Output, consumption and investment expand more in this scenario, but the overall effect on inflation is now negative, so the interest rate declines following the Taylor rule.

Figure 6: An Immigration Shock with Stronger Labor Supply Effects

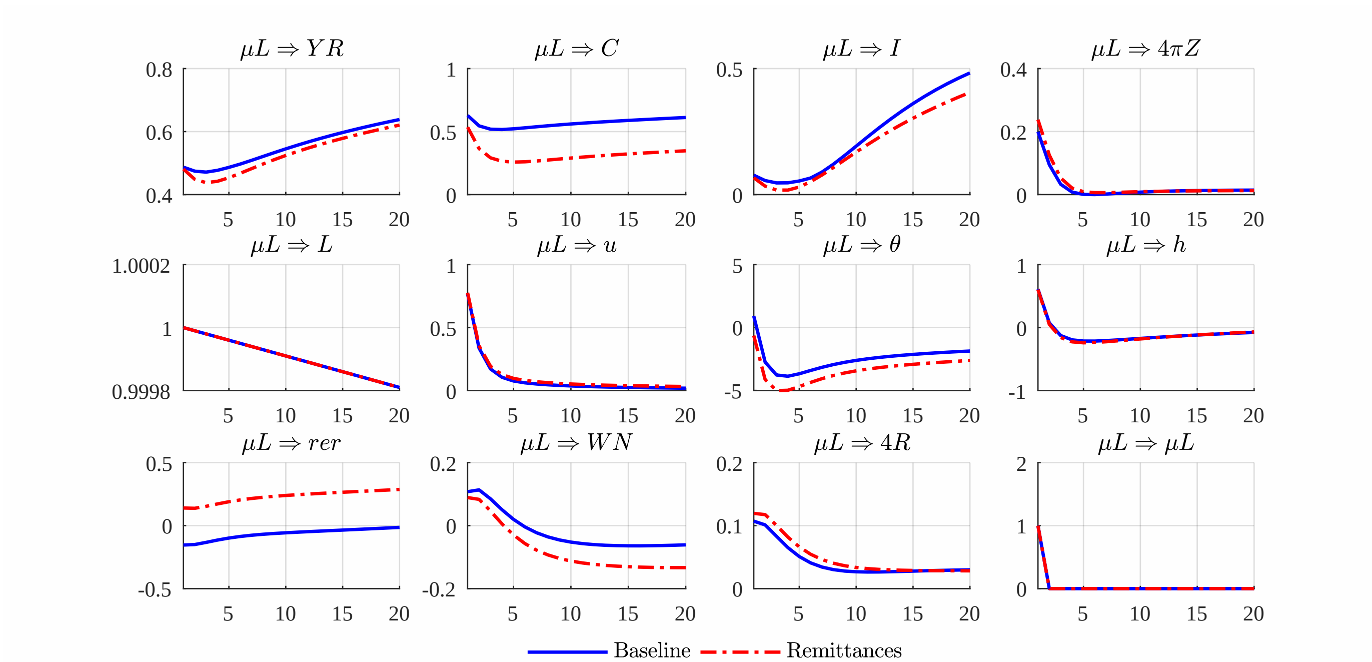


Note: Impulse responses to an immigration shock. Blue lines show responses under the baseline parameterization. Dashed red lines consider immigrants that supply more labor at any given wage or, in other words, are willing to accept jobs for lower wages than natives. See the note to figure 5 for more details.

Figure 7 considers the possibility that immigrants send a fraction of their income as remittances to their home country.[19] This feature has little effect on the response of inflation and the interest rate to the immigration shock, since it generates opposing forces on inflation that, under our parameterization, nearly cancel each other out. On the one hand, immigrants have less disposable income, which lowers consumption demand and pushes inflation downwards. On the other hand, the outflow of capital from the small open economy leads to a depreciation of the exchange rate, which turns imported goods more expensive in local currency, pushing inflation upwards. Allowing for the possibility of remittances, under our parameterization, does not materially affect the way in which inflation and monetary policy respond to immigration. The response of the exchange rate and import prices in this simulation is consistent with the empirical findings of Furlanetto and Robstad (2017) for Norway. They find a positive impact of immigration on prices mainly driven by a depreciating exchange rate. When we allow for remittances, aggregate consumption increases substantially less than in the baseline case without remittances. Despite this, inflation is, if anything, slightly higher than in the baseline results. The depreciating exchange rate not only increases inflation due to its direct effect on the price of imported goods, but the change in relative prices also reallocates consumption demand to domestic goods, which, all else equal, sustains their price.

[19] As mentioned in our discussion of the calibration, immigrants send 17% of their labor income as remittances in this scenario.

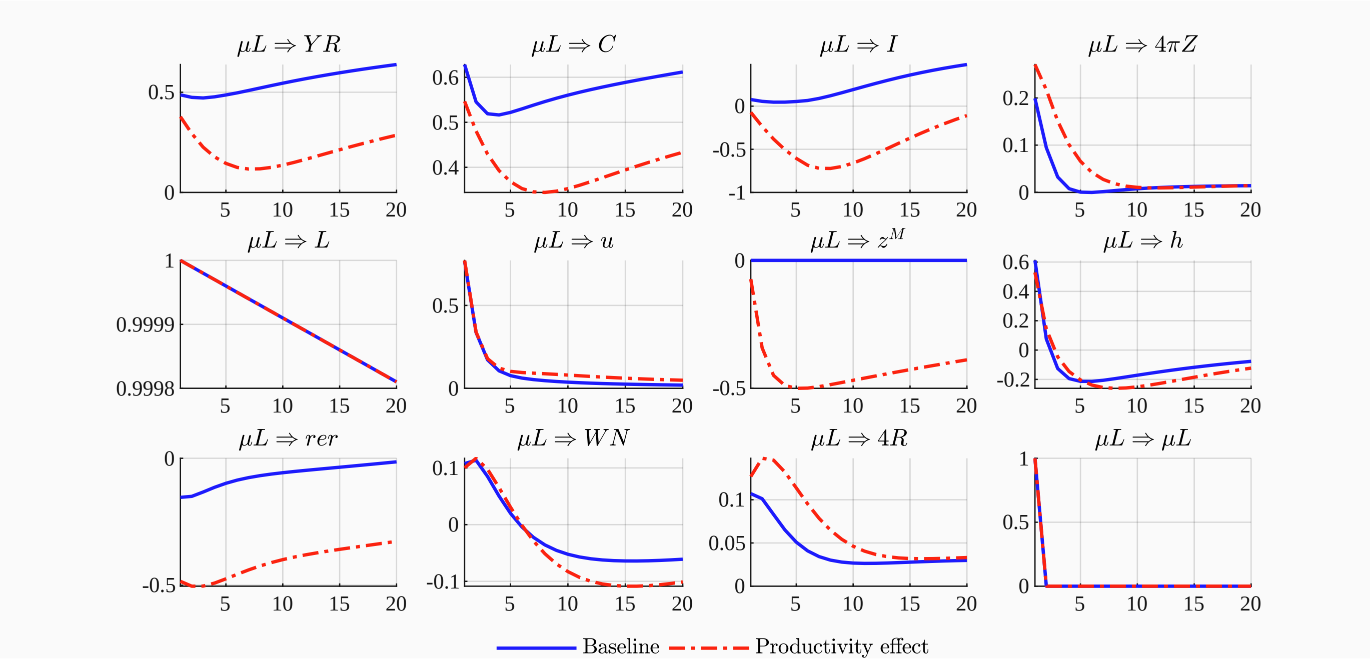
Figure 7: Allowing for Remittances



Note: Impulse responses to an immigration shock. Blue lines show responses under the baseline parameterization. Dashed red lines allow for the possibility of immigrants send 17% of their income as remittances to their home country. See the note to figure 5 for more details.

We have modeled immigrants and natives that have the same productivity (and skills), motivated, as previously mentioned, by the fact that, in Chile, immigrants and natives have similar education levels. An interesting possibility is that, despite this similarity, immigrants experience an underemployment spell—a period in which they cannot fully exercise the productivity associated to their skill level. Arias and Guerra-Salas (2019) consider this feature of the adaptation of immigrants to the local labor market in an OLG model with workers of heterogeneous skills. Although our framework does not feature skill heterogeneity, we simulate this underemployment spell as a transitory decline in average labor productivity, as previously mentioned. Figure 8 compares the baseline effects of an immigration shock with a scenario of lower average productivity in the economy (see the graph in the second row and third column). Output, consumption, and investment expand to their permanently higher levels, but do so more slowly than in the baseline case. Lower productivity puts upward pressure on firms' marginal costs, leading to higher prices. Inflation is higher even though the labor market is more slack due to lower vacancy posting (not shown in the figure). Following the Taylor rule, the central bank raises the interest rate more persistently than in the baseline case.

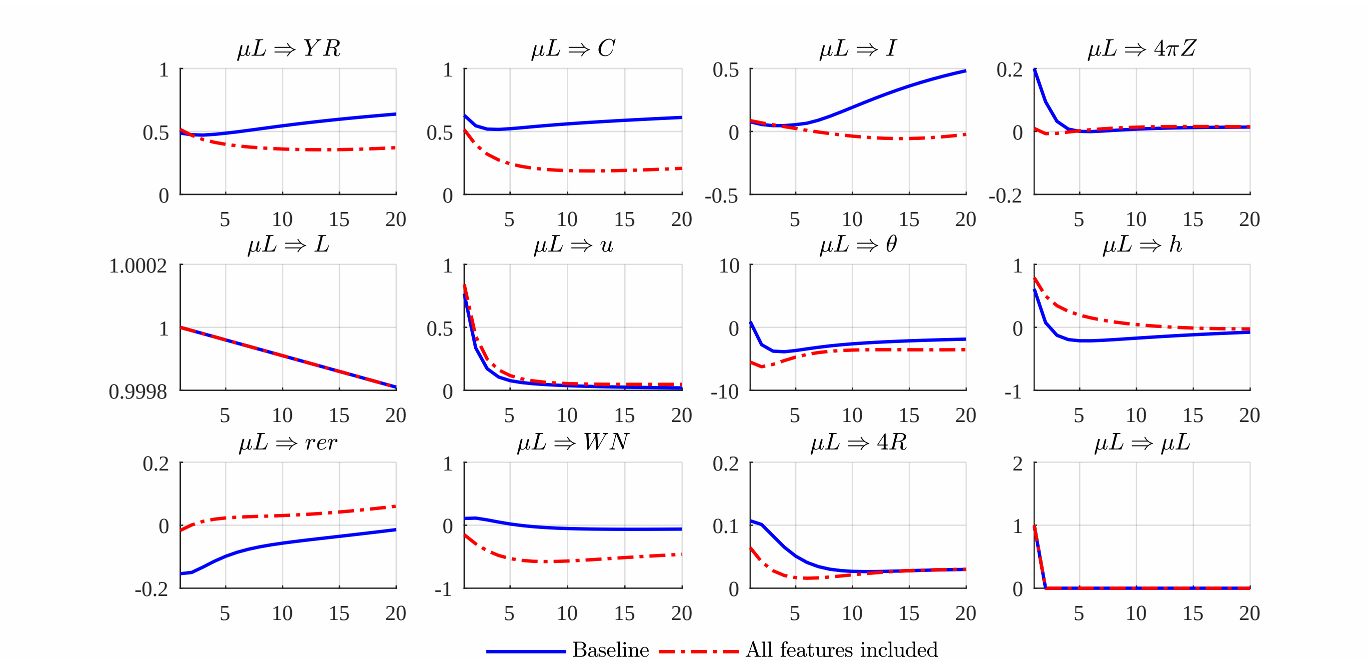
Figure 8: Immigrant Underemployment: Lower Average Labor Productivity



Note: Impulse responses to an immigration shock. Blue lines show responses under the baseline parameterization. Dashed red lines feature a reduction in average labor productivity that aims to capture a situation in which immigrants experience a transitory underemployment spell. See the note to figure 5 for more details.

To conclude this section, and for completeness, figure 9 considers the effect of an immigration shock with all the features previously discussed—a stronger labor supply effect, remittances, and lower average labor productivity. The figure also shows the baseline impulse-responses. The response of inflation, our main variable of interest, is practically nil when all features are considered. This happens because, under our parameterization, the stronger labor supply effect, which pulls inflation downwards, is about the same size as the demand effect and labor productivity effects, which push inflation upwards. Our results, therefore, show that the effects of immigration on inflation depend crucially on the way immigrants adjust to the local labor market: whether they are willing to supply more labor than natives at a given wage, and whether they are able to find jobs that match their skill level.

Figure 9: Dynamic Effects of an Immigration Shock



Note: Impulse responses to an immigration shock. Blue lines show responses under the baseline parameterization. Dashed red lines include all the additional features previously discussed: a stronger labor supply effect, remittances, and lower average labor productivity. See the note to figure 5 for more details.

5 Conclusion

An immigration shock generates inflationary pressures through a demand channel, and disinflationary pressures through a labor supply channel. We have studied these effects, as well as the response of a central bank following a Taylor rule, in a small open economy New Keynesian model with search frictions in the labor market. A baseline parameterization for Chile, an emerging country that has experienced a substantial immigration flow in recent years, suggests the demand channel dominates the labor supply channel, so an immigration shock leads to an increase in inflation and the interest rate.

These effects would be reversed, i.e. the labor supply channel would dominate the demand channel, if it were the case, for example, that immigrants were willing to accept lower wages than natives for a given job, or put another way, to offer more labor at any given wage. This stronger labor supply effect would lead to a decrease in inflation and the interest rate. We also find that, under our parameterization, allowing immigrants to send a fraction of their income as remittances has little effect on inflation and the response of monetary policy. Remittances generate opposing forces on inflation that nearly cancel each other out: an exchange rate depreciation pushes inflation up, but weaker consumption pushes inflation down. In our last experiment, we simulate a reduction in average labor productivity that would result from a situation in which newly arrived immigrants experience an underemployment spell: a period in which they cannot fully exercise the productivity associated to their skill level (which we assume to be similar to natives following empirical evidence). In this case, firms face cost pressures that lead to higher inflation.

Our analysis of the effects of immigration does not benefit from hindsight. By focusing on the immigration that Chile has experienced in the recent past, it is concerned with the possible outcomes of an ongoing phenomenon, rather than a conclusive analysis of an historical phenomenon. We believe this approach is useful to policymakers, who face the difficult task of evaluating the state of the economy in real time, and responding with policy decisions under substantial uncertainty. Our framework contributes to that challenge, since it facilitates the quantification of the different channels through which immigration affects the economy.

In future work, it would be useful to study the optimal design of monetary policy when the economy is subject to immigration shocks. Our framework is adequate for that task, as it features search frictions in a labor market at which immigrants arrive as unemployed members of the labor force. Optimal policy would consider the inefficiencies associated to the adjustment of immigrants to the local labor market.

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Appendix

The appendix contains the complete set of equilibrium conditions and the steady state, but in addition to this paper, we refer the reader to García et al. (2019), who describe the model without the extension that allows for exogenous immigration shocks.

A Equilibrium Conditions

The variables in uppercase that are not prices contain a unit root in equilibrium due to the presence of the non-stationary productivity shock A_t. We need to transform these variables to have a stationary version of the model. To do this, with the exceptions we enumerate below, lowercase variables denote the uppercase variable divided by A_{t-1} (e.g. $c_t \equiv \frac{C_t}{A_{t-1}}$). There are two exceptions: the Lagrange multiplier Λ_t that is multiplied by A_{t-1}^{σ} (i.e. $\lambda_t \equiv \Lambda_t A_{t-1}^{\sigma}$), for it decreases along the balanced growth path, and the parameter $\psi_t^U = \Psi_t^U / A_{t-1}^{\sigma}$, which we need to define in order to derive a stationary equilibrium in the labor market.

The rational expectations equilibrium of the stationary version of the model is the set of sequences for the endogenous variables such that for given initial values and exogenous variables, and assuming

$$\tilde{c}_t \sim \log N\left(\mu_{\tilde{c}}, \sigma_{\tilde{c}}^2\right),$$

the following conditions are satisfied:

$$\widehat{c}_t^{s,R} = \left[\left(1 - o_{\widehat{C}}\right)^{\frac{1}{\eta_{\widehat{C}}}} \left(\hat{c}_t^R - \varsigma \hat{c}_{t-1}^R / a_{t-1}\right)^{\frac{\eta_{\widehat{C}}-1}{\eta_{\widehat{C}}}} + o_{\widehat{C}}^{\frac{1}{\eta_{\widehat{C}}}} \left(\hat{c}_t^G\right)^{\frac{\eta_{\widehat{C}}-1}{\eta_{\widehat{C}}}}\right]^{\frac{\eta_{\widehat{C}}}{\eta_{\widehat{C}}-1}} \tag{EE.1}$$

$$\widehat{c}_t^{s,NR} = \left[\left(1 - o_{\widehat{C}}\right)^{\frac{1}{\eta_{\widehat{C}}}} \left(\hat{c}_t^{NR} - \varsigma \hat{c}_{t-1}^{NR} / a_{t-1}\right)^{\frac{\eta_{\widehat{C}}-1}{\eta_{\widehat{C}}}} + o_{\widehat{C}}^{\frac{1}{\eta_{\widehat{C}}}} \left(\hat{c}_t^G\right)^{\frac{\eta_{\widehat{C}}-1}{\eta_{\widehat{C}}}}\right]^{\frac{\eta_{\widehat{C}}}{\eta_{\widehat{C}}-1}} \tag{EE.2}$$

$$\hat{c}_t^G = \frac{c_t^G}{L_t^{\gamma^G}} \tag{EE.3}$$

$$\tau_t^L = \tau_t^{UFA} + \tau_t^W \tag{EE.4}$$

$$\hat{k}_t^{S,R} = \bar{u}_t \frac{\hat{k}_{t-1}^R}{a_{t-1}\gamma_t^L} \tag{EE.5}$$

$$\phi_{\bar{u}}(\bar{u}_t) = \frac{r^k}{\Phi_{\bar{u}}}\left(e^{\Phi_{\bar{u}}(\bar{u}_t - 1)} - 1\right) \tag{EE.6}$$

$$\hat{k}_t^R = (1-\delta)\frac{\hat{k}_{t-1}^R}{a_{t-1}\gamma_t^L} + \left(1 - \frac{\Phi_I}{2}\left(\gamma_t^L \frac{\hat{i}_t^R}{\hat{i}_{t-1}^R} a_{t-1} - a\right)^2\right)\varpi_t \hat{i}_t^R \tag{EE.7}$$

$$\left(1 + \tau_t^C\right)\lambda_t^R = \left(\widehat{c}_t^{s,R}\right)^{-\sigma}\left(\frac{\left(1 - o_{\widehat{C}}\right)\widehat{c}_t^{s,R}}{\hat{c}_t^R - \varsigma \hat{c}_{t-1}^R / a_{t-1}}\right)^{\frac{1}{\eta_{\widehat{C}}}} \tag{EE.8}$$

$$\lambda_t^R = \frac{\beta}{a_t^\sigma} R_t E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\pi_{t+1}} \right\} \tag{EE.9}$$

$$\lambda_t^R = \frac{\beta}{a_t^\sigma} R_t^* \xi_t E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\pi_{t+1}^S \lambda_{t+1}^R}{\pi_{t+1}} \right\} \tag{EE.10}$$

$$q_t = \frac{\beta}{a_t^\sigma} E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} \left[\begin{array}{c} r_{t+1}^K \bar{u}_{t+1} \left(1 - \tau_{t+1}^K\right) + q_{t+1}(1-\delta) \\ + p_{t+1}^I \left[\tau_{t+1}^K \delta - \phi_{\bar{u}}(\bar{u}_{t+1}) \left(1 - \tau_{t+1}^K\right) \right] \end{array} \right] \right\} \tag{EE.11}$$

$$\begin{aligned} \frac{p_t^I}{q_t} = & \left[\left(1 - \frac{\Phi_I}{2} \left(\frac{\hat{i}_t^R \gamma_t^L}{\hat{i}_{t-1}^R} a_{t-1} - a \right)^2 \right) - \Phi_I \left(\frac{\hat{i}_t^R \gamma_t^L}{\hat{i}_{t-1}^R} a_{t-1} - a \right) \frac{\hat{i}_t^R \gamma_t^L}{\hat{i}_{t-1}^R} a_{t-1} \right] \varpi_t \\ & + \frac{\beta}{a_t^\sigma} E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} \frac{q_{t+1}}{q_t} \Phi_I \left(\frac{\hat{i}_{t+1}^R \gamma_{t+1}^L}{\hat{i}_t^R} a_t - a \right) \left(\frac{\hat{i}_{t+1}^R \gamma_{t+1}^L}{\hat{i}_t^R} a_t \right)^2 \varpi_{t+1} \right\} \end{aligned} \tag{EE.12}$$

$$\bar{u}_t = 1 + \frac{\log\left(r_t^K / r^K \right) - \log\left(p_t^I\right)}{\Phi_{\bar{u}}} \tag{EE.13}$$

$$\xi_t = \bar{\xi} \exp\left[-\psi \left(\frac{rer_t b_t^*}{p_t^Y y_t} - \frac{rer\, b^*}{p^Y y} \right) + \frac{\zeta_t^O - \zeta^O}{\zeta^O} + \frac{\zeta_t^U - \zeta^U}{\zeta^U} \right] \tag{EE.14}$$

$$b_t^* = b_t^{Pr^*} + b_t^{G^*} \tag{EE.15}$$

$$\left(1 + \tau_t^C\right) \lambda_t^{NR} = \left(\widehat{c}_t^{s,NR}\right)^{-\sigma} \left(\frac{\left(1 - o_{\widehat{C}}\right) \widehat{c}_t^{s,NR}}{\hat{c}_t^{NR} - \varsigma \hat{c}_{t-1}^{NR}/a_{t-1}} \right)^{\frac{1}{\eta_{\widehat{C}}}} \tag{EE.16}$$

$$\left(1 + \tau_t^C\right) \hat{c}_t^{NR} = \left(1 - \tau_t^L\right) w_t n_t h_t + u_t ub + \hat{tr}_t^{NR} - \hat{t}_t^{NR} \tag{EE.17}$$

$$\Theta_t^R = \widetilde{\chi}_t^R \left(\nabla_{t-1}^H\right)^\sigma \left(\widehat{c}^s \left(\hat{c}_t^R - \varsigma \hat{c}_{t-1}^R / a_{t-1}, \hat{c}^G\right)\right)^{-\sigma} \tag{EE.18}$$

$$\Theta_t^{NR} = \widetilde{\chi}_t^{NR} \left(\nabla_{t-1}^H\right)^\sigma \left(\widehat{c}^s \left(\hat{c}_t^{NR} - \varsigma \hat{c}_{t-1}^{NR} / a_{t-1}, \hat{c}^G\right)\right)^{-\sigma} \tag{EE.19}$$

$$\widetilde{\chi}_t^R = \left(\widetilde{\chi}_{t-1}^R\right)^{1-\nu} \left(\nabla_{t-1}^H\right)^{-\sigma\nu} \left(\widehat{c}^s \left(\hat{c}_t^R - \varsigma \hat{c}_{t-1}^R / a_{t-1}, \hat{c}^G\right)\right)^{\sigma\nu} \tag{EE.20}$$

$$\widetilde{\chi}_t^{NR} = \left(\widetilde{\chi}_{t-1}^{NR}\right)^{1-\nu} \left(\nabla_{t-1}^H\right)^{-\sigma\nu} \left(\widehat{c}^s \left(\hat{c}_t^{NR} - \varsigma \hat{c}_{t-1}^{NR} / a_{t-1}, \hat{c}^G\right)\right)^{\sigma\nu} \tag{EE.21}$$

$$\gamma_t^L = \frac{L_t}{L_{t-1}} \tag{EE.22}$$

$$L_t = L1_t \times L2_t \tag{EE.23}$$

$$L1_t = \left(L1_{t-1}\right)^{\rho_{L1}} \mu_t^{L1} \tag{EE.24}$$

$$L2_t = \left(L2_{t-1}\right)^{\rho_{L2}} \mu_t^{L2} \tag{EE.25}$$

$$N_t = (1 - \rho_t) \left(N_{t-1} + m_{t-1} v_{t-1}^{1-\mu} U_{t-1}^{\mu} \right) \tag{EE.26}$$

$$m_t = \frac{\bar{m}\left(U_t^L + \lambda_M U_t^M\right)}{U_t} \tag{EE.27}$$

$$L_t = N_t + U_t \tag{EE.28}$$

$$L_t^L = L\left(1 - \omega_M\right) \tag{EE.29}$$

$$L_t^M = L_t - L_t^L \tag{EE.30}$$

$$s_t = m_t \left(\frac{v_t}{U_t}\right)^{1-\mu} \tag{EE.31}$$

$$s_t^L = \frac{s_t \bar{m}}{m_t} \tag{EE.32}$$

$$U_t^L = \left[1 - s_t^L\left(1 - \rho_t\right)\right] U_{t-1}^L + \rho_t N_{t-1}^L \tag{EE.33}$$

$$N_t^L = L_t^L - U_t^L \tag{EE.34}$$

$$N_t^M = N_t - N_t^L \tag{EE.35}$$

$$e_t = m_t \left(\frac{v_t}{U_t}\right)^{-\mu} \tag{EE.36}$$

$$\rho_t = \rho_t^x + \left(1 - \rho_t^x\right)\rho_t^n \tag{EE.37}$$

$$\rho_t^n = 1 - F(\bar{c}_t) = 1 - \Phi\left(\frac{\ln \bar{c}_t - \mu_{\tilde{c}}}{\sigma_{\tilde{c}}}\right) \tag{EE.38}$$

$$n_t = \frac{N_t}{L_t} \tag{EE.39}$$

$$u_t = \frac{U_t}{L_t} \tag{EE.40}$$

where Φ is the standard normal c.d.f.

$$c_t = \left[\left(1 - \kappa_O - \kappa_A\right)^{\frac{1}{\eta_C}} \left(c_t^Z\right)^{\frac{\eta_C - 1}{\eta_C}} + \kappa_O^{\frac{1}{\eta_C}} \left(c_t^O\right)^{\frac{\eta_C - 1}{\eta_C}} + \kappa_A^{\frac{1}{\eta_C}} \left(c_t^A\right)^{\frac{\eta_C - 1}{\eta_C}}\right]^{\frac{\eta_C}{\eta_C - 1}} \tag{EE.41}$$

$$c_t^Z = \left(1 - \kappa_O - \kappa_A\right)\left(p_t^Z\right)^{-\eta_C} c_t \tag{EE.42}$$

$$c_t^O = \kappa_O \left(p_t^O\right)^{-\eta_C} c_t \tag{EE.43}$$

$$c_t^A = \kappa_A \left(p_t^A\right)^{-\eta_C} c_t \tag{EE.44}$$

$$c_t^Z = \left[\left(1 - o_Z\right)^{\frac{1}{\eta_Z}} \left(x_t^{Z,H}\right)^{\frac{\eta_Z - 1}{\eta_Z}} + o_Z^{\frac{1}{\eta_Z}} \left(x_t^{Z,F}\right)^{\frac{\eta_Z - 1}{\eta_Z}}\right]^{\frac{\eta_Z}{\eta_Z - 1}} \tag{EE.45}$$

$$x_t^{Z,F} = o_Z \left(p_t^F / p_t^Z\right)^{-\eta_Z} c_t^Z \tag{EE.46}$$

$$x_t^{Z,H} = \left(1 - o_Z\right)\left(p_t^H / p_t^Z\right)^{-\eta_Z} c_t^Z \tag{EE.47}$$

$$c_t^A = z_t^A \left[(1-o_A)^{\frac{1}{\eta_A}} \left(x_t^{A,H} \right)^{\frac{\eta_A - 1}{\eta_A}} + o_A^{\frac{1}{\eta_A}} \left(x_t^{A,F} \right)^{\frac{\eta_A - 1}{\eta_A}} \right]^{\frac{\eta_A}{\eta_A - 1}} \tag{EE.48}$$

$$x_t^{A,F} = \left(z_t^A \right)^{\eta_A - 1} o_A \left(p_t^F / p_t^A \right)^{-\eta_A} c_t^A \tag{EE.49}$$

$$x_t^{A,H} = \left(z_t^A \right)^{\eta_A - 1} (1-o_A) \left(p_t^H / p_t^A \right)^{-\eta_A} c_t^A \tag{EE.50}$$

$$c_t^G = \left[(1-o_{CG})^{\frac{1}{\eta_{CG}}} \left(x_t^{CG,H} \right)^{\frac{\eta_{CG} - 1}{\eta_{CG}}} + o_{CG}^{\frac{1}{\eta_{CG}}} \left(x_t^{CG,F} \right)^{\frac{\eta_{CG} - 1}{\eta_{CG}}} \right]^{\frac{\eta_{CG}}{\eta_{CG} - 1}} \tag{EE.51}$$

$$x_t^{CG,F} = o_{CG} \left(p_t^F / p_t^{CG} \right)^{-\eta_{CG}} c_t^G \tag{EE.52}$$

$$x_t^{CG,H} = (1-o_{CG}) \left(p_t^H / p_t^{CG} \right)^{-\eta_{CG}} c_t^G \tag{EE.53}$$

$$i_t^f = \left[(1-o_I)^{\frac{1}{\eta_I}} \left(x_t^{I,H} \right)^{\frac{\eta_I - 1}{\eta_I}} + o_I^{\frac{1}{\eta_I}} \left(x_t^{I,F} \right)^{\frac{\eta_I - 1}{\eta_I}} \right]^{\frac{\eta_I}{\eta_I - 1}} \tag{EE.54}$$

$$x_t^{I,F} = o_I \left(p_t^F / p_t^I \right)^{-\eta_I} i_t^f \tag{EE.55}$$

$$x_t^{I,H} = (1-o_I) \left(p_t^H / p_t^I \right)^{-\eta_I} i_t^f \tag{EE.56}$$

$$i_t^f = i_t + \phi_{\bar{u}} \left(\bar{u}_t \right) \frac{k_{t-1}}{a_{t-1}} \tag{EE.57}$$

$$i_t^{Co,f} = \left[(1-o_{Co})^{\frac{1}{\eta_{Co}}} \left(x_t^{Co,H} \right)^{\frac{\eta_{Co} - 1}{\eta_{Co}}} + o_{Co}^{\frac{1}{\eta_{Co}}} \left(x_t^{Co,F} \right)^{\frac{\eta_{Co} - 1}{\eta_{Co}}} \right]^{\frac{\eta_{Co}}{\eta_{Co} - 1}} \tag{EE.58}$$

$$x_t^{Co,F} = o_{Co} \left(p_t^F / p_t^{ICo} \right)^{-\eta_{Co}} i_t^{Co,f} \tag{EE.59}$$

$$x_t^{Co,H} = (1-o_{Co}) \left(p_t^H / p_t^{ICo} \right)^{-\eta_{Co}} i_t^{Co,f} \tag{EE.60}$$

$$i_t^G = \left[(1-o_{IG})^{\frac{1}{\eta_{IG}}} \left(x_t^{IG,H} \right)^{\frac{\eta_{IG} - 1}{\eta_{IG}}} + o_{IG}^{\frac{1}{\eta_{IG}}} \left(x_t^{IG,F} \right)^{\frac{\eta_{IG} - 1}{\eta_{IG}}} \right]^{\frac{\eta_{IG}}{\eta_{IG} - 1}} \tag{EE.61}$$

$$x_t^{IG,F} = o_{IG} \left(p_t^F / p_t^{IG} \right)^{-\eta_{IG}} i_t^G \tag{EE.62}$$

$$x_t^{IG,H} = (1-o_{IG}) \left(p_t^H / p_t^{IG} \right)^{-\eta_{IG}} i_t^G \tag{EE.63}$$

$$\begin{aligned} f_t^H \quad = \quad & \left(\tilde{p}_t^H \right)^{-\epsilon_H} y_t^H mc_t^H + \frac{\beta}{a_t^{\sigma-1}} \theta_H \\ & \times E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} \left(\frac{g_t^{\Gamma^H}}{\pi_{t+1}} \frac{\tilde{p}_t^H}{\tilde{p}_{t+1}^H} \right)^{-\epsilon_H} \left(\frac{p_t^H}{p_{t+1}^H} \frac{\left(1 + \tau_{t+1}^C \right)}{\left(1 + \tau_t^C \right)} \right)^{-1-\epsilon_H} f_{t+1}^H \right\} \end{aligned} \tag{EE.64}$$

$$
\begin{aligned}
f_t^H &= \left(\tilde{p}_t^H\right)^{1-\epsilon_H} y_t^H \left(\frac{\epsilon_H - 1}{\epsilon_H}\right) + \frac{\beta}{a_t^{\sigma-1}}\theta_H \\
&\quad \times E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} \left(\frac{g_t^{\Gamma^H}}{\pi_{t+1}} \frac{\tilde{p}_t^H}{\tilde{p}_{t+1}^H}\right)^{1-\epsilon_H} \left(\frac{p_t^H}{p_{t+1}^H} \frac{\left(1+\tau_{t+1}^C\right)}{\left(1+\tau_t^C\right)}\right)^{-\epsilon_H} f_{t+1}^H \right\}
\end{aligned} \tag{EE.65}
$$

$$
1 = (1-\theta_H)\left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \theta_H \left(\frac{p_{t-1}^H}{p_t^H} \frac{g_{t-1}^{\Gamma^H}}{\pi_t} \frac{\left(1+\tau_t^C\right)}{\left(1+\tau_{t-1}^C\right)}\right)^{1-\epsilon_H} \tag{EE.66}
$$

$$
mc_t^H = \frac{p_t^{\widetilde{H}}}{p_t^H} \tag{EE.67}
$$

$$
g_t^{\Gamma^H} = \pi_t^{\vartheta_H} \pi^{1-\vartheta_H} \tag{EE.68}
$$

$$
\begin{aligned}
f_t^F &= \left(\tilde{p}_t^F\right)^{-\epsilon_F} y_t^F mc_t^F + \frac{\beta}{a_t^{\sigma-1}}\theta_F \\
&\quad \times E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} \left(\frac{g_t^{\Gamma^F}}{\pi_{t+1}} \frac{\tilde{p}_t^F}{\tilde{p}_{t+1}^F}\right)^{-\epsilon_F} \left(\frac{p_t^F}{p_{t+1}^F} \frac{\left(1+\tau_{t+1}^C\right)}{\left(1+\tau_t^C\right)}\right)^{-1-\epsilon_F} f_{t+1}^F \right\}
\end{aligned} \tag{EE.69}
$$

$$
\begin{aligned}
f_t^F &= \left(\tilde{p}_t^F\right)^{1-\epsilon_F} y_t^F \left(\frac{\epsilon_F - 1}{\epsilon_F}\right) + \frac{\beta}{a_t^{\sigma-1}}\theta_F \\
&\quad \times E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} \left(\frac{g_t^{\Gamma^F}}{\pi_{t+1}} \frac{\tilde{p}_t^F}{\tilde{p}_{t+1}^F}\right)^{1-\epsilon_F} \left(\frac{p_t^F}{p_{t+1}^F} \frac{\left(1+\tau_{t+1}^C\right)}{\left(1+\tau_t^C\right)}\right)^{-\epsilon_F} f_{t+1}^F \right\}
\end{aligned} \tag{EE.70}
$$

$$
1 = (1-\theta_F)\left(\tilde{p}_t^F\right)^{1-\epsilon_F} + \theta_F \left(\frac{p_{t-1}^F}{p_t^F} \frac{g_{t-1}^{\Gamma^F}}{\pi_t} \frac{\left(1+\tau_t^C\right)}{\left(1+\tau_{t-1}^C\right)}\right)^{1-\epsilon_F} \tag{EE.71}
$$

$$
mc_t^F = \frac{p^{M*} rer_t}{p_t^F} \tag{EE.72}
$$

$$
g_t^{\Gamma^F} = \pi_t^{\vartheta_F} \pi^{1-\vartheta_F} \tag{EE.73}
$$

$$
\begin{aligned}
f_t^{H*} &= \left(\tilde{p}_t^{H*}\right)^{-\epsilon_{H*}} y_t^{H*} mc_t^{H*} + \frac{\beta}{a_t^{\sigma-1}}\theta_{H*} \\
&\quad \times E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} \frac{rer_{t+1}}{rer_t} \left(\frac{g_t^{\Gamma^{H*}}}{\pi_{t+1}^*} \frac{\tilde{p}_t^{H*}}{\tilde{p}_{t+1}^{H*}}\right)^{-\epsilon_{H*}} \left(\frac{p_t^{H*}}{p_{t+1}^{H*}}\right)^{-1-\epsilon_{H*}} f_{t+1}^{H*} \right\}
\end{aligned} \tag{EE.74}
$$

$$
\begin{aligned}
f_t^{H*} &= \left(\tilde{p}_t^{H*}\right)^{1-\epsilon_{H*}} y_t^{H*} \left(\frac{\epsilon_{H*} - 1}{\epsilon_{H*}}\right) + \frac{\beta}{a_t^{\sigma-1}}\theta_{H*} \\
&\quad \times E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} \frac{rer_{t+1}}{rer_t} \left(\frac{g_t^{\Gamma^{H*}}}{\pi_{t+1}^*} \frac{\tilde{p}_t^{H*}}{\tilde{p}_{t+1}^{H*}}\right)^{1-\epsilon_{H*}} \left(\frac{p_t^{H*}}{p_{t+1}^{H*}}\right)^{-\epsilon_{H*}} f_{t+1}^{H*} \right\}
\end{aligned} \tag{EE.75}
$$

$$1 = (1-\theta_{H*})\left(\tilde{p}_t^{H*}\right)^{1-\epsilon_{H*}} + \theta_{H*}\left(\frac{p_{t-1}^{H*}}{p_t^{H*}}\frac{g_{t-1}^{\Gamma^{H*}}}{\pi_t^*}\right)^{1-\epsilon_{H*}} \tag{EE.76}$$

$$mc_t^{H*} = \frac{p_t^{\widetilde{H}}}{rer_t p_t^{H*}} \tag{EE.77}$$

$$g_t^{\Gamma^{H*}} = \left(\pi_t^*\right)^{\vartheta_{H*}}\left(\pi^*\right)^{1-\vartheta_{H*}} \tag{EE.78}$$

$$y_t^{\widetilde{H}} = z_t\left[(1-o_O)^{\frac{1}{\eta_O}}\left(x_t^{\widetilde{Z}}\right)^{\frac{\eta_O-1}{\eta_O}} + o_O{}^{\frac{1}{\eta_O}}\left(x_t^O\right)^{\frac{\eta_O-1}{\eta_O}}\right]^{\frac{\eta_O}{\eta_O-1}} \tag{EE.79}$$

$$x_t^{\widetilde{Z}} = (z_t)^{\eta_O-1}(1-o_O)\left(\frac{mc_t^{\widetilde{Z}}}{p_t^{\widetilde{H}}}\right)^{-\eta_O} y_t^{\widetilde{H}} \tag{EE.80}$$

$$x_t^O = (z_t)^{\eta_O-1} o_O\left(\frac{p_t^O}{p_t^{\widetilde{H}}}\right)^{-\eta_O} y_t^{\widetilde{H}} \tag{EE.81}$$

$$y_t^{\widetilde{Z}} = \left(\widetilde{k}_t\right)^{\alpha}\left(a_t z_t^M \nabla_t^H N_t h_t\right)^{1-\alpha} \tag{EE.82}$$

$$log(z_t^M) = \rho_{ZM} log(z_{t-1}^M) - \alpha_{ZM}\left(\frac{N_t^M}{N_t} - \frac{N_{t-1}^M}{N_{t-1}}\right) \tag{EE.83}$$

$$\widetilde{k}_t = \alpha\left(\frac{r_t^{\widetilde{K}}}{mc_t^{\widetilde{Z}}}\right)^{-1} y_t^{\widetilde{Z}} \tag{EE.84}$$

$$\widetilde{k}_t = \left[(1-o_{KG})^{\frac{1}{\eta_{KG}}}\left(k_t^S\right)^{\frac{\eta_{KG}-1}{\eta_{KG}}} + o_{KG}{}^{\frac{1}{\eta_{KG}}}\left(\frac{k_{t-1}^G}{a_{t-1}}\right)^{\frac{\eta_{KG}-1}{\eta_{KG}}}\right]^{\frac{\eta_{KG}}{\eta_{KG}-1}} \tag{EE.85}$$

$$k_t^S = (1-o_{KG})\left(\frac{r_t^K}{r_t^{\widetilde{K}}}\right)^{-\eta_{KG}} \widetilde{k}_t \tag{EE.86}$$

$$\frac{\nabla_{t-1}^H p_t^H \Omega_v}{e_t} = \frac{\beta}{a_t^{\sigma-1}} E_t\left\{\frac{\varrho_{t+1}}{\varrho_t}\frac{\lambda_{t+1}^R}{\lambda_t^R}(1-\rho_{t+1})\left(mc_{t+1}^{\widetilde{Z}}(1-\alpha)\frac{y_{t+1}^{\widetilde{Z}}}{N_{t+1}} - w_{t+1}h_{t+1} - p_{t+1}^H h_{t+1}^C + \frac{\nabla_t^H p_{t+1}^H \Omega_v}{e_{t+1}}\right)\right\} \tag{EE.87}$$

$$\nabla_{t-1}^H p_t^H \bar{c}_t = mc_t^{\widetilde{Z}}(1-\alpha)\frac{y_t^{\widetilde{Z}}}{N_t} - w_t h_t + \frac{\nabla_{t-1}^H p_t^H \Omega_v}{e_t} \tag{EE.88}$$

$$h_t^C = \nabla_{t-1}^H \frac{\exp\left(\mu_{\tilde{c}} + \frac{\sigma_{\tilde{c}}^2}{2}\right)\Phi\left(\frac{\ln \bar{c}_t - \mu_{\tilde{c}} - \sigma_{\tilde{c}}^2}{\sigma_{\tilde{c}}}\right)}{1-\rho_t^n} \tag{EE.89}$$

$$h_t = \left[\frac{mc_t^{\widetilde{Z}} (1-\alpha)^2 \frac{y_t^{\widetilde{Z}}}{N_t}}{\frac{\psi_t^U \kappa_t}{(1-\tau_t^L)} \left(\nabla_{t-1}^H\right)^{1-\sigma}} \right]^{\frac{1}{1+\phi}} \tag{EE.90}$$

$$\begin{aligned} w_t^n h_t = \varphi^U \left[mc_t^{\widetilde{Z}} (1-\alpha) \frac{y_t^{\widetilde{Z}}}{N_t} - p_t^H h_t^C + \nabla_{t-1}^H \frac{p_t^H \Omega_v}{e_t} \right] \\ + \frac{\left(1-\varphi^U\right)}{(1-\tau_t^L)} \left[ub + \psi_t^U \kappa_t \left(\nabla_{t-1}^H\right)^{1-\sigma} \frac{h_t^{1+\phi}}{1+\phi} - (1-s_t)\, \sigma_t^U \right] \end{aligned} \tag{EE.91}$$

$$\sigma_t^U = \left(1-\omega^U\right) E_t \left\{ \frac{\beta}{a_t^\sigma} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} (1-\rho_{t+1})\, s_{t+1}^{R,U} \right\} + \omega^U E_t \left\{ \frac{\beta}{a_t^\sigma} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^{NR}}{\lambda_t^{NR}} (1-\rho_{t+1})\, s_{t+1}^{NR,U} \right\} \tag{EE.92}$$

$$s_t^{R,U} = \ (1-\tau_t^L) w_t^n h_t - \frac{\Theta_t^R \kappa_t \left(\nabla_{t-1}^H\right)^{1-\sigma} \frac{h_t^{1+\phi}}{1+\phi}}{\lambda_t^R} - ub + (1-s_t)\, E_t \left\{ \frac{\beta}{a_t^\sigma} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^R}{\lambda_t^R} (1-\rho_{t+1})\, s_{t+1}^{R,U} \right\} \tag{EE.93}$$

$$s_t^{NR,U} = \ (1-\tau_t^L) w_t^n h_t - \frac{\Theta_t^{NR} \kappa_t \left(\nabla_{t-1}^H\right)^{1-\sigma} \frac{h_t^{1+\phi}}{1+\phi}}{\lambda_t^{NR}} - ub + (1-s_t)\, E_t \left\{ \frac{\beta}{a_t^\sigma} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^{NR}}{\lambda_t^{NR}} (1-\rho_{t+1})\, s_{t+1}^{NR,U} \right\} \tag{EE.94}$$

$$\kappa_t = \kappa_t^x + \kappa_t^M \tag{EE.95}$$

$$\kappa_t^M = \rho^\kappa \kappa_{t-1}^M - \alpha_\kappa \left(\frac{L_t^M}{L_t} \right) \tag{EE.96}$$

$$\pi_t w_t = \varkappa_W \Gamma_{t-1}^W \frac{w_{t-1}}{a_{t-1}} \frac{(1+\tau_t^C)}{(1+\tau_{t-1}^C)} + (1-\varkappa_W) \pi_t w_t^n \tag{EE.97}$$

$$\Gamma_t^W = \left(a \frac{\nabla_t^H}{\nabla_{t-1}^H} \right)^{\alpha_W} (a)^{1-\alpha_W}\, \pi_t^{\vartheta_W} \pi^{1-\vartheta_W} \tag{EE.98}$$

$$\psi_t^U = \left(1-\omega^U\right) \frac{\Theta_t^R}{\lambda_t^R} + \omega^U \frac{\Theta_t^{NR}}{\lambda_t^{NR}} \tag{EE.99}$$

$$y_t^{Co} = z_t^{Co} \left(\bar{u}_t^{Co} \frac{k_{t-1}^{Co}}{a_{t-1}} \right)^{\alpha Co} \left(a_t \nabla_t^{Co} \overline{L} \right)^{1-\alpha Co} \tag{EE.100}$$

$$k_t^{Co} = (1-\delta_{Co}) \frac{k_{t-1}^{Co}}{a_{t-1}} + \left(1 - \frac{\Phi_I^{Co}}{2} \left(\frac{i_{t-N_{Co}+1}^{ACo}}{i_{t-N_{Co}}^{ACo}} a_{t-N_{Co}} - a \right)^2 \right) \frac{i_{t-N_{Co}+1}^{ACo} a_t}{\prod_{i=1}^{N_{Co}} a_{t+1-i}} \varpi_{t-N_{Co}+1}^{Co} \tag{EE.101}$$

$$i_t^{Co} = \varphi_0^{Co} i_t^{ACo} + \frac{\varphi_1^{Co} i_{t-1}^{ACo}}{a_{t-1}} + \frac{\varphi_2^{Co} i_{t-2}^{ACo}}{a_{t-1} a_{t-2}} + \ldots + \frac{\varphi_{N_{Co}-1}^{Co} i_{t-N_{Co}+1}^{ACo}}{a_{t-1} a_{t-2} \ldots a_{t-N_{Co}+1}} \tag{EE.102}$$

$$\phi_{\bar{u}}^{Co} \left(\bar{u}_t^{Co} \right) \equiv \frac{r^{K,Co}}{\Phi_{\bar{u}}^{Co}} \left(e^{\Phi_{\bar{u}}^{Co} \left(\bar{u}_t^{Co} - 1 \right)} - 1 \right) \tag{EE.103}$$

$$i_t^{Co,f} = i_t^{Co} + \phi_{\bar{u}}^{Co} \left(\bar{u}_t^{Co} \right) \frac{k_{t-1}^{Co}}{a_{t-1}} \tag{EE.104}$$

$$cf_t^{Co} = rer_t p_t^{Co*} y_t^{Co} - p_t^{ICo} i_t^{Co.f} \tag{EE.105}$$

$$\lambda_t^{Co} = \left[1 - \tau_t^{Co}\left(1 - \chi^{Co}\right)\right] rer_t p_t^{Co*} \tag{EE.106}$$

$$q_t^{Co} = E_t\left\{\frac{\beta}{a_t^{\sigma}}\frac{\varrho_{t+1}}{\varrho_t}\frac{\lambda_{t+1}^R}{\lambda_t^R}\left[\lambda_{t+1}^{Co}\alpha^{Co}\frac{y_{t+1}^{Co}}{k_t^{Co}}a_t + q_{t+1}^{Co}(1-\delta_{Co}) - p_{t+1}^{ICo}\phi_{\bar{u}}^{Co}\left(\bar{u}_{t+1}^{Co}\right)\right]\right\} \tag{EE.107}$$

$$\begin{aligned}
0 = {} & E_t\left\{\frac{\sum_{j=0}^{N^{Co}-1}\beta^j\frac{\varrho_{t+j}}{\varrho_t}\frac{\lambda_{t+j}^R}{\lambda_t^R}\left(\frac{a_{t-1}}{\prod_{i=0}^{j}a_{t+i-1}}\right)^{\sigma}\varphi_j^{Co}p_{t+j}^{ICo}}{\left(\beta^{N^{Co}-1}\right)\frac{\varrho_{t+N^{Co}-1}}{\varrho_t}\frac{\lambda_{t+N^{Co}-1}^R}{\lambda_t^R}\left(\frac{a_{t-1}}{\prod_{i=0}^{N^{Co}-1}a_{t+i-1}}\right)^{\sigma}q_{t+N^{Co}-1}^{Co}}\right\} \\
& - \left[\left(1 - \frac{\Phi_I^{Co}}{2}\left(\frac{i_t^{ACo}}{i_{t-1}^{ACo}}a_{t-1} - a\right)^2\right) - \Phi_I^{Co}\left(\frac{i_t^{ACo}}{i_{t-1}^{ACo}}a_{t-1} - a\right)\frac{i_t^{ACo}}{i_{t-1}^{ACo}}a_{t-1}\right]\varpi_t^{Co} \\
& - E_t\left\{\begin{array}{l} \beta\frac{\varrho_{t+N^{Co}}}{\varrho_{t+N^{Co}-1}}\frac{\lambda_{t+N^{Co}}^R}{a_{t+N^{Co}}^{\sigma}\lambda_{t+N^{Co}-1}^R}\frac{q_{t+N^{Co}}^{Co}}{q_{t+N^{Co}-1}^{Co}} \\ \times\Phi_I^{Co}\left(\frac{i_{t+1}^{ACo}}{i_t^{ACo}}a_t - a\right)\left(\frac{i_{t+1}^{ACo}}{i_t^{ACo}}a_t\right)^2\varpi_{t+1}^{Co} \end{array}\right\}
\end{aligned} \tag{EE.108}$$

$$r_t^{K,Co} = \lambda_t^{Co}\alpha^{Co}\frac{y_t^{Co}}{\bar{u}_t^{Co}k_{t-1}^{Co}}a_{t-1} \tag{EE.109}$$

$$\bar{u}_t^{Co} = 1 + \frac{\log\left(r_t^{K,Co}/r^{K,Co}\right) - \log\left(p_t^{ICo}\right)}{\Phi_{\bar{u}}^{Co}} \tag{EE.110}$$

$$g_t = p_t^{CG}c_t^G + p_t^{IG}i_t^G + tr_t^G + \left(rer_t p_t^{O*} - p^O\right)o_t \tag{EE.111}$$

$$t_t = \alpha^T p_t^Y y + (1 - I_{rule})\,\varepsilon^T\left(rer_t b^{G^*} + b^G - rer_t b_t^{G^*} - b_t^G\right) \tag{EE.112}$$

$$L_t\omega\hat{t}_t^{NR} = \omega_G t_t \tag{EE.113}$$

$$L_t(1-\omega)\,\hat{t}_t^R = (1-\omega_G)\,t_t \tag{EE.114}$$

$$\tau_t = \tau_t^C c_t + \tau_t^W w_t N_t h_t + \tau_t^K\left[r_t^K\bar{u}_t - p_t^I\left(\delta + \phi_{\bar{u}}\left(\bar{u}_t\right)\right)\right]\frac{k_{t-1}}{a_{t-1}} + \tau_t^D d_t + (1-\chi^{Co})\tau_t^{Co}(cf_t^{Co} + p_t^{ICo}i_t^{Co,f}) + t_t \tag{EE.115}$$

$$b_t^G + rer_t b_t^{G^*} = R_{t-1}\frac{b_{t-1}^G}{\pi_t a_{t-1}} + R_{t-1}^*\xi_{t-1}\frac{rer_t b_{t-1}^{G*}}{\pi_t^* a_{t-1}} + \tau_t + \chi^{Co}cf_t^{Co} - g_t \tag{EE.116}$$

$$rer_t b_t^{G^*} = \alpha^D\left(rer_t b_t^{G^*} + b_t^G\right) \tag{EE.117}$$

$$\widetilde{g}_t = \left(\widetilde{g}_t^{rule}\right)^{I_{rule}}\left(\widetilde{g}_t^{exo}\right)^{1-I_{rule}} \tag{EE.118}$$

$$\widetilde{g}_t^{exo} = g\xi_t^G \tag{EE.119}$$

$$\widetilde{g}_t^{rule} = \frac{(R_{t-1}-1)b_{t-1}^G}{\pi_t a_{t-1}} + \frac{(R_{t-1}^*\xi_{t-1}-1)rer_t b_{t-1}^{G*}}{\pi_t^* a_{t-1}} + \tau_t - \gamma^D\breve{\tau}_t + \chi^{Co}\left(cf_t^{Co} - \gamma^D\breve{cf}_t^{Co}\right) - \bar{s}_B p_t^Y y_t \tag{EE.120}$$

$$\breve{\tau}_t = \tau_t - \widetilde{\tau}_t \tag{EE.121}$$

$$\widetilde{\tau}_t = \tau_t^C c + \tau_t^W wNh + \tau_t^K \left(r^K \overline{u} - p^I \left(\delta + \phi_{\bar{u}} \left(\bar{u} \right) \right) \right) k/a + \tau_t^D d + (1-\chi^{Co}) \tau_t^{Co} (cf^{Co} + p^{ICo} i^{Co,f}) + t \quad \text{(EE.122)}$$

$$\breve{cf}_t^{Co} = cf_t^{Co} - \widetilde{cf}_t^{Co} \quad \text{(EE.123)}$$

$$\widetilde{cf}_t^{Co} = rer_t \widetilde{p}_t^{Co*} y_t^{Co} - p_t^{ICo} i_t^{Co,f} \quad \text{(EE.124)}$$

$$\log\left(\widetilde{p}_t^{Co*}\right) = \frac{1}{40} E_t \sum_{i=1}^{40} \log\left(p_{t+i}^{Co*}\right) \quad \text{(EE.125)}$$

$$p_t^{CG} c_t^G = \alpha_{CG} \widetilde{g}_t \xi_t^{CG} \quad \text{(EE.126)}$$

$$tr_t^G = \left(1 - \alpha_{CG} - \alpha_{IG} - \frac{tr^{UFA}}{g}\right) \widetilde{g}_t \xi_t^{TR} + tr_t^{UFA} \quad \text{(EE.127)}$$

$$L_t \omega \hat{tr}_t^{NR} = \omega_G \left(tr_t^G - tr_t^{UFA}\right) \quad \text{(EE.128)}$$

$$L_t \left(1-\omega\right) \hat{tr}_t^R = \left(1-\omega_G\right) \left(tr_t^G - tr_t^{UFA}\right) \quad \text{(EE.129)}$$

$$k_t^G = \left(1-\delta_G\right) \frac{k_{t-1}^G}{a_{t-1}} + \frac{i_{t-N^G+1}^{AG} a_t}{\prod_{i=1}^{N^G} a_{t+1-i}} \quad \text{(EE.130)}$$

$$i_t^G = \varphi_0 i_t^{AG} + \frac{\varphi_1 i_{t-1}^{AG}}{a_{t-1}} + \frac{\varphi_2 i_{t-2}^{AG}}{a_{t-1} a_{t-2}} + \ldots + \frac{\varphi_{N^G-1} i_{t-N^G+1}^{AG}}{a_{t-1} a_{t-2} \ldots a_{t-N^G+1}} \quad \text{(EE.131)}$$

$$p_t^{IG} i_t^{AG} = \alpha_{IG} E_t \left[\sum_{j=0}^{N^G-1} \varphi_j \frac{\widetilde{g}_{t+j} \prod_{i=0}^{j} a_{t+i-1}}{a_{t-1}}\right] \xi_t^{IG} \quad \text{(EE.132)}$$

$$p_t^O = \left(\left(p^O\right)^{1-\alpha_O} \left(p_{t-1}^O\right)^{\alpha_O}\right)^{\rho_O} \left(rer_t p_t^{O^*}\right)^{1-\rho_O} \xi_t^O \quad \text{(EE.133)}$$

$$R_t = \left(R_{t-1}\right)^{\rho_R} \left[\overline{R}_t \left(\frac{\widetilde{\pi}_t}{\overline{\pi}_t}\right)^{\alpha_\pi} \left(\frac{y_t^D}{y_{t-1}^D}\right)^{\alpha_y}\right]^{1-\rho_R} \exp(\varepsilon_t^R) \quad \text{(EE.134)}$$

$$\widetilde{\pi}_t = \left[\left(\pi_t^Z\right)^{\alpha_{\pi Z}} \left(\pi_t\right)^{1-\alpha_{\pi Z}}\right]^{1-\alpha_{\pi E}} \left[\left(E_t \pi_{t+4}^Z\right)^{\alpha_{\pi Z}} \left(E_t \pi_{t+4}\right)^{1-\alpha_{\pi Z}}\right]^{\alpha_{\pi E}} \quad \text{(EE.135)}$$

$$y_t^D = y_t - x_t^L - \phi_{\bar{u}} \left(\bar{u}_t\right) \frac{k_{t-1}}{a_{t-1}} - \phi_{\bar{u}}^{Co} \left(\bar{u}_t^{Co}\right) \frac{k_{t-1}^{Co}}{a_{t-1}} \quad \text{(EE.136)}$$

$$\pi_t^Z = \frac{p_t^Z}{p_{t-1}^Z} \pi_t \quad \text{(EE.137)}$$

$$b_t^{UFA} = \tau_t^{UFA} w_t h_t N_t - U_t ub + tr_t^{UFA} + R_{t-1} \frac{b_{t-1}^{UFA}}{\pi_t a_{t-1}} \quad \text{(EE.138)}$$

$$tr_t^{UFA} = \overline{tr}^{UFA} + \varepsilon^{UFA} \left(b^{UFA} - b_t^{UFA}\right) \quad \text{(EE.139)}$$

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^S \pi_t^*}{\pi_t}, \quad \text{(EE.140)}$$

$$x_t^{H*} = \left[x_{t-1}^{H*}\right]^{\rho_{XH*}} \left[o^* \left(p_t^{H*}\right)^{-\eta^*} y_t^*\right]^{1-\rho_{XH*}} \xi_t^{XH*} \tag{EE.141}$$

$$y_t^* = a_t z_t^* \tag{EE.142}$$

$$\nabla_t^H = \left(\frac{a}{a_t} \nabla_{t-1}^H\right)^{1-\Gamma^H} \tag{EE.143}$$

$$\nabla_t^{Co} = \left(\frac{a}{a_t} \nabla_{t-1}^{Co}\right)^{1-\Gamma^{Co}} \tag{EE.144}$$

$$\pi_t^{F*} = \frac{f_t^*}{f_{t-1}^*} \pi_t^* \tag{EE.145}$$

$$1 = \left(\frac{\pi^*}{\pi_t^*}\right)^{1-\Gamma^*} \left(f_t^*\right)^{\Gamma^*} \xi_t^* \tag{EE.146}$$

$$p_t^{Co*} = \left(\frac{\pi^*}{\pi_t^*} p_{t-1}^{Co*}\right)^{1-\Gamma^{Co*}} \left(f_t^*\right)^{\Gamma^{Co*}} \xi_t^{Co*} \tag{EE.147}$$

$$p_t^{O*} = \left(\frac{\pi^*}{\pi_t^*} p_{t-1}^{O*}\right)^{1-\Gamma^{O*}} \left(f_t^*\right)^{\Gamma^{O*}} \xi_t^{O*} \tag{EE.148}$$

$$p_t^{M*} = \left(\frac{\pi^*}{\pi_t^*} p_{t-1}^{M*}\right)^{1-\Gamma^{M*}} \left(f_t^*\right)^{\Gamma^{M*}} \xi_t^{M*} \tag{EE.149}$$

$$\pi_t^{Co*} = \frac{p_t^{Co*}}{p_{t-1}^{Co*}} \pi_t^* \tag{EE.150}$$

$$\pi_t^{O*} = \frac{p_t^{O*}}{p_{t-1}^{O*}} \pi_t^* \tag{EE.151}$$

$$\pi_t^{M*} = \frac{p_t^{M*}}{p_{t-1}^{M*}} \pi_t^* \tag{EE.152}$$

$$c_t = L_t \left[\omega \hat{c}_t^{NR} + (1-\omega) \hat{c}_t^R\right] \tag{EE.153}$$

$$k_t = L_t (1-\omega) \hat{k}_t^R \tag{EE.154}$$

$$k_t^S = L_t (1-\omega) \hat{k}_t^{S,R} \tag{EE.155}$$

$$i_t = L_t (1-\omega) \hat{i}_t^R \tag{EE.156}$$

$$b_t^{Pr} = L_t (1-\omega) \hat{b}_t^R \tag{EE.157}$$

$$b_t^{Pr*} = L_t (1-\omega) \hat{b}_t^{R*} \tag{EE.158}$$

$$d_t = L_t (1-\omega) \hat{d}_t^R \tag{EE.159}$$

$$y_t^H = x_t^H \tag{EE.160}$$

$$x_t^H = x_t^{Z,H} + x_t^{A,H} + x_t^{CG,H} + x_t^{I,H} + x_t^{ICo,H} + x_t^{IG,H} + x_t^L, \tag{EE.161}$$

$$y_t^F = x_t^F \tag{EE.162}$$

$$x_t^F = x_t^{Z,F} + x_t^{A,F} + x_t^{CG,F} + x_t^{I,F} + x_t^{ICo,F} + x_t^{IG,F}, \tag{EE.163}$$

$$y_t^{H*} = x_t^{H*} \tag{EE.164}$$

$$x_t^L = h_t^C n_t + \nabla_{t-1}^H \Omega_v v_t \tag{EE.165}$$

$$y_t^{\tilde{H}} = y_t^H \Delta_t^H + y_t^{H*} \Delta_t^{H*} \tag{EE.166}$$

$$m_t^* = y_t^F \Delta_t^F \tag{EE.167}$$

$$\Delta_t^H = (1-\theta_H)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \theta_H \left(\frac{p_{t-1}^H}{p_t^H} \frac{g_{t-1}^{\Gamma^H}}{\pi_t} \frac{\left(1+\tau_t^C\right)}{(1+\tau_{t-1}^C)}\right)^{-\epsilon_H} \Delta_{t-1}^H \tag{EE.168}$$

$$\Delta_t^F = (1-\theta_F)\left(\tilde{p}_t^F\right)^{-\epsilon_F} + \theta_F \left(\frac{p_{t-1}^F}{p_t^F} \frac{g_{t-1}^{\Gamma^F}}{\pi_t} \frac{\left(1+\tau_t^C\right)}{(1+\tau_{t-1}^C)}\right)^{-\epsilon_F} \Delta_{t-1}^F \tag{EE.169}$$

$$\Delta_t^{H*} = (1-\theta_{H*})\left(\tilde{p}_t^{H*}\right)^{-\epsilon_{H*}} + \theta_{H*} \left(\frac{p_{t-1}^{H*}}{p_t^{H*}} \frac{g_{t-1}^{\Gamma^{H*}}}{\pi_t^*}\right)^{-\epsilon_{H*}} \Delta_{t-1}^{H*} \tag{EE.170}$$

$$o_t = c_t^O + x_t^O \tag{EE.171}$$

$$y_t^C = c_t + p_t^{CG} c_t^G + p_t^I i_t^f + p_t^{ICo} i_t^{Co,f} + p_t^{IG} i_t^G + p_t^H x_t^L \tag{EE.172}$$

$$tb_t = rer_t\left(p_t^{H*} y_t^{H*} + p_t^{Co*} y_t^{Co} - p_t^{M*} m_t^* - p^{O*} o_t\right) \tag{EE.173}$$

$$y_t = c_t + c_t^G + i_t^f + i_t^G + i_t^{Co,f} + x_t^L + y_t^{H*} + y_t^{Co} - m_t^* - o_t \tag{EE.174}$$

$$p_t^Y y_t = y_t^C + tb_t \tag{EE.175}$$

$$d_t = p_t^Y y_t - rer_t p_t^{Co*} y_t^{Co} + o_t\left(rer_t p_t^{O*} - p_t^O\right) - r_t^K k_t^S - w_t n_t h_t - p_t^H x_t^L \tag{EE.176}$$

$$rer_t\left(b_t^* - \frac{b_{t-1}^*}{\pi_t^* a_{t-1}}\right) = rer_t \frac{b_{t-1}^*}{\pi_t^* a_{t-1}}\left(R_{t-1}^* \xi_{t-1} - 1\right) + tb_t + rer_t ren_t^* \tag{EE.177}$$

$$ren_t^* = (1-\omega)\, ren_t^{R*} + \omega ren_t^{NR*} - \left(1-\chi^{Co}\right) \frac{cf_t^{Co} - \tau_t^{Co}(cf_t^{Co} + p_t^{ICo} i_t^{Co,f})}{rer_t} \tag{EE.178}$$

$$ren_t^{R*} = \frac{\overline{ren}^{R*} \xi_t^{ren}}{(L_t/L)} - \lambda_M\left(\frac{w_t h_t n_t}{rer_t}\right) \frac{N_t^M}{N_t} \tag{EE.179}$$

$$ren_t^{NR*} = -\lambda_M\left(\frac{w_t h_t n_t}{rer_t}\right) \frac{N_t^M}{N_t} \tag{EE.180}$$

$$b_t^{Pr} + b_t^G + b_t^{UFA} = 0 \tag{EE.181}$$

The exogenous processes for

$X = \left\{a, \pi^*, R^*, \varpi, \varrho, \xi_t^G, \xi_t^{CG}, \varpi^{Co}, \xi_t^{IG}, \xi_t^{pO}, \xi_t^{PCo*}, \xi_t^{PM*}, \xi_t^{PO*}, \xi_t^{TR}, \xi_t^{REN}, \xi_t^{Y^{H*}}, z, z^A, z^{Co}, \zeta^O, \zeta^U, z^*, \kappa^x, m, \rho^x\right\}$

are $\log\left(X_t/\bar{X}\right) = \rho_X \log\left(X_{t-1}/\bar{X}\right) + \varepsilon_t^X$, where the ε_t^X are i.i.d. shocks, $\rho_X \in (0,1)$ and $\bar{X} > 0$.

B Steady State

By assuming a stationary population size, and by normalizing $L = 1$, the steady state remains unmodified from the baseline XMAS specification.

We show how to compute the steady state for given values of h,π^S,p^O,p^{Co},p^H, $s^{OC} = p^O c^O / c$,$\nabla^{k^{Co} y^{Co}} = \frac{q^{Co} k^{Co}}{rer \times p^{Co*} y^{Co}}$,$s^{iCo} = p^{ICo} i^{Co} / \left(p^Y y\right)$,$s^{Co} = rer \times p^{Co*} y^{Co} / \left(p^Y y\right)$,$s^{cg} = p^{CG} c^G / \left(p^Y y\right)$,$s^{ig} = p^{IG} i^G / \left(p^Y y\right)$,$s^{trG} = tr^G / \left(p^Y y\right)$, $s^{tb} = tb / \left(p^Y y\right)$, $def^{UFA} = \frac{(1-n) ub - \tau^{UFA} whn}{\tau^{UFA} whn}$, $s^{CA} = rer \times b^* \left(1 - \frac{1}{a\pi^*}\right) / \left(p^Y y\right)$, u, $\rho = p^{E,U} / \left(1 - p^{U,E}\right)$, $s_{\rho^x} = \rho^x / \rho$, and e and with the parameters $\overline{R}^*$,$\overline{\pi}^*$,$\overline{\chi}^{pO*}$,$\overline{\chi}^{pCo*}$,$\overline{\kappa}$,κ_O,ub,o_{KG},$\overline{z}^{Co}$,α_{Co},δ_{Co},α_{CG},c ,ren^{R*} , ρ, ρ^x, v, $\mu_{\tilde{C}}$ and Ω_v determined endogenously, while the values of the remaining parameters are taken as given.

From the exogenous processes for

$$X^* = \left\{a, \varpi, \varrho, \xi_t^{CG}, \xi_t^G, \varpi^{Co}, \xi_t^{IG}, \xi_t^{PM*}, \xi_t^{pO}, \xi_t^{REN}, \xi_t^{TR}, \xi_t^{Y^{H*}}, z, z^A, z^{Co}, \zeta^O, \zeta^U, z^*\right\}$$

we have that $X = \overline{X}$.

The steady state for the remaining endogenous variables is defined as the set of values for which all equations below hold. The system of equations is solved numerically. Starting from arbitrary values for $utCo$,k^G, h^C, k^S, $r^{\widetilde{K}}$, y_{VA}^{H*}, o^C, and o^H, we iterate repeatedly through the set of equations until finding a fixed point.[20]

$$\tau^L = \tau^W + \tau^{UFA} \tag{SS.1}$$

$$a^{Co} = a \tag{SS.2}$$

$$a^H = a \tag{SS.3}$$

$$\nabla^{Co} = 1 \tag{SS.4}$$

$$\nabla^H = 1 \tag{SS.5}$$

$$\Theta_t^R = 1 \tag{SS.6}$$

$$\Theta_t^{NR} = 1 \tag{SS.7}$$

$$\tilde{p}^H = 1 \tag{SS.8}$$

$$\tilde{p}^F = 1 \tag{SS.9}$$

$$\kappa^M = 0 \tag{SS.10}$$

$$z^M = 1 \tag{SS.11}$$

$$\Delta^H = (\tilde{p}^H)^{-\epsilon_H} \tag{SS.12}$$

[20] This is a rather fast process, usually converging after around 25 iterations for a tolerance of 10^{-8} on the root of the sum squared differences.

$$\Delta^F = (\tilde{p}^H)^{-\epsilon_F} \tag{SS.13}$$

$$mc^H = \frac{\epsilon_H - 1}{\epsilon_H}\tilde{p}^H \tag{SS.14}$$

$$mc^F = \frac{\epsilon_F - 1}{\epsilon_F}\tilde{p}^F \tag{SS.15}$$

$$p^{\tilde{H}} = mc^H p^H \tag{SS.16}$$

$$mc^{\tilde{Z}} = \left[\frac{\left(z \times p^{\tilde{H}}\right)^{1-\eta_O} - o_O\left(p^O\right)^{1-\eta_O}}{1 - o_O}\right]^{\frac{1}{1-\eta_O}} \tag{SS.17}$$

$$\pi = \bar{\pi}. \tag{SS.18}$$

$$g^{\Gamma^H} = \pi. \tag{SS.19}$$

$$g^{\Gamma^F} = \pi. \tag{SS.20}$$

$$R = a^{\sigma}\pi/\beta. \tag{SS.21}$$

$$\tilde{p}^{H*} = 1 \tag{SS.22}$$

$$\Delta^{H*} = \left(\tilde{p}^{H*}\right)^{-\epsilon_{H*}} \tag{SS.23}$$

$$mc^{H*} = \frac{\epsilon_{H*} - 1}{\epsilon_{H*}}\tilde{p}^{H*} \tag{SS.24}$$

$$\kappa_O = s^{OC}\left(p^O\right)^{-\eta_C} \tag{SS.25}$$

p^A is then obtained numerically from:

$$(1-\kappa_O-\kappa_A)\left((1-o_Z)\left(p^H\right)^{1-\eta_Z} + o_Z\left[\frac{\left(p^A\right)^{1-\eta_A} - (1-o_A)\left(p^H\right)^{1-\eta_A}}{o_A}\right]^{\frac{1-\eta_Z}{1-\eta_A}}\right)^{\frac{1-\eta_C}{1-\eta_Z}} + \kappa_O\left(p^O\right)^{1-\eta_C} + \kappa_A\left(p^A\right)^{1-\eta_C} = \tag{SS.26}$$

$$p^F = \left[\frac{\left(p^A\right)^{1-\eta_A} - (1-o_A)\left(p^H\right)^{1-\eta_A}}{o_A}\right]^{\frac{1}{1-\eta_A}} \tag{SS.27}$$

$$p^Z = \left((1-o_Z)\left(p^H\right)^{1-\eta_Z} + o_Z\left(p^F\right)^{1-\eta_Z}\right)^{\frac{1}{1-\eta_Z}} \tag{SS.28}$$

$$rer = mc^F p^F \tag{SS.29}$$

$$p^{H*} = \frac{p^H mc^H}{rer \times mc^{H*}} \tag{SS.30}$$

$$p^I = \left[(1 - o_I)\left(p^H\right)^{1-\eta_I} + o_I\left(p^F\right)^{1-\eta_I}\right]^{\frac{1}{1-\eta_I}} \tag{SS.31}$$

$$p^{CG} = \left[(1 - o_{CG})\left(p^H\right)^{1-\eta_{CG}} + o_{CG}\left(p^F\right)^{1-\eta_{CG}}\right]^{\frac{1}{1-\eta_{CG}}} \tag{SS.32}$$

$$p^{IG} = \left[(1 - o_{IG})\left(p^H\right)^{1-\eta_{IG}} + o_{IG}\left(p^F\right)^{1-\eta_{IG}}\right]^{\frac{1}{1-\eta_{IG}}} \tag{SS.33}$$

$$p^{ICo} = \left[(1 - o_{Co})\left(p^H\right)^{1-\eta_{Co}} + o_{Co}\left(p^F\right)^{1-\eta_{Co}}\right]^{\frac{1}{1-\eta_{Co}}} \tag{SS.34}$$

$$f^* = \left(\frac{1}{\xi^*}\right)^{\frac{1}{\Gamma^*}} \tag{SS.35}$$

$$p^{O*} = \frac{p^O}{rer}\left(\xi^O\right)^{\frac{1}{1-\rho_O}} \tag{SS.36}$$

$$p^{Co*} = \frac{p^{Co}}{rer} \tag{SS.37}$$

$$\xi^{Co*} = \left(\frac{p^{Co^*}}{f^*}\right)^{\Gamma^{Co*}} \tag{SS.38}$$

$$\xi^{O*} = \left(\frac{p^{O*}}{f^*}\right)^{\Gamma^{O*}} \tag{SS.39}$$

$$\lambda^{Co} = \left(1 - \tau^{Co}\left(1 - \chi^{Co}\right)\right) rer \times p^{Co*} \tag{SS.40}$$

$$\overline{u} = 1 - \frac{\log\left(p^I\right)}{\Phi_{\bar{u}}} \tag{SS.41}$$

$$q = \frac{p^I}{\varpi} \tag{SS.42}$$

$$r^K = \frac{q\left(\frac{a^\sigma}{\beta} - 1 + \delta\right) - p^I \tau^K \delta}{(1 - \tau^K)\left(\bar{u} - \frac{p^I}{\Phi_{\bar{u}}}\left[\exp\left(\Phi_{\bar{u}}\left(\bar{u} - 1\right)\right) - 1\right]\right)} \tag{SS.43}$$

$$\phi^{\overline{u}} = \frac{r^K}{\Phi_{\bar{u}}}\left[\exp\left(\Phi_{\bar{u}}\left(\bar{u} - 1\right)\right) - 1\right] \tag{SS.44}$$

$$R^* = a^\sigma \pi / (\beta \pi^S \xi). \tag{SS.45}$$

$$\pi^* = \pi / \pi^S \tag{SS.46}$$

$$\pi^{F*} = \pi^* \tag{SS.47}$$

$$\pi^{Co*} = \pi^* \tag{SS.48}$$

$$\pi^{O*} = \pi^* \tag{SS.49}$$

$$\pi^{M*} = \pi^* \tag{SS.50}$$

$$p^{M*} = \left(\xi^{M*}\right)^{\frac{1}{\Gamma^{M*}}} f^* \tag{SS.51}$$

$$n = 1 - u \tag{SS.52}$$

$$\rho^x = s_{\rho^x}\rho \tag{SS.53}$$

$$\rho^n = \frac{\rho - \rho^x}{1 - \rho^x} \tag{SS.54}$$

$$\mu_{\tilde{c}} = \log\left[\frac{h^C\left(1-\rho^n\right)}{\Phi\left(\Phi^{-1}\left(1-\rho^n\right) - \sigma_{\tilde{c}}\right)\nabla^H}\right] - \frac{\sigma_{\tilde{c}}^2}{2} \tag{SS.55}$$

$$\bar{c} = \exp\left(\sigma_{\tilde{c}}\Phi^{-1}\left(1-\rho^n\right) + \mu_{\tilde{c}}\right) \tag{SS.56}$$

$$v = \frac{n\rho}{e\left(1-\rho\right)} \tag{SS.57}$$

$$\bar{m} = e\left(\frac{v}{u}\right)^{\mu}\left(\frac{U^L + \lambda_M U^M}{U}\right)^{-1} \tag{SS.58}$$

$$m = \frac{\bar{m}\left(U^L + \lambda_M U^M\right)}{U} \tag{SS.59}$$

$$s = m\left(\frac{v}{u}\right)^{1-\mu} \tag{SS.60}$$

$$L^L = L\left(1 - \omega_M\right) \tag{SS.61}$$

$$L^M = L - L^L \tag{SS.62}$$

$$s^L = \bar{m}\left(\frac{v}{u}\right)^{1-\mu} \tag{SS.63}$$

$$U^L = \frac{L^L\rho}{s^L\left(1-\rho\right)+\rho} \tag{SS.64}$$

$$N^L = L^L - U^L \tag{SS.65}$$

$$N^M = N - N^L \tag{SS.66}$$

$$o_{KG} = \frac{k^G}{k^S + k^G} \tag{SS.67}$$

$$\widetilde{k} = \left[\left(1 - o_{KG}\right)^{\frac{1}{\eta_{KG}}}\left(k^S\right)^{\frac{\eta_{KG}-1}{\eta_{KG}}} + o_{KG}{}^{\frac{1}{\eta_{KG}}}\left(k^G/a\right)^{\frac{\eta_{KG}-1}{\eta_{KG}}}\right]^{\frac{\eta_{KG}}{\eta_{KG}-1}} \tag{SS.68}$$

$$k = \frac{k^S a}{\overline{u}} \tag{SS.69}$$

$$i = k\left(\frac{1-(1-\delta)/a}{\varpi}\right) \tag{SS.70}$$

$$y^{\widetilde{Z}} = \left(\widetilde{k}\right)^{\alpha} \left(a\nabla^{H} nh\right)^{1-\alpha} \tag{SS.71}$$

$$y^{\widetilde{H}} = z\left[(1-o_O)^{\frac{1}{\eta_O}} \left(y^{\widetilde{Z}}\right)^{\frac{\eta_O-1}{\eta_O}} + o_O{}^{\frac{1}{\eta_O}} \left(x^{O}\right)^{\frac{\eta_O-1}{\eta_O}}\right]^{\frac{\eta_O}{\eta_O-1}} \tag{SS.72}$$

$$\gamma^{W} = a\pi \tag{SS.73}$$

$$w = \frac{1}{h}\left[a^{1-\sigma}\beta(1-\rho)\ \ p^{H}\left(\nabla^{H}\overline{c} - h^{C}\right) + mc^{\widetilde{Z}}\left(1-\alpha\right)\frac{y^{\widetilde{Z}}}{n}\ \ - p^{H}\nabla^{H}\overline{c}\right] \tag{SS.74}$$

$$w^{n} = \ \ w \tag{SS.75}$$

$$\Omega_v \ \ = \frac{e}{\nabla^{H}p^{H}}\left[wh + \nabla^{H}p^{H}\overline{c} - mc^{\widetilde{Z}}\left(1-\alpha\right)\frac{y^{\widetilde{Z}}}{n}\right] \tag{SS.76}$$

$$x^{L} = h^{C}n + \nabla^{H}\Omega_v v \tag{SS.77}$$

$$\Psi^{U}\kappa = \frac{mc^{\widetilde{Z}}\left(1-\alpha\right)^{2}\frac{y^{\widetilde{Z}}}{n}}{\frac{h^{\frac{1}{1+\phi}}}{(1-\tau^{L})}\left(\nabla^{H}\right)^{1-\sigma}} \tag{SS.78}$$

$$ub = \ \ \frac{(1-\tau^{L})}{(1-\varphi^{U})}\left[w^{n}h - \varphi^{U}\left(mc^{\widetilde{Z}}\left(1-\alpha\right)\frac{y^{\widetilde{Z}}}{n} - p^{H}h^{C} + \frac{p^{H}\nabla^{H}s\Omega_v}{e}\right)\right] - \Psi^{U}\kappa\left(\nabla^{H}\right)^{1-\sigma}\ \frac{h^{1+\phi}}{1+\phi} \tag{SS.79}$$

$$o = c^{O} + x^{O} \tag{SS.80}$$

$$i^{f} = i + \phi^{\overline{u}}\frac{k}{a} \tag{SS.81}$$

$$p^Y y = \frac{p^H y^{\tilde{H}} + p^O c^O - rer \times p^{O*} o + y_{VA}^{H*} + p^F \left(1 - mc^F \Delta^F\right) \Theta_1}{1 - s^{Co} - p^F \left(1 - mc^F \Delta^F\right) \Theta_2} \tag{SS.82}$$

where

$$\begin{aligned}
\Theta_1 = & -o_Z \left(\frac{p^F}{p^Z}\right)^{-\eta_Z} (1 - \kappa_O - \kappa_A) \left(p^Z\right)^{-\eta_C} \left(p^I i^f + p^{ICo} ut^{Co} + p^H x^L\right) \\
& - o_A \left(\frac{p_t^F}{p_t^A}\right)^{-\eta_A} \kappa_A \left(p^A\right)^{-\eta_C} \left(p^I i^f + p^{ICo} ut^{Co} + p^H x^L\right) \\
& + o_I \left(\frac{p^F}{p^I}\right)^{-\eta_I} i^f \\
& + o_{Co} \left(\frac{p^F}{p^{ICo}}\right)^{-\eta_{Co}} ut^{Co} \\
\Theta_2 = & \; o_Z \left(\frac{p^F}{p^Z}\right)^{-\eta_Z} (1 - \kappa_O - \kappa_A) \left(p^Z\right)^{-\eta_C} \left(1 - s^{tb} - s^{cg} - s^{ig} - s^{iCo}\right) \\
& + o_A \left(\frac{p^F}{p^A}\right)^{-\eta_A} \kappa_A \left(p^A\right)^{-\eta_C} \left(1 - s^{tb} - s^{cg} - s^{ig} - s^{iCo}\right) \\
& + o_{CG} \left(\frac{p^F}{p^{CG}}\right)^{-\eta_{CG}} \frac{s^{cg}}{p^{CG}} \\
& + o_{IG} \left(\frac{p^F}{p^{IG}}\right)^{-\eta_{IG}} \frac{s^{ig}}{p^{IG}} \\
& + o_{Co} \left(\frac{p_t^F}{p_t^{ICo}}\right)^{-\eta_{Co}} \frac{s_t^{iCo}}{p^{ICo}}
\end{aligned}$$

$$tb = s^{tb} p^Y y \tag{SS.83}$$

$$i^G = s^{ig} \frac{p^Y y}{p^{IG}} \tag{SS.84}$$

$$i^{Co} = \frac{s^{iCo} p^Y y}{p^{ICo}} \tag{SS.85}$$

$$tr^G = s^{tr} p^Y y \tag{SS.86}$$

$$i^{AG} = \left({}^{i^G}\!/\!_{\varphi_0}\right) \frac{1 - \left({}^{\rho^\varphi}\!/\!_a\right)}{1 - \left({}^{\rho^\varphi}\!/\!_a\right)^{N^G}} \tag{SS.87}$$

$$i^{aCo} = \left({}^{i^{Co}}\!/\!_{\varphi_0^{Co}}\right) \frac{1 - \left({}^{\rho^{\varphi Co}}\!/\!_a\right)}{1 - \left({}^{\rho^{\varphi Co}}\!/\!_a\right)^{N^{Co}}} \tag{SS.88}$$

$$c^G = s^{cg}\frac{p^Y y}{p^{CG}} \tag{SS.89}$$

$$q^{Co} = p^{ICo}\varphi_0^{Co}\frac{1-\left(\rho^{\varphi Co}\frac{\pi}{R}\right)^{N^{Co}}}{1-\rho^{\varphi Co}\frac{\pi}{R}}\left(\frac{R}{\pi}\right)^{N^{Co}-1} \tag{SS.90}$$

$$y^{Co} = s^{Co}p^Y y/\left(rer \times p^{Co*}\right) \tag{SS.91}$$

$$k^{Co} = \nabla^{k^{Co},y^{Co}}y^{Co}\frac{rer \times p^{Co*}}{q^{Co}} \tag{SS.92}$$

$$\delta_{Co} = \frac{i^{aCo}}{k^{Co}a^{N^{Co}-2}} - a + 1 \tag{SS.93}$$

$$\bar{u}^{Co} = 1 - \frac{\log\left(p^{ICo}\right)}{\Phi_{\bar{u}}^{Co}} \tag{SS.94}$$

$$\alpha_{Co} = \frac{k^{Co}q^{Co}\left(\frac{R}{\pi}-(1-\delta_{Co})\right)}{a\lambda^{Co}y^{Co}\left[1-\frac{p^{ICo}}{\Phi_{\bar{u}}^{Co}\bar{u}^{Co}}\left(e^{\Phi_{\bar{u}}^{Co}(\bar{u}^{Co}-1)}-1\right)\right]} \tag{SS.95}$$

$$r^{K,Co} = \left[1-\tau^{Co}\left(1-\chi^{Co}\right)\right] rer \times p^{Co*}\alpha^{Co}\frac{y^{Co}}{\bar{u}^{Co}k^{Co}}a \tag{SS.96}$$

$$\phi_{\bar{u}}^{Co} = \frac{r^{K,Co}}{\Phi_{\bar{u}}^{Co}}\left(e^{\Phi_{\bar{u}}^{Co}\left(\bar{u}^{Co}-1\right)}-1\right) \tag{SS.97}$$

$$i^{Co,f} = i^{Co} + \phi_{\bar{u}}^{Co}\frac{k^{Co}}{a} \tag{SS.98}$$

$$z^{Co} = y^{Co}\left(\frac{\bar{u}^{Co}k^{Co}}{a}\right)^{-\alpha Co}\left(a\nabla^{Co}\overline{L}\right)^{\alpha Co-1} \tag{SS.99}$$

$$cf^{Co} = rer \times p^{Co*}y^{Co} - p^{ICo}i^{Co,f} \tag{SS.100}$$

$$ren^{R*} = \frac{ren^* - \omega ren^{NR*} + \frac{(1-\chi^{Co})\left(cf^{Co}-\tau^{Co}\left(cf^{Co}+p^{ICo}i^{Co,f}\right)\right)}{rer}}{1-\omega} \tag{SS.101}$$

$$ren^* = \frac{p^Y y}{rer}\left[\frac{s^{CA}\left(a\pi^* - R^*\xi\right)}{\left(a\pi^*-1\right)} - s^{tb}\right] \tag{SS.102}$$

$$ren^{NR*} = -\lambda_M\left(\frac{whn}{rer}\right)\frac{N^M}{N} \tag{SS.103}$$

$$c = p^Y y - p^I i^f - p^{ICo}i^{Co,f} - p^{CG}c^G - p^{IG}i^G - tb - p^H x^L \tag{SS.104}$$

$$c^A = \kappa_A\left(p^A\right)^{-\eta_C} c \tag{SS.105}$$

$$x^{A,H} = \left(z^A\right)^{\eta_A-1}(1-o_A)\left(\frac{p^H}{p^A}\right)^{-\eta_A} c^A \tag{SS.106}$$

$$x^{A,F} = \left(z^A\right)^{\eta_A - 1} o_A \left(\frac{p^F}{p^A}\right)^{-\eta_A} c^A \tag{SS.107}$$

$$c^Z = (1 - \kappa_O - \kappa_A) \left(p^Z\right)^{-\eta_C} c \tag{SS.108}$$

$$y^C = c + p^I i^f + p^{ICo} i^{Co,f} + p^{CG} c^G + p^{IG} i^G + p^H x^L \tag{SS.109}$$

$$x^{Z,H} = (1 - o_Z) \left({}^{p^H}/{}_{p^Z}\right)^{-\eta_Z} c^Z \tag{SS.110}$$

$$x^{Z,F} = o_Z \left({}^{p^F}/{}_{p^Z}\right)^{-\eta_Z} c^Z \tag{SS.111}$$

$$x^{I,H} = (1 - o_I) \left(\frac{p^H}{p^I}\right)^{-\eta_I} i^f \tag{SS.112}$$

$$x^{I,F} = o_I \left(\frac{p^F}{p^I}\right)^{-\eta_I} i^f \tag{SS.113}$$

$$x^{CG,H} = (1 - o_{CG}) \left(\frac{p^H}{p^{CG}}\right)^{-\eta_{CG}} c^G \tag{SS.114}$$

$$x^{CG,F} = o_{CG} \left(\frac{p^F}{p^{CG}}\right)^{-\eta_{CG}} c^G \tag{SS.115}$$

$$x^{IG,H} = (1 - o_{IG}) \left(\frac{p^H}{p^{IG}}\right)^{-\eta_{IG}} i^G \tag{SS.116}$$

$$x^{IG,F} = o_{IG} \left(\frac{p^F}{p^{IG}}\right)^{-\eta_{IG}} i^G \tag{SS.117}$$

$$x^{Co,H} = (1 - o_{Co}) \left(\frac{p^H}{p^{ICo}}\right)^{-\eta_{Co}} i^{Co,f} \tag{SS.118}$$

$$x^{Co,F} = o_{Co} \left(\frac{p^F}{p^{ICo}}\right)^{-\eta_{Co}} i^{Co.f} \tag{SS.119}$$

$$x^H = x^{Z,H} + x^{I,H} + x^{Co,H} + x^{CG,H} + x^{IG,H} + x^{A,H} + x^L \tag{SS.120}$$

$$y^H = x^H \tag{SS.121}$$

$$f^H = mc^H \left(\tilde{p}^H\right)^{-\epsilon_H} y^H / (1 - \beta a^{1-\sigma} \theta_H) \tag{SS.122}$$

$$y^{H*} = \left(y^{\tilde{H}} - y^H \Delta^H\right) / \Delta^{H*} \tag{SS.123}$$

$$x^{H*} = y^{H*} \tag{SS.124}$$

$$x^F = x^{Z,F} + x^{I,F} + x^{Co,F} + x^{CG,F} + x^{IG,F} + x^{A,F} \tag{SS.125}$$

$$y^F = x^F \tag{SS.126}$$

$$f^F = mc^F \left(\tilde{p}^F\right)^{-\epsilon_F} y^F / (1 - \beta a^{1-\sigma} \theta_F) \tag{SS.127}$$

$$m^* = y^F \Delta^F \tag{SS.128}$$

$$y = c + i^f + i^{Co} + c^G + i^G + y^{H*} + y^{Co} - m^* - o + x^L \tag{SS.129}$$

$$p^Y = \left(y^C + tb\right) / y \tag{SS.130}$$

$$\overline{tr}^{UFA} = (1-n)\, ub - \tau^{UFA} wnh \tag{SS.131}$$

$$b^{UFA} = \frac{\left(\tau^{UFA} wnh - (1-n)\, ub + \overline{tr}^{UFA}\right)}{1 - \frac{R}{\pi a}} \tag{SS.132}$$

$$tr^{UFA} = \overline{tr}^{UFA} \tag{SS.133}$$

$$tr^{NR} = \frac{\omega_G}{\omega} \left(tr^G - tr^{UFA}\right) \tag{SS.134}$$

$$tr^R = \frac{1 - \omega_G}{1 - \omega} \left(tr^G - tr^{UFA}\right) \tag{SS.135}$$

$$b^* = \frac{s_{CA} \times p^Y y \times a\pi^*}{rer\,(a\pi^* - 1)} \tag{SS.136}$$

$$y^* = a \times z^* \tag{SS.137}$$

$$o^* = \left(y^{H*} / y^*\right) \left(p^{H*}\right)^{\eta^*}. \tag{SS.138}$$

$$i_t^R = \frac{i_t}{1 - \omega} \tag{SS.139}$$

$$k_t^R = \frac{k_t}{1 - \omega} \tag{SS.140}$$

$$k_t^{S,R} = \frac{k_t^S}{1 - \omega} \tag{SS.141}$$

$$g = p^{CG} c^G + p^I i^G + tr^G \tag{SS.142}$$

$$\widetilde{g}^{rule} = g \tag{SS.143}$$

$$\widetilde{g}^{exo} = g \tag{SS.144}$$

$$\widetilde{g} = g \tag{SS.145}$$

$$\alpha^{CG} = \frac{p^{CG} c^G}{g \xi^{CG}} \tag{SS.146}$$

$$\alpha^{IG} \quad = \quad \frac{p^{IG} i^{G}}{g \xi^{IG}} \tag{SS.147}$$

$$d = p^{Y} y - rer \times p^{Co*} y^{Co} - r^{K} k^{S} - wnh - p^{H} x^{L} \tag{SS.148}$$

$$b^{G} = \frac{-s^{def} p^{Y} y}{\left(1 - {}^{1}\!/_{a\pi}\right) + \frac{\alpha^{D}}{1-\alpha^{D}} \left(1 - {}^{1}\!/_{a\pi^{*}}\right)} \tag{SS.149}$$

$$b^{Pr} = -b^{G} - b^{UFA} \tag{SS.150}$$

$$b^{G^{*}} = \left(\frac{\alpha^{D}}{1 - \alpha^{D}} \right) \frac{b^{G}}{rer} \tag{SS.151}$$

$$b^{Pr*} = b^{*} - b^{G^{*}} \tag{SS.152}$$

$$b_{t}^{R} = \frac{b_{t}^{Pr}}{1 - \omega} \tag{SS.153}$$

$$b_{t}^{R*} = \frac{b_{t}^{Pr*}}{1 - \omega} \tag{SS.154}$$

$$d^{R} = \frac{d}{1 - \omega} \tag{SS.155}$$

$$\tau' = \tau^{C} c + \tau^{W} wnh + \tau^{K} \left[r^{K} \bar{u} - p^{I} \left(\delta + \frac{r^{K}}{\Phi_{\bar{u}}} \left[\exp\left(\Phi_{\bar{u}} \left(\bar{u} - 1 \right) \right) - 1 \right] \right) \right] \frac{k}{a} + \tau^{D} d + \tau^{Co} \left(1 - \chi^{Co} \right) \left(cf^{Co} + p^{ICo} i^{Co,f} \right) \tag{SS.156}$$

$$t \quad = \quad g + b^{G} \left(1 - \frac{R}{a\pi} \right) + b^{G*} rer \left(1 - \frac{\xi R^{*}}{a\pi^{*}} \right) - \tau' - \chi^{Co} cf^{Co} \tag{SS.157}$$

$$t^{NR} = \left(\frac{\omega_{G}}{\omega} \right) t \tag{SS.158}$$

$$t^{R} = \left(\frac{1 - \omega_{G}}{1 - \omega} \right) t \tag{SS.159}$$

$$\alpha_{T} = \frac{t}{p^{Y} y} \tag{SS.160}$$

$$\tau = \tau' + t \tag{SS.161}$$

$$\widetilde{\tau} = \tau \tag{SS.162}$$

$$\widetilde{p}^{Co*} = p^{Co*} \tag{SS.163}$$

$$\bar{s}_{B} = -s^{def} \tag{SS.164}$$

$$g_t^{\Gamma^{H*}} = \pi^* \tag{SS.165}$$

$$c^{NR} = \frac{\left(1-\tau^L\right) whn + ub \times u + tr^{NR} - \frac{\omega^G}{\omega} t}{1+\tau^C} \tag{SS.166}$$

$$c^R = \frac{c - \omega c^{NR}}{1-\omega} \tag{SS.167}$$

$$o_{\hat{C}} = \frac{c^G}{c^R - \varsigma \frac{c^R}{a} + c^G} \tag{SS.168}$$

$$\hat{c}^R = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^R - \varsigma \frac{c^R}{a} \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + o_{\hat{C}}^{\frac{1}{\eta_{\hat{C}}}} \left(c^G \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{SS.169}$$

$$\hat{c}^{NR} = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^{NR} - \varsigma \frac{c^{NR}}{a} \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + o_{\hat{C}}^{\frac{1}{\eta_{\hat{C}}}} \left(c^G \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{SS.170}$$

$$\lambda^R = \frac{\left(\hat{c}^R\right)^{-\sigma}}{1+\tau^C} \left(\frac{(1-o_{\hat{C}})\, \hat{c}^R}{c^R - \varsigma \frac{c^R}{a}} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{SS.171}$$

$$\lambda^{NR} = \frac{\left(\hat{c}^{NR}\right)^{-\sigma}}{1+\tau^C} \left(\frac{(1-o_{\hat{C}})\, \hat{c}^{NR}}{c^{NR} - \varsigma \frac{c^{NR}}{a}} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{SS.172}$$

$$\widetilde{\chi}^R = \left(\nabla^H\right)^{-\sigma} \left(\hat{c}^R\right)^{\sigma} \tag{SS.173}$$

$$\widetilde{\chi}^{NR} = \left(\nabla^H\right)^{-\sigma} \left(\hat{c}^{NR}\right)^{\sigma} \tag{SS.174}$$

$$\Psi^U = (1-\omega) \frac{\Theta^R}{\lambda^R} + \omega \frac{\Theta^{NR}}{\lambda^{NR}} \tag{SS.175}$$

$$\kappa = \bar{\kappa} = \frac{mc^{\widetilde{Z}} (1-\alpha)^2 \frac{y^{\widetilde{Z}}}{n}}{\frac{\Psi^U h^{1+\phi}}{(1-\tau^L)} \left(\nabla^H\right)^{1-\sigma}} \tag{SS.176}$$

$$s^{R,U} = \frac{(1-\tau^L) w^n h - \Theta^R \kappa \left(\nabla^H\right)^{1-\sigma} \frac{h^{1+\phi}}{1+\phi} / \lambda^R - ub}{1-(1-s)\frac{\beta}{a^{\sigma-1}}(1-\rho)} \tag{SS.177}$$

$$s^{NR,U} = \frac{(1-\tau^L) w^n h - \Theta^{NR} \kappa \left(\nabla^H\right)^{1-\sigma} \frac{h^{1+\phi}}{1+\phi} / \lambda^{NR} - ub}{1-(1-s)\frac{\beta}{a^{\sigma-1}}(1-\rho)} \tag{SS.178}$$

$$\sigma^U = \frac{\beta}{a^{\sigma-1}} (1-\rho) \left[\left(1-\omega^U\right) s^{R,U} + \omega^U s^{NR,U} \right] \tag{SS.179}$$

$$f^{H*} = \frac{\left(\tilde{p}^{H*}\right)^{-\epsilon_{H*}} y^{H*} mc^{H*}}{1 - \beta a^{1-\sigma} \theta_{H*}} \tag{SS.180}$$

$$\breve{\tau} = 0 \tag{SS.181}$$

$$\breve{cf}^{Co} = 0 \tag{SS.182}$$

$$\tilde{cf}^{Co} = rer \times p^{Co*} y^{Co} - p^{ICo} I^{Co,f} \tag{SS.183}$$

$$y^D = y - x^L - i^f + i - i^{Co,f} + i^{Co} \tag{SS.184}$$

$$ut^{Co} = \frac{\phi_{\bar{u}}^{Co} k^{Co}}{a}$$

$$k^G = \frac{i^{AG}/a^{N_G - 1}}{1 - \left(1-\delta^G\right)/a} \tag{SS.185}$$

$$h^C = \frac{s^{Hc} p^Y y}{n \times p^H} \tag{SS.186}$$

$$r^{\widetilde{K}} = \alpha \frac{mc^{\tilde{Z}}}{\widetilde{k}} y^{\widetilde{Z}} \tag{SS.187}$$

$$k^S = \left(\frac{r^{\tilde{K}}}{r^K}\right)^{\eta_{KG}} \tilde{k}\left(1 - o_{KG}\right) \tag{SS.188}$$

$$y_{VA}^{H*} = p^{H*} rer \times y^{H*} - p^H y^{H*} \tag{SS.189}$$

$$c^O = \kappa_O \left(p^O\right)^{-\eta_C} c \tag{SS.190}$$

$$o^H = o_O \left(\frac{p^O}{p^H mc^H}\right)^{-\eta_O} y^{\tilde{H}} \tag{SS.191}$$

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