

Central Banks Going Long*

Ricardo Reis

LSE

November 2017

Abstract

Many central banks have at times focussed on long-term interest rates as either their main policy tool or their goal. This paper describes three historical episodes when this happened, in the US in 1942-51, in the UK in the 1960s, and in Japan since 2016. Each differed in important ways. Using a standard model of inflation dynamics, this paper discusses how these policies compare to each other, and how they affected the ability of the central bank to keep inflation on target. A complementary methodological contribution is so re-state the classic problem of inflation control using interest-rate rules in a continuous-time setting where shocks follow diffusions so that monetary policy can be integrated with models of the term structure of interest rates.

JEL codes: E31, E52, E58.

Keywords: Taylor rule, yield curve, pegs, ceilings, affine models.

*Contact: r.a.reis@lse.ac.uk. I am grateful for the research assistance of Arthur Taburet and the financial support of a grant to the LSE FMG from Swiss Re.

1 Introduction

Long-term interest rates play an ambiguous role in the operation of modern monetary policy. In principle, a central bank could issue reserves and make loans at any one arbitrary maturity, and use its lending and deposit facilities together with open-market operations to target the market interest rate at this maturity. Yet, almost all central banks choose very short maturities, from the traditional focus on the overnight Federal Funds rate by the Federal Reserve, to the one-week main refinancing operations of the European Central Bank.¹ Even though the main policy rate across most central banks has been the overnight deposit rates, when policymakers discuss the effect of new monetary policies, from forward guidance to quantitative easing, they commonly state their impact on longer-term interest rates as a proof of success.

From the perspective of the targets of monetary policy, the Federal Reserve Act of 1913 that created the Federal Reserve set the monetary policy objective to be: “... to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates.” It was only after the Treasury Fed accord of 1951 that the Fed dropped the third of these objectives, and has since referred to itself as having a “dual mandate”. At the same time, after the financial crisis when interest rate spreads across different financial instruments spiked, the Fed and many other central banks have emphasized a goal of financial stability.

During the late 1940s, the long-term interest rate played a crucial role in U.S. monetary policy, both in its operations and in its goals. The Federal Reserve was not unique in this regard, as the Bank of England focussed policy in part on long-term interest rates following the recommendations of the 1959 Radcliffe report. More recently, since September of 2016, the Bank of Japan has followed a policy of “yield curve control” announcing a target of 0% for the 10-year government yield.

This paper describes the context behind each of these three historical experiments, and analyzes their role in determining inflation through the lenses of a model of inflation with interest-rate rules. Each of these experiments was different, but each went well beyond just using the long-term interest rate as one of many indicators of the state of economy when setting policy. Central banks went long, significantly changing the composition of their balance sheets and adapting their procedures to focus monetary policy on long-term interest rates. In the context of interest-rate rules, the long-term interest rate was not just one more variable on the right-hand side, but crossed to the left-hand side of the policy rule. Motivated

¹An exception is the Swiss National Bank, whose target interest rate is a 3-month money-market rate.

by these historical episodes, this paper discusses different ways in which the familiar model of monetary policy can integrate long-term interest rates with an effect on the dynamics of inflation. While each case is different, the results brought together suggest that focussing on long-term interest rates leads to more volatile and less anchored inflation.

The economic analysis in this paper requires linking the dynamics of inflation, short-term nominal interest rates, and long-term yields. There is an extensive literature on the yield curve and inflation, including several tractable models and extensions that have successfully fit the data.² A barrier to merging them with the study of inflation is that they are mostly set in continuous time with shocks that follow diffusions, while most work in monetary economics uses linearized models in discrete time.³ To overcome this barrier, this paper re-sets the classic inflation control problem in continuous model. This may prove to be useful in other contexts. Methodologically, it pushes forward a research agenda promoted by Brunnermeier and Sannikov (2017), who argue that bringing continuous-time tools to macroeconomics will allow models to better incorporate endogenous risk premia and financial frictions.

Sections 2 to 4 of the paper set up the model, solve for the dynamics of inflation, and characterize the dynamics of the yield curve. Sections 5-7 discuss the three case studies of central banks going long and apply the model to understand them. Section 8 concludes.

2 Controlling inflation in continuous time

I first describe the choices facing the private sector, then the central bank's policies, and finally define the equilibrium interaction between the two. Section 2.4 provides general-equilibrium microfoundations for the model.

2.1 The private sector

A representative household chooses how much to save in a real riskless bond that, in exchange for one unit of consumption today, returns for sure $R_t^{(s)}$ units of consumption s periods from now. Letting m_t denote the marginal utility of consumption at date t , then the optimal holdings of this bond must satisfy the Euler equation:

$$m_t = \mathbb{E}_t \left(m_{t+s} R_t^{(s)} \right). \quad (1)$$

²See Piazzesi (2010) for a survey, and Smith and Taylor (2009) for an estimated model closer to the one in this paper.

³Jones and Kulish (2013) and McGough, Rudebusch and Williams (2005) are the closest papers in the literature in their treatment of long rates, but they work with linearized discrete-time new Keynesian models.

Buying an extra unit of the bond lowers utility by the left-hand side of this equation, but is expected to raise it by the right-hand side. At the optimum, the net effect must be zero.

Taking the limit as s becomes a time interval dt that is infinitesimally close to zero, and since $R_t^{(s)}$ is known at date t , gives the continuous-time version of this equation:

$$\mathbb{E}_t \left(\frac{dm_t}{m_t} \right) = -r_t dt, \quad (2)$$

where r_t is the return on an instantaneous bond. Using the language of the Ramsey model, this equation states that marginal utility must decline at the same rate as the safe return to savings at an intertemporal optimum.

I assume that the utility function of the households is time separable and has constant relative risk aversion. Therefore: $m_t = \beta^t c_t^{-\gamma}$, where $\beta \in (0, 1)$ is the discount factor, c_t is consumption, and $\gamma > 0$ is the coefficient of relative risk aversion. As in baseline new Keynesian economies, there is no capital or investment; therefore markets clear when consumption equals output y_t .

The key assumption in this economy is that prices are flexible, so the classical dichotomy holds. As illustrated in Woodford (2003), Cochrane (2011), or more recently in Hall and Reis (2016), the economic problem of pinning down the price level using interest-rate rules is conceptually unchanged if there are nominal rigidities and a Phillips curve. Adding nominal rigidities complicates the expressions and requires linearizing the equilibrium conditions, but the qualitative conclusions on when is inflation pinned down remain unchanged.

Given this assumption, it is then a mere simplification to further assume that output is exogenous, as in an endowment economy. (For the unconvinced readers, section 2.4 endogenizes the evolution of output as a function of technology shocks.) In particular, I assume that output follows a random walk, that has normally distributed innovations with standard deviation σ_y , and a stochastic mean growth rate g_t .⁴ This trend in turn follows a stationary autoregressive process, with long-run mean \bar{g} , speed of mean reversion κ_g , and normal shocks with standard deviation σ_g . In continuous-time notation, this is compactly written as:

$$\frac{dy_t}{y_t} = g_t dt + \sigma_y dz_t^y \quad (3)$$

$$dg_t = -\kappa_g (g_t - \bar{g}) dt + \sigma_g dz_t^g \quad (4)$$

⁴To be more precise, time is continuous, y_t is a stochastic variable defined on a filtered probability space, and z_t^y is an adapted Brownian motion in this space. The same applies to all other stochastic variables in this paper.

The shocks are independent Wiener processes, so they are normally distributed with mean zero and variance $\mathbb{E}_0((z_t^y)^2) = \mathbb{E}_0((z_t^g)^2) = t$.

To solve for the real interest rate, note that market clearing in the goods market implies that: $m_t = \beta^t y_t^{-\gamma}$. Using Ito's lemma to take time derivatives of this expression:

$$\frac{dm_t}{m_t} = [\ln \beta - \gamma g_t + 0.5\gamma(\gamma + 1)\sigma_y^2]dt - \gamma\sigma_y dz_t^y. \quad (5)$$

Then, the Euler equation gives the solution for the real interest rate:

$$r_t = \gamma g_t - \ln \beta - 0.5\gamma(\gamma + 1)\sigma_y^2. \quad (6)$$

The first two terms on the right-hand side are the standard ones from the Ramsey model: higher growth rates or more patient households increase the equilibrium real interest rate. The third term captures the precautionary savings effect that more uncertainty on output induces the consumer to save more and this lowers the real interest rate in equilibrium. A virtue of working in continuous time is that this precautionary savings term is present (in discrete-time linearized setups, it is zero) and it is linear, so studying it does not require the use of high-order numerical approximation.

Collecting all the results gives the real equilibrium:

Lemma 1. *In the real equilibrium in this economy, $c_t = y_t$, and the marginal utility of consumption and the real interest rate are given by:*

$$\frac{dm_t}{m_t} = -r_t dt - \gamma\sigma_y dz_t^y \quad (7)$$

$$dr_t = -\kappa_g(r_t - \bar{r})dt + \gamma\sigma_g dz_t^g. \quad (8)$$

where $\bar{r} = \gamma\bar{g} - \ln \beta - 0.5\gamma(\gamma + 1)\sigma_y^2$.

2.2 The central bank

Central banks take deposits from banks, commonly called reserves. This liability is crucial in the modern monetary system, because people make electronic payments using cards and other means of payments issued by banks. Therefore, there are large gross cross-bank liabilities every day. Reserves are the settlement currency used by the banks to clear these transactions between themselves.

If the deposits at the central bank have maturity s , then the usual central-bank policy is

to promise a safe nominal return of $I_t^{(s)}$ per unit of currency held as reserves. Central banks can also make loans to banks. For simplicity, I assume that the central bank's deposits and loans have the same maturity at the same interest rate. In reality, central banks typically charge a higher interest rate on their loans, and these have a higher maturity than reserves, which tend to be overnight.

The central bank is independent, and its dividend rule is to rebate net profits every instant to the fiscal authority. By the result in Hall and Reis (2015), the central bank is therefore always solvent, as its reserves satisfy a no-Ponzi scheme condition.

Because the central bank is the monopoly issuer of reserves, it can perfectly choose $I_t^{(s)}$. The private agents in the economy, represented by the representative consumer, would want to hold these deposits according to the optimality condition:

$$\mathbb{E}_t \left(\frac{m_{t+s} I_t^{(s)}}{p_{t+s}} \right) = \frac{m_t}{p_t}. \quad (9)$$

The price level p_t appears because reserves are the unit of account in the economy. Moreover, as is true in most central banks, the central bank promises a nominal return on its reserves. In the extreme case where reserves are instantaneous deposits, then the differential version of this condition is:

$$\mathbb{E}_t \left(\frac{d(m_t/p_t)}{(m_t/p_t)} \right) = -i_t dt, \quad (10)$$

where i_t is the nominal interest rate on an instantaneous deposit at the central bank.

Following a long line of work, I assume that the central bank adopts a feedback rule for the choice of the interest rate. The first component of this rule is a constant inflation target π^* .⁵ A strict reading of the mandate of most central banks sets π^* to a constant equal to 2% at an annual rate.⁶

The central bank then responds to any deviation of actual inflation dp_t/p_t from this target, by raising interest rates by an amount $\phi \geq 0$ in the next instant of time. The assumption that this is positive corresponds to the famous Taylor principle (since in a one-unit period of time this corresponds to $e^\phi \geq 1$.)

Most central banks, however, do not engage in such strict inflation targeting, but rather adopt a policy of flexible inflation targeting. Any given period, they target an inflation rate different from π^* depending on the state of the economy. This is optimal in many models

⁵Letting the target vary over time deterministically would make no difference to the results.

⁶For instance, if a one-unit period in the model corresponds to one week, then $\pi^* = 0.02/52$.

of nominal rigidities.⁷ As a result, interest rates rise and fall to push inflation above or below the strict inflation target temporarily in order to obtain welfare gains by stabilizing real output.

Moreover, when inflation is on target, then the nominal interest rate must mimic changes in real interest rates. Yet, most central banks find it difficult to measure the right real interest and respond to it instantly, or more generally to track the state of the business cycle. Errors in measurement lead to changes in interest rates.

Finally, almost no central bank follows a rule, but rather chooses a path for monetary policy from the aggregation of the opinions of different committee members. As opinions, or even the composition of the committee changes, this will lead to changes in interest rates.

Whether it is in response to desires to stabilize real fluctuations, due to mis-measurement of the actual state of the business cycle, or because of monetary policy shocks, then even if inflation is at π^* , nominal interest rates may vary. I capture the combination of all these factors through a random nominal interest rate target, or intercept, x_t that also follows a Markov process with long-run mean \bar{x} .

Finally, central banks smooth interest rates at a rate $\rho > 0$.

Combining all these ingredients, and assuming for now that the central bank sets policy in terms of the instantaneous interest rate on reserves, gives the monetary policy rule:

$$d(i_t - x_t) = -\rho(i_t - x_t)dt + \phi \left(\frac{dp_t}{p_t} - \pi^* dt \right). \quad (11)$$

2.3 The equilibrium

Because the classical dichotomy holds in this economy, all the real variables are already pinned down. What remains to determine is the price level. A rational expectations equilibrium is a path for the price level $\{p_t \in \mathfrak{R}^+ : t \geq 0\}$ given the real equilibrium in lemma 1 and the monetary policy rule in equation (11).

Without any further conditions, in a standard monetary model like this one, the rational expectations equilibrium is indeterminate. Mathematically, the interest-rate rule makes inflation a differential equation but fails to provide a boundary condition for it. Following a long literature on new Keynesian economics and DSGE models of monetary policy, I focus on a narrower definition of equilibrium:

Proposition 1. *A bounded homoskedastic Markov equilibrium is a function for expected*

⁷See Woodford (2010) or Ball, Mankiw and Reis (2005).

inflation $\bar{\pi}(r, x) : \mathfrak{R}^2 \rightarrow \mathfrak{R}$ and three constants, $\alpha_y, \alpha_g, \alpha_r$ such that:

$$\frac{dp_t}{p_t} = \bar{\pi}(r_t, x_t)dt + \alpha_y \sigma_y dz_t^y + \alpha_g \sigma_g dz_t^g + \alpha_r \sigma_r dz_t^r, \quad (12)$$

equations (7)-(8) and (11) hold, and expected inflation satisfies:

$$\lim_{T \rightarrow \infty} \mathbb{E}_t (e^{-\epsilon(T-t)} \pi(r_T, x_T)) = 0 \quad (13)$$

for any $\epsilon > 0$.

There are three restrictions imposed in this definition relative to the rational-expectations equilibrium. First, since the state of the economy is captured by the real interest rate and the nominal interest rate, and (r_t, x_t) follows a Markov process, the restriction to look only at Markov equilibrium is natural. At the same time, it rules out the possibility that sunspots drive inflation. Second, since all variances are independent of time, the definition imposes that the variance of inflation also did not depend on time. Therefore, the responses to shocks stacked in the vector $Z_t = (z_t^y, z_t^g, z_t^x)$ are given by a vector of constants $\alpha = (\alpha_y \sigma_y, \alpha_g \sigma_g, \alpha_x \sigma_x)$ rather than by three functions of the state vector. I conjecture that allowing for sunspot shocks by letting inflation depend also on $\alpha_\epsilon dz_t^\epsilon$, or allowing the responses of inflation to shocks to depend on (r_t, x_t) would actually make no difference: in equilibrium, $\alpha_\epsilon = 0$ and the other α 's would not depend on the state of the economy.

More important is the assumption of boundedness. Cochrane (2011) provides a scathing critique of this assumption as an equilibrium selection device. It is not micro-founded since it does not follow from optimal behaviour or market clearing conditions. Moreover, it plays an important role in pinning down inflation. The long literature on interest-rate rules has proposed other related boundary conditions, Obstfeld and Rogoff (1983) being a famous example, and there is also an extensive literature with other monetary policies to control the price level (Reis, 2016b). I follow Woodford (2003) and the extensive literature after it in maintaining this assumption because there is little in the analysis that brings any new light to the issues involved.

Given the stochastic process for marginal utility in equation (7), and for prices in equation (12), one can use Ito's lemma to calculate the expected rate of change in m_t/p_t . By the Euler equation (10), this is equal to the instantaneous nominal interest rate. This gives a modified Fisher equation as a no arbitrage condition between nominal and real bonds:

$$i_t = r_t + \bar{\pi}_t - \alpha' \alpha - \gamma \sigma_y^2 \alpha_y. \quad (14)$$

As usual, the nominal interest rate is equal to the sum of the real interest rate and expected inflation, the two first terms on the right-hand side, respectively. However, shocks to inflation introduce two extra terms. First, because of the convexity of returns, more variable inflation subtract from the realized real returns on nominal bonds. Second, there is an inflation risk premium. If positive shocks to inflation come at times when the marginal utility of consumption is high, then nominal bonds will have a realized return that is lower when returns are more valuable. Thus, holding a nominal bond comes with risk, and so it must pay a higher nominal interest rate to compensate for this risk. The focus on a homoskedastic equilibrium makes this risk premium constant, which is counterfactual. Allowing for heteroskedasticity in the growth rate of output or in the shocks to monetary policy would easily lead to a time-varying risk premium, and future work should explore its role.

2.4 Where does the price level come from?

Because reserves are the unit of account in the economy, their real value is, by definition $1/p_t$. It is the absence of arbitrage between private bonds and reserves at the central bank that pins down the price level. Outside of equilibrium, if the price level was too high, then reserves would cost less, which would make banks want to sell private bonds and deposit more reserves at the central bank. As the supply of reserves is fixed by the central bank this “excess demand” for reserves would make their value fall, which comes through the price level rising back to equilibrium.

This description of equilibrium may strike some readers as odd in two ways. First, output was taken as exogenous, as in an endowment economy. Second, there was no mention of goods’ prices. Both of these features resulted from not having any mention of firms selling goods and setting prices. This section shows how introducing these makes no difference.

Assume that the representative agent solves the following problem:

$$\max_{\{c_{t,j}, l_t\}} \mathbb{E}_0 \int_0^\infty \beta^t \left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{l_t^{1+\psi}}{1+\psi} \right) dt \quad (15)$$

$$\text{s.t.} \quad c_t = \left(\int_0^1 c_{t,j}^{\frac{\mu}{\mu-1}} dj \right)^{\frac{\mu-1}{\mu}}, \quad (16)$$

$$d(b_t + v_t) = \left(i_t - \frac{dp_t}{p_t} \right) v_t + r_t b_t + (w_t l_t + z_t - c_t) dt. \quad (17)$$

The representative household chooses its consumption of a continuum of varieties ($c_{t,j}$) and

hours worked (l_t) for a real wage (w_t) to maximize expected discounted utility, subject to its preference for different varieties and to a flow budget constraint where labor and investment income is complemented with dividends from firms (z_t). For simplicity, this assumes only instantaneous bonds (b_t) and reserves (v_t), but allowing for higher maturities would not change the argument.

The optimal behavior of consumers is then characterized by the two Euler equations already presented in equations (2) and (10), its flow of resources combined with a transversality condition, and finally the optimality condition for labor supply:

$$c_t^\gamma l_t^\psi = w_t. \quad (18)$$

The real wage is equal to the marginal rate of substitution between labor and consumption.

A continuum of monopolistic firms operate a technology $y_{t,j} = a_t l_{t,j}$ to produce each variety of good subject to the common productivity a_t . Using their monopoly power, the optimal price they charge is a markup over costs:

$$\frac{p_{t,j}}{p_t} = \mu w_t. \quad (19)$$

A general equilibrium of this economy is a situation where households and firms behave optimally, and all market clear. There is a market for labor, so that $l_t = \int l_{t,j} dj$. In the goods market, $c_{t,j} = y_{t,j}$ leading to $c_t = y_t$. Finally, the supply of real bonds and nominal reserves are both zero on net: $b_t = v_t = 0$.

This economy maps exactly into the price determination problem defined before. To see this, note that because prices are flexible, the symmetry of the problem leads to $p_{t,i} = p_t$. It then follows from combining equations (18) and (19) that:

$$y_t^{\gamma+\psi} = \mu a_t^{1+\gamma}. \quad (20)$$

Therefore, given an exogenous stochastic process for technology such that a_t is a random walk in logs with a stochastic stationary trend, this maps exactly into the assumption on y_t . The model is fully microfounded with firms that choose prices.

The process by which an equilibrium price level is attained in the economy can be explained differently. If the price level is hypothetically too high, the private agents realize that the return on savings in reserves at the central bank is high. They therefore cut consumption to save more. But, as they cut consumption, this lowers the demand for goods, leading firms

to want to cut their prices, making the price level fall back to equilibrium.

In Walrasian general equilibrium economies, either this story or the one at the start of this subsection, are equally valid. All markets, for savings, for bonds, for reserves, for goods, and for labor must jointly clear, so excess demand or supply in any one of them comes with excess demand or supply in all others. Firms are choosing prices, and households are responding to them by consuming more or less, by saving more or less, and by depositing more funds at the central bank or not, all together and at once.

3 The dynamics of inflation

This section solves the mathematical problem set out in the previous section: to solve for the dynamics of inflation in equation (12), subject to the equilibrium Fisher equation (14), the policy Taylor rule in equation (11), and the boundary condition in equation (13).

3.1 Expected inflation

Taking time differences of the Fisher equation gives:

$$di_t = dr_t + d\bar{\pi}_t. \quad (21)$$

In turn, using the Fisher equation to replace i_t in the Taylor rule, and the dynamics of inflation to replace for dp_t/p_t gives:

$$di_t = dx_t - \rho(r_t + \bar{\pi}_t - \alpha'\alpha + \gamma\sigma_y^2\alpha_y - x_t)dt + \phi(\bar{\pi}_t dt + \alpha' dZ_t - \pi^* dt). \quad (22)$$

Equating the right-hand sides of the previous two equations and rearranging gives the law of motion for expected inflation:

$$d\bar{\pi}_t = (\phi - \rho)(\bar{\pi}_t - \pi^*)dt - d\varepsilon_t - \rho\varepsilon_t dt + \phi\alpha' dZ_t. \quad (23)$$

This expression defined a new variable: $\varepsilon_t = r_t - x_t + \pi^* - \alpha'\alpha - \gamma\sigma_y^2\alpha_y$. For now, take this as being just a convenient way to collect terms in what would otherwise be a long and messy expression.

Take expectations of the differential equation, and let hatted variables denote the expected value of actual variables, e.g., $d\hat{\varepsilon}_t = \mathbb{E}_t(d\varepsilon_t)$. Expected inflation then evolves accord-

ing to:

$$d(\hat{\pi}_t + \hat{\varepsilon}_t - \pi^*) + (\rho - \phi)(\hat{\pi}_t + \hat{\varepsilon}_t - \pi^*)dt = -\phi\hat{\varepsilon}_t dt \quad (24)$$

This is a standard ordinary differential equation that has the solution

$$\hat{\pi}_t = \pi^* - \hat{\varepsilon}_t + e^{-(\phi-\rho)(T-t)}\hat{\pi}_t + \int_t^T \phi e^{-(\phi-\rho)(s-t)}\hat{\varepsilon}_s ds \quad (25)$$

Taking the limits as T goes to infinity, noting that $\hat{\pi}_t = \bar{\pi}_t$, and imposing the boundary condition gives the solution for expected inflation:

$$\bar{\pi}_t = \pi^* + \left(\frac{\rho}{\phi - \rho}\right)\varepsilon_t + \int_0^\infty \phi e^{-(\phi-\rho)s} \mathbb{E}_t(\varepsilon_{t+s} - \varepsilon_t) ds \quad (26)$$

as long as $\phi > \rho$.

The condition $\phi > \rho$ is the famous Taylor condition, adapted to account for interest-rate smoothing. Intuitively, as long as the central bank commits to raising interest rates when expected inflation increases from target, then from the Fisher equation, this will raise expected inflation. But because this further raises inflation the next instant, it leads to a new rise in interest rates, and a further rise in expected inflation. Equation (24) shows why this is so. As long as the Taylor condition holds, then expected inflation becomes an explosive process since positive deviations from target lead to further increases in the gap between expected inflation and target. If private agents in the economy rule out from their expectations these infinite forward-looking possibilities where inflation explodes at an accelerating pace, as captured in equation (13), then this disciplines their initial inflation expectations to not deviate from target. Thus the determinacy of inflation.

3.2 The deviations of expected inflation from target

Inflation deviates from target due to the terms on the right-hand side of equation (26). Recall that:

$$\varepsilon_t = (r_t + \pi^* - \alpha'\alpha - \gamma\sigma_y^2\alpha_y) - x_t. \quad (27)$$

If $\varepsilon_t = 0$ at all dates, then inflation will always be on target. An omniscient, long-lived, and inflation-nutter central bank would perfectly control inflation by choosing x_t to mimic one-to-one movements in real interest rates. Because x_t would be perfectly negatively correlated with r_t , the only state variable in the economy would be the real equilibrium, and monetary policy would introduce no extra source of uncertainty to any nominal variable.

But, a central bank that has trouble tracking and measuring r_t in real time, that wants to use interest rates to have inflation deviate from the target in order to stimulate economic activity, or that in its deliberative process changes its views on the appropriate policy, will not be able or willing to set x_t to keep inflation at π^* at all dates. Extending the model to have time-varying risk premium would make it even more unlikely that the central bank would be able to measure in real time changes in $\sigma_y^2\alpha_y$ and adjust the interest rate to these changes.

The opposite case is one in which the setting of nominal interest rates by monetary policy is independent of the real interest rate. In this case, we can take x_t to follow an exogenous process:

$$dx_t = -\kappa_x(x_t - \bar{x})dt + \sigma_x dz_t^x, \quad (28)$$

where the shocks dz_t^x are independent from the shocks to output, dz_t^y and dz_t^g .

Using the stochastic processes for real interest rates and policy interest rates in equations (8) and (28), one can evaluate the expectations and the integral in equation (26).⁸ The final solution for expected inflation is:

$$\begin{aligned} \bar{\pi}(r_t, x_t) = & \pi^* + \left(\frac{\rho}{\phi - \rho} \right) (\bar{r} + \pi^* - \alpha'\alpha - \gamma\sigma_y^2\alpha_y - \bar{x}) \\ & + \left(\frac{\rho - \kappa_g}{\kappa_g + \phi - \rho} \right) (r_t - \bar{r}) - \left(\frac{\rho - \kappa_x}{\kappa_x + \phi - \rho} \right) (x_t - \bar{x}). \end{aligned} \quad (29)$$

Expected inflation is a linear function of the two state variables.

The first line of this equation has the intercept for inflation. A central bank that cannot fully keep track of movements in real interest rates or in inflation risk premia still has to figure out what these are on average and set its average interest rate appropriately. In times when secular changes in productivity may have led to changes in safe real rates, or when the long-run inflation risk premium may be changing due to financial crises, this normal interest rate to which monetary policy should converge is not easy to assess, but it plays a crucial role in keeping inflation on target.

The second line of the expression above shows the sensitivity of expected inflation to the state of the economy. Depending on the persistence of interest-rate changes, shocks to monetary policy can raise or lower expected inflation. This is to be expected, since of course forever higher nominal interest rates unambiguously raise inflation since they correspond to an effective increase in the inflation target. A different question is whether actual inflation

⁸Simply recall that $\mathbb{E}_t(r_{t+s} - r_t) = (\bar{r} - r_t)(1 - e^{-\kappa_r s})$ and likewise for $\mathbb{E}_t(x_{t+s})$, and that $\int_0^\infty e^{-\phi t} dt = 1/\phi$.

rises or falls with positive shocks to nominal interest rates. I turn to this question next.

3.3 Shocks to inflation

The final step is to solve for inflation's response to shocks in the vector α . Subtracting equation (23) from equation (24):

$$d\bar{\pi}_t - d\hat{\pi}_t = d\hat{\varepsilon}_t - d\varepsilon_t + \phi\alpha'dZ_t \quad (30)$$

Using the solution for expected inflation in equation (29) and the definition of ε_t in equation (27), this equation becomes:

$$\begin{aligned} & \left(\frac{\rho - \kappa_g}{\kappa_g + \phi - \rho} \right) \gamma \sigma_g dz_t^g - \left(\frac{\rho - \kappa_x}{\kappa_x + \phi - \rho} \right) \sigma_x dz_t^x \\ & = -\gamma \sigma_g dz_t^g + \sigma_x dz_t^x + \phi \alpha_y \sigma_y dz_t^y + \phi \alpha_g \sigma_g dz_t^g + \phi \alpha_x \sigma_x dz_t^x. \end{aligned} \quad (31)$$

This equation must hold for all realizations of the shocks. Therefore, the solution is:

$$\alpha_x = -\frac{1}{\kappa_x + \phi - \rho} \quad (32)$$

$$\alpha_r = \frac{\gamma}{\kappa_g + \phi - \rho} \quad (33)$$

$$\alpha_y = 0. \quad (34)$$

The first interesting result is that a positive shock to the nominal interest rate lowers actual inflation. The effect is smaller the more aggressive is the Taylor rule coefficient response in future periods, the less persistent is the shock, and the more interest rates are smoothed. The higher is the variance of these monetary policy shocks, the higher will be the variance of inflation deviations from target.

The second result is that permanent shocks to output that do not move real interest rates have no effect on inflation. Similarly, sunspot nominal shocks that do not move real interest rates would have no effect on inflation. Moreover since all the responses to shocks depend on parameters that are time-invariant, the equilibrium has a constant variances of shocks to inflation. This justifies the conjecture that restricting attention to Markov homoskedastic equilibrium is not limiting.

A summary of the results from this model is in the next proposition:

Proposition 1. *The bounded homoskedastic Markov equilibrium has expected inflation $\bar{\pi}(x_t, r_t)$ given by equation (29) and the response to shocks α given by equations (32)-(34).*

4 Equilibrium interest rates

With inflation determined, the Fisher equation gives right away the equilibrium dynamics of the overnight interest rate. The next lemma states it formally.

Lemma 2. *In equilibrium, the instantaneous nominal interest rate is:*

$$i_t = \theta_0 + \theta_x x_t + \theta_r r_t \quad (35)$$

where $\theta_0 = [\phi/(\phi - \rho)](\pi^* - \alpha'\alpha) + [\phi\kappa_x/(\phi - \rho)(\kappa_x + \phi - \rho)]\bar{x} - [\phi\kappa_g/(\phi - \rho)(\kappa_g + \phi - \rho)]\bar{r}$ and $\theta_x = (\kappa_x - \rho)/(\kappa_x + \phi - \rho)$ and $\theta_r = \phi/(\kappa_g + \phi - \rho)$.

In this simple model, the nominal interest rate is an affine function of the two state variables, the state of the real economy and the stance of monetary policy. Therefore, the model fits into the general family of affine models of the term structure (Piazzesi, 2010). The key result from this literature then follows:

Lemma 3. *Define the yield on the bond as $i_t^{(s)} = \log(I_t^{(s)})/s$. In equilibrium, it is:*

$$i_t^{(s)} = \delta_0(s) + \delta_i(s)i_t + \delta_x(s)x_t \quad (36)$$

where $\delta_i(s) = (1 - e^{-\kappa_g s})(\kappa_g + \phi - \rho)/(\kappa_g \phi)$, and the coefficients $\delta_0(s), \delta_x(s)$ solve a set of Riccati equations given in the appendix.

This relation between long and short rates results from the absence of arbitrage along the yield curve. A central bank could not set a separate exogenous target for the short and the long rates that disrespects the equation in this lemma. Otherwise, if $i_t^{(s)}$ is larger than the expression in the lemma, private banks and investors would all want to deposit long-dated reserves at the central bank and want to hold zero instantaneous reserves. If the inequality flipped, so would the balance sheet of the central bank suddenly, from long to short reserves. The central bank, pushed from one corner to the next, would have to adjust its assets correspondingly and quickly, otherwise it would be exposed to losses that could endanger its solvency.

For large maturities, s is large so $\delta_i(s)$ is expected to be quite small as long as the shocks to real interest rates are not very persistent (so κ_g is not too small). This says that temporary

changes in short-term interest rates move long rates less than one-to-one. From a different perspective, if a central bank is targeting the long rate, then any policy decision to change it will have a large impact on short-term interest rates. Because policy decisions tend to be infrequent and lumpy, say every so many weeks for 25 basis points, this implies that using a long rate as the tool for policy would come with intense speculation on short bonds in the days before.

Another implication of lemma 3 is that long-term interest rates are linear functions of the instantaneous interest rate.⁹ This affine property of the model is very convenient on many accounts. First, this class of models has been extensively taken to the data on interest rates of different maturities. Second, it has been extended in different directions. One could for instance consider shocks to the long-run growth rate of the economy, akin to news shocks, or stochastic volatility in the growth rate mapping into uncertainty shocks, and so incorporate these two recent popular business-cycle literatures into the determination of inflation and the study of long-term interest rate policies.

Third, we can easily incorporate other state variables. For instance, Greenwood, Hanson and Vayanos (2016) introduce limits to arbitrage in the bond market so that there are two extra linear factors corresponding to the actual bond holdings by the central bank and their expected mean at different maturities. In their model, when central banks go long in the sense of buying government bonds of different maturities, they affect long-term interest rates. In this paper instead central banks go long directly by choosing the value of the long-term interest rate. Merging the two models would provide a rich theory for how quantitative easing policies can affect inflation.

5 The United States pre accord: 1942-1951

The behavior of the Federal Reserve during the Great Depression is one of the most studied in monetary history. In turn, modern analyses of U.S. monetary policy almost exclusively focus on the behavior of the Fed after the Treasury-Fed accord on March 4, 1951, described by Friedman and Schwartz (1963) as: “Few episodes in American monetary history have attracted so much attention in the halls of Congress and in academic quarters, alike.” Considerably less attention has been spent on the period that goes from World War II to the

⁹Smith and Taylor (2009) impose this linearity and obtain a related result to lemma 3 to focus on how changes in ϕ affect $\delta_x(s)$.

accord. This was a period when the Federal Reserve went long.¹⁰

5.1 Pegging interest rates

The U.S. entered World War II on the 8th of December of 1941, following the attack on Pearl Harbor. As almost always happens when a country enters a major war, the primary goal of economic policy became the financing of large war expenditures, and the Treasury was its leading executor. The Federal Reserve was a subordinate, with monetary policy given the role of ensuring that the banks that it regulated and the financial markets in which it intervened would provide a steady demand for the government bonds. While the Treasury officially managed the public debt, the Federal Reserve was supposed to ensure that the government bonds were sold at a favorable price.

The particular approach by the United States during this time was announced in April of 1942. One part of this policy was that the Fed stood ready to buy and sell 90-day Treasury bills at a fixed rate of $3/8\%$. The T-bill rate then became the effective policy rate. Certificates of deposit could be discounted at rates that still changed from time to time to respond to demands in the banking sector, but the peg on the T-bill rate was the focus of policy. Correspondingly, Treasury bills, not reserves became the major liquid asset on the balance sheet of banks. Knowing that these could be bought and sold from the Fed at a fixed price at any time, banks did not need reserves, for T-bills were just as liquid.

While much has been made of the policy pegging interest rates, this actually lasted for a relatively short period of time. The Federal Reserve continuously clashed with the Treasury about raising the T-bill rate, especially at the end of the War when inflation accelerated. Eventually, in July of 1947, the Fed raised the T-bill rate after striking a bargain with the Treasury that involved the payment to the Treasury of a significant share of the net income it had accumulated. Further increases immediately followed, so that by December the bill rate was 1% , and one year later by the end of 1948 it was set at 1 and $1/8\%$.

Between 1949 and 1951, there was an intense political struggle between the Treasury, partly backed by the president, and the Fed, which at times seems worthy of a political drama TV series (Hetzl and Leach, 2001). It started with the FOMC statement in June of 1949 statement, stating an intended change in the interpretation of the mandate to keep a peg on interest rates. A crucial shock then arrived in 1950 with the intensification of the Korean war. Real interest rates rose as a response, and large government deficits were

¹⁰Standard references for the history of the Federal Reserve are Friedman and Schwartz (1963) and Meltzer (2010).

expected. Moreover the anticipation of price controls, led to a sharp increase in inflation, mostly for durable goods. On one side, the Treasury became nervous about keeping the peg on the price of its debt, especially given the prospect of another long conflict. On the other side, the Fed worried that to keep its interest rates low, it would have to issue reserves to buy more government bonds, and that this would fuel credit and inflation. In 1951, in testimony to Congress, the chairman of the Federal Reserve system unequivocally stated that: *“As long as the Federal Reserve is required to buy government securities at the will of the market for the purpose of defending a fixed pattern of interest rates established by the Treasury, it must stand ready to create new bank reserves in unlimited amount. This policy makes the entire banking system, through the action of the Federal Reserve System, an engine of inflation. (U.S. Congress 1951, p. 158)”*.

The Treasury-Fed accord of March 4, 1951 declared a truce between the Treasury and the Fed. In spite of having little legal force, and in itself stating little of substance, Fed chairman Martin masterfully interpreted it in a way that affirmed the independence of the Fed from the Treasury from the onwards. One fundamental implication of the accord for the conduct of U.S. monetary policy was that supporting the national debts was no longer an objective for monetary policy, which became concentrated on macroeconomic and price stability. Another implication was that the peg on the bill rate was lifted and the Fed gained full autonomy over the setting of interest rates.

There was a third implication of the Treasury-Fed accord. Since then and all the way until the adoption of quantitative easing in 2008, the Federal Reserve focussed its attention on short-term interest rates and conducted the bulk of its open market operations using Treasury bills. This was not the case before the accord.

5.2 Ceiling policy

While the peg for the T-bill rate gets much of the attention, it only lasted for five years. More persistent, and arguably more significant was a different part of the March 1942 policy. The Fed announced an explicit ceiling of 2.5% for the 10-year yield, which was kept for 11 years, only being abandoned in March of 1953. Friedman and Schwartz (1963) argue that, unlike the peg on the bill rate, the Fed was in general favorable to this policy. The bond support program, as it was called, had originated intellectually within the Fed.

While the War lasted, the yield on Treasury bills was low relative to the yield on longer-dated Treasury bonds. As a result, banks were happy to hold bonds earning higher returns, exchanging them for Treasury bills at the Fed whenever they needed liquidity. The Fed

rarely needed to intervene, and its assets mostly consisted of Treasury bills.

This changed between 1945 and 1948. The Treasury started issuing many more long-term bonds with the goal of delaying the payment of the wartime debt. Yields rose, reaching 2.37% in November of 1947 and forcing the Fed to step in with a large-scale purchase of bonds to keep the ceiling unbroken. In December 24 of that year, the Fed released a mere suggestion that it might allow for small deviations from the ceiling, and the long-term yield immediately jumped to 2.45%, demonstrating the active role the Fed was playing in the the bond market. As the slope of the yield curve shrank, the private sector shifted the composition of its portfolio towards Treasury bills. Correspondingly, the maturity of the Fed’s bond portfolio expanded.

Noticeably, while between 1947 and 1950 the Fed raised the bill rate several times, and wanted to raise them more and more often, it always stayed committed to the ceiling on the bond rate. In fact, in October 16 of 1947, the Board of governors wrote a letter to the Treasury secretary where in the process of defending the change in the bill rate, it stated: *“We can assure you that these actions will not affect the maintenance of the 2 1/2 percent rate for the outstanding long-term government bonds.”*

This changed with the Korean War. The flattening of the yield curve intensified the pressure for the balance sheet to grow and become longer. In 1950, chairman Eccles advocated a relaxation of the ceiling on bond yields, but was strongly opposed by president Truman who, having imposed wage controls in 1951, was adamant that long-term mortgage rates would not increase. Moreover, the Treasury warned of a large financial crisis in bond markets if the ceiling was dropped. Following the accord, the Fed did not explicitly abandon its interest rate ceiling, doing so only a full two years later, in March of 1953. Only then did the Fed start selling bonds at a fast pace. Intellectual and policy support for a “bills only” policy with regards to the Fed’s balance sheet arose, and remained for many years to come, as the Fed moved completely away from its going long policy.

5.3 A hard peg

To study the effect of the Federal Reserve policy, let ι denote the ceiling on the long-term interest rate $i_t^{(s)}$. While policy was set in terms of the 90-day rate, I will for simplicity model it as setting the instantaneous rate; the next section on the Bank of England’s policy will cover the use of interest rates of different maturities as the main policy tool.

With a peg on the bill rate, as was the supposedly agreed policy, then inflation is indeterminate. This corresponds to $\phi = 0$ and to a constant x_t in the model. In this case,

combining the Fisher equation in (14) with the policy rule, now gives:

$$\bar{\pi}_t = \bar{x} - r_t + \alpha' \alpha + \gamma \sigma_y^2 \alpha_y. \quad (37)$$

Since r_t is a stationary process, this satisfies the boundedness condition. Therefore, this equation is the sole condition with which to pin down the evolution of inflation. This is one equation in three variables, expected inflation and the response of inflation to shocks to output and to the real interest rate. The result is indeterminacy.

This is not the classic indeterminacy result of Sargent and Wallace (1975). In a deterministic model, $\alpha = 0$, so the equation above uniquely pins down inflation. Sargent and Wallace (1975) instead emphasized that the initial price level is indeterminate. With uncertainty, there is another form of indeterminacy (Nakajima and Polemarchakis, 2005). Monetary policy can at best pin down a risk-adjusted measure of expected inflation, but how this breaks down between expected inflation and the actual inflation response to shocks is indeterminate.

Figure 1a plots a phase diagram to provide further intuition. The Fisher equation is a linear relation between interest rates and inflation with slope 1 and a vertical intercept at the level of the real interest rate. The policy rule with an interest rate peg is a horizontal line. Starting from the steady state where $r_t = \bar{r}$ and $x_t = \bar{x}$, the dynamics of inflation and interest rates consist of movements along the Fisher equation given by the arrows in the diagram.

Clearly, the system is globally stable: after a shock to the real economy (which shifts the Fisher relation) or a shock to monetary policy (which shifts the policy rule), inflation will converge back to the new intersection of the two lines. The boundedness condition puts no restriction on equilibrium. Yet, with risk, the change in inflation in response to the shocks (α) affects the location of the Fisher relation. Therefore, there are multiple possible combinations of inflation and its responsiveness to shocks that are consistent with equilibrium.

5.4 The Fed going long

If this \bar{x} policy was strongly enforced and expected to be maintained forever, then $i_t^{(s)} = \bar{x}$ as well, so the ceiling would never bind. Going long would have been immaterial. Yet, the supposed hard peg only lasted for a little over 5 years; by comparison the policy rate was unchanged in the United States for 7 years, between December of 2009 to December of 2016. An alternative interpretation of the policy at the time is that the Fed followed a feedback rule

for interest rates, as in equation (11), but with a very high extent of interest-rate smoothing (low ρ) and a relatively low sensitivity of interest rates to inflation (low ϕ). In this case, the interest rate ceiling can bind, and this affects the evolution of inflation.

Figure 1b shows the phase diagram for this case. The Fisher equation is the same, but now the policy rule in equation (11) is an upward-sloping line with slope $\phi/\rho > 1$. Since being below the policy rule implies that interest rates rise, and above means that they fall, the dynamics are now given by the arrow. The dynamic system is clearly unstable, and inflation must stay always at the intersection of the two lines, otherwise it would diverge to infinity violating the boundedness condition. Given any shock to monetary policy that shifts the policy equation, or to real interest rates that shifts the Fisher equation, the equilibrium inflation will always jump to be right at the intersection of the two.

With a ceiling on the long interest rates, the policy rule is different. For simplicity of the graph, consider the case where all shocks are zero, so that $i_t^{(s)} = \int_t^s i_j dj$ and take the intercept in the policy rule to be consistent with the inflation target: $x_t = \bar{x} = r + \pi^*$. The monetary policy is to follow the feedback rule unless it implies a violation of the constraint $i_t^{(s)} \leq \iota$. If so, the interest rate is unchanged. Figure 1c plots this case.

Starting from a point where the short-term rate equals the long-term rate, the policy resembles a peg in response to positive shocks to inflation. Given the monotonicity of the interest rate in the dynamic system, it then follows that the interest rate will be at the bound for all levels of inflation between point H and the point where the ceiling intersects the unbounded policy rule. There are now two equilibria: the previous unstable one, with inflation on target at point L, and the new globally stable equilibrium at point H, with high persistently high inflation.

This model allows us to make sense of the conflict between the Fed and the Treasury in the late 40s and of the dynamics of inflation at the time. At first, the economy was close to the L equilibrium. The Fed would make small adjustments to the bill rate (changes in x_t) to stay at the equilibrium with low inflation. These adjustments had to be negotiated with the Treasury, but they were essential since if the shocks pushed for higher inflation and the Fed was not quick to raise x_t and shift the policy rule, it risked being to the right of the L point and entering an escape dynamics towards the H equilibrium. Post 1945, when these positive inflation shocks happened, the political tension between Fed and the Treasury was therefore large, and centred on the level of the T-bill rate.

In 1950, the the intensification of the Korea War implied a a large increase in r_t shifting the Fisher equation upwards. Controlling inflation would require a sharp increase in the bill

rate x_t to shift the policy rule upwards as well and keep inflation on target. The tension intensified and the accord had to follow. The ceiling played a crucial role because as the two upwards sloping line segments shift upwards, the range over which the policy rule is horizontal to the right of the L point gets larger. Translating this into economics, as real and nominal rates rise, the yield curve flattens, and this reduces the room for further shocks to not make the ceiling put a binding constraint on short rates.

By 1953, it is clear that the Fed must let go of the ceiling for, even with control of short rates and potentially a more aggressive policy in the form of a higher ϕ , still the danger that a future shock would start dynamics towards the H point was real. The statements by the Fed at the time, of fearing that the policy of pegging the long rate would lead to inflation getting out of hand are justified by this simple model. The ceiling put a strain on monetary policy because any mistake in setting x_t too low, would lead the economy to enter a stable path where inflation monotonically rises and converge to the high inflation equilibrium in point H. Abandoning the ceiling was the way to prevent the high-inflation stable equilibrium from becoming the dominant reality in the United States.

According to the model, the way in which the Fed went long at the behest of the Treasury was ultimately unsustainable. It created a high inflation equilibrium that might have been reached and set its stable roots in the U.S. were it not for the strong intervention of the Fed to break its ties with the Treasury.

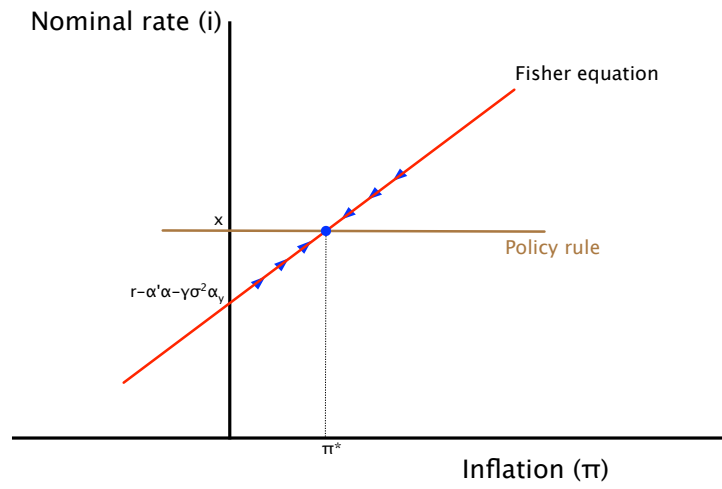
6 The Radcliffe commission and UK monetary policy in the 1960s

On May 3 of 1957, the chancellor of the exchequer set up a “Committee on the Working of the Monetary System,” headed by Lord Radcliffe. Its official goal was “to inquire into Britain’s monetary and credit mechanism and to make recommendations”. It deliberated over more than two years, questioning more than 200 witnesses and receiving more than one hundred special memoranda, until the final report was presented in August of 1959.

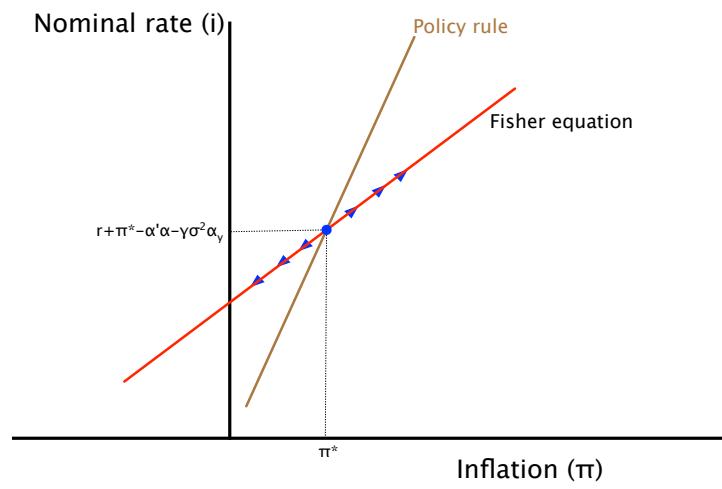
The Radcliffe report’s purported to explain how monetary policy worked and how should it work. Unsurprisingly, it attracted both strong support as well as violent disagreement across the globe. In the academic world, in 1960 alone, there were special articles in the *American Economic Review*, the *Journal of Finance*, and the *Review of Economics and Statistics* devoted to the report. The prominent monetary economist Anna Schwartz for many years argued that the report was misguided (Schwartz, 1987). Its policy principles

Figure 1: Phase diagrams for inflation with different policy rules

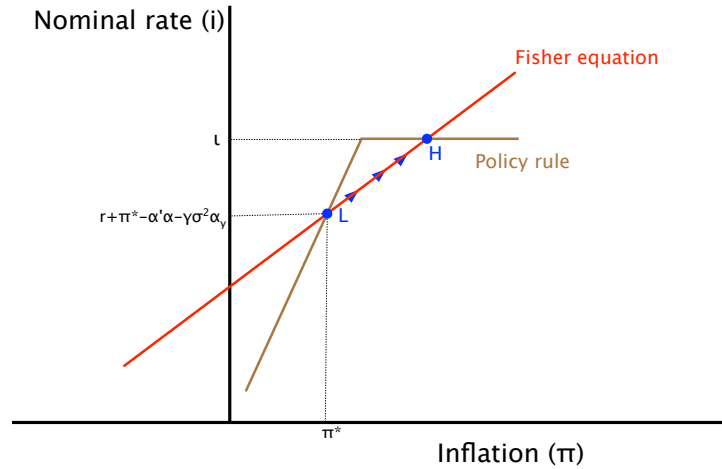
(a) Hard peg



(b) Short-rate policy



(c) Long ceiling policy



explicitly guided the Bank of England’s monetary policy during the 1960s, and were arguably influential into the 1970s, so that the report plays a central role in any history of the Bank of England in the XXth century.¹¹

6.1 Prelude: monetary policy in the 1950s

Throughout the 1950s, the UK economy was still recovering from the devastating effects of World War II. There were many direct economic controls in place and a large stock of public debt outstanding. The maturity of that debt was low relative to what had been typical, leading to constant pressure to refinance bonds that were coming due.

The Bank of England was not independent, operating under control of the Treasury. Reducing unemployment was the dominant goal of economic policy, and following the dominance of Keynesian thought, fiscal policy directed to controlling aggregate demand was perceived as the best way to achieve it. Monetary policy was left with two main roles. First, to manage international reserves and prevent fluctuations in the value of the exchange rate, and second, to keep interest rates low in order to help in the management of the public debt. As a result, almost all of the changes in the main policy rate—the bank rate at which the Bank of England lends to banks—came in response to international shocks that affected the exchange rate. The Bank affected the interest rate on Treasury bills through purchase and sale operations in bond markets.

As in the United States, there was an explicit target for the 10-year rate on government bonds of 2.5%. However, it was implemented quite differently. If investors required higher returns to buy the bonds, the Treasury simply refused to sell them. As a result, when during the 1950s, the Bank increased interest rates at times in response to some foreign shock, the market for long-term gilts would dry up, and the Treasury would issue mostly Treasury bills.

Politically, the Bank was criticized whenever it increased interest rates insofar as this caused difficulties in managing the national debt. Moreover, raising bill rates with fixed long rates made short bonds more attractive relative to long bonds, undermining the national goal of extending the maturity of the stock of government debt. More generally, the Bank’s focus on short-term interest rates was seen as misguided. The dominant intellectual Keynesian view demanded an emphasis on long-term interest rates, and especially on lowering them through what was known as “cheap money” in order to stimulate investment. Following large sudden increases in the bank rate in 1954-55 to stop an outflow of international reserves,

¹¹Good references for monetary policy in the period before and after the report are Allen (2014), Capie (2010) and Dimsdale (1991).

and the 1957 election of prime minister McMillan, the Radcliffe committee was formed to clarify the role of the Bank of England.

This was also a time of fervent debate in academic circles. The interviews leading to the Radcliffe report involved most prominent macroeconomists in the United Kingdom, and a variety of strong views were espoused. Treasury officials saw macroeconomic policy through the lenses of a tradeoff between unemployment and inflation, in the spirit of the Phillips curve. Students of the gold standard thought that the central bank should be solely in charge of setting an interest rate to affect currency markets. More dominant was the view that credit policies were the main tool for a central bank to affect financial markets, while only a minority argued that inflation should be the target for monetary policy, or that money was important. The Radcliffe commission became a public arena where competing views of monetary policy were openly contrasted.

6.2 The report's view of monetary policy

While the report was unanimously approved by its members, it did not offer a clear list of conclusions and recommendations. Still, most contemporary readers summarized its contribution in a list of five points. The first four of these have attracted much academic attention already. They were: first, the recommendation that monetary policy has many different goals, sprayed throughout the report without any clear discussion of policy tradeoffs, and no clear connection beyond the fact that central bank actions could in principle be relevant to each of them. Second, the downplay of monetary aggregates or more generally of the role of money in affecting macroeconomic outcomes due to the combination of a view that velocity is infinitely elastic and a preference for a broader and looser concept of “liquidity” as the relevant influence on aggregate demand and inflation. Third, the preference for explicit credit policies and controls as the tool that the Bank of England should use to complement the role of fiscal policy in steering aggregate demand. Fourth, a conventional and unremarkable discussion of the role of international reserves and exchange rate volatility.

The fifth conclusion concerned the role of interest rates, especially at longer maturities, in monetary policy. This is the part of the Radcliffe report relevant for this paper. It is the more grounded and clearly argued of the five main points, because it builds and expanded on the 1945 National Debt Enquiry. Unlike the targets for liquidity, which were never concretely implemented, the advice on interest rates was influential in the setting of Bank of England policy in the 1960s.

The Radcliffe report saw the management of the public debt as a fundamental goal of

monetary policy. This was to be done by setting interest rates at many different maturities since policymakers: “...must have and must consciously exercise a positive policy about interest rates, long as well as short, and about the relationship between them.” The quantities of government bonds held at different maturities would then be decided in markets according to investors’ demand. The Radcliffe report implicitly rejected the no-arbitrage view of the term structure, and was closer instead to a clientele perspective, where in each maturity separately, the central bank could choose a price, and market forces would determine the finite quantity that cleared the market.

The report went further and dismissed the idea that in setting interest rates, the central bank would have a significant effect on aggregate demand. It likewise dismissed a connection between money and interest rates. Finally, it was critical of the Bank of England for focussing only on short-term interest rates, and blamed the failings of monetary policy in the previous decade on its neglect of active management of long-term interest rates.

6.3 The Bank of England going long

Throughout the 1960s, U.K. monetary policy devoted itself first to stabilizing the exchange rate and capital flows through the setting of short-term interest rates, and second to managing the yield curve and the cost of government financing through the setting of long-term interest rates. The Radcliffe report urged the central bank to estimate the “right level” for interest rates. Managing the issuance of bonds of different maturities, using credit controls, regulating banks, and adjusting the bank rate were all tools used to ensure that a steady demand for government bonds materialized at the desired target. While the Bank never explicitly embraced focussing on one particular long-term interest rate, it announced a target for the 10-year gilt yield in 1962.

The closer description of the interest-rate policy during this time in perhaps as a peg of $i_t^{(s)}$ around an exogenous ι_t . Inflation was not a target for monetary policy, and changes in ι_t were infrequent and slow, often driven by political compromises with the Treasury and changing views on the need to stimulate investment. The going long policy consisted of focussing monetary policy operations on a long interest rate and choosing this somewhat independently of inflation or aggregate demand. The central bank focused instead on devising a target for the long rate, ι_t . Following Radcliffe, in the discussions on the right value for ι_t at the time were exogenous source of shocks to inflation. With this policy, an analysis similar to the one in the previous section would generate an indeterminate inflation.

It is not a big stretch though to instead model the Bank of England’s policy as a feedback

rule for the long rate:

$$d(i_t^{(s)} - \iota_t) = -\rho(i_t^{(s)} - \iota_t)dt + \phi_\iota \left(\frac{dp_t}{p_t} - \pi^* dt \right), \quad (38)$$

with a small ϕ_ι and a large extent of smoothing ρ . The history of policy decisions at the time has some episodes where an increase in inflation expectations is followed by a discussion of whether to adjust the target for long-term interest rates.

Lemma 3 maps long-term interest rates into the instantaneous rate. From this map follows the result:

Lemma 4. *The policy rule for long-term interest rates in equation (37) leads to inflation dynamics as in proposition 1 with $\phi = \phi_\iota/\delta_i(s)$ and*

$$x_t = \frac{\iota_t - \delta_0(s)}{\delta_i(s) + \delta_x(s)} \quad (39)$$

The proof is as follows. Conjecture that the policy rule leads to a rule for instantaneous rates as in equation (11) for some ϕ and some x_t . Then, from lemma 3, we know that $i_t^{(s)} - \iota_t = \delta_0(s) + \delta_i(s)i_t + \delta_x(s)x_t - \iota_t = \delta_i(s)(i_t - (\iota_t - \delta_0(s) - \delta_x(s)x_t)/\delta_i(s))$. The conjecture will be verified if $\phi = \phi_\iota/\delta_i(s)$ and if

$$\delta_i(s)x_t = \iota_t - \delta_0(s) - \delta_x(s)x_t. \quad (40)$$

Rearranging gives the expression in the lemma.

In a sense, all central banks follow a rule of this type, as few are able to set a truly instantaneous interest rate, but instead set overnight or one week interest rates. For these short maturities, $\delta_0(s)$ and $\delta_x(s)$ are close to zero, while $\delta_i(s)$ is close to 1. The result in lemma 4 shows that the analysis of inflation in section 3 then applies with no modifications to these actual policies. Likewise, the discussion of the Fed's ceiling policy in the previous section neglected the fact that the Fed during the 1940s used the 90-day T-bill as its policy rate, but the lemma above shows that qualitatively one would obtain the same phase diagram and conclusions.

When the central bank goes long, instead, s is large and so $\delta_i(s)$ is small. Section 3 discussed three main determinants of inflation, which using 4 we can now apply to a policy of targeting long-term interest rates.

First, it must be that $\phi^d > \delta_i(s)\rho$ for inflation to be determinate. Since $\delta_i(s) < 1$, the condition for determinacy is therefore less stringent than it was for shorter rates.

Second, section 3 noted that it takes a precise setting of \bar{x} to make inflation equal its target on average. This translates into an average target for the long-term rate \bar{i} that follows the formula in the lemma above. To calculate accurately the real interest rate and the inflation risk premium, the policymaker must also now understand all the determinants of long-term yields, from their long-run average to their sensitivity to each shock. The problem is harder.

The third result in section 3 was that the variance of inflation depended on the variance of the interest rate. Given uncertainty on the parameters that determine the yield curve, setting the interest rate exactly to keep inflation on target (a choice of ι_t to hit $\varepsilon_t = 0$) appears to be harder. Moreover, the insistence on lowering as much as possible the burden of paying for the national debt and the reluctance in linking interest rates to the evolution of inflation suggests that ι_t was not chosen to attempt to keep ε_t close to zero. Finally, exogenous shocks to ι_t may lead to a larger impact on inflation if $\delta_i(s) + \delta_x(s) < 1$.

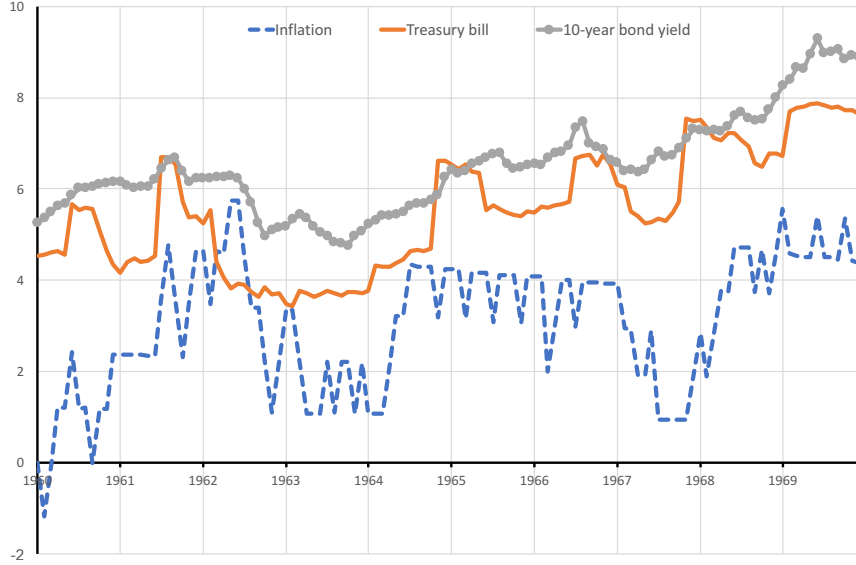
In conclusion, using long-term rates as the policy tool is consistent with controlling inflation and involves similar considerations as using short-term rates. In fact, the one-to-one map between long and short and long rates in lemma (3) implies that the set of equilibria that a going long policy can achieve is the same as the set of equilibria that an equivalent policy for the short rate can achieve, in the sense of lemma (4). However, uncertainty on the shape of the yield curve suggest that this strategy likely comes with higher level and variability of inflation as well as nominal interest rates.

Figure 2 shows the path of interest rates and inflation during the 1960s in the UK. Interest rates crept up from 1965 onwards, revealing the failure to pin them down at a natural rate. While, as the figure showed, the slope of the yield curve was small, the fiscal problems of the government persisted and intensified eventually ending in a request for an IMF loan a few years later. Towards the end of the decade, inflation started accelerating, and by the early 1970s the Bank of England stopped going long, with the strategy deemed a failure.

7 The Bank of Japan's yield curve control, 2016-...

Since 1985, annual core CPI inflation in Japan has only exceeded 2% in two years. This remarkable period of an almost constant price level came together with equally low inflation expectations and low growth in output per capita. In response to the fears of deflation, to the disappointing economic performance, and to the Asian financial crisis, in 1997 the Bank of Japan gradually went long, making this policy explicit at the end of 2016.

Figure 2: U.K. interest rates and inflation, 1960-70



7.1 Unconventional monetary policy

Since 1997, the BoJ's policy framework combined balance-sheet and interest-rate policy as follows. Between July of 1996 and March of 1999, the BoJ expanded the size of its balance sheet by issuing reserves in order to provide liquidity to its financial markets in response to the Asian financial crisis. The market for reserves was saturated and the Friedman rule achieved in the market for reserves as is desirable (Reis, 2016a). Next, starting from March of 2001, the BoJ gradually introduced quantitative easing by committing to buy government bonds and to lend to banks in horizons that gradually rose all the way to 3 months in order to raise inflation. The interest-rate policy was clearly laid out in the February 12th directive of 1999, which stated that: *“The Bank of Japan will provide more ample funds and encourage the uncollateralized overnight call rate to move as low as possible. To avoid excessive volatility in the short-term financial markets, the Bank of Japan will, by paying due consideration to maintaining market function, initially aim to guide the above call rate to move around 0.15%, and subsequently induce further decline in view of the market developments.”* The BoJ repeatedly used forward guidance to state its intention to keep the overnight rate low until inflation expectations rose.

The first stage of this policy was unsuccessful insofar as the price level barely moved between 1997 and 2010, and inflation expectations stayed tightly anchored at 0. In a second stage, between 2010 and 2016, the BoJ rolled out a new policy: qualitative and quantitative easing (QQE). In September of 2010, it introduced an asset purchase program that included

buying not only government bonds but also commercial paper, asset-backed commercial paper, exchange-traded funds, and real estate investment trusts, and U.S. government bonds (TBs, CPs, ABCPs, CBs, ETFs, J-RIETs). In April of 2013, it committed to buying 60-70 trillion yen of these assets per year, and in October of 2014 it expanded this to more than 80 trillion yen per year. The balance sheet not only rapidly grew but especially changed in its composition, becoming significantly more varied.

The second stage produced an increase in the rate of core inflation, from close to -2% in 2010 to slightly above 1% in 2015. Yet, after an initial jump in inflation in 2013, rising by 1.5% in a little over one year, inflation fell again throughout the second half of 2014 and 2015, so that by the middle of 2016, inflation was back to -0.5%. Consensus inflation expectations started falling since the mid to end of 2015, far from their intended 2% inflation target.

7.2 Yield curve control

In September of 2016, the BoJ announced the introduction of a new policy framework: “Quantitative and Qualitative Easing with Yield Curve Control.” Qualitative and quantitative easing refers to expansions of the size of the balance sheet by issuing reserves and using them to buy a variety of government bonds and private securities thus exposing the BoJ’s net income to many different forms of market risk.

Yield curve control refers to announcing a target not just for the overnight central bank rate, but also for an intended yield on the the 10-year government bond rate and, in the future, potentially other maturities as well. The BoJ announced a desired target of 0% for the 10-year government bond rate, while the target for the overnight rate was -0.1% . This was implemented by adjusting the purchase programs of bonds at the 10-year maturity to stay near the target. Moreover, the BoJ restated its authority to buy these bonds or accepts deposits at this long maturity in potentially large amounts. Effectively, the BoJ therefore used open market operations and put a floor and ceiling on the 10-year rate, in much the same way that central banks have traditionally tried to steep the interbank overnight rate. The policy has been successful in keeping the 10-year rate very close to 0% since then.

At the same time, the BoJ has kept short-term interest rates at zero, and from January of 2016 onwards at -0.1% , while reinforcing a state-contingent commitment to keep them there until inflation overshoots the 2% inflation target.

7.3 The Bank of Japan going long

In targeting more than one interest rate, the BoJ's policy framework has resemblances with the discussions and policies followed by the Federal Reserve in the 1940s and the Bank of England in the 1960s. However, the BoJ did not put a ceiling, but targeted directly the 10-year rate. Moreover, it did not abdicate from continuing to set an precise target for the short-term rate at the same time.

If the BoJ set its two interest rate targets independently from each other, or was perceived as doing so, then this would potentially create arbitrage opportunities across the yield curve. The fact that its balance sheet did not become dominated by only short-term or only long-term securities, suggests that investors perceive it to be setting the two rates in a consistent manner. Moreover, the BoJ's policy statements suggest that it will move its two interest-rate targets in tandem in the future, preventing the large portfolio shifts across maturities that the Fed in the 1940s and the Bank in the 1950 went through.

I model this as a policy rule for the slope of the yield curve:

$$d(i_t^{(s)} - i_t - \iota_t) = -\rho(i_t^{(s)} - i_t - \iota_t) + \phi_\iota \left(\frac{dp_t}{p_t} - \pi^* dt \right). \quad (41)$$

Identical steps to those in lemma 4 show that

Lemma 5. *The policy rule for the slope of the yield curve in equation (38) leads to inflation dynamics as in proposition 1 with $\phi = \phi_\iota / (\delta_i(s) - 1)$ and*

$$x_t = \frac{\iota_t - \delta_0(s)}{\delta_i(s) + \delta_x(s) - 1}. \quad (42)$$

Taking again the relevant case where s is large so $\delta_i(s)$ is small, the outcomes under a slope policy differ significantly from those under a long policy in one aspect. Pinning down inflation requires that $\phi > \rho$ and the higher is ϕ , the lower the variability of inflation in response to shocks. Using the result in the lemma, this requires ϕ_ι to be negative, and significantly so. That is, the model suggests that the BoJ should commit to increasing its 10-year yield target by less than its overnight interest rate when inflation increases. Controlling inflation requires flattening the yield curve targeted as inflation rises.

In the last few years, as central banks around the world have struggled to raise inflation towards its target, they have employed forward guidance and quantitative easing to flatten the yield curve. This was meant to stimulate inflation to rise closer to target. The Bank of Japan's policy is the most extreme follower of this policy through its explicit yield curve

target enforcing that the slope of the yield curve is a mere 0.1% because inflation is so far below target. The rule above suggests instead that with inflation well below target, the yield curve should be steeper than usual to push inflation upwards. In the model, monetary policy is stimulative when short-term rates are lower relative to usual because this steepens the yield curve. Stimulating inflation requires low overnight rates today that are expected to rise in the future. Policies that flatten the yield curve achieve the opposite effect on inflation.

8 Conclusion

In the past decade, it became common among policymakers to discuss monetary policies in terms of their impact on long-term interest rates. For instance, in her survey of the conduct of monetary policy and the role of quantitative easing by the Federal Reserve during the crisis, chair Janet Yellen (2017) wrote: “For this reason, the Committee turned to asset purchases to help make up for the shortfall by putting additional downward pressure on longer-term interest rates.” The Bank of Japan has gone further by announcing an explicit 0% target for the 10-year rate. Central banks have been going long by increasingly focussing on longer-term interest rates than before.

This paper went back in history to discuss and contrast the Japanese policy with the experience of the Bank of England in the 1960s and the Federal Reserve in the 1940. While in each of these episodes, long-term interest rates took centre stage, they did so in quite different ways. The results in this paper suggest that each of these three going long policies as implemented were unsuccessful at controlling inflation.

The analysis required linking long-term, short-term interest rates, and inflation in a tractable way that keeps the effects of uncertainty and risk premiums. This suggested setting the problem of inflation control in an economy where shocks follow continuous-time diffusions. There are likely many other applications where this formulation may prove to be useful in alternative to the standard discrete-time linearized approach that dominates the literature.

References

- Allen, William A.** 2014. *Monetary Policy and Financial Repression in Britain, 1951-59*. Palgrave MacMillan.
- Ball, Laurence, N. Gregory Mankiw, and Ricardo Reis.** 2005. “Monetary Policy for Inattentive Economies.” *Journal of Monetary Economics*, 52(4): 703–725.
- Brunnermeier, Markus K., and Yuliy Sannikov.** 2017. “Macro, Money and Finance: A Continuous-Time Approach.” In *Handbook of Macroeconomics*. Vol. 2B, 1497–1546. North-Holland.
- Capie, Forrest.** 2010. *The Bank of England: 1950s to 1979*. Cambridge University Press.
- Cochrane, John H.** 2011. “Determinacy and Identification with Taylor Rules.” *Journal of Political Economy*, 119(3): 565–615.
- Dimsdale, N. H.** 1991. “British Monetary Policy since 1945.” In *The British Economy since 1945*. , ed. N.F.R. Crufts and N.W.C. Woodwood. Oxford: Clarendon Press.
- Friedman, Milton, and Anna J. Schwartz.** 1963. *A Monetary History of the United States, 1867–1960*. Princeton University Press.
- Greenwood, Robin, Samuel G. Hanson, and Dimitri Vayanos.** 2016. “Forward Guidance in the Yield Curve: Short Rates versus Bond Supply.” In *Monetary Policy through Asset Markets: Lessons from Unconventional Measures and Implications for an Integrated World*. Vol. 24. 1 ed., , ed. Elías Albagli, Diego Saravia and Michael Woodford, Chapter 02, 011–062. Central Bank of Chile.
- Hall, Robert E., and Ricardo Reis.** 2015. “Maintaining Central-Bank Solvency under New-Style Central Banking.” *NBER Working Paper 21173*.
- Hall, Robert E., and Ricardo Reis.** 2016. “Achieving Price Stability by Manipulating the Central Bank’s Payment on Reserves.” *NBER working paper 22761*.
- Hetzl, Robert L., and Ralph Leach.** 2001. “The Treasury-Fed Accord: A New Narrative Account.” *FRB Richmond Economic Quarterly*, 87(1): 33–56.
- Jones, Callum, and Mariano Kulish.** 2013. “Long-term interest rates, risk premia and unconventional monetary policy.” *Journal of Economic Dynamics and Control*, 37(12): 2547 – 2561.
- McGough, Bruce, Glenn D. Rudebusch, and John C. Williams.** 2005. “Using a long-term interest rate as the monetary policy instrument.” *Journal of Monetary Economics*, 52(5): 855 – 879.
- Meltzer, Allan.** 2010. *A History of the Federal Reserve, Volume 1: 1913-1951*. University of Chicago Press.
- Nakajima, Tomoyuki, and Herakles Polemarchakis.** 2005. “Money and Prices Under Uncertainty.” *Review of Economic Studies*, 72(1): 223–246.
- Obstfeld, Maurice, and Kenneth Rogoff.** 1983. “Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out?” *Journal of Political Economy*, 91(4): 675–687.

- Piazzesi, Monika.** 2010. “Affine Term Structure Models.” In *Handbook of Financial Econometrics*. 691–766. North-Holland.
- Reis, Ricardo.** 2016a. “Funding Quantitative Easing to Target Inflation.” In *Designing Resilient Monetary Policy Frameworks for the Future*. Jackson Hole Symposium:Federal Reserve Bank of Kansas City.
- Reis, Ricardo.** 2016b. “How do central banks control inflation? A guide to the perplexed.” Detailed lecture notes.
- Sargent, Thomas J, and Neil Wallace.** 1975. “Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule.” *Journal of Political Economy*, 83(2): 241–254.
- Schwartz, Anna J.** 1987. *Money in Historical Perspective*. University of Chicago Press.
- Smith, Josephine M., and John B. Taylor.** 2009. “The term structure of policy rules.” *Journal of Monetary Economics*, 56(7): 907–917.
- Woodford, Michael.** 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton, NJ:Princeton University Press.
- Woodford, Michael.** 2010. “Optimal Monetary Stabilization Policy.” In *Handbook of Monetary Economics*. Vol. 3, , ed. Benjamin M. Friedman and Michael Woodford, Chapter 14, 723 – 828. Elsevier: North Holland.