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Accounting for Nature in Economic Models*

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Abstract
We build a two-block general equilibrium model that accounts for Nature by including, alongside man-made capital, natural capital defined as a variety of ecosystem goods and services essential to economic activity. Natural capital is unevenly distributed, displays critical thresholds or 'tipping points' beyond which the ecosystem is irreversibly altered, and contributes to the evolution of productivity. We show that: (1) when natural capital is abundant, it is optimal to deplete some and conserve some, but less depletion should occur if there is a critical threshold beyond which Nature is irreversibly altered; (2) subsidizing the conservation of Nature makes long-run growth stronger and more sustainable in Nature-rich and Nature-poor countries alike, but implies lower consumption globally in the short run.

Resumen
Construimos un modelo de equilibrio general de dos bloques que da cuenta de la Naturaleza incluyendo, junto con el capital físico, el capital natural definido como una variedad de bienes y servicios ecosistémicos esenciales para la actividad económica. El capital natural está distribuido de manera desigual, muestra umbrales críticos o "puntos de inflexión" más allá de los cuales el ecosistema se altera irreversiblemente y contribuye a la evolución de la productividad. Mostramos que: (1) cuando el capital natural es abundante, la asignación de equilibrio es agotar parcialmente la naturaleza, pero debería producirse un agotamiento menor si existe un umbral crítico, ya que más allá de ese umbral la Naturaleza se altera irreversiblemente; (2) subsidiar la conservación de la naturaleza aumenta el crecimiento a largo plazo y lo hace más sostenible tanto en los países ricos como en los pobres en naturaleza, pero implica un menor consumo en el corto plazo.

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1 Introduction

Science indicates that climate change and the loss of biological diversity are twin problems that should be tackled together (Pörtner et al. (2021); Dasgupta (2021); Toensmeier et al. (2020)). At the same time, biodiversity loss has now been widely identified as a potential source of significant economic and financial risk (NGFS (2023)). Yet, while economic models embedding climate change have been circulating since the early 1990s (Nordhaus (1991)), the notion of biophysical limits to growth has not yet taken root in modern macroeconomics, and models accounting for natural capital are less common and popular. As a result, prevalent economic theory still assumes that economic agents have access to boundless natural resources and bottomless sinks for waste products, thereby eliminating the need for an explicit discussion of economic growth within a natural world.

This paper fills this gap by bringing natural capital and the implications of its finiteness for production into a standard general equilibrium economic model. The model has two blocs or regions to capture the fact that, in the real world, natural capital is unevenly distributed across countries.

In our analysis, natural capital is defined as the stock of Nature providing Earth's system of ecosystem services, namely, the many and varied benefits to human economies associated with a healthy natural environment like carbon sink services, freshwater, oxygen, and food. As in Dasgupta (2021), in our set up natural capital is thus a key driver of total factor productivity: a larger stock of natural capital provides more abundant ecosystem services, which in turn expands the output that can be produced for each unit of labor and man-made capital given the underlying rate of technological progress. This assumption is fully in line with recent evidence of the strong dependency of economic activity on Nature (IUCN (2021); Batini (2021)).

Since, contrary to man-made capital, natural capital appreciates when left untouched, in our model we assume that it is possible to 'invest' in natural capital through ecosystem conservation, which basically requires ensuring that natural capital is protected from extractive uses or man-made degradation. It follows that, in our set up, growth is partly endogenous, in the sense that investment in natural capital can be a contributor to economic growth.

We derive two main results. First, when natural assets are abundant, it is optimal to deplete some and conserve some, but less depletion should occur if there is a critical threshold beyond which Nature is irreversibly altered. Second, accordingly, subsidizing the accumulation of natural capital delivers stronger and more sustainable growth in the longer run in both Nature-rich and Nature-poor countries. The paper is organized as follows: Section 2 summarizes the literature linking economics and Nature. The model is described in Section 3. Section 4 discusses model calibration and dynamics. Section 5 introduces an example of Nature-positive green policies. Section 6 concludes.

2 Economic models and Nature

Modern growth models, like the Solow-Swan 1956 neoclassical growth model (Solow (1956)), are silent about the natural foundation of production. According to this class of models, capital goods and human labor combine to produce a commodity output, but no land is required to produce it, and no materials nor energy are needed in the production process. As Solow himself (1956, p. 67) remarked: 'The production function is homogeneous of first degree. This amounts to assuming no scarce non-augmentable resource like land.'[1]

In the 1970s, in response to emerging resource constraints from global energy price shocks and rising pollution, attempts were made to integrate natural resources (as distinct from natural capital) among factors of production and growth in economic models. Initial endeavors focused on including a 'non-renewable resource' factor among traditional ones (labor, man-made capital) in order to explore rules for the exploitation of natural resources compatible with constant per capita consumption into the indefinite future (Solow (1974); Stiglitz (1974); Dasgupta and Heal (1974); and Hartwick (1977)).

Further attempts, embracing definitions of resources more strongly correlated with ecological economics definitions of natural capital, included work to embed limits to sustained growth from increased pollution (seen as a phenomenon degrading the natural environment) through pollution-reducing technological change (see, for example, Tahvonen and Kuuluvainen (1991); Bovenberg and Smulders (1995); Howitt and Aghion (1998); and more recently, Brock and Taylor (2010) and Hassler et al. (2016)).

In parallel, Brander and Taylor (1997) analyzed the dynamic system of population interactions with natural resources, finding that an excessive rate of exploitation of stocks of resources tends to generate cycles in both population and natural capital. Dalton et al. (2005) extended the model to technological change dependent on institutional parameters showing, for example, that institutions that favor strong property rights tend to bias technological change toward resource conservation rather than encourage or enable resource depletion.

Others have tried to model natural capital as a renewable resource (Hinterberger et al. (1997); Bringezu et al. (2003); Fischer-Kowalski et al. (2011)), examining how to link material production and consumption to the pace of anthropogenic degradation of natural capital.[2] Recently, Albagli and Vial (2023) tried to assess the role

[1] In classical mainstream economics, 'natural capital' would usually be classified as "land" distinct from traditional "capital", and land, together with labor and capital would be one of the three factors of production. Among the three, land was the "original and inexhaustible gift of nature." In modern mainstream economics, however, land comprises all naturally occurring resources as well as geographic land. Examples include geographical locations, mineral deposits, forests, fish stocks, atmospheric quality, geostationary orbits, and portions of the electromagnetic spectrum. Supply of these resources is normally assumed to be fixed even if many of these are renewable resources.

[2] Subsequent interpretations tried to define natural capital more comprehensively equating it to

of economic growth and population in biodiversity loss, proposing alternative growth pathways that would ensure conservation. In line with earlier analysis of limits to growth (for example by Schumacher (2011); Meadows et al. (1972), Meadows et al. (2004), Costanza and Daly (1992)) this research de facto suggests that production must be decoupled from resource use to ensure sustainable use of natural capital. Yet, both empirical and theoretical evidence shows that it is unprecedented and technino-logically impossible to decouple economic growth from the growth of material and energy use at the scale and in the time needed to stabilize the Earth system (Ward et al. (2016); Parrique et al. (2019)).

To further explore this dilemma, Kornafel and Telega (2020) embed natural capital, intended as a renewable resource, in a neoclassical growth closed-economy model to explore bounds to economic growth. Their model features no behavioral equations and no direct role for Nature in the production function, but innovates upon previous models because it characterizes natural capital as a source of social services beyond those typically provided by a 'normal' renewable resource that is a mere input of production. Kornafel and Telega (2020) also assume that produced goods and natural capital are complements, so that when economies grow, natural capital depreciates alongside. They find stable equilibria when: (i) the initial stock of natural capital is large; (ii) the growth rates of capital and technological progress are strong enough given the assumed elasticity of material intensity of production relative to the elasticity of material intensity of technology; (iii) investments in natural capital are large enough to maintain the stock of natural capital at a level compatible with the complementarity requirements of continuous production given assumed technologies.

Dasgupta (2021)'s famous review of the economics of biodiversity puts natural capital in the production function both as a *flow* of extracted provisioning service (like timber or fish), and as a *stock* supplying ecosystem services essential to production (like climate regulation or soil regeneration)–a modeling device to capture the fact that "the human economy is embedded in the biosphere" (Dasgupta (2021), p. 144). Ecosystem services are complementary, implying limits on converting natural capital services into output, and therefore limits to global growth. Limits are assumed to the ability of Nature of regenerating herself; and if natural capital is over-degraded, the economy is no longer able to operate and produce. The review shows that subject to these assumptions, multiple stationary equilibria exist for different combinations of various types of capital (man-made, human and natural), but these depend on the current size of each of these stocks.

Our analysis follows in the footsteps of Kornafel and Telega (2020) and Dasgupta (2021), but introduces natural capital as a bounded input in the supply side of a fully-specified two-bloc/two-sector dynamic stochastic general equilibrium (DSGE) to study the impact of biophysical limits to economic production[3] in the types of

the sum of the stock of renewable, nonrenewable, replenishable and cultivated natural capital (see, for example, Aronson et al. (2007)).

[3]And, accordingly, the study of "offset" markets and environmental trading schemes as in (May

models that are typically used in the design of macroeconomic policies.

Since climate is a specific type of natural capital, our work is also related in aims to the literature that focuses on the economic impact of varying global temperatures. This encompasses the seminal DICE model (Nordhaus and Sztorc (2013)) and its evolutions (Golosov et al. (2014); Lontzek et al. (2015); Barrage (2020)), as well as large-scale Integrated Assessment Models (IAMs) and Computable General Equilibrium (CGE) models (Château et al. (2014)). But while those models are primarily concerned with the economic damages that arise from higher temperatures, our model is meant to capture all damages associated with the exploitation of ecosystems (Balboni et al. (2023)).[4]

Finally, our paper is also broadly connected to the ongoing interdisciplinary effort that tries to integrate ecosystem services valuations into economic models, like the GTAP-InVEST framework (Johnson et al. (2023)). Compared to ours, that line of research, however, tends to model production in considerable more detail, while, at the same time, greatly simplifies model dynamics and expectations by assuming a static equilibrium.[5]

3 A two-bloc model with natural capital

3.1 General specification

We consider a two-bloc version of the neoclassical growth model. Time is discrete and infinite. The two blocs, Home (H) and Foreign (F), are populated by households and firms. We assume that there is a total of N_t (N_t^*) households in the H (F) bloc at a given time. The blocs trade with each other and differ in size and production structure. Importantly, only H is endowed with natural capital.[6] The model is free from nominal and real frictions. Financial markets are complete. Finally, both blocs have a fiscal authority that collects lump-sum taxes (and distributes subsidies) from (and to) households. Below we spell out the problems of the representative households and firms in the two blocs.[7]

et al. (2017))).

[4]An exception is offered by Bastien-Olvera and Moore (2021) that modify a DICE model to allow climate change to reduce the stock of Nature through a damage function but using a simpler specification of natural capital and focus on the impact of changes to such stock to the social cost of carbon.

[5]An advance to account for dynamics is provided by the recursive formulation of the GTAP-InVEST, which is solved as sequence of comparative static equilibria (akin to myopic expectations) and allows for an endogenous process of capital accumulation.

[6]This assumption is simplifying but helps mimic the world uneven distribution of natural capital.

[7]Throughout we adopt the following notation conventions: foreign variables are denoted by an asterisk (for example, foreign consumption of an household is defined as c_t^*).[8] Also, while nominal (aggregate) prices (quantities) are denoted in capital letters, real prices in terms of final consumption goods (individual quantities) are represented using small letters (for example nominal wages in H are given by W, while deflated wages are given by w; similarly each H household stock of man-made

3.1.1 Households

Households in both blocs supply labor, consume an aggregate consumption good (made of H and F consumption goods) and save through internationally traded financial assets and by renting domestic man-made capital to domestic firms. Each household in the H bloc maximizes her utility $U(\bullet) = \frac{1}{1-\sigma}(\bullet)^{1-\sigma}$, where σ is the inverse of the elasticity of intertemporal substitution, subject to her budget constraint:

$$\max_{c_t, \tilde{a}_{t+1}, k_{t+1}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right\}$$

s.t.

$$P_t c_t + P_{H,t} k_{t+1} + P_t \tilde{a}_{t+1} \leq W_t + (R_t^K - \delta) P_{H,t} k_t + R_t P_t \tilde{a}_t + P_t t_t + P_t o_t \tag{1}$$

where, using H bloc notation, W_t is the nominal wage; $\tilde{a}_{t+1}$ is the financial asset purchased at t; R_t is the gross real interest rate on that asset; k_{t+1}, R_{t+1}^K and δ are the corresponding (gross) investment in domestic physical capital, the (gross) return to man-made capital and the depreciation rate; P_t indicates the price of the final consumption good and $P_{H,t}$ refers to the price of the home consumption good. Furthermore, t_t, o_t denote a fiscal transfer and lump-sum profits accruing from ownership of firms, respectively.[9] Finally, $\mathbb{E}_t$ is the time t expectations operator, and $\beta \in (0, 1)$ is the discount factor. The problem of the household in the F bloc is analogous.

3.1.2 Firms

The production process of the two blocs reflects the respective endowment of natural capital. H is endowed with natural capital and produces both "green" and "brown" intermediate goods, which require natural capital as an input of production; both blocs must buy these goods to produce. Both blocs produce final goods combining the purchased (and aggregated) intermediate goods together with hired labor and rented man-made capital; the technologies used to aggregate the intermediate goods and produce the final good are symmetric between the two regions. We assume that producing green goods (e.g. harvesting forest food) does not dent the stock of natural capital while producing brown goods (e.g. extracting timber from a forest) does. In the end, households in both blocs consume an aggregate of H- and F-produced final consumption goods.

H's intermediate green and brown goods sectors are modeled as a continuum of production units.[10] H firms own a certain stock of natural capital ($k_{N,t}$) and employ

capital is given by k, while the aggregate economy-wide H stock is represented by K).

[9]The assumption of complete markets guarantees that consumption is equated across households. The complete asset market equilibrium can be achieved in a simple asset market in which only nominal bonds are traded; see Benigno and Benigno (2002), Clarida et al. (2002).

[10]This is normalized to 1 in the absence of entry or exit decisions.

green- and brown-intermediate-good production Cobb-Douglas (CD) technologies to produce:

$$y_{g,t} = A_{g,t}(k^g_{N,t})^{\alpha_g} n_{g,t}^{1-\alpha_g} \tag{2}$$

$$y_{b,t} = A_{b,t}(k^b_{N,t})^{\alpha_b} n_{b,t}^{1-\alpha_b} \tag{3}$$

where $k^g_{N,t}$ is the amount of natural capital that is conserved between two consecutive periods, $k^b_{N,t}$ is the amount of natural capital that is exploited to produce the brown intermediate good, and $n_{g,t}, n_{b,t}$ are the amounts of labor employed in the green and brown technologies, respectively. Each period, the following identity holds:

$$k_{N,t} = k^b_{N,t} + k^g_{N,t} \tag{4}$$

that is, the initial stock of natural capital ($k_{N,t}$) is either conserved or exploited. We assume that $A_{i,t} = A^0_{i,t}(K^g_{N,t})^{\phi_i}$, for $i \in \{b, g\}$, to capture that productivity also depends on the aggregate "economy-wide" stock of conserved natural capital, where $\phi_i \geq 0$ is a parameter. $A^0_{i,t}$ is an exogenous productivity process, which we assume common across firms of the same sector (e.g., brown or green). As we explain in more detail in Section 3.2, the stock of natural capital is generally assumed to evolve as a function of parameters and endogenous variables:

$$k_{N,t+1} = k_{N,t+1}(k^b_{N,t}, k^g_{N,t}, r_N, CT, CC, z_t) \tag{5}$$

The regeneration rate r_N is a parameter affecting the speed at which natural capital accumulates when left untouched; we assume that it is fixed. CC indicates the upper ecological limit of natural capital i.e. the maximum amount of k_N that can be sustained in steady state considering the resources available in the ecosystem in which capital resides. This maximum level proxies what ecologists call 'environmental carrying capacity'. Finally, CT refers to the critical threshold for natural capital, or its 'tipping point': reducing k_N below this threshold alters the ecosystem irreversibly because it affects the ability of natural capital to regenerate itself. Usually CC and CT are assumed to exert two opposite forces upon the ability of capital to accumulate: the closer the stock of natural capital is to CT, the lower its accumulation rate, with the rate turning negative when the stock of natural capital falls below CT; similarly, the closer k_N is to CC the slower its accumulation rate, as the stock approaches its long-run pristine level. Reflecting these forces, the faster accumulation rate is attained when k_N lies somewhere between these two extremes.

Following the environmental literature (see for example, Barbier et al. (1991), for the case of deforestation), we assume variable costs associated with the production of brown goods. These costs arise from exploiting the stock of natural capital and are interpreted as a dead-weight loss (akin to more standard adjustment costs usually employed in DSGE models) for the firm. The cost function is assumed to be quadratic and dependent on the remaining stock of natural capital:

$$\kappa(k_{N,t}, k_{N,t}^b) = e^{-\frac{b_1}{2}\left(1-\frac{k_{N,t}^b}{k_{N,t}}\right)^2} \tag{6}$$

Hence, given an existing stock of natural capital, the cost faced by the firm is increasing in the amount of natural capital that is exploited. Total real costs are a function of b output and are equal to $\kappa(k_{N,t}, k_{N,t}^b)y_{b,t}$.[11]. The maximization problem can be summarized as:

$$\max_{n_{g,t},n_{b,t},k_{N,t}^g,k_{N,t}^b,k_{N,t+1}} \quad P_{g,t}A_{g,t}(k_{N,t}^g)^{\alpha_g}n_{g,t}^{1-\alpha_g} + P_{b,t}A_{b,t}(k_{N,t}^b)^{\alpha_b}n_{b,t}^{1-\alpha_b}(1-\kappa(k_{N,t}, k_{N,t}^b))$$
$$- W_{g,t}n_{g,t} - W_{b,t}n_{b,t}$$
$$s.t. \quad (4), \quad (5), \quad (6)$$

H and F firms in the intermediate goods aggregator sector purchase the g and b goods and combine them using Constant Elasticity of Substitution (CES) technologies; using H bloc (symmetrical to the F bloc, mutatis mutandis) the technology is given by:

$$I_t = \left(\omega_g I_{g,t}^{\frac{\rho-1}{\rho}} + \omega_b I_{b,t}^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{7}$$

Where $I_t, I_{g,t}, I_{b,t}$ denote the intermediate composite good and the green and brown intermediate inputs, respectively, and where $\rho > 0, \omega_g + \omega_b = 1$; the resulting maximization problem is then given by:

$$\max_{I_t,I_{b,t},I_{g,t}} P_{I,t}I_t - P_{b,t}I_{b,t} - P_{g,t}I_{g,t} \tag{8}$$
$$\text{s.t.} \quad (7)$$

H and F firms in the final consumption goods sector produce final consumption goods for each bloc purchasing the intermediate goods, hiring labor and renting man-made capital using a CD technology. In the case of the H bloc (and F bloc, mutatis mutandis) the algebraic specification is given by:

$$Y_{H,t} = A_{H,t}K_t^{\zeta_K}N_{H,t}^{\zeta_N}I_t^{1-\zeta_N-\zeta_K} \tag{9}$$

where $N_{H,t}$ is the labor employed and A_H is the exogenously given TFP. The optim-

[11] In the main text we also refer to this expression as $\kappa(k_{N,t}, k_{N,t}^g)$, which follows from substituting $k_{N,t}^b = k_{N,t} - k_{N,t}^g$

ization problem is:

$$\max_{Y_{H,t},I_t,N_{H,t},K_t} \quad P_{H,t}Y_{H,t} - P_{I,t}I_t - W_{H,t}N_{H,t} - P_{H,t}(R_t^K - 1)K_t \tag{10}$$
$$\text{s.t.} \quad (9)$$

Finally, H and F competitive firms in the aggregate home and foreign final goods, combine final goods produced in H and F into an aggregate final consumption good using a CD technology. In the case of the H bloc (and F bloc, mutatis mutandis) the resulting specification is given by:

$$C_t = C_{H,t}^{1-\gamma} C_{F,t}^{\gamma} \tag{11}$$

where $C_t, C_{H,t}, C_{F,t}$ denote the aggregate consumption basket, the aggregate final home good and the aggregate final foreign good. The maximization is then:

$$\max_{C_{H,t},C_{F,t}} P_tC_t - P_{H,t}C_{H,t} - P_{F,t}C_{F,t}$$
$$\text{s.t.} \quad (11) \tag{12}$$

3.2 Modeling natural capital

This subsection explains in greater detail our modeling strategy for the evolution of natural capital.[12] In line with Dasgupta and Mäler (2004), D'Alessandro (2007) and Kornafel and Telega (2019), in studying our model dynamics we concentrate on two versions of this general specification: one with and one without a critical threshold. These two versions have a well established tradition in the study of fisheries management (Clark (2006), and conservation more in general Clark (2010)), and thus allow us to conceptualize the dynamic resource-harvesting problem that economic agents face when deciding how much of the natural resource to exploit for production and how much to keep in place for (possible) future use.

3.2.1 Natural capital with no critical threshold

In this version the stock of natural capital can always recover to its original, carrying capacity level, no matter what amount of depletion occurs between periods. In particular, in this version, the stock of natural capital depends non-linearly on its "background" or "natural" regeneration rate, which in turn depends on how far the existing stock is from its carrying capacity level CC, as well as on the amount that

[12] We focus on the aggregate economy-wide stock since we rule out idiosyncratic shocks and also assume that the dynamics governing the stock are identical across all its natural capital stock owners.

is exploited for production:

$$K_{N,t+1} = K_{N,t}\left(1 + r_N\left(1 - \frac{K_{N,t}}{CC}\right)\right) - K^b_{N,t} \tag{13}$$

We call the rate at which natural capital accumulates (or decumulates) through the impact of its own regeneration given the beginning of period existing stock k_N's*accumulation rate*:

$$A_{N,t} \equiv r_N K_{N,t}\left(1 - \frac{K_{N,t}}{CC}\right) \tag{14}$$

In other words, the rate of accumulationdepends on Nature'scarrying capacity CC, and it declines as the stock of natural capital approaches CC, as follows:

Proposition 1. *$A_{N,t}$ is monotonously increasing in $K_{N,t}$ for $K_{N,t} < CC/2$ and monotonously decreasing in $K_{N,t}$ for $K_{N,t} > CC/2$; that is, as the stock approaches its carrying capacity, the speed of regeneration diminishes (converging to zero in the limit, as $K_{N,t}$ approaches CC). Moreover, $\frac{\partial^2 A_{N,t}}{\partial K^2_{N,t}} < 0$ for $K_{N,t} \in (0, CC)$.*

Proof. See Appendix A.3. □

3.2.2 Natural capital with a critical threshold

Since the ability of natural capital to recover may change when natural capital exceeds CT, we consider also a second version of the general specification which makes the evolution of natural capital dependent on such threshold. Assuming that the level of CT is known to the agents in the economy, the equation for natural capital under this specification becomes:

$$K_{N,t+1} = K_{N,t}\left(1 + r_N\left(1 - \frac{K_{N,t}}{CC}\right)\left(\frac{K_{N,t}}{CT} - 1\right)\right) - K^b_{N,t} \tag{15}$$

In this case, once $K_{N,t} < CT$, the existing stock of natural capital converges progressively to zero. The accumulation/decumulation rate is then given by:

$$A_{N,t} \equiv r_N K_{N,t}\left(1 - \frac{K_{N,t}}{CC}\right)\left(\frac{K_{N,t}}{CT} - 1\right) \tag{16}$$

In other words, in the presence of a critical threshold, the rate at which natural capital accumulates/decumulates depends not only on CC and r_N but now also on CT. The proposition below analyzes the sign of $A_{N,t}$ in the case in which there is a critical threshold.

Proposition 2. *For values of $K_{N,t} \in (CT, CC)$, $A_{N,t}$ is monotonously increasing if* *and only if*

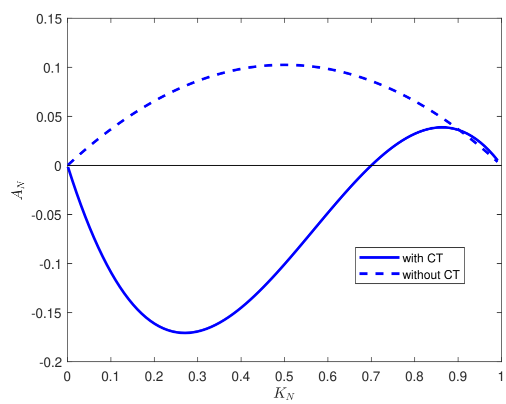
$$1 - 2K_{N,t}/CC + \frac{K_{N,t}}{CC}\left(\frac{CC - K_{N,t}}{K_{N,t} - CT}\right) > 0 \tag{17}$$

and monotonously decreasing otherwise. Moreover, $\frac{\partial^2 A_{N,t}}{\partial K_{N,t}^2} < 0$. *Importantly, the second term in the expression introduces a tension which ultimately determines the sign of* $\frac{\partial A_{N,t}}{\partial K_{N,t}}$, *and which will depend on the distance of* K_N *from* CC *and* CT*; for example, the closer the stock of natural capital is to* CT *(from above) the stronger is the positive pressure coming from being far from* CC*, but also, at the same time, the stronger is the negative pressure coming from being close to* CT*. The maximum in the interval (*CT*,*CC*) reflects the balancing out of this tension.*

Proof. See Appendix A.3. □

To help understand what these two alternative specifications entail for K_N in practice, Figure 1 plots the rate at which natural capital evolves (that is, k_N's accumulation rate A_N) with or without CT, normalizing the value of CC to 1. In line with the above propositions, the figure shows that, in the absence of a critical threshold, the accumulation rate of natural capital is always positive and increases before decreasing in proximity of natural capital's maximum sustainable level, CC (namely, A_N is always above zero in the interval $(0, 1)$, increasing for $K_N < CC/2$ and decreasing for $K_N > CC/2$) . Conversely, in presence of a critical threshold, A_N is negative for values below CT, but positive and increasing for a range of values between CT and CC before converging to zero as K_N approaches CC.

Figure 1: Nature Accumulation Rates (A_N)



Note: $CC = 1, CT = 0.7$. $r_N = 1.4$ when assuming a CT, and $r_N = 0.4$ otherwise.

In practice, under both specifications, the accumulation rate of natural capital remains uncertain because parametric shocks to each specification may affect the evolution of natural capital. To capture this we go one step further in modeling K_N and postulate that there are shocks that affect multiplicatively k_N's accumulation rate. Specifically we define a stationary shock process:

$$ln\ (z_{t+1}) = \rho^N ln\ (z_t) + \sigma_\epsilon \epsilon_{t+1} \tag{18}$$

where $\sigma_t > 0$, $|\rho^N| \leq 1$ and $\epsilon_{t+1} \sim \mathcal{N}(0,1)$. We thus re-write the law of motion of natural capital (in the absence of critical threshold) as:

$$K_{N,t+1} = K_{N,t}\left(1 + z_t r_N \left(1 - \frac{K_{N,t}}{CC}\right)\right) - K^b_{N,t} \tag{19}$$

We adopt the same approach when modeling natural capital in the presence of a critical threshold. The multiplicative assumption implies that the greater A_N, the larger the uncertainty that the agents or the social planner face when making optimal decisions, due to higher possible realizations of the shock.[13] Importantly the (log) formulation of the shock implies that the accumulation rate cannot turn negative following the realization of a bad shock. This is a simplifying assumption, which we adopt to contain the studied equilibria within the economically sustainable region (to the right of CT).

3.3 Aggregation and market clearing

We allow for aggregate shocks in the evolution of natural capital, but rule out idiosyncratic shocks. Then, assuming that each atomistic unit is indexed by i, the aggregate production, stock of natural capital, amount of labor associated with the green and brown technologies, consumption levels, and physical and financial assets are given

[13]In more complex specifications one could assume that σ_ϵ is time-varying. For example, as the stock of nature approaches the CT of the economy, there could be greater variability in the persistence and size of the shocks associated with the regeneration rate, due for example to an increase over time in the frequency and magnitude of extreme natural events. The inclusion of a shock over the regeneration rate could also be interpreted as a parsimonious way to model uncertainty over property rights; for example by setting $\epsilon_{t+1} \sim \mathcal{TN}_{(-\infty,0)}(0,1)$, where $\mathcal{TN}_{(a,b)}(.)$ represents a truncated normal distribution on the interval (a,b), it is possible interpret the (negative) shocks as originating from expropriation of (part) of the newly formed amount of Nature.

by:

$$Y_{g,t} = \int_0^1 y_{g,t}(i)di, \quad Y_{b,t} = \int_0^1 y_{b,t}(i)di, \quad K_{N,t} = \int_0^1 k_{N,t}(i)di,$$
$$K^g_{N,t} = \int_0^1 k^g_{N,t}(i)di, \quad K^b_{N,t} = \int_0^1 k^b_{N,t}(i)di, \quad N_{b,t} = \int_0^1 n_{b,t}(i)di,$$
$$N_{g,t} = \int_0^1 n_{g,t}(i)di, \quad C_t = \int_0^{N_t} c_t(i)di, \quad C^*_t = \int_0^{N^*_t} c^*_t(i)di,$$
$$\tilde{A}^*_t = \int_0^{N^*_t} \tilde{a}^*_t(i)di, \tilde{A}_t = \int_0^{N_t} \tilde{a}_t(i)di, K_t = \int_0^{N_t} k_t(i)di, K^*_t = \int_0^{N^*_t} k^*_t(i)di$$

We also assume that the Law of One Price (LOP) holds and that there is no home bias in consumption so that:

$$P_{H,t} = P^*_{H,t}E_t \tag{20}$$
$$P_{F,t} = P^*_{F,t}E_t \tag{21}$$
$$P_{b,t} = P^*_{b,t}E_t \tag{22}$$
$$P_{g,t} = P^*_{g,t}E_t \tag{23}$$

where E_t denotes the nominal exchange rate. From the LOP (and assuming no home bias, $\gamma = \gamma^*$) it follows that $P_t = P^*_t E_t$. Combining the budget constraint equations with the households' consumption optimizing conditions we have:

$$p^*_{F,t}(Y_{F,t} - (K^*_{t+1} - (1-\delta)K^*_t)) = \gamma(C^*_t + C_t) \tag{24}$$
$$p_{H,t}(Y_{H,t} - (K_{t+1} - (1-\delta)K_t)) = (1-\gamma)(C^*_t + C_t) \tag{25}$$

Notice that from the LOP it follows that $P^*_{F,t}/P^*_t = P_{F,t}/P_t$. From the above two conditions we can establish a clearing expression for world output:

$$C_t + C^*_t + p_{H,t}(K_{t+1} - (1-\delta)K_t) + p^*_{F,t}(K^*_{t+1} - (1-\delta^*)K^*_t) = p_{H,t}Y_{H,t} + p^*_{F,t}Y_{F,t} \tag{26}$$

Net financial assets demand equals 0:

$$\tilde{A}_{t+1} + \tilde{A}^*_{t+1} = 0 \tag{27}$$

Labor markets clearing in H implies:

$$N_t = N_{H,t} + N_{g,t} + N_{b,t} \tag{28}$$

Also, the nominal wage is the same across sectors

$$W_t \equiv W_{H,t} = W_{g,t} = W_{b,t} \tag{29}$$

3.4 Competitive Equilibrium

The competitive equilibrium in the two-bloc model is summarized by:
a consumption allocation for the domestic and foreign households, (c_t, c_t^*); a saving allocation for the domestic and foreign households, $(\tilde{a}_t, \tilde{a}_t^*, k_{t+1}, k_{t+1}^*)$; an allocation for the home intermediate goods firms $(n_{g,t}, n_{b,t}, k_{N,t}^g, k_{N,t}^b, k_{N,t+1})$; an allocation for the home final and intermediate goods firm $(N_{H,t}, I_t, I_{b,t}, I_{g,t}, K_t)$; an allocation for the final goods aggregator firm $(C_{H,t}^*, C_{F,t}^*, C_{H,t}, C_{F,t})$; an allocation for the foreign final and intermediate goods firms $(N_t^*, I_t^*, I_{b,t}^*, I_{g,t}^*, K_t^*)$; relative prices and returns $(p_{b,t}, p_{g,t}, p_{H,t}, p_{F,t}, p_{I,t}, w_{g,t}, w_{b,t}, w_t, w_t^*, R_t, R_t^K, R_t^*, R_t^{K*}, E_t)$; all of which satisfy the domestic and foreign households optimization problem and the home and foreign goods firms optimization problems and which clear goods, assets, and labor markets for all t, given a spending and transfer policy t_t, o_t, t_t^*, o_t^* and given the exogenous processes $A_{b,t}^0, A_{g,t}^0, A_{H,t}, A_{F,t}, N_t, N_t^*$.

3.5 FOCs, shadow price of Nature and Hotelling condition

We report the solution of the Competitive Equilibrium in Appendix A.1. A special case of First Order Condition (FOC) arises from solving the intermediate goods firms problem pertains to the shadow price of natural capital, which we call μ_t. Based on the intermediate goods maximization problem's FOC with respect to $k_{N,t}^g$, this is given by:[14]

$$\mu_t = p_{b,t}\alpha_b \frac{y_{b,t}}{(k_{N,t} - k_{N,t}^g)}(1 - \kappa(k_{N,t}, k_{N,t}^g)) - p_{b,t} y_{b,t} \frac{\partial \kappa(k_{N,t}, k_{N,t}^g)}{\partial k_{N,t}^g} - p_{g,t}\alpha_g \frac{y_{g,t}}{k_{N,t}^g} \quad (30)$$

Notice that

$$\frac{\partial y_{b,t}}{\partial k_{N,t}^g} < 0; \quad \frac{\partial \kappa(k_{N,t}, k_{N,t}^g)}{\partial k_{N,t}^g} < 0 \quad (31)$$

It follows that the shadow price of natural capital can be interpreted as the difference between two absolute magnitudes: the marginal cost from foregoing an additional unit of k_N in the production of brown intermediate goods (first term in the above expression); and the marginal benefit associated with preserving one additional unit of k_N for the production of green intermediate goods (last two terms in the above expression). This means that in our set up, the shadow price of k_N is positive when there exists an trade-off between exploitation and conservation of the stock of natural capital.

Crucially, k_N's shadow price must satisfy an intertemporal condition. This relates the current shadow price with expected future benefits from changes in the amount of natural capital. Following from the FOC with respect to $k_{N,t+1}$, this Hotelling

[14]We keep the general specification for the exploitation costs $\kappa(\bullet)$ to streamline the presentation.

condition, for the cases with and without CT is:

$$\mu_t = \mathbb{E}_t \Bigg[\Delta_{t,t+1} \Bigg(\mu_{t+1} \frac{\partial A_{N,t+1}(.)}{\partial k_{N,t+1}} + p_{b,t+1} \alpha_b \frac{y_{b,t+1}}{(k_{N,t+1} - k^g_{N,t+1})} (1 - \kappa(k_{N,t+1}, k^g_{N,t+1})) - \\ p_{b,t+1} y_{b,t+1} \frac{\partial \kappa(k_{N,t+1}, k^g_{N,t+1})}{\partial k_{N,t+1}} \Bigg) \Bigg] \tag{32}$$

where $\Delta_{t,t+1}$ is the stochastic discount factor of the households (owners of the firms). In line with this condition, assuming $\mu_t > 0$, the pace of natural capital accumulation/decumulation rate thus depends on two forces: first, the future expected change in k_N accumulation rate, which in turn depends on the level of the natural capital stock relative to CC (and CT if this is assumed); and second, the expected future marginal economic value of changes due to next perod's natural capital stock. Accordingly, for example, with no CT, and natural capital initially above $CC/2$, the expected change in the regeneration rate from a marginal increase in the stock of natural capital is negative (Proposition 1), while the marginal future expected benefit is positive.

3.6 Social planner problem

The competitive, decentralized solution and the Social Planner's (SP) solution are the same when there are no externalities associated with Nature, if the SP's aim is to maximize global welfare subject to world resources constraints and the dynamics of natural capital putting equal weight to the welfare of both blocs. Thus, also under a SP, global welfare depends on domestic and foreign consumption (C, C^*) through the utility functions $U(\bullet)$ of households in H and F, and does not have a specific preference nor target for a certain level of the natural capital stock. The SP problem then reads:

$$\max_{\Psi} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\chi U(C_t) + (1-\chi) U(C^*_t)) \tag{33}$$

s.t.

$$Y_{F,t} = C^*_{F,t} + C_{F,t} + K^*_{t+1} - (1-\delta^*) K^*_t \tag{34}$$

$$Y_{H,t} = C_{H,t} + C^*_{H,t} + K_{t+1} - (1-\delta) K_t \tag{35}$$

$$Y_{b,t}(1 - \kappa(K_{N,t}, K^b_{N,t})) = I^*_{b,t} + I_{b,t} \tag{36}$$

$$Y_{g,t} = I^*_{g,t} + I_{g,t} \tag{37}$$

$$K_{N,t} = K^g_{N,t} + K^b_{N,t} \tag{38}$$

where the F bloc's resources constraint is given by (34), the H bloc's resources constraint by (35), the intermediate brown goods clearing condition by (36) and the intermediate green goods clearing condition by (37); also, the problem takes into account the law of motion of natural capital, summarized by equation (13) or (15), the definitions of consumption, equations (11)) (and an equivalent one for the F bloc), the definitions of final goods output and intermediate goods, equations (7) and (9) (and two equivalent ones for the F bloc), the aggregate definitions of green and brown intermediate goods, which directly follow from aggregation of equations (2) and (3), the labor market clearing equation (28), and the non-negativity constraints on consumption. The parameter χ is a Pareto weight. Before solving it is convenient to substitute several definitions in the planning problem presented above, so that eventually the SP optimizes only over a set of variables, Ψ, defined as:

$$\{K_{t+1}, K^*_{t+1}, K_{N,t+1}, K^g_{N,t}, N^g_t, N^b_t, C_{H,t}, C^*_{H,t}, C_{F,t}, C^*_{F,t}, I_{g,t}, I_{b,t}, I^*_{g,t}, I^*_{b,t}\}$$

We define the shadow value of the foreign output resource constraint as $\lambda_1 \geq 0$, the shadow value of the home output resource constraint as $\lambda_2 \geq 0$, the shadow values associated with the intermediate inputs b as λ_3, the shadow values associated with the intermediate inputs g as λ_4 and the shadow values on the natural resources constraints as μ^{SP}.

The FOCs associated with the macroeconomic variables are standard and are presented in Appendix A.2. Importantly, the SP now internalizes the impact of Nature on TFP, which means that the shadow price of Nature and the Euler conditions need to be adjusted accordingly.

4 Model calibration and dynamics

In this section we calibrate and solve the macroeconomic model assuming no externalities from Nature and show how Nature is efficiently managed intertemporally.

4.1 Calibration

We assume that one period corresponds to 5 years, in line with the literature that focuses on modeling long-run climate economics phenomena. As explained below, we align common macroeconomic parameters with prior existing empirical literature; and experiment with parameters governing natural capital basing our priors on environmental research in connection to each parameter, since the economic literature on this is scarce or non-existent.

4.1.1 Macroeconomic parameters

We use World Bank data on total resources rents as a share of GDP to distinguish between the two blocs or regions. Specifically, using the full sample available in the

dataset, we include in the H bloc countries where the share of total natural resources rents in the data is equal to, or above, the 70^{th} percentile of the full sample (which corresponds, approximately, to a share of natural resources equal or above approx 7% of GDP). Conversely, to calibrate the F bloc we include all the remaining countries, with the exclusion of "low income" and "lower middle income" countries.[15]
We calibrate the size of each bloc using population data from the Penn World tables as of 2019, normalizing the foreign population to 1, which gives $N^*_{t0} = 1$ and $N_{t0} = 1.54$. The households inverse of the intratemporal elasticity of substitution and the discount factor are set within standard ranges, namely 2 and 0.985, respectively (see for example Golosov et al. (2014)).

Regarding the shares of intermediate inputs, labor and capital in the production of final consumption goods, we follow results from regressions in Baptist and Hepburn (2013) which, while fit to U.S. data, can be used as a rough starting point sufficient for our numerical illustrations. Looking at the regressions of gross output (e.g., value added + intermediate inputs) on capital, labor and intermediate inputs, under the assumption of CRS, we take the average across industries (excluding those industries where the coefficients are not reported as being statistically significant), which gives $\zeta^*_K = 0.11, \zeta^*_N = 0.25$. We then use these values as they are and apply them to the F bloc, which, being mostly comprised of advanced countries, is de facto closer to the U.S. economy. For the H bloc we tilt the share of labor upwards, coherently with the larger labor shares observed in emerging economies. In practice, we simulate a deterministic steady state (without growth) of the model with the exogenous CT, and set the labor share so as to roughly match the capital-value added ratio as it appears in the Penn Tables as of 2019. This requires increasing the labor share (ζ_N) to 0.30 of gross output, which is still a number smaller than the labor share of GDP in the Penn World Tables (0.52, in 2019). For the labor shares in the production functions associated with intermediate inputs in H, we directly follow the Penn World tables, using economy-wide labor shares, as of 2019. Similarly, we calculate the depreciation rates of physical capital using the Penn tables.

Regarding the intratemporal rate of substitution between intermediate inputs, g, b, previous literature has highlighted a vast range of estimates, depending on whether the elasticity refers to inter- and or intra-industry substitution (see for example Saito (2004)) with estimates varying widely (see Peter and Ruane (2023) for the case of India and Blaum et al. (2018) for the case of France) highlighting the difficulty in pinning down a value for our parsimonious model; in our exercises we use a value equal to 1.5, so as to emphasize imperfect substitution (this value is below 2.4, which is the value reported for domestic and imported intermediates by Blaum et al. (2018) but above the value of 0.9 proposed by Peter and Ruane (2023) to characterize

[15]Overall, the Foreign block includes 32 High Income (OECD) countries, 19 High Income (non-OECD) countries, and 34 Upper Middle income countries, while the Home block includes 10 high income countries (OECD and non-OECD), 21 Lower Middle Income, 21 low income and 17 upper middle income countries, according to the WB income classification.

the elasticity between energy, material and services inputs).

Another difficulty consists in calibrating the share of green and brown inputs in the intermediate goods production function; because our definition of green production encompasses all types of goods that are generated in such a way as to preserve the existing stock of natural capital between periods, there are no official statistics that we can directly rely on. Common knowledge is that this type of production represents a minor share of imported inputs across countries. In our benchmark exercises we set this value equal to 10% of the aggregate intermediate input, but it is possible to explore sensitivity of our main results, by considering production functions that allow for both higher and lower shares. A low share of green goods allows to incorporate well the trade-off that a social planner is likely to face, conditional on the current economic structure. That is, in an economy where the majority of production relies on natural capital exploitation, shifting economic output to the green sector is costly, as it requires, ceteris paribus, a larger share of labor relocation from the other sectors than what it would be if both types of intermediates held the same weight.

In the cost function that dictates the exploitation of natural capital we set $b_1 = 10$, which implies that full exploitation of the existing stock of natural capital roughly requires a total cost equal to produced output (that is $Y_{b,t}(1 - \kappa(K_{N,t}, K_{N,t})) \approx 0$).

Table 1 summarizes the calibration of the macroeconomic parameters.

Table 1: Calibration of Economic Parameters

Parameters	*Value*	*Explanation*
σ	2	HHs inverse Intratemporal Elast. of Subst.
β	0.985	Annual HHs Discount Factor
δ	0.05	Annual Depreciation rate H
δ^*	0.04	Annual Depreciation rate F
ζ_N	0.30	Labor Share in H Production
ζ_N*	0.25	Labor Share in F production
ζ_K	0.11	Capital Share in H production
ζ_K*	0.11	Capital Share in F production
ω_g	0.10	share of g intermediate inputs in H production
ω_g^*	0.10	Share of g intermediate inputs in F production
ρ	1.50	Intratemporal Elast. of H intermediate inputs
ρ^*	1.50	Intratemporal Elast. of F intermediate inputs
α_b	0.48	Share of Natural Capital in b production
α_g	0.48	Share of Natural Capital in g production
ϕ_g, ϕ_b	**	Elasticity of intermed. output to stock of K_N
b_1	10	Parameters in Cost Function of h firms
γ	0.50	Share F consumption goods
χ	0.50	Pareto Weight

Note: ** We will initially set the value of ϕ_g, ϕ_b equal to zero, and only assume a positive value when studying green policies.

4.1.2 Environmental parameters

Environmental parameters include the intrinsic regeneration rate, the levels of the carrying capacity (CC) and critical threshold (CT) of natural capital, and the elasticity of output to the aggregate stock of natural capital (ϕ_g, ϕ_b) as summarized in Table 3. Calibrating the dynamics of natural capital through a single metric is hard. This is because ecosystems are characterized by an enormous variety of goods and services, each with different attributes, regeneration rates, and each conventionally measured in a variety of different units. For these reasons, even though we will describe the though process associated with calibrating our parameters, we will also allow for explicit uncertainty in the intrinsic regeneration rate that governs the accumulation of natural capital.

In this exercise, to set the CT level, we rely on current estimates of what is considered a safe decline in biodiversity levels, and ecosystems more generally. The case of the Atlantic rainforest is on point, in the sense that, as shown by research, the forest itself when in a self-sustainable state can recycle much of the rain that falls on it, generating a self preserving cycle. Research suggests that removing as little as 30% of the forest cover can impede this self-perpetuating stabilizing cycle. Without this active restoration system in place, the system can then flip to another state, such as a savannah grassland (see Nepstad et al. (2007); Salati (1987); Farley (2008)). This 30% threshold seems also to apply in the case of fisheries where it is estimated that the highest average catch that can be continuously taken from a stock under average environmental conditions is approximately equal to 30% of the population (Bousquet et al. (2008); Thorpe et al. (2015)); it is important to emphasize the lack of consensus across researchers: some scientist go as far as to say that already a 10% loss in biodiversity might be considered unsafe (see Newbold et al. (2016)), even though other authors are much less pessimistic, setting safe limits as low as 30% of the original biodiversity richness (Steffen et al. (2015)).

Normalizing the level of CC to 1 we set $CT = 0.7$. This means that, when studying the dynamics with an exogenous CT, starting from a stock size normalized to 1, a decline exceeding 30% of the original stock activates the critical threshold, flipping the system into another state.

To calibrate the regeneration rate r_N, we follow Poorter et al. (2021), who analyzed 2,200 patches of forest in West Africa and Central and South America, including areas of the Atlantic and Amazon rainforests. The main results of the study suggest that soil richness was restored about 10 years after deforestation and after a period between 25-60 years, the forests' structure and function had fully returned. However, biodiversity took longer to fully return, at an average of 120 years. We consider the 60 years mark to calibrate the regeneration rate in our dynamic equation of natural capital.

Using the law of motion of natural capital, with $CC = 1, CT = 0.7$ (if we assume a CT level) and setting $K_N^g = K_N$ (that is assuming natural regeneration and no further exploitation), this means that, starting from a value of $K_{N,t0}$, close but above the CT level of 0.7 we can endogenously back-out a level of r_N that allows for full natural capital restoration within 60 years. Table 2 summarizes different values of r_N, given a range of starting levels of $K_{N,t0}$ "close enough" to 0.7. As expected, how close

Table 2: Calibration of intrinsic regeneration rate (r_N)

$K_{N,t0}$	with CT	Without CT
0.701	2.8	0.43
0.75	1.4	0.42
0.80	1.2	0.40
0.85	1.0	0.38

the initial assumed level of $K_{N,t0}$ is to CT determines how significant the influence on the implied value of r_N is, due to the non-linear dynamics that characterize the law of motion of natural capital. In the simulations below we take an agnostic approach and calculate the regeneration rates that are coherent with a grid of initial levels of $K_{N,t0}$ going from 0.701 (very close to the CT) to 0.851 (still relatively close to the CT) with increments of 0.01. Specifically, we calculate the value of r_N consistent with each starting level of $K_{N0} \in [0.701, 0.851]$ and then compute an unweighted average of this statistic. The computed mean is approximately equal to $r_N = 0.4$ for the model without CT and $r_N = 1.4$ for the model with CT.

Table 3: Calibration of Environmental Parameters

Parameters	*Value*	*Explanation*
CC	1	Carrying Capacity
CT	0.7	Critical Threshold
r_N	1.4; 0.4	Intrinsic regen. rate
σ_ϵ	0.03	StD. of regen. rate shocks
ρ^N	0.95	Persistence shocks

Finally, regarding the uncertainty associated with the accumulation rate of natural capital, we assume small ($\sigma_\epsilon = 0.03$) but persistent ($\rho^N = 0.95$) shocks; also, when solving for the optimal policy functions, we assume that the shocks take values $\epsilon_{t+1,1} = 0, \epsilon_{t+1,2} = \sigma\sqrt{\frac{3}{2}}, \epsilon_{t+1,3} = -\sigma\sqrt{\frac{3}{2}}$ with probability $\frac{2\sqrt{\pi}}{3}$ for the first realization and $\frac{\sqrt{\pi}}{6}$ for the second and third realizations (that is, we assume a three-node Gauss-Hermite quadrature method to account for uncertainty).

4.2 Model dynamics

In this section we study how optimal economic decisions are affected by different initial levels of natural capital and by natural capital's own dynamics. To this end we solve the for the global solution of the model using dynamic programming, in line with the recent literature focusing on climate economics (see for example, Fried et al. (2022) and Lemoine and Traeger (2016), among many others).

Given an initial stock of H and F capital (both physical and natural), the agents decide how to allocate economic resources across sectors, choose how much physical capital to accumulate, and decide how much of the available stock of natural capital to conserve or exploit depending on the existence of a CT. These decisions are taken simultaneously and are coherent with maximizing the expected discount sum of world consumption.

We start by assuming that there is no Total Factor Productivity (TFP) growth and that there is no externality from the stock of Nature. We look at four natural capital scenarios: (1) initially abundant K_N, no CT; (2) initially abundant K_N, with CT; (3) critically depleted K_N, no CT; and finally, (4) critically depleted K_N, with CT. In all cases we trace the behavior of both natural capital and key macro-variables as the economy evolves starting from the given levels of the state variables toward the model long-run stationary equilibrium.

We then move to a world with positive TFP growth. We look again at the four above cases, and trace once more the evolution of the economy. Results from this analysis are detailed below.

4.2.1 Model dynamics with no technological growth

Assuming no TFP growth,[16] we consider the case in which the economy starts with abundant natural capital (i.e. with K_N initially near CC, namely set to a value equal to 0.95, that is a value to the right of the level that maximizes the natural accumulation rate, see Figure 1). We then look at when the economy starts with a critically depleted stock of natural capital (K_N initially close to its CT).[17] In both cases we contemplate what happens when there is or not a CT.[18]

Abundant K_N. What happens to natural capital and production when K_N is close to its carrying capacity? To answer this question the first thing to note is that in our set up it is not optimal to maintain K_N close to its CC level–independently of whether there is or not a CT –because this requires setting aside a large portion of K_N for conservation, which in turn implies reducing substantially the production of brown goods both presently and in the future; and brown goods are the dominant

[16]Unless stated otherwise, we simulate the model setting to zero the realizations of the exogenous uncertainty shocks on the regeneration rate.

[17]To gauge what representative values of CC might be, we used the Biodiversity Intactness Index, which is an estimated percentage of the original number of species and their abundance that remains in any given area, despite human impacts (see Scholes and Biggs (2005)). Specifically, we computed a GDP-weighted average of the index across the H group of countries covered by the Index. In 2014 (the latest year of historical data available), the calculation gives 0.72 for the group of H, a value aligned to the approximate critical thresholds levels mentioned above.

[18]The initial stock of physical capital is assumed to be equal across versions (model without and with CT) of the model; we calibrate K_{t0}, K^*_{t0} assuming $K_{N,t0} = 0.75$, and in such a way that in the first period of the model with CT, the capital-output ratios in the two blocs are roughly matching the data as of 2019. We normalize the initial TFP level of F to 1 and set the relative TFP level of H equal to 0.61, following the Penn World Table data on TFP level data, as of 2019.

input in the production of aggregate intermediate goods. Besides, the output trade-off when K_N is close to CC is particularly stark because–both with and without a CT–the rate at which K_N accumulates is low, as in the proximity of CC this rate declines as natural capital increases (see Figure 1).[19]

It follows that when the economy finds itself with a high level of K_N close to CC, the SP, discounting future consumption at rate β, efficiently chooses to lower the K_N gradually over time, as shown in the upper panel of Figure 2. This way, the dynamics of natural capital lead to a smooth decline of brown production, and one that is consistent with the evolution of its relative marginal value over time. Eventually, K_N converges to a lower level than the initial one, in line with the law governing K_N accumulation rate for both the CT and no-CT model versions (see Figure 2). At such lower but stationary level, K_N is both "technologically" optimal in the sense that it is in line with a level of brown and green productions which maximizes expected discount utility of the households in both blocs.

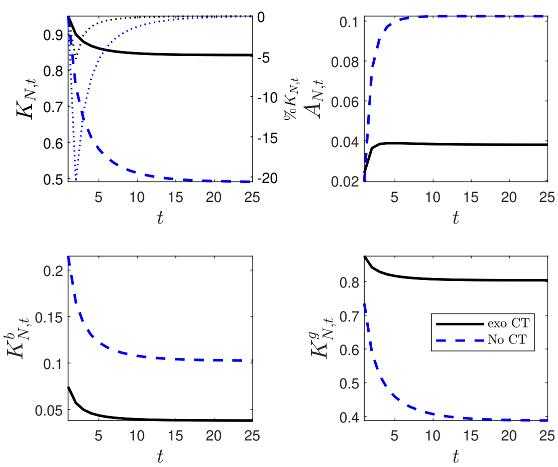
There are differences in the way the model evolves away from a starting abundant K_N state when there is a CT versus the case where there is no CT. Intuitively, K_N declines *less* when there is a critical threshold. This is because, in this case, it is not efficient to push K_N below CT and reach a lower level of natural capital than the low one already in place. So in the presence of a CT, the stationary level of natural capital is relatively higher than in the absence of a CT. This implies that it is optimal to exploit K_N less (bottom panel of Figure 2), in turn meaning that less brown output is produced compared with the case when there is no CT. One immediate consequence of a declining level of brown and green output, is that physical investment also declines. But since the speed of physical disinvestment is higher than the speed at which output declines, in equilibrium consumption can (temporarily) increase before falling (upper panel of Figure 3).

One final distinction between the CT case and the no-CT case when the economy starts with a large endowment of K_N relates to the response of green output. When there is no CT green output falls significantly, as a direct consequence of progressive natural capital depletion; this outcome is costly but still efficient due to the relatively small share that green production plays in generating the final consumption goods. By contrast, when there is a CT, green output marginally increases; this is possible because, despite a lower level of conserved K_N, there is a sufficiently strong re-allocation of labor from the brown intermediate and final good consumption producing sectors towards the green intermediate good producing sector, making relatively more green production possible. This outcome is efficient because the existence of a CT prevents to optimally exploit as much K_N as in the unconstrained equilibrium. As a result, the marginal productivity of labor in the brown sector cannot grow as much as it would, were the equilibrium unconstrained, which also means that relative

[19] In other words, in order to keep the stock of K_N from falling, the SP would need to earmark increasing amounts of K_N for conservation as K_N approaches CC; something clearly inconsistent with a stationary equilibrium where both brown and green production are positive.

marginal productivity of the green sector is relatively higher, ultimately generating the observed expansion in green output.

Figure 2: Natural Capital Dynamics

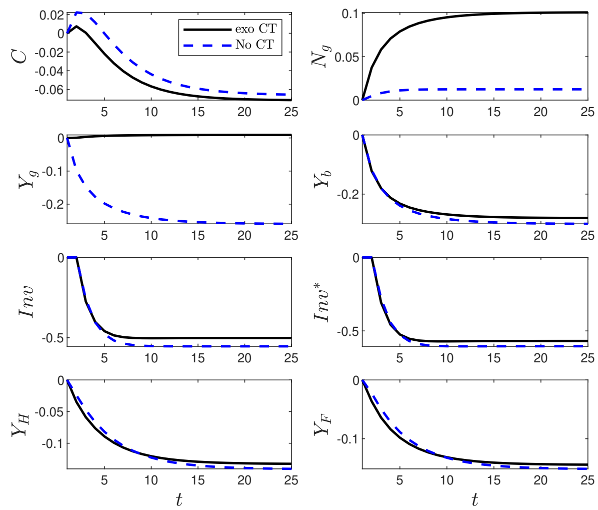


Note: CC=1, CT=0.7, $K_{N,t0} = 0.95$. $K_{N,t}$ (top left) and $A_{N,t}$ (top right). Blue dashed lines refer to the No-CT case, while black solid lines refer to the CT case. Blue and black dotted lines in the top left panel (right hand side y-axis) represent corresponding percentage changes between periods in the stock of $K_{N,t}$, in the two cases, respectively.

Critically Depleted K_N. To compare model dynamics in the case when the economy starts with a K_N close to CT relative to the case in which it is close to CC, we compute initial values of key macroeconomic variables in the two cases. Table 4 reports such values, showing that in both cases, a world with compromised natural assets is a world characterized by significantly lower economic activity than a world in which natural capital is initially abundant. The discrepancy in activity is visibly larger in the presence of a CT than in the absence of it, with the exception of green production and green labor, since when K_N is close to its tipping point, it becomes optimal to shift production to the green sector.

Figure, 4 describes the evolution of K_N when the model starts from this critically depleted K_N state. Like in the case of abundant initial K_N, natural capital adjusts to its economically efficient long-run levels compatible with the presence or absence of a CT, and these levels coincide with the level attained when the starting point of K_N was close to CC. The *rate* at which K_N accumulates in this scenario is much lower, however, when a CT is present. The reason is that Nature's ability to regenerate itself starting from its CT is much lower than when a CT exists, but K_N starts close to its carrying capacity. This insight is coherent with the observation that when an

Figure 3: Growth Rates of Macroeconomic Variables



Note: CC=1, CT=0.7, $K_{N,t0} = 0.95$. Blue dashed lines refer to the No-CT case, while black solid lines refer to the CT case. Each variable is expressed in terms of the percentage change with respect to its initial level at $t0$. For example, looking at N_g in the CT scenario, the figure suggests that in the long run the level of green labor is approx. 10% larger than its initial level. The variable $Inv_t = K_{t+1} - (1-\delta)K_t, Inv_t^* = K_{t+1}^* - (1-\delta)K_t^*$.

ecosystem is nearing its tipping point, recovery times from even from small shocks, such as a small drought or warmer-than-usual winter, are slower than when K_N is far from tipping points (Scheffer et al. (2012); Scheffer et al. (2015)).

What about the evolution of macroeconomic variables when the economy starts with a low K_N? If the economy starts close to the CT, it is clearly optimal to conserve more natural assets than when the economy starts with abundant K_N. Incremental additions to K_N according to this plan allow *both* more exploitation *and* conservation. When there is a CT, brown output can expand (Figure 5), allowing for an overall increase in consumption through time from, however, a lower initial starting point relative to the abundant K_N case. This happens because, with K_N initially close to CT, it is optimal at first to reduce brown output and divert labor resources to green production–in order to raise the level of K_N from its critically-depleted state. But as the economy moves away from its tipping point, it becomes increasingly inefficient for the social planner to sacrifice brown production to favor green production. In fact, as the economy moves progressively closer to its higher, long-run level of K_N, the social planner prefers to increase brown production while decreasing green production. The rate of decline in green production is relatively muted, and this is possible because, moving away from CT, the rate at which Nature can regenerate itself increases, and thus raises the endogenous accumulation rate, which in turn allows the social planner

Table 4: Percentage Diff. (%) in levels at $t0$

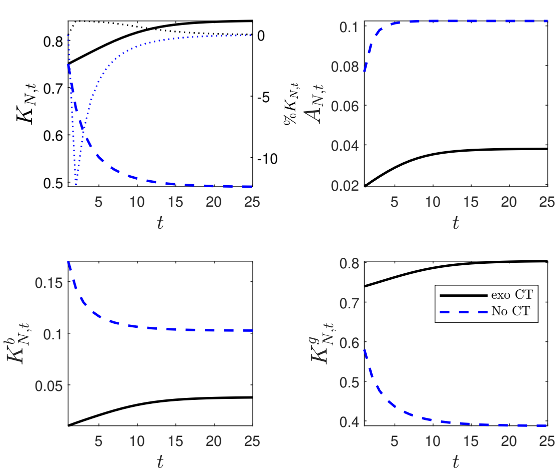
	Exo CT	No CT
C	-32.3	-4.9
Y_b	-61.9	-10.7
Y_g	8.0	-10.7
N_b	-4.1	0.0
N_g	35.5	0.0
Inv	-90.5	-17.2
Inv^*	-85.6	-16.6
Y_H	-39.0	-6.5
Y_F	-41.6	-7.0

Note: The table shows differences in percent of selected variables' values at time $t0$ in a ¨critically depleted¨ K_N scenario versus an ¨abundant¨ K_N scenario in the case of CT (column 2) and no-CT (column 3). For example, consumption at time $t0$ is about 32 percent lower when natural capital is critically depleted relative to the case when it is abundant. In the case of Inv, Inv^* the values refer to time $t0 + 1$.

to count on more K_N in the future while still allowing for more K_N accumulation in the near term–a virtuous cycle, which ends as the change in the accumulation rate tends to zero.

Figure 5 also shows that when there is no critical threshold, there are no large qualitative differences in the dynamics of output (and its components) compared to the scenario in which the model starts with abundant K_N.

Figure 4: Natural Capital Dynamics

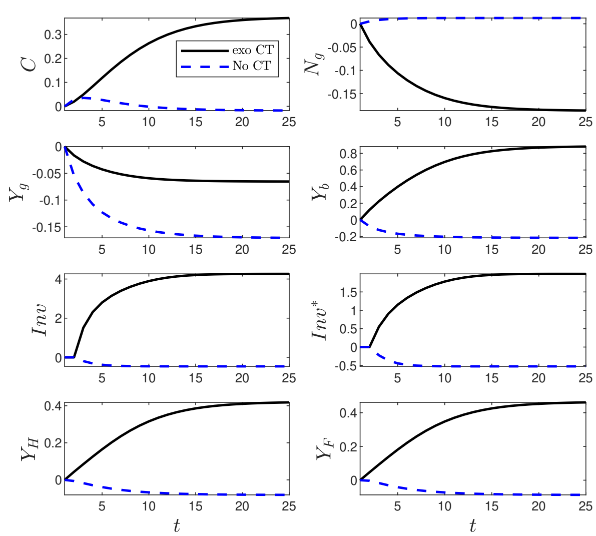


Note: CC=1, CT=0.7, $K_{N,t0} = 0.75$. See Note in Figure 2.

4.2.2 Model dynamics with technological growth

We now turn to a version of the model with positive technological growth. In this case the policy functions are no longer stationary, but rather incorporate, in each period, expected future optimal plans that in turn depend on future realizations of technological growth (which we assume deterministic) and shocks to K_N. To handle these new modeling assumptions we employ the general framework developed by Maliar et al. (2020) that contemplates time-in-homogeneity in optimal value and decision functions, while accounting for the possibility that the model possesses no well-defined balanced growth path. This way we can embed anticipatory effects (in our case due to technological growth), which would otherwise be unaccounted for when using a naive solution approach that relies on a stationary solution (see also Cooley et al. (1984) for a critique of using a naive-style solution that assumes stationary policy functions in growth models).

Figure 5: Growth Rates of Macroeconomic Variables



Note: CC=1, CT=0.7, $K_{N,t0} = 0.75$. See Note in Figure 3.

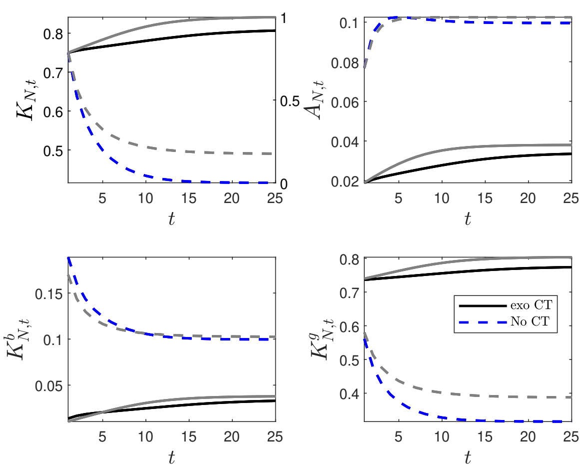
The growth processes are formalized following standard specifications used in climate and environmental models. More precisely we follow a specification used by Barrage and Nordhaus (2023):

$$A_{j,t} = A_{j,t-1}/(1 - g_{0,j}exp(-g_{1,j}\tilde{N}(t-1))) \tag{39}$$

for $j \in \{H, F, g, b\}$, where $g_{0,j}, g_{1,j}$ are parameters and $\tilde{N}$ corresponds to the number of years in each period (which in the present context is equal to 5).

We calibrate the initial exogenous productivity levels of H, F by matching the relative average TFP in 2019 between the two groups (at current PPP). To calibrate the exogenous growth rates of productivity $A_{H,t}$ and $A_{F,t}$ we follow work in Christensen et al. (2018), which distinguishes between high, middle and low income regions. As a baseline we use the same growth rates between the two groups by setting $g_{0,j} = 0.082, g_{1,j} = 0.0072$, as done in Barrage and Nordhaus (2023) in the context of a world economy. Regarding A_g, A_b we make the simplifying assumption that $A_{b,t} = A_{g,t} = A_{H,t}\ \forall t$. We assume that after 15 periods (corresponding to 75 years) the economy reaches a steady growth rate. Also, starting in period 20, we assume a stationary solution, in which we revert to the naive ("stationary policy function") approach where the agents think that next period level of productivity remains fixed forever.

Figure 6: Natural Capital Dynamics



Note: CC=1, CT=0.7, $K_{N,t0} = 0.75$. $K_{N,t}$ (Top left) and $A_{N,t}$ (top right). Dashed blue line plots model responses in the No-CT case, while solid black line plots responses for CT case. The figure also reports the evolution of variables under zero technological growth, with the solid gray lines corresponding to the CT scenario, and the dashed gray lines corresponding to the No-CT scenario, respectively.

As a natural extension of the stationary exercises presented above we consider a scenario where the initial stock of K_N is close to its critical threshold level (e.g., $K_{N,t0} = 0.75$) and compare the model dynamics with CT and without CT. As shown in Figure 6, when we embed positive technological growth, K_N declines faster and more significantly than in the absence of technological growth, when we assume the absence (presence) of a CT. This is because when the SP determines allocations, it now accounts for the fact that, for a given K_N conserved today, the marginal benefit that arises from producing brown goods tomorrow–i.e. goods which entail depleting K_N–is relatively higher compared to a scenario without technological growth. This in turn generates an equilibrium where it is optimal to deplete relatively more K_N (or equivalently, in the CT scenario, to conserve relatively less K_N), a finding consistent with the idea that economic growth increases the demand of material resources, thus requiring more use of K_N, as highlighted in Kornafel and Telega (2020).

Looking at macroeconomic developments, this dynamic resembles qualitatively the scenario without TFP growth: even with TFP growth, in the presence of a CT it is optimal to contain the accumulation of physical capital (that is, to invest less), compared to a scenario without a CT (Figures 7). In fact, here both physical capital and the level of final goods are permanently lower, reflecting the trade-off between allocating more labor to the green sector while maintaining enough labor to keep production of the intermediate good sector unaltered. Quantitatively, results underline the need of a slower accumulation of physical capital both in the H and in the F bloc both in the medium and long run when K_N- is initially depleted and TFP growth is assumed uniform across blocs and sectors. [20]

5 An example of Nature-positive green policy

So far, we assumed that there are no externalities associated with K_N, making the SP solution isomorphic to that from the Competitive Equilibrium (CE). In this section we relax this assumption by contemplating a direct impact of K_N accumulation on aggregate TFP in both the green and brown intermediate sectors. In particular, we assume that this is given by:

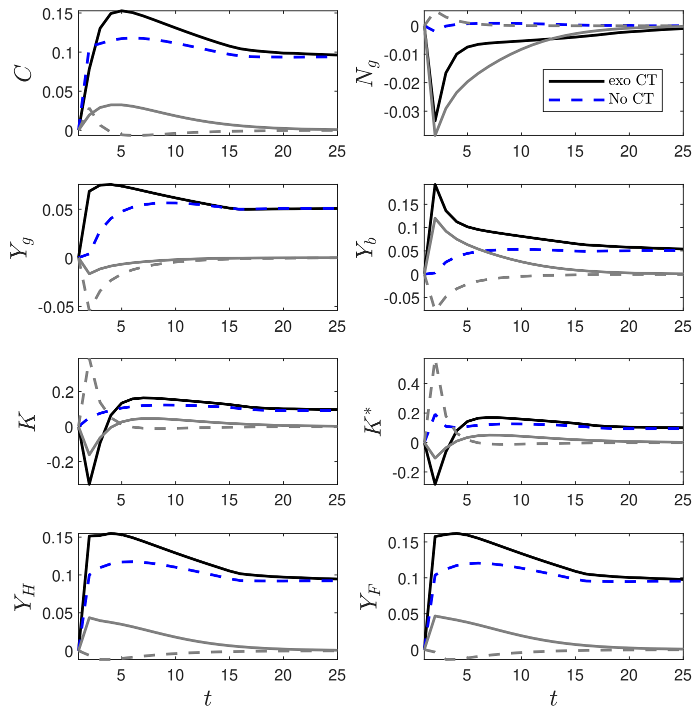
$$A_{b,t} = A^0_{b,t}(K^g_{N,t})^{\phi_b} \tag{40}$$

$$A_{g,t} = A^0_{g,t}(K^g_{N,t})^{\phi_g} \tag{41}$$

where $K^g_{N,t}$ represents the economy-wide K_N, $\phi_g > 0, \phi_b > 0$ describe the elasticity of output to such stock, and $A^0_{b,t}, A^0_{g,t}$ represent the exogenous component of TFP; through this change we (partly) "endogeneize" growth (to the extent that TFP is going to depend, in part, on the sum of conservation choices made by atomistic firms) and open the door to economic policies that, in the context of a CE, can help replicate the SP outcomes. To this end, we consider the case of a domestic subsidy on the H bloc's K_N, which resolves the externality.

[20] Specifically, the model dynamics under the assumption of positive TFP growth suggest that a SP would slow domestic (foreign) capital in the range of -6.6% (-7.4%) per year on average over the course of the next 10 years, and of -1.6% (-1.7%) per year on average over the course of the next 25 years.

Figure 7: Growth Rates in Macro Variables



Note: CC=1, CT=0.7, $K_{N,t0} = 0.75$. Dashed blue line plots model response in No-CT case, while solid black line plots response for CT case. The figure also reports the evolution of variables under zero technology growth, with the gray solid line corresponding to the CT scenario, while the gray dashed lines corresponding to the No-CT scenario, respectively. Each variable is expressed in growth rates with respect to the previous period.

Comparing the SP and CE shadow prices of K_N it is immediate to realize that in a decentralized economy the Hotelling equation of the agents does not incorporate the benefits accruing from a larger amount of conserved nature via TFP. The difference is that the SP internalizes that there are benefits from having a larger K_N, through the marginal change in the productivity of both brown and green production. Figure 8 illustrates this numerically, summarizing how these differences affect the accumulation of K_N (assuming $\phi_g = \phi_b = 0.05$). In particular, the upper panel shows that K_N accumulation is stronger when there is a CT and the SP internalizes the TFP externality (and symmetrically, K_N decumulation is less pronounced when there is no CT and the SP internalizes the TFP externality); the lower panel shows that the SP can enable K_N accumulation, by reducing exploitation and fostering conservation. Eventually, with a CT, the SP can achieve both higher exploitation *and* conservation; this is because, as explained before, with a CT, and K_N increasing, the regeneration rate also increases, which allows to expand brown production even further.

Even though our model assumes private ownership of K_N, so that there is no *"tragedy of the commons"*, the externality leads to prescriptions similar to an open-access environment since each producer is ignoring the impact of its own marginal depletion on aggregate outcomes. (Xepapadeas (2009)).

Last, it is important to stress the differences in K_N accumulation rates across the CE and SP worlds (upper right panel of Figure 8). With a CT, the accumulation rate of K_N is larger with a SP than in a CE, and this happens because this rate is increasing in the stock of K_N, when the economy starts near its CT (as also shown in Proposition 2); symmetrically, when there is no CT, the accumulation rate is relatively lower because the accumulation rate is decreasing in the level of K_N for values of K_N above $CC/2$ (as also shown in Proposition 2). Notably, in both cases there is *front-loading* and the speed of accumulation (decumulation) of K_N is stronger (milder) during the first periods, as demonstrated by the non-monotonic evolution of the percentage differences in accumulation rates (which have a maximum in the 2^{nd} period when assuming no CT and a minimum in the 9^{th} period when assuming a CT).

This discussion suggests that a tax authority in H could establish a (lump-sum financed) subsidy offered to the intermediate goods firms on the amount of conserved nature ($k^g_{N,t}$). This subsidy can be expressed in nominal units (that is in the bloc "currency"), as similarly done in the case of carbon taxes, which are usually expressed in terms of $ per ton of CO_2. In our case, the relevant measure is the "unit of Nature", or "unit of K_N", where the interpretation of Nature might differ depending on the relevant context. We formalize the idea of a H green subsidy in the proposition below:

Proposition 3. *A nominal, lump-sum financed, subsidy to the intermediate goods firms equal to* $P_t \tau_t k^g_{N,t}$ *where* τ_t

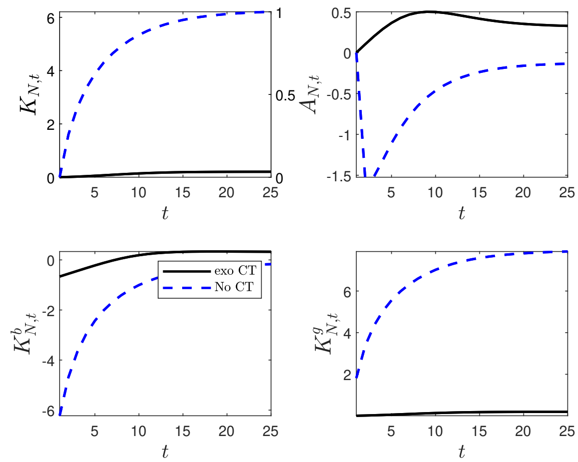
$$\tau_t = p_{b,t}\phi_b \frac{y_{b,t}}{K^g_{N,t}}(1-\kappa(k_{N,t}, k^g_{N,t})) + p_{g,t}\phi_g \frac{y_{g,t}}{K^g_{N,t}} \tag{42}$$

offsets the TFP-related externality associated with K_N*.*

Proof. See Appendix A.3. □

The assumption is that even though K_N is not directly traded in economic markets, it can be valued in nominal terms. This idea, although somewhat *ad hoc*, is coherent with the principle that "the price" of Nature increases as K_N goes down. In our framework, this is ensured to the extent that the amount of consumption goods (and hence their prices) is indirectly determined by K_N itself, through the production of intermediate inputs. In other words, as K_N diminishes, both the prices of consumption and K_N should increase, reflecting scarcity.

Figure 8: Natural Capital (% Differences SP and CE)



Note: Each period corresponds to 5 years. The Figure shows the percentage differences (%) in the allocation of K_N between the SP and the CE. In the SP case we assume that $\phi_g = \phi_b = 0.05$.

Figure 9 provides a numerical illustration of how such subsidy should be set inter-temporally in an economy without exogenous TFP growth. In particular, the figure shows both the subsidy rate and the total receipt as a percentage of GDP, or:

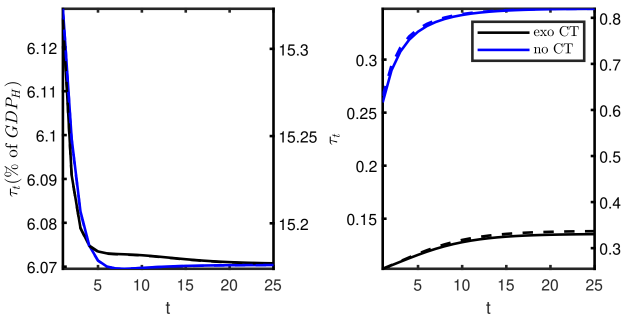
$$\frac{\tau_t K^g_{N,t}}{p_{H,t} Y_{H,t} - p_{I,t} I_t} \tag{43}$$

where small letters denote relative prices, and we substract intermediate inputs from gross output (Y_H).[21] We compare two scenarios: in the first one $\phi_g = \phi_b = 0.02$, while in the second one $\phi_g = \phi_b = 0.05$. We focus on the policy-relevant case of an initial K_N equal to 0.75. Overall the numerical illustration shows that the subsidy rate is increasing in both the CT and no-CT cases consistently with the results presented in Figure 8. Also, the subsidy is significantly smaller in the CT model, reflecting the much more limited possibility to raise K^g_N beyond what was already done in the CE. In terms of GDP both subsidies decline through time as K_N gradually converges toward its steady state.

We complement these results with those from the model with TFP growth. Figure 10 considers the case where $\phi_g = \phi_b = 0.05$. As expected, and in line with results in Figure 6, TFP growth reduces the efficient levels of the subsidy during the first years, because with TFP growth K_N accumulates more slowly when there is a CT and vice versa. Eventually the subsidy, in percentage of GDP, settles at a relatively higher

[21] All prices are normalized with respect to the price of the final consumption good.

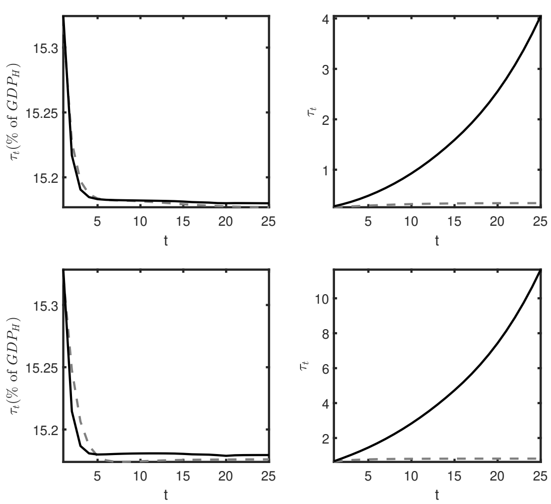
Figure 9: Optimal Green Subsidy



Note: Each period corresponds to 5 years. Left axis (solid lines) correspond to the case with $\phi_g = \phi_b = 0.02$, while right axis (dashed lines) correspond to the case with $\phi_g = \phi_b = 0.05$. The tax rate is expressed in % terms.

level as the economy converges to an equilibrium characterized by a lower K_N. The right panel of Figure 10 indicates that as the economy grows the marginal tax rate increases accordingly.

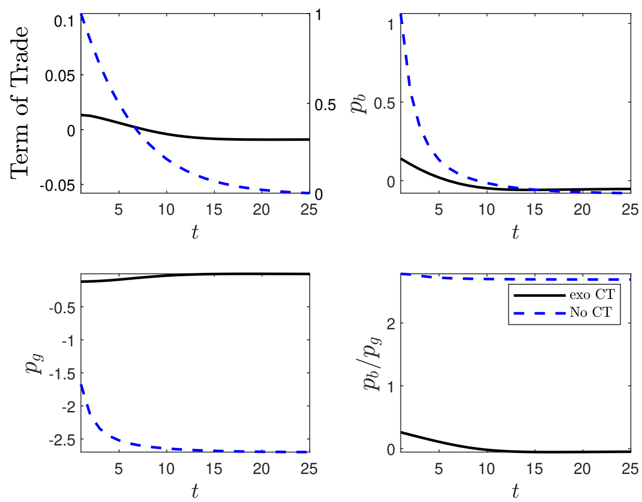
Figure 10: Optimal Green Subsidy
(Economy with Technological Growth)



Note: Each period corresponds to 5 years. Solid lines correspond to the case with TFP growth while the dashed lines correspond to the case without TFP growth. The top panel corresponds to the case with a CT and the bottom panel to the case without a CT. The tax rate is expressed in % terms. We assume $\phi_g = \phi_b = 0.05$.

Introducing a subsidy on the accumulation of K_N affects the prices in both economies, but this effect varies over time (Figure 11). In particular, promoting conservation initially shifts labor toward the green sector, which expands. In turn, an increase in the relative production of the green intermediate goods then requires a decline in the relative price $p_{g,t}$, while the relative price of the brown goods, $p_{b,t}$, increases. This effect reverses as the economy approaches its long-run steady state where a relatively higher K_N allows for both more green and brown production. The terms of trade, defined as $p_{F,t}/p_{H,t}$ also expand initially before contracting in the long run.[22]

Figure 11: Relative Prices (% Differences SP and CE)



Note: Each period corresponds to 5 years. Solid lines refer to the case without CT, while dashed lines refer to the case with a CT. We assume $\phi_g = \phi_b = 0.05$ and no exogenous technological growth.

What are the welfare gains associated with conservation subsidies? To answer this question, in line with Lucas (1987), we quantify the proportional increase in consumption that would make the households indifferent between the economy with and without green policies. To do so we define the aggregate welfare gains of the households at $t0$ as the value ð that makes the following equality hold:

$$\mathbb{E}_0\left[\sum_{t=t0}^{\infty}\beta^t\frac{\left((1+ð)C_t^W\right)^{1-\sigma}-1}{1-\sigma}\right]=\mathbb{E}_0\left[\sum_{t=t0}^{\infty}\beta^t\frac{\left(C_t^{WG}\right)^{1-\sigma}-1}{1-\sigma}\right] \tag{44}$$

[22]This follows from the initial decline in production of the final goods, which is relatively stronger in the F country, where intermediate inputs constitute a higher share of gross output compared to labor and man-made capital.

where C_t^W denotes the world consumption, C_t^{WG} denotes the world consumption when the intervention is introduced and $t0$ denotes the period when we start computing the gains.[23]

We find that welfare gains from these subsidies are more contained in the presence than in the absence of a CT, with specific values depending on the assumed elasticity of output to K_N and the presence or absence of TFP growth. Future welfare gains are positive, and significantly larger for both versions with and without a CT for a range of elasticities. Results are summarized in Figure 12 below for a range of elasticities values (assumed common between the two sectors).

Welfare gains with a CT are close to zero because it is hard to boost K_N with a low $K_N$regeneration rate; while foregoing the already relatively small K_N comes at large output costs.

More generally, welfare gains are more contained when calculated from $t0 = 0$, since the optimal conservation policy, which requires exploiting less K_N, demands a reduction of today, time $t0 = 0$, consumption. This strategy can nevertheless improve welfare since it allows the economy, through a relatively higher accumulation of K_N, to eventually reach an equilibrium where there is a larger (stable) K_N, making possible higher future consumption in both blocs. This trade-off is well illustrated in Figure 13 for the case without TFP growth and $\phi_g = \phi_b = 0.05$. In particular, the figure highlights how, in the absence of a CT the policymaker can foster conservation much more effectively, by significantly tilting consumption forward. Overall, the policy prescription emerging from this exercise aligns well with the previous literature on limits to growth, suggesting that under finite K_N, policy should optimally attempt to contain present-day levels of material consumption. Our exercise suggests that it is possible to do so without hurting economic welfare.

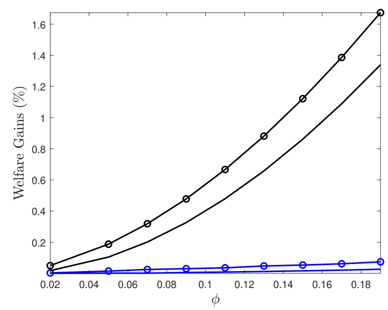
6 Conclusion

Nature is a blind spot in economics. But Nature is an asset just as man made capital. Should natural capital be accounted for in economic models and decision making?

In this paper we attempted to answer these questions by embedding for the first time natural capital–defined as a variety of ecosystem goods and services essential to economic activity– alongside man-made capital in a standard stochastic, two-bloc growth model. We assumed that natural capital is an input to production in both blocs, and that like in the real world, natural capital is unevenly distributed, displays critical thresholds or 'tipping points' beyond which the ecosystem is irreversibly altered, and becomes incapable of regenerating itself, and may contribute to the evolution of total factor productivity. We used this set up, calibrated using

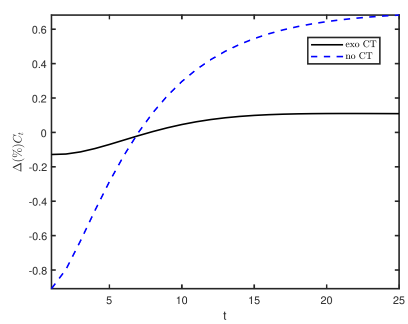
[23] To highlight the trade-off between present and future consumption, we compute "present-day" welfare gain (that is, computed taking the sum from $t = 0$ onward in equation (44)) but also future welfare gains (by computing the sum from $t = 1$ onward in equation (44)). In this way, even though our framework does not allow for overlapping generations, we attempt to communicate some insights over the time varying welfare impact of these policies.

Figure 12: Welfare Gains of a Conservation Subsidy (%)



Note: The dark and dotted continuous lines refer to the "future" welfare gains in the scenario without a CT, while the dark continuous lines refer to the "present-day" welfare gains. A similar description applies to the light lines in the case with a CT.

Figure 13: Consumption (% Differences SP and CE)



Note: Each period corresponds to 5 years. The solid (dashed) line corresponds to the percentage difference in world consumption between the SP and the CE assuming a CT (absence of a CT). $\phi_g = \phi_b = 0.05$.

prior empirical estimates from economic and ecological studies, to study the efficient allocation of economic and natural resources across time and space.

In particular, we examined how natural capital and economic variables evolved towards their long-run equilibria starting from different states of the world: a world still rich in natural assets and a world in which these assets have been critically depleted. We showed that when natural assets are abundant, it is optimal to use some and conserve some, but less depletion should occur if there is a critical threshold beyond which Nature is irreversibly altered and left unable to regenerate itself. We also showed that if production can push Nature to a level beyond which it can no longer regenerate itself, producing and consuming less initially makes growth sustainable and stronger long run in both Nature-rich and Nature-poor regions. When Nature is assumed to generate an externality through TFP, we showed that economic policy can steer the world economy towards its long run efficient equilibrium levels.

We conclude that accounting for natural capital when setting economic policy is key to ensure future economic sustainability and maximum growth levels globally. Our analysis opens the doors to further policy relevant applications, such as the study of the economic impact of greening production and the welfare costs of delaying such transition. Our analysis can be extended in several directions, for example, to study a world with weak and imperfect property rights, instead of perfect property rights as in our case. Weak property rights could lead to an equilibrium where it is optimal to deplete more Nature, as owners anticipate the possibility of losing the endowment in future. Another interesting extension may involve studying how optimal resource management is affected by alternative specifications for the law of motion of natural capital. We leave these extensions for future work.

References

Albagli, E., Vial, J. 2023. Biodiversity and economic growth: Something must give.Technical Report 70, Economic Policy Papers Banco Central de Chile.

Aronson, J., Milton, S. J., Blignaut, J. N. 2007. Restoring natural capital: Definitions and rationale..

Balboni, C., Berman, A., Burgess, R., Olken, B. A. 2023. The economics of tropical deforestation. Annual Review of Economics, 15, 723–754.

Baptist, S., Hepburn, C. 2013. Intermediate inputs and economic productivity. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 371, p. 20110565.

Barbier, E. B., Burgess, J. C., Markandya, A. 1991. The economics of tropical deforestation. Ambio, 55–58.

Barrage, L. 2020. The fiscal costs of climate change. In AEA Papers and Proceedings, 110, 107–112, American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203.

Barrage, L., Nordhaus, W. D. 2023. Policies, projections, and the social cost of carbon: Results from the dice-2023 model.Technical report, National Bureau of Economic Research.

Bastien-Olvera, B. A., Moore, F. C. 2021. Use and non-use value of nature and the social cost of carbon. Nature Sustainability, 4, 101–108.

Batini, N. 2021. The Economics of Sustainable Food: Smart Policies for Health and the Planet. Island Press.

Benigno, G., Benigno, P. 2002. Implementing monetary cooperation through inflation targeting. Available at SSRN 303983.

Blaum, J., Lelarge, C., Peters, M. 2018. The gains from input trade with heterogeneous importers. American Economic Journal: Macroeconomics, 10, 77–127.

Bousquet, N., Duchesne, T., Rivest, L.-P. 2008. Redefining the maximum sustainable yield for the schaefer population model including multiplicative environmental noise. Journal of Theoretical Biology, 254, 65–75.

Bovenberg, A. L., Smulders, S. 1995. Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model. Journal of public Economics, 57, 369–391.

Brander, J. A., Taylor, M. S. 1997. International trade between consumer and conservationist countries. Resource and energy economics, 19, 267–297.

Bringezu, S., Schütz, H., Moll, S. 2003. Rationale for and interpretation of economy-wide materials flow analysis and derived indicators. Journal of Industrial Ecology, 7, 43–64.

Brock, W. A., Taylor, M. S. 2010. The green solow model. Journal of Economic Growth, 15, 127–153.

Château, J., Dellink, R., Lanzi, E. 2014. An overview of the oecd env-linkages model: version 3.

Christensen, P., Gillingham, K., Nordhaus, W. 2018. Uncertainty in forecasts of long-run economic growth. Proceedings of the National Academy of Sciences, 115, 5409–5414.

Clarida, R., Galı, J., Gertler, M. 2002. A simple framework for international monetary policy analysis. Journal of monetary economics, 49, 879–904.

Clark, C. W. 2006. Fisheries bioeconomics: why is it so widely misunderstood? Population Ecology, 48.

Clark, C. W. 2010. Mathematical bioeconomics: the mathematics of conservation. 91, John Wiley & Sons.

Cooley, T. F., LeRoy, S. F., Raymon, N. 1984. Econometric policy evaluation: Note. The American Economic Review, 74, 467–470.

Costanza, R., Daly, H. E. 1992. Natural capital and sustainable development. Conservation biology, 6, 37–46.

D'Alessandro, S. 2007. Non-linear dynamics of population and natural resources: The emergence of different patterns of development. Ecological economics, 62, 473–481.

Dalton, T. R., Coats, R. M., Asrabadi, B. R. 2005. Renewable resources, property-rights regimes and endogenous growth. Ecological Economics, 52, 31–41.

Dasgupta, P. 2021. The Economics of Biodiversity: the Dasgupta Review.. HM Treasury.

Dasgupta, P., Heal, G. 1974. The optimal depletion of exhaustible resources. The review of economic studies, 41, 3–28.

Dasgupta, P., Mäler, K.-G. 2004. The economics of non-convex ecosystems: introduction. Springer.

Farley, J. 2008. The role of prices in conserving critical natural capital. Conservation Biology, 22, 1399–1408.

Fischer-Kowalski, M., Krausmann, F., Giljum, S., Lutter, S., Mayer, A., Bringezu, S., Moriguchi, Y., Schütz, H., Schandl, H., Weisz, H. 2011. Methodology and indicators

of economy-wide material flow accounting: State of the art and reliability across sources. Journal of Industrial Ecology, 15, 855–876.
Fried, S., Novan, K., Peterman, W. B. 2022. Climate policy transition risk and the macroeconomy. European Economic Review, 147, p. 104174.
Golosov, M., Hassler, J., Krusell, P., Tsyvinski, A. 2014. Optimal taxes on fossil fuel in general equilibrium. Econometrica, 82, 41–88.
Hartwick, J. M. 1977. Intergenerational equity and the investing of rents from exhaustible resources. The american economic review, 67, 972–974.
Hassler, J., Krusell, P., Nycander, J. 2016. Climate policy. Economic Policy, 31, 503–558.
Hinterberger, F., Luks, F., Schmidt-Bleek, F. 1997. Material flows vs.natural capital': What makes an economy sustainable? Ecological economics, 23, 1–14.
Howitt, P., Aghion, P. 1998. Capital accumulation and innovation as complementary factors in long-run growth. Journal of Economic Growth, 111–130.
IUCN. 2021. Iucn programme 2021-2024.
Johnson, J. A., Baldos, U. L., Corong, E., Hertel, T., Polasky, S., Cervigni, R., Roxburgh, T., Ruta, G., Salemi, C., Thakrar, S. 2023. Investing in nature can improve equity and economic returns. Proceedings of the National Academy of Sciences, 120, p. e2220401120.
Kornafel, M., Telega, I. 2019. Natural capital in economic models. Statistical Review, 65, 253–270.
Kornafel, M., Telega, I. 2020. Dynamics of natural capital in neoclassical growth model. International Journal of Sustainable Economy, 12, 1–24.
Lemoine, D., Traeger, C. P. 2016. Ambiguous tipping points. Journal of Economic Behavior & Organization, 132, 5–18.
Lontzek, T. S., Cai, Y., Judd, K. L., Lenton, T. M. 2015. Stochastic integrated assessment of climate tipping points indicates the need for strict climate policy. Nature Climate Change, 5, 441–444.
Lucas, R. E. 1987. Models of business cycles. Basil Blackwell. New York.
Maliar, L., Maliar, S., Taylor, J. B., Tsener, I. 2020. A tractable framework for analyzing a class of nonstationary markov models. Quantitative Economics, 11, 1289–1323.
May, J., Hobbs, R. J., Valentine, L. E. 2017. Are offsets effective? an evaluation of recent environmental offsets in western australia. Biological Conservation, 206, 249–257.
Meadows, D. H., Meadows, D. H., Randers, J., Behrens III, W. W. 1972. The limits to growth: a report to the club of rome (1972). Google Scholar, 91, p. 2.
Meadows, D., Randers, J., Meadows, D. 2004. Limits to growth: The 30-year update. Chelsea Green Publishing.
Nepstad, D. C., Tohver, I. M., Ray, D., Moutinho, P., Cardinot, G. 2007. Mortality of large trees and lianas following experimental drought in an amazon forest. Ecology, 88, 2259–2269.

Newbold, T., Hudson, L. N., Arnell, A. P., Contu, S., De Palma, A., Ferrier, S., Hill, S. L., Hoskins, A. J., Lysenko, I., Phillips, H. R. et al. 2016. Has land use pushed terrestrial biodiversity beyond the planetary boundary? a global assessment. Science, 353, 288–291.

NGFS. 2023. Nature-related financial risks: a conceptual framework to guide action by central banks and supervisors. Technical Document.

Nordhaus, W. D. 1991. To slow or not to slow: the economics of the greenhouse effect. The economic journal, 101, 920–937.

Nordhaus, W., Sztorc, P. 2013. Dice 2013r: Introduction and user's manual. Yale University and the National Bureau of Economic Research, USA.

Parrique, T., Barth, J., Briens, F., Kuokkanen, A., Spangenberg, J. 2019. Evidence and arguments against green growth as a sole strategy for sustainability. European Environmental Bureau.

Peter, A., Ruane, C. 2023. The aggregate importance of intermediate input substitutability.Technical report, National Bureau of Economic Research.

Poorter, L., Craven, D., Jakovac, C. C., Van Der Sande, M. T., Amissah, L., Bongers, F., Chazdon, R. L., Farrior, C. E., Kambach, S., Meave, J. A. et al. 2021. Multidimensional tropical forest recovery. Science, 374, 1370–1376.

Pörtner, H.-O., Scholes, R. J., Agard, J., Archer, E., Arneth, A., Bai, X., Barnes, D., Burrows, M., Chan, L., Cheung, W. L. W. et al. 2021. Scientific outcome of the ipbes-ipcc co-sponsored workshop on biodiversity and climate change.

Saito, M. 2004. Armington elasticities in intermediate inputs trade: a problem in using multilateral trade data. Canadian Journal of Economics/Revue canadienne d'économique, 37, 1097–1117.

Salati, E. 1987. The forest and the hydrological cycle. The geophysiology of Amazonia.

Scheffer, M., Carpenter, S. R., Dakos, V., van Nes, E. H. 2015. Generic indicators of ecological resilience: inferring the chance of a critical transition. Annual Review of Ecology, Evolution, and Systematics, 46, 145–167.

Scheffer, M., Carpenter, S. R., Lenton, T. M., Bascompte, J., Brock, W., Dakos, V., Van de Koppel, J., Van de Leemput, I. A., Levin, S. A., Van Nes, E. H. et al. 2012. Anticipating critical transitions. science, 338.

Scholes, R. J., Biggs, R. 2005. A biodiversity intactness index. Nature, 434, 45–49.

Schumacher, E. F. 2011. Small is beautiful: A study of economics as if people mattered. Random House.

Solow, R. M. 1956. A contribution to the theory of economic growth. The quarterly journal of economics, 70.

Solow, R. M. 1974. Intergenerational equity and exhaustible resources. The review of economic studies, 41, 29–45.

Steffen, W., Richardson, K., Rockström, J., Cornell, S. E., Fetzer, I., Bennett, E. M., Biggs, R., Carpenter, S. R., De Vries, W., De Wit, C. A. et al. 2015. Planetary boundaries: Guiding human development on a changing planet. Science, 347.

Stiglitz, J. 1974. Growth with exhaustible natural resources: efficient and optimal growth paths. The review of economic studies, 41, 123–137.
Tahvonen, O., Kuuluvainen, J. 1991. Optimal growth with renewable resources and pollution. European Economic Review, 35, 650–661.
Thorpe, R. B., Le Quesne, W. J., Luxford, F., Collie, J. S., Jennings, S. 2015. Evaluation and management implications of uncertainty in a multispecies size-structured model of population and community responses to fishing. Methods in Ecology and Evolution, 6, 49–58.
Toensmeier, E., Mehra, M., Frischmann, C., Foley, J. 2020. Farming our way out of the climate crisis. A Drawdown Primer.
Ward, J. D., Sutton, P. C., Werner, A. D., Costanza, R., Mohr, S. H., Simmons, C. T. 2016. Is decoupling gdp growth from environmental impact possible? PloS one, 11, p. e0164733.
Xepapadeas, A. 2009. Ecological economics: principles of economic policy design for ecosystem management. Levin S. Princeton University Press, Guide to Ecology, 740–747.

A Appendix

A.1 Competitive Equilibrium- First Order Conditions

In this section we present a full characterization of the Competitive Equilibrium (CE) outcome, which we use to analyze green policies in the main text.

Households and aggregator firms: The first order conditions from the aggregator and households problem in the H bloc can be summarized as:

$$p_{H,t}C_{H,t} = (1-\gamma)C_t \tag{45}$$

$$p_{F,t}C_{F,t} = \gamma C_t \tag{46}$$

$$\lambda_t = U'(c_t) \tag{47}$$

$$\beta\mathbb{E}_t\left[U'(c_{t+1})R_{t+1}\right] = U'(c_t) \tag{48}$$

$$\beta\mathbb{E}_t\left[U'(c_{t+1})\frac{p_{H,t+1}}{p_{H,t}}(R^K_{t+1}-\delta)\right] = U'(c_t) \tag{49}$$

Equivalent conditions hold for the F bloc.

Foreign Firms: From the above programs presented in the main text we characterize the following equilibrium relationships:

$$I^*_{g,t} = \left(\frac{P^*_{g,t}\omega^*_b}{P^*_{b,t}\omega^*_g}\right)^{-\rho} I^*_{b,t} \tag{50}$$

$$P^*_{F,t}\frac{\partial Y_{F,t}}{\partial I^*_t} = P^*_{I,t} \tag{51}$$

$$P^*_{F,t}\frac{\partial Y_{F,t}}{\partial N^*_t} = W^*_t \tag{52}$$

$$P^*_{F,t}\frac{\partial Y_{F,t}}{\partial K^*_t} = P^*_{F,t}(R^{*K}_t - 1) \tag{53}$$

$$P^*_{I,t} = \left((\omega^*_g)^{\rho^*}(P^*_{g,t})^{1-\rho^*} + (\omega^*_b)^{\rho*}(P^*_{b,t})^{1-\rho^*}\right)^{\frac{1}{1-\rho^*}} \tag{54}$$

$$I^*_{g,t} = \left(\frac{P^*_{g,t}}{\omega^*_g P^*_{I,t}}\right)^{-\rho^*} I^*_t \tag{55}$$

Home Firms: Normalizing the firms problem by P_t and substituting out $k^b_{N,t}$, the first order conditions for the intermediate goods firms are:

$$p_{g,t}(1-\alpha_g)y_{g,t}/n_{g,t} = w_{g,t} \tag{56}$$

$$p_{b,t}(1-\alpha_b)y_{b,t}/n_{b,t}(1-(k_{N,t}, k^g_{N,t})) = w_{b,t} \tag{57}$$

$$\mu_t = p_{b,t}\alpha_b\frac{y_{b,t}}{(k_{N,t}-k^g_{N,t})}(1-\kappa(k_{N,t}, k^g_{N,t})) - p_{b,t}y_{b,t}\frac{\partial\kappa(k_{N,t}, k^g_{N,t})}{\partial k^g_{N,t}} - p_{g,t}\alpha_g\frac{y_{g,t}}{k^g_{N,t}} \tag{58}$$

$$\begin{aligned}\mu_t =&\mathbb{E}_t\Bigg[\Delta_{t,t+1}\Bigg(\mu_{t+1}\frac{\partial A_{N,t+1}}{\partial k_{N,t+1}} + p_{b,t+1}\alpha_b\frac{y_{b,t+1}}{(k_{N,t+1}-k^g_{N,t+1})}(1-\kappa(k_{N,t+1}, k^g_{N,t+1}))- \\ &p_{b,t+1}y_{b,t+1}\frac{\partial\kappa(k_{N,t+1}, k^g_{N,t+1})}{\partial k_{N,t+1}}\Bigg)\Bigg]\end{aligned} \tag{59}$$

From the programs of the representative final goods and intermediate goods inputs firms we have the following equilibrium relationships:

$$I_{g,t} = \left(\frac{P_{g,t}\omega_b}{P_{b,t}\omega_g}\right)^{-\rho} I_{b,t} \tag{60}$$

$$P_{H,t}\frac{\partial Y_{H,t}}{\partial I_t} = P_{I,t} \tag{61}$$

$$P_{H,t}\frac{\partial Y_{H,t}}{\partial N_{H,t}} = W_t \tag{62}$$

$$P_{H,t}\frac{\partial Y_{H,t}}{\partial K_t} = P_{H,t}(R_t^K - 1) \tag{63}$$

$$P_{I,t} = \left(\omega_g^\rho (P_{g,t})^{1-\rho} + \omega_b^\rho (P_{b,t})^{1-\rho}\right)^{\frac{1}{1-\rho}} \tag{64}$$

$$I_{g,t} = \left(\frac{P_{g,t}}{\omega_g P_{I,t}}\right)^{-\rho} I_t \tag{65}$$

A.2 Social Planner Problem - First Order Conditions

In this section we summarize the First Order Conditions associated with the Social Planner Program that was presented in the main text. The first order conditions associated with consumption (C_F, C_F^*, C_H, C_H^*) are given by:

$$FOC(C_{F,t}):\ \chi\frac{\partial U'(C_t)}{\partial C_{F,t}} = \lambda_{1,t};\quad FOC(C_{F,t}^*):\ (1-\chi)\frac{\partial U'(C_t^*)}{\partial C_{F,t}^*} = \lambda_{1,t} \tag{66}$$

$$FOC(C_{H,t}):\ \chi\frac{\partial U'(C_t)}{\partial C_{H,t}} = \lambda_{2,t},\quad FOC(C_{H,t}^*):\ (1-\chi)\frac{\partial U'(C_t^*)}{\partial C_{H,t}^*} = \lambda_{2,t} \tag{67}$$

The equations characterize the relationship between marginal consumption and the shadow values of output. The first order conditions associated with I_{b^*}, I_b are:

$$FOC(I_{b,t}^*):\ \lambda_{1,t}\frac{\partial Y_{F,t}}{\partial I_{b,t}^*} = \lambda_{3,t};\quad FOC(I_{b,t}):\ \lambda_{2,t}\frac{\partial Y_{H,t}}{\partial I_{b,t}} = \lambda_{3,t} \tag{68}$$

which we can combine:

$$\lambda_{1,t}\frac{\partial Y_{F,t}}{\partial I_{b,t}^*} = \lambda_{2,t}\frac{\partial Y_{H,t}}{\partial I_{b,t}} \tag{69}$$

This expression equates the marginal value of the intermediate input b between the two blocs. Similarly the FOCs for I_{g^*}, I_g give:

$$FOC(I_{g,t}^*):\ \lambda_{1,t}\frac{\partial Y_{F,t}}{\partial I_{g,t}^*} = \lambda_{4,t};\quad FOC(I_{g,t}):\ \lambda_{2,t}\frac{\partial Y_{H,t}}{\partial I_{g,t}} = \lambda_{4,t} \tag{70}$$

which can be similarly combined, giving:

$$\lambda_{1,t}\frac{\partial Y_{F,t}}{\partial I^*_{g,t}} = \lambda_{2,t}\frac{\partial Y_{H,t}}{\partial I_{g,t}} \tag{71}$$

The first order conditions with respect to N^b_t and N^g_t (after subsituting out the labor market clearing condition) are given by:

$$FOC(N^b_t): \quad -\lambda_{2,t}\frac{\partial Y_{H,t}}{\partial N^b_t} = \lambda_{3,t}\frac{\partial Y_{b,t}}{\partial N^b_t}(1-\kappa(K_{N,t},K^g_{N,t})) \tag{72}$$

$$FOC(N^g_t): \quad -\lambda_{2,t}\frac{\partial Y_{H,t}}{\partial N^g_t} = \lambda_{4,t}\frac{\partial Y_{g,t}}{\partial N^g_t} \tag{73}$$

The first order conditions with respect to K_{t+1}, K^*_{t+1} produce standard Euler conditions:

$$FOC(K^*_{t+1}): \; \beta\mathbb{E}_t\Big[\lambda_{1,t+1}\Big(\frac{\partial Y_{F,t+1}}{\partial K^*_{t+1}} + (1-\delta^*)\Big)\Big] = \lambda_{1,t} \tag{74}$$

$$FOC(K_{t+1}): \; \beta\mathbb{E}_t\Big[\lambda_{2,t+1}\Big(\frac{\partial Y_{H,t+1}}{\partial K_{t+1}} + (1-\delta)\Big)\Big] = \lambda_{2,t} \tag{75}$$

Turning to natural capital, the first order condition with respect to $K^g_{N,t}$ reads as:

$$FOC(K^g_{N,t}): \; \mu^{SP}_t = -\lambda_{3,t}\Big(\frac{\partial Y_{b,t}}{\partial K^g_{N,t}}\Big(1-\kappa(K_{N,t},K^g_{N,t})\Big) - Y_{b,t}\frac{\partial\kappa(K_{N,t},K^g_{N,t})}{\partial K^g_{N,t}}\Big) - \lambda_{4,t}\frac{\partial Y_{g,t}}{\partial K^g_{N,t}} \tag{76}$$

Finally the FOC with respect to to $K_{N,t+1}$ is:

$$FOC(K_{N,t+1}): \; \mu^{SP}_t = \beta\mathbb{E}_t\Bigg[\mu^{SP}_{t+1}\frac{\partial A_{N,t+1}}{\partial K_{N,t+1}} + \lambda_{3,t+1}\Big(\frac{\partial Y_{b,t+1}}{\partial K_{N,t+1}}(1-\kappa(K_{N,t+1},K^g_{N,t+1})) - \\ Y_{b,t+1}\frac{\partial\kappa(K_{N,t+1},K^g_{N,t+1})}{\partial K_{N,t+1}}\Big)\Bigg] \tag{77}$$

A.3 Proofs

Proof of Proposition 1. Simply compute the derivative of $A_{N,t}$ with respect to $K_{N,t}$ which gives:

$$r_N\Big(1-\frac{K_{N,t}}{CC}\Big) - r_N K_{N,t}\frac{1}{CC}$$

This expression is positive for $K_{N,t} < CC/2$ and negative for $K_{N,t} > CC/2$

□

Proof of Proposition 2. When accounting for a CT, the change in the regeneration rate is equal to:

$$\frac{\partial A_{N,t}}{\partial K_{N,t}} = r_N(1 - \frac{K_{N,t}}{CC})(\frac{K_{N,t}}{CT} - 1) - r_N(\frac{K_{N,t}}{CT} - 1)\frac{K_{N,t}}{CC} + r_N(1 - \frac{K_{N,t}}{CC})\frac{K_{N,t}}{CT}$$

in turn the above expression (we assume $CC > K_{N,t} > CT$) is positive if and only if

$$1 - \frac{K_{N,t}}{CC} - \frac{K_{N,t}}{CC} + \frac{1 - K_{N,t}/CC}{K_{N,t}/CT - 1}\frac{K_{N,t}}{CT}$$

which can be re-arranged as

$$1 - 2K_{N,t}/CC + \frac{K_{N,t}}{CC}\left(\frac{CC - K_{N,t}}{K_{N,t} - CT}\right) > 0$$

□

Proof of Proposition 3. Define μ_t^{CE} as the shadow price of nature in the CE equilibrium. To show that τ_t is sufficient to internalize the TFP effect of nature, notice that the FOC with respect to $k_{N,t}^g$ in a CE is equal to (after applying the subsidy):

$$\mu_t^{CE} = -p_{b,t}\frac{\partial y_{b,t}}{\partial k_{N,t}^g}\left(1 - \kappa(k_{N,t}, k_{N,t}^g)\right) - p_{b,t}y_{b,t}\frac{\partial \kappa(k_{N,t}, k_{N,t}^g)}{\partial k_{N,t}^g} - p_{g,t}\frac{\partial y_{g,t}}{\partial k_{N,t}^g} - \tau_t$$

Equivalency with the SP equilibrium follows directly by replacing τ_t with the values presented in the main text and aggregating the resulting expression. □

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